I217: Functional Programming

8. A Programming Language Processor – Interpreter

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Roadmap

- Minila
- Interpreter

Minila

Minila (a *Mini-la*nguage) is an imperative programming language.

The data types available are natural numbers only.

Some operations of natural numbers, such as addition and multiplication, are available.

Program constructs are assignment statements ($x := e\ ;$) , if statements ($\textbf{if}\ e\ \{s_1\}\ \textbf{else}\ \{s_2\}$), while statements ($\textbf{while}\ e\ \{s\}$), for statements ($\textbf{for}\ i\ e_1\ e_2\ \{s\}$), and sequential composition statements ($s_1\ s_2$).

Minila

A Minila program that computes the factorial of 10:

$$
\begin{array}{l}
x := \mathrm{n}(1)\ ; \\
y := \mathrm{n}(1)\ ; \\
\textbf{while}\ y < \mathrm{n}(10) \mid\mid y === \mathrm{n}(10)\ \{ \\
\quad x := x * y\ ; \\
\quad y := y + \mathrm{n}(1)\ ; \\
\}
\end{array}
$$

A natural number n is expressed as $\mathrm{n}(n)$ in Minila

$===$ checks if two natural numbers are equal.

As the program terminates, the factorial of 10 is stored in x.

Minila

For the program, the interpreter returns the following:

((x , 3628800) | ((y , 11) | empEnv))

This is what is called an *environment*. This environment says that 3628800 is stored in x and 11 is stored in y.

The compiler generates the following:

push(1) | store(x) | push(1) | store(y) | load(y) | push(10) | lessThan | load(y) | push(10) | equal | or | jumpOnCond(2) | jump(10) | load(x) | load(y) | multiply | store(x) | load(y) | push(1) | add | store(y) | bjump(17) | quit | iln)

When the virtual machine executes the list of instructions, returns the same environment as that returned by the interpreter.

Minila

Expressions in Minila are inductively defined as follows:

- ✓ Natural numbers are expressions; Natural numbers in Minila are expressed as n(0), n(1), n(2), …;
- ✓ Variables are expressions;
- ✓ If e_1 and e_2 are expressions, so are the following:

$e_1 + e_2$ $e_1 - e_2$ $e_1 * e_2$ e_1 / e_2 $e_1 \% e_2$

$e_1 === e_2$ $e_1 =!= e_2$ $e_1 < e_2$ $e_1 > e_2$

$e_1 \&\& e_2$ $e_1 \| e_2$ (e_1)

Minila

Precedence order of the operators is as follows:

Operators	
*, /, %	high
+, –	
=e=, !=e=, <, >	
&&	
||	low

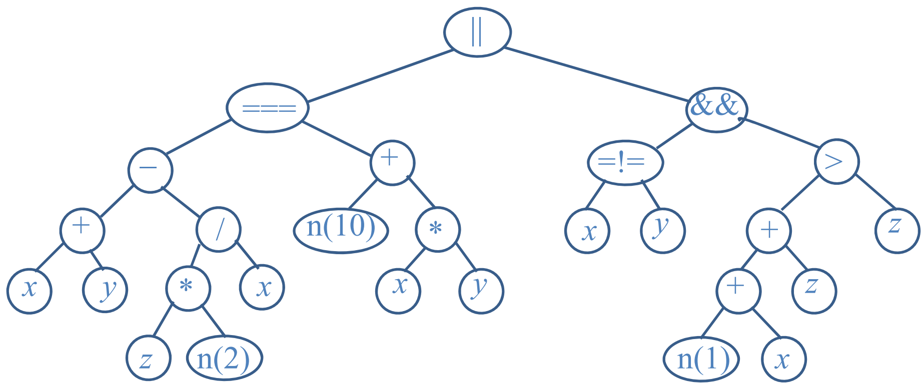
All the operators are left associative.

Minila

$x + y - z * \mathrm{n}(2) / x === \mathrm{n}(10) + x * y \,||\, x =!= y \,\&\&\, \mathrm{n}(1) + x + y > z$

is parsed as follows:

$(((x + y) - ((z * \mathrm{n}(2)) / x)) === (\mathrm{n}(10) + (x * y))) \,||\, ((x =!= y) \,\&\&\, (((\mathrm{n}(1) + x) + y) > z))$



Minila

```
mod* VAR principal-sort Var {
  [Var]
}
```

Terms whose sorts are Var are variables in Minila.

For example,

```
ops x y z tmp : -> Var .
```

x, y, tmp and z are variables in Minila.

Minila

```
mod! EXP {  pr(VAR)  pr(NAT)
  [Var < Exp]
  op n : Nat -> Exp {constr} .
  op _+_ : Exp Exp -> Exp {constr prec: 33 l-assoc} .
  op _-_ : Exp Exp -> Exp {constr prec: 33 l-assoc} .
  op _*_ : Exp Exp -> Exp {constr prec: 31 l-assoc} .
  op _/_ : Exp Exp -> Exp {constr prec: 31 l-assoc} .
  op _%_ : Exp Exp -> Exp {constr prec: 31 l-assoc} .
```

l-assoc specifies that the binary operator is left associative.

prec: *n* specifies the precedence of the operator, where *n* is a natural number. The greater *n* is, the weaker the precedence of the operator is.

Minila

```
  op _===_ : Exp Exp -> Exp {constr prec: 40 l-assoc} .
  op _=!=_ : Exp Exp -> Exp {constr prec: 40 l-assoc} .
  op _<_ : Exp Exp -> Exp {constr prec: 40 l-assoc} .
  op _>_ : Exp Exp -> Exp {constr prec: 40 l-assoc} .
  op _&&_ : Exp Exp -> Exp {constr prec: 50 l-assoc} .
  op _||_ : Exp Exp -> Exp {constr prec: 55 l-assoc} .
}

open EXP .
 ops x y z : -> Var .
 red x + y - z * n(2) / x === n(10) + x * y || x =!= y
     && n(1) + x + y > z .
close
```


Minila

Statements in Minila are inductively defined as follows:

- ✓ estm is a statement.
- ✓ If x is a variable and e is an expression, the following is a statement: $x := e\ ;$
- ✓ If e is an expression and s_1 and s_2 are statements, the following is a statement: **if** e $\{s_1\}$ **else** $\{s_2\}$
- ✓ If e is an expression and s is a statement, the following is a statement: **while** e $\{s\}$
- ✓ If i is a variable, e_1 and e_2 are expressions and s is a statement, the following is a statement: **for** i e_1 e_2 $\{s\}$
- ✓ If s_1 and s_2 are statements, the following is a statement: $s_1\ s_2$

Minila

```
mod! STM {
 pr(EXP)
 [Stm]
 op estm : -> Stm {constr} .
 op _:=_; : Var Exp -> Stm {constr} .
 op if_{_}else{_} : Exp Stm Stm -> Stm {constr} .
 op while_{_} : Exp Stm -> Stm {constr} .
 op for_ _ _{_} : Var Exp Exp Stm -> Stm {constr} .
 op _ _ : Stm Stm -> Stm {constr prec: 60 id: estm l-assoc} .
}
```

A program in Minila is a statement.

Interpreter

How to interpret (or compute) expressions that may have varaibles

What is called an *environment* (or *store*) is used.

An environment is a table from the set of variables to values (natural numbers in Minila) stored in those variables.

How to interpret (or compute) a variable is to look up the value (natural number) associated to the variable in a given environment.

Interpreter

Given an expression *e* that may have variables and an environment *env*, evalExp interprets (or computes) *e* with *env* and returns the result.

```
op evalExp : Exp Env&Err -> Nat&Err .

vars E E1 E2 : Exp .
var V : Var .
var EV : Env .
var N : Nat .

eq evalExp(E,errEnv) = errNat .
```

If the environment is an error, then it returns errNat for any expressions.

Interpreter

```
eq evalExp(n(N),EV) = N .
```

If the expression is a natural number n(N), it returns N regardless of any valid environments.

```
eq evalExp(V,EV) = lookup(EV,V) .
```

If the expression is a variable V, it looks up the value (natural number) associated with V in the environment and returns the value that may be errNat.

```
eq evalExp(E1 + E2,EV) = evalExp(E1,EV) + evalExp(E2,EV) .
```

If the expression is E1 + E2, it interprets E1 and E1 with the same environment and returns what is obtained by adding the two results. The result of interpreting E1 (or E2) with the environment may be errNat. If that is the case, the result of the addition is also errNat.

Interpreter

```
eq evalExp(E1 === E2,EV)
   = if evalExp(E1,EV) == errNat or evalExp(E2,EV) == errNat
     then {errNat}
     else {if evalExp(E1,EV) == evalExp(E2,EV) then {1} else {0}} .
```

If the expression is E1 === E2, it interprets E1 and E1 with the same environment and returns errNat if at least one of the results is errNat; otherwise it returns 1 if the results are the same and 0 if the results are different.

Note that 0 is used as false and non-zero natural numbers are used as true in Minila.

Interpreter

For the remaining operators, equations can be described likewise for evalExp.

Interpreter

Given a program in Minila, interpret interprets the program and returns the environment at the time when the program terminates. It may return errEnv if something wrong, such as division by zero, happens.

```
op interpret : Stm -> Env&Err .
vars S S1 S2 : Stm .
eq interpret(S) = eval(S,empEnv).
```

Given a program in Minila and an environment, eval interprets the program with the environment and returns the environment at the time when the program terminates.

Initially, the environment is empty.

Interpreter

```
op eval : Stm Env&Err -> Env&Err .
eq eval(S,errEnv) = errEnv .
```

If the environment is errEnv, eval returns errEnv regardless of the program.

```
eq eval(estm,EV) = EV .
```

If the program is estm, then it returns the environment.

Interpreter

eq eval(V := E ;,EV) = evalAssign(V,evalExp(E,EV),EV) .

If the program is V := E ;, then it uses evalAssig to interpret the program (a single assignment) with the environment.

The expression E is interpreted with the environment.

op evalAssign : Var Nat&Err Env&Err -> Env&Err .

If the second argument is errNat and/or the third argument is errEnv, it returns errEnv.

Otherwise, it updates the environment by associating the second argument (a natural number) with the first argument (an environment) and returns the updated environment.

Interpreter

eq eval(if E {S1} else {S2},EV) = evalIf(evalExp(E,EV),S1,S2,EV) .

If the program is if E {S1} else {S2}, then it uses evalIf to interpret the program (a single if statement) with the environment.

The expression E is interpreted with the environment.

op evalIf : Nat&Err Stm Stm Env&Err -> Env&Err .

If the first argument is errNat and/or the fourth argument is errEnv, it returns errEnv.

Otherwise, if the first argument is 0, then it interprets the third argument (a statement) with the environment and returns the environment obtained by the interpretation; if the first argument is not 0, then it interprets the second argument (a statement) with the environment and returns the environment obtained by the interpretation.

Interpreter

eq eval(while E {S1},EV) = evalWhile(E,S1,EV) .

If the program is while E {S1}, then it uses evalWhile to interpret the program (a single while statement) with the environment.

op evalWhile : Exp Stm Env&Err -> Env&Err .

If the third argument is errEnv, it returns errEnv.

Otherwise, it interprets the first argument (an expression) with the environment and if the result is errNat, then it returns errEnv; if the result is 0, then it returns the current environment; if the result is not 0, then it interprets(*1) the second argument (a statement) with the current environment and interprets(*2) the while statement with the environment obtained by the interpretation (*1), returning the environment obtained by the interpretation (*2).

Interpreter

for V E1 E2 {S1}

is equivalent to

```
V := E1 ;
while V < E2 || V === E2 {
  S1
  V := V + s(1) ;
}
```

This can be used to describe the equation to interpret the for statement with a given environment.

eq eval(for V E1 E2 {S1},EV) = ...

Interpreter

How to interpret a sequential composition statement S1 S2 with a given environment is as follows: S1 is interpreted with the environment, and then S2 is interpreted with the environment obtained by the first interpretation; the environment obtained by the second interpretation is returned as the result.

eq eval(S1 S2,EV) = … .

Exercises

1. Complete the interpreter and do some tests for the interpreter.

Appendices

Interpreting the program

```
x := n(1) ;
for y n(1) n(10) {
  x := y * x ;
}
```

returns the environment

```
((x , 3628800) | ((y , 11) | empEnv)):Env
```


Appendices

Interpreting the program

```
x := n(24) ;  y := n(30) ;
while y =!= n(0) {
  z := x % y ;  x := y ;  y := z ;
}
```

returns the environment

```
((x , 6) | ((y , 0) | ((z , 0) | empEnv))):Env
```

Appendices

Interpreting the program

```
x := n(20000000000000000) ;  y := n(0) ;  z := x ;
while y =!= z {
  if ((z - y) % n(2)) === n(0) {
    tmp := y + (z - y) / n(2) ;
  } else { tmp := y + ((z - y) / n(2)) + n(1) ;  }
  if tmp * tmp > x {  z := tmp - n(1) ; }
  else {  y := tmp ;  }
}
```

returns the environment

((x , 20000000000000000) | ((y , 141421356) | ((z , 141421356) | ((tmp , 141421356) | empEnv)))):Env