

# The CTRV (Constant turn rate and velocity) model

$$x = \begin{bmatrix} p_x \\ p_y \\ v \\ \psi \\ \dot{\psi} \end{bmatrix}$$

## Initial state and covariance matrix

Initialized by LIDAR

Given

$$z = \begin{bmatrix} p_x \\ p_y \end{bmatrix}$$

Initial state is

$$x = \begin{bmatrix} p_x \\ p_y \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Initial P is

$$P = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1000 & 0 & 0 \\ 0 & 0 & 0 & 1000 & 0 \\ 0 & 0 & 0 & 0 & 1000 \end{pmatrix}$$

Initialized by RADAR

Given

$$z = \begin{bmatrix} \rho \\ \varphi \\ \dot{\rho} \end{bmatrix}$$

Initial state is

$$x = \begin{bmatrix} \rho \cos(\varphi) \\ \rho \sin(\varphi) \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Initial P is

$$P = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1000 & 0 & 0 \\ 0 & 0 & 0 & 1000 & 0 \\ 0 & 0 & 0 & 0 & 1000 \end{pmatrix}$$

Generate Sigma point

$$X_{k|k} = \begin{bmatrix} x_{k|k} & x_{k|k} + \sqrt{(\lambda + n_x)P_{k|k}} & x_{k|k} - \sqrt{(\lambda + n_x)P_{k|k}} \end{bmatrix}$$

## UKF augmentation

Augmented state

$$x = \begin{bmatrix} p_x \\ p_y \\ v \\ \psi \\ \dot{\psi} \\ v_a \\ v_{\ddot{\psi}} \end{bmatrix}$$

Augmented Covariance matrix

$$P_{a,k|k} = \begin{bmatrix} P_{k|k} & 0 \\ 0 & Q \end{bmatrix}$$

## Sigma point prediction

if  $\dot{\psi}_k$  is not zero

$$State = x_{k+1} = x_k + \begin{bmatrix} \frac{v_k}{\dot{\psi}_k} (\sin(\psi_k + \dot{\psi}_k \Delta t) - \sin(\psi_k)) \\ \frac{v_k}{\dot{\psi}_k} (-\cos(\psi_k + \dot{\psi}_k \Delta t) + \cos(\psi_k)) \\ 0 \\ \dot{\psi}_k \Delta t \\ 0 \end{bmatrix} + \begin{bmatrix} \frac{1}{2} (\Delta t)^2 \cos(\psi_k) v_{a,k} \\ \frac{1}{2} (\Delta t)^2 \sin(\psi_k) v_{a,k} \\ \Delta t \cdot v_{a,k} \\ \frac{1}{2} (\Delta t)^2 \cdot v_{\ddot{\psi},k} \\ \Delta t \cdot v_{\ddot{\psi},k} \end{bmatrix}$$

$$State = x_{k+1} = x_k + \begin{bmatrix} v_k \cos(\psi_k) \Delta t \\ v_k \sin(\psi_k) \Delta t \\ 0 \\ \dot{\psi}_k \Delta t \\ 0 \end{bmatrix} + \begin{bmatrix} \frac{1}{2} (\Delta t)^2 \cos(\psi_k) v_{a,k} \\ \frac{1}{2} (\Delta t)^2 \sin(\psi_k) v_{a,k} \\ \Delta t \cdot v_{a,k} \\ \frac{1}{2} (\Delta t)^2 \cdot v_{\ddot{\psi},k} \\ \Delta t \cdot v_{\ddot{\psi},k} \end{bmatrix}$$

## Predicted mean and covariance

Weight

$$w_i = \frac{\lambda}{\lambda + n_a}, i = 1$$

$$w_i = \frac{\lambda}{2(\lambda + n_a)}, i = 2 \dots n_a$$

Predicted Mean

$$x_{k+1|k} = \sum_{i=1}^{n_\sigma} w_i X_{k+1|k}$$

Predicted Covariance

$$P_{k+1|k} = \sum_{i=1}^{n_\sigma} w_i (X_{k+1|k,i} - x_{k+1|k}) (X_{k+1|k,i} - x_{k+1|k})^T$$

## Measurement model

State Vector

$$x = \begin{bmatrix} p_x \\ p_y \\ v \\ \psi \\ \dot{\psi} \end{bmatrix}$$

# LIDAR

## Measurement Vector

$$z = \begin{bmatrix} p_x \\ p_y \end{bmatrix}$$

## Measurement Model

$$z_{k+1|k} = h(x_{k+1}) + w_{k+1}$$
$$z_{k+1|k} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} p_x \\ p_y \\ v \\ \psi \\ \dot{\psi} \end{bmatrix} + w_{k+1}$$
$$R = E(w_k \cdot w_k^T) = \begin{bmatrix} \sigma_{p_x}^2 & 0 \\ 0 & \sigma_{p_y}^2 \end{bmatrix}$$

# RADAR

## Measurement Vector

$$z = \begin{bmatrix} \rho \\ \varphi \\ \dot{\rho} \end{bmatrix}$$

## Measurement Model

$$z_{k+1|k} = h(x_{k+1}) + w_{k+1}$$
$$z_{k+1|k} = \begin{bmatrix} \sqrt{p_x^2 + p_y^2} \\ \arctan(\frac{p_y}{p_x}) \\ \frac{p_x \cos(\psi) v + p_y \sin(\psi) v}{\sqrt{p_x^2 + p_y^2}} \end{bmatrix} + w_{k+1}$$
$$R = E(w_k \cdot w_k^T) = \begin{bmatrix} \sigma_\rho^2 & 0 & 0 \\ 0 & \sigma_\varphi^2 & 0 \\ 0 & 0 & \sigma_{\dot{\rho}}^2 \end{bmatrix}$$

## Predicted measurement model

$$z_{k+1|k} = \sum_{i=1}^{n_\sigma} w_i Z_{k+1|k,i}$$

## Predict Covariance

$$S_{k+1|k} = \sum_{i=1}^{n_\sigma} w_i (Z_{k+1|k,i} - z_{k+1|k}) (Z_{k+1|k,i} - z_{k+1|k})^T + R$$

## UKF Update

Cross-correlation Matrix

$$T_{k+1|k} = \sum_{i=1}^{n_\sigma} w_i (X_{k+1|k,i} - x_{k+1|k}) (Z_{k+1|k,i} - z_{k+1|k})^T$$

Kalman gain K

$$K_{k+1|k} = T_{k+1|k} S_{k+1|k}^{-1}$$

Update State

$$x_{k+1|k+1} = x_{k+1|k} + K_{k+1|k} (z_{k+1} - z_{k+1|k})$$

Covariance Matrix Update

$$P_{k+1|k+1} = P_{k+1|k} + K_{k+1|k} S_{k+1|k} K_{k+1|k}^T$$