The CTRV (Constant turn rate and velocity) model

$$x = \begin{bmatrix} p_x \\ p_y \\ v \\ \psi \\ \dot{\psi} \end{bmatrix}$$

Initial state and covariance matrix

Initialized by LIDAR

Given

$$z = \begin{bmatrix} p_x \\ p_y \end{bmatrix}$$

Initial state is

$$x = \begin{bmatrix} p_x \\ p_y \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Initial P is

$$P = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1000 & 0 & 0 \\ 0 & 0 & 0 & 1000 & 0 \\ 0 & 0 & 0 & 0 & 1000 \end{pmatrix}$$

Initialized by RADAR

Given

$$z = \begin{bmatrix} \rho \\ \varphi \\ \dot{\rho} \end{bmatrix}$$

Initial state is

$$x = \begin{bmatrix} \rho\cos(\varphi) \\ \rho\sin(\varphi) \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Initial P is

$$P = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1000 & 0 & 0 \\ 0 & 0 & 0 & 1000 & 0 \\ 0 & 0 & 0 & 0 & 1000 \end{pmatrix}$$

Generate Sigma point

$$X_{k|k} = \left[x_{k|k} \quad x_{k|k} + \sqrt{(\lambda + n_x)} P_{k|k} \quad x_{k|k} - \sqrt{(\lambda + n_x)} P_{k|k} \right]$$

UKF augmentation

Augmented state

$$x = \begin{bmatrix} p_x \\ p_y \\ v \\ \psi \\ \dot{\psi} \\ v_a \\ v_{\ddot{\psi}} \end{bmatrix}$$

Augmented Covariance matrix

$$P_{a,k|k} = \begin{bmatrix} P_{k|k} & 0 \\ 0 & Q \end{bmatrix}$$

Sigma point prediction

if $\dot{\psi}_k$ is not zero

$$State = x_{k+1} = x_k + \begin{bmatrix} \frac{v_k}{\dot{\psi}_k}\left(\sin\left(\psi_k + \dot{\psi}_k \Delta t\right) - \sin(\psi_k)\right) \\ \frac{v_k}{\dot{\psi}_k}\left(-\cos\left(\psi_k + \dot{\psi}_k \Delta t\right) + \cos(\psi_k)\right) \\ 0 \\ \dot{\psi}_k\ \Delta t \\ 0 \end{bmatrix} + \begin{bmatrix} \frac{1}{2}(\Delta t)^2 \cos(\psi_k)\, v_{a,k} \\ \frac{1}{2}(\Delta t)^2 \sin(\psi_k)\, v_{a,k} \\ \Delta t\ \cdot v_{a,k} \\ \frac{1}{2}(\Delta t)^2\ \cdot v_{\ddot{\psi},k} \\ \Delta t\ \cdot v_{\ddot{\psi},k} \end{bmatrix}$$

$$State = x_{k+1} = x_k + \begin{bmatrix} v_k \cos(\psi_k)\Delta t \\ v_k \sin(\psi_k)\Delta t \\ 0 \\ \dot{\psi}_k\ \Delta t \\ 0 \end{bmatrix} + \begin{bmatrix} \frac{1}{2}(\Delta t)^2 \cos(\psi_k)\, v_{a,k} \\ \frac{1}{2}(\Delta t)^2 \sin(\psi_k)\, v_{a,k} \\ \Delta t\ \cdot v_{a,k} \\ \frac{1}{2}(\Delta t)^2\ \cdot v_{\ddot{\psi},k} \\ \Delta t\ \cdot v_{\ddot{\psi},k} \end{bmatrix}$$

Predicted mean and covariance

Weight

$$w_i = \frac{\lambda}{\lambda + n_a}, i = 1$$

$$w_i = \frac{\lambda}{2(\lambda + n_a)}, i = 2 \dots n_a$$

Predicted Mean

$$x_{k+1|k} = \sum_{i=1}^{n_\sigma} w_i X_{k+1|\,k}$$

Predicted Covariance

$$P_{k+1|k} = \sum_{i=1}^{n_\sigma} w_i \left(X_{k+1|k,i} - x_{k+1|k}\right)\left(X_{k+1|k,i} - x_{k+1|k}\right)^T$$

Measurement model

State Vector

$$x = \begin{bmatrix} p_x \\ p_y \\ v \\ \psi \\ \dot{\psi} \end{bmatrix}$$

LIDAR

Measurement Vector

$$z = \begin{bmatrix} p_x \\ p_y \end{bmatrix}$$

Measurement Model

$$z_{k+1|k} = h(x_{k+1}) + w_{k+1}$$

$$z_{k+1|k} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} p_x \\ p_y \\ v \\ \psi \\ \dot{\psi} \end{bmatrix} + w_{k+1}$$

$$R = E(w_k \cdot w_k^T) = \begin{bmatrix} \sigma_{p_x}^2 & 0 \\ 0 & \sigma_{p_y}^2 \end{bmatrix}$$

RADAR

Measurement Vector

$$z = \begin{bmatrix} \rho \\ \varphi \\ \dot{\rho} \end{bmatrix}$$

Measurement Model

$$z_{k+1|k} = h(x_{k+1}) + w_{k+1}$$

$$z_{k+1|k} = \begin{bmatrix} \sqrt{p_x^2 + p_y^2} \\ \arctan(\frac{p_y}{p_x}) \\ \frac{p_x \cos(\psi)\, v + p_y \sin(\psi)\, v}{\sqrt{p_x^2 + p_y^2}} \end{bmatrix} + w_{k+1}$$

$$R = E(w_k \cdot w_k^T) = \begin{bmatrix} \sigma_\rho^2 & 0 & 0 \\ 0 & \sigma_\varphi^2 & 0 \\ 0 & 0 & \sigma_{\dot{\rho}}^2 \end{bmatrix}$$

Predicted measurement model

$$\mathrm{z_{k+1|k}} = \sum_{i=1}^{n_\sigma} w_i Z_{k+1|k,i}$$

Predict Covariance

$$S_{k+1|k} = \sum_{i=1}^{n_\sigma} w_i \left(Z_{k+1|k,i} - z_{k+1|k}\right)\left(Z_{k+1|k,i} - z_{k+1|k}\right)^T + R$$

UKF Update

Cross-correlation Matrix

$$T_{k+1|k} = \sum\nolimits_{i=1}^{n_\sigma} w_i (X_{k+1|k,i} - x_{k+1|k})(Z_{k+1|k,i} - z_{k+1|k})^T$$

Kalman gain K

$$K_{k+1|k} = T_{k+1|k} S_{k+1|k}^{-1}$$

Update State

$$x_{\mathrm{k+1|k+1}} = x_{k+1|k} + K_{k+1|k}(z_{k+1} - z_{k+1|k})$$

Covariance Matrix Update

$$P_{\mathrm{k+1|k+1}} = P_{k+1|k} + K_{k+1|k} S_{k+1|k} K_{k+1|k}^T$$

Normalized Inovation Squared

$$\varepsilon = \left(z_{k+1} - z_{k+1|k}\right)^T \cdot S_{k+1|k}^{-1} \cdot (z_{k+1} - z_{k+1|k})$$