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Multiaxial high-cycle fatigue life prediction under random spectrum loadings

Haoyang Wei[a], Patricio Carrion[b,c], Jie Chen[a], Anahita Imanian[d], Nima Shamsaei[b,c], Nagaraja Iyyer[d], Yongming Liu[a,*]

[a] School for Engineering of Matter, Transport and Energy, Arizona State University, Tempe, AZ 85287, USA
[b] National Center for Additive Manufacturing Excellence (NCAME), Auburn University, Auburn, AL 36849, USA
[c] Department of Mechanical Engineering, Auburn University, Auburn, AL 36849, USA
[d] Technical Data Analysis, Inc., Falls Church, VA 22042, USA

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ABSTRACT

A multiaxial fatigue life prediction model under general multiaxial random loadings is proposed in this paper. First, a brief review for existing multiaxial fatigue models is given with a special focus is on the Liu- Mahadevan critical plane concept, which can be applied to both brittle and ductile materials. Next, the new model development based on the Liu-Mahadevan critical plane concept for random loading is presented. The key concept is to use two-steps to identify the critical plane: identify the maximum damage plane due to normal stress and calculate the critical plane orientation with respect to the maximum damage plane due to normal stress. Multiaxial rain-flow cycle counting method with mean stress correction is used to estimate the damage on the critical plane. Equivalent stress transformation is proposed to convert the multiaxial random load spectrum to an equivalent constant amplitude spectrum. The equivalent stress is then used for fatigue life predictions. The proposed model is validated with both literature and in-house testing data generated using an Al 7075-T6 alloy under various random uniaxial and multiaxial spectrums. Comparison between experimental and predicted fatigue life lives shows good agreements; thus, demonstrating efficacy of the proposed model. Finally, concluding remarks and future work based on the results obtained are discussed.

1. Introduction

Multiaxial fatigue models can be classified into four major categories: stress-based [1–3], strain-based [4,5], energy-based [6,7], and fracture mechanics-based approaches [8]. Employed under any of these categories, there is a criterion that offers a physical interpretation of the fatigue damage by relating the crack orientation (initiation and propagation) to a plane of critical loading, known as the “critical plane approach”.

The basic concept of the critical plane approach is to use the stress/strain components on a plane to calculate the fatigue damage of material under general multiaxial cyclic loadings. Many models assume the maximum shear stress plane as the critical plane, which is suitable for ductile failure [9,10]. Other models assume the maximum normal stress range plane as the critical plane, which is mostly suitable for brittle failure [11–13]. However, when these two failure modes mix, or the material is neither ductile nor brittle, it is difficult to select an appropriate model. Furthermore, material's failure mode can also change with respect to the fatigue life regime (i.e. low-cycle or high-cycle) [14]. Several attempts trying to solve this issue have been proposed in the past. One successful approach is to let the critical plane change its orientation for different failure modes, i.e., along the maximum normal stress range plane for brittle materials and along the maximum shear stress range plane for ductile materials. The concept was initially proposed using an empirical function [15–17].

Liu and Mahadevan [18,19] proposed an analytical solution for the critical plane orientation based on the material ductility, known as Liu-Mahadevan critical plane concept. The concept was first applied using the stress-based approach for high-cycle fatigue [18] and was later extended for low-cycle fatigue using a strain-based model [19]. Extensive model validations for this concept have been performed for both brittle and ductile materials at the material and component level. However, since only the strain terms are used in the model [19], it cannot include the out-of-phase hardening behaviour explicitly. Out-of-phase loading causes rotation of the direction of the principal stresses. The rotation brings additional hardening to the material due to the

* Corresponding author.
E-mail address: yongming.liu@asu.edu (Y. Liu).

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activation of more slip planes compared to other types of loading. Thus, the rotation causes an increase in stress response and thus reduces the fatigue life. In order to compensate for out-of-phase hardening, some empirical parameters are used [19,20]. Calibration of these parameters requires several out-of-phase multiaxial fatigue tests.

Recently, Wei and Liu [21] proposed an energy-based model integrating the Liu-Mahadevan critical plane concept. The energy-based model can be applied to a wide range of materials (both brittle and ductile) under both proportional and non-proportional loadings without the need of calibration parameters. This is because that both stress and strain terms are used in the energy-based criteria and the out-of-phase hardening effect is automatically included. Only constant amplitude loading is considered in [21]. In this paper, the Liu-Mahadevan critical plane concept is extended for random fatigue loading.

The main new development considers the damage accumulation under general random multiaxial loadings in the Liu-Mahadevan model framework. Many existing fatigue criteria for random loading are in either time domain [22] or frequency domain [23]. Usually time domain approaches are based on a cycle counting method and a cumulative damage rule. Rainflow cycle counting method is widely used for analysis and simplification of random fatigue load paths. Amongst other cycle counting models proposed, a new cycle counting procedure for normal stress on critical plane model was introduced by Carpinteri et al. [24]. Across all the fatigue damage accumulation rules, the linear damage accumulation rule (LDR), also known as the Miner's rule, is probably the most commonly used due to simplicity [25]. For a critical plane-based multiaxial fatigue model, enumeration of all possible plane orientation is usually used for damage evaluation. The maximum damage along a certain plane is identified. This is certainly possible, but usually very time consuming as exhaust search is required. In addition, cycle counting for both normal and shear stress may be required.

In the current study, the authors attempt to develop a multiaxial fatigue model for random loading by integrating the Liu-Mahadevan critical plane approach along with Rainflow counting and Miner's damage rule. For computational efficiency, a maximum damage plane is estimated first and an equivalent stress transformation is proposed to determine fatigue life.

The remaining of the paper is organized as follows. First, a brief review of the Liu-Mahadevan model concept is given. Extension from constant amplitude loading to random spectrum loading is done by replacing the maximum normal stress amplitude plane with the maximum normal damage plane. Derivation of this maximum damage plane is discussed in detail. Next, an equivalent constant stress transformation of the random multiaxial spectrum loading is proposed for the fatigue life prediction. Following this, both literature data and in-house testing data are used to validate the proposed methodology. Finally, discussion and conclusions are provided based on the proposed method.

2. Proposed methodology for fatigue life prediction related to HCF under multiaxial random spectrum

2.1. Brief review of Liu-Mahadevan model under constant amplitude loading

The Liu-Mahadevan critical plane approach for stress-based fatigue life prediction under constant multiaxial loading condition [18] is given in Eq. (1);

$$\sqrt{\left(\frac{\sigma_{a,c}}{f_{-1}}\right)^2 + \left(\frac{\tau_{a,c}}{t_{-1}}\right)^2 + k\left(\frac{\sigma_{a,c}^H}{f_{-1}}\right)^2} = \beta \tag{1}$$

where $\sigma_{a,c}$, $\tau_{a,c}$, $\sigma_{a,c}^H$ are the normal, shear, and hydrostatic stress amplitudes acting on the critical plane, respectively. f_{-1} and t_{-1} are uniaxial and torsional fatigue stress limits for fully-reversed constant amplitude loading, respectively. Parameters k and β are material constants determined from uniaxial and pure torsional fully-reversed tests. Eq. (1) represents the summation of the damage caused by normal, shear and hydrostatic stress components. Most of critical plane approaches assume that the critical plane only depends on the stress state, which

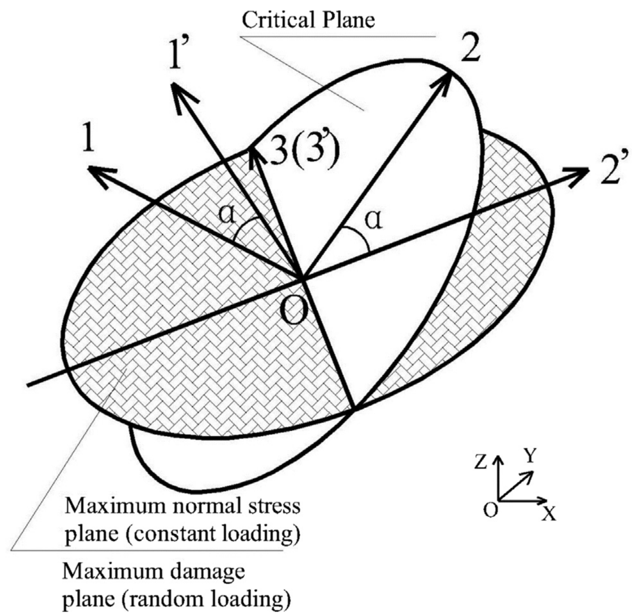

Fig. 1. Orientation of maximum damage plane and critical plane.

Table 1
Liu-Mahadevan material parameters.

Material property	$s=\frac{t_{-1}}{f_{-1}} \leqslant 1$	$s=\frac{t_{-1}}{f_{-1}} > 1$
α	$\cos(2\alpha) = \frac{-2+\sqrt{4-4\times(1/s^2-3)\times(5-1/s^2-4s^2)}}{2\times(5-1/s^2-4s^2)}$	$\alpha = 0^\circ$
k	k= 0	$k= 9\left[\left(\frac{t_{-1}}{f_{-1}}\right)^2 - 1\right]$
β	$\beta = \sqrt{\cos^2(2\alpha)s^2 + \sin^2(2\alpha)}$	$\beta = 1$

Fig. 2. Experimental testing setup.

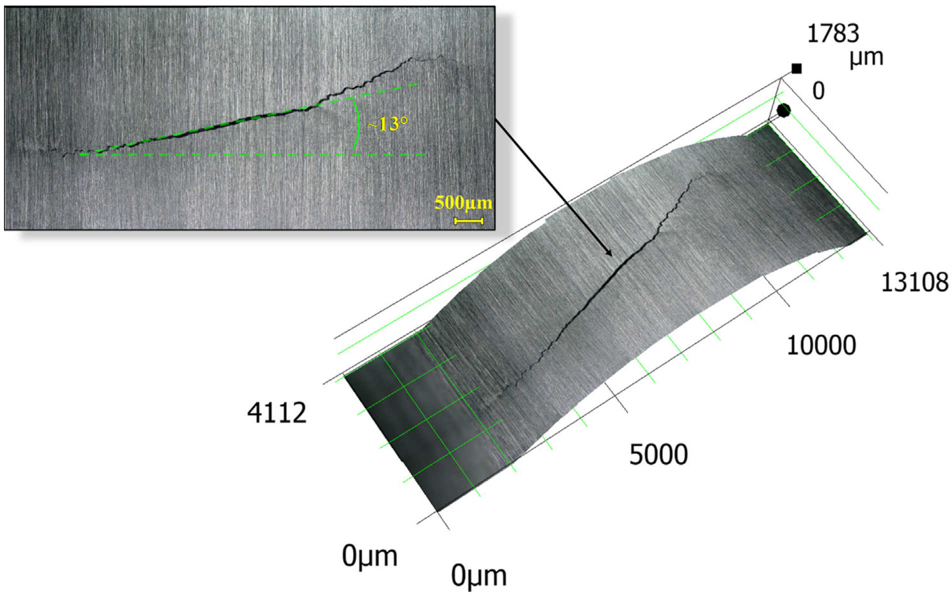

Fig. 3. Optical microscope image of fatigue failure under multiaxial random loading and crack orientation.

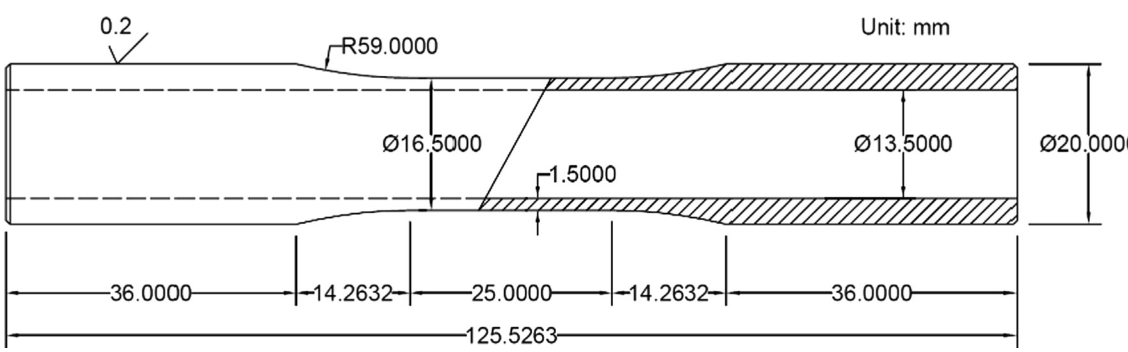

Fig. 4. Specimen design for multiaxial fatigue testing.

indicates that such models account the fatigue damage accumulation in the same way for different materials under the same stress state. Their applicability generally depends on the material's properties. However, in the Liu-Mahadevan critical plane concept, the critical plane does not only depend on the stress state but also on the material properties. The critical plane is theoretically determined by minimizing the damage introduced by the hydrostatic stress amplitude, which has almost no applicability limitations with respect to different metals. The key concept to achieve this wide applicability of the model is to let the critical plane rotate with respect to the material ductility. A schematic illustration is shown in Fig. 1. The critical plane has angle α with respect to the maximum normal stress plane. The angle α is zero for brittle materials and is 45 degrees for ductile materials. Instead of empirically interpolating the angle α between these two extremes, an analytical solution is obtained (see details in [18]). The results for model parameters are shown in Table 1 [18].

The Liu-Mahadevan fatigue prediction model under constant loading has been validated by different materials with a wide range of ductility, from extremely brittle steels to ductile steels [18].

2.2. Proposed model for arbitrary random loading

The above discussion and review are only for constant amplitude loading. It should be noted that the relationship of critical plane with maximum normal stress amplitude plane is no longer valid under random loading as the final fracture plane may not be the same as the maximum stress amplitude plane. Now the question is: how to define the critical plane under general random loadings?

The definitions of the fatigue fracture plane and the critical plane should be clarified first before determining the critical plane orientation. In the fatigue model under constant amplitude loading proposed by Liu and Mahadevan [18,19], the fatigue fracture plane refers to the crack plane observed at the macro level. The critical plane is not an actual crack plane, but it is a material plane on which the fatigue damage is evaluated. The two planes may or may not coincide with each other. The fatigue fracture plane is assumed to be the plane, which experiences the maximum normal stress amplitude. The critical plane orientation may differ from the fatigue fracture plane for different materials, which depends on material ductility.

In the proposed model, it is assumed that, under variable loading condition, the fatigue fracture plane is the maximum damage plane, and damage occurs due to the normal stress amplitude acting on the above plane. Normal stress spectrum on all possible orientations is computed and then the plane with maximum damage due to normal stress amplitude is found using the rainflow counting algorithm and Miner's rule. Next, the proposed study replaces the maximum normal stress amplitude plane (under constant amplitude loading) with the maximum normal damage plane (under random loading) for all calculation steps in Section 2.1. Note that the maximum normal damage plane definition is the same as that of the maximum normal stress amplitude plane under constant amplitude loading cases. Thus, the proposed extension can be considered as generalization of the Liu-Mahadevan model concept. The maximum damage plane under random loading (the maximum normal stress plane under constant amplitude

Table 2
Standard fatigue spectrums.

Name	Purpose	Structural details	Description of load history	Block size (cycles)	Equiv. usage	No. of load levels	Ref
FELIX	Helicopters fixed rotors	Rotor blade bending	Blocks of cycles with different amplitude and mean stress levels	2.3×10^6	140	33	[31]
FALSTAFF	Fighter aircraft	Wing root	Maneuver dominated spectrum, moderate fluctuation of mean stress	18,000	200	32	[32]
Mini TWIST	Shortened version of TWIST	Wing root bending moment	Omission of low gust load cycles	62,000	4000	20	[33]

loading) and the critical plane are shown in Fig. 1.

For the proposed fatigue model under variable loading, all parameters from material properties keep the same value as those under constant amplitude loadings (i.e., Table 1). However, the formula of the fatigue model under constant loading cannot be directly applied because, under a variable amplitude loading, the stress components on the critical plane are not constant. There are two approaches to solve this issue. One is to calculate the cumulative fatigue damage cycle-by-cycle, which could be time consuming, specifically for random high cycle fatigue loading. The other one is to transfer a variable loading condition to an equivalent constant loading condition, then the equivalent stress components are used following the same procedure described in [18]. The detailed procedure for computing the equivalent stress on the critical plane is shown below.

First, fatigue damage of normal stress D_c^{σ} and shear stress D_c^{τ} on the critical plane can be computed using the Miner's rule and rainflow accounting algorithm as is presented in Eq. (2);

$$\begin{cases} D_c^{\sigma} = \sum_{i=1}^{N_\sigma} \frac{1}{f(\sigma_{a,c}^i)} \\ D_c^{\tau} = \sum_{i=1}^{N_\tau} \frac{1}{f(\tau_{a,c}^i)} \end{cases} \tag{2}$$

where N_σ and N_τ are number of cycles of normal and shear stress spectrum on critical plane based on the rainflow counting algorithm, respectively. The term $f(\sigma)$ is the fatigue life function of the normal or shear stress on the critical plane, which computes the number of cycles under a given loading case σ. It should be noticed that the fatigue life function is fitted by uniaxial and pure torsional S-N curves. For example, $f(\sigma)$ could be in power form (Basquin's model) given by Eq. (3):

$$f(\sigma) = \mathrm{A} \times \sigma^B \tag{3}$$

Thus, the term $\frac{1}{f(\sigma^i)}$ is the fatigue damage in the i$^{\text{th}}$ cycle of normal or shear loading. Eq. (2) represents a summation of the fatigue damage of a random spectrum with zero mean stress.

Next, the mean stress effect of arbitrary random spectrum is included. It is well-known that the mean normal stress has an important effect on fatigue life. Generally, tensile mean stress reduces the fatigue life, while compressive mean stresses are said to increase fatigue life or considered to be neutral. The effect of the mean shear stress is small [26], and it is neglected in this paper. The mean stress term σ_m refers to the mean normal stress. Under constant amplitude loading conditions, the mean stress value is defined as the algebraic mean of the maximum and minimum stress value in one cycle. For the random loading, mean stress is determined by the average of normal stress spectrum on the maximum damage plane at i$^{\text{th}}$ cycle [27,28], presented in the integral form in Eq. (4):

$$\sigma_m^i = \frac{\int \sigma_D^i(t)dt}{t^i} \tag{4}$$

Mean stress is introduced into the fatigue model with a correction factor $\left(1 + \eta\frac{\sigma_m}{\sigma_y}\right)$, where η is a material coefficient, which can be obtained from uniaxial fatigue test with the same mean stress ratio, and σ_y is the yield stress. It should be noticed that σ_m is defined as acting on the maximum damage plane. Thus, the damage of normal and shear stress on the critical plane can be calculated using the Miner's rule based on the rainflow accounting algorithm, given in Eq. (5) as:

$$\begin{cases} D_c^{\sigma} = \sum_{i=1}^{N_\sigma} \frac{1}{f\left[\left(1+\eta\frac{\sigma_m{}^i}{\sigma_y}\right)\sigma_{a,c}^i\right]} \\ D_c^{\tau} = \sum_{i=1}^{N_\tau} \frac{1}{f\left[\left(1+\eta\frac{\sigma_m{}^i}{\sigma_y}\right)\tau_{a,c}^i\right]} \end{cases} \tag{5}$$

where $\sigma_m{}^i$ is mean stress at i$^{\text{th}}$ cycle, $\sigma_{a,c}^i$ and $\tau_{a,c}^i$ are normal and shear

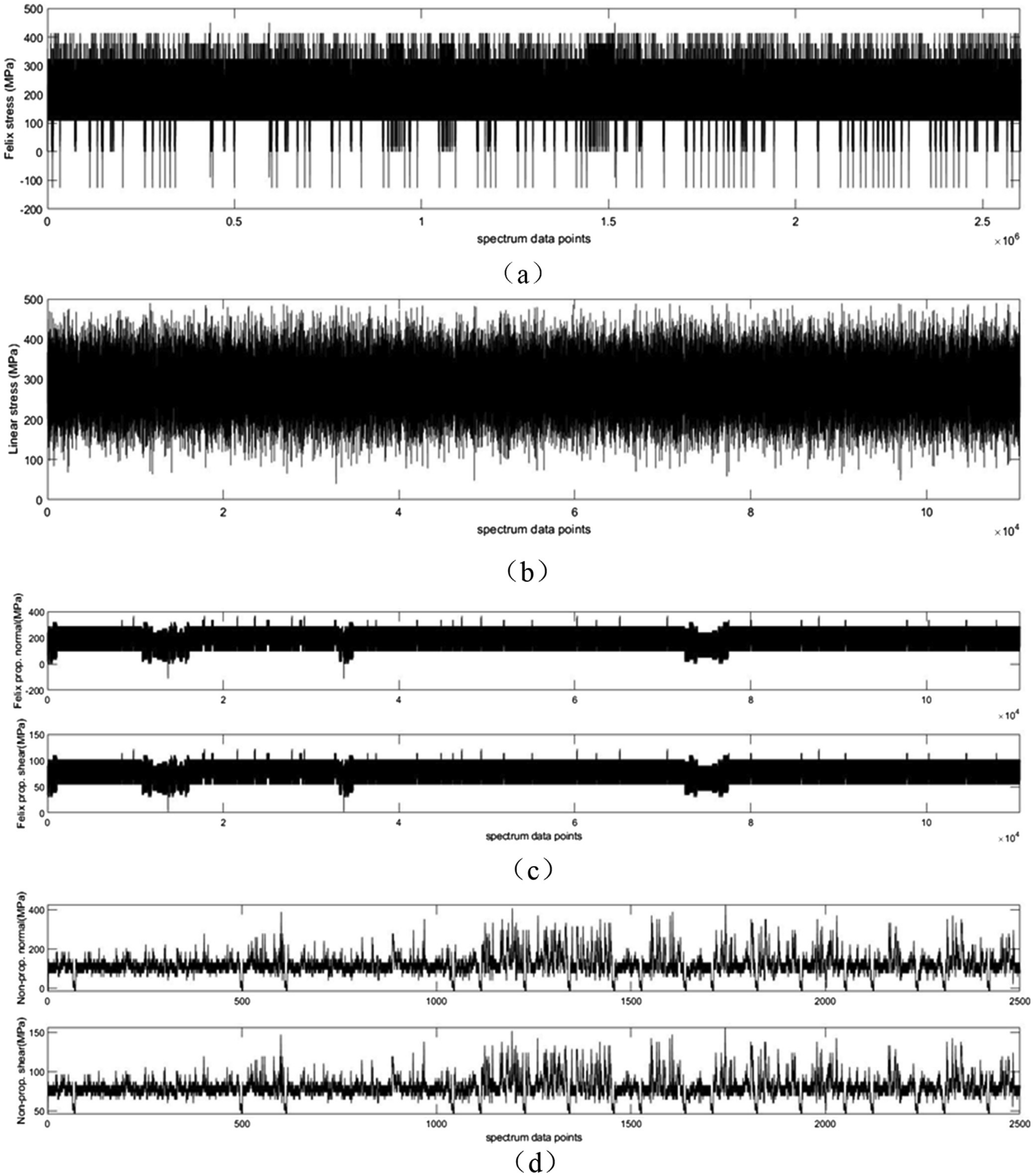

Fig. 5. Generated spectrums for fatigue experiments (a) Axial spectrum based on FELIX, (b) Linear spectrum based on AR(2), (c) Proportional spectrum based on FELIX, and (d) Non-proportional spectrum based on FALSTAFF.

stress amplitude on critical plane at ith cycle, respectively.

Following this, an equivalent stress transformation is proposed. The main objective of this paper is to convert a complex multiaxial variable amplitude load path to a simpler constant amplitude multiaxial loading condition in order to predict the fatigue life using the Liu-Mahadevan model. Equivalent stress transformation from random loading to constant loading has been investigated for fatigue crack propagation [29], which is based on crack growth rate equivalence. In this Section, an equivalent stress transformation for fatigue life prediction is proposed, which is based on fatigue damage equivalence on the critical plane.

For stationary variable amplitude loading, damage from normal stress amplitude D_c^{σ}, shear stress amplitude D_c^{τ}, and hydrostatic stress amplitude D_c^{H} on the critical plane for one block is investigated using the rainflow accounting algorithm and Miner's rule. Mean stress correction factor is also included. The equivalent normal stress $\sigma_{a,c}^{eq}$ and the equivalent shear stress $\tau_{a,c}^{eq}$ are defined as they have the same normal

Table 3
Experimental testing summary.

Test No.	Load	Number of reversals	Test No.	Load	Number of reversals
1	Multiaxial (Proportional)	1,340,500	7	Uniaxial	5,956,555
2	Multiaxial (Proportional)	1,748,119	8	Uniaxial	1,971,920
3	Multiaxial (Non-proportional)	1,221,030	9	Uniaxial	2,905,820
4	Multiaxial (Proportional)	2,553,210	10	Uniaxial	1,196,807
5	Multiaxial (Non-proportional)	405,053	11	Uniaxial	2,112,921
6	Uniaxial	736,909			

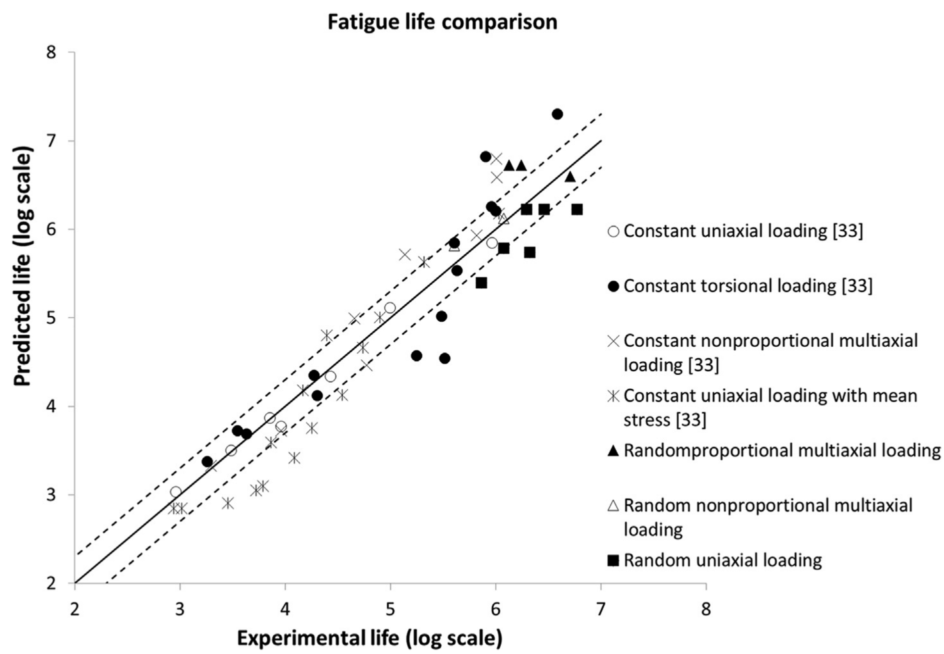

Fig. 6. Comparison of model prediction with experimental results.

and shear damage on the critical plane with a given equivalent number of loading cycles, which can be expressed using damage mechanics concept given in Eq. (6):

$$\begin{cases} D_c^{\sigma} = N^{eq} \times \dfrac{1}{f(\sigma_{a,c}^{eq})} \\ D_c^{\tau} = N^{eq} \times \dfrac{1}{f(\tau_{a,c}^{eq})} \end{cases} \tag{6}$$

where N^{eq} is the number of loading cycles for equivalent normal and shear stresses. For consistency, N^{eq} is assumed to be the total number of cycles or reversals of tensile spectrum from the rainflow accounting algorithm. According to the definition of equivalent stress, the damage components on the critical plane in Eq. (5) and Eq. (6) are equal. Thus, combining Eq. (5) and Eq. (6), there are two equations and two unknowns, $\sigma_{a,c}^{eq}$ and $\tau_{a,c}^{eq}$, which can be solved:

$$\begin{cases} \sigma_{a,c}^{eq} = f^{-1}\left(\dfrac{N^{eq}}{D_c^{\sigma}}\right) = f^{-1}\left(N^{eq} \Bigg/ \sum_{i=1}^{N_\sigma} \dfrac{1}{f\left(\left(1+\eta\frac{\sigma_m^i}{\sigma_y}\right)\sigma_{a,c}^i\right)}\right) \\ \tau_{a,c}^{eq} = f^{-1}\left(\dfrac{N^{eq}}{D_c^{\tau}}\right) = f^{-1}\left(N^{eq} \Bigg/ \sum_{i=1}^{N_\tau} \dfrac{1}{f\left(\left(1+\eta\frac{\sigma_m^i}{\sigma_y}\right)\tau_{a,c}^i\right)}\right) \end{cases} \tag{7}$$

The equivalent hydrostatic stress $\sigma_{a,c}^{eq,H}$ can be computed from the average of the hydrostatic stress spectrum since it is not dependent on the critical plane orientation. Therefore, a random loading block can be converted to a constant loading condition with equivalent normal and shear stresses, $\sigma_{a,c}^{eq}$, $\tau_{a,c}^{eq}$, on the critical plane with N^{eq} number of loading cycles. Comparing with the original random loading condition, the equivalent constant loading condition has the same orientation of the critical plane and the same fatigue damage on the critical plane. Thus, the equivalent normal, shear and hydrostatic stresses can be used for fatigue life prediction as is shown in the constant completed loading cases by Liu and Mahadaven [18]. Eq. (1) is modified by replacing the constant amplitude loading with the equivalent constant amplitude loading as is shown in Eq. (8):

$$\sqrt{\left(\frac{\sigma_{a,c}^{eq}}{f_{-1}}\right)^2 + \left(\frac{\tau_{a,c}^{eq}}{t_{-1}}\right)^2 + k\left(\frac{\sigma_{a,c}^{eq,H}}{f_{-1}}\right)^2} = \beta \tag{8}$$

where $\sigma_{a,c}^{eq}$ $\tau_{a,c}^{eq}$ and $\sigma_{a,c}^{eq,H}$ are the equivalent normal, shear, and hydrostatic stress amplitudes acting on the critical plane, respectively.

Finally, life evaluation can be performed following the similar procedure shown in [18]. Eq. (8) can be rewritten as Eq. (9):

$$\frac{1}{\beta}\sqrt{(\sigma_{a,c}^{eq})^2 + \left(\tau_{a.c}^{eq}\frac{f_{-1}}{t_{-1}}\right)^2 + k(\sigma_{a,c}^{eq,H})^2} = f_{-1} \tag{9}$$

The left-hand side of Eq. (9) can be treated as an effective stress term and can be related to the uniaxial S-N curve for fatigue life prediction. Thus, the fatigue model for variable loading condition of finite life N is expressed in Eq. (10) as follows:

$$\frac{1}{\beta}\sqrt{(\sigma_{a,c}^{eq})^2 + \left(\tau_{a.c}^{eq}\frac{f_{-1}}{t_{-1}}\right)^2 + k(\sigma_{a,c}^{eq,H})^2} = f_N \tag{10}$$

where N means the predicted fatigue life under the equivalent constant loading. Eq. (10) is the general formulation for finite life prediction under general random multiaxial loading spectrums.

3. Experimental testing

3.1. Experimental setup

Fatigue testing was conducted and the test system used was an MTS 809 close-loop servohydraulic axial-torsion load frame with a load capability of 100 kN and 1100 N m. Axial/Torsion tests were conducted at a frequency of 5 Hz, using different sinusoidal load spectrums in force and torque control. The experimental setup and test system are presented in Fig. 2. The load spectrum was repeated until failure, which was defined as either a 5% increase in the maximum displacement or angle amplitude; or the appearance of a visible crack length of 2–5 mm on surface. A representative image of the observed fatigue failure and resultant crack orientation is presented in Fig. 3. Duplicate tests for proportional loading conditions were performed to verify the results obtained.

3.2. Specimen and load spectrum design

Testing material is Al-7075-T6. The specimen design follows ASTM standard E2207-15 (shown in Fig. 4). A tubular thin-walled specimen geometry is used to minimize the stress gradient effect across the thickness direction.

Nonlinear, nonstationary standard fatigue spectrums were generated. Table 2 lists the characteristics of the selected spectrums. A linear nonstationary fatigue spectrum, was also generated using Auto Regressive (2) (AR(2)) process, where the stationarity of spectrum was tested via Hinch method [30].

The equation used for linear nonstationary spectrum generation is the following one:

$$x(n) = a_1(n)x(n-1) - a_2x(n-2) + \eta \tag{11}$$

where

$$a_1(n) = 2\cos\left(\frac{2\pi}{T(n)}\right)e^{-\frac{1}{T}},\ a_2 = e^{-\frac{2}{T}} \tag{12}$$

$$T(n) = T_e + M_T \sin\left(\frac{2\pi t}{T_{\mathrm{mod}}}\right) \tag{13}$$

with $M_T = 6$ for non-stationary conditions and $\eta \sim N(0, 1)$.

Fatigue spectrums for uniaxial, in-phase and out-of-phase axial-torsion test cases are shown in Fig. 5. FELIX and linear spectrums are shown in Fig. 5 (a) and (b), respectively. Non-proportional loading spectrum was generated based on FALSTAFF spectrum. Fig. 5 (c) and (d) show the proportional and non-proportional fatigue spectrums.

Fatigue test results of all finished specimens are presented in Table 3, which includes the test number, the load spectrum used, and the number of reversals to failure.

4. Model validation

The proposed fatigue model is applied to predict the fatigue life of Al-7075-T6 under uniaxial and multiaxial variable amplitude loading conditions. The basic material properties used are from the open literature [34]. Young's modulus is 71GPa; Poisson's ratio is 0.3; Yield stress is 503 MPa. The proposed fatigue model predicts fatigue live under multiaxial constant loading condition based on experiment data from the literature [34] and under random loading based on our in-house fatigue testing data. In order to validate the variable fatigue model, the predicted fatigue lives are compared with the experimentally obtained fatigue lives. The comparison is shown in Fig. 6.

The solid line indicates that the predicted results are identical with the experimental results, i.e., the perfect fit line. The dashed lines are the bounds of fatigue life scatter with a factor of 2. The comparison considers different loading conditions including uniaxial and multiaxial loading, proportional and non-proportional loading. As can be seen in the figure, the predicted results are in good agreement with the experimental results not only under constant loading but also under random loading with or without mean stress.

5. Conclusion and future work

In this paper, a stress-based fatigue model under arbitrary random multiaxial loading is proposed using the Liu-Mahadeven critical plane concept and an equivalent stress transformation method. To validate the proposed model, Al-7076-T6 specimens are tested under different loading conditions. It is shown that the proposed model predictions have satisfactory accuracy without systematic errors for both random uniaxial loading and multiaxial loading. Several major conclusions can be drawn from the present study:

(1). The fracture plane is determined under variable loading condition by using the maximum damage plane concept.

(2). The proposed equivalent stress transformation can reduce a random loading case to a simple constant loading case for easy fatigue life prediction, which avoids cycle-by-cycle calculation of fatigue damage.

(3). The current model validation shows that the linear damage accumulation rule, i.e., the Miner's rule, provides satisfactory results for the investigated random spectrum.

The current investigation focuses on the high cycle fatigue (HCF). Future work to extend the concept to low cycle fatigue (LCF) and to mixed HCF + LCF is needed. Linear damage accumulation is shown to provide satisfactory results, and the used spectrum are not highly nonstationary, such as high-low and low-high step loading. Further confirmation of the applicability of linear damage accumulation rule is needed under other types of multiaxial random loading. Probabilistic modeling considering both material randomness and loading randomness needs further study. Special attention should be paid on the uncertainty quantification, and modeling with limited number of fatigue data is needed as it would be very time consuming and expensive to design and repeat the testing of fall possible combinations (e.g., loading conditions, material surface finish).

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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