# Homework 1

**ECE 269: Linear Algebra and Applications**
**Homework #1**
**Instructor: Behrouz Touri**
**Due Date: ~~Thursday Jan 16th~~ Friday Jan 17th, 8 pm (submission through Gradescope)**

1. *Review of prior Linear Algebra: Properties of Matrix multiplication.* Let $A, B \in \mathbb{R}^{n\times n}$ be regular $n \times n$ real-valued matrices ($n \geq 2$). Prove or disprove the following claims:
   (a) $AB = BA$.
   (b) If $AB = 0$, then $A = 0$ or $B = 0$.
   (c) If $A^k = 0$ for all $k > 1$, then $A = 0$.
   (d) If $A'A = 0$, then $A = 0$.

2. *Some set theory.* Let $A, B, C$ be three sets.
   (a) Show that if $A \subseteq B$, then $A - C \subseteq B - C$ where $A - C = A \cap C^c$.
   (b) Is this true or not: $A \cup (B \cap C) = (A \cup B) \cap C$.

3. *Finite field $GF(4)$.* As mentioned in the class, a field is a set equipped with two tables (operations), an addition table, and a multiplication table that are related through the distributive law.
   (a) Construct those tables for the set $\mathbb{F} = \{0, 1, a, b\}$ where 0 is the unity of the additive table and 1 is the unity of the multiplication table.
   (b) Given your answer in part (a), solve $a \cdot x + 1 = b$ (i.e., find $x \in \mathbb{F}$ that satisfies this identity).

4. *Field Properties.* Let $(\mathbb{F}, +, \cdot)$ be a field with the additive identity element $z$ and multiplicative identity element $o$. Prove the following.
   (a) For all $a \in \mathbb{F}$, $z \cdot a = z$.
   (b) Show that if $\mathbb{F}$ is finite, and $a \neq z$, then $a^q = o$ for some $q \geq 1$.

5. *Subspaces.* Let $\mathcal{V}$ and $\mathcal{W}$ be subspaces of a vector space. Which of the following is also a subspace?
   (a) *Minkowski sum* $\mathcal{V} + \mathcal{W} = \{v + w : v \in \mathcal{V},\ w \in \mathcal{W}\}$.
   (b) $\mathcal{V} \cap \mathcal{W}$.
   (c) $\mathcal{V} \cup \mathcal{W}$.

   For each case, either verify that it is a subspace or prove otherwise.

6. *Bases.* Find a basis for each of the following subspaces of $\mathbb{R}^4$.
   (a) All vectors whose components are equal.

(b) All vectors whose components sum to zero.

(c) All vectors orthogonal to both $\begin{bmatrix}1 & 0 & 1 & 0\end{bmatrix}'$ and $\begin{bmatrix}0 & 1 & 0 & 1\end{bmatrix}'$.

(d) All vectors spanned by $\begin{bmatrix}1 & 1 & 0 & 0\end{bmatrix}'$, $\begin{bmatrix}0 & 1 & 1 & 0\end{bmatrix}'$, $\begin{bmatrix}0 & 0 & 1 & 1\end{bmatrix}'$, and $\begin{bmatrix}1 & 0 & 0 & 1\end{bmatrix}'$

Repeat parts (a)–(d) for $\mathbb{Z}_2^4$ instead of $\mathbb{R}^4$.