Name: ____________________

ECE 269: Linear Algebra and Applications, Midterm
Instructor: Behrouz Touri

- Policy:
  - ***No electronics***
  - Closed book
  - All paper & scratch paper provided:
    - feel free to use back of each page as a scratch paper,
    - there is an extra page provided as a blank page at the back of the exam sheet,
    - if you need extra papers, ask the proctors,
    - write your name on top of each provided paper,
    - all the papers need to be returned to the proctors (including cheat-sheets).
  - ***You must show all work to receive credit for a solution...even if final answer is correct.***
  - The exam carries 6 extra points (out of 100).
- There are 3 equally weighted questions.
- ***Please do not forget to write down your name at the top of each page.***

- In all of the following questions, $(\mathbb{F}, +, \cdot)$ is a field.

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1. For each of the following statements, determine whether it is true or false. Explain your answer. Correct answers without explanation carry no point.

   a. There exists a 2×3 matrix $A \in \mathbb{R}^{2\times 3}$, such that there are two matrices B and C with $BA = I_{3\times 3}$ and $AC = I_{2\times 2}$ (here, $I_{n\times n}$ is the $n \times n$ identity matrix).

   b. If columns of the matrix $A \in \mathbb{F}^{n\times n}$ are independent, then the columns of A^2 are also independent.

c. For an inner-product space V with the inner product $\langle \cdot, \cdot \rangle$, if S is a basis and $\langle x, v \rangle = 0$ for all $v \in S$, then $x = 0$.

d. For all matrices $A \in \mathbb{F}^{n \times n}$, $\text{rank}(A) = \text{rank}(A^T A)$.

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2. Prove the following statements.

   (a) We say that a matrix $A \in \mathbb{F}^{n\times n}$ is a lower-triangular matrix if $A_{ij} = 0$ for $j > i$. Show that if A is an invertible lower-triangular matrix, then all its diagonal elements are non-zero.

   (b) Suppose that $\mathbb{F}$ is a finite-field. Show that if A is invertible, then $A^k = I$ for some $k \geq 1$.

   (c) Is the statement in Part (b) still true for an infinite field? Justify your answer.

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3. Consider the space of finite-energy real-valued functions $L_2([0,1])$, i.e., the space of functions $f : [0,1] \to \mathbb{R}$ with $\int_0^1 f^2(x)dx < \infty$.

   (a) Show that the mapping $\langle f(x), g(x) \rangle := \int_0^1 f(x)g(x)dx$ is an inner-product in this space.

   (b) Consider the three polynomials $p_1(x) = 1$, $p_2(x) = x$, and $p_3(x) = x^2$. Using the Gram-Schmidt procedure, find out three orthonormal polynomials $q_1(x), q_2(x), q_3(x)$, such that $\mathrm{span}(\{p_1(x), \ldots, p_i(x)\}) = \mathrm{span}(\{q_1(x)), \ldots, q_i(x)\}$ for $i = 1, 2, 3$.

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