ECE 269: Linear Algebra and Applications

Lecture 1: Introduction

Behrouz Touri (email: btouri@ucsd.edu, office: EBU1 6408)

University of California San Diego

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Course Logistics

This course: Teaching crew :-)

- Behrouz Touri (instructor)
  - Office: EBU1 (JH) 6408
  - Email: btouri@ucsd.edu
  - Office hours: Tuesdays 11:00 am to 12:00 pm (in-person)
  - Research interest: control theory, dynamics over networks, stochastic systems

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  - Email: miz152@ucsd.edu
  - Office hours: Fridays 11 am-12 pm (In person)
- Anand Kumar (TA)
  - Office: Jacobs Hall (EBU1) 5101C
  - Email: ank029@ucsd.edu
  - Office hours: Thursdays 11:00 am to 12:00 pm (in-person)

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Academic Integrity

- Any cheating in any parts of the course leads to zero grade in the respective grade item.

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- Example: copying Problem 1 in HW 1 results in 0 out of 15% of HW grade for the course.

Intro to Linear Algebra

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- The study of linear objects and structures

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- Example topics:
    (i) What scalar, addition, scalar multiplication means?
    (ii) Moving beyond Euclidean spaces
    (iii) What vectors can be achieved by linear transformations?
    (iv) What happens when we combine linear transformations?
    (v) ...

Applications of Linear Algebra

- Machine Learning: classification problem

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- Control theory: controllability
- Coding theory: introducing efficient redundant bits
- Probability theory: card shuffling
- Optimization theory: pretty much anything! :-)

Applications of Linear Algebra: Machine Learning

Real-world problem:

- Given the movie ratings of a number of women and men and knowing their genders, can we determine the gender of users with unspecified gender based on their preferences?

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A Mathematical approach to this problem:

- Suppose we are given two sets of vectors $A = \{x_1, \ldots, x_n\}$ and $B = \{y_1, \ldots, y_m\}$ in $\mathbb{R}^n$, representing the ratings of the users in each group, where n is the selected number of movies.

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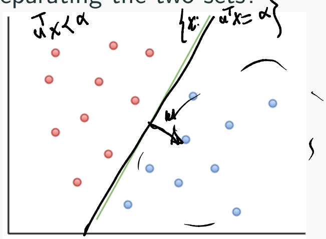

$u^T x > \alpha$

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- Can we find a linear subspace separating the two sets?
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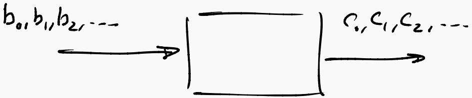

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Applications of Linear Algebra: Markov Chains

$$\overset{s_i}{} \qquad \overset{s_j}{} \qquad P_{ij} = P(\text{output} = s_j \mid \text{input} = s_i)$$

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- Suppose that we are given a non-negative matrix P such that rows and columns of P add to 1.
- The above question is very closely related to the following problem:
  Suppose that $P^k \to A$ as $k \to \infty$. Then, what can we say about the speed of convergence.

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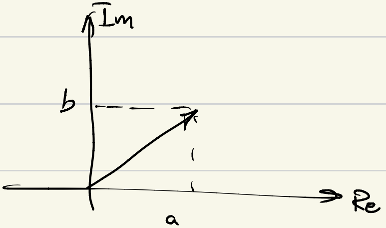
$j = \sqrt{-1}$

set of complex numbers $= \mathbb{C} = \{a+bj \mid a,b \in \mathbb{R}\} \equiv \{(a,b) \mid a,b \in \mathbb{R}\}$

$c=(a+bj)$: $Re(c) = a$

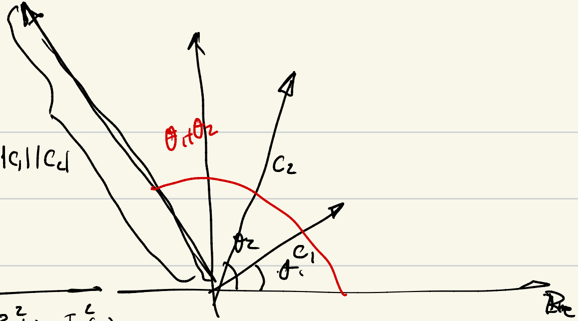
$Im(c) = b$



$c_1 = a_1+b_1 j$, $c_2 = a_2+b_2 j \Rightarrow$ $c_1+c_2 = (a_1+a_2)+(b_1+b_2)j$

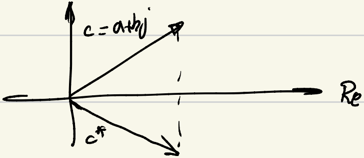
$c_1 \times c_2 = (a_1a_2 - b_1b_2) + (a_1b_2 + a_2b_1)j$

$(a_1+b_1j)(a_2+b_2j) = a_1a_2 + a_1b_2 j + a_2b_1 j + b_1b_2 \underbrace{j^2}_{-1} = (a_1a_2-b_1b_2) + (a_1b_2+a_2b_1)j$



notation: $|c| = \sqrt{Re^2(c) + Im^2(c)}$

for $c \in \mathbb{C}$ ("a+bj"), we define $c^* = \bar{c} := a-bj$



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- We use capital letters A, B, Γ, etc., to denote matrices (most often).
- For a set $\mathcal{X}$, the set of $m \times n$ matrices with entries in $\mathcal{X}$ is denoted by $\mathcal{X}^{m \times n}$.

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  d. $\mathbb{C} := \{(a + bj) \mid a, b \in \mathbb{R}\}$ the set of complex numbers
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- For a set $\mathcal{X}$, the set of $m \times n$ matrices with entries in $\mathcal{X}$ is denoted by $\mathcal{X}^{m \times n}$.
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- For a matrix A, we denote its transpose by A' or A^T.

Fields: A field $(F, +, \cdot)$ such that

set

$\cdot, + : F \times F \to F$

$+$ needs to:

(*): $a+b = b+a$

(i): $\exists z \in F : a+z = z+a = a \quad \forall a \in F$

there exists for all

(ii): $\forall a \in F, \exists! b \in F : a+b = b+a = z$

there exists a unique element

(iii): $(a+b)+c = a+(b+c) \quad \forall a,b,c \in F$

i.e., $(F, +)$ is an Abelian (Commutative) group

$(F^*, \cdot)$ is also an Akebian group

$\Downarrow$

$F - \{z\}$

identity element of $+$