### Enumeration rules

This document is part of the online appendix to the paper titled "Optimizing Navigational Graph Queries" and details the *enumeration rules* that were not presented in the paper. To this end, a few helpful definitions are given in section 1, the additional enumeration rules are presented in section 2 - 5 and the topic of projection (push-down) is discussed in section 6.

#### 1 Definitions

**Definition 1** Let  $Q(\overline{x}) \leftarrow R_1(\overline{y_1}), ..., R_n(\overline{y_n})$  be a query. The set of query variables  $Y_Q$  for Q is defined as:

$$Y_Q = \bigcup_{1 \le i \le n} \overline{y_i}$$

**Definition 2** Let  $y_1, y_2$  be query variables and  $\theta \in \{=, \neq, <, >, \leq, \geq\}$  a comparator. A literal predicate  $y_1 \theta y_2$  is a predicate that evaluates to true or false for any pair of bindings of  $y_1$  and  $y_2$ .

**Definition 3** Let  $Q(\overline{x}) \leftarrow R_1(\overline{y_1}), ..., R_n(\overline{y_n}), y_{1,1} \theta_1 y_{1,2}, ..., y_{m,1} \theta_m y_{m,2}$  be a query. The set of literal predicates  $T_Q$  for Q is defined as:

$$T_Q = \bigcup_{1 \le j \le m} \{ y_{j,1} \, \theta_j \, y_{j,2} \}$$

## 2 Edges rule

This rule applies to abstractions of the form  $\Box(Q(\overline{x}) \leftarrow E(s,e,t))$ . That is, it solves the recursive sub-problem which asks for all triples (s,e,t) that represent edges in the data graph. With  $i \in \mathbb{N}$  as a fresh identifier to distinguish multiple instances of E, this rule outputs exactly one plan  $\mathcal{P}$ . If  $\overline{x} = \{s, e, t\}$  then  $\mathcal{P} = (\{E(i)\}, E(i))$ . However, when  $\overline{x} \subset \{s, e, t\}$  a projection operator is necessary and  $\mathcal{P} = \{\Pi(\overline{x}, E(i)), E(i)\}, \Pi(\overline{x}, E(i))$ .

# 3 Properties rule

This rule applies to abstractions of the form  $\Box(Q(\overline{x}) \leftarrow P(o,k,v))$ . That is, it solves the recursive sub-problem which asks for all triples (o,k,v) that represent an object's property values in the data graph. With  $i \in \mathbb{N}$  as a fresh identifier to distinguish multiple instances of P, this rule outputs exactly one plan  $\mathcal{P}$ . If  $\overline{x} = \{o, k, v\}$  then  $\mathcal{P} = (\{P(i)\}, P(i))$ . However, when  $\overline{x} \subset \{o, k, v\}$  a projection operator is necessary and  $\mathcal{P} = \{\Pi(\overline{x}, P(i)), P(i)\}, \Pi(\overline{x}, P(i))$ .

### 4 Union rule

This rule applies to abstractions of the form:

$$\Box(Q(\overline{x}) \leftarrow B_1$$

$$Q(\overline{x}) \leftarrow B_n$$

where  $B_1$  through  $B_n$  are non-recursive. That is, it solves the recursive sub-problem which asks for the union of the evaluations of  $B_1$  through  $B_n$  projected to  $\overline{x}$ . The abstraction is split into n sub-problems of the form  $Q_i(\overline{x}) \leftarrow B_i$  for  $1 \le i \le n$  and a union operator is constructed as the parent of each of these abstractions. Hence, the rule outputs exactly one plan of the form:

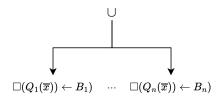


Figure 1: Union plan

### 5 Selection rule

This rule applies to abstractions of the form  $\Box(Q(\overline{x}) \leftarrow R_1(\overline{y_1}), ..., R_n(\overline{y_n}), y_{1,1} \theta_1 y_{1,2}, ..., y_{m,1} \theta_m y_{m,2})$ . That is, it solves the recursive sub-problem which asks for those tuples that satisfy a set of topological constraints captured by  $R_1, ..., R_n$  and a set of literal constraints captured by  $y_{1,1} \theta_1 y_{1,2}, ..., y_{m,1} \theta_m y_{m,2}$ . Let T' be a set of literal predicates defined as follows:

$$T' = \{ y_1 \,\theta \, y_2 \,|\, y_1 \,\theta \, y_2 \in T_Q \wedge \exists_{1 \leq i, j \leq n} \,|\, i \neq j \wedge y_1 \in \overline{y_i} \wedge y_2 \in \overline{y_j} \}$$

That is, T' is the subset of  $T_Q$  consisting only of all those literal predicates that reference variables  $y_1, y_2$  that come from different  $\overline{y_i}, \overline{y_j}$ . These are the literal predicates that cannot be *pushed down* (e.g., below a join). This rule applies *only* to queries Q for which  $T' \neq \emptyset$ . The set T' is constructed and a selection operator is instantiated over it. The predicates in T' are removed from the input abstraction and the resulting abstraction is added as a child of the selection operator. Hence, the result outputs exactly one plan of the form:

$$egin{aligned} \sigmaig(T'ig) \ & \downarrow \ & igcup \ ig(Q(\overline{x}) \leftarrow R_1(\overline{y_1}),...,R_n(\overline{y_n}),T_Q-T'ig) \end{aligned}$$

Figure 2: Selection plan

Additional projection may be required when the set of variables referenced in T' contains variables that are not in  $\overline{x}$ .

## 6 Projection

There is no enumeration rule that deals exclusively with projection. Instead, all rules are defined in such as way as to facilitate maximum projection push-down. That is, the join, seeding and selection rules all ensure that the output schemas  $\bar{x}$  of the abstractions they instantiate are minimized (i.e., contain no variables that are not required higher up in the query plan). Projection operators are only instantiated once further push-down is no longer possible, for example as parents of leaf operators E(i) or P(i) or as parents of join operators.