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Current multiplication during relativistic electron-beam propagation in plasma

Frank W. Chambers

Lawrence Livermore Laboratory, University of California, Livermore, California 94550

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During the passage of a highly relativistic electron beam through a plasma, the net current can be greater than the beam current. This current increase can be caused by momentum transferred from the beam to the plasma via the two-stream instability. The current gain at saturation of the two-stream instability is computed, and the theoretical results agree well with a one-dimensional computer simulation of the phenomenon. An equation is derived for the total plasma current, including both the current driven by the two-stream instability and the current driven by the beam-induced electric field.

INTRODUCTION

Relativistic electron beams propagating in a low-pressure gas[1] have shown net current exceeding the beam current by as much as a factor of three. This gain in current can occur if momentum is transferred from the relativistic beam electrons to the plasma electrons. Classical Coulomb collisions are insufficient to drive the observed plasma currents, but the two-stream interaction, which produces a large-amplitude plasma oscillation, does drive such a dc plasma current. We find that the driven-current density J_{ts} can be related to the field amplitude $E_0[E(x,t)=E_0\cos(kx-\omega_p t)]$ of the oscillation at the wavenumber k and frequency $\omega_p[\omega_p^2 = 4\pi n_p e^2/m$ (n_p is the plasma density, $-e$ is the charge of the electron, m is the electron mass)], as

$$J_{ts} = -n_p e \frac{(eE_0/m\omega_p)^2}{2(\omega_p/k)} = -n_p e \frac{v_{hf}^2}{2v_\phi}. \qquad (1)$$

Here, $v_{hf} \equiv eE_0/m\omega_p$, and $v_\phi = \omega_p/k$ is the mode phase velocity.

For the homogeneous, one-dimensional cold beam with velocity v_b, the two-stream driven wave at $\omega=\omega_p$ and $k=\omega_p/v_b$ grows exponentially with growth rate $\mathrm{Im}(\omega)$ given by

$$\frac{\mathrm{Im}(\omega)}{\omega_p} = \frac{\sqrt{3}}{2\gamma}\left(\frac{\omega_{pb}^2}{2\omega_p^2}\right)^{1/3}, \qquad (2)$$

where $\omega_{pb}^2 = 4\pi n_b e^2/m$, $\gamma^2 = 1/(1-\beta^2)$, n_b is the beam electron density, and $\beta = v_b/c$. The wave amplitude saturates at a level[2]

$$\frac{E_0^2}{8\pi} = 1.5\,\frac{S}{(1+1.5S)^{5/2}}\gamma n_b mc^2, \qquad (3a)$$

where

$$S \equiv \beta^2\gamma(n_b/2n_p)^{1/3}. \qquad (3b)$$

The numerical constants in (3a) were received from Thode[2] and differ slightly from those of Thode and Sudan.[2] The resulting current gain at saturation is

$$G = \frac{J_{ts}+J_b}{J_{b0}} = \frac{I_n}{I_{b0}} = 1.0 + \frac{1.5\gamma S}{(1+1.5S)^{5/2}}, \qquad (4)$$

where J_b is the beam current density at saturation, J_{b0} is the beam current density in the absence of current multiplication, I_{b0} is the corresponding initial beam current, and I_n is the net current at saturation ($I_n = I_p + I_b$, where I_p and I_b are the plasma and beam currents, respectively). One-dimensional simulations using the computer code EMI[3] and various beam plasma parameters yield excellent agreement with the analytic predictions.

To include induction effects, we reformulate the current multiplication in terms of a force law both in the kinetic equation framework, where the driving nonlinear force term is quasi-linear diffusion, and in the fluid equation framework. From the force-law equation we obtain an equation that includes two-stream and inductive effects in relating the plasma and beam current

$$\frac{\partial I_p}{\partial t} = -\frac{L\pi a^2}{c^2}\frac{\omega_p^2}{4\pi}\frac{\partial(I_p+I_b)}{\partial t} + \frac{\partial I_{ts}}{\partial t} - \nu_c I_p, \qquad (5)$$

where L is the dimensionless inductance, a is the beam radius, ν_c is the appropriate collision frequency for plasma electrons. The two-stream driven current is $I_{ts} = J_{ts}\pi a^2$.

MOMENTUM TRANSFER AND CURRENT GAIN

In a beam of highly relativistic electrons of energy $E_e = \gamma mc^2$, if each electron loses an amount of energy $\Delta E_e = -\Delta\gamma mc^2$ and a corresponding amount of momentum $\Delta p = -\Delta\gamma mc/\beta$, the change in beam electron current I_b is

$$\Delta I_b/I_{b0} = -\Delta\gamma/\beta^2\gamma^3 \approx 0. \qquad (6)$$

If the momentum is transferred to a group of nonrelativistic plasma electrons, then these electrons will gain in plasma current I_p by

$$\Delta I_p/I_{b0} = \Delta\gamma/\beta^2. \qquad (7)$$

Thus if $\gamma \gg 1$, then $\beta \approx 1$ and the loss in beam current is negligible compared with the gain in plasma current, so that the net current increases. The gain G is given by

$$G = I_n/I_{b0} = (I_p+I_b)/I_{b0} \approx 1.0 + \Delta\gamma. \qquad (8)$$

If the beam electrons are slowed to nonrelativistic speeds, the maximum possible net gain is $G=\gamma$.

This calculation assumes conservation of momentum between beam and plasma electrons. No momentum appears in the electromagnetic fields because we are dealing with electrostatic plasma modes and we assume that there is insufficient time for momentum transfer

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to the ions. The beam energy lost goes both into the electric fields of the modes excited and into the plasma kinetic energy. To simultaneously conserve energy and momentum in one dimension, our calculation must allow each beam electron to interact with two or more plasma electrons. To achieve the beam slowing and momentum transfer, two mechanisms are considered: classical collisions and the two-stream instability.

MOMENTUM TRANSFER VIA COLLISIONS

The rate at which the parallel beam momentum $p_{\parallel}$ changes because of Coulombic collisions with the plasma electrons is[4]

$$\frac{d(p_{\parallel}/mc)}{dt} = -\frac{4\pi n_p e^2}{m}\frac{e^2}{mc^2}\frac{1}{c}\ln\left(\frac{b_{\max}}{b_{\min}}\right) = -\omega_p^2\tau_e\ln\left(\frac{b_{\max}}{b_{\min}}\right). \tag{9}$$

Here, $b_{\max}$ and $b_{\min}$ are the maximum and minimum radii, respectively, between which the Coulomb-collision model is valid, but their actual values are of no consequence. The time $\tau_e(=e^2/mc^3 = 9.4\times 10^{-24}$ sec) is the time required for light to cross an electron. Considering realistic plasma densities, we find that the momentum transfer to plasma electrons is negligible. One can also intuit this result from the more familiar change in beam energy with distance (dE/dx), which results mainly from the collision of the beam electrons with atomic electrons of the gas molecules. The small values of dE/dx that permit the beam to travel many times its own length before stopping indicate that the beam loses little of its energy or momentum per unit volume of gas. One expects the beam to lose even less momentum to the free plasma electrons (there are fewer free electrons than bound electrons). Hence, classical collisions, or drag, cannot account for current multiplication.

MOMENTUM TRANSFER VIA TWO-STREAM MODE

The beam traveling through a plasma will produce a large-amplitude plasma wave via the two-stream instability.[5] The presence of this large-amplitude wave requires that the plasma have a net time-averaged momentum, or current, which can be seen as follows. The instantaneous current density J in the plasma can be expanded in plasma density n and velocity v as

$$J = -e(n_0v_0 + n_0v_1 + n_1v_0 + n_1v_1 + n_0v_2 + n_2v_0 \ldots). \tag{10}$$

Assuming the presence of a large-amplitude plasma oscillation at ω_p and using the fluid equations, one finds

$$n_1 = \frac{kE_0\sin(kx-\omega_p t)}{4\pi e} \tag{11}$$

and

$$v_1 = \frac{\omega_p}{k}\frac{n_1}{n_p} = \frac{\omega_p E_0\sin(kx-\omega_p t)}{4\pi e n_p}. \tag{12}$$

Time-averaging J over one period eliminates the n_0v_1 and n_0v_2 terms in (10) and the only surviving term is

$$J_{ts} = -\frac{en_1v_1}{2} = \frac{-en_p(eE_0/m\omega_p)^2}{2(\omega_p/k)}. \tag{13}$$

Writing Eq. (13) in terms of a plasma drift velocity $v_d = J_{ts}/(-en_p)$ gives

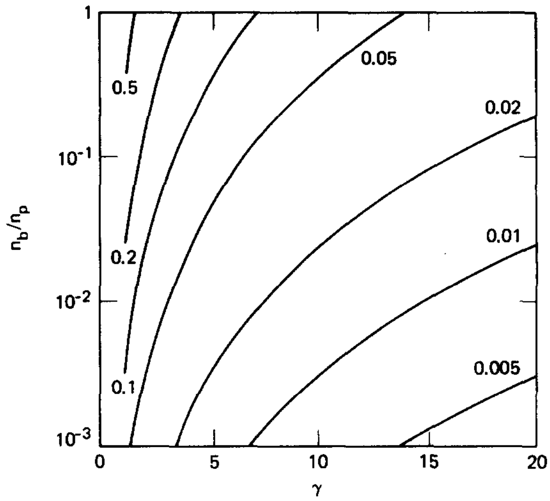

FIG. 1. Contours of constant growth rate Im(ω) at $k=\omega_p/v_b$ for the collisionless two-stream interaction in n_b/n_p and γ.

$$v_d = \frac{(eE_0/m\omega_p)^2}{2(\omega_p/k)} = \frac{v_{hf}^2}{2v_\phi}. \tag{14}$$

Thus, associated with the plasma wave, there is a net direct current or momentum in the plasma. The dynamics of how this current is driven will be considered later when we derive an equation for the net current. This two-stream-driven current is not linear ($J_{ts}\propto E_0^2$); it arises because the plasma electron-density and velocity perturbations are in phase. In the usual beam-plasma interactions, the current carried by the background plasma because of the plasma wave is exactly cancelled by the current lost from the beam in driving the plasma wave. A relativistic beam, being more rigid, will not lose current, and hence current multiplication can occur.

CURRENT GAIN AT SATURATION

Having related the plasma current to the mode electric field, we consider the time development of the mode resulting from the two-stream interaction. For the one-dimensional, homogeneous, and collisionless case of a cold beam and plasma with $k=\omega_p/v_b$, the two-stream growth rate is given by Eq. (2).

We plot contours of constant Im(ω)/ω_p for n_b/n_p vs γ in Fig. 1. The mode amplitude grows exponentially according to Eq. (2) until saturation by beam-electron trapping occurs. During this entire time, the plasma behavior is well described by linear theory. The mode amplitude at saturation is given by Eq. (3). Using this value of E_0^2 in Eq. (13) for $J_{ts}=J_p$ yields the current gain given in Eq. (4).

Figure 2 shows plots of constant S for n_b/n_p vs γ and $(E_0^2/8\pi)/\gamma n_b mc^2$ vs S. In Fig. 3 the predicted current-gain contours are plotted. The fraction of beam energy density converted to wave energy as seen in Fig. 2(a) increases with S until $S=0.45$. Above this value, the efficiency of energy transfer decreases with increasing S. This variation in the two-stream efficiency is also seen in the gain-contour plot of Fig. 3. For values of S below 0.4, we can approximate $G=1.5\gamma S = 1.5\beta^2\gamma^2(n_b/$

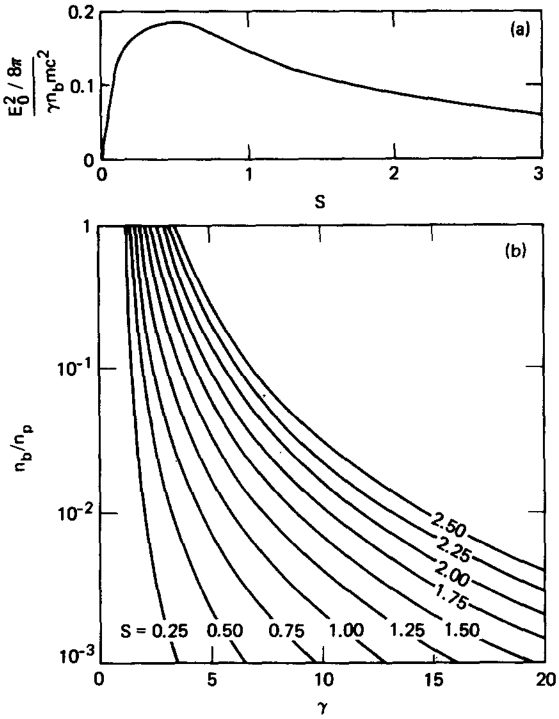

FIG. 2. Plots of Eq. (3) for determining E_0^2 as a function of n_b/n_p and γ. (a) Fraction of beam energy in the two-stream mode vs constant-strength parameter S. (b) Contours of S in n_b/n_p and γ.

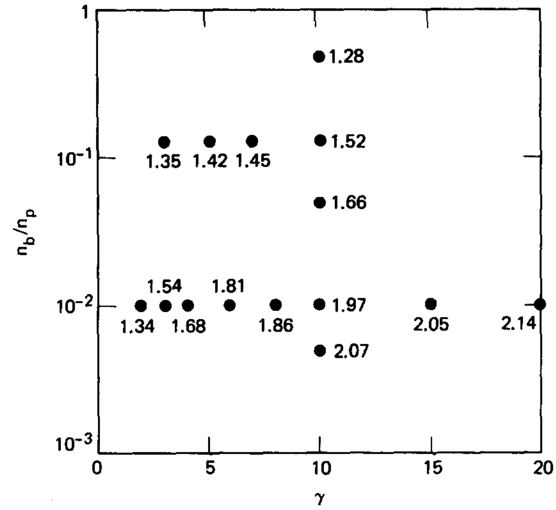

FIG. 4. Observed simulation gains G in n_b/n_p and γ.

$2n_p)^{1/3}$, so the gain is quite sensitive to γ but relatively insensitive to n_b/n_p. When $S > 1.5$, the gain is relatively insensitive to γ. The reduced efficiency of the interaction as S increases approximately cancels the increase in beam energy with γ such that the net energy and momentum transferred to the plasma are nearly independent of γ. In comparing Figs. 1 and 3, note that the high-γ, low-n_b/n_p region with the highest current gains also has the lowest growth rates relative to the ω_p and hence requires a longer time to reach saturation.

SIMULATION

We have used the relativistic electron-simulation code EM1 to study current multiplication.[3] This is a one and one-half dimensional code with an electromagnetic field solver and a relativistic particle pusher. We do not apply a magnetic field nor a transverse electric field, so our use of the code is effectively one dimensional. Periodic boundary conditions are employed, so the spatially averaged longitudinal electric field vanishes. For our application, the unstable mode of the beam-plasma system at $k = \omega_p/v_b$ is given a moderate initial amplitude, and the mode amplitude and beam and plasma currents are observed as functions of time. The relation between current and mode amplitude given by Eq. (13) holds for the plasma during the simulation. The plasma remains linear; a large-amplitude wave is present, but no true heating takes place. The mode amplitude grows at the linear growth rate given by Eq. (2) until it saturates by trapping at the levels predicted in Eq. (3). The observed current gains at saturation are plotted in Fig. 4 and tabulated in Table I. The simulation agrees well with theory.

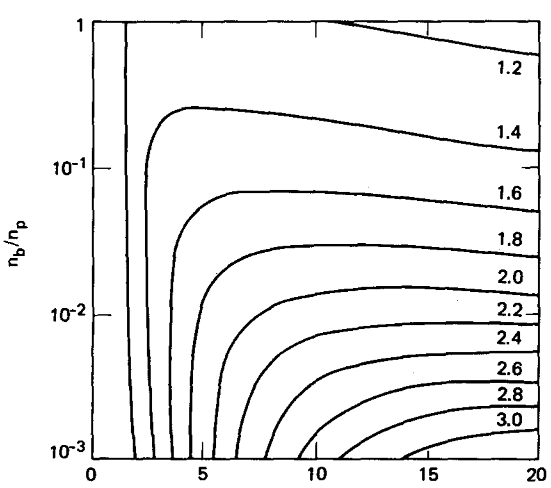

FIG. 3. Contours of constant gain G in n_b/n_p and γ.

TABLE I. Current gains predicted and observed using the simulation, for various values of γ and n_b/n_p.

γ	n_b/n_p	Predicted gain	Observed gain
10.0	0.5	1.27	1.28
10.0	0.10	1.51	1.52
10.0	0.05	1.66	1.66
10.0	0.01	2.08	1.97
10.0	0.005	2.28	2.07
3.0	0.125	1.44	1.35
5.0	0.125	1.49	1.42
7.0	0.125	1.49	1.45
10.0	0.125	1.48	1.47
2.0	0.01	1.34	1.34
3.0	0.01	1.56	1.54
4.0	0.01	1.71	1.68
6.0	0.01	1.91	1.81
8.0	0.01	2.02	1.86
10.0	0.01	2.08	1.97
15.0	0.01	2.12	2.05
20.0	0.01	2.11	2.14
30.0	0.01	2.04	2.25

We do not consider in detail the long-term evolution of the current for several reasons. The simulation becomes increasingly expensive and unreliable at longer times. Bouncing of trapped beam particles persists for long times, so, for the relativistic two-stream interaction, the electrons do not reach a simple asymptotic state.[6] Over several ion plasma periods, ion modes will be excited via the oscillating two-stream instability. However, this time can be long compared with the beam pulse time. For our purposes, two other effects are probably more important: the large electric fields produced will ionize the background gas, and the spatial inhomogeneity of the beam-generated plasma will alter the temporal instability evolution.

DYNAMICS OF CURRENT MULTIPLICATION

To include collisional and inductive effects for the finite beam in a gas, we derive a force equation for the low-frequency motion ($\partial/\partial t \ll \omega_p$) of the plasma. We derive this equation from the kinetic equation with quasi-linear diffusion, as done by Sudan[7] for ion beams, and from the fluid equation. From the force equation, we derive an equation for the net plasma current.

The equation describing the evolution of an averaged electron distribution function $f(v, t)$ in the presence of large-amplitude mode is

$$\frac{\partial f}{\partial t} - \frac{eL}{mc^2}\frac{\partial I_n}{\partial t}\frac{\partial f}{\partial v} - \frac{\partial}{\partial v} D \frac{\partial f}{\partial v} - \nu_c (f - f_M) = 0 , \tag{15}$$

where we have taken for the induced E_z field

$$E_z = -\frac{L}{c^2}\frac{\partial I_n}{\partial t}, \quad L = \ln\left(1 + \frac{b^2}{a^2}\right) . \tag{16}$$

In these equations, L is computed for a Bennett beam profile of Bennett radius a inside a conducting cylinder of radius b. This value of E_z is correct only along the beam axis, but we shall ignore effects of the radial profiles in E_z. Quasi-linear diffusion is described by the diffusion coefficient

$$D(v) = \frac{e^2}{m^2}\sum_k \frac{|E_{0k}|^2 \gamma_k}{(\omega_k - kv)^2 + \gamma_k^2}, \tag{17}$$

where the summation is over mode wave number k and ω_k and γ_k are the real and imaginary parts of the mode frequency. Collisional relaxation to a Maxwellian f_M is included in the ν_c term of Eq. (15). Assuming the electrons are drifting at a velocity v_d, we expand D to first order in v about v_d. Then, assuming that $\omega(\approx \omega_p)$ is much greater than γ_k and v_d is less than v_b, taking $\int v\, dv$ of Eq. (15) yields a force equation for v_d:

$$\frac{\partial v_d}{\partial t} = \frac{eL}{mc^2}\frac{\partial I_n}{\partial t} + \frac{e^2}{m^2}\sum_k \frac{|E_{0k}|^2 2\gamma_k k}{\omega_p^3} - \nu_c v_d . \tag{18}$$

In evaluating the diffusion term we assume $v_d \ll \omega_p/k$. Note that $I_n (I_n = I_p + I_b)$ is a function of v_d.

Equation (18) can also be derived from a fluid approach by computing a space-time-averaged force on an electron in an oscillating and exponentially growing high-frequency electric field. For an electric field of the form $E = E_0 \exp(\gamma t) \cos(kx - \omega_p t)$ with the particle orbit computed to first order in E, averaging the force due to the electric field seen by the particle in space and time gives for the average force F

$$F = m\frac{(eE_0/m\omega_p)^2 2\gamma}{2(\omega_p/k)} . \tag{19}$$

The force equation including the average high-frequency field and inductive and resistive forces is

$$\frac{\partial v_d}{\partial t} = \frac{eL}{mc^2}\frac{\partial I_n}{\partial t} + \frac{(eE_0/m\omega_p)^2 2\gamma}{2(\omega_p/k)} - \nu_c v_d . \tag{20}$$

This agrees with our previous result (note that $\sum_k |E_{0k}|^2 = E_0^2/2$).

To obtain an equation for the plasma current, we assume a uniform distribution of particles and fields out to some radius a. Multiplying the force equation by $-en_p\pi a^2 m$ and noting that $I_p = -en_p\pi a^2 v_d$, $I_{ts} = -en_p(eE_0/m\omega_p)^2/2(\omega_p/k)$, and $\partial I_{ts}/\partial t = 2\gamma I_{ts}$, we find

$$\frac{\partial I_p}{\partial t} = -\frac{L\pi a^2\omega_p^2}{c^2 4\pi}\frac{\partial I_n}{\partial t} + \frac{\partial I_{ts}}{\partial t} - \nu_c I_p . \tag{21}$$

This is the usual circuit equation for currents driven by electron beams, including inertial, inductive, two-stream, and collisional effects but not including charge-separation effects. Because $I_p = I_n - I_b$ and $\tau_m \equiv L\pi a^2\omega_p^2/4\pi c^2 \nu_c$, the equation becomes

$$\frac{1}{\nu_c}\frac{\partial}{\partial t}(I_n - I_b) = \tau_m \frac{\partial I_n}{\partial t} - \frac{1}{\nu_c}\frac{\partial I_{ts}}{\partial t} - (I_n - I_b) . \tag{22}$$

The quantity I_{ts} is just the two-stream-driven current in a homogeneous, collisionless plasma as calculated in Eq. (13). In the limit $I_{ts} = 0$ and $\tau_m \nu_c \gg 1$, we recover the usual circuit equation showing the buildup of the net current to equal the beam current in a time on the order of τ_m. In the $L = 0$, $\nu_c = 0$ limit, we find $I_p = I_{ts}$. This limit applies to our one-dimensional simulation. In the most interesting case, where collisions and the beam-induced return current are neglected, we find that the fraction of two-stream current actually appearing in the plasma is given by

$$I_p = \frac{I_{ts}}{1 + La^2\omega_p^2/4c^2} . \tag{23}$$

For large plasma densities, the induction term $La^2\omega_p^2/4c^2$ can result in a significant reduction in the observed current.

DISCUSSION

The current multiplication phenomenon studied here is analogous to anomalous resistivity phenomena often studied. Anomalous resistivity results in the loss of directed beam energy and current (usually electrons) to the background (usually ions) through a turbulent wave spectrum. During current multiplication, the beam electrons experience anomalous resistivity when they transfer energy and momentum to the plasma electrons. Because of relativistic effects, the plasma electrons are much lighter than the beam electrons. Therefore, the plasma-current gain greatly exceeds the beam-current loss, resulting in current multiplication.

As pointed out by Sudan,[7] a similar effect will occur in an ion beam propagating in a gas. Here, the ratio of the beam-particle mass to the plasma-electron mass can be quite large. Because the beam charge is opposite to the plasma-electron charge, the effect is now current reversal. Inductive effects will decrease this reversed electron current, but the final net current will be in the opposite direction to the beam current, leading to beam defocusing. To prevent this catastrophic result, one must propagate an ion beam at sufficiently high gas pressure and with sufficiently wide beam-velocity spread to stabilize the two-stream mode.[5]

Experimental observations of current multiplication at the Lawrence Livermore Laboratory[1] have shown gains of about 1.3 to 1.5 for an FX-25-diode beam with $\gamma \approx 3$, and gains of 2.0 and higher for the astron-accelerator beam. In the experiment, the plasma is produced by ionization of the gas, so n_p varies from zero to several hundred times the beam density. These gains, which probably occur when n_b/n_p is about 0.1 to 0.01, agree reasonably well with the predictions of Fig. 3. From our work, induction appears capable of greatly reducing the current gain, but this effect is not experimentally observed. The theory presented here lacks two-dimensional effects on the two-stream interaction and spatial effects such as charge separation. It also lacks long-term description of the two-stream evolution, including the development of ion turbulence. Such turbulence might lead to anomalous resistivity, which might limit the induced currents. The space and time variations of beam and plasma density are not considered. This theory does, however, provide a basic mechanism for driving current multiplication.

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