$\int I_8^{SO(5)}$ gives:

$\mathrm{Tr}(R^3) = 2(g-1)(2\varepsilon_1^3 z - 6\varepsilon_1 z - 3\varepsilon_1^2 + 11)$,

$\mathrm{Tr}(R) = -2(g-1)(1-2z\varepsilon_1)$

$\Rightarrow$ central charges:

$$a = \frac{3(g-1)(6\varepsilon_1^3 z - 20\varepsilon_1 z - 9\varepsilon_1^2 + 34)}{16}$$

Performing a-maximization we find $\varepsilon_1 = \frac{3-\sqrt{9+40z^2}}{6z}$

$$\rightarrow \quad a = (g-1)\,\frac{9(-3+\sqrt{9+40z^2}) + 8z^2(54+5\sqrt{9+40z^2})}{96\,z^2}$$

$$c = (g-1)\,\frac{9(-3+\sqrt{9+40z^2} + z^2(432+44\sqrt{9+40z^2})}{96\,z^2}$$

For $z=1$, these simplify to:

$$a = \frac{187}{24}(g-1), \qquad c = \frac{97}{12}(g-1)$$

$\rightarrow$ matches 4d result

$$a = \frac{1}{24}\left((g-1)187 + 78s\right), \quad c = \frac{1}{12}\left((g-1)97 + 42s\right)$$

obtained by gluing trinions together

Now observe:

$$a(g=0, s=3) = \frac{47}{24}, \quad c(g=0, s=3) = \frac{29}{12}$$

→ matches anomalies of orbifold theory !

Let us check this :

- anomalies of chiral fields :

$$a_\chi(R) = \frac{3}{32}\left(3(R-1)^3 - (R-1)\right),$$

$$c_\chi(R) = \frac{1}{32}\left(9(R-1)^3 - 5(R-1)\right)$$

- anomalies of vector fields for gauge group $G$:

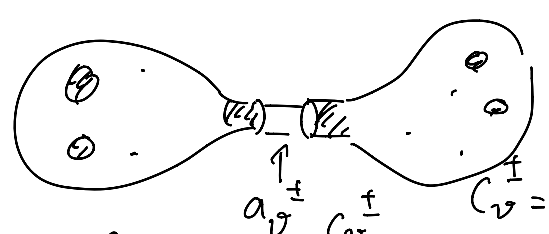
$$a_v = \frac{3}{16}\dim G, \quad c_v(G) = \frac{1}{8}\dim G$$

- superconformal R-charge :

$$R_c(R, q_\beta, q_\gamma, q_t) = R + \ell_1 q_\beta + \ell_2 q_\gamma + \ell_3 q_t$$

→ $\ell_1, \ell_2, \ell_3$ are to be determined through a-maximization

- anomalies introduced when gluing punctures:



$a_v^\pm, c_v^\pm$

$\pm$: color of puncture

$$a_v^\pm = a_v(SU(2)\times SU(2)) + 4a_\chi(R_c(1,1,\mp 1,-1)) + 4a_\chi(R_c(1,-1,\pm 1,-1))$$

$$c_v^\pm = c_v(SU(2)\times SU(2)) + 4c_\chi(R_c(1,1,\mp 1,-1)) + 4c_\chi(R_c(1,-1,\pm 1,-1))$$

compute anomalies of orbifold theory:

$$\begin{aligned} a_{\text{orb}} &= a_{\bar{v}} + 8\Big(a_\chi\big(R_c(\tfrac{1}{2},1,0,\tfrac{1}{2})\big) + a_\chi\big(R_c(\tfrac{1}{2},-1,0,\tfrac{1}{2})\big) \\ &\qquad + a_\chi\big(R_c(\tfrac{1}{2},0,-1,\tfrac{1}{2})\big) + a_\chi\big(R_c(\tfrac{1}{2},0,1,\tfrac{1}{2})\big)\Big) \\ &= -\frac{3}{8}\left(3\ell_3^3 + 9\ell_3^2 + 36\,\ell_1\ell_2\ell_3 - 7\ell_3 + 18(\ell_1^2+\ell_2^2) - 4\right) \end{aligned}$$

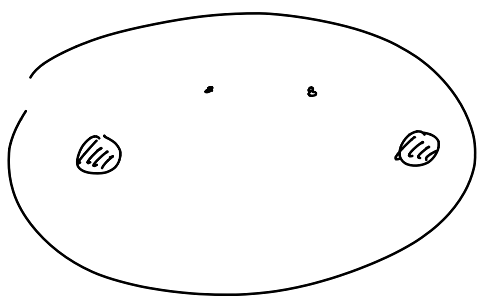
Maximizing $a_{\text{orb}}$ as a function of $\ell_i$ gives $\ell_i = (0,0,\frac{1}{3})$ and thus $a_{\text{orb}} = \frac{47}{24}$

For $c$ we find:

$$\begin{aligned} c_{\text{orb}} &= c_{\bar{v}} + 8\Big(c_\chi\big(R_c(\tfrac{1}{2},1,0,\tfrac{1}{2})\big) + c_\chi\big(R_c(\tfrac{1}{2},-1,0,\tfrac{1}{2})\big) \\ &\qquad + c_\chi\big(R_c(\tfrac{1}{2},0,-1,\tfrac{1}{2})\big) + c_\chi\big(R_c(\tfrac{1}{2},0,1,\tfrac{1}{2})\big)\Big) \\ &= -\frac{1}{8}\left(9\ell_3^3 + 27\ell_3^2 + 108\,\ell_1\ell_2\ell_3 - 17\ell_3 + 54(\ell_1^2+\ell_2^2) - 17\right) \end{aligned}$$

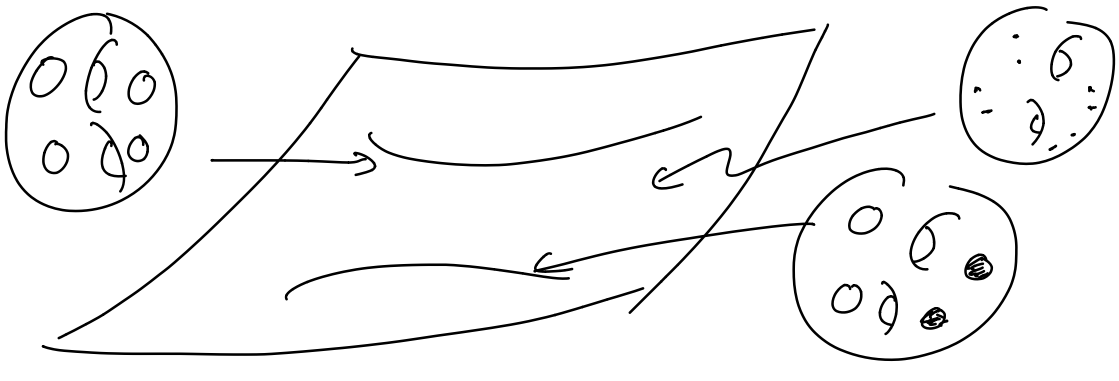
$$\longrightarrow \quad c_{\text{orb}} = \frac{29}{12}$$

<u>Recall:</u>



orbifold theory

## Interpretation:



$\mathcal{M}_{g,s}$

there are special loci on conformal manifold where 2 minimal punctures combine into a maximal puncture !

## Conformal manifold of orbifold theory

special point:

symmetry $H = su(8)_1 \times su(8)_2 \times su(2) \times u(1)_t$

$\mathcal{Q}_1 = \{ Q_1^+, Q_2^-, Q_1'^+, Q_2'^+ \}$ in 8 of $su(8)_1$

$\mathcal{Q}_2 = \{ Q_2^-, Q_2^+, Q_1'^-, Q_2'^+ \}$ in 8 of $su(8)_2$

$su(2)$ rotates the two $\Phi$'s.

Under $u(1)_t$ $\Phi$'s have charge $-1$,

all other fields charge $\frac{1}{2}$

marginal operators : $\lambda\, Q_1 \cdot \Phi \cdot Q_2$ (superpotential)

two gauge couplings: $g_1, g_2$

couplings $\lambda$ are singlets of $U(1)_t$

and transform as $(8, 8, 2)$ under $SU(8)_1 \times SU(8)_2 \times SU(2)$

$\to$ dimension of $\mathcal{M}_c$ is dim. of quotient

$$(\lambda, g_1, g_2) / H$$

$\to$ count indep. hol. invariants:

- baryons (invariants under 2 $SU(8)$'s) form $9$ of $SU(2)$
- 6 indep. $SU(2)$ invariants built from baryons

$\to$ dim $\mathcal{M}_c = 6$