§ 4.3 Anomalies from six dimensions

6d (1,0) SCFT as $\mathbb{Z}_2$ orbifold of A_1 (2,0)-theory.

$\longrightarrow$ flavor group $SO(7)$

anomaly polynomial:

$$I_8^{SO(7)} = \frac{11\, C_2^2(R)}{12} - \frac{C_2(R)\, p_1(T)}{24} + \frac{C_2(SO(7))_8\, p_1(T)}{24}$$

$$- \frac{C_2(R)\, C_2(SO(7))_8}{2} + \frac{7 C_2^2(SO(7))_8}{48}$$

$$- \frac{C_4(SO(7))_8}{6} + \frac{29 p_1^2(T) - 68 p_2(T)}{2880}$$

Want to compute central charges of 4d theory

1) topological twist:

$$\text{6d Lorentz group } SO(5,1) \longrightarrow \underbrace{SO(3,1)}_{\circlearrowright \mathbb{R}^4} \times \underbrace{SO(2)_s}_{\circlearrowright \Sigma_g}$$

$$SU(2)_R \longrightarrow U(1)_R \quad , \quad SO(2)_s \longrightarrow SO(2)_s - U(1)_R$$

Decomposing supercharges, we find

$$2_R \otimes 4_L \longrightarrow \left(1_{\frac{1}{2}} + 1_{-\frac{1}{2}}\right) \otimes \left(2_{\frac{1}{2}} + 2'_{-\frac{1}{2}}\right) \longrightarrow \underline{2_0 + 2'_0} + 2_1 + 2'_{-1}$$

$SO(2)_s - U(1)_R$ charge

$\longrightarrow$ half the supercharges survive

$\longrightarrow \mathcal{N}=1$ SUSY in 4d

2) Integrate anomaly polynomial

decomposition of characterictic classes:

4d tangent bundle: $p_1(T)'$, $p_2(T)'$

tangent bundle of Σ_g: t

$U(1)_R$-bundle : $c_1(F)$

$$\to \quad p_1(T) = t^2 + p_1(T)'$$
$$p_2(T) = t^2 p_1(T)' + p_2(T)'$$

For the R-symmetry, we get:

$$1 + c_1(R)x + c_2(R)x^2 = (1+xn_1)(1+xn_2)$$

$$\to c_1(R) = n_1 + n_2, \quad c_2(R) = n_1 n_2$$

We have $n_2 = -n_1 = \underset{\substack{\uparrow \\ \text{4d } U(1)\ R\text{-symmetry}}}{c_1(F)}$

We need $\underbrace{n_1 + n_2 + t = 0}_{\text{top. twist}}$ to preserve SUSY

$\to$ shift $n_2 \to n_2 - t$

Thus we set : $n_1 = -c_1(F)$, $n_2 = c_1(F) - t$

$$\to \int_{\Sigma_g} I_8^{SO(7)} = \frac{11(g-1)}{3} c_1^3(F) + \frac{g-1}{12} c_1(F) p_1(T)' + (g-1) c_1(F) c_2(SO(2))_8 , \quad (*)$$

where we used the Gauss–Bonnet theorem:

$$\int t = 2(1-g)$$

Compare to 4d anomaly polynomial of Weyl fermion with SO(7) global symmetry and $U(1)_R$ symmetry bundles:

$$I_6 = \frac{\mathrm{Tr}(R^3)}{6} c_1^3(F) - \frac{\mathrm{Tr}(R)}{24} c_1(F) p_1(T)'$$

$$- \mathrm{Tr}\left(R F^2_{SO(7)}\right) c_1(F) c_2(SO(7)) \qquad (**)$$

where we have defined $c_2(SO(7))_r = T_r\, c_2(SO(7))$, for T_r the 2nd Casimir of representation r

Comparing $(*)$ and $(**)$, we find:

$\mathrm{Tr}(R^3) = 2L(g-1)$, $\mathrm{Tr}(R) = -2(g-1)$,

and $\mathrm{Tr}\left(R F^2_{SO(7)}\right) = -(g-1)$, where we used

$T_8 = 1$

→ central charges are:

$$a = \frac{3}{32}\left(3\,\mathrm{Tr}\,R^3 - \mathrm{Tr}\,R\right) = 51$$

$$c = \frac{1}{32}\left(9\,\mathrm{Tr}\,R^3 - 5\,\mathrm{Tr}\,R\right) = \frac{13}{2}(g-1)$$

→ numbers match results from Trinions for $G^{max} = SO(7)$

Models with $G^{max} = SO(5) \times U(1)$

turn on non-trivial flux for $U(1) \subset SO(7)$

$$\to \quad SO(7) \longrightarrow \underbrace{U(1) \times SO(5)}_{\text{global symmetry}}$$

8 of $SO(7)$ decomposes as

$$8 \longrightarrow 4_{\frac{1}{2}} + 4_{-\frac{1}{2}}$$

We have :

$$8 - C_2(SO(7))_8 + \frac{1}{12}\left(C_2^2(SO(7))_8 - 2C_4(SO(7))_8\right) = ch(SO(7)_8)$$

$$= ch\left(u(1)_{\frac{1}{2}} \otimes usp(4)_4 \oplus u(1)_{-\frac{1}{2}} \otimes usp(4)_4\right)$$

$$= ch(u(1)_{\frac{1}{2}})\, ch(usp(4)_4) + ch(u(1)_{-\frac{1}{2}})\, ch(usp(4)_4)$$

$$= \left(1 + \frac{C_1(u(1)_a)}{2} + \frac{C_1^2(u(1)_a)}{8} + \frac{C_1^3(u(1)_a)}{48} + \frac{C_1^4(u(1)_a)}{384}\right)$$

$$\times \left(1 - \frac{C_1(u(1)_a)}{2} + \frac{C_1^2(u(1)_a)}{8} + \frac{C_1^3(u(1)_a)}{48} + \frac{C_1^4(u(1)_a)}{384}\right)$$

$$\times \left(4 - C_2(usp(4))_4 + \frac{1}{12}\left(C_2^2(usp(4))_4 - 2C_4(usp(4))_4\right)\right),$$

Comparing forms of equal dimension, we find:

$$C_2(SO(7))_8 = -C_1^2(u(1)_a) + 2C_2(usp(4))_4, \quad C_4(SO(7))_8$$

$$= 3\frac{C_1^4(u(1)_a)}{8} - \frac{1}{2}C_1^2(u(1)_a)\, C_2(usp(4))_4 + C_2^2(usp(4))_4$$

$+ 2C_4(usp(4))_4 .$

Inserting into the anomaly polynomial, we get

$$I_8^{so(5)} = \frac{11 C_2^2(R)}{12} - \frac{C_2(R)\, p_1(T)}{24} - \frac{c_1^2(u(1)_a)\, p_1(T)}{24}$$
$$+ \frac{C_2(R)\, c_1^2(u(1)_a)}{2} - C_2(R)\, C_2(usp(4))_4$$
$$+ \frac{C_2(usp(4))_4\, p_1(T)}{12} + \frac{c_1^4(u(1)_a)}{12}$$
$$- \frac{c_1^2(u(1)_a)\, C_2(usp(4))_4}{2} + \frac{5 C_2^2(usp(4))_4}{12}$$
$$- \frac{C_4(usp(4))_4}{3} + \frac{29 p_1^2(T) - 69 p_2(T)}{2880}$$

set $c_1(u(1)_a) = -z\,t + \varepsilon_1 c_1(F) + c_1'(u(1)_a)$

flux through Σ_g 6d R-sym. 4d global $u(1)$-sym.

The flux z is quantized s.t. $\int c_1(u(1)_a) = \frac{n}{q}$, where n is an integer and q is the smallest charge in the game.

Integrating the anomaly polynomial, we obtain:

$$\int_{\Sigma_g} I_8^{SO(5)} = \left(\frac{2\varepsilon_1^3 z - 6\varepsilon_1 z - 3\varepsilon_1^2 + 1}{3}\right)(g-1)\, c_1^3(F)$$

$$+ \frac{(g-1)(1-2z\varepsilon_1)}{12} c_1(F) p_1(T)' + 2(g-1)(1-z\varepsilon_1) c_1(F)\, c_2(usp(4))_4$$

$$+ 2(g-1)(\varepsilon_1^2 z - z - \varepsilon_1) c_1^2(F)\, c_1'(u(1)_a) + \frac{2(g-1) z c_1'^3(u(1)_a)}{3}$$

$$+ (g-1)(2z\varepsilon_1 - 1) c_1(F) c_1'^2(u(1)_a) - \frac{(g-1) z c_1'(u(1)_a) p_1(T)'}{6}$$

$$- 2(g-1) z c_1'(u(1)_a)\, c_2(usp(4))_4$$