§1.1 6d (2,0) SCFT's

(2,0) SCA in 6d : $Osp(8|4)$

bosonic subalgebra: $\underbrace{SO(6,2)}_{\text{conformal algebra}} \oplus \underbrace{\overset{SO(5)}{\overset{\shortparallel}{Sp(4)_R}}}_{\text{R-symmetry}}$

Representation is given by abelian tensor-multiplet in 6d:

- Real scalars Φ^I $(I = 1, \dots, 5)$ in 5 of $SO(5)_R$. They satisfy $\Box \Phi^I = 0$ and have $\Delta_\Phi = 2$
- Weyl fermions in 4 of $SO(5,1)$ Lorentz algebra and 4 of $SO(5)_R$ subject to symplectic Weyl reality condition $Q_{i\alpha} = \Omega_{ij} (C\sigma_0^T)_\alpha^{\ \beta} Q^\dagger_{j\beta}$
  Scaling dimension: $\Delta_\psi = \frac{5}{2}$
- A real, self-dual three-form $H = *H$
  $\rightarrow$ field strength of two-form gauge field B.
  $\rightarrow$ $H = dB$ with $dH = d*H = 0$
  Scaling dim: $\Delta_H = 3$

(2,0) SCFT posses no relevant or marginal operators $\longrightarrow$ no SUSY preserving deformations

String theory construction:

- Compactify type IIB string theory on ADE singularity $\mathbb{C}^2/\Gamma_{\mathfrak{g}}$ where $\mathfrak{g}$ is Lie algebra of ADE type $\to$ denote resulting theory by $\mathcal{T}_{\mathfrak{g}}$

$\to$ locally characterized by a real Lie algebra $\mathfrak{g} = \bigoplus_i \mathfrak{g}_i$ where $\mathfrak{g}_i$ is either $\mathfrak{u}(1)$ or a compact, simple Lie algebra of ADE type

- $\mathfrak{g} = \mathfrak{u}(r)$ can be obtained as world-volume theory of r M5-branes in 11d M-theory

Moduli space of vacua:

- In flat Minkowski space $\mathbb{R}^{5,1}$, $\mathcal{T}_{\mathfrak{g}}$ has moduli space of vacua: parametrized by $\langle \Phi^I \rangle$

$$\mathcal{M}_{\mathfrak{g}} = (\mathbb{R}^5)^{r_{\mathfrak{g}}} / \mathcal{W}_{\mathfrak{g}} ,$$

where $r_{\mathfrak{g}}$ and $\mathcal{W}_{\mathfrak{g}}$ are rank and Weyl group of $\mathfrak{g}$ $\to$ low-energy dynamics described by $r_{\mathfrak{g}}$ Abelian tensor multiplets (ATM's) valued in Cartan of $\mathfrak{g}$ "tensor branch"

$\to$ Conformal and $SO(5)_R$-symmetry are spontaneously broken

- At boundaries of moduli space: SCFT $\mathcal{T}_{\mathfrak{h}}$ with $\mathfrak{h} \subset \mathfrak{g}$ semisimple subalgebra with $r_{\mathfrak{h}} < r_{\mathfrak{g}}$ and $r_{\mathfrak{g}} - r_{\mathfrak{h}}$ ATM's

The tensor branch in 6d:

Restrict to breaking patterns $\mathfrak{g} \to \mathfrak{h} \oplus \mathfrak{u}(1)$

$\mathfrak{h}$ is obtained from $\mathfrak{g}$ by deleting a node in its Dynkin diagram (adjoint Higgsing)

- general properties of $\mathcal{L}_{\text{tensor}}$:

$$\mathcal{L}_{\text{free}} = -\frac{1}{2} \sum_{I=1}^{5} (\partial_\mu \Phi^I)^2 - \frac{1}{2} H \wedge * H + (\text{Fermions})$$

self-duality implies: $H \wedge * H = 0$

however, $\mathcal{L}_{\text{free}}$ formally correct

- example

consider $\mathfrak{g} = \mathfrak{su}(2)$

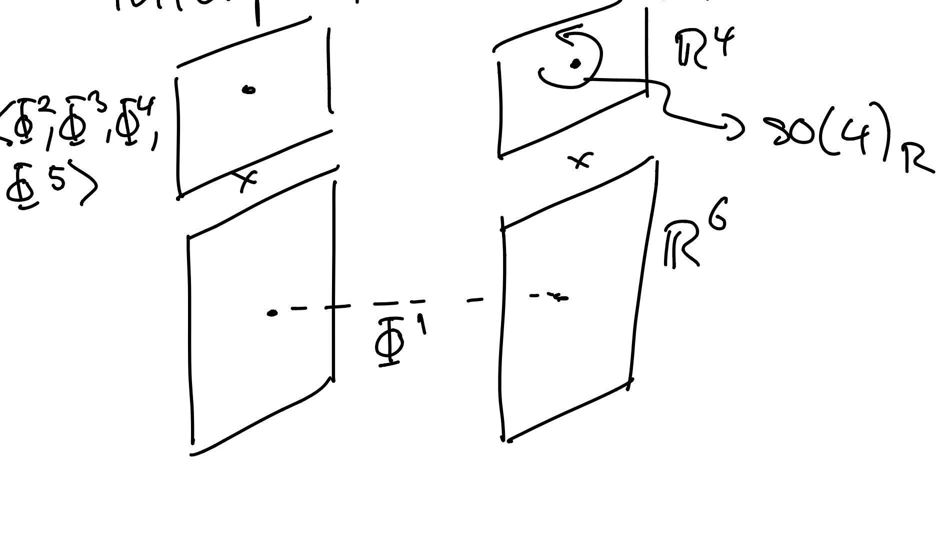
$\to$ adjoint Higgsing gives $\mathfrak{su}(2) \to \mathfrak{u}(1)$

by turning on scalar expectation value $\langle \Phi^1 \rangle \neq 0$

and $\langle \Phi^I \rangle = 0$ for $I \neq 1$

$\to$ R-symmetry is broken to $SO(4)_R$

interpretation in M-theory:



Compactification to 5d:

Central assumption (motivated from string theory):

$$6d \ (2,0) \ \text{SCFT} \ \mathcal{T}_{\mathfrak{g}}$$

$$\Big\downarrow \ S^1_R \ \text{(spacial circle with radius } R\text{)}$$

effective 5d theory below KK-scale $\frac{1}{R}$:

$\mathcal{N}=2$ SYM with gauge algebra $\mathfrak{g}$ and gauge coupling $g^2 \sim R$

effective 5d Lagrangian:

- gauge field: $A = A_\mu dx^\mu$, $\mathfrak{g}$-valued
- field strength $F = \frac{1}{2} F_{\mu\nu} dx^\mu \wedge dx^\nu$
  $= dA - i A \wedge A$, $\mathfrak{g}$-valued
- scalars ϕ^I in 5 of $SO(5)_R$, $\mathfrak{g}$-valued (R-symmetry is preserved by circle comp.)
- symplectic Majorana fermions in fundamentals of $SO(4,1)_L$ and $SO(5)_R$
- A and ϕ^I have mass dimensions 1, Fermions have $\Delta = \frac{3}{2}$
- Lie algebra $\mathfrak{g}$ decomposes into Cartan subalgebra $\mathfrak{t}_{\mathfrak{g}}$ and root vectors e_α

we have:

$$[h, e_\alpha] = \alpha(h)\, e_\alpha \quad \forall\, h \in t_{\mathfrak{g}}$$

where the real functional $\alpha \in t_{\mathfrak{g}}^*$ is the root associated with e_α

→ set of all roots comprises root system $\Delta_{\mathfrak{g}} \subset t_{\mathfrak{g}}^*$

Coroots $h_\alpha \in t_{\mathfrak{g}}$ satisfy

$$[e_\alpha, e_{-\alpha}] = h_\alpha, \quad [h_\alpha, e_{\pm\alpha}] = \pm 2 e_{\pm\alpha}$$

Define normalized, positive-definite trace

$$\mathrm{Tr}_{\mathfrak{g}} = \frac{1}{2h^\vee_{\mathfrak{g}}} \mathrm{Tr}_{adj}$$

where $h^\vee_{\mathfrak{g}}$ is dual Coxeter number.

→ induces positive-definite metric $\langle \cdot, \cdot \rangle_{\mathfrak{g}}$ on Cartan subalgebra

$$\langle h, h' \rangle \equiv \mathrm{Tr}_{\mathfrak{g}}(h h'), \quad h, h' \in t_{\mathfrak{g}}$$

⇒ 5d effective Lagrangian:

$$\mathcal{L}_0^{(5)} = -\frac{1}{2g^2} \mathrm{Tr}_{\mathfrak{g}} \Big(F \wedge * F + \sum_{I=1}^{5} D_\mu \phi^I D^\mu \phi^I - \frac{1}{8} \sum_{I,J} [\phi^I, \phi^J]^2 \Big)$$

$$+ \text{(Fermions)} + \text{(higher derivatives)}$$

where $D = d - i[A, \cdot]$

In 5d, $\Delta(g^2) = -1$ (dimension of length)

$\rightarrow$ scale-invariance of 6d theory gives $g^2 \sim R$

At origin of Coulomb branch $\phi^I = 0$, $I = 1, \dots, 5$

5d $\mathcal{N}=2$ SYM admits instanton-solitons

mass $\sim \frac{n}{g^2}$, where n is instanton number

$$\frac{1}{8\pi^2} \int_{S^4} \mathrm{Tr}_{\mathfrak{g}} (F \wedge F) \in \mathbb{Z}$$

as $g^2 \sim R \longrightarrow$ interpret as massive KK-modes of 6d theory

$$\rightarrow g^2 = 4\pi^2 R$$

BPS states on the Coulomb branch

Coulomb branch : $\langle \phi^I \rangle \in \mathbb{R}^5 \otimes \mathfrak{t}_{\mathfrak{g}} / W_{\mathfrak{g}}$

where $\mathbb{R}^5$ transforms in 5 of $SO(5)_R$

$\rightarrow$ at generic points:

- r Abelian vector multiplets with scalars φ_i^I and field strengths f_i
- $\phi^I = \sum_{i=1}^{r_{\mathfrak{g}}} h_i \varphi_i^I$, $F = \sum_{i=1}^{r_{\mathfrak{g}}} h_i f_i$
- We have commutation relations

$$[h_i, h_j] = 0, \quad [e_{+i}, e_{-j}] = \delta_{ij} h_j, \quad [h_i, e_{\pm j}] = \pm C_{ij} e_{\pm j}$$

Cartan matrix

→ leading two-derivative effective action

$$\mathcal{L}^{(5)}_{\text{Coulomb}} = -\frac{1}{2g^2}\,\Omega_{ij}\left(f_i \wedge * f_j + \sum_{I=1}^{5} \partial_\mu \varphi_i^I \partial^\mu \varphi_j^I\right)$$

$$+ \,(\text{Fermions}) + \cdots$$

where $\Omega_{ij} = \mathrm{Tr}_{\mathfrak{g}}(h_i h_j) = \langle h_i, h_j \rangle_{\mathfrak{g}}$