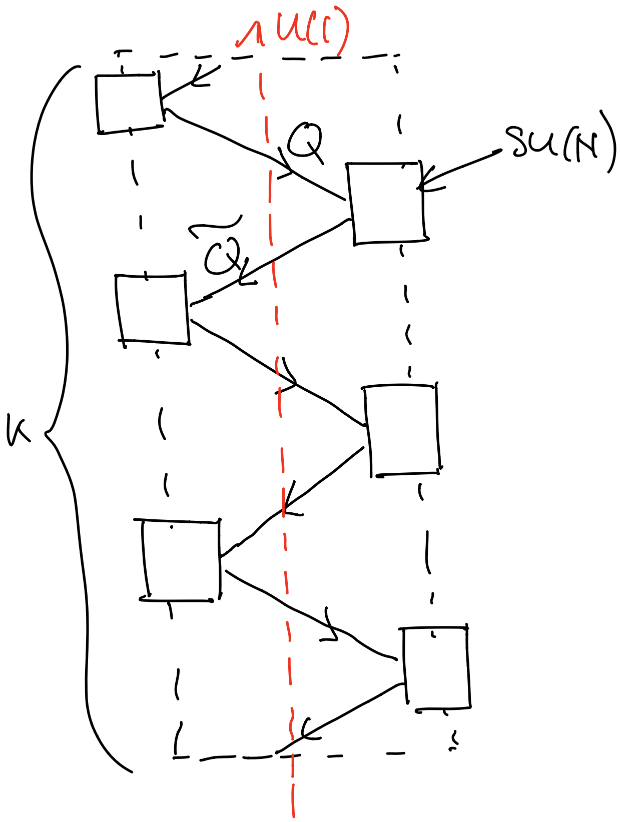
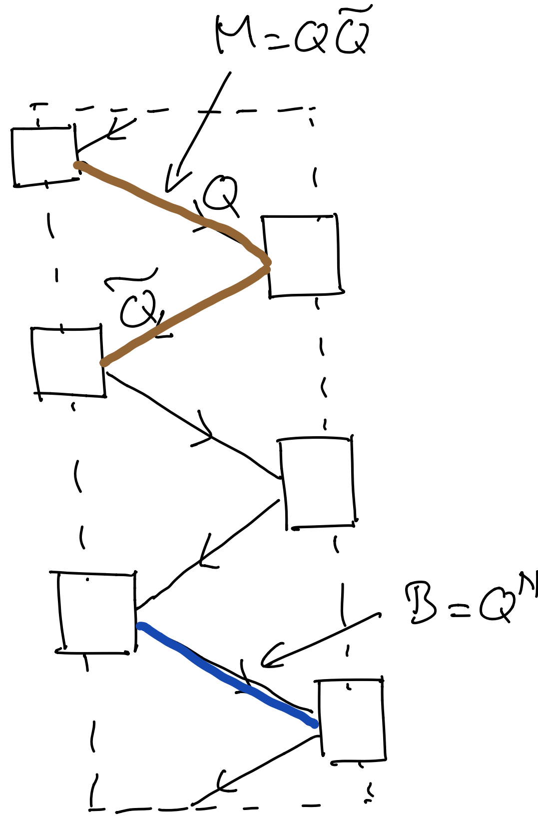
Free trinion:





Operators:

- mesonic: $M^b_\ell = Q_\ell \tilde{Q}_\ell$ and $M^a_\ell = Q_\ell \tilde{Q}_{\ell-1}$ — charged under maximal puncture sym.
- baryonic: $B = \sum Q_\ell^N$ and $\sum \tilde{Q}_\ell^N$ — charged under minimal puncture symmetries

$\longrightarrow$ K mesonic operators for every maximal puncture, 2K baryonic operators for every minimal puncture charges:

	$U(1)_t$	$U(1)_s^{k-1} \oplus_c U(1)_\gamma^{k-1}$	$U(1)_s^{k-1} \ominus_c U(1)_\gamma^{k-1}$
M	$+1$	charged	not charged

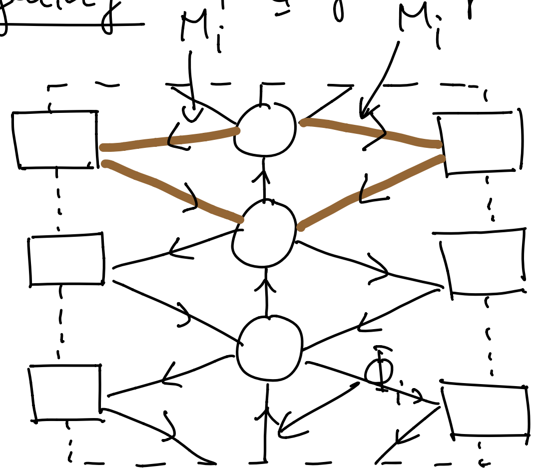
$U(1)_s^{k-1} \oplus_c U(1)_\gamma^{k-1}$: diagonal subgroup of $U(1)_s^{k-1}$ and $U(1)_\gamma^{k-1}$

denoted by "colour": $\oplus_c$ $\mathbb{Z}_k$-valued

for $k=N=2$: abelian symmetry $U(1)_s^{k-1} \oplus_c U(1)_\gamma^{k-1}$ enhances to $SU(2) \times U(1)$

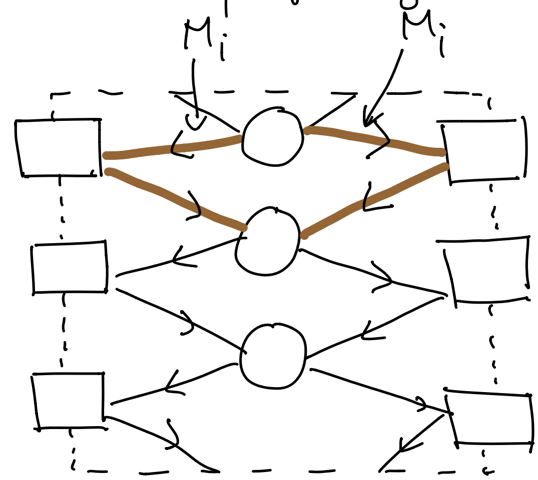
(reason from 6d: internal group enhances to $SO(7)$)

gluing: Φ-gluing



$W = \Phi \cdot M - M' \cdot \Phi$

S-gluing



$W = M \cdot M'$

→ linear quiver with 2 maximal and 2 minimal punctures

conformal manifold ⊃ positions of minimal punctures

dualities: exchange of minimal punctures

gluing of 2 maximal punctures:

→ torus with only minimal punctures

Basic example:

gluing of 2 maximal punctures of free trinion → affine $\mathcal{N}=2$ quiver with K nodes coupled to K singlet chiral fields (1 minimal puncture)

Removing minimal puncture gives:

$U(1)_\alpha \times U(1)_t \times U(1)_\beta^{K-1}$ symmetry

$\rightarrow$ conformal manifold:

1 complex str. def (of T^2)

+ $K-1$ $SU(K)$ holonomies

+ 1 $U(1)_t$ holonomy

<u>rules:</u>

glue along max puncture of <u>same</u> $\mathbb{Z}_K$-color

color differs by number of min punctures

$\rightarrow$ if not multiple of K, we are gluing different colors

$\rightarrow$ breaks some symmetries

<u>closing of punctures:</u>

give vacuum expectation values to Meson operators $\rightarrow$ $SU(N)^K$ is broken to subgroup

2 M5-branes on A_1-singularity

case $k = N = 2$:

$SU(k) \times SU(k) \times U(1)$ enhances to $SO(7)$

flux:

Can turn on flux on Riemann surface for abelian subgroup $L = U(1)^r$ of $SO(7)$

$\longrightarrow$ conformal manifold

$=$ commutant of L in $SO(7) = G^{max}$

possible values:

G^{max}	$U(1)^3$	$SU(2)U(1)^2$	$SU(2)_{diag}U(1)$	$SU(2)SU(2)U(1)$
L	$U(1)^3$	$U(1)^2$	$U(1)^2$	$U(1)$
$\mathcal{F}$	(a,b,c)	$(a,0,b)/(0,a,b)$	$(a,\pm a,b)$	$(a,0,0)/(0,a,0)$

and

G^{max}	$\widetilde{SO(5)}U(1)$	$SO(5)U(1)$	$SU(3)U(1)$	$SO(7)$
L	$U(1)$	$U(1)$	$U(1)$	$\emptyset$
$\mathcal{F}$	$(a,\pm a,0)$	$(0,0,a)$	$(a,0,\pm a)/$ $(0,a,\pm a)$	$(0,0,0)$

3 Cartans of $SO(7)$: $U(1)_\beta \times U(1)_\gamma \times U(1)_t$

characters:

- adjoint of $SO(7)$:

$$21_{SO(7)} = 1 + 10_{SO(5)} + \left(t^2 + \frac{1}{t^2}\right) 5_{SO(5)}$$

where

- $10_{SO(5)} = 3_{SU(2)_1} + 3_{SU(2)_2} + 2_{SU(2)_1} 2_{SU(2)_2}$
- $5_{SO(5)} = 1 + 2_{SU(2)_1} 2_{SU(2)_2}$
- $3_{SU(2)_1} = 1 + \frac{1}{\beta^4} + \beta^4$, $3_{SU(2)_2} = 1 + \frac{1}{\gamma^4} + \gamma^4$,

  $2_{SU(2)_1} = \frac{1}{\beta^2} + \beta^2$, $2_{SU(2)_2} = \frac{1}{\gamma^2} + \gamma^2$

Decompose $SO(7) \to SO(6)$

$$SO(6) \hookleftarrow SO(3) \times SO(3) = SU(2) \times SU(2)$$

as $SU(2)_{\beta/\gamma} \times SU(2)_{\beta\gamma}$

$\to$ $SO(7)_{adj} \to 15 + 6$ of $SO(6)$:

$$15 = \left(1 + \frac{\gamma^2}{\beta^2} + \frac{\beta^2}{\gamma^2}\right) + \left(1 + \frac{1}{\gamma^2\beta^2} + \beta^2\gamma^2\right)$$
$$+ \left(1 + \frac{\gamma^2}{\beta^2} + \frac{\beta^2}{\gamma^2}\right)\left(1 + \frac{1}{\gamma^2\beta^2} + \beta^2\gamma^2\right)$$
$$6 = \left(1 + \frac{\gamma^2}{\beta^2} + \frac{\beta^2}{\gamma^2}\right) + \left(1 + \frac{1}{\gamma^2\beta^2} + \beta^2\gamma^2\right)$$