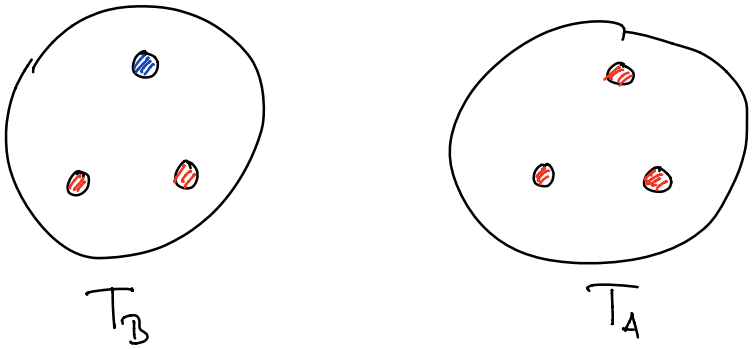
The $G^{max} = SU(2)_{diag}\, U(1)^2$ models

$SO(7) \longrightarrow SO(5) \times U(1) \longrightarrow SU(2)_{diag}\, U(1)^2$

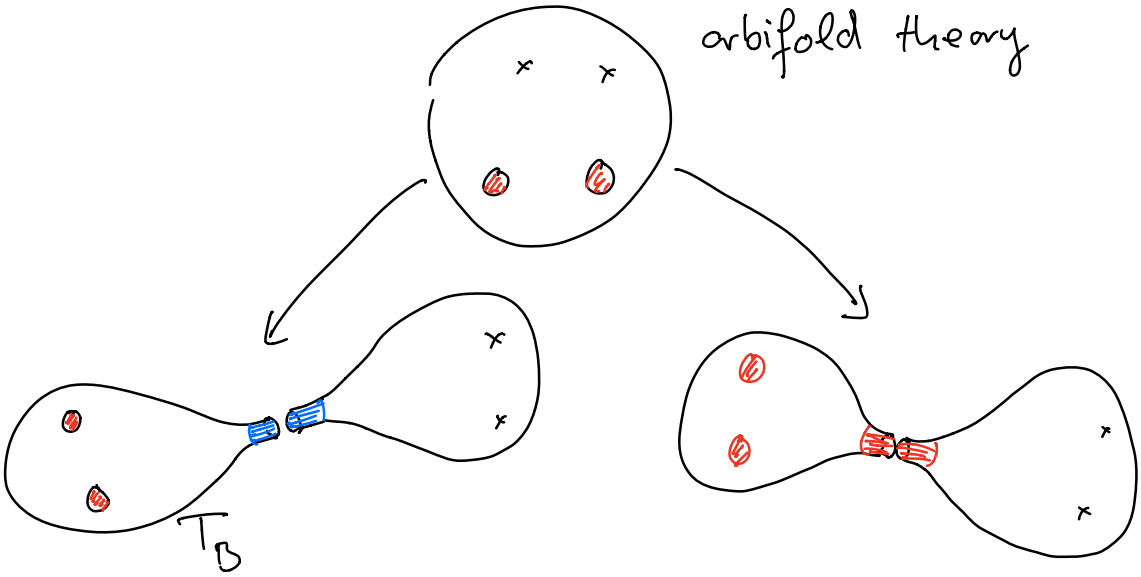
2 different colors for maximal punctures:

$SU(2)_{5/2}$ or $SU(2)_{5/1}$

$\longrightarrow$ 2 different trinions:

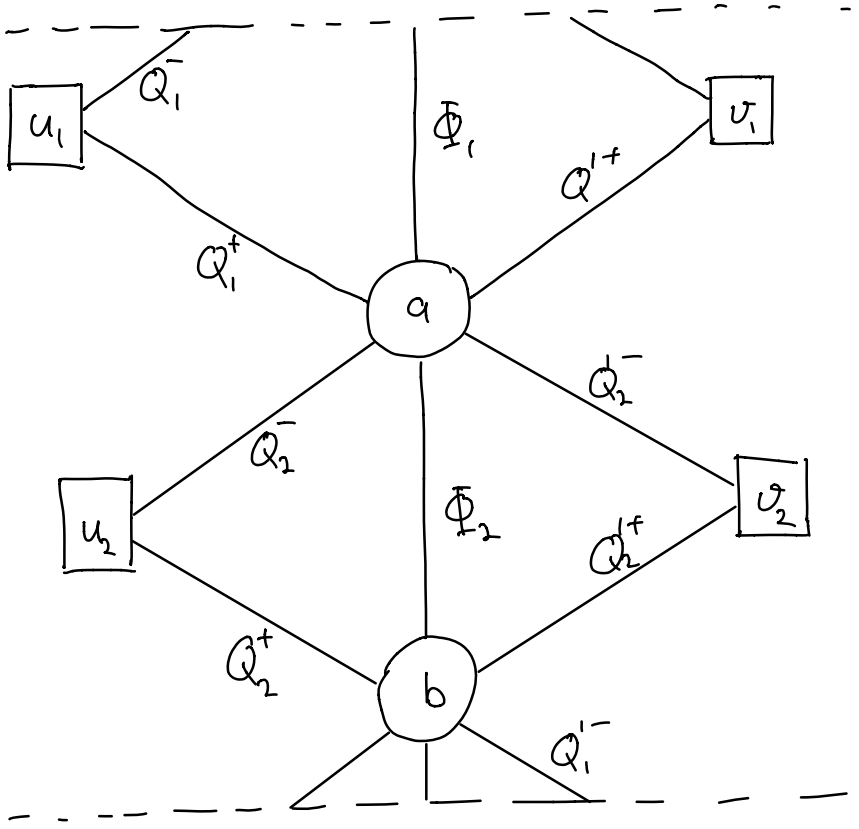


obtained from 2 IR descriptions of orbifold theory:



1) The $\mathbb{Z}_2$ orbifold of the $\mathcal{N}=2$ SYM

gauge group : $SU(2)_a \times SU(2)_b$



	$SU(2)_a$	$SU(2)_b$	$U(1)_\beta$	$U(1)_\alpha$	$SU(2)_1^u$	$SU(2)_2^u$	$SU(2)_1^v$	$SU(2)_2^v$	β	γ	t	R
									$U(1)$-sym.			
Q_1^+	2	1	−1	0	2	1	1	1	1	0	1/2	2/3
Q_1^-	1	2	1	0	2	1	1	1	0	1	1/2	2/3
Q_2^+	1	2	−1	0	1	2	1	1	−1	0	1/2	2/3
Q_2^-	2	1	1	0	1	2	1	1	0	−1	1/2	2/3
Q'^+_1	2	1	0	1	1	1	2	1	0	1	1/2	2/3
Q'^-_1	1	2	0	−1	1	1	2	1	1	0	1/2	2/3
Q'^+_2	1	2	0	1	1	1	1	2	0	−1	1/2	2/3
Q'^-_2	2	1	0	−1	1	1	1	2	−1	0	1/2	2/3
Φ_1	2	2	0	0	1	1	1	1	−1	−1	−1	2/3
Φ_2	2	2	0	0	1	1	1	1	1	1	−1	2/3

- each gauge node has six flavors
  $\longrightarrow N_f = 3N_c \longrightarrow \beta$-function vanishes
- one exactly marginal deformation
  $\longrightarrow \mathcal{M}_{conf}$
- assume that $\exists$ infinite coupling cusp on $\mathcal{M}_{conf}$ with enhancement $U(1)_{s/a} \to SU(2)$
  Under exchange $U(1)_a \longleftrightarrow U(1)_s$
  some mesons are invariant, some map to each other : $Q_i^+ Q_i'^+ \longleftrightarrow Q_i^- Q_i'^-$

table of mesons and baryons :

	$U(1)_{a/s}$	$U(1)_{a s}$	$SU(2)_1^u$	$SU(2)_2^u$	$SU(2)_1^v$	$SU(2)_2^v$	$U(1)_{B/S}$	$U(1)_{Y/0}$
$M_-^u = Q_1^+ Q_2^-$	0	0	2	2	1	1	0	-1
$M_+^u = Q_1^- Q_2^+$	0	0	2	2	1	1	0	1
$M_+^v = Q_1'^+ Q_2'^-$	0	0	1	1	2	2	0	1
$M_-^v = Q_1'^- Q_2'^+$	0	0	1	1	2	2	0	-1
$B_{\ell;+-} = (Q_\ell^+)^2$	$+1$	-1	1	1	1	1	$(-1)^{\ell+1}$	$(-1)^\ell$
$B_{\ell;--} = (Q_\ell'^-)^2$	-1	-1	1	1	1	1	$(-1)^{\ell+1}$	$(-1)^\ell$
$B_{\ell;-+} = (Q_\ell^-)^2$	-1	$+1$	1	1	1	1	$(-1)^{\ell+1}$	$(-1)^{\ell+1}$
$B_{\ell;++} = (Q_\ell'^+)^2$	$+1$	$+1$	1	1	1	1	$(-1)^{\ell+1}$	$(-1)^{\ell+1}$
$T^{111} = Q_1^\pm Q_1'^\pm$	± 1	0	2	1	2	1	1	0
$T^{122} = Q_2^\pm Q_2'^\pm$	± 1	0	1	2	1	2	-1	0
$T^{212} = Q_1^\pm Q_2'^\mp$	0	∓ 1	2	1	1	2	0	0
$T^{221} = Q_2^\pm Q_1'^\mp$	0	∓ 1	1	2	2	1	0	0

2) IR dual descriptions A

We seek IR dual of orbifold theory which is $SU(2)$ gauging of an SCFT T_A.

- flavor sym.:
  $SU(N)_u^2 \times SU(N)_v^2 \times SU(N)_z^2 \times U(1)_\beta \times U(1)_\gamma \times U(1)_t$
- operators:
  $M^u_\pm$, $M^v_\pm$, and $M^z_\pm$
  ($U(1)$-charges of M^z as for M^u and M^v)

gauge $SU(2)_{z_1}$-symmetry of T_A and add fields:

	$SU(2)_z$	$U(1)_\beta$	$U(1)_\alpha$	$U(1)_R$	$U(1)_\beta$	$U(1)_\gamma$	$U(1)_t$
$q^{(\pm)}$	2	± 1	∓ 1	0	-1	-1	0
$\Phi^{1(\pm)}$	2	∓ 1	∓ 1	$2/3$	± 1	∓ 1	-1
$B_{1,\pm\pm}$	1	0	± 2	$4/3$	$\lvert\mp 1\rvert$	$\lvert\pm 1\rvert$	1
$B_{1,\mp\pm}$	1	± 2	0	$4/3$	$\lvert\mp 1\rvert$	$\lvert\pm 1\rvert$	1
T_0	1	0	0	2	2	2	0
$(M^z_{\mp 2})^{\pm 1}$	2	$\pm_1 \lvert$	$\pm_1 \lvert$	$4/3$	$\pm_2 \lvert$	$\mp_2 \lvert$	1

superpotential:

$$W \supset q^{(-)}\Phi^{1(+)}B_{1,-+} + q^{(+)}\Phi^{1(+)}B_{1,++} + q^{(-)}\Phi^{1(-)}B_{1,--} + q^{(+)}\Phi^{1(-)}B_{1,+-} + q^{(+)}q^{(-)}T_0 + \Phi^{1(+)}(M_+^z)^+ + \Phi^{1(-)}(M_-^z)^-$$

- Under duality $M^u_{\pm}$, $M^v_{\pm}$, and $\mathcal{B}_{1,ab}$ map as names suggest
- Baryons $\mathcal{B}_{2,ab}$ of theory I) map as:

$$\mathcal{B}_{2,+-} \longrightarrow q^{(-)}_{+}(M^z_{+})^{-}, \quad \mathcal{B}_{2,-+} \longrightarrow q^{(+)}_{+}(M^z_{-})^{+},$$

$$\mathcal{B}_{2,++} \longrightarrow q^{(-)}_{+}(M^z_{-})^{+}, \quad \mathcal{B}_{2,--} \longrightarrow q^{(+)}_{+}(M^z_{+})^{-}$$

All 't Hooft anomalies of theory I) and 2) match!

The T_A SCFT:

- add following fields to both sides of duality I) $\longleftrightarrow$ 2):

	$SU(2)_w$	$U(1)_\delta$	$U(1)_\alpha$	$U(1)_R$	$U(1)_\beta$	$U(1)_\gamma$	$U(1)_t$
$\tilde{q}^{(\pm)}$	2	± 1	∓ 1	0	1	1	0
$b_{1,\pm\pm}$	1	0	∓ 2	$\frac{2}{3}$	-1 ± 1	-1 ∓ 1	-1
$b_{1,\mp\pm}$	1	∓ 2	0	$\frac{2}{3}$	-1 ± 1	-1 ∓ 1	-1
t_0	1	0	0	2	-2	-2	0

and superpotential:

$$\Delta W = \tilde{q}^{(-)}\tilde{q}^{(+)} t_0 + b_{1,\mp\mp}\mathcal{B}_{1,\mp\mp} + b_{1,\mp\pm}\mathcal{B}_{1,\mp\pm}$$

- tune to enhance to $SU(2)_{d/s}$

  in theory 2) $SU(2)$ is broken by SP-terms
  → turn them off
  in theory 1): go to strong coupling point

- gauge $SU(2)_{d/s}$
  → give vev to gauge inv. mesonic operators breaking $SU(2)_z$ (Higgsing)
  → theory T_A remains coupled to $\tilde{\Phi}'$ fields through SP

- remove extra fields:
  add SP $\tilde{\Phi}^{1(\pm)}\Phi^{1(\pm)}$ → integrate out $\tilde{\Phi}^{1(\pm)}$

→ T_A SCFT has flavor sym.: