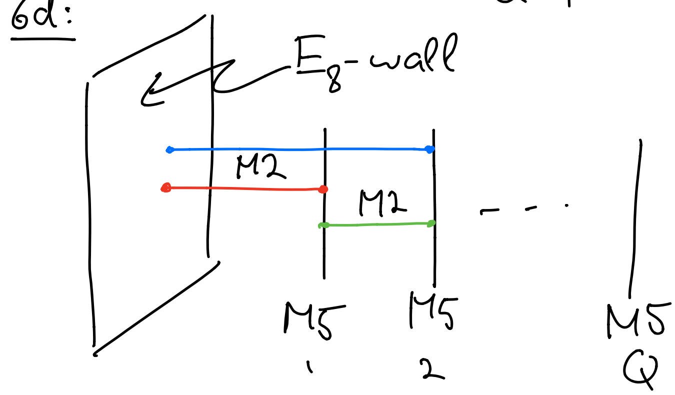
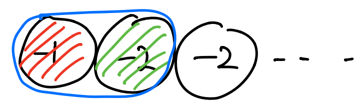
E-string theory

Let us consider the E-string theory with

$$\mathcal{T}_{E\text{-string}} = 1\underbrace{22\cdots2}_{Q-1}$$

6d:





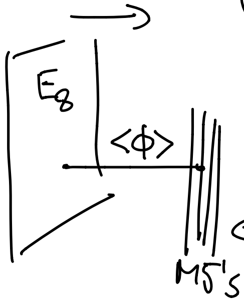
$\Big\downarrow$ compactification on S^1

5d:

$\mathcal{N}=1$ $Usp(Q)$ theory with
anti-sym hyper
+ 8 fundamental hyper-multiplets $\rightarrow$ $SO(16)$ flavor sym.

Giving vev $\langle\phi\rangle$ to adjoint scalar in VM

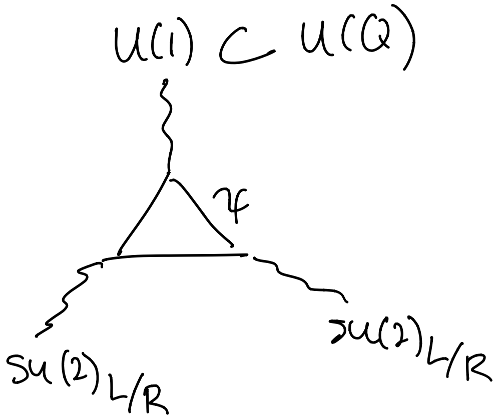
$\longrightarrow$ $Usp(Q) \longrightarrow U(Q)$



all M5's on top of each other

position parametrized by free hyper

strategy to compute anomaly polynomial: 5d CS



ψ:

- Q hypers in $SO(16)$, $q_{U(1)} =$

$SU(2)_R$ • $Q^2 + Q$ vectors, $q_{U(1)} = 2$

$SU(2)_L$ • $Q^2 - Q$ hypers, $q_{U(1)} = 2$

$$\Rightarrow \quad A \wedge F \wedge F$$

$$= \frac{1}{2} A \wedge \Big[Q\Big(\frac{\mathrm{Tr}\,F^2}{2} + \frac{16 p_1(T)}{24}\Big) + 2(Q^2 - Q)\frac{1}{2}\Big(c_2(L) - \frac{2p_1(T)}{24}\Big)$$

$$- 2(Q^2 + Q)\frac{1}{2}\Big(c_2(R) + \frac{2p_1(T)}{24}\Big)\Big]$$

$$= \Omega A \wedge \Big(\frac{Q}{2}\chi_4(N) + I_4\Big)$$

where $\Omega = Q$ and

$$I_4 = \frac{1}{4}\Big(\mathrm{Tr}\,F^2 + p_1(T) + p_1(N)\Big)$$

and

$$\chi_4(N) = c_2(L) - c_2(R), \quad p_1(N) = -2\big(c_2(L) + c_2(R)\big)$$

$$\Rightarrow I^{\text{rank}Q\text{ E-string}} = I^{Q\text{ M5s}} + \frac{Q^2}{2}\chi_4(N) I_4 + Q\Big(\frac{1}{2}I_4^2 - I_8\Big)$$

contains contribution of free hyper-mult.:

$$I_{free} = \frac{7p_1(T)^2 - 4p_2(T)}{5760} + \frac{c_2(L)\,p_1(T)}{48} + \frac{c_2(L)^2}{24}$$

Tensor branches with gauge multiplets

6d $\mathcal{N}=(1,0)$

$\xrightarrow{\text{tensor branch}}$

- t VMs with gauge group G_A, $A=1,\dots,t$
- t free tensor-mult. (scalars give coupling constants of G_A)
- bifundamental matte

<u>Note:</u> $t_{VM} = t_{TM} \rightarrow$ we do not give vev to E-string tensors

→ "One-loop" anomaly:

$$I^{\text{one-loop}} = \sum_A I^{\text{vec}}_{F_A} + \sum_{A,B} I^{\text{matter}}_{F_A, F_B} + t\, I^{\text{tensor}}$$

$$\cup$$

$$-\frac{1}{32} C^{AB}\, \mathrm{Tr}\, F_A^2\, \mathrm{Tr}\, F_B^2 - \frac{1}{4} \underset{\substack{\uparrow \\ \text{background flavor} \\ \text{\& gravity fields}}}{X^A}\, \mathrm{Tr}\, F_A^2$$

→ needs to cancell gauge anomalies by Green-Schwarz contribution $\frac{1}{2} \underset{\substack{\nearrow \\ \text{intersection matrix}}}{\Omega^{ij}} I_i I_j$

$$\longrightarrow \quad I_i = \frac{1}{4} d_i^A \operatorname{Tr} F_A^2 + (\Omega^{-1})_{ij} (d^{-1})^j_A X^A , \qquad (*)$$

$$d_i^A d_j^B \Omega^{ij} = c^{AB} \qquad (t_{VM} = t_{TM} \to d_i^A \text{ invertible})$$

giving

$$\frac{1}{2} \Omega^{ij} I_i I_j = \frac{1}{32} c^{AB} \operatorname{Tr} F_A^2 \operatorname{Tr} F_B^2 + \frac{1}{4} X^A \operatorname{Tr} F_A^2 + \frac{1}{2} (c^{-1})_{AB} X^A X^B$$

Use $$I^{vec} = -\frac{1}{24}\left(\frac{3}{4}\omega_G (\operatorname{Tr} F^2)^2 + 6 h^\vee_G \operatorname{Tr} F^2 c_2(R) + d_G c_2(R)^2\right)$$

where $\frac{3\omega_G}{4}$ is the coefficient converting $\operatorname{tr}_{adj} F^4$ to $(\operatorname{Tr} F^2)^2$, $h^\vee_G$ and d_G are dual Coxeter numbers and dimension of G

<u>Consistency condition:</u>

$$dH_i = I_i$$

$(*)$: instanton of $\frac{1}{4} \operatorname{Tr} F_A^2 \longrightarrow$ string in 6d with charge $q_i = d_i^A$

$$\Rightarrow c^{AB} = \Omega^{ij} d_i^A d_j^B = \langle d^A, d^B \rangle_{6d} \in \mathbb{Z}$$

by 6d charge quantization

$\rightarrow$ strong constraints on theory

Green-Schwarz terms for F-theory models

start with type IIB supergravity on

$\mathbb{R}^{5,1} \times \mathcal{B}$ ← base of F-theory construction

Curves C_a are wrapped by 7-branes giving rise to gauge/flavor symmetries

strings in 6d are obtained by D3-branes wrapping C_a

Notation: a runs over all curves

i runs over compact curves

$$F_5 = H_i \wedge \omega^i \,, \quad \omega^i \text{ dual to } C_i$$

$$dF_5 = Z \longrightarrow dH_i = I_i \,, \quad \eta^{ij} I_j = -\int_{\mathcal{B}} Z \wedge \omega^i$$

where $\eta^{ij} = -\int_{\mathcal{B}} \omega^i \wedge \omega^j = -C_i \cdot C_j$

$$\rightarrow I^{GS} = -\frac{1}{2}\int_{\mathcal{B}} Z^2 = \frac{1}{2}\eta^{ij} I_i I_j$$

note $\eta^{ij} = \Omega^{ij}$

10d GS-term:

$$Z = \frac{1}{4} c_1(B) \wedge p_1(T) + \frac{1}{4} \sum_a \omega^a \, \mathrm{Tr}\, F_a^2$$

where F_a is field strength on 7-branes wrapping C_a.

$$\Rightarrow \eta^{ij} I_j = \frac{1}{4}\left(\eta^{ia} \mathrm{Tr}\, F_a^2 - K^i p_1(T)\right),$$

$$K^i := \int_B c_1(B) \wedge \omega^i = 2 - \eta^{ii}$$

up to term proportional to $c_2(R)$

to cancel mixed gauge-$SU(2)_R$ anomalies, we write

$$\eta^{ij} I_j = \frac{1}{4}\left(\eta^{ia} \mathrm{Tr}\, F_a^2 - K^i p_1(T)\right) + y^i c_2(R)$$

$\to$ determine y^i

$$I^{GS} \supset \frac{1}{4} y^i \, \mathrm{Tr}\, F_i^2 \, c_2(R)$$

for cycles C_i with non-trivial gauge group G_i, we conclude: $y^i = h^\vee_{G_i}$

For -1, -2 curves we cannot determine y_i

<u>Solution</u>: shrink these curves giving rank 1 and 2 E-string theories

Consider blowing-down a -1 curve C_A in B which intersects with C_{A-1} and C_{A+1}

$\longrightarrow \hat{B}$ with $p: B \to \hat{B}$ the blow-down map

homology cycles $\hat{C}$ in $\hat{B}$ and C in B are related by

$$p^* [\hat{C}_i] = \begin{cases} [C_i] + [C_A] & i = A-1, A+1 \\ [C_i] & i \neq A-1, A, A-1 \end{cases}$$

$$\Rightarrow \hat{\eta}^{ij} = -\hat{C}^i \cdot \hat{C}^j = \begin{cases} \eta^{ii} - 1 & i = j = A \pm 1 \\ -1 & (i,j) = (A-1, A+1), \\ & (A+1, A-1) \\ \eta^{ij} & \text{otherwise} \end{cases}$$

$$d\hat{H}_i = \hat{I}_i,$$

$$\hat{I}_i = \begin{cases} I^i + I^A & i = A+1, A-1 \\ I^i & \text{otherwise} \end{cases}$$

$$(I^i = \eta^{ij} I_j)$$

$$\Rightarrow \hat{I}^{GS} = \frac{1}{2} \hat{\eta}^{ij} \hat{I}_i \hat{I}_j = \frac{1}{2} (\hat{\eta}^{-1})_{ij} \hat{I}^i \hat{I}^j,$$

$$I^{tot} = I^{\text{one-loop}} + I^{GS} = \hat{I}^{\text{one-loop}} + \hat{I}^{GS}$$