4d $\mathcal{N}=1$ SCFT's

We want to look at compactifications on Riemann surfaces to 4d $\mathcal{N}=1$ theories

Let us review some facts about 4d $\mathcal{N}=1$ SCFT's:

- β-function:

$$\beta_{8\pi^2/g_p^2} = \frac{\partial}{\partial \log M} \frac{8\pi^2}{g_p^2} = \frac{3T_2(\mathrm{adj}) - \sum_i T_2(r_i)(1-\gamma_i(g))}{1 - \frac{g_p^2 T_2(\mathrm{adj})}{8\pi^2}}$$

where M is the energy scale,

$\mathrm{tr}\, T_{r_i}^a T_{r_i}^b = T_2(r_i)\,\delta^{ab}$: quadratic Casimir

$\gamma(g_p)$: anomalous dimension of Φ_i

sum is over matter fields

normalization : $T_2(\square) = \frac{1}{2}$, $T_2(\mathrm{adj}) = N$

- chiral primary operator $\mathcal{O}$ with dimension $D[\mathcal{O}]$ has R-charge

$$R[\mathcal{O}] = \frac{2}{3} D[\mathcal{O}] = \frac{2}{3}\left(D_{uv}[\mathcal{O}] + \frac{\gamma[\mathcal{O}]}{2}\right)$$

$$\rightarrow 3T_2[\mathrm{adj}] - \sum_i T_2(r_i)(1-\gamma_i(g_p))$$
$$= 3T_2(\mathrm{adj}) + 3\sum_i R_i T_2(r_i) = 3\,\mathrm{tr}\, R\, T^a T^b$$

Consider a non-Lagrangian theory which has a flavor symmetry with current superfield $\mathcal{J}^a$

$$\rightarrow \quad \mathcal{L} \supset 2\int d^4\theta\, \mathcal{J}^a \mathcal{V}^a + (\text{terms for gauge invariance})$$

$\mathcal{J}^a$ is a real linear superfield :

$$D^2 \mathcal{J}^a = \bar{D}^2 \mathcal{J}^a = 0$$

containing j^a_μ

OPE :
$$j^a_\mu(x)\, j^b_\nu(0) = \frac{3K_G}{4\pi^4}\delta^{ab}\frac{x^2 g_{\mu\nu} - 2x_\mu x_\nu}{x^6} + \frac{2}{\pi^2} f^{abc}\frac{x_\mu x_\nu\, x\cdot j^c(0)}{x^6} + \cdots$$

K_G is called central charge of the flavor sym.

n free chiral multiplets have $K_{U(n)} = 1$

For $G \subset U(n)$ we have

$$K_G = 2\sum T_2(r_i)$$

where $n = \sum_i r_i$

For $G \subset H$: $K_{G\subset H} = I_{G\hookrightarrow H}\, K_H$

G: weakly gauged; H: flavor; $I_{G\hookrightarrow H}$: embedding index

β-function receives contributions at one-loop and higher-loop:

- one-loop ($\gamma_i = 0$):

$$\beta_{\text{one-loop}} = 3T_2(\text{adj}) - \sum_i T_2(r_i) - \frac{K_G}{2}$$

Define

$$3\,\mathrm{tr}_{\text{non-Lagrangian}} R T^a T^b = -K \delta^{ab}$$

$$\rightarrow \beta_{8\pi^2/g^2} = 3\,\mathrm{tr}\, R T^a T^b = 3T_2(\text{adj}) + 3\sum_i R_i T_2(r_i) - K$$

Examples:

R-symmetry of $\mathcal{N}=2$ SCFT: $SU(2) \times U(1)$ and R-sym. of $\mathcal{N}=1$ SCFT:

$$R_{\mathcal{N}=1} = \frac{1}{3} \underbrace{R_{\mathcal{N}=2}}_{U(1)_R} + \frac{4}{3} \underbrace{I_3}_{\text{Cartan of } SU(2)_R}$$

- $\mathcal{N}=2$ theories:

superalgebra enforces

$$\mathrm{tr}\, R_{\mathcal{N}=2} T^a T^b = -\frac{K_G}{2} \delta^{ab}$$

for any flavor sym. G

Setting $k = k_G/2 \longrightarrow$ exact β-function for $\mathcal{N}=1$ stops at one-loop

- Argyres-Seiberg theory:

consider $SU(2)$ $\mathcal{N}=2$ gauge theory with one hypermultiplet

Take $SU(2) \subset E_6$ flavor of Minahan-Nemeshansky SCFT

$$\rightarrow \beta = 3T_2(adj) + 3\sum_i R_i T_2(r_i) - \frac{k_G}{2}$$

use $T_2(SU(2)) = 2$

$T_2(\square) = \frac{1}{2}$

$k_{E_6} = 6$, $I_{SU(2)\hookrightarrow E_6} = 1$

$$= 3 \cdot 2 - 2 - 1 - 3 = 0$$

consider $SU(2) \subset E_7$ MN SCFT $(k_{SU(2)\subset E_7} = 8)$

$$\rightarrow \beta = 3 \cdot 2 - 2 - 4 = 0$$

- Mass deformed Argyres-Seiberg theory

add $\delta W = m\Phi^2$, where Φ is chiral superfield inside $\mathcal{N}=2$ VM

$\rightarrow R_{IR} = \frac{1}{2} R_{\mathcal{N}=2} + I_3 = \frac{3}{2} R_{\mathcal{N}=1} - I_3$

$$K = \frac{3}{4} K_G$$

$\rightarrow \beta$ - function of $SU(2)$ is

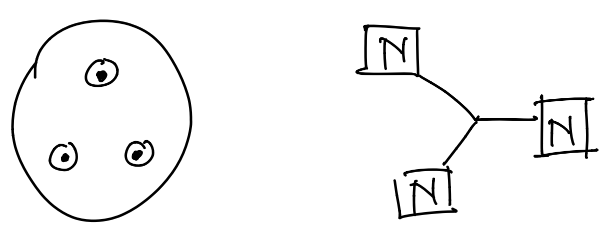
$$\beta = 3 \cdot 2 - \frac{3}{2} - \frac{3}{4} \cdot 6 = 0$$

<u>T_N theory</u>

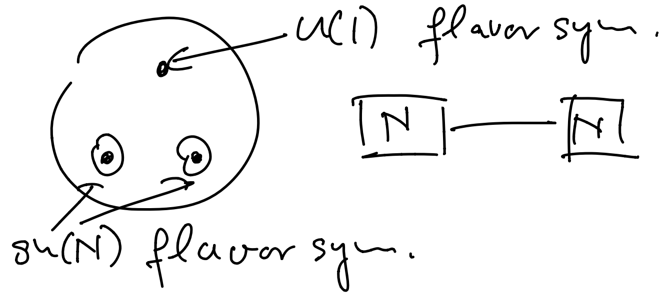
This is an $\mathcal{N}=2$ SCFT with no marginal couplings and flavor sym. $\supset SU(N)^3$

- T_2 is the theory of eight free chiral multiplets Q_{ijk}
- T_3 is MN SCFT

T_N theory is obtained by wrapping N M5-branes on a sphere with 3 maximal punctures



By comparison, a bifundamental of $SU(N) \times SU(N)$ arises by wrapping N M5-branes on sphere with 2 maximal punctures and 1 simple puncture:



T_N theory can be used as "building blocks" by gauging their flavor symmetries: