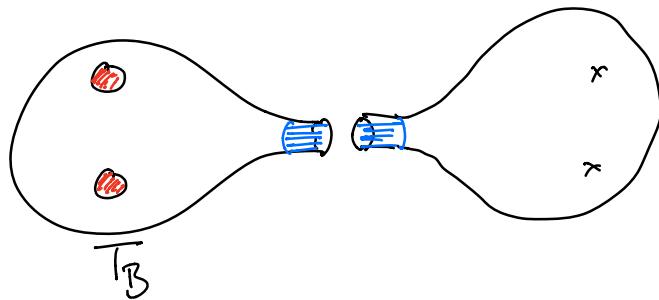


IR dual description B

in terms of $\text{su}(2)$ gauging of SCFT T_B



- flavor symmetry:

$$\text{su}(N)_u^2 \times \text{su}(N)_v^2 \times \text{su}(N)_z^2 \times U(1)_S \times U(1)_R \times U(1)_T$$

- operators:

$$\underbrace{M_{\pm}^u, M_{\pm}^v, \text{ and } M_{\pm}^z}_{\text{are in fund.}}$$

of $\text{su}(2)_{R/S}$,

have $U(1)_T$ charge +1,

$U(1)_S$ charge 0,

R-charge $\frac{4}{3}$

is in fund. of
 $\text{su}(2)_{R/S}$

$U(1)_T$ charge +1,

$U(1)_{R/S}$ charge 0,

R-charge $\frac{4}{3}$

Have to gauge $\text{su}(2)_z$ symmetry of T_B
and add following fields:

	$SU(2)_z$	$U(1)_S$	$U(1)_K$	$U(1)_R$	$U(1)_{JS}$	$U(1)_Y$	$U(1)_t$
$q^{(\pm)}$	2	± 1	∓ 1	0	1	-1	0
$\Phi'^{(\pm)}$	2	∓ 1	∓ 1	$\frac{2}{3}$	∓ 1	∓ 1	-1
$B_{2,\pm -}$	1	-1 ∓ 1	-1 ± 1	$\frac{4}{3}$	-2	0	1
$B_{1,\pm +}$	1	1 ∓ 1	1 ± 1	$\frac{4}{3}$	0	2	1
T_o	1	0	0	2	-2	2	0
$(M_{\mp 2}^2)^{\pm 1}$	2	± 1	± 1	$\frac{4}{3}$	$\mp_2)$	$\mp_2)$	1

together with superpotential

$$W \supset q^{(+)} \Phi'^{(+)} B_{1,++} + q^{(-)} \Phi'^{(+)} B_{1,-+} + q^{(-)} \Phi'^{(-)} B_{2,--} \\ + q^{(+)} \Phi'^{(-)} B_{2,+-} + q^{(+)} q^{(-)} T_o + \Phi'^{(+)} (M_+^2)^\dagger \\ + \Phi'^{(-)} (M_-^2)^\dagger$$

Under the duality, the operator M_\pm^u , M_\pm^v , and $B_{1,a+}$, $B_{2,a-}$ map as the names suggest and

$$B_{1,+} \rightarrow q^{(-)} (M_+^2)^\dagger, \quad B_{2,-} \rightarrow q^{(+)} (M_-^2)^\dagger,$$

$$B_{2,++} \rightarrow q^{(-)} (M_-^2)^\dagger, \quad B_{1,--} \rightarrow q^{(+)} (M_+^2)^\dagger$$

The T_B SCFT

Want to isolate T_B -theory

- add matter

	$SU(2)_\sim$	$U(1)_S$	$U(1)_\alpha$	$U(1)_R$	$U(1)_B$	$U(1)_S$	$U(1)_t$
$\tilde{q}^{(\pm)}$	2	± 1	∓ 1	0	-1	1	0
$b_{1,\pm -}$	1	1 ∓ 1	1 ∓ 1	$\frac{2}{3}$	2	0	-1
$b_{2,\pm +}$	1	-1 ∓ 1	-1 ∓ 1	$\frac{2}{3}$	0	-2	-1
t_0	1	0	0	2	2	-2	0

- superpotential

$$\Delta W = \tilde{q}^{(-)} \tilde{q}^{(+)} t_0 + b_{1,\pm +} \mathcal{B}_{1,\pm +} + b_{1,\pm -} \mathcal{B}_{1,\pm -}$$

→ makes b_1 and \mathcal{B}_1 fields massive
and removes them from theory

- tune to enhance to $SU(2)_{\alpha/8}$

set corresponding 8P-terms to zero

$$q^{(-)} \Phi^{(-)} \mathcal{B}_{2,-} + q^{(+)} \Phi^{(+)} \mathcal{B}_{2,+} \rightarrow 0$$

on orbifold side: go to infinite coupling point

- gauge $SU(2)_{\alpha/S}$:

this gauge sector has $N_f = 2$

→ all symmetries are non anomalous
of moduli space

→ description in terms of gauge invariant
mesonic operators with non-zero vev

→ Higgs the $SU(2)_Z$ gauge symmetry

→ left with T_B coupled to Φ' fields
through superpotential

- remove extra fields:

add on both sides of duality fields ϕ'
and couplings $\cdot \Phi'^{(\pm)} \phi^{1(\mp)}$

→ flavor symmetry:

$$SU(2)_W \times SU(2)_{\alpha/S} \times (SU(2))_u^2 \times (SU(2))_d^2 \times U(1)_f \times U(1)_Y \times U(1)_S$$

will denote $w, \alpha, \delta \rightarrow w_1, w_2$

Supersymmetric index and anomalies

T_A and T_B correspond to compactifications of
type $G^{max} = SU(2)_{\text{diag}} U(1)^2$

Want to compute supersymmetric index

Definition:

- counts BPS operators of theory
→ gives information about marginal, relevant and irrelevant deformations
- computed as trace over Hilbert space on S^3 :

$$I(p, q; u) = \text{Tr}_{H_{S^3}} (-1)^F p^{j_1 + j_2 - \frac{1}{2}r} q^{j_1 - j_2 - \frac{1}{2}r} \prod_{\alpha \in \mathcal{F}} u_\alpha^{q_\alpha}$$

- j_i are Cartans of $SO(4) \sim SU(2)_1 \times SU(2)_2$
isometry of S^3

- r is $U(1)_R$ R-symmetry

- q_α correspond to $U(1)$ global flavor symmetry charges

Index of free chiral field of R-charge R

is given by:

$$I_x = \prod_{i,j=0}^{\infty} \frac{1 - p^{1-\frac{R}{2}+i} q^{1-\frac{R}{2}+j} u^{-1}}{1 - p^{\frac{R}{2}+i} q^{\frac{R}{2}+j} u}$$

$$= T((pq)^{\frac{R}{2}} u; pq) = T_c((pq)^{\frac{R}{2}} u)$$

When $SU(N)_2$ flavor symmetry of a theory with index $I(\tilde{z})$ is gauged, we get

$$I = \frac{(q; q)^{N-1} (p; p)^{N-1}}{N!} \oint_{z=1}^{N-1} \prod_{i=1}^{N-1} \frac{dz_i}{2\pi i z_i} \prod_{i \neq j} T_e(z_i/z_j) I(\tilde{z})$$

with $(z; q) = \prod_{i=0}^{\infty} (1 - zq^i)$

The result for the T_A theory is:

$$\begin{aligned} I_A = 1 &+ \left(\frac{2}{\beta^2 \gamma^2 f^2} + \frac{\beta^2 \gamma^2}{f^2} + t(2u_1 8_v + 2v_1 8_s + 2w_1 8_c) \right. \\ &+ t \beta^2 \gamma^2 2u_1 2v_1 2w_1 (pq)^{\frac{2}{3}} \\ &+ (\beta^2 \gamma^2 (3u_1 + 3v_1 + 3w_1) + \frac{1}{\beta^2 \gamma^2} 28 - 28 - 3u_1 - 3v_1 \\ &- 3w_1 - (-1)(pq) + \dots \end{aligned}$$

notation:

$$8_v = 2r/\beta 2u_2 + 2v_2 2w_2, \quad 8_s = 2r/\beta 2v_2 + 2u_2 2w_2,$$

$$8_c = 2r/\beta 2w_2 + 2v_2 2u_2, \quad 28 = 3r/\beta + 3w_2 + 3u_2 + 3v_2$$

→ flavor symmetry enhanced: $+ 2w_2 2u_2 2v_2 r/\beta$

$$SO(8) \times SU(2)_{u_1} \times SU(2)_v \times SU(2)_{w_1} \times U(1)_t \times U(1)_{\beta}$$

marginal defns : at order pq

relevant defns : at lower order

→ no exactly marginal operators

Index of T_B :

$$\begin{aligned}
 I_{T_B} = & 1 + \left(\frac{1}{\beta^2 \gamma^2 t^2} + \frac{\beta^2 \gamma^2}{t^2} + \frac{\beta^2}{\gamma^2 t^2} \right. \\
 & + t^2 \beta \gamma (2u_1^1 2u_2^2 + 2u_3^1 2u_3^2) + t^2 \gamma \beta 2u_1^1 2u_1^2 \\
 & + t \gamma^2 2u_1^1 2u_3^1 2u_2^1 + t \frac{1}{\beta^2} 2u_1^1 2u_3^2 2u_2^2 \\
 & \left. + t^2 u_1^2 (2u_3^2 2u_2^1 + 2u_3^1 2u_2^2) \right) (pq)^{\frac{2}{3}} \\
 & + (\beta^2 \gamma^2 (3u_1^1 + 3u_3^1) + \frac{1}{\beta^2 \gamma^2} (3u_2^2 + 3u_3^2)) \\
 & + \frac{1}{\beta} 2u_1^2 \left(\beta^4 2u_3^1 2u_2^1 + \frac{1}{\gamma^4} 2u_2^2 2u_3^2 \right) + \frac{\beta^2}{\gamma^2} 3u_2^2 \\
 & + \frac{\gamma^2}{\beta^2} 3u_1^1 + \beta^4 + \frac{1}{\gamma^4} - \frac{\gamma^2}{\beta^2} - \frac{1}{\beta} 2u_1^2 (2u_3^1 2u_2^1 + 2u_3^2 2u_2^2) \\
 & - \sum_{i=1}^3 \sum_{l=1}^2 3u_i^l (-1 - 1 - 1) (pq) + \dots
 \end{aligned}$$

→ flavor symmetry does not enhance here

→ no marginal operators

Genus g index from T_A :

$$\begin{aligned}
 I_{g,s} = & 1 + \left(\frac{2g-2+s}{\beta^2 \gamma^2} 3\gamma/\beta + 3g-3+s + (3\gamma/\beta + 1 + 1) \right) g \\
 & - \gamma/\beta - 1 - 1) pq + \left(\sum_{i=1}^3 ((\gamma/\beta^2 - 1) \beta u_i^{(i)} + \left(\frac{1}{\gamma \beta^2} - 1 \right) \beta u_i^{(i)}) \right) pq \\
 & + \left(\frac{2g-2+s}{t^2} \right) \beta^2 \gamma^2 + \frac{4g-4+2s}{t^2 \beta^2 \gamma^2} + \frac{3g-3+s}{t^2} 3\gamma/\beta + 2\gamma/\beta \sum_{i=1}^3 2u_i^{(i)} 2u_i^{(i)} \\
 & \cdot (pq)^{2/\beta}
 \end{aligned}$$

$$\rightarrow \dim M_{g,1,5}^{\mathfrak{su}(2)_{\text{diag}} U(1)^2}$$

$$= 3g - 3 + 5 + 3(g-1) + 2g$$

$(3g-3+5$ exactly marginal couplings,

additional $(3g/2 + 1 + 1)g$ marginal operators,

only $3(g-1) + 2g$ of these are exactly marginal)

$$= 3g - 3 + 5 + \dim(\mathfrak{su}(2)_{\text{diag}} U(1)^2)(g-1 + \frac{5}{2})$$

$$- \frac{5}{2} \dim(\mathfrak{su}(2)_{\text{diag}} U(1)^2) + \dim U(1)^2$$