## § 2.1 Superconformal Multiplets and Supersymmetric Deformations

{Qi, Qj}~0, i,j=1,...,NQ

Superconformal multiplet: QlV,

where 0414 lmax, lmax 4NQ

Long multiplets no null-states

Vis irreducible under Lovente- and R-sym.

at level  $l: (\Lambda^l R_Q) \otimes V$  (x)

where Ra is the Lorentz- and R-sym rep of Q

- -> unique top component: lmax = dim 2a = Na
- → Yoventz and R-syn. singlet dimension:  $\Delta_2 + \frac{1}{2}N_Q$
- · Racah-Speiser algorithm:
  - select highest-weight state Vh.w. EV with respect to L- and R-sym.
  - at each level l, consider all sequences of l supercharges acting an Vn.w.

Qi,Qi, ... Qie Vi.w.

add L- and R-sym, weights of Q; to

those of Vn.w.

-> set of RS trial weights was at level e

-> one-to-one correspondence between irreducible reps. of Ge) and RS trial weights was

- when V is too small, the bijection can fail

Example:

consider long multiplets in 3d N=1 SCFTs

-> R-sym. is trivial, L-sym. is su(2)

supercharges  $Q_{\chi}(x=1)$  transform as L-doublet notation:  $R_{Q} = [1]$ 

If L-rep. of V is  $[n] \longrightarrow n$ -index symmetric spinor  $V_{(x_1, \dots, x_n)}$  with  $x_{i=1,\dots,n} = t$ 

 $\ell=0$ :  $V_{n,\omega}=V_{++\cdots+},\qquad W_{RS}^{(0)}=\{[n]\}$ 

l = 1:  $Q_{+} V_{h.u.}, Q_{-} V_{h.u.}, \quad W_{RS}^{(1)} = \{[u+1], [u-1]\}$ 

 $\ell = 1$ :  $Q_{+}Q_{-}V_{h.w.}$ ,  $V_{RS}^{(2)} = \{[n]\}$ 

For n21, we have a match between wer and irreps of conformal primaries

For n=0 only [1] rep. appears at level 1 [-1] is removed by RS algorithm RS trial states do not coincide with true highest weight states of corr. rep.:

true highest weight at l=1 of [n-1] is

Q\_ V++...+ - Q+ V\_++...+ \$ Q\_V++...+

(RS algorithm by passes full Clebsch-Gordan problem)

 $Q - V_{+++---} = \frac{1}{2} \left[ \text{highest weight} \left( \left[ n-1 \right] \right) + \text{h.w.} \left( \left[ n+1 \right] \right) \right]$ 

-> Q\_ does not take us from [n] to [n-1] transition l -> l+1;

suppose 0 is conformal primary at level l Q6 -> 6' at level 41

6' < R<sub>0</sub> ⊗ O

However: 01 night not be in the image of Q!

- 1) due to Fermi-statistics or if multiplet has hull-states
- 2) O' occurs at level l+1, but transition does not occur!

example:

consider long multiplet in 2d N=4 SCFT

-> Q' transform in trifundamental [1](1)

of su(1) x su(2) 2 and su(2) L-sym. take VE [0](0;0) to be a singlet €[0](01.0) true highest weights:  $l=1: S = (Q_{+}^{++} Q_{-}^{--} + Q_{+}^{--} Q_{+}^{++} Q_{+}^{-+} Q_{+}^{-+} Q_{+}^{+-}) V$ l=3: O= Q++ S ∈ [1](1:1)  $\ell=4:$   $G'=Q_{+}^{++}Q_{+}^{+-}Q_{-}^{-+}Q_{-}^{++}V\in[2]^{(2;2)}$ Note that Q: 6 -> 0' does not occur because  $Q_{+}^{ff}O = (Q_{+}^{ff})^{2}S = 0$  by Ferm; statistics although  $[1]^{(1)} \in \mathbb{R}_{\mathbb{Q}} \otimes \mathcal{O} = [1]^{(1)} \otimes [1]^{(1)}$ short multiplets: possess mull states - must be removed from representation We have the following possibilities for top components:

D'Manifest top components:

conformal primary O with QG = descendant

i.e. no conformal primary at level 1+1

in tensor product 2000

examples: -all primarics at level lmax
- universal mass deformation in Id
discussed in last lecture

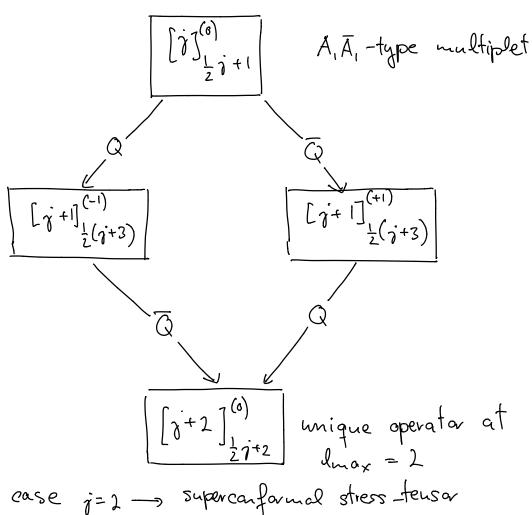
2) "Accidental top components":

there are conformal primaries at level It but they not in the image of Q level & We will focus on class )

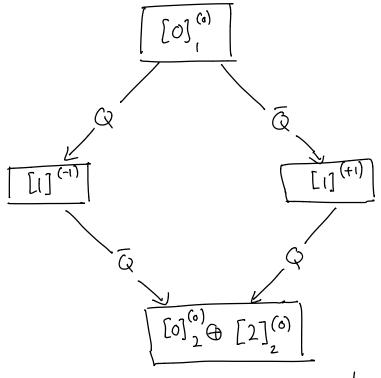
Consider a generic short multiplet in 3d W=2

Q, , Q, carry u(i) R charges -1 and +1

-> [1] (-1) \( \Pi \) [1] 1/2



example with two top components A, A, [0] (6).



[2] (6); conserved flavor current

[0](6): Zoventz-invariant flavor mass deformation