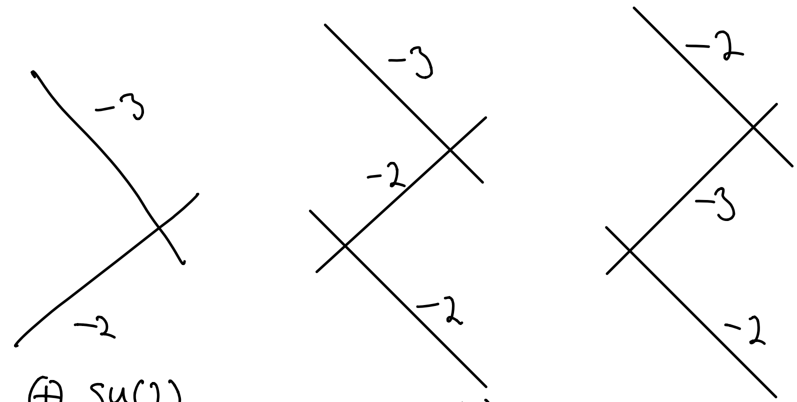
Non-Higgsable clusters

$\underline{\quad -m \quad}$

$su(3)$, $SO(8)$, f_4

e_6, e_7, e_8



$_2 \oplus su(2)$ $\quad$ $g_2 \oplus su(2)$ $\quad$ $su(2) \oplus so(7) \oplus su(2)$

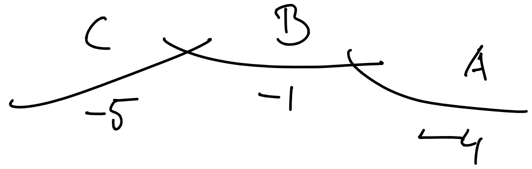
Table 2:

Diagram	Algebra	matter	(f, g, Δ)	ΔT_{max}
-3	$su(3)$	0	$(2,2,4)$	$1/3$
-4	$SO(8)$	0	$(2,3,6)$	1
-5	f_4	0	$(3,4,8)$	$16/9$
-6	e_6	0	$(3,4,8)$	$8/3$
-7	e_7	$\frac{1}{2}\,56$	$(3,5,9)$	$57/16$
-8	e_7	0	$(3,5,9)$	$9/2$
-12	e_8	0	$(4,5,10)$	$25/3$
$-3, -2$	$g_2 \oplus su(2)$	$(7+1, \frac{1}{2}2)$	$(2,3,6), (1,2,3)$	$3/8$
$-3, -2, -2$	$g_2 \oplus su(2)$	$(7+1, \frac{1}{2}2)$	$(2,3,6), (2,2,4), (1,1,2)$	$5/12$
$-2, -3, -2$	$su(2) \oplus so(7) \oplus su(2)$	$(1, 8, \frac{1}{2}2) + (\frac{1}{2}2, 8, 1)$	$(1,2,3), (2,4,6), (1,2,3)$	$1/2$

Connecting Clusters with (-1)-curves

Clusters can be connected by (-1)-curves

but not always:



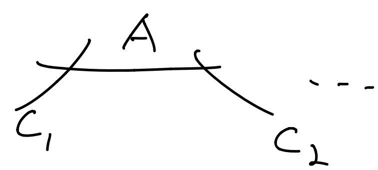
$\deg(f,g,\Delta)\big|_B = (3,4,8)$ $\quad \deg(f,g,\Delta)\big|_C = 2,2,4$

$\rightarrow \deg\big|_{C\cap B} \geq (4,6,12)$

$\rightarrow$ intersection point is too singular

For a (-1)-curve A we have:

$$-nK = aA + \sum_i c_i C_i + X$$



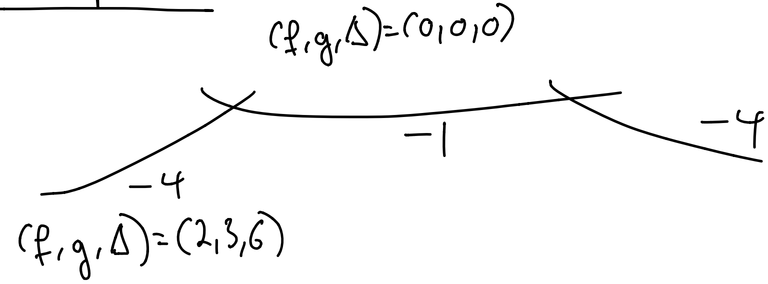
where $X\cdot A \geq 0$, $C_i \geq 0$

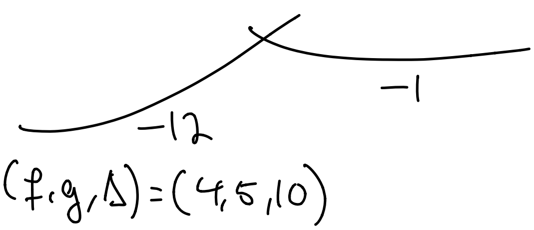
$$\Rightarrow -nK\cdot A \underset{\substack{\uparrow \\ \text{adj. formula}}}{=} n = -a + \sum_i c_i p_i + X\cdot A$$

$X\cdot A \geq 0 \rightarrow a \geq \sum_i c_i p_i - n$

For $\sum_i c_i p_i \leq n$ this condition can be satisfied with $a=0$ $\rightarrow$ (-1)-curve A does not affect gauge group + matter content of th.

Examples:





Bounding the number of tensors

- theories without vector multiplets

  $\rightarrow$ no irreducible divisors with $C\cdot C < -2$

  $\Rightarrow -K\cdot C = 2-m \geq 0$

bound: $T < 10$ (for trivial gauge group G)

proof:

$$-K = \sum_i l_i C_i \,, \quad l_i > 0$$

if $K\cdot K < 0 \rightarrow$ for some i : $-K\cdot C_i < 0$
$\rightarrow$ non-abelian gauge group factor

$$\Rightarrow K\cdot K = 9 - T \geq 0 \quad \text{so } T < 10$$

Also possible to understand from gravitational anomaly cancellation: $H - V = 273 - 29T$

- bound for given gauge algebra

example: consider $SO(8)$ gauge factor associated with (-4)-curve C

since $-12K = 6C + X$ and $-12K \cdot C = -24$

$$\longrightarrow 144K^2 = 36C^2 + X^2 = -144 + X^2$$

where we used $X \cdot C = 0$

since $X^2 \geq 0$ $(G = SO(8)) \rightarrow K^2 > -1$

$$\Rightarrow \mathfrak{g} = SO(8) \rightarrow T \leq 10$$

$\rightarrow$ each additional $SO(8)$ summand raises bound on T by 1.

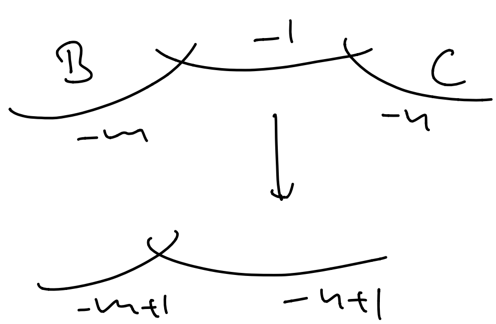
see table 2 for other NHC's.

<u>Bounds on linear chains of curves</u>

Consider linear chains of divisors C_i with $C_i \cdot C_{i+1} = 1$

example: $(---, -4, -1, -4, -1, -4, \cdots)$

blow-down:



$$(\cdots, -4, -1, -4, -1, \cdots)$$

$$\downarrow$$

$$(---, -2, -2, -2, \cdots)$$

$\rightarrow$ 3 properties:

1) Each link allowed in F-th

2) locally maximal, self-intersections cannot be increased without violating property 1)

3) blow-down of all (-1)-curves possible
$\rightarrow$ removes all non-abelian gauge groups

F-theory consistency $\rightarrow$ blow-down all the way to $\mathbb{F}_m$ or $\mathbb{P}^2$

start with a $(-1,-4,-1,\dots,-1,-4,-1)$-configuration with N (-4)-curves

$\rightarrow$ order $\sim \underset{(-1)}{N} + \underset{(-2)}{N} = 2N$ blow-downs

necessary

$\rightarrow$ $T \sim 2N \leq 9 + N$

$\rightarrow$ $N \sim 9$

3 other possibilities:

x_6: $(\dots, -6, -1, -3, -1, -6, -1, -3, \dots)$

x_8: $(\dots, -8, -1, -2, -3, -2, -1, -8, -1, -2, -3, -2, \dots)$

x_{12}: $(\dots, -12, -1, -2, -2, -3, -1, -5, -1, -3, -2, -2, -1, -12$

For X_6, the gauge algebra is

$$N(e_6 \oplus su(3)) \quad \text{or} \quad (N \pm 1)e_6 \oplus N(su(3))$$

From table 2 $\Delta T(e_6) = \frac{8}{3}$, $\Delta T(su(3)) = \frac{1}{3}$

$\rightarrow \Delta T = 3N$

$$(\underbrace{-\overset{1}{6}, -1, -3, -1}, \cdots, \underbrace{-\overset{N}{6}, -1, -3, -1})$$

$\xrightarrow{\text{3 blow-down's}}$

$$\underset{1}{(-2)} \cdots \underset{N}{(-2)}$$

$\rightarrow T \sim \mathcal{O}(4N) \rightarrow T \sim 4N \leq 3N + 9$

$\rightarrow N \sim \mathcal{O}(9)$

X_8: $e_7 \oplus (su(2) \oplus so(7) + su(2)) \longrightarrow \Delta T = 5N$

$\rightarrow T \sim 6N \leq 5N + 9 \rightarrow N \sim \mathcal{O}(9)$

X_{12}: $e_8 \oplus f_4 \oplus 2(g_2 \oplus su(2)) \rightarrow \Delta T = 10\,\frac{17}{18}$

$\xrightarrow{\text{11 blow-downs}}$ (-2)-curve

$\rightarrow \mathfrak{g} = (N+1)e_8 \oplus N(f_4) \oplus 2N(g_2 \oplus su(2))$

$\rightarrow T \leq 9 + N \times \frac{197}{18} + \frac{25}{3}$ ← from e_8 at end

$12 N_{max} \cong \frac{52}{3} + \frac{197}{18} N_{max} \Rightarrow N < N_{max} = 16.4$

$\rightarrow N = 16$, $T = 193$ largest known T !