§1.2 6d (1,0) SCFT's

Relevant SCA: $osp(2,6|2)$ "(1,0) algebra"

bosonic subgroups: $SO(2,6)$ and $Sp(1) \cong SU(2)$

"conformal symmetry" "R-symmetry"

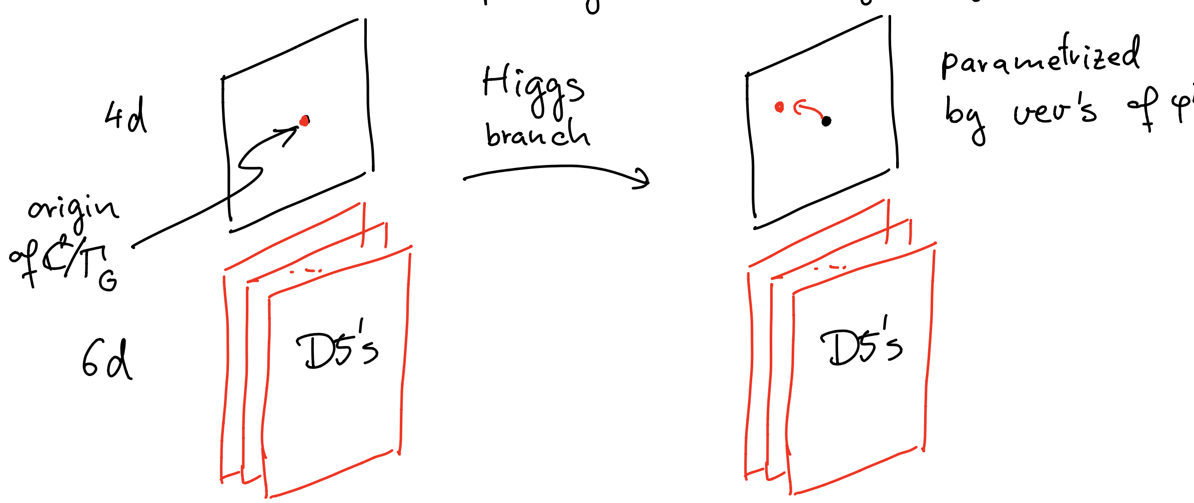
field content:

- Hypermultiplet: $\varphi^{1,\dots,4}$ + fermions
- Vectormultiplet: A_μ + fermions
- tensor multiplet: $B^+_{\mu\nu}$, ϕ + fermions

tensor branch: $\langle\phi\rangle > 0$

Example:

Consider N D5 branes probing an ADE singularity in type IIB



tensor branch: blow-up of ADE singularity

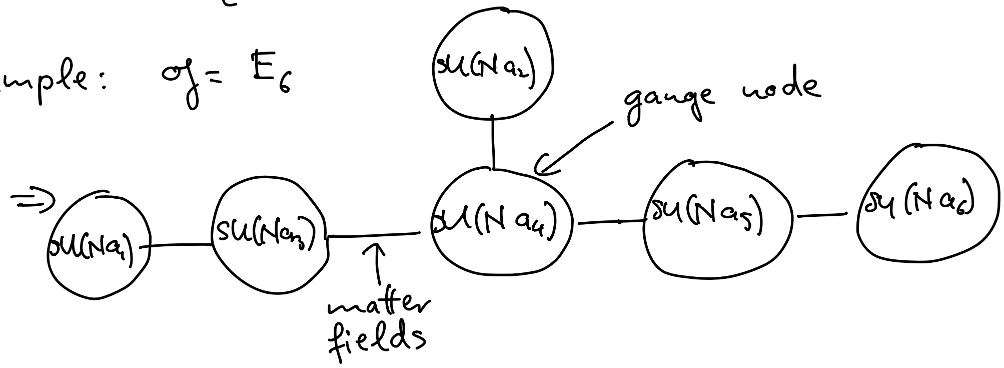
$\longrightarrow$ r (rank of $\mathfrak{g}$) tensor multiplets

VM's give rise to gauge group $\prod_{i=1}^{r} SU(Na_i)$

with matter multiplets in reps. $\frac{1}{2}\sum_{\mu,\nu} a_{\mu\nu}(\square_\mu, \overline{\square}_\nu)$

where $a_{\mu\nu} = \begin{cases} 1 & \text{if } \mu \text{ and } \nu \text{ are linked in Dynkin diag} \\ 0 & \text{otherwise} \end{cases}$

Example: $\mathfrak{g} = E_6$



$$\Rightarrow \quad a_{\mu\nu} = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \qquad \text{"adjacency matrix"}$$

<u>effective action</u>:

$$2\pi \int \eta^{ij} \Big(-\tfrac{1}{2} d\phi_i \wedge * d\phi_j - \tfrac{1}{2} dB_i \wedge * dB_j + \phi_i \big(\tfrac{1}{4} \mathrm{Tr}\, F_j \wedge * F_j \big) + B_i \big(\tfrac{1}{4} \mathrm{Tr}\, F_j \wedge F_j \big) \Big) \qquad (*)$$

due to anomaly cancellation

where $\eta^{ij} = \mathrm{tr}(H^i H^j)$ with H^i being the Cartan generators of $\mathfrak{g} \to \eta^{ij}$ is Cartan matrix of $\mathfrak{g}$

- anomaly cancellation:

  I_8 should be representable as

  $$I_8 = \frac{1}{2} \Omega_{ij} X^i X^j$$

  with $X^i = b^i_a \, \mathrm{tr}\, F_a^2$ (summation over a implied)

  (closed and gauge invariant)

  cancellation due to "Green–Schwarz" mechanism:

  $$A = \int_{\mathbb{R}^{5,1}} \Omega_{ij} B^i X^j$$

where $H^i = dB^i + b^i_a\, \omega^a_{3Y}$

$\uparrow$ Yang-Mills CS-form

$$\longrightarrow \delta_\Lambda (A+S) = \int \Omega_{ij} (\delta_\Lambda B^i) X^j + \int I_6^1(\Lambda) = 0$$

$\uparrow$ from descent formalism

- supersymmetry: the term $\phi_i \left(\frac{1}{4} \mathrm{Tr}\, F_j \wedge * F_j\right)$ in the action (*) is due to $\mathcal{N}=(1,0)$ supersymmetry (ϕ_i and B_i are in the same multiplet)

<u>dimensional reduction:</u>

$$6d: \quad \phi_i, B_i$$

$$\downarrow S^1_R$$

$$5d: \quad \Phi_i = 2\pi R \phi_i, \quad A_{i,\mu} = 2\pi R B_{i,5}$$

$$S \longrightarrow \int \eta^{ij} \Big(-\frac{1}{2R} \left(d\Phi_i \wedge * d\Phi_j + dA_i \wedge * dA_j \right) + 2\pi\, \Phi_i \left(\frac{1}{4} \mathrm{Tr}\, F_j \wedge * F_j \right) + 2\pi A_i \left(\frac{1}{4} \mathrm{Tr}\, F_j \wedge F_j \right) \Big)$$

<u>Compactification on T^2 to 4d:</u>

$\rightarrow$ 4d conformal quiver gauge theory

$$\rho_i = \mathrm{Vol}(T^2)(i\phi_i + B^i_{45})$$

running of gauge couplings

$$\beta_i = 2\pi i \frac{d\rho_i}{d \log \Lambda} = -2N_i + \sum_{e:\, t(e)=i} N_{s(e)} + \sum_{e:\, s(e)=i} N_{t(e)}$$

Λ: energy scale $\qquad$ N_i: rank of node i

e: edge of quiver

$t(e)$: target node

$s(e)$: source node

$$\Rightarrow \beta_i = \sum_j (-2\delta_{ij} + a_{ij}) N_j = -\sum_j C_{ij} N_j$$

Cartan matrix

demand $\beta_i = 0 \ \forall i$ (conformal invariance)

$\Rightarrow N_j = N a_j$, where a_j are Dynkin indices of node i

for example: for E_6 we have $\vec{a} = \begin{pmatrix} 1 \\ 2 \\ 2 \\ 3 \\ 2 \\ 1 \end{pmatrix}$

<u>What happens at the SCFT fixed point?</u>

Let us rewrite the 5d action as follows:

introduce $\Phi_G = 2\pi H^i \Phi_i$, $A_G = 2\pi H^i A_i$

$$\rightarrow S = \int \Big(-\frac{1}{g_G^2} \mathrm{tr}(d\Phi_G \wedge * d\Phi_G + F_G \wedge * F_G) + \mathrm{tr}(H^j \Phi_G)\left(\tfrac{1}{4}\mathrm{Tr} F_j \wedge * F_j\right) + \mathrm{tr}(H^j A_G)\left(\tfrac{1}{4}\mathrm{Tr} F_j \wedge F_j\right)\Big)$$

where $8\pi^2/g_G^2 = R^{-1}$

where on tensor branch $\langle \Phi_G \rangle \neq 0$

at origin of t.b. $\Phi_G = 0$

$\rightarrow$ expect gauge group G to be restored

$\rightarrow$ 5d SCFT $S^{5d}\{G\}$ coupled to G gauge field

flavor sym.

then $S^{5d}\{G\} \xrightarrow{\text{mass def.}}$ 5d quiver th.

mass def.: $\langle \Phi_G \rangle = m_G$ hypermultiplets get mass

$$\rightarrow \frac{8\pi^2}{g_i^2} = \mathrm{tr}(H^i m_G)$$

§ 2. Classification of 6d (1,0) SCFT's

We seek a geometric classification

$\longrightarrow$ given by F-theory (12d) on elliptic CY_3

§ 2.1 Introduction to F-theory

What is F-theory?

typ IIB supergravity has $\mathcal{N}=2$ SUSY in 10d
(32 SUSY gen.)

bosonic fields:

$\tau := C_0 + ie^{-\phi}$, ϕ is the dilaton $(\underset{\substack{\uparrow \\ \text{string coupling constant}}}{g_{IIB}} = e^{\phi})$

$G_3 := F_3 - \tau H_3$

$\tilde{F}_5 := F_5 - \frac{1}{2} C_2 \wedge H_3 + \frac{1}{2} B_2 \wedge F_3$

$F_p := dC_{p-1} \quad (p=1,3,5), \quad H_3 := dB_2$

action:

$$S_{IIB} = \frac{2\pi}{l_s^8} \Big[\int d^{10}x \sqrt{-g}\, R - \frac{1}{2} \int \frac{1}{(\mathrm{Im}\tau)^2} d\tau \wedge * d\bar{\tau} \qquad (1)$$

$$+ \frac{1}{\mathrm{Im}\tau} G_3 \wedge * \bar{G}_3 + \frac{1}{2} \tilde{F}_5 \wedge * \tilde{F}_5 + C_4 \wedge H_3 \wedge F_3 \Big]$$

l_s is string length

supplemented with the self-duality constraint

$* \tilde{F}_5 = \tilde{F}_5$ after varying action

$l_s = 2\pi \sqrt{\alpha'}$

tension of D_p-branes : $T_{D_p} = \frac{2\pi}{l_s^{p+1}}$

Action (1) is invariant under:

$$\tau \longrightarrow \frac{a\tau + b}{c\tau + d} \qquad \tilde{F}_5 \longrightarrow \tilde{F}_5 \qquad (2)$$

$$\begin{pmatrix} H \\ F \end{pmatrix} \longrightarrow \begin{pmatrix} d & c \\ b & a \end{pmatrix} \begin{pmatrix} H \\ F \end{pmatrix} \qquad \underset{\substack{\uparrow \\ \text{metric}}}{g_{MN}} \longrightarrow g_{MN}$$

$\to$ as if obtained from "12d theory" on $T^2 = S^1_A \times S^1_B$
with compl. str. τ and

$$\begin{array}{ccc} & 12d\ \tilde{F}_4 & \\ \swarrow^{S^1_A} & & \searrow^{S^1_B} \\ F_3 & & F_4 \end{array}$$

$\to$ $SL(2,\mathbb{Z})$ symmetry (2) becomes symmetry of T^2.

$\to$ F-theory as 12 theory

Some hurdles:

- no (1,11)d supergravity theory
- where is the size modulus of T^2?

Solution:

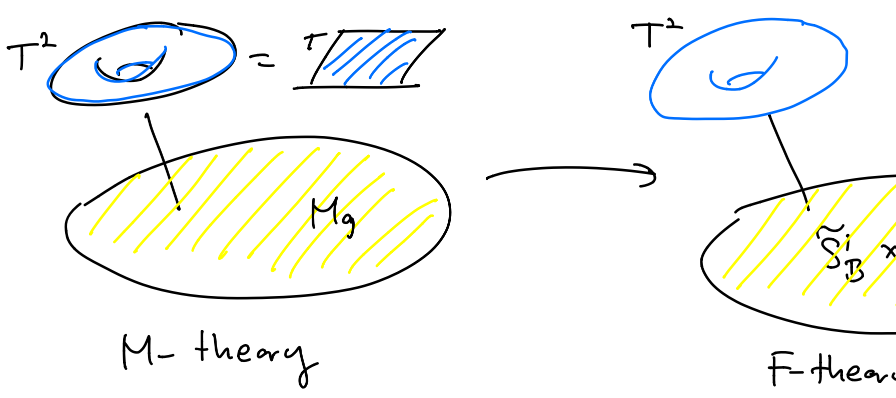
start with M-theory (11d)

$$\begin{array}{l} \text{M-theory (11d)} \\ \quad \Big\downarrow\ T^2 = S^1_A \times S^1_B,\ R_A \to 0 \\ \text{type IIA on } \mathbb{R}^9 \times S^1_B \\ \quad \Big\downarrow\ \text{T-duality along } S^1_B \\ \text{type IIB on } \mathbb{R}^9 \times \tilde{S}^1_B \\ \\ \text{typ IIB on } \mathbb{R}^{10} \end{array}$$

$R_A, R_B \to 0$

$\to$ can be done fiber-wise

$\to$ extend the procedure to varying τ



$Vol(T^2) = \frac{1}{\tilde{R}_B}$

Take M_9 to be $\mathbb{R}^5 \times B_4$

$\to$ to preserve SUSY need $T^2 \hookrightarrow X$ to be CY

$$\begin{array}{c} T^2 \hookrightarrow X \\ \downarrow \\ B_4 \end{array}$$

<u>Example:</u> K3

$B_4 = \mathbb{P}^1 \times \mathbb{R}^2$, i.e. F-theory on $\mathbb{R}^{1,7} \times K3$

K3 eq. : $y^2 = x^3 + f(u,v) x z^4 + g(u,v) z^6$ $\quad (*)$

where $x, y, z, u, v \in \mathbb{C} / \sim$

$$(u, v, x, y, z) \sim (\lambda u, \lambda v, \lambda^4 x, \lambda^6 y, z)$$
$$\sim (u, v, \mu^2 x, \mu^3 y, \mu z)$$

where $\mu, \lambda \in \mathbb{C}^* = \mathbb{C} \setminus \{0\}$, and $(u,v) \neq (0,0)$, $(x,y,z) \neq (0,0,0)$

$f(\lambda u, \lambda v) = \lambda^8 f(u,v)$,

$g(\lambda u, \lambda v) = \lambda^{12} g(u,v)$

consistent

<u>rule:</u> deg of $(*)$ = sum of weights

λ: $12 = 1+1+4+6+0$

μ: $6 = 0+0+2+3+1$

$\to$ total space is Calabi-Yau

projection $\pi : K3 \to \mathbb{P}^1 : (x,y,z,u,v) \mapsto (u,v)$

$\mathbb{P}^1 = \{(u,v) \neq (0,0) \mid (u,v) \simeq (\lambda u, \lambda v)\}$

at fixed $z \equiv 1$, $v \equiv 1$, we get

$(*) \to y^2 = x^3 + f(u) x + g(u) \quad (**)$

$\to$ equation of elliptic curve in (x,y,z) space

deg $= 6$ $=$ sum of weights $= 2+3+1 = 6$

Relation to $T^2 = \mathbb{C}/(\mathbb{Z}+\tau\mathbb{Z})$

holomorphic coordinate: $z = x + \tau y$

for $P \in T^2$: $z(P) = \int_0^P \Omega_1$, $\Omega_1 = dz$

For T^2 descriped by $(**)$,

$$\Omega_1 = \frac{c\,dx}{y}, \qquad c = \text{const.}$$

then
$$\tau = \frac{\oint_B \Omega_1}{\oint_A \Omega_1}$$

$\rightarrow$ ambiguity in basis choice is $SL(2,\mathbb{Z})$

How to compute τ?

Use j-invariant!

$$j(\tau) = \frac{4\cdot(24f)^3}{\Delta}, \qquad \Delta = 27g^2 + 4f^3$$

and $j(\tau)$ is $SL(2,\mathbb{Z})$ modular invariant

$$j(\tau) = e^{-2\pi i\tau} + 744 + \mathcal{O}(e^{2\pi i\tau})$$

$\Delta = 0$ is "discriminant locus"

$\rightarrow \deg(\Delta) = 24 \rightarrow 24$ zero's on $\mathbb{P}^1 \rightarrow$ denote by u_i, $i = 1, \ldots, 24$

We have: $j(\tau(u)) \sim \frac{1}{u - u_i}$ near u_i

$\rightarrow \tau(u) \simeq \frac{1}{2\pi i}\ln(u - u_i)$

for $u \rightarrow u_i$: $\tau \rightarrow i\infty$ (ratio of A- and B-cycle of T^2 vanishes)

Since $\tau = C_0 + \frac{i}{g_{IIB}}$, this is "weak coupling" limit:

$g_{IIB} \rightarrow 0$