§4. Compactifications to 4d

6d $\mathcal{N}=(1,0)$ SCFT's

$$\Big\downarrow \Sigma_g \text{ Riemann surface}$$

4d $\mathcal{N}=1$ SCFT

after shrinking area of Σ_g to zero

6d SCFT's can have global symmetries: $\mathcal{T}_{6d}[G]$

$\longrightarrow$ turn on gauge background along Σ_g : $F_{\mu\nu}$

SUSY condition: $\underset{\substack{\uparrow \\ \text{spinor}}}{\delta\lambda} = F_{\mu\nu}\gamma^{\mu\nu}\cdot\varepsilon + \underset{\substack{\uparrow \\ \text{susy variation}}}{D\varepsilon} = 0$

2 possibilities:

1) D-term breaks G to abelian subgroup

2) $D=0 \longrightarrow F=0$ flat bundle on Σ_g

1) Assign G holonomies for each A_i, B_j cycles of Σ_g with : $\prod_{i=1}^{g}[A_i, B_i] = 1$ "gluing condition"

$\longrightarrow$ $(g-1)\dim_{\mathbb{C}} G$ dimensional moduli space

2) $F_{z\bar{z}}\,\bar{\varepsilon}\,\gamma^{z\bar{z}}\,\varepsilon + D = F_{z\bar{z}}\,\varepsilon^{z\bar{z}} + D = 0$

along Riemann surface

$\rightarrow$ solved by $F_{z\bar{z}} = \text{const.}\, \varepsilon_{z\bar{z}}$

allowed constants are quantized as:

$$\frac{1}{2\pi}\int_{\Sigma} F = c_1(F) \in \mathbb{Z}$$

can add $g-1$ addition flat connections

$\rightarrow$ for each $n \in \mathbb{Z}$ and each $u(1) \subset G$

get g complex moduli

More general possibility:

fix L abelian subgroup of G

$\rightarrow$ commutant: $L \times G' \subset G$,

fix $c_1(L) \in \mathbb{Z}^{\dim(L)} \rightarrow g \cdot \dim(L)$ abelian moduli

$+ (g-1)\dim(G')$ non-abelian moduli

expected dimension of 4d $\mathcal{N}=1$ conformal manifold $\mathcal{M}_g$:

$$\dim \mathcal{M}_g = (3g-3) + g\cdot\dim(L) + (g-1)\dim(G')$$

Example 1:

6d (2,0) SCFT with $SO(5)_R$ R-symmetry

We have $SU(2)_L \times SU(2)_R = SO(4) \subset SO(5)$

$$\begin{array}{c} SU(2)_R \\ \cup \\ SO(2)_R \text{ of 4d } \mathcal{N}=1 \\ \shortparallel \\ t_1 + t_2 \\ SO(2)\times SO(2) \subset SO(4) \end{array}$$

$\rightarrow$ flavor symmetry $G = SU(2)_L$

1) turn on flat $SU(2)_L$ bundle on Σ_g:

$\dim \mathcal{M}_g = (3g-3) + (g-1)\cdot 3 = 6g-6$

2) Choose abelian subgroup $L \subset G$: $U(1) \subset SU(2)_L$

$\rightarrow$ turn on flux characterized by integer $c_1(L) = n$

$\rightarrow \dim \mathcal{M}_g = (3g-3) + g = 4g-3$

Example 2:

Consider E-string $\mathcal{N}=(1,0)$ SCFT in 6d

$\rightarrow$ global symmetry: $G = E_8$

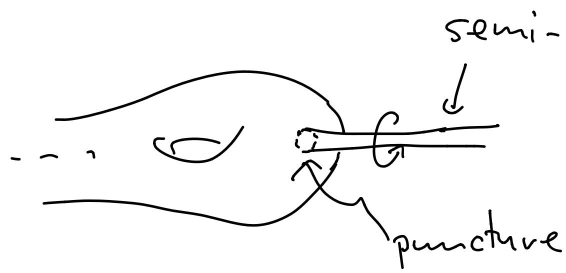
1) choose $G' = G = E_8 \rightarrow$ moduli space of flat E_8 bundles on Σ_g

2) or abelian subgroup $L \subset E_8$
with $\dim L = 1, \dots, 8$
$\to$ numerous possibilities for each dimension

Contrast this with $\mathcal{O}(-12)$ SCFT
$\to$ no global symmetries : $G = 0$
$\to$ unique choice

<u>Adding punctures:</u>

s punctures $\longrightarrow$ positions add s complex moduli



$\big\downarrow$ reduction on circle + of holonomy for G

5d theory
+ mass-parameters
$\to$ G is broken to $P \subset G$
inequivalent choices : G/P
$\to$ each puncture adds complex dim
$\frac{1}{2} \dim (G/P)$ moduli
holonomies in the bulk: $P^{max} = G^{max} \cap P$

where $G^{max} = L \times G'$

$\rightarrow$ dimension of 4d conformal manifold:

$$\dim \mathcal{M}_g = (3g-3+s) + (g-1)\dim(G^{max}) + \dim(L) + \frac{1}{2}\sum_i \dim\left(\frac{G^{max}}{P_i^{max}}\right)$$

$$= 3g-3+s + \left(g-1+\frac{s}{2}\right)\dim(G^{max}) + \dim(L) - \frac{1}{2}\sum_i \dim(P_i^{max})$$

<u>M5 branes probing ADE compactified on Σ_g</u>

N M5 branes probing $\mathbb{C}^2/\Gamma_K$

$\rightarrow$ resulting theory has $G = K_L \times K_R$ global symmetry

focus on $K = SU(k)$

For $N=1$: 6d SCFT is hyper in $(k,\bar{k})_{+1}$ representation of $SU(k)_L \times SU(k)_R \times U(1)$

Can turn on non-abelian bundles for $SU(k)_L \times SU(k)_R$ along cycles of Σ_g and abelian bundles from $U(1)$