§2.2 Classification of 6d (1,0) SCFT's

Example 1:

6d (2,0) SCFT's have simple classification in F-theory:

$$CY = B \times T^2, \quad \text{where } B = \mathbb{C}^2/\Gamma$$
$$\Gamma \subset SU(2) \text{ discrete}$$

$\longrightarrow$ resolution gives (−2)-curves intersecting according to ADE type

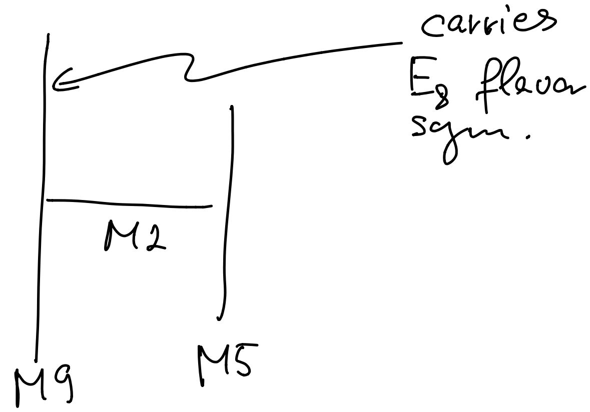
Example 2:

E-string theory is 6d (1,0) SCFT with E_8 flavor symmetry:

F-th.:

$$\begin{tikzcd} T^2 \arrow[r, hook] & CY \arrow[d] \\ \mathcal{O}(-1) \arrow[r, hook] & B \arrow[d] \\ & \mathbb{P}^1 \end{tikzcd}$$

M-th.:



"decompactify fiber of $\mathbb{F}_1$"

Conformal fixed point: shrink $\mathbb{P}^1$ to zero

Similarly, taking $\mathcal{B}$ to be

$$\begin{tikzcd} \mathcal{O}(-n) \arrow[r] & \mathcal{B} \arrow[d] \\ & \mathbb{P}^1 \end{tikzcd} \qquad n = 1,2,3,4,5,6,7,8,12$$

gives "minimal 6d SCFT's"

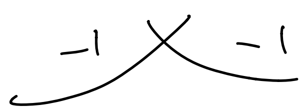
<u>Building blocks of 6d SCFT's</u>

introduce curves Σ_i with negative self-intersection and "adjacency matrix":

$$A_{ij} = -(\Sigma_i \cap E_j)$$

all Σ_i contractible $\longrightarrow$ A_{ij} positive definite

consider counter example:

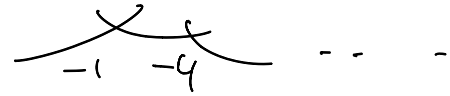


$$\longrightarrow A_{ij} = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

$$\mathrm{eig}(A) = (2,0) \quad ↯$$

another property from non-Higgsable clusters:

"no loops"



tree-like

$\xrightarrow{\text{blow-down}}$ $\mathbb{C}^2/\Gamma$, $\Gamma \subset U(2)$ discrete consider $C = x_1 \cdots x_r, \quad x_i = -(\Sigma_i \cap \Sigma_i)$

blow-down gives orbifold $\mathbb{C}^2/\Gamma$:

$$(z_1, z_2) \mapsto (\omega z_1, \omega^q z_2)$$

where $\omega = e^{2\pi i/p}$ and $\frac{p}{q} = x_1 - \cfrac{1}{x_2 \cdots \frac{1}{x_r}}$

For $-n$ theories:

$$(z_1, z_2) \longmapsto (\omega z_1, \omega z_2) \text{ for } \omega = \exp(2\pi i/n)$$

For clusters with >1 curves:

cluster:	3,2	3,2,2	2,3,2
p/q	$5/2$	$7/3$	$8/5 = 2 - \cfrac{1}{3 - \frac{1}{2}}$

elliptic curve T^2 is non-trivially fibered over $\mathbb{C}^2/\Gamma$ $\longrightarrow$ CY geometry

<u>example</u>: $(\mathbb{C}^2 \times T^2)/\mathbb{Z}_n$

holomorphic 3-form $\Omega = dz_1 \wedge dz_2 \wedge \lambda$

hol. 1-form on T^2

$$(z_1, z_2, \lambda) \mapsto (\omega z_1, \omega z_2, \omega^{-2}\lambda)$$

ω^{-2} need to be of order 1, 2, 3, 4, 6 ($n=5, 7$ not of this type!)

Some examples:

$C = 3\,1\,3$

$$\rightarrow \quad A_C = \begin{pmatrix} 3 & -1 & 0 \\ -1 & 1 & -1 \\ 0 & -1 & 3 \end{pmatrix} \rightarrow \text{positive definite}$$

on the other hand $C = 13131$ is not!

$$13131 \xrightarrow{\text{blow-down}} 1221 \rightarrow 11 \quad ↯$$

$\rightarrow$ not all allowed configurations are simultaneously contractible!

Classification of SCFT bases:

blow-down: $[\Sigma] \rightarrow [\Sigma] + \underset{(-1)\text{-curve}}{[E]} = [\Sigma_{new}]$

$$\Rightarrow [\Sigma_{new}]\cdot[\Sigma_{new}] = \underbrace{[\Sigma]\cdot[\Sigma]}_{=-(n-1)} + 2[\Sigma]\cdot[E] + [E]\cdot[E]$$

$$C \xrightarrow{\text{blow-down}} C' \rightarrow \cdots \rightarrow C_{end}$$

$\rightarrow$ "minimal SCFT's" (for example $B_{end} = \mathbb{C}^2$ and NHC's)

Want to show: $\mathcal{B}_{end} = \mathbb{C}^2/\Gamma$,

$\Gamma \subset U(2)$ discrete and of the form

1) $A(x_1, \dots, x_r)$ for $C_{end} = x_1 \cdots x_r$

2) $D(y|x_1, \dots, x_e)$ for $C_{end} = $

	2	
2	y	$x_1 \cdots x_e$

$A(x_1, \dots, x_r)$ is cyclic of order p with generators: $(z_1, z_2) \mapsto (\omega z_1, \omega^q z_2)$, $\omega = e^{2\pi i/p}$

$$p/q = x_1 - \cfrac{1}{x_2 - \cdots \frac{1}{x_r}}$$

$D(y|x_1, \dots, x_e)$ is generated by cyclic group

$$A(x_e, \dots, x_1, 2y-2, x_1, \dots, x_e)$$

<u>and</u> Λ of order 4: $(z_1, z_2) \mapsto (z_2, -z_1)$:

$$D(y|x_1, \dots, x_e) \simeq \langle \Lambda, A(x_e, \dots, x_1, 2y-2, x_1, \dots, x_e) \rangle$$

<u>Algorithm for minimal resolution:</u>

1) Check all pairs of neighbors $x_i x_{i+1}$
if $\not\subset$ NHC's $\xrightarrow{\text{blow-up}} x_i^{(1)} | x_{i+1}^{(1)}$
iterate through entire graph

2) Check gauging condition $\mathfrak{g} \oplus \mathfrak{g}' \subset \mathfrak{e}_8$

if violated blow-up

3) Keep repeating until configuration of NHC's connected by (−1)-curves is reached.

example 1: $C_{end} = 33$

1) violated $\Big\downarrow$ blow-up

414

2) satisfied: $so(8) \oplus so(8) \subset e_8$

$\downarrow$

stop

example 2: $C_{end} = 44$

1) violated $\Big\downarrow$ blow-up

515

2) violated: $f_4 \oplus f_4 \not\subset e_8$

$\downarrow$ blow-up

6125

1) violated $\Big\downarrow$ blow-up

61316

1) satisfied : NHC's connected by (-1)'s

2) satisfied : $e_6 \oplus su(3) \subset e_8$

$\Big\downarrow$

stop