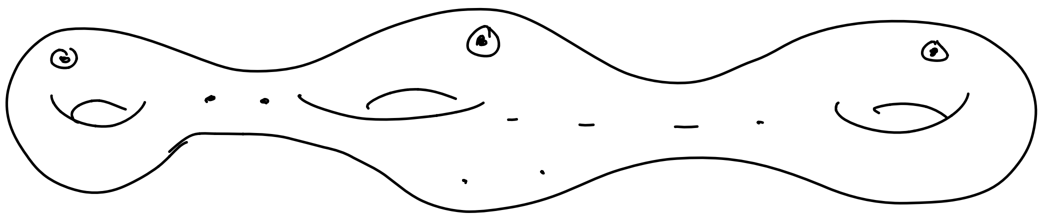
Want to look at compactifications of $k=N=2$ theory on Riemann surfaces $\Sigma_{g,s}$:



Punctures and gluings

maximal punctures

$\rightarrow SU(N)^2$ factor into 4d global symmetry

have color, sign, and orientation

color: group unbroken by puncture

$$\rightarrow SU(2)_{diag}\, U(1)^2 \subset SO(7)$$
$$\cap \qquad \checkmark$$
$$SO(5)\times U(1)$$

$\rightarrow$ 3 different choices : $U(1)_t,\ U(1)_\beta,\ U(1)_\gamma$

restrict to $SO(5)\times U(1)_t \subset SO(7)$

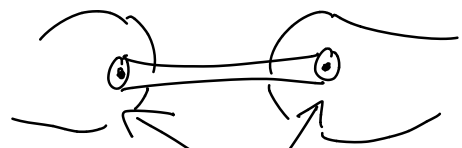
$$\cup$$
$$SU(2)\times SU(2) \rightarrow 2 \text{ choices}$$

$$\rightarrow \quad P_1 = SU(2)_{\gamma/\beta}\times U(1)_{\beta\gamma}\times U(1)_t$$
$$P_2 = SU(2)_{\beta\gamma}\times U(1)_{\beta/\gamma}\times U(1)_t$$

orientation: ordering of two $SU(N)$ factors
$(SU(2)_a, SU(2)_b)$ or $(SU(2)_b, SU(2)_a)$

sign : two different embeddings
$U(1)_t \subset SO(7)$ (related by cplx conjugation)

gluing:

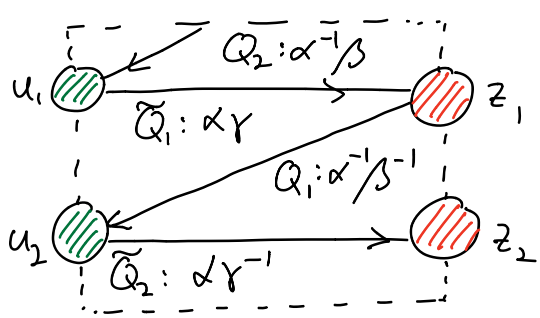


opposite orientation
same color

- same sign:
  use Φ-gluing : $W = \Phi \cdot M - M' \cdot \Phi$
- opposite sign:
  Use S-gluing: $W = M \cdot M'$

<u>Closing a maximal puncture</u>:

give vev to meson operators!



$M_i := Q_i \tilde{Q}_i$ has charges $u_1^{\pm 1} u_2^{\pm 1} t \left(\frac{\beta}{\gamma}\right)^{\pm 1}$

$\langle M_i \rangle \neq 0 \longrightarrow$ breaks $SU(2)_{u_1} \times SU(2)_{u_2}$ to $U(1)_\delta$ subgroup

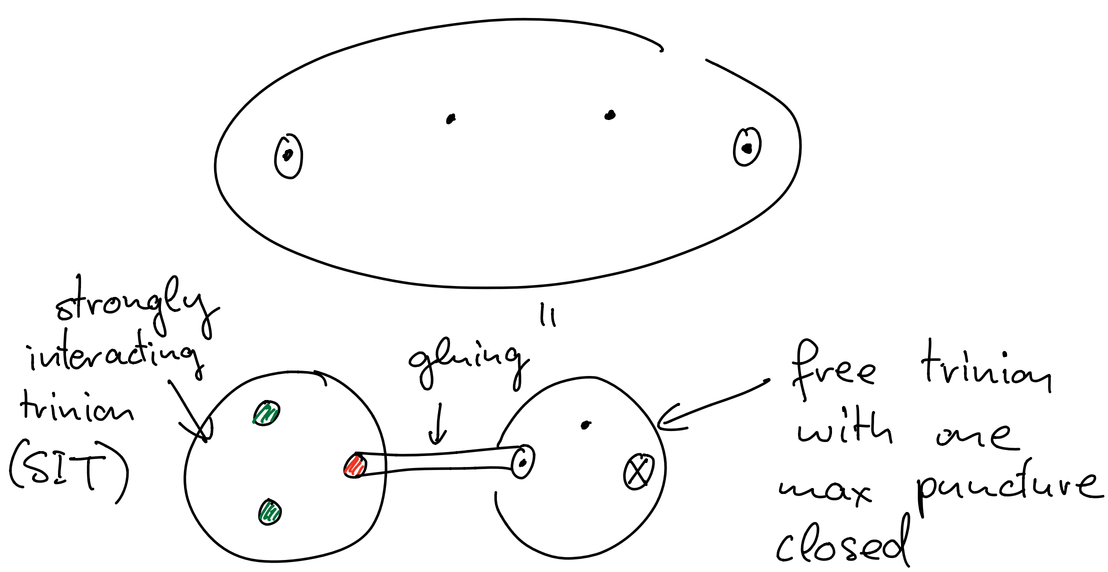
consider first $\langle Q_1 \tilde{Q}_1 \rangle \neq 0$ with $(u_1 u_2)^{-1} t \frac{\gamma}{\beta}$

set $(u_1, u_2) = \left(t^{\frac{1}{2}} \frac{\gamma}{\delta}, t^{\frac{1}{2}} \frac{\delta}{\beta}\right)$ with δ the fugacity of a $U(1)_\delta$

and $z_1 = \alpha \delta$ (Q_1 has charge z_1, $\tilde{Q}_1$ has charge z_1^{-1})

$\longrightarrow$ $SU(2)_{z_1}$ is Higgsed and only $SU(2)_{z_2}$ is left

reinterpret 4-punctured sphere:

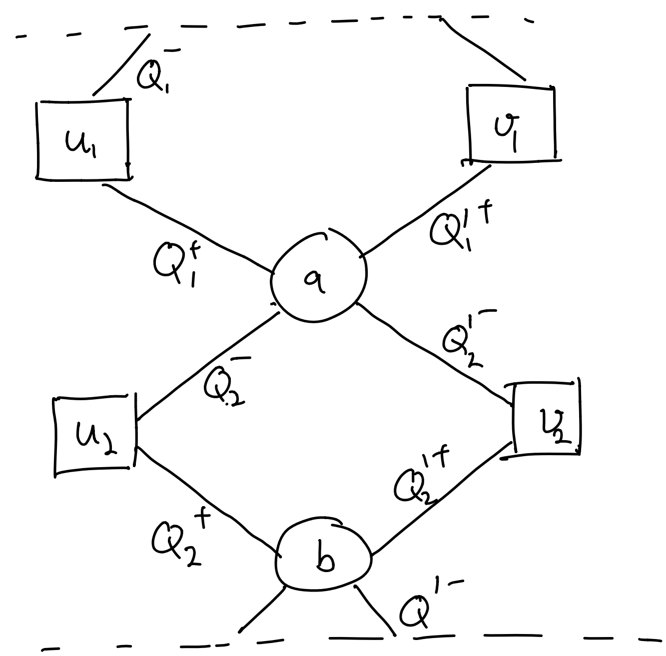


SIT can be used as building block to assemble arbitrary genus g Riemann surface!

The $G^{max} = SO(7)$ models

reduce 6d (1,0) T_K^N with $N = K = 2$ on Riemann surface with <u>no</u> SO(7) flux

Consider 2 free trinions glued using S-gluing:



→ turn on quartic superpotential couplings

→ gives conformal manifold with same superconformal R-symmetries and conformal anomalies

global symmetry on generic points:

$$SU(2)^2 \times SU(2)^2 \times U(1)_t \times U(1)_\alpha \times U(1)_\delta \times U(1)_\gamma \times U(1)_\beta$$

$U(1)_{\alpha_1}$ $U(1)_\delta$ correspond to minimal punctures

$U(1)_t$, $U(1)_\beta$, $U(1)_\gamma$ are the three Cartans of $SO(7)$

At special points on conformal manifold

$U(1)_\alpha \times U(1)_\delta$ enhances to $SU(2)_{\alpha/\delta} \times SU(2)_{\delta\alpha}$

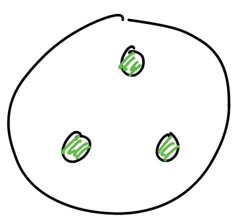
and $U(1)_{\gamma/\beta}$ enhances to $SU(2)_{\beta/\gamma}$

$\rightarrow$ 7 $SU(2)$-factors

$\rightarrow$ enhance to E_7

Denote $(\alpha/\delta, \delta\alpha)$ as (ω_1, ω_2)

$\rightarrow$ $SU(N)_u^2$, $SU(N)_v^2$, $SU(N)_w^2$ appear symmetrically



$\longrightarrow$ can be used as building block to obtain higher genus Riemann surface

dimension of conformal manifold:

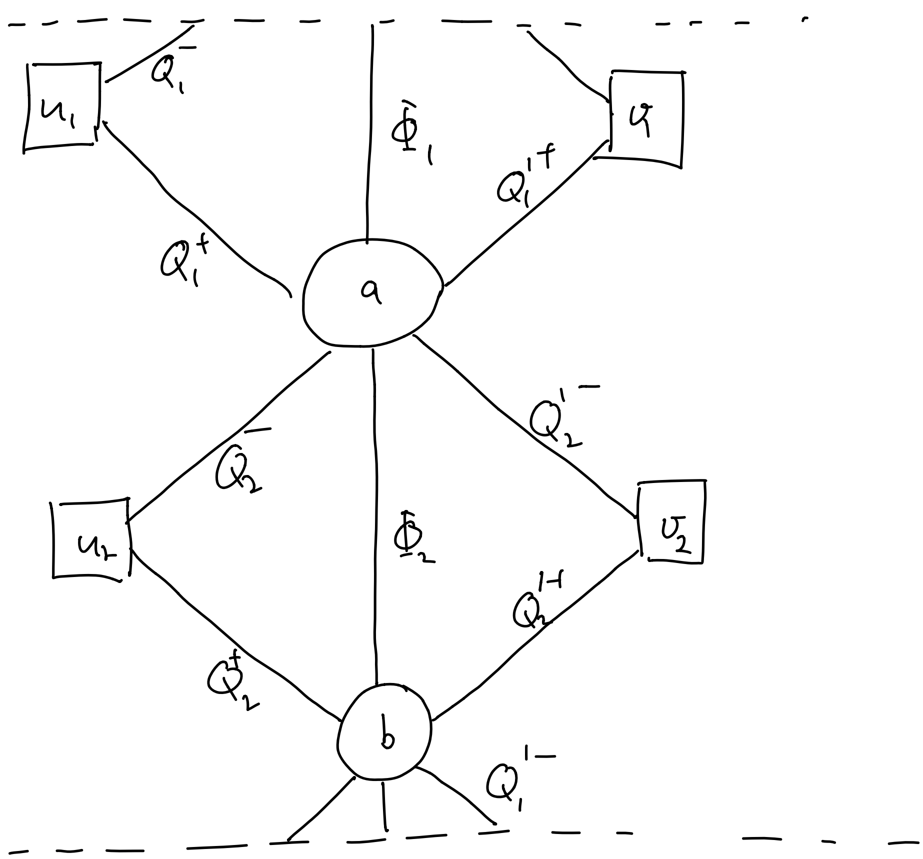
$$\dim \mathcal{M}_{g,s}^{SO(7)} = \left(g-1+\frac{s}{2}\right)\dim(SO(7))$$
$$-\frac{s}{2}\dim(SU(2)_{diag}\, U(1)^2) + 3g-3+s$$

The $G^{max} = SO(5) \times U(1)$ models

turn on $U(1)_t$ flux

$\longrightarrow$ $SO(5) \times U(1)_t$ symmetry on some locus of conformal manifold

Compactification on sphere with 2 max and 2 min punctures gives:



superpotential:

$$W = Q_1^+ Q_1^- \Phi_1 - Q_2^+ Q_2^- \Phi_2 + Q_1'^+ Q_1'^- \Phi_1 - Q_2'^+ Q_2'^- \Phi_2$$

$\longrightarrow$ enhancement to $SU(4) \times SU(4) \times U(1)_t \times U(1)_{\alpha\delta} \times U(1)_{\gamma\beta}$