Bounded Mark-Price Dynamics in Trade.xyz Equity Perpetuals

@ymsynn

November 2, 2025

Abstract

Hyperliquid, through HIP-3[1], enables all entities to deploy perpetuals on their infrastructure, with the only requirement being to stake HYPE tokens. The deployer chooses the trading pair as well as the oracle source. Trade.xyz (referred to as **xyz** in this paper) was the first to deploy a HIP-3 market to mainnet, with the XYZ100 pair (an index on NASDAQ). Their goal is to bring stocks on-chain through perpetuals that leverage crypto-native mechanisms. The first weeks of trading have been highly encouraging, with more than \$100M in trading volume and \$70M in open interest.

In this study, we investigate xyz's HIP-3 mark-price bounding mechanism[2] for equity perpetual swaps during weekend off-market periods. This is a key design feature in xyz's stock perpetuals: when the equity market is open, the oracle source is a real-time feed from traditional markets. When the market is closed, xyz uses an internal oracle computation.

Through a discrete-time simulation of orderbook and position dynamics, we analyze the potential impact of black swan scenarios on the design. We follow every design aspect present in both HIP-3 documentation and **xyz**'s documentation[3]. For other aspects of the simulation, we make assumptions that are explicitly listed later in this paper. This study highlights the emergence of hidden insolvencies, long–short parity distortions, and leverage asymmetries specific to equity markets with trading hour constraints when a black swan event occurs during a closed market period. Our results demonstrate that under the bounded mark price mechanism, system-level bad debt accumulates invisibly during weekends until Monday market reopen triggers price synchronization.

Contents

[1]`https://hyperliquid.gitbook.io/hyperliquid-docs/hyperliquid-improvement-proposals-hips/hip-3-builder-deployed-perpetuals`
[2]`https://docs.trade.xyz/xyz-perps-specification/equity-perpetuals#internal-pricing`
[3]`https://docs.trade.xyz/trading/liquidations-and-auto-deleveraging`

1 Introduction and Context

xyz[4] offers perpetual swap contracts for equity assets (stocks), enabling traders to gain leveraged exposure to US equities 24/7, including weekends when traditional markets are closed. Unlike crypto perpetuals that trade continuously, equity perpetuals face a unique challenge: the underlying reference markets (NYSE, NASDAQ) close Friday evening and reopen Monday morning, creating potential **weekend gaps** where news or global events can cause significant price movements while US markets are offline.

To protect users against price manipulation during these off-market periods, **xyz** implements a **Bounded Mark Price**[5] design, which constrains the mark price derived from the internal oracle to move no more than $\frac{1}{x_{\max}}\%$ from the last known oracle price (Friday close), where $x_{\max}$ denotes the maximum leverage allowed for the asset. This mechanism prevents sudden liquidation cascades from potentially stale or manipulated price feeds during weekends.

However, such constraints create a fundamental tension: while the bounded mark price stabilizes liquidations and PnL calculations, the *true* market price may diverge significantly. When markets reopen Monday morning, price synchronization may trigger concentrated liquidations.

[4] `https://trade.xyz`
[5] `https://docs.trade.xyz/xyz-perps-specification/equity-perpetuals#internal-pricing`

1.1 Research Motivation

This study analyzes the behavior of such a bounded mark mechanism during black swans, specifically:

- **Hidden insolvencies:** positions that appear safe under bounded mark but carry latent bad debt exposure
- **Leverage asymmetry:** unequal maximum leverage for longs vs shorts during bounded periods
- **Liquidation concentration:** delayed price discovery leading to Monday morning liquidation cascades

We develop a discrete-time simulation framework capturing extreme weekend gap scenarios and quantifying bad debt emergence patterns.

—

2 System Design, Notations, and Concerns

2.1 Notation

Let:

$$
\begin{aligned}
P_o(t) &: \text{oracle price at time } t \\
P_m(t) &: \text{mark price (bounded) at time } t \\
\tilde{P}_m(t) &: \text{unbounded mark price (orderbook reference)} \\
\Delta_{max} &: \text{maximum allowed relative deviation from initial oracle } (= \frac{1}{x_{\max}}\%) \\
\delta_{max} &: \text{maximum price change per update (e.g. } 0.01 = 1\%) \\
C_i(t) &: \text{collateral of position } i \text{ at time } t \\
N_i(t) &: \text{notional size of position } i \text{ at time } t \\
L_i &: \text{liquidation price of position } i \\
x_i &: \text{leverage of position } i \text{ (fixed at opening)} \\
x_{\max} &: \text{maximum leverage allowed for the asset} \\
E_i &: \text{entry price of position } i \\
\text{PnL}_i(t) &: \text{profit/loss of position } i \text{ at time } t \\
\text{BDR}_{\text{realized}}(t) &: \text{realized bad debt ratio at time } t \text{ (\% of total collateral)} \\
\text{BDR}_{\text{unrealized}}(t) &: \text{unrealized bad debt ratio at time } t \text{ (\% of total collateral)}
\end{aligned}
$$

2.2 Key Assumptions

While we follow all design aspects specified in HIP-3 documentation[6] and **xyz**'s documentation[7], certain implementation details are not explicitly specified. For these aspects, we make assumptions

[6] `https://hyperliquid.gitbook.io/hyperliquid-docs/hyperliquid-improvement-proposals-hips/hip-3-builder-deployed-perpetuals`

[7] `https://docs.trade.xyz`

that are listed below and clearly marked so readers can understand which behaviors are protocol-specified versus simulation choices.

This simulation represents a *naive* but *valid* market model. While we do not claim it captures all possible market behaviors, we ensure that all aspects of the **xyz** DEX mechanics are faithfully followed. This makes our simulation one valid realization among the infinite space of possible market outcomes—one that *could* occur in practice, though actual outcomes will vary based on trader behavior, liquidity dynamics, and external events.

1. **Trading bounds policy:** When the orderbook mid price falls outside the mark price bounds during the bounded period, the documentation does not specify whether trading should be allowed or blocked. We test both scenarios:

    - **Scenario A (primary results):** Trading (opening/closing positions) is *allowed* even when mid price is outside bounds. This reflects a permissive policy where traders can still access the market.
    - **Scenario B (sensitivity analysis):** Trading is *blocked* when mid price is outside bounds. This reflects a conservative policy that prevents potential manipulation or unhealthy price discovery.

    Main results are presented for Scenario A, with sensitivity analysis for Scenario B provided where relevant.

2. **Constant liquidity around mid price:** We model orderbook depth as continuously available around the mid price, with exponential decay based on mark–oracle divergence. This represents a *best-case scenario* for liquidity availability. Our assumption is optimistic: removing liquidity would only *increase* liquidation costs and bad debt accumulation, making our results conservative (lower bound) on actual bad debt.

3. **Instant orderbook price adjustment:** The orderbook bid/ask prices adjust toward the unbounded mark price $\tilde{P}_m(t)$.

4. **No slippage on executions:** All trades (opening, closing, liquidations) execute at best bid/ask without slippage. In practice, large liquidations would experience price impact, increasing realized bad debt. This assumption again makes our bad debt estimates *conservative* (understated).

5. **Positions aggregated into liquidation buckets:** We group positions sharing the same liquidation price into buckets, treating them as a single aggregate position. This simplification ignores individual position heterogeneity but preserves liquidation map dynamics.

6. **Probabilistic order flow:** Volume at each tick is split probabilistically between opening/closing and long/short sides, with configurable biases for OI increase/decrease/neutral. We do not model:

    - Strategic trader behavior (momentum following, contrarian strategies)
    - Information asymmetry (insider trading, news events)
    - Market maker inventory management

7. **Average leverage:** We do not model participants' average leverage using empirical data. It is reasonable to assume that most market participants with smaller position sizes use

maximum leverage, while sophisticated traders with larger positions use lower leverage for risk management. In the simulation, we sample leverage values from $[\![1, x_{\max}]\!]$ with probability proportional to the leverage value itself, such that leverage i has i times more probability of being chosen than leverage 1. The probability mass function is:

$$P(i) = \frac{i}{\sum_{j=1}^{x_{\max}} j} = \frac{2i}{x_{\max}(x_{\max}+1)}, \quad \forall i \in [\![1, x_{\max}]\!] \tag{1}$$

8. **No circuit breakers or trading halts:** The simulation continues regardless of extreme bad debt, price gaps, or liquidation cascades.
9. **Random walk for unbounded mark price:** During the closed market period, the unbounded mark price follows a random walk pattern toward the target path price, with stochastic noise. We do not model:
   - Market microstructure (limit order book depth, bid–ask bounce)
   - Volatility clustering (GARCH effects)
   - Regime changes (volatility spikes during news events)
10. **Matched liquidation flow:** When a position is liquidated, there is no cascade effect as we assume orderbook liquidity.
11. **Isolated margin mode:** For ease of simulation, we consider that all market participants use isolated margin rather than cross margin. This means each position has its own collateral allocation and margin requirements, independent of other positions held by the same trader. In reality, cross margin allows traders to share collateral across positions, potentially reducing margin requirements but also creating interdependencies between positions.

These assumptions collectively create a *controlled environment* where we can isolate the effects of bounded mark price mechanics without confounding factors from strategic behavior, liquidity shocks, or external market dynamics.

2.3 Dual Mark Price System

In our simulation, we keep track of two mark prices, under the assumption that there is no restriction on orderbook order placing and trading. **Bounded Mark Price** $P_m(t)$ (used for liquidations and PnL):

$$P_m(t+\Delta t) = \min\left(\max\left(P_m(t) + \text{step}(t),\, P_o(0)(1-\Delta_{max})\right),\, P_o(0)(1+\Delta_{max})\right) \tag{2}$$

where the step toward target is:

$$\text{step}(t) = \min\left(\max\left(\frac{P_{\text{target}}(t) - P_m(t)}{T_{\text{remain}}/\Delta t},\, -\delta_{max} P_m(t)\right),\, +\delta_{max} P_m(t)\right) \tag{3}$$

This dual bound ensures:

1. $|P_m(t) - P_o(0)| \leq \Delta_{max} \cdot P_o(0)$ (deviation from initial oracle)
2. $|P_m(t+\Delta t) - P_m(t)| \leq \delta_{max} \cdot P_m(t)$ (rate limit per update)

Unbounded Mark Price $\tilde{P}_m(t)$ (used for orderbook pricing):

$$\tilde{P}_m(t) = P_{\text{target}}(t) + \eta(t) \tag{4}$$

where $\eta(t)$ represents small stochastic noise to create realistic price movements. The orderbook follows $\tilde{P}_m(t)$ while liquidations are based on $P_m(t)$.

2.4 Position Mechanics

2.4.1 Opening Positions

When a position i opens at time t_0 with notional N_i and leverage x_i:

$$C_i(t_0) = \frac{N_i}{x_i} \tag{5}$$

$$E_i = P_{\text{exec}}(t_0) \quad \text{(execution price from orderbook)} \tag{6}$$

The liquidation price is fixed at opening:

$$L_i^{\text{long}} = E_i \left(1 - \frac{1}{x_i}\right) \tag{7}$$

$$L_i^{\text{short}} = E_i \left(1 + \frac{1}{x_i}\right) \tag{8}$$

2.4.2 PnL Calculation

For a position with current mark price $P_m(t)$:

$$\text{PnL}_i^{\text{long}}(t) = \frac{P_m(t) - E_i}{E_i} \cdot N_i(t) \tag{9}$$

$$\text{PnL}_i^{\text{short}}(t) = \frac{E_i - P_m(t)}{E_i} \cdot N_i(t) \tag{10}$$

2.4.3 Partial Closes and Collateral Adjustment

When a position partially closes amount ΔN at price P_{close}:

$$\text{PnL}_\Delta = \frac{P_{\text{close}} - E_i}{E_i} \cdot \Delta N \quad \text{(for longs)} \tag{11}$$

$$C_{\text{released}} = \frac{\Delta N}{N_i(t)} \cdot C_i(t) \tag{12}$$

The remaining collateral is adjusted:

$$C_i(t^+) = C_i(t) - C_{\text{released}} + \text{PnL}_\Delta \tag{13}$$

If $\text{PnL}_\Delta < -C_{\text{released}}$, bad debt occurs:

$$\text{BadDebt}_{\text{voluntary}} = |\text{PnL}_\Delta| - C_{\text{released}} \tag{14}$$

2.5 Leverage Constraints During Bounded Periods

When $P_m(t) \neq \tilde{P}_m(t)$, new positions face solvency-based leverage limits. These constraints arise from the requirement that a newly opened position must not be immediately insolvent when marked at the bounded mark price $P_m(t)$, even though execution occurs at the market price $\tilde{P}_m(t)$.

For shorts when $\tilde{P}_m < P_m$ (market price below mark):

When a short position is opened at market price $\tilde{P}_m$ but marked at the higher bounded price P_m, there is an immediate unrealized loss. The position remains solvent only if this loss does not exceed the collateral.

For a short position with notional N, leverage x, and collateral $C = \frac{N}{x}$:

- Entry price: $E = \tilde{P}_m$
- Immediate unrealized PnL (when marked at P_m): $\text{PnL}^{\text{short}} = \frac{E-P_m}{E} \cdot N = \frac{\tilde{P}_m - P_m}{\tilde{P}_m} \cdot N < 0$
- Solvency condition: $\text{PnL} > -C$, i.e., $\text{PnL} > -\frac{N}{x}$

Solving for the maximum leverage:

$$\frac{\tilde{P}_m - P_m}{\tilde{P}_m} \cdot N > -\frac{N}{x} \tag{15}$$

$$\frac{P_m - \tilde{P}_m}{\tilde{P}_m} < \frac{1}{x} \tag{16}$$

$$x < \frac{\tilde{P}_m}{P_m - \tilde{P}_m} = \frac{1}{\frac{P_m}{\tilde{P}_m} - 1} = \frac{1}{1 - \frac{\tilde{P}_m}{P_m}} \tag{17}$$

Therefore:

$$x_{\max}^{\text{short}} = \frac{1}{1 - \frac{\tilde{P}_m}{P_m}} \tag{18}$$

For longs when $P_m < \tilde{P}_m$ (mark price below market):

When a long position is opened at market price $\tilde{P}_m$ but marked at the lower bounded price P_m, there is an immediate unrealized loss. The position remains solvent only if this loss does not exceed the collateral.

For a long position with notional N, leverage x, and collateral $C = \frac{N}{x}$:

- Entry price: $E = \tilde{P}_m$
- Immediate unrealized PnL (when marked at P_m): $\text{PnL}^{\text{long}} = \frac{P_m-E}{E} \cdot N = \frac{P_m - \tilde{P}_m}{\tilde{P}_m} \cdot N < 0$
- Solvency condition: $\text{PnL} > -C$, i.e., $\text{PnL} > -\frac{N}{x}$

Solving for the maximum leverage:

$$\frac{P_m - \tilde{P}_m}{\tilde{P}_m} \cdot N > -\frac{N}{x} \tag{19}$$

$$\frac{\tilde{P}_m - P_m}{\tilde{P}_m} < \frac{1}{x} \tag{20}$$

$$x < \frac{\tilde{P}_m}{\tilde{P}_m - P_m} = \frac{1}{1 - \frac{P_m}{\tilde{P}_m}} \tag{21}$$

Therefore:

$$x_{\max}^{\text{long}} = \frac{1}{1 - \frac{P_m}{\tilde{P}_m}} \tag{22}$$

These constraints prevent opening positions that would be immediately underwater upon opening.

2.6 Identified Concerns

1. **Hidden insolvencies:** Longs (resp. shorts) remain unliquidated when mark is capped below (resp. above) oracle, deferring realized losses until the bound lifts.
2. **Leverage asymmetry:** During bounded periods, longs may open with higher leverage than shorts (or vice versa), creating collateral imbalance.
3. **Parity distortion:** Long–short PnL parity is temporarily broken, restored only after ADL or insurance intervention.

—

3 Experimental Setup and Assumptions

3.1 Simulation Overview

We design a discrete-time simulator that models a typical **xyz** equity perpetual (e.g., AAPL-PERP, TSLA-PERP) over a weekend period:

- **Friday 16:00 EST:** US equity markets close, external oracle price freezes
- **Weekend (Sat/Sun):** Bounded mark price period with $\Delta_{\max} = \frac{1}{x_{\max}}$ (**xyz** constraint)
- **Monday 01:00 EST:** External oracle starts working again and updates to new price, mark price unbounds

The simulation captures orderbook dynamics, position entry/exit, liquidation processing, and bad debt accumulation specific to equity perpetuals during this bounded period.

3.2 Simulation Scenarios

We conduct two primary simulation scenarios to assess the systemic risk of **xyz**'s HIP-3 mechanism under different market conditions and policy settings:

1. **Realistic Market Nuke (Permissive Trading):** A synthetic stress test simulating a severe weekend market crash, with trading *allowed* even when the orderbook mid price falls outside mark price bounds. This represents a permissive policy where traders maintain access to the market despite price discovery divergence. This scenario helps quantify bad debt accumulation under maximum market exposure.
2. **Realistic Market Nuke (Restrictive Trading):** The same synthetic stress test, but with trading *blocked* when the orderbook mid price is outside mark price bounds. This represents a conservative policy that prevents potential manipulation or unhealthy trading during extreme divergence. Comparing this to Scenario 1 isolates the impact of trading bounds enforcement on bad debt formation.

Each scenario is configured with:

- Configurable OI dynamics (increase/decrease/neutral probabilities)
- Weighted leverage sampling (bias toward higher leverage)
- Funding rate calculation following HyperCore specification

Detailed parameter sets and results for each scenario are presented in Section 5.

3.3 Liquidation Processing

According to Hyperliquid documentation[8], liquidations occur when account equity falls below maintenance margin requirements. During bounded periods, liquidations are restricted as the mark price is constrained. After market reopen, when the mark price becomes unbounded, positions are liquidated when:

- Position equity $(C_i(t) + \text{UnrealizedPnL}_i(t)) < \text{MaintenanceMargin}_i$
- Maintenance margin is defined as $\text{MaintenanceMargin}_i = N_i(t)/(2 \times x_{\max})$
- Positions with unrealized bad debt $(\text{PnL}_i(t) < -C_i(t))$ are not liquidated by this mechanism—they are handled by ADL instead

Upon liquidation, the position closes at the current orderbook best bid (for longs) or best ask (for shorts), without slippage (see assumptions). Liquidation bad debt occurs when:

$$\text{BadDebt}_{\text{liq},i} = \max(0, |\text{PnL}_i| - C_i) \tag{23}$$

where PnL_i is the realized PnL from partial closes.

3.4 Bad Debt Categorization

We distinguish two types of bad debt:

Realized Bad Debt (cumulative, never resets):

$$\text{BD}_{\text{realized}}(t) = \sum_{i \in \text{voluntarily closed}} \text{BadDebt}_i + \sum_{j \in \text{liquidated}} \text{BadDebt}_j \tag{24}$$

where bad debt from voluntary closes occurs when a position partially or fully closes with losses exceeding the released collateral, and bad debt from liquidations occurs when liquidation execution doesn't cover the full loss.

Unrealized Bad Debt (mark-to-market, resets after ADL):

$$\text{BD}_{\text{unrealized}}(t) = \sum_{k \in \text{open}} \max(0, |\text{PnL}_k(t)| - C_k(t)) \tag{25}$$

These are mutually exclusive: a position contributes to unrealized bad debt while open, and to realized bad debt once closed or liquidated.

To facilitate comparison across scenarios with different total collateral levels, we normalize bad debt by total system collateral, defining the **Bad Debt Ratio**:

$$\text{BDR}_{\text{realized}}(t) = \frac{\text{BD}_{\text{realized}}(t)}{\sum_i C_i(t)} \times 100\% \tag{26}$$

$$\text{BDR}_{\text{unrealized}}(t) = \frac{\text{BD}_{\text{unrealized}}(t)}{\sum_i C_i(t)} \times 100\% \tag{27}$$

where $\sum_i C_i(t)$ represents the total collateral across all open positions at time t. This ratio expresses bad debt as a percentage of total system collateral, providing a scale-invariant metric for systemic risk assessment.

[8] `https://hyperliquid.gitbook.io/hyperliquid-docs/trading/liquidations`

3.5 Auto-Deleveraging (ADL)

According to **xyz** documentation[9], ADL serves as a safeguard mechanism that ensures system solvency when positions become under-collateralized but cannot be liquidated through normal market mechanisms.

ADL is checked for every position individually, not based on system-wide total collateral. When a position's total PnL (realized plus unrealized) falls below negative collateral (i.e., $\text{PnL}_i(t) < -C_i(t)$), indicating the position value is negative, ADL triggers. The system socializes these losses by force-closing profitable positions on the opposite side, with their profits used to offset the underwater position's bad debt.

For position i at time t:

- ADL triggers when: $\text{PnL}_i(t) < -C_i(t)$ (position value is negative)
- ADL mechanism: Force-close profitable positions on the *opposite side* (if position i is a long with negative value, close profitable shorts; if position i is a short with negative value, close profitable longs)
- The profits from force-closed positions are used to cover the bad debt of the underwater position

Note: ADL should only trigger when the market is unbounded (after market reopen), as during bounded periods, positions cannot be liquidated based on unrealized PnL that exceeds the mark price bounds.

3.6 Model Limitations

Beyond the explicit assumptions listed in Section 3 (see Subsection 2.2), our simulation has inherent limitations:

1. **Discrete-time approximation:** We simulate at fixed intervals (default 2.5 seconds), not continuous time. This may miss rapid cascades that occur between ticks.
2. **Internal Pricing implementation:** During closed market periods (when external oracle inputs are unavailable), we implement **xyz**'s Internal Pricing mechanism[10] using an exponentially weighted moving average (EWMA) with impact price difference (IPD):

$$S_t = \beta_t S_{t-} + (1 - \beta_t) x_t \tag{28}$$

$$\beta_t = \exp\left(-\frac{\Delta t^*}{\tau}\right), \quad \tau = 8 \text{ hours} \tag{29}$$

$$x_t = S_{t-} + \text{IPD}_t \tag{30}$$

$$\text{IPD}_t = \max(P_{\text{impactBid}} - S, 0) - \max(S - P_{\text{impactAsk}}, 0) \tag{31}$$

where $P_{\text{impactBid}}$ and $P_{\text{impactAsk}}$ are average execution prices to trade a configured impact notional on the bid and ask sides. With our simplified orderbook model (best bid/ask + depth), we approximate impact prices: if depth is sufficient, $P_{\text{impact}} \approx \text{best\_price}$; otherwise we estimate slippage based on depth availability. This allows the oracle to slowly adjust based on DEX trading activity, creating a feedback loop between orderbook prices and oracle updates.

[9] https://docs.trade.xyz/trading/liquidations-and-auto-deleveraging

[10] https://docs.trade.xyz/xyz-perps-specification/equity-perpetuals#internal-pricing

During open market periods, the external oracle feed takes precedence. The mark price path in scenarios guides where the oracle should converge during closed periods, but actual oracle updates follow the Internal Pricing EWMA mechanism rather than jumping directly to target prices. This implementation follows **xyz**'s specification with $\Delta t^* = \min(\Delta t, c\tau)$ and $c = 0.1$ (so $1 - \beta_t \leq 1 - e^{-0.1} \approx 9.5\%$ per update), ensuring the oracle remains responsive to market activity while being robust to irregular updates and market halts.

3. **Parametric orderbook depth:** Depth is modeled via exponential decay, not derived from actual limit order book simulation. Real orderbooks exhibit discrete levels, order clustering, and dynamic depth adjustments.

4. **Static leverage constraints:** Leverage limits are computed based on mark–market divergence but do not adjust dynamically with volatility or funding rates (beyond the solvency-based constraints we implement).

These limitations are accepted trade-offs for computational tractability and clarity of mechanism analysis. Future work could extend the simulation to address these constraints.

3.7 Visualization and Heatmaps

The simulation produces several key visualizations:

1. Liquidation Heatmap (Unbounded): Shows the evolution of liquidation price distribution over time, revealing:

- Dense horizontal bands during bounded period (positions cluster at mark bounds)
- Sudden disappearance of bands at market reopen (liquidation cascade)
- Mark price bounds (dashed red lines) stopping at reopen time

2. Bounded Liquidation Heatmap: Shows liquidation prices *as traders perceive them* during bounded period:

- Liquidation prices capped at $[P_o(0)(1 - \Delta_{max}), P_o(0)(1 + \Delta_{max})]$
- Creates illusion of safety—most positions appear far from liquidation
- Reveals true exposure only at market reopen

3. Bad Debt Evolution: Two-series plot showing bad debt ratios (as percentage of total collateral):

- Realized bad debt ratio (solid line): monotonically increasing, never resets, expressed as percentage of total system collateral
- Unrealized bad debt ratio (dashed line): brief spike at reopen, then drops to near-zero, expressed as percentage of total system collateral
- ADL trigger events (vertical orange lines)

4. Open Interest Dynamics: Shows position count fluctuations driven by stochastic OI model, with visible drop at liquidation events.

—

4 Historical Context and Equity Market Gaps

To assess realism, we analyze historical weekend gaps in US equity markets (2020–2024), particularly focusing on:

- **COVID-19 Period (Mar 2020):** Multiple 5-10% weekend gaps as pandemic news broke
- **Earnings Announcements:** Companies reporting after Friday close causing Monday gaps
- **Geopolitical Events:** Weekend news triggering Monday open gaps (e.g., Ukraine conflict, OPEC decisions)

Figure 1 demonstrates the magnitude of weekend returns for TSLA, showing that significant moves during weekend periods are not uncommon.

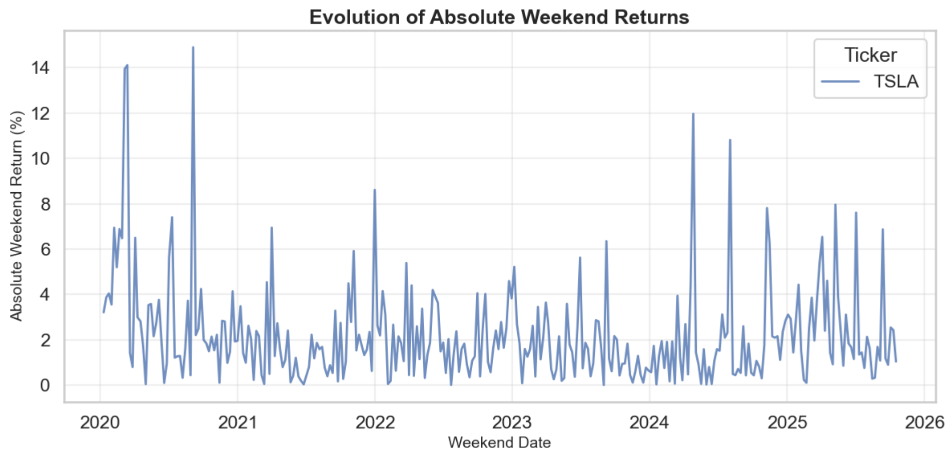

Figure 1: TSLA absolute weekend returns, demonstrating the frequency and magnitude of weekend price movements that occur while US equity markets are closed.

Our simulation scenarios replicate these gap magnitudes.
—

5 Simulation Results and Discussion

5.1 Scenario Results

This section presents detailed results for each simulation scenario. Summary tables and key visualizations are included for comparative analysis.

Disclaimer: For computational reasons, the simulation is run on a 3-hour closed window rather than over 2 days as in reality. All results are condensed but proportional, except funding.

5.1.1 Realistic Market Nuke (Permissive Trading)

This experiment models a synthetic 30% market crash during a bounded weekend period (10x max leverage, $\pm 10\%$ mark bound). Trading remains *permitted* even when the orderbook mid price falls outside the bounded mark range, isolating system behavior when participants continue trading despite mark–market divergence.

Leverage Dynamics. Figure 2 shows the evolution of notional-weighted average leverage. Initially both sides cluster around 13–14×. As the unbounded market price drifts below the bounded mark, shorts face tight solvency constraints (see Section 2.4), declining to under 5×, while longs remain near 13×. This asymmetric deleveraging reflects the bounded mark mechanism enforcing collateral discipline on overvalued positions (shorts), while longs can sustain higher leverage, creating latent exposure revealed at bound release.

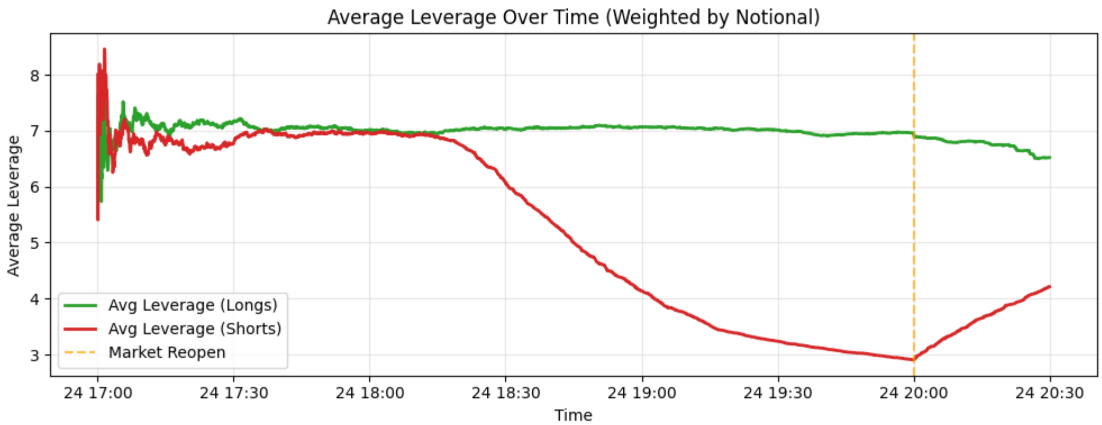

Figure 2: Average leverage over time (weighted by notional). Long leverage (green) remains stable around 13–14×, while short leverage (red) declines to below 5× as market divergence grows. Dashed line marks market reopen.

Bad Debt Formation and Realization. During the bounded period, long-side insolvencies cannot crystallize as the mark price remains constrained above market price. Unrealized bad debt ratio grows gradually (Figure 3, dashed line) as positions become underwater when marked to true market $\tilde{P}_m$, representing latent exposure that appears liquid under the bounded scheme but would be insolvent at market prices. The unrealized bad debt ratio reaches approximately 20–25% of total system collateral by market reopen. At market reopen, the mark price synchronizes with the external oracle, triggering ADL within one tick, where profitable shorts are force-closed to cover insolvent longs.

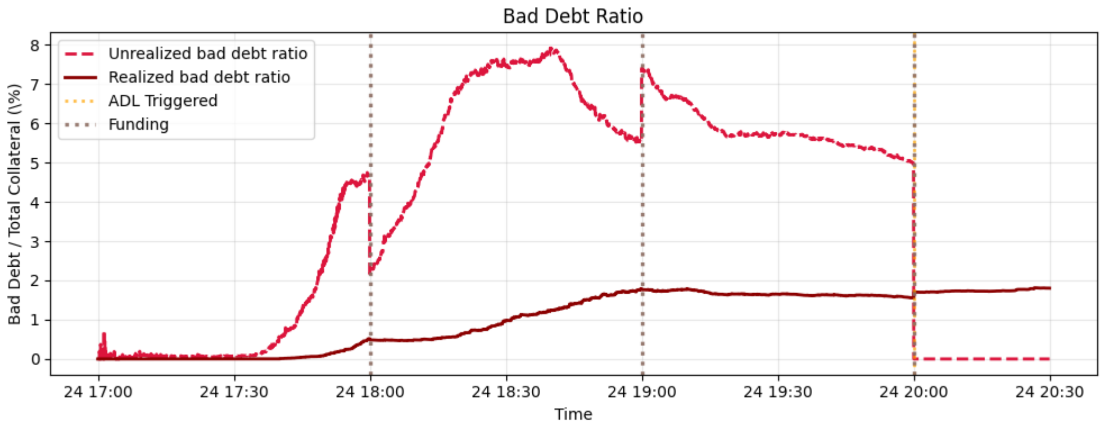

Figure 3: Unrealized (dashed) and realized (solid) bad debt ratios over time, expressed as percentage of total system collateral. Bad debt accumulates invisibly during the bounded period and is instantaneously crystallized upon market reopen, triggering ADL.

The sharp, discontinuous jump illustrates that bounded mark mechanisms defer insolvency recognition until unbounding, creating a period where solvency metrics appear stable while exposure accumulates.

Funding Rate Inversion. Figure 4 shows the evolution of annualized funding rates during the same interval. As the market price declines far below the bounded mark, shorts increasingly dominate funding payments to maintain exposure, driving the rate sharply negative (below –400% annualized). This extreme inversion is an endogenous signal of imbalance: longs appear profitable and thus demand funding, even though they would be insolvent if marked to the true price. Once the bound lifts and ADL resets positions, funding rates revert immediately to neutral, confirming that the dislocation was purely a mark-induced artifact.

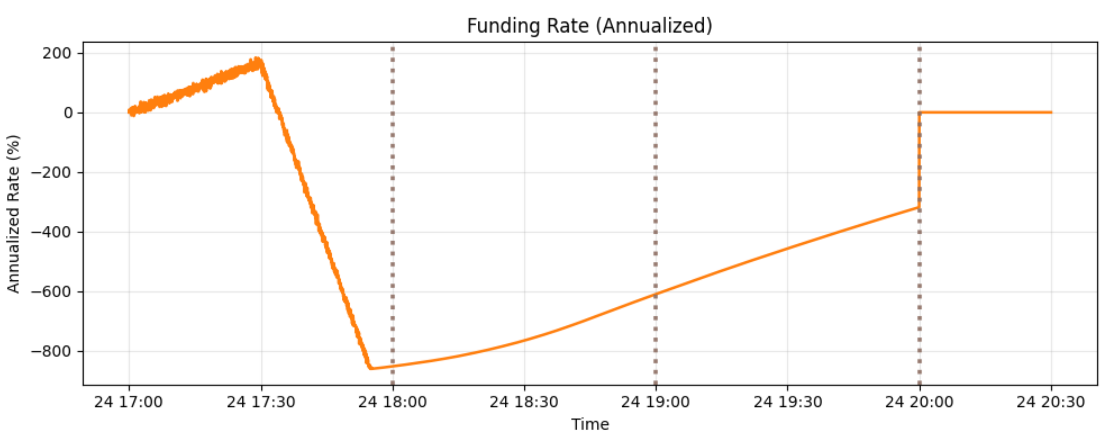

Figure 4: Funding rate (annualized) inversion during bounded period. Shorts overpay to maintain exposure as mark divergence widens, leading to extreme negative rates that normalize post-reopen.

Bounded vs. Unbounded Liquidation Maps. Figures 5 and 6 illustrate liquidation concentration patterns. During the bounded phase, liquidation prices cluster at mark bounds, creating an illusion of stability while exposure drifts. At reopen, all positions beyond true liquidation thresholds liquidate simultaneously, producing the vertical stripe corresponding to the bad debt spike.

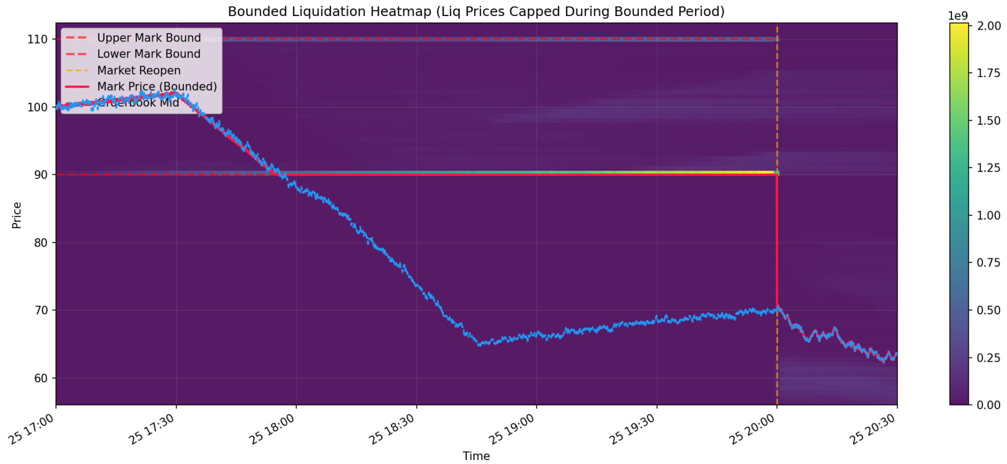

Figure 5: Bounded liquidation heatmap. Liquidations appear capped at artificial mark bounds, masking true exposure during bounded period.

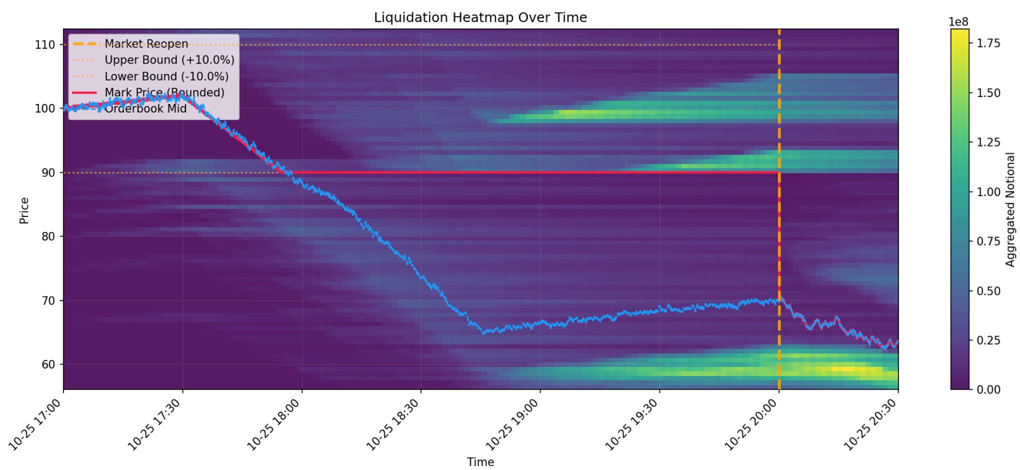

Figure 6: Unbounded liquidation heatmap. At market reopen, mark price aligns with orderbook mid, triggering a concentrated liquidation cascade.

Interpretation. The bounded mark mechanism transfers volatility from *time* to *state*: constrained moves during the bounded period concentrate into a realization event at bound release. The permissive trading policy maximizes latent bad debt accumulation through endogenous buildup of inconsistency between bounded valuation and true market exposure.

5.1.2 Realistic Market Nuke (Restrictive Trading)

This scenario applies the same synthetic 30% crash simulation as Scenario 1, but with trading *blocked* when the orderbook mid price falls outside the mark price bounds. This restrictive policy prevents traders from opening or closing positions once price divergence exceeds the bounded mark range, effectively halting market activity during extreme divergence.

The key finding is that under the restrictive trading policy, **no bad debt accumulates** (bad debt ratio remains at 0% throughout). By preventing trading once the orderbook mid price falls outside the mark bounds, the system avoids the formation of new positions at economically distorted prices. This eliminates the latent insolvency exposure that builds up in the permissive scenario, demonstrating that trading bounds enforcement is an effective risk mitigation strategy.

In contrast, the permissive trading scenario (Scenario 1) shows unrealized bad debt ratios reaching 20–25% of total collateral by market reopen, demonstrating the significant risk accumulation when trading continues despite mark–market divergence.

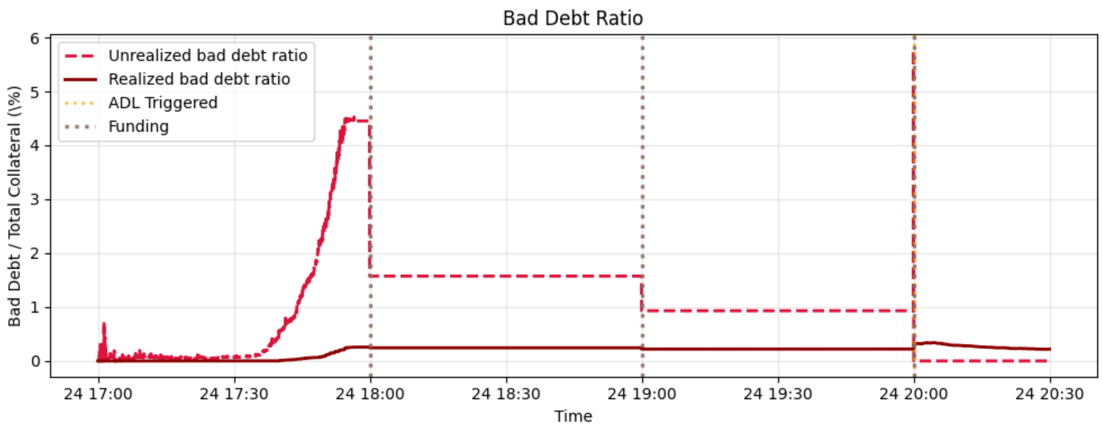

Figure 7: Bad debt ratios under restrictive trading policy remain at 0% throughout the simulation, demonstrating the effectiveness of trading bounds enforcement. This contrasts with the permissive scenario where unrealized bad debt ratios reach 20–25% of total collateral.

Bounded vs. Unbounded Liquidation Maps. Figures 8 and 9 show both the bounded and unbounded liquidation heatmaps for the restrictive trading scenario. Figure 8 illustrates how liquidation prices are capped at mark bounds during the restricted trading period. Unlike the permissive scenario, no new positions form once price divergence occurs, preventing the accumulation of underwater positions that would trigger liquidations at market reopen. Figure 9 shows the true liquidation prices without bounds, revealing the actual exposure that would be liquidated if the market were unbounded.

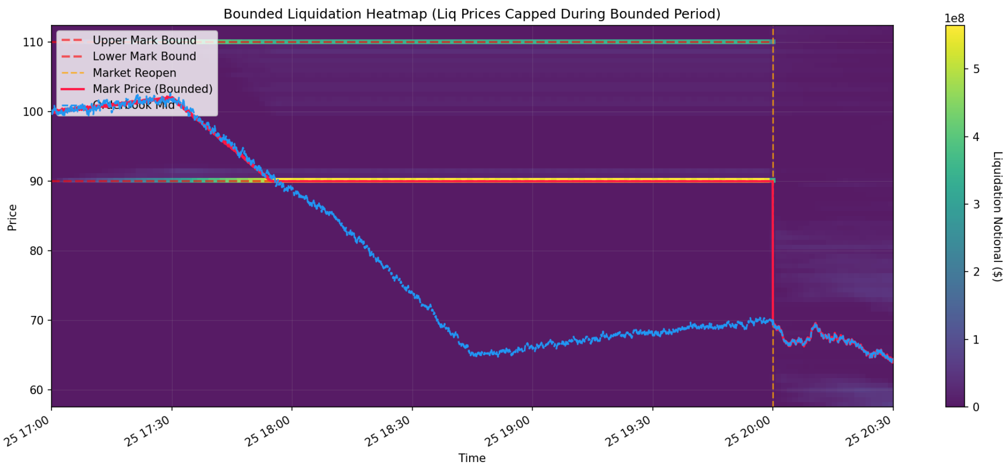

Figure 8: Bounded liquidation heatmap under restrictive trading. Liquidation prices are capped at mark bounds, and trading halts prevent new position formation once divergence occurs, eliminating bad debt accumulation.

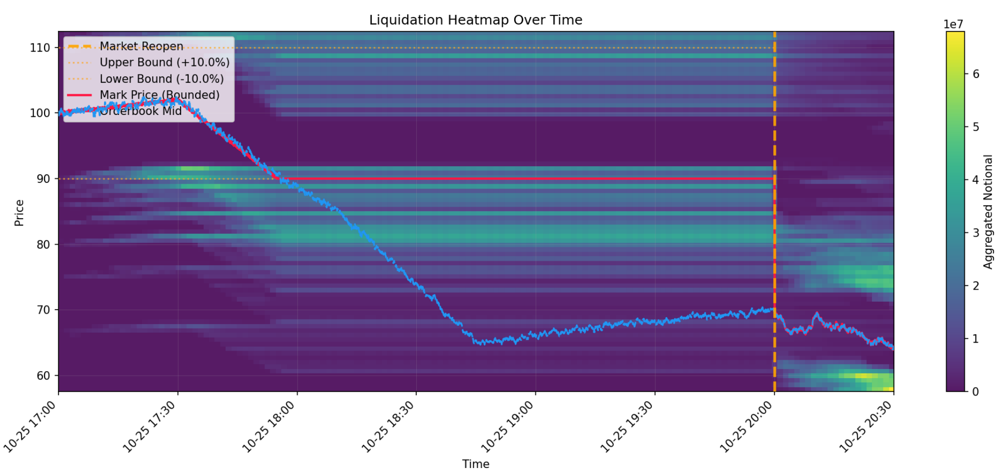

Figure 9: Unbounded liquidation heatmap under restrictive trading. Shows actual liquidation prices without bounds, demonstrating that trading restrictions prevent accumulation of underwater positions.

This demonstrates that enforcing trading bounds eliminates bad debt, though at the cost of reduced market accessibility during extreme events.

5.2 Bad Debt Evolution and Timing

Our simulation reveals a characteristic bad debt pattern:

Phase 1 - Bounded Period (t_0 to t_{reopen}):

- Unrealized bad debt gradually increases as positions become underwater when marked to true market price $\tilde{P}_m$
- While the bounded mark price P_m remains constrained, positions appear safe under the bounded scheme but carry latent bad debt exposure

- Positions accumulate at bounded prices, unaware of true market exposure
- Liquidation map shows dense clustering near mark bounds

Phase 2 - Market Reopen ($t = t_{\text{reopen}}$):

- Oracle price synchronizes: $P_o(t_{\text{reopen}}) = \tilde{P}_m(t_{\text{reopen}})$
- Mark price becomes unbounded, allowing it to align with the true market price
- Unrealized bad debt spikes sharply as positions are now marked to true prices
- ADL triggers immediately when unrealized bad debt is detected, socializing losses rather than converting them to realized bad debt

Phase 3 - Post-Reopen ($t > t_{\text{reopen}}$):

- Realized bad debt remains stable (only from voluntary closes and liquidations before ADL)
- Unrealized bad debt is socialized by ADL, resetting to zero through force-closing profitable positions
- New positions open at true market prices

This creates the observation: **unrealized bad debt accumulates during the bounded period, with a portion potentially realized if trading is allowed, then spikes at market reopen where ADL socializes losses rather than converting them to realized bad debt**. The bounded mark mechanism defers bad debt recognition until the bound lifts, at which point ADL intervenes to socialize the losses among profitable counterparties.

5.3 Parity Breakdown and Restoration

While notional parity ($\text{OI}_{\text{long}} = \text{OI}_{\text{short}}$) is preserved by exchange design, economic parity breaks under insolvency.

Ideal Case:

$$\sum_{i \in \text{longs}} \text{PnL}_i + \sum_{j \in \text{shorts}} \text{PnL}_j = 0 \tag{32}$$

With Bad Debt:

$$\sum_{i \in \text{longs}} \text{PnL}_i + \sum_{j \in \text{shorts}} \text{PnL}_j = -\text{BD}_{\text{realized}}(t) - \text{BD}_{\text{unrealized}}(t) \tag{33}$$

The parity deficit equals the system's total bad debt. This is restored through:

- **ADL**: Unrealized bad debt socialized to profitable counterparties
- **Insurance Fund**: Realized bad debt absorbed by protocol reserves

Our simulation shows parity errors can exceed 10% of total collateral during extreme bounded periods.

—

6 Risk Mitigation Strategies

Based on our findings, two conclusions arise:

6.1 Dynamic Leverage Constraints

To avoid the average leverage asymmetry, a real-time dynamic cap could be implemented:

$$x_{\max}^{\text{dynamic}} = \min\left(x_{\text{protocol}}, \frac{1}{1 - |\tilde{P}_m - P_m|/P_m}\right) \tag{34}$$

This prevents asymmetry of collateral during a stressed market.

6.2 Block Trading Out of Bounds

As demonstrated by the stark difference between Scenario A (permissive trading) and Scenario B (restrictive trading), preventing trading when the orderbook mid price falls outside the mark price bounds is an effective mechanism to prevent bad debt accumulation. Comparing the bad debt ratios between the two scenarios reveals a stark contrast: the permissive trading scenario exhibits unrealized bad debt ratios reaching 20–25% of total system collateral by market reopen, while the restrictive trading scenario maintains bad debt ratios at 0% throughout. This 20–25 percentage point difference highlights the critical impact of trading policy on systemic risk accumulation during bounded periods.

When trading is blocked during bounded periods, the protocol enforces transparent risk limits for both long and short positions:

For the losing side: Positions opened before the bound is exceeded can lose up to their full collateral, resulting in maximum loss:

$$\text{MaxLoss}_i = -C_i(0) = -\frac{N_i}{x_i} \tag{35}$$

For the winning side: Positions are subject to ADL when the bound lifts, which socializes unrealized bad debt from underwater positions. The maximum realized PnL for a winning position is constrained by the mark price bound:

For a **short position** when price declines (winning side):

$$\text{MaxPnL}_{\text{winning}}^{\text{short}} = \frac{E_i - P_m^{\min}}{E_i} \cdot N_i = \frac{E_i - P_o(0)(1 - \Delta_{\max})}{E_i} \cdot N_i \tag{36}$$

where $P_m^{\min} = P_o(0)(1 - \Delta_{\max})$ is the lower mark price bound.

For a **long position** when price increases (winning side):

$$\text{MaxPnL}_{\text{winning}}^{\text{long}} = \frac{P_m^{\max} - E_i}{E_i} \cdot N_i = \frac{P_o(0)(1 + \Delta_{\max}) - E_i}{E_i} \cdot N_i \tag{37}$$

where $P_m^{\max} = P_o(0)(1 + \Delta_{\max})$ is the upper mark price bound.

However, when ADL triggers, profitable positions on the winning side are force-closed to cover underwater positions on the losing side, effectively reducing their realized PnL. The actual maximum PnL accounting for ADL is:

$$\text{MaxPnL}_{\text{winning}}^{\text{ADL}} = \text{MaxPnL}_{\text{winning}} - \text{ADL}_{\text{socialized}} \tag{38}$$

where $\text{ADL}_{\text{socialized}}$ represents the portion of profits socialized through ADL to cover bad debt from underwater positions.

Blocking trading when bounds are exceeded provides transparency: participants know that during bounded periods, the losing side risks total collateral loss while the winning side's profits are capped by the bound and subject to ADL socialization. This creates symmetric risk awareness and prevents the accumulation of latent bad debt that would otherwise crystallize at market reopen.

This would imply to add a systematic ADL-like process at end of bounded period.

—

7 Conclusion

Through discrete-time simulation of **xyz**'s bounded mark price mechanism for equity perpetuals, we demonstrate that bounded mark mechanisms effectively prevent manipulation but create temporal risk concentration: unrealized bad debt accumulates during bounded periods as positions become underwater when marked to true market price, then spikes at market reopen where ADL socializes losses rather than converting them to realized bad debt.

Our analysis reveals that leverage asymmetry, hidden insolvencies, and parity distortions are quantifiable and predictable characteristics of bounded mark price systems during extreme market events, creating a fundamental tension between price stability (achieved through bounds) and economic reality (true market prices).