Robust Human Capital Investment under Risk and Ambiguity

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Introduction

Introduction: Human Capital Investment

Human capital investment decisions have long term consequences and involve a substantial degree of uncertainty

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Caveat: Treatment of uncertainty remains sparse and mostly restricted to risk (Hartog and Diaz-Serrano, 2013)

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Uncertainty includes risk and ambiguity

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- Probabilities either do not exist or are not known exactly

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Ambiguity-aversion has already been documented in

- Lab experiments
  (Ahn, Choi, Gale, and Kariv, 2010; Hey and Pace, 2014; Carbone, Dong, and Hey, 2017)
- Static real-life situations
  (Easley and O'Hara, 2009; Berger, Bleichrodt, and Eeckhoudt, 2013)

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Educational and occupational choices are subject to risk and ambiguity

Introduction: *Robust* Human Capital Investment

Contributions in this presentation

1. **Incorporation of robust decision-making** into the workhorse model of human capital investment (Keane and Wolpin, 1997)

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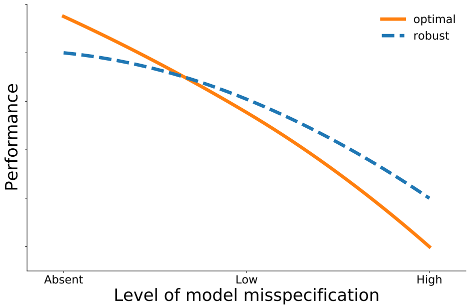

Figure 1. Stylized illustration robust decision-making

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1. **Incorporation of robust decision-making** into the workhorse model of human capital investment (Keane and Wolpin, 1997)
2. **Out-of-sample validation** outside the support of the estimation sample

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Contributions in this presentation

1. **Incorporation of robust decision-making** into the workhorse model of human capital investment (Keane and Wolpin, 1997)
2. **Out-of-sample validation** outside the support of the estimation sample
   - Preliminary result: Robust human capital model leads to a much better out-of-sample performance

Economic Model

Economic Model: Sequential Decision Making

Sequential decision making under uncertainty

Notation

- State $s_t \in \mathcal{S}$ of economic environment
- Action $a_t \in \mathcal{A}$ from set of admissible alternatives
- Policy $\pi = \left(a_1^\pi(s_1), \dots a_T^\pi(s_T)\right) \in \Pi$

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Sequential decision making under uncertainty

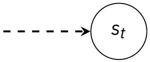

Figure 1. Timing of events

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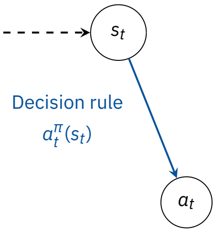

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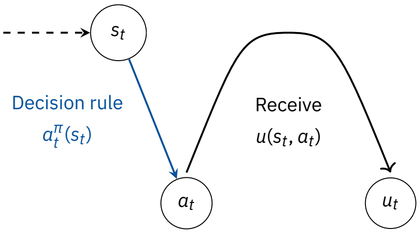

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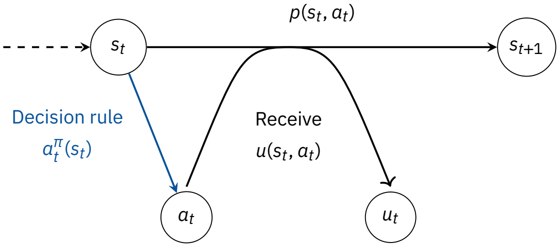

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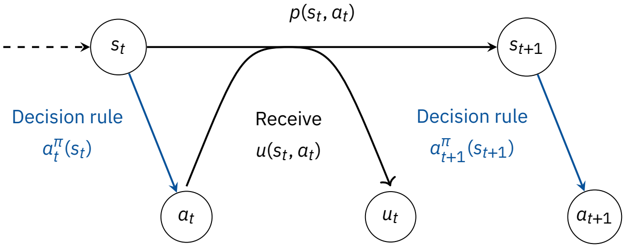

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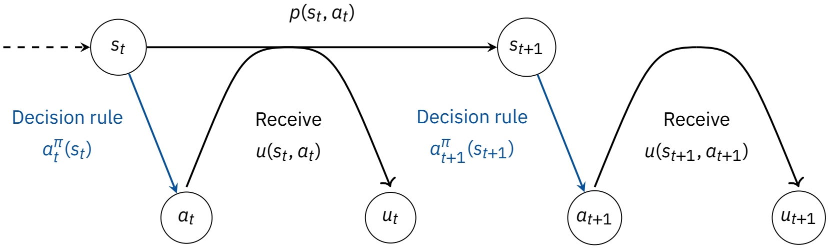

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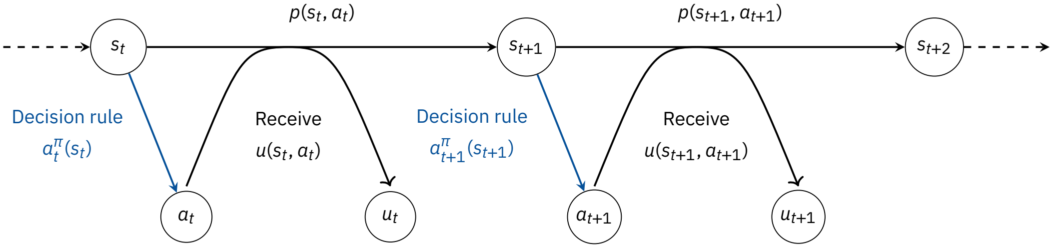

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Beliefs about the transition probabilities

- Risk-only: Unique (objective) transition probability distribution $p(s_t, a_t)$
- Ambiguity: Some transition probability distribution $p(s_t, a_t) \in \mathcal{P}(s_t, a_t)$

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Multiple-prior paradigm: Multiple beliefs, i.e. set of probability distributions

- We use **maxmin expected utility** preferences (Gilboa and Schmeidler, 1989)

Mathematical Framework

Mathematical Framework: Backward Induction Algorithm

Algorithm for robust Markov decision process – Implementation via `respy`

for $t = T, \dots, 1$ **do**

  if t == T **then**

$$v_T^{\pi^*}(s_T) = \max_{a_T \in \mathcal{A}} \left\{ u(s_T, a_T) \right\} \qquad \forall\, s_T \in \mathcal{S}$$

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$$v_t^{\pi^*}(s_t) = \max_{a_t \in A} \left\{ u(s_t, a_t) + \min_{p \in \mathcal{P}(s_t, a_t)} \delta\, \mathrm{E}_p \left[v_{t+1}^{\pi^*}(s_{t+1}) \,\middle|\, s_t \right] \right\}$$

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robust Bellman equation

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   and set

$$a_t^{\pi^*}(s_t) = \arg\max_{a_t \in \mathcal{A}} \left\{ u(s_t, a_t) + \min_{p \in \mathcal{P}(s_t, a_t)} \delta\, \mathrm{E}_p \left[v_{t+1}^{\pi^*}(s_{t+1}) \,\middle|\, s_t \right] \right\}$$

  end if

end for

Mathematical Framework: Ambiguity Set

Figure 1. Stylized illustration ambiguity set, where *D* denotes a probability divergence measure (Pardo, 2005)

$$\mathcal{P}(p_0, \eta) \equiv \left\{ p : D(p \,||\, p_0) \leq \eta \right\}$$

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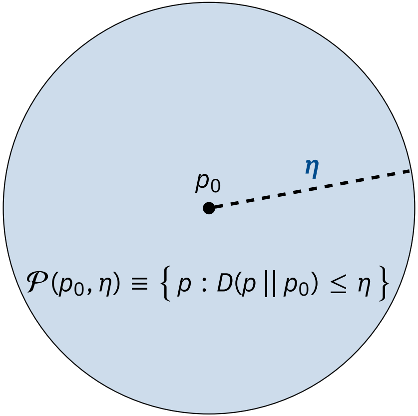

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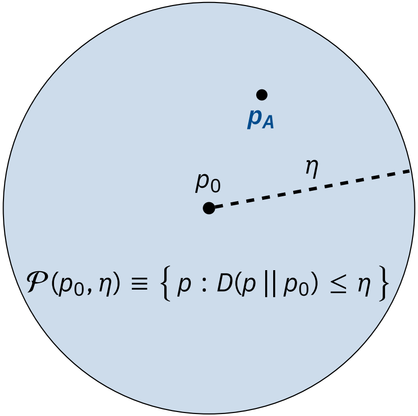

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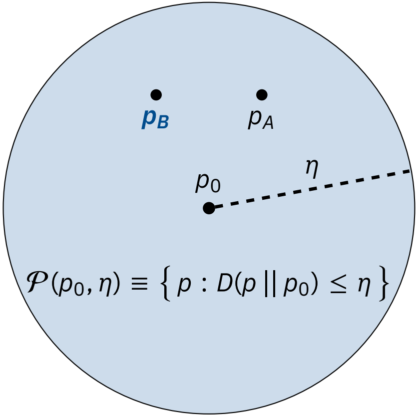

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Mathematical Framework: Worst Case Probabilities

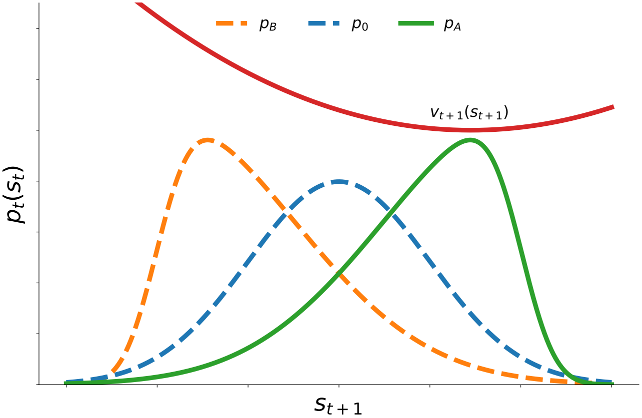
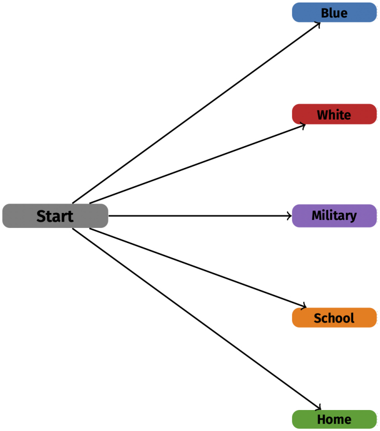


Figure 2. Selection of worst case probability distribution, implemented via **`robupy`**

Computational Implementation

Workhose model: Keane and Wolpin (1997)



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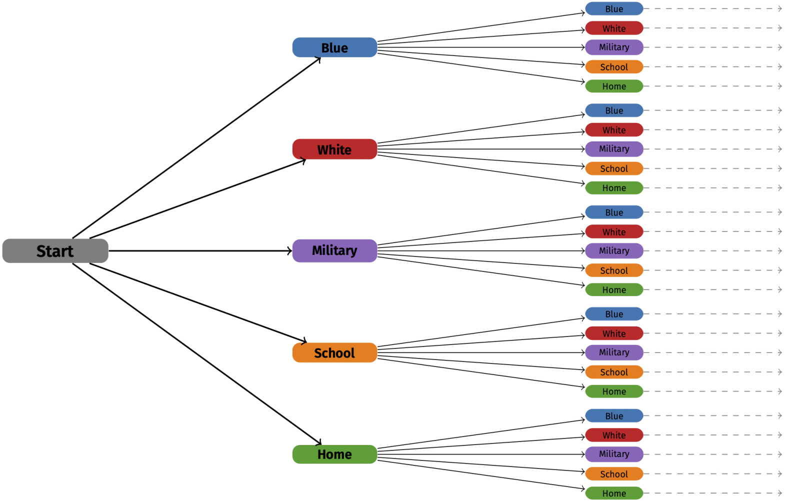

Computational Implementation: Operationalization Model

Structure of utility functions

$$u(\cdot) \;=\; \begin{cases} w_a(\boldsymbol{k}_t, h_t, t, a_{t-1}, e_{j,a}, \varepsilon_{a,t}) & \text{if} \quad a_t \in \{1, 2, 3\} \\ \xi_a(\boldsymbol{k}_t, h_t, t, a_{t-1}, e_{j,a}, \varepsilon_{a,t}) & \text{if} \quad a_t \in \{4, 5\} \end{cases}$$

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Work experience $\boldsymbol{k_t}$ and years of completed schooling h_t evolve deterministically

$$\begin{aligned} k_{a,t+1} &= k_{a,t} + \mathbb{I}[a_t = a] \quad \text{if} \quad a \in \{1, 2, 3\} \\ h_{t+1} &= h_t \quad + \mathbb{I}[a_t = 4] \end{aligned}$$

Computational Implementation: Operationalization Model

Productivity and taste shocks

$$\boldsymbol{\varepsilon}_t \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}) \equiv \mathcal{N}_{\mathbf{0}}$$

Unrestricted covariance matrix, serially uncorrelated

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Operationalize ambiguity set with Kullback-Leibler divergence D_{KL} (Kullback and Leibler, 1951)

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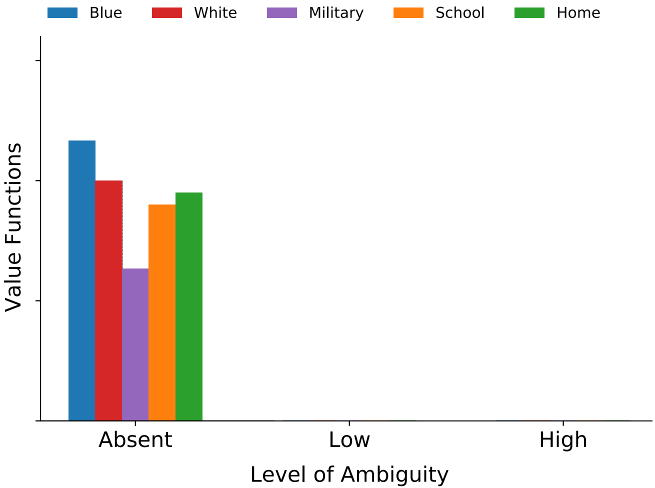
Special case $\eta = 0$ leads to decision-making under risk, i.e. $\boldsymbol{\varepsilon}_t \sim \mathcal{N}_{\mathbf{0}}$

Computational Implementation: Admissible Value Functions



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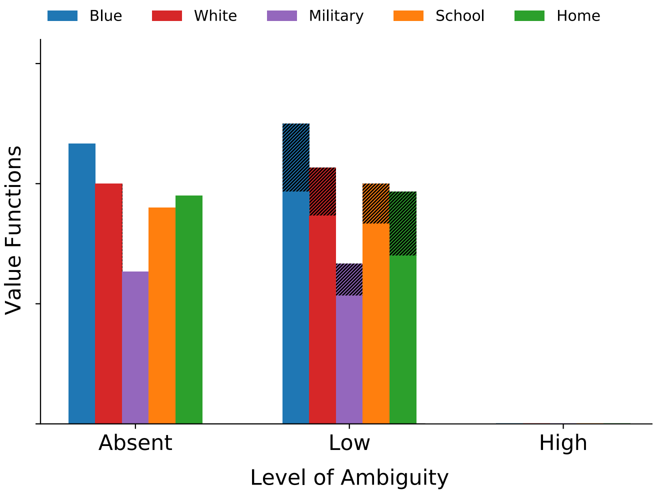

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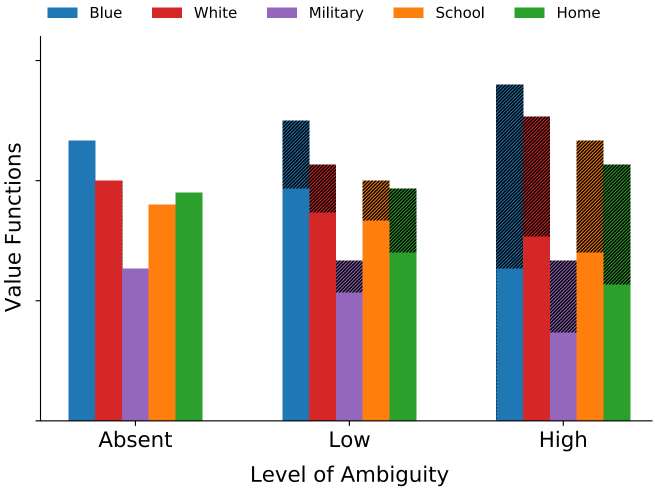

Data

Data: Estimation and Validation Sample

Estimation sample: Original Keane and Wolpin (1997) data set (11 periods of NLSY79)

- Estimate model parameters via Method of Simulated Moments procedure

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- Estimate model parameters via Method of Simulated Moments procedure

Validation sample: Extended Keane and Wolpin (1997) data set (35 periods of NLSY79)

- Used for out-of-sample validation exercise only

Results

Results: Model Credibility

Within–sample: Robust human capital model ($\eta = 1.60$) achieves best fit

- Simulate choice probabilities
- Compare against simulated choice probabilities from risk-only model ($\eta = 0.00$)
- Compare against observed choice probabilities from estimation sample

Results: Model Credibility

Within–sample fit

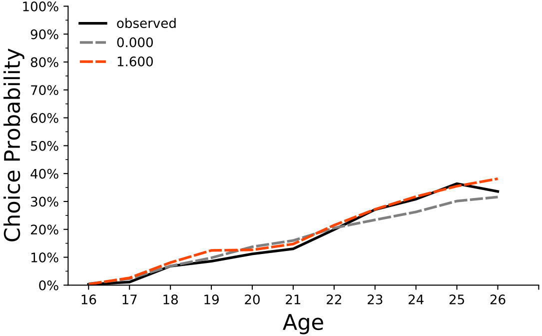

Figure 4. Choice probabilities white-collar

Results: Model Credibility

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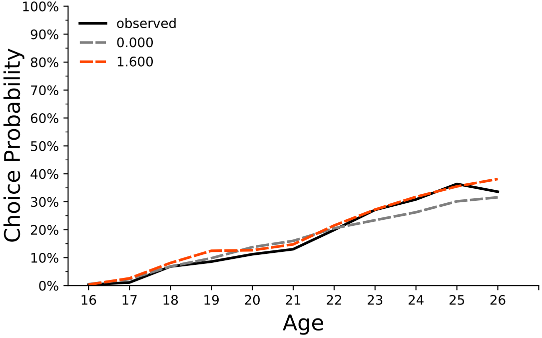

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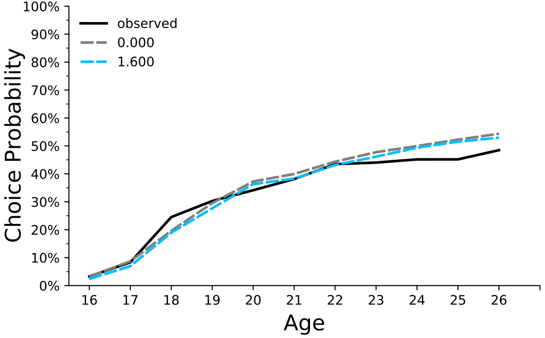

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Out–of–sample: Robust human capital model ($\eta = 1.60$) achieves best fit

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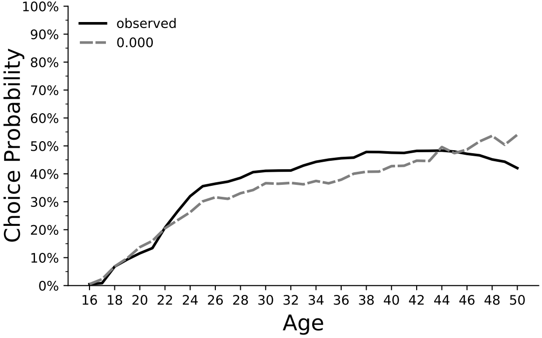
- Compare against observed choice probabilities from **validation sample**

Results: Model Credibility

Out–of–sample fit



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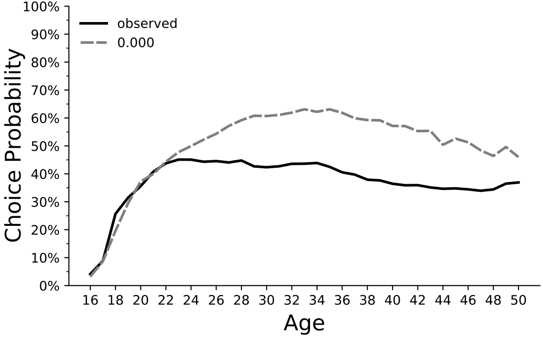
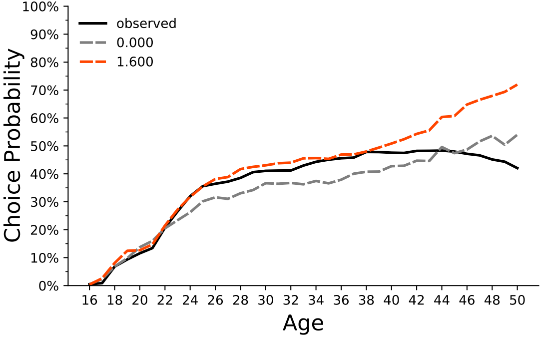


Figure 7. Choice probabilities blue-collar

Results: Model Credibility

Out–of–sample fit



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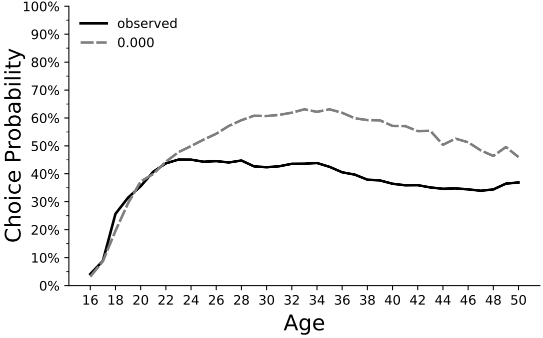

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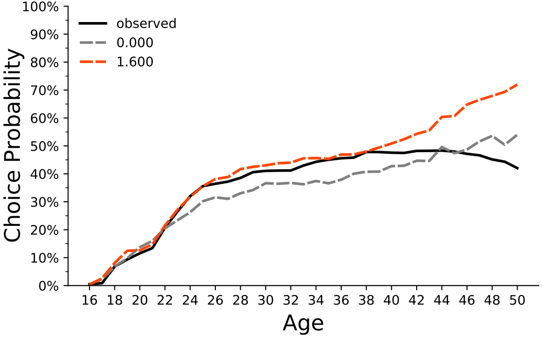

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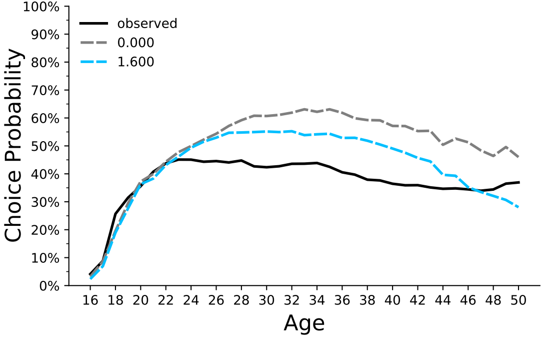

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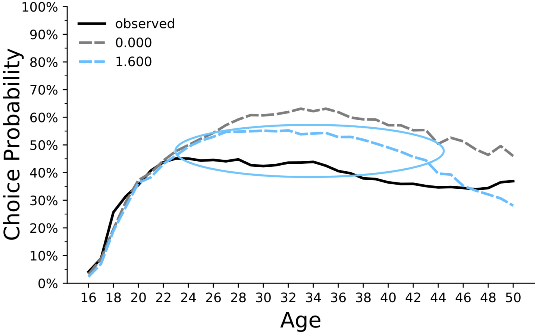


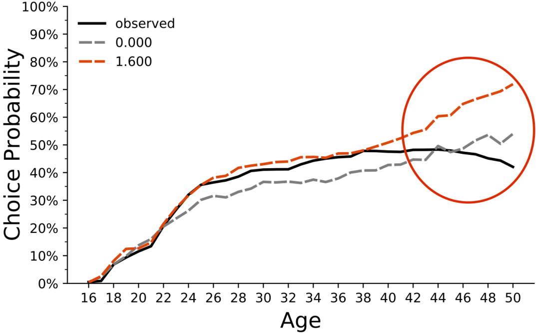
Figure 6. Model reconciliation

Data-driven explanation of the remaining gap: **Residual category home**

- **Health-related** factors (Hokayem and Ziliak, 2014; Capatina, 2015; Blundell, Britton, Costa Dias, and French, 2016)
- **Employment-inhibiting** effects, e.g. incarceration (Mueller-Smith, 2015; Bhuller, Dahl, Loken, and Mogstad, 2020)

Results: Model Credibility

Out–of–sample fit



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- **Health-related** factors and retirement (Hokayem and Ziliak, 2014; Capatina, 2015; Blundell, Britton, et al., 2016)
- **Structural breaks** that replaced white-collar with blue-collar work around the year 2000 (Beaudry, Green, and Sand, 2016)

Conclusion

Conclusion: Summary

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1. We have generalized the standard human capital investment model to account for risk and ambiguity by relying on results from robust optimization
   - Our extension is computationally tractable and a framework for further applications
2. Our out-of-sample validation reveals shortcomings of the risk-only approach and shows a much better ouf-of-sample fit of the robust human capital model
3. Ambiguity leads to novel economic model mechanisms and different policy responses

Thank you!

References

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Appendix: Bellman Optimality Equations

Bellman Optimality Equations

- Compute $\pi^* = \left(a_1^{\pi^*}(s_1), \ldots, a_T^{\pi^*}(s_T)\right)$ by solving inductively defined single-stage problems

Appendix: Bellman Optimality Equations: Bellman Optimality Equations

Bellman Optimality Equations

- Compute $\pi^* = \left(a_1^{\pi^*}(s_1), \ldots, a_T^{\pi^*}(s_T)\right)$ by solving inductively defined single-stage problems
- **Standard** human capital investment model: At each s_t retrieve

$$a_t^{\pi^*}(s_t) = \arg\max_{a_t \in \mathcal{A}} \left\{ u(s_t, a_t) + \delta \, \mathrm{E}_{p^{\pi^*}} \left[v_{t+1}^{\pi^*}(s_{t+1}) \,\middle|\, s_t \right] \right\}$$

Bellman Optimality Equations

- Compute $\pi^* = \left(a_1^{\pi^*}(s_1), \ldots, a_T^{\pi^*}(s_T)\right)$ by solving inductively defined single-stage problems
- **Robust** human capital investment model: At each s_t retrieve

$$a_t^{\pi^*}(s_t) = \underset{a_t \in \mathcal{A}}{\arg\max} \left\{ u(s_t, a_t) + \delta \min_{p \in \mathcal{P}^{\pi}(s_t, a_t)} \mathrm{E}_{p^{\pi^*}} \left[v_{t+1}^{\pi^*}(s_{t+1}) \,\middle|\, s_t \right] \right\}$$

Bellman Optimality Equations

- Compute $\pi^* = \left(a_1^{\pi^*}(s_1), \ldots, a_T^{\pi^*}(s_T)\right)$ by solving inductively defined single-stage problems
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Solution

- Choose $P(s_t, a_t)$ such that it satisfies rectangularity condition (Epstein and Schneider, 2003, Definition 3.1)
- Implementation of backward induction procedure for robust Bellman equations (Iyengar, 2005; Nilim and El Ghaoui, 2005)

Appendix: Estimation and Validation Data Set

Appendix: Estimation Data Set

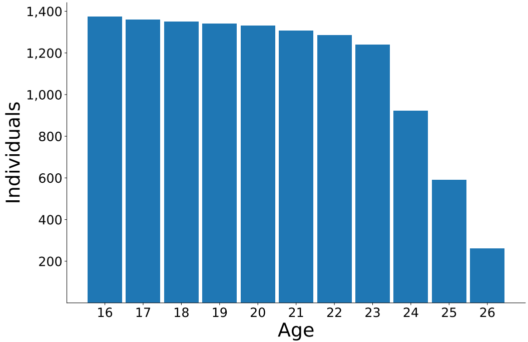
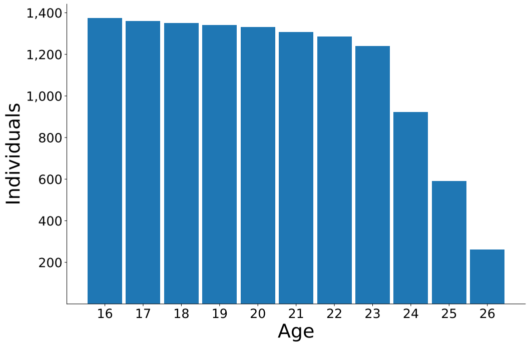


Figure 7. Sample size by age

Appendix: Estimation Data Set



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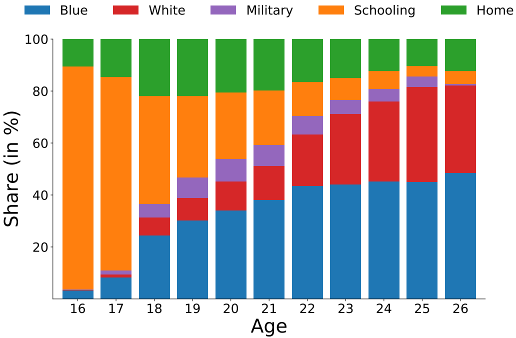

Figure 8. Observed choices by age

Appendix: Estimation Data Set

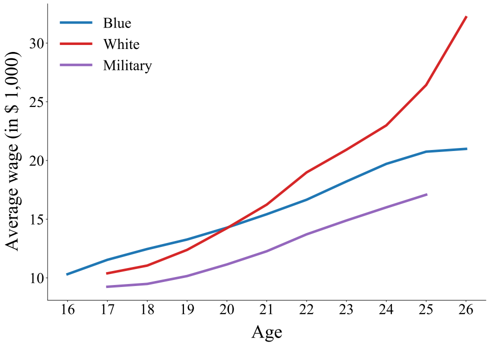

Figure 7. Observed wages by age

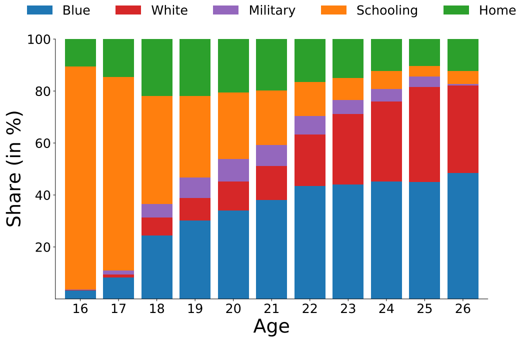

Figure 8. Observed choices by age

Appendix: Validation Data Set

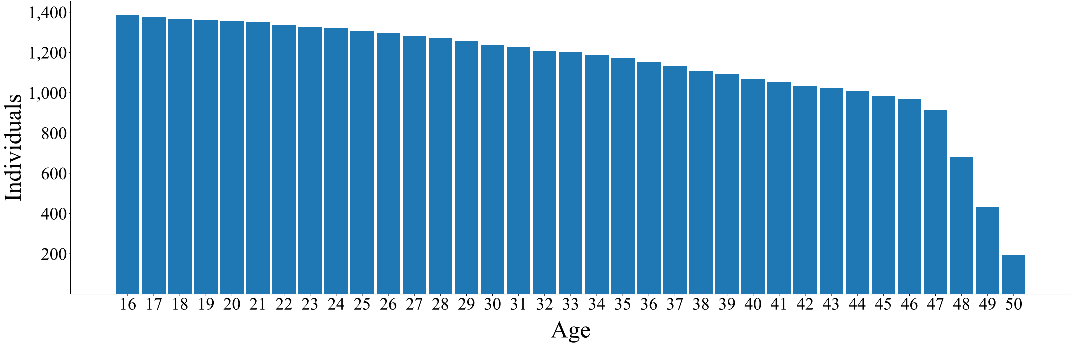

Figure 9. Sample size by age

Appendix: Validation Data Set

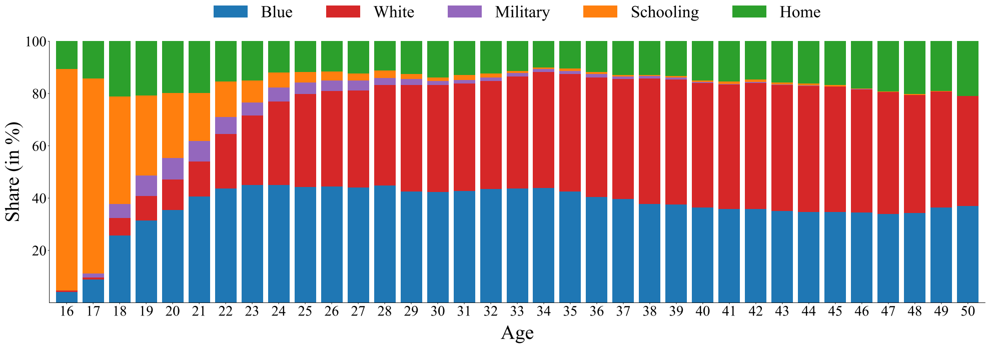

Figure 10. Observed choices by age

Appendix: Validation Data Set

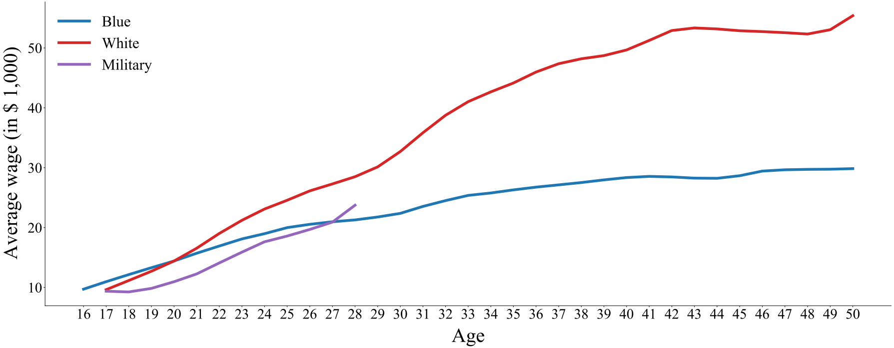

Figure 11. Observed wages by age

Appendix: Policy Evaluation

Appendix: Policy Evaluation

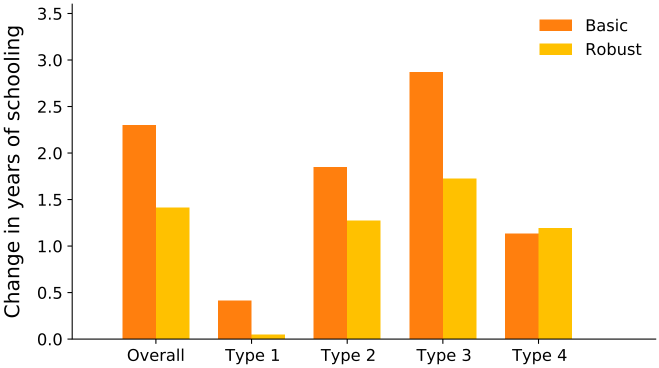

Figure 12. Effect $2, 000 college tuition subsidy

Main Insight: Sluggish policy response in case of ambiguity – tuition subsidy less effective

Appendix: Comparative Statics

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Main idea: Increase risk by dispersing distribution of productivity and taste shocks

Example for two-dimensional normal distribution

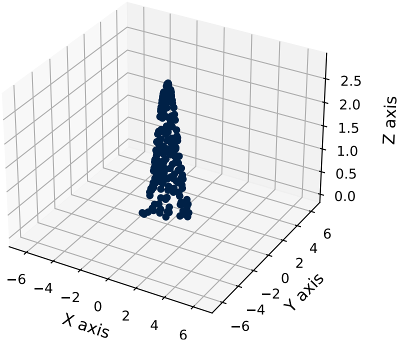
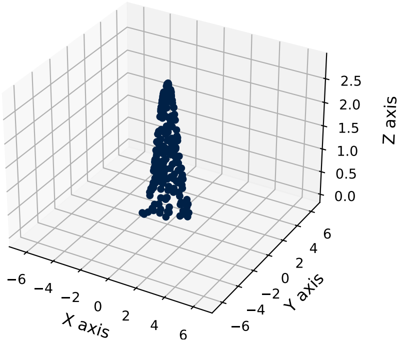


Figure 13. Baseline case

Appendix: Comparative Statics

Main idea: Increase risk by dispersing distribution of productivity and taste shocks

Example for two-dimensional normal distribution



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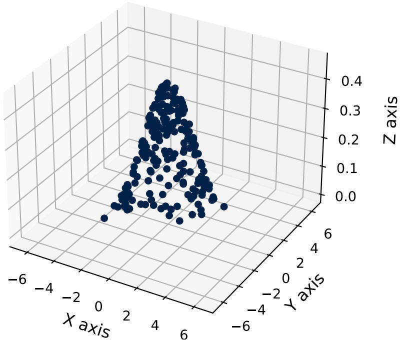

Figure 14. Dispersed case

Appendix: Comparative Statics

Main idea: Increase risk by dispersing distribution of productivity and taste shocks

Implementation

Calibrate **dispersion factor** $\varphi(\eta)$ such that

$$D_{\mathrm{KL}}\left(\mathcal{N}\left(\mathbf{0}, \boldsymbol{\Sigma}_{\boldsymbol{\eta}}\right) \,||\, \mathcal{N}\left(\mathbf{0}, \boldsymbol{\Sigma}\right)\right) = \eta,$$

where D_{KL} is the Kullback and Leibler (1951) divergence

Appendix: Comparative Statics

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Implementation

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where D_{KL} is the Kullback and Leibler (1951) divergence

Direct **link between ambiguity and risk**

- Can we replace ambiguity with risk?

Appendix: Comparative Statics – Choice Shares under Risk and Ambiguity

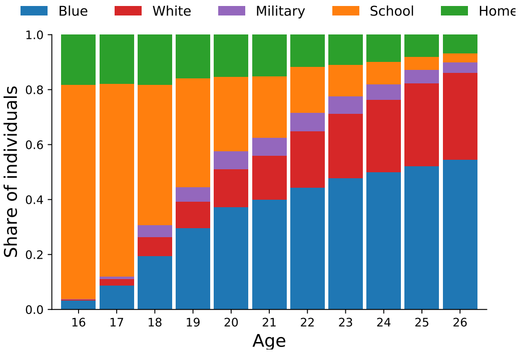

Figure 13. Standard model under risk

Appendix: Comparative Statics – Choice Shares under Risk and Ambiguity

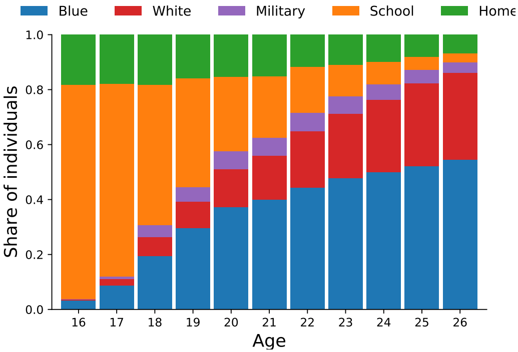

Figure 13. Standard model under risk

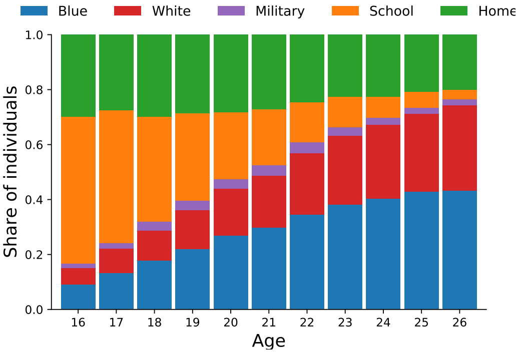

Figure 14. Standard model with **increased risk**

Insight: Home acts as an absorbing career

Appendix: Comparative Statics – Choice Shares under Risk and Ambiguity

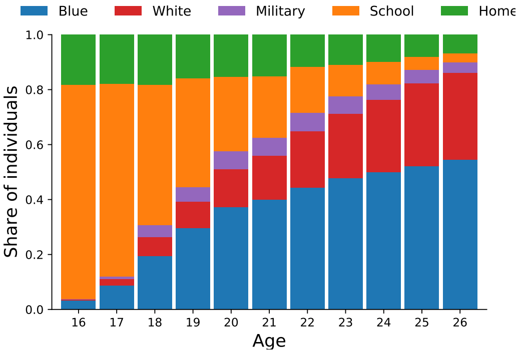

Figure 13. Standard model under risk

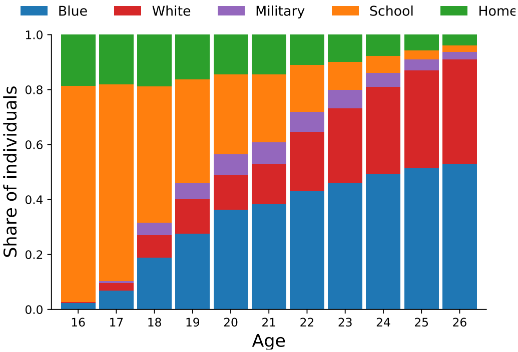

Figure 14. Robust human capital model

Insight: Initial schooling and white-collar occupation act as insurance