Supplementary File of ‘Efficient Resource Allocation in Cooperative Co-evolution for Large-scale Global Optimization’

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Contents

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TABLE I: The average fitness values ± standard deviations on the CEC'2010 and CEC'2013 functions over 25 independent runs. The significant best results are in bold font (Wilcoxon rank sum test with Holm p-value correction, α=0.05). R^+, R^- and p-value are obtained through multiple-problem analysis by the Wilcoxon test between CCFR-I (U=D_i) and its competitors.

	CEC'2010 Functions		
F	CCFR-I (U=D_i)	CCFR-I (U=2D_i)	CCFR-I (U=10D_i)
f_1	1.20e-05±4.89e-06	1.31e-05±5.19e-06	1.68e-05±6.54e-06$\uparrow$
f_2	**2.75e+01±5.25e+00**	5.13e+01±5.04e+00$\uparrow$	1.52e+02±7.22e+00$\uparrow$
f_3	**4.56e+00±4.63e-01**	5.56e+00±4.63e-01$\uparrow$	8.10e+00±4.65e-01$\uparrow$
f_4	8.33e+10±6.16e+10	8.69e+10±4.68e+10	1.06e+11±4.31e+10$\uparrow$
f_5	7.23e+07±1.32e+07	7.32e+07±1.22e+07	9.12e+07±1.74e+07$\uparrow$
f_6	7.74e+05±7.15e+05	7.83e+05±8.28e+05	7.28e+05±8.51e+05
f_7	**1.49e-03±2.47e-04**	1.66e-03±2.78e-04$\uparrow$	2.14e-03±3.90e-04$\uparrow$
f_8	3.19e+05±1.08e+06	6.38e+05±1.46e+06	9.57e+05±1.70e+06$\uparrow$
f_9	9.38e+06±1.18e+06	8.81e+06±1.05e+06	1.05e+07±1.44e+06$\uparrow$
f_{10}	1.41e+03±1.01e+02	1.42e+03±7.83e+01	1.61e+03±1.10e+02$\uparrow$
f_{11}	1.03e+01±2.71e+00	9.72e+00±2.11e+00	1.00e+01±2.59e+00
f_{12}	**1.17e+00±4.57e+00**	4.72e+00±1.75e+01$\uparrow$	7.49e+00±2.30e+01$\uparrow$
f_{13}	3.18e+02±9.91e+01	3.25e+02±1.01e+02	4.03e+02±9.45e+01$\uparrow$
f_{14}	2.48e+07±2.85e+06	2.48e+07±2.85e+06	2.48e+07±2.85e+06
f_{15}	2.81e+03±1.31e+02	2.81e+03±1.31e+02	2.81e+03±1.31e+02
f_{16}	2.01e+01±2.62e+00	2.01e+01±2.62e+00	2.01e+01±2.62e+00
f_{17}	9.78e+00±1.09e+01	9.78e+00±1.09e+01	9.78e+00±1.09e+01
f_{18}	1.14e+03±1.82e+02	1.14e+03±1.82e+02	1.14e+03±1.82e+02
f_{19}	1.16e+06±9.47e+04	1.16e+06±9.47e+04	1.16e+06±9.47e+04
f_{20}	1.01e+09±8.96e+08	1.01e+09±8.96e+08	1.01e+09±8.96e+08
R^+	—	168.0	170.0
R^-	—	42.0	40.0
p-value	—	2.66e-02	1.71e-02
	CEC'2013 Functions		
F	CCFR-I (U=D_i)	CCFR-I (U=2D_i)	CCFR-I (U=10D_i)
f_1	1.30e-05±3.18e-06	1.40e-05±3.49e-06	1.80e-05±4.65e-06$\uparrow$
f_2	**5.51e-01±1.47e+00**	5.33e+01±1.70e+01$\uparrow$	3.14e+02±2.05e+01$\uparrow$
f_3	2.00e+01±3.06e-07	**2.00e+01±3.23e-07**$\downarrow$	2.00e+01±3.89e-04$\uparrow$
f_4	4.50e+07±1.66e+07	5.26e+07±2.22e+07	7.47e+07±2.31e+07$\uparrow$
f_5	2.53e+06±2.67e+05	2.47e+06±3.75e+05	2.62e+06±3.88e+05
f_6	1.06e+06±1.19e+03	**1.06e+06±1.30e+03**$\downarrow$	1.07e+06±1.64e+03$\uparrow$
f_7	8.60e+06±1.90e+07	9.94e+06±2.64e+07	1.04e+07±1.85e+07
f_8	9.61e+09±1.59e+10	9.61e+09±1.59e+10	9.61e+09±1.59e+10
f_9	1.85e+08±2.79e+07	1.84e+08±2.70e+07	1.84e+08±2.73e+07
f_{10}	9.47e+07±1.86e+05	9.46e+07±3.84e+05	**9.43e+07±3.44e+05**$\downarrow$
f_{11}	3.25e+08±3.24e+08	2.53e+08±3.33e+08	3.28e+08±3.38e+08
f_{12}	6.00e+08±7.09e+08	6.00e+08±7.09e+08	6.00e+08±7.09e+08
f_{13}	9.28e+08±5.33e+08	9.28e+08±5.33e+08	9.28e+08±5.33e+08
f_{14}	2.14e+09±2.11e+09	2.14e+09±2.11e+09	2.14e+09±2.11e+09
f_{15}	8.25e+06±3.28e+06	8.25e+06±3.28e+06	8.25e+06±3.28e+06
R^+	—	49.5	89.5
R^-	—	70.5	30.5
p-value	—	6.25e-01	1.60e-01

The symbols $\uparrow$ and $\downarrow$ denote that the CCFR-I (U=D_i) algorithm performs significantly better than and worse than this algorithm at a 0.05 significance level by the Wilcoxon rank sum test, respectively.

I. The Sensitivity Study of the Parameter U of CCFR

Table I summarizes the results of CCFR-I with different values of the parameter U (see Eq. (6a) in the paper) on the CEC'2010 and CEC'2013 large-scale functions [1], [2]. D_i is the dimensionality of a subcomponent.

For the functions with separable variables (i.e., the CEC'2010 functions f_1–f_{13} and the CEC'2013 functions f_1–f_7), the smaller value of U, the better performance of CCFR in general. This is because CCFR with a small value of U can stop early evolution for the stagnant subpopulations. It can save more computational resources on the separable and stagnant variables than CCFR with a larger value of U. Therefore, we use U=D_i as the default setting of U. For the functions without separable variables, the subpopulations hardly enter the stagnant state, so there is no difference between CCFR-I with different values of U. Overall, CCFR-I with different values of U had similar performances on most of the CEC'2010 and CEC'2013 functions.

II. The Scale-up Study on CCFR

We use the block-rotated ellipsoid function [3] to study the performance of CCFR-I, CBCC1-I, CBCC2-I and CC-I with the scale-up dimensionality of the function and the number of subcomponents. The dimensionality of the function ranges from

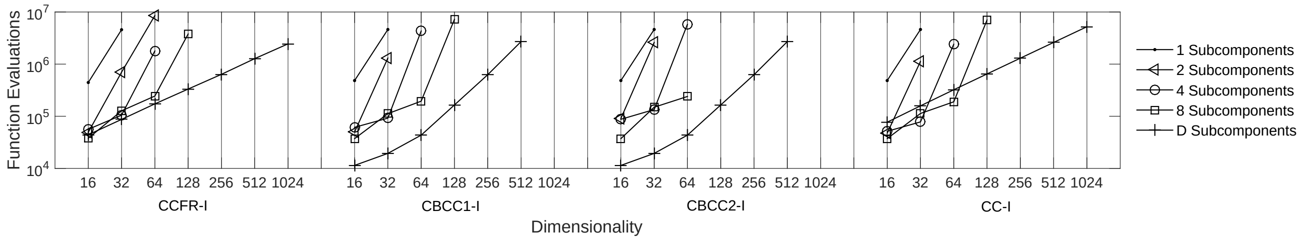


Fig. 1: For the block-rotated ellipsoid function with variant numbers of subcomponents, the average function evaluations used by CCFR-I, CBCC1-I, CBCC2-I and CC-I over successful runs out of 10 runs.

2^4 to 2^{10}. The numbers of subcomponents are $\{1,2,4,8,D\}$, where D is the dimensionality. Within 10^7 function evaluations, if the best overall objective value is smaller than a target value (i.e., 0.1) in a run, CCFR-I stops running and this run is considered successful. Fig. 1 shows the average number of function evaluations over successful runs out of 10 runs. CCFR-I can reach the target value in 10^7 function evaluations when there are less than 64 variables in a subcomponent. When the number of variables in a subcomponent is equal to or smaller than 8, the number of function evaluations linearly increases as the dimensionality of the function and the number of subcomponents increase. When there are more than 8 variables in a subcomponent, the number of function evaluations increases rapidly and linearly as the dimensionality of the function and the number of subcomponents increase. It can be seen in Fig. 1 that CBCC1-I, CBCC2-I and CC-I have similar performances to CCFR-I, but for CCFR-I, as the dimensionality of the function and the number of subcomponents increase, the number of function evaluations increases less rapidly than other three CC frameworks.

III. The Performance of CCFR with Groupings

In order to study the effect of decomposition on the performance of CCFR, we test CCFR with two grouping methods (DG [4] and IDG2 [5]). DG is a differential grouping method with a theoretical foundation, which is able to group the interacting variables with a high accuracy. In DG, the parameter ϵ was set to 10^{-3}, which was recommended in [4]. IDG2 is an improved variant of DG, which is able to group the interacting variables better than DG. Table II summarizes the grouping results of IDG2 and DG.

Table III summarizes the optimization results of CCFR, CBCC1 [6], CBCC2 [6] and DECC [4] with IDG2 and DG. Note that, for the algorithms with IDG2 and DG, the function evaluations spent by grouping (see the ‘FEs’ column in Table II) are counted into the entire function evaluations. The multiple-problem analysis results show that CCFR-IDG2 and CCFR-DG performed better than other peer algorithms on the CEC’2010 and CEC’2013 functions.

CCFR-DG performed significantly better than other peer algorithms with DG on most separable functions f_1–f_3. For the almost partially separable functions (the CEC’2010 functions f_4–f_{18}; the CEC’2013 functions f_4–f_{11}), the difference between results of the algorithms with DG is not significant. For the CEC’2010 functions f_7, f_8 and f_{13}, DG is not able to identify the interdependence between variables. There is interdependence between the subcomponents formed by DG. CCFR-DG performed worse than CBCC1-DG and DECC-DG by several orders of magnitude. This indicates that if there is interdependence between the subcomponents, it may be a good way to optimize each subcomponent one by one.

CCFR-IDG2 significantly outperformed other peer algorithms on most separable functions f_1–f_3 by several orders of magnitude. CCFR-IDG2 outperformed other peer algorithms on most partially separable functions (the CEC’2010 functions f_4–f_{18}; the CEC’2013 functions f_4–f_{11}). For the partially separable functions on which CCFR-IDG2 performed worse, the difference between results of CCFR and other peer algorithms is not significant. But for the functions on which CCFR-IDG2 performed better, the difference is significant. For the nonseparable functions (the CEC’2010 functions f_{19}–f_{20}; the CEC’2013 functions f_{12}–f_{15}), all variables are grouped into one subcomponent. Therefore, there is no significant difference between the algorithms with IDG2 on these nonseparable functions.

For most functions, the algorithms with IDG2 performed better than the ones with DG. This is because IDG2 can identify the interdependence between variables with higher accuracies than DG. The multiple-problem analysis results show that compared with DG, IDG2 made CCFR performed much better than other peer algorithms. The performances of CCFR-IDG2 and CCFR-DG do not differ greatly on most of the functions where CCFR-IDG2 performed worse than CCFR-DG. But for most of the functions where CCFR-IDG2 performed better than CCFR-DG, CCFR-IDG2 outperformed greatly CCFR-DG by several orders of magnitude due to its higher grouping accuracy for nonseparable variables on these functions (i.e., the CEC’2010 functions f_7, f_8, f_{13} and f_{18}; the CEC’2013 functions f_4, f_7, f_8 and f_{11}). The experimental results show that the performance of CCFR is dependent on the decomposition method. A high grouping accuracy, especially for the nonseparable variables, can improve the performance of CCFR.

TABLE II: The grouping results on the CEC'2010 and CEC'2013 functions. The values of IDG2 and DG are separated by "/". The bold font denotes IDG2 performed better than DG; the gray background denotes IDG2 performed worse than DG.

CEC'2010 Functions

F	Sep Vars	Non-Sep		IDG2 / DG ($\epsilon = 10^{-3}$)						
		Vars	Groups	FEs	Sep			Non-sep		
					Formed Vars	Captured Vars	Accuracy	Formed Subcomponents	Captured Subcomponents	Accuracy
f_1	1000	0	0	500501 / 1001000	1000 / 1000	1000 / 1000	100.0% / 100.0%	0 / 0	0 / 0	100.0% / 100.0%
f_2	1000	0	0	500501 / 1001000	1000 / 1000	1000 / 1000	100.0% / 100.0%	0 / 0	0 / 0	100.0% / 100.0%
f_3	1000	0	0	500501 / 1001000	0 / 1000	0 / 1000	0.0% / 100.0%	1 / 0	0 / 0	100.0% / 100.0%
f_4	950	50	1	500501 / 14554	950 / 33	950 / 33	**100.0% / 3.5%**	1 / 10	1 / 1	100.0% / 100.0%
f_5	950	50	1	500501 / 905450	950 / 950	950 / 950	100.0% / 100.0%	1 / 1	1 / 1	100.0% / 100.0%
f_6	950	50	1	500501 / 906332	854 / 950	854 / 950	89.9% / 100.0%	2 / 1	1 / 1	100.0% / 100.0%
f_7	950	50	1	500501 / 67742	950 / 248	950 / 248	**100.0% / 26.1%**	1 / 4	1 / 0	**100.0% / 0.0%**
f_8	950	50	1	500501 / 23286	950 / 134	950 / 133	**100.0% / 14.0%**	1 / 5	1 / 0	**100.0% / 0.0%**
f_9	500	500	10	500501 / 270802	500 / 500	500 / 500	100.0% / 100.0%	10 / 10	10 / 10	100.0% / 100.0%
f_{10}	500	500	10	500501 / 272958	500 / 500	500 / 500	100.0% / 100.0%	10 / 10	10 / 10	100.0% / 100.0%
f_{11}	500	500	10	500501 / 270640	0 / 501	0 / 500	0.0% / 100.0%	11 / 10	10 / 9	**100.0% / 90.0%**
f_{12}	500	500	10	500501 / 271390	500 / 500	500 / 500	100.0% / 100.0%	10 / 10	10 / 10	100.0% / 100.0%
f_{13}	500	500	10	500501 / 50328	500 / 131	500 / 107	**100.0% / 21.4%**	10 / 34	10 / 0	**100.0% / 0.0%**
f_{14}	0	1000	20	500501 / 21000	0 / 0	0 / 0	100.0% / 100.0%	20 / 20	20 / 20	100.0% / 100.0%
f_{15}	0	1000	20	500501 / 21000	0 / 0	0 / 0	100.0% / 100.0%	20 / 20	20 / 20	100.0% / 100.0%
f_{16}	0	1000	20	500501 / 21128	0 / 4	0 / 0	100.0% / 100.0%	20 / 20	20 / 16	**100.0% / 80.0%**
f_{17}	0	1000	20	500501 / 21000	0 / 0	0 / 0	100.0% / 100.0%	20 / 20	20 / 20	100.0% / 100.0%
f_{18}	0	1000	20	500501 / 39624	0 / 78	0 / 0	100.0% / 100.0%	20 / 50	20 / 0	**100.0% / 0.0%**
f_{19}	0	1000	1	500501 / 2000	0 / 0	0 / 0	100.0% / 100.0%	1 / 1	1 / 1	100.0% / 100.0%
f_{20}	0	1000	1	500501 / 155430	0 / 33	0 / 0	100.0% / 100.0%	1 / 241	1 / 0	**100.0% / 0.0%**

CEC'2013 Functions

F	Sep Vars	Non-Sep		IDG2 / DG ($\epsilon = 10^{-3}$)						
		Vars	Groups	FEs	Sep			Non-sep		
					Formed Vars	Captured Vars	Accuracy	Formed Subcomponents	Captured Subcomponents	Accuracy
f_1	1000	0	0	500501 / 1001000	1000 / 1000	1000 / 1000	100.0% / 100.0%	0 / 0	0 / 0	100.0% / 100.0%
f_2	1000	0	0	500501 / 1001000	1000 / 1000	1000 / 1000	100.0% / 100.0%	0 / 0	0 / 0	100.0% / 100.0%
f_3	1000	0	0	500501 / 1001000	0 / 1000	0 / 1000	0.0% / 100.0%	1 / 0	0 / 0	100.0% / 100.0%
f_4	700	300	7	500501 / 15792	700 / 40	700 / 40	**100.0% / 5.7%**	7 / 13	7 / 3	**100.0% / 58.3%**
f_5	700	300	7	500501 / 527026	700 / 707	700 / 700	100.0% / 100.0%	7 / 10	7 / 6	**100.0% / 66.7%**
f_6	700	300	7	500501 / 579848	0 / 752	0 / 700	0.0% / 100.0%	8 / 5	7 / 3	**100.0% / 50.0%**
f_7	700	300	7	500501 / 11452	700 / 64	700 / 64	**100.0% / 9.1%**	7 / 10	7 / 0	**100.0% / 0.0%**
f_8	0	1000	20	500501 / 22682	200 / 4	0 / 0	100.0% / 100.0%	18 / 25	18 / 14	**80.0% / 65.0%**
f_9	0	1000	20	500501 / 17650	0 / 0	0 / 0	100.0% / 100.0%	20 / 20	20 / 20	100.0% / 100.0%
f_{10}	0	1000	20	500501 / 48650	0 / 152	0 / 0	100.0% / 100.0%	20 / 18	20 / 14	**100.0% / 65.0%**
f_{11}	0	1000	20	500501 / 9102	0 / 1	0 / 0	100.0% / 100.0%	20 / 18	20 / 0	**100.0% / 0.0%**
f_{12}	0	1000	1	500501 / 149894	0 / 50	0 / 0	100.0% / 100.0%	1 / 222	1 / 0	**100.0% / 0.0%**
f_{13}	0	905	1	409966 / 18786	0 / 0	0 / 0	100.0% / 100.0%	1 / 20	1 / 0	**100.0% / 0.0%**
f_{14}	0	905	1	409966 / 26698	0 / 0	0 / 0	100.0% / 100.0%	1 / 19	1 / 0	**100.0% / 0.0%**
f_{15}	0	1000	1	500501 / 2000	0 / 0	0 / 0	100.0% / 100.0%	1 / 1	1 / 1	100.0% / 100.0%

IV. Comparison between CCFR-IDG2 and Non-CC Algorithms

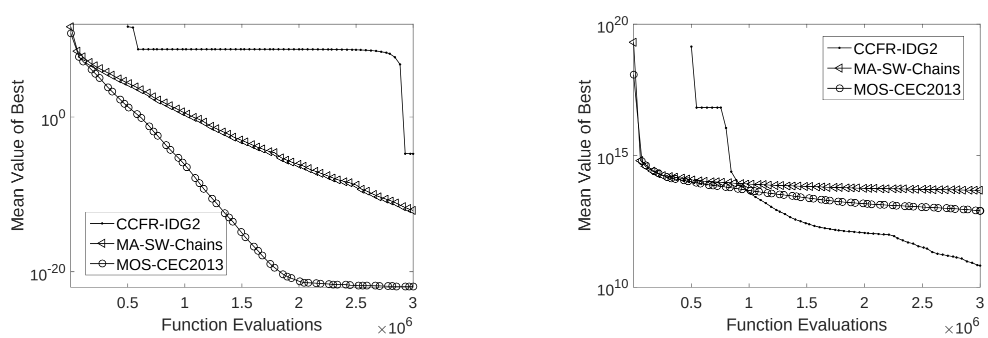
Table IV summarizes the results of CCFR-IDG2, MA-SW-Chains [7] and MOS-CEC2013 [8]. For the partially separable functions (the CEC'2010 functions f_4–f_{18}; the CEC'2013 functions f_4–f_{11}) on which CCFR-IDG2 performed better than MA-SW-Chains, the difference between the results of CCFR-IDG2 and MA-SW-Chains is very significant. But for the partially separable functions on which CCFR-IDG2 performed worse than MA-SW-Chains, the difference is not significant except for the CEC'2010 function f_{12}. CCFR-IDG2 performed worse than MOS-CEC2013 on most of the CEC'2010 and CEC'2013 functions. For the nonseparable functions (the CEC'2010 functions f_{19}–f_{20}; the CEC'2013 functions f_{12}–f_{15}), CCFR-IDG2 optimized all decision variables together and performed significantly worse than MA-SW-Chains and MOS-CEC2013. This indicates that the optimizer used by CCFR-IDG2 (i.e., SaNSDE) is inferior to MA-SW-Chains and MOS-CEC2013. The multiple-problem analysis shows that CCFR-IDG2 performed worse than MA-SW-Chains and MOS-CEC2013 on the CEC'2013 functions. This may be because that the optimizer used by CCFR-IDG2 is worse than MA-SW-Chains and MOS-CEC2013. The previous experimental results have shown that for a given optimizer (i.e., SaNSDE), CCFR is superior to other peer algorithms.

Fig. 2 shows the convergence behavior of CCFR-IDG2, MA-SW-Chains and MOS-CEC2013. Because CCFR-IDG2 spends 500501 function evaluations grouping the decision variables, in Fig. 2 the convergence lines of CCFR-IDG2 start from 500502 function evaluations. For the separable function f_1, CCFR-IDG2 optimized each separable variable one by one and converged slowly, but when CCFR-IDG2 finished evolving the last variable with the largest weight value, the best overall objective value dropped sharply. f_8 is a partially separable function with imbalance between subcomponents. For f_8, compared with MA-SW-

TABLE III: The average fitness values ± standard deviations on the CEC'2010 and CEC'2013 functions over 25 independent runs. The significant best results are in bold font (Wilcoxon rank sum test with Holm p-value correction, α=0.05). R^+, R^- and p-value have similar meanings as in Table I.

CEC'2010 Functions								
F	CCFR-IDG2	CBCC1-IDG2	CBCC2-IDG2	DECC-IDG2	CCFR-DG	CBCC1-DG	CBCC2-DG	DECC-DG
f_1	**1.6e-05±6.5e-06**	1.7e+07±2.1e+07↑	1.7e+07±2.1e+07↑	1.7e+07±2.1e+07↑	4.8e+08±9.8e+07	2.9e+07±3.1e+07↓	2.9e+07±3.1e+07↓	2.9e+07±3.1e+07↓
f_2	**1.7e+02±8.6e+00**	4.7e+03±4.8e+02↑	4.7e+03±4.8e+02↑	4.7e+03±4.8e+02↑	**3.2e+02±1.7e+01**	4.7e+03±4.8e+02↑	4.7e+03±4.8e+02↑	4.7e+03±4.8e+02↑
f_3	1.2e+01±3.7e-01	1.2e+01±3.7e-01	1.2e+01±3.7e-01	1.2e+01±3.7e-01	**1.1e+01±3.8e-01**	1.2e+01±3.7e-01↑	1.2e+01±3.7e-01↑	1.2e+01±3.7e-01↑
f_4	1.3e+11±7.5e+10	7.4e+10±4.8e+10↓	1.1e+11±2.9e+10	8.9e+10±4.6e+10↓	4.3e+10±1.6e+10	3.5e+11±2.0e+11↑	5.1e+10±3.1e+10	7.8e+11±5.5e+11↑
f_5	9.2e+07±1.6e+07	6.8e+07±1.1e+07↓	6.8e+07±9.4e+06↓	6.7e+07±1.0e+07↓	4.9e+08±2.4e+07	6.9e+07±1.0e+07↓	6.9e+07±1.0e+07↓	6.9e+07±1.1e+07↓
f_6	6.8e+05±7.1e+05	1.1e+06±7.9e+05↑	1.1e+06±6.9e+05↑	6.4e+05±6.8e+05	1.1e+07±7.5e+05	1.3e+06±6.4e+05↓	1.3e+06±6.4e+05↓	**8.1e+05±7.2e+05**↓
f_7	**2.0e-03±3.5e-04**	7.9e+04±1.0e+04↑	1.1e+05±1.8e+04↑	4.2e+04±1.2e+04↑	2.7e+07±7.0e+07	1.1e+05±8.5e+04↓	7.6e+09±6.6e+09↑	**6.0e+04±3.3e+04**↓
f_8	**3.2e+05±1.1e+06**	8.8e+05±1.6e+06↑	1.1e+06±1.7e+06↑	5.2e+05±1.3e+06↑	2.6e+08±1.9e+08	**4.6e+06±8.8e+06**↓	6.3e+07±6.0e+07↓	1.5e+07±2.3e+07↓
f_9	1.3e+07±1.7e+06	2.1e+07±2.2e+07	4.4e+09±7.0e+08↑	5.4e+07±6.4e+07↑	1.1e+07±1.4e+06	1.8e+07±2.1e+07	1.8e+07±2.1e+07	3.3e+07±2.0e+07↑
f_{10}	**1.8e+03±1.4e+02**	3.4e+03±1.7e+02↑	4.6e+03±7.7e+02↑	4.3e+03±1.8e+02↑	**1.6e+03±1.2e+02**	3.2e+03±1.7e+02↑	3.2e+03±1.7e+02↑	4.1e+03±1.7e+02↑
f_{11}	**2.0e+01±3.3e+00**	2.4e+01±2.4e+00↑	2.5e+01±2.3e+00↑	2.3e+01±2.1e+00↑	**1.1e+01±2.5e+00**	2.3e+01±2.2e+00↑	2.3e+01±2.1e+00↑	2.3e+01±2.7e+00↑
f_{12}	**2.0e+01±2.2e+01**	2.6e+04±7.4e+03↑	3.7e+04±9.7e+03↑	2.3e+04±8.8e+03↑	**4.6e+00±6.9e+00**	2.2e+04±6.3e+03↑	2.2e+04±6.3e+03↑	1.9e+04±7.3e+03↑
f_{13}	**5.3e+02±1.0e+02**	2.6e+04±7.8e+03↑	3.9e+04±6.2e+03↑	2.5e+04±7.8e+03↑	2.8e+06±9.2e+05	**5.8e+03±4.4e+03**↓	1.6e+04±7.8e+03↓	8.7e+03±3.9e+03↓
f_{14}	**3.1e+07±3.3e+06**	3.5e+07±2.6e+06↑	9.5e+09±5.2e+08↑	3.3e+07±2.7e+06↑	**2.5e+07±2.9e+06**	2.8e+07±2.1e+06↑	2.8e+07±2.1e+06↑	2.7e+07±2.2e+06↑
f_{15}	**3.2e+03±1.5e+02**	4.4e+03±1.5e+02↑	4.6e+03±1.7e+02↑	4.4e+03±1.9e+02↑	**2.8e+03±1.3e+02**	4.0e+03±1.5e+02↑	4.0e+03±1.5e+02↑	4.0e+03±1.6e+02↑
f_{16}	2.0e+01±2.6e+00	1.9e+01±3.2e+00	2.0e+01±3.4e+00	2.0e+01±4.0e+00	2.4e+01±4.3e+00	2.0e+01±3.4e+00↓	2.1e+01±3.1e+00	2.1e+01±3.4e+00
f_{17}	**6.7e+01±8.7e+01**	1.3e+02±8.9e+01↑	7.2e+02±3.4e+02↑	8.0e+01±5.2e+01↑	1.1e+01±1.1e+01	3.6e+01±4.9e+01↑	3.6e+01±4.9e+01↑	2.4e+01±3.7e+01
f_{18}	1.4e+03±1.9e+02	1.3e+03±1.9e+02	1.7e+03±2.4e+02↑	1.2e+03±1.5e+02↓	**1.3e+08±9.9e+07**	6.9e+09±2.3e+09↑	1.4e+10±2.0e+09↑	2.1e+10±3.9e+09↑
f_{19}	1.3e+06±1.0e+05	1.3e+06±1.0e+05	1.3e+06±1.0e+05	1.3e+06±1.0e+05	1.2e+06±9.5e+04	1.2e+06±9.5e+04	1.2e+06±9.5e+04	1.2e+06±9.5e+04
f_{20}	2.0e+09±1.8e+09	2.0e+09±1.8e+09	2.0e+09±1.8e+09	2.0e+09±1.8e+09	**3.1e+07±6.6e+06**	1.4e+10±2.7e+09↑	1.6e+08±1.5e+08↑	3.3e+10±5.9e+09↑
R^+	—	165.0	174.0	153.0	—	123.0	137.0	123.0
R^-	—	45.0	36.0	57.0	—	87.0	73.0	87.0
p-value	—	2.51e-02	1.00e-02	7.31e-02	—	5.02e-01	2.32e-01	5.02e-01
CEC'2013 Functions								
F	CCFR-IDG2	CBCC1-IDG2	CBCC2-IDG2	DECC-IDG2	CCFR-DG	CBCC1-DG	CBCC2-DG	DECC-DG
f_1	**1.8e-05±4.5e-06**	4.6e+07±1.3e+08↑	4.6e+07±1.3e+08↑	4.6e+07±1.3e+08↑	4.8e+08±6.9e+07	6.2e+07±1.3e+08↓	6.2e+07±1.3e+08↓	6.2e+07±1.3e+08↓
f_2	**3.6e+02±2.1e+01**	2.1e+04±1.0e+03↑	2.1e+04±1.0e+03↑	2.1e+04±1.0e+03↑	**7.4e+02±4.0e+01**	2.1e+04±1.0e+03↑	2.1e+04±1.0e+03↑	2.1e+04±1.0e+03↑
f_3	2.1e+01±1.2e-02	2.1e+01±1.2e-02	2.1e+01±1.2e-02	2.1e+01±1.2e-02	**2.0e+01±6.0e-07**	2.1e+01±1.1e-02↑	2.1e+01±1.1e-02↑	2.1e+01±1.1e-02↑
f_4	**9.6e+07±4.0e+07**	2.2e+08±6.0e+07↑	6.6e+10±5.6e+09↑	2.9e+08±9.7e+07↑	9.1e+10±5.6e+10	8.7e+10±5.1e+10	4.6e+11±2.8e+11↑	8.3e+10±4.7e+10
f_5	2.8e+06±3.2e+05	2.6e+06±4.3e+05	2.5e+06±4.7e+05↓	3.0e+06±4.7e+05	3.0e+06±5.2e+05	2.8e+06±3.6e+05	2.6e+06±4.4e+05↓	3.3e+06±4.0e+05↑
f_6	1.1e+06±1.0e+03	1.1e+06±1.7e+03↓	1.1e+06±1.8e+03↓	1.1e+06±1.6e+03↓	1.1e+06±1.6e+03	1.1e+06±2.1e+03↓	1.1e+06±1.5e+03↓	1.1e+06±2.3e+03↓
f_7	2.0e+07±2.9e+07	2.2e+07±2.6e+07	9.9e+07±3.7e+08	2.4e+07±3.8e+07	1.4e+08±9.7e+07	1.2e+08±3.9e+07	1.6e+10±1.4e+10↑	1.4e+08±7.1e+07
f_8	**6.6e+10±9.5e+10**	2.3e+13±1.6e+13↑	1.1e+12±1.7e+11↑	7.4e+13±5.8e+13↑	1.6e+15±1.0e+15	2.0e+15±1.5e+15	5.9e+15±4.3e+15↑	2.0e+15±1.4e+15
f_9	**1.9e+08±2.8e+07**	2.6e+08±4.0e+07↑	2.3e+08±3.0e+07↑	3.0e+08±5.7e+07↑	**1.9e+08±2.8e+07**	2.5e+08±3.8e+07↑	2.2e+08±2.9e+07↑	2.9e+08±5.2e+07↑
f_{10}	9.5e+07±1.8e+05	9.4e+07±2.8e+05↓	9.4e+07±2.5e+05↓	9.5e+07±3.0e+05↓	9.5e+07±3.1e+05	9.4e+07±6.1e+05↓	9.4e+07±6.6e+05↓	9.4e+07±2.4e+05↓
f_{11}	4.2e+08±3.4e+08	5.0e+09±1.5e+10	7.3e+10±1.2e+11↑	2.8e+09±1.1e+10	**2.8e+10±6.0e+10**	4.5e+10±6.1e+10↑	5.2e+12±3.7e+12↑	4.7e+10±5.7e+10↑
f_{12}	1.6e+09±1.6e+09	1.6e+09±1.6e+09	1.6e+09±1.6e+09	1.6e+09±1.6e+09	**8.0e+07±8.3e+06**	6.0e+10±8.3e+09↑	6.6e+08±1.3e+08↑	1.2e+11±1.4e+10↑
f_{13}	1.2e+09±6.0e+08	1.2e+09±6.0e+08	1.2e+09±6.0e+08	1.2e+09±6.0e+08	**2.0e+09±1.0e+09**	4.0e+09±1.5e+09↑	4.1e+10±2.7e+10↑	6.3e+09±1.9e+09↑
f_{14}	3.4e+09±3.1e+09	3.5e+09±3.2e+09	3.5e+09±3.2e+09	3.5e+09±3.2e+09	7.4e+09±8.5e+09	1.3e+10±1.2e+10↑	5.0e+11±1.2e+12↑	8.9e+09±6.8e+09
f_{15}	9.8e+06±3.7e+06	9.9e+06±3.7e+06	9.9e+06±3.7e+06	9.9e+06±3.7e+06	8.3e+06±3.3e+06	8.3e+06±3.3e+06	8.3e+06±3.3e+06	8.3e+06±3.3e+06
R^+	—	107.0	107.0	112.0	—	80.0	99.0	91.0
R^-	—	13.0	13.0	8.0	—	40.0	21.0	29.0
p-value	—	5.37e-03	5.37e-03	1.53e-03	—	2.77e-01	2.56e-02	8.33e-02

The symbols ↑ and ↓ have similar meanings as in Table I.



(a) f_1 (b) f_8

Fig. 2: The average convergence graph over 25 independent runs on the selected CEC'2013 functions.

Chains and MOS-CEC2013, in the beginning of evolutionary process, CCFR-IDG2 converged very slowly. But when the first evolutionary cycle ended (about 0.8×10^6 function evaluations), CCFR-IDG2 started to allocate most computational resources to the subpopulation which made the greatest improvement of the best overall objective value. CCFR-IDG2 converged much faster than MA-SW-Chains and MOS-CEC2013. This indicates that if the optimizer used by CCFR-IDG2 (i.e., SaNSDE) performs well on a function, CCFR might outperform MA-SW-Chains and MOS-CEC2013 on this function.

To show a better performance of CCFR-IDG2, we replaced SaNSDE with a better optimizer (i.e., CMAES [9]). Table V summarizes the results of CCFR-IDG2 with the CMAES optimizer. CCFR-IDG2 with CMAES significantly outperformed MA-SW-Chains on almost CEC'2010 and CEC'2013 functions. CCFR-IDG2 with CMAES performed significantly better than MOS-CEC2013 by several orders of magnitude on most of the partially separable functions (the CEC'2010 functions f_4–f_{18};

TABLE IV: The average errors ± standard deviations on the CEC'2010 and CEC'2013 functions over 25 independent runs. The significant best results are in bold font (Wilcoxon rank sum test with Holm p-value correction, α=0.05). R^+, R^- and p-value have similar meanings as in Table I.

CEC'2010 Functions			
F	CCFR-IDG2	MA-SW-Chains	MOS-CEC2013
f_1	1.62e-05±6.55e-06	3.88e-14±3.59e-14↓	**0.00e+00±0.00e+00**↓
f_2	1.73e+02±8.62e+00	8.63e+02±5.84e+01↑	**0.00e+00±0.00e+00**↓
f_3	1.22e+01±3.66e-01	**5.41e-13±2.13e-13**↓	1.65e-12±6.73e-14↓
f_4	1.26e+11±7.50e+10	2.94e+11±9.32e+10↑	**1.56e+10±6.02e+09**↓
f_5	**9.15e+07±1.61e+07**	1.75e+08±1.03e+08↑	1.11e+08±2.25e+07↑
f_6	6.85e+05±7.05e+05	3.52e+04±1.72e+05	**1.22e-07±6.43e-08**↓
f_7	2.04e-03±3.45e-04	3.30e+02±1.40e+03	**0.00e+00±0.00e+00**↓
f_8	3.19e+05±1.08e+06	9.28e+06±2.36e+07↑	**1.95e+00±8.03e+00**↓
f_9	1.34e+07±1.68e+06	1.45e+07±1.59e+06	**3.46e+06±4.49e+05**↓
f_{10}	**1.81e+03±1.43e+02**	2.06e+03±1.19e+02↑	3.78e+03±1.47e+02↑
f_{11}	**1.99e+01±3.26e+00**	3.69e+01±8.24e+00↑	1.91e+02±4.07e-01↑
f_{12}	2.03e+01±2.23e+01	3.19e-06±5.32e-07↓	**0.00e+00±0.00e+00**↓
f_{13}	5.26e+02±1.04e+02	1.09e+03±6.29e+02↑	7.14e+02±5.68e+02
f_{14}	3.08e+07±3.35e+06	3.34e+07±3.37e+06↑	**9.80e+06±6.03e+05**↓
f_{15}	3.18e+03±1.51e+02	**2.69e+03±9.75e+01**↓	7.44e+03±1.84e+02↑
f_{16}	**2.01e+01±2.62e+00**	1.08e+02±1.51e+01↑	3.82e+02±1.55e+01↑
f_{17}	6.72e+01±8.68e+01	1.26e+00±9.45e-02↓	**2.83e-07±7.97e-08**↓
f_{18}	1.37e+03±1.93e+02	1.87e+03±5.79e+02↑	1.54e+03±7.46e+02
f_{19}	1.28e+06±1.01e+05	2.85e+05±1.74e+04↓	**2.91e+04±2.14e+03**↓
f_{20}	1.97e+09±1.83e+09	1.05e+03±7.59e+01↓	**3.52e+02±4.43e+02**↓
R^+	—	143.0	73.0
R^-	—	67.0	137.0
p-value	—	1.56e-01	2.32e-01
CEC'2013 Functions			
F	CCFR-IDG2	MA-SW-Chains	MOS-CEC2013
f_1	1.77e-05±4.52e-06	8.49e-13±1.09e-12↓	**1.27e-22±7.41e-23**↓
f_2	**3.64e+02±2.06e+01**	1.22e+03±1.14e+02↑	8.32e+02±4.48e+01↑
f_3	2.07e+01±1.21e-02	2.14e+01±5.62e-02↑	**9.18e-13±5.12e-14**↓
f_4	**9.56e+07±4.03e+07**	4.58e+09±2.46e+09↑	1.74e+08±7.87e+07↑
f_5	2.80e+06±3.18e+05	**1.87e+06±3.06e+05**↓	6.94e+06±8.85e+05↑
f_6	1.06e+06±1.05e+03	1.01e+06±1.53e+04↓	**1.48e+05±6.43e+04**↓
f_7	2.03e+07±2.94e+07	3.45e+06±1.27e+06	**1.62e+04±9.10e+03**↓
f_8	**6.63e+10±9.52e+10**	4.85e+13±1.02e+13↑	8.00e+12±3.07e+12↑
f_9	1.89e+08±2.83e+07	**1.07e+08±1.68e+07**↓	3.83e+08±6.29e+07↑
f_{10}	9.48e+07±1.82e+05	9.18e+07±1.06e+06↓	**9.02e+05±5.07e+05**↓
f_{11}	4.17e+08±3.43e+08	2.19e+08±2.98e+07	**5.22e+07±2.05e+07**↓
f_{12}	1.56e+09±1.58e+09	1.25e+03±1.05e+02↓	**2.47e+02±2.54e+02**↓
f_{13}	1.21e+09±6.00e+08	1.98e+07±1.82e+06↓	**3.40e+06±1.06e+06**↓
f_{14}	3.39e+09±3.06e+09	1.36e+08±2.11e+07↓	**2.56e+07±7.94e+06**↓
f_{15}	9.82e+06±3.69e+06	5.71e+06±7.57e+05↓	**2.35e+06±1.94e+05**↓
R^+	—	34.0	41.0
R^-	—	86.0	79.0
p-value	—	1.51e-01	3.03e-01

The symbols ↑ and ↓ have similar meanings as in Table I.

the CEC'2013 functions f_4–f_{11}).

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TABLE V: The average errors ± standard deviations on the CEC'2010 and CEC'2013 functions over 25 independent runs. The significant best results are in bold font (Wilcoxon rank sum test with Holm p-value correction, α=0.05). R^+, R^- and p-value have similar meanings as in Table I.

CEC'2010 Functions			
F	CCFR-IDG2 (CMAES)	MA-SW-Chains	MOS-CEC2013
f_1	4.01e-03±1.41e-03	3.88e-14±3.59e-14↓	**0.00e+00±0.00e+00**↓
f_2	5.41e+02±4.80e+01	8.63e+02±5.84e+01↑	**0.00e+00±0.00e+00**↓
f_3	1.02e+00±3.98e-01	**5.41e-13±2.13e-13**↓	1.65e-12±6.73e-14↓
f_4	**4.31e+02±6.35e+02**	2.94e+11±9.32e+10↑	1.56e+10±6.02e+09↑
f_5	9.84e+07±1.38e+07	1.75e+08±1.03e+08↑	1.11e+08±2.25e+07
f_6	9.38e+00±6.70e-01	3.52e+04±1.72e+05↑	**1.22e-07±6.43e-08**↓
f_7	2.15e-04±4.49e-05	3.30e+02±1.40e+03	**0.00e+00±0.00e+00**↓
f_8	**6.76e-03±4.94e-03**	9.28e+06±2.36e+07↑	1.95e+00±8.03e+00↑
f_9	**1.39e+05±2.04e+04**	1.45e+07±1.59e+06	3.46e+06±4.49e+05↑
f_{10}	**1.31e+03±6.41e+01**	2.06e+03±1.19e+02↑	3.78e+03±1.47e+02↑
f_{11}	**2.35e-01±4.08e-01**	3.69e+01±8.24e+00↑	1.91e+02±4.07e-01↑
f_{12}	1.90e-07±3.64e-07	3.19e-06±5.32e-07↑	**0.00e+00±0.00e+00**↓
f_{13}	3.58e+02±1.08e+02	1.09e+03±6.29e+02↑	7.14e+02±5.68e+02
f_{14}	**2.61e-19±3.26e-20**	3.34e+07±3.37e+06↑	9.80e+06±6.03e+05↑
f_{15}	**2.04e+03±8.22e+01**	2.69e+03±9.75e+01↑	7.44e+03±1.84e+02↑
f_{16}	**8.07e-13±2.60e-14**	1.08e+02±1.51e+01↑	3.82e+02±1.55e+01↑
f_{17}	**7.42e-24±1.63e-25**	1.26e+00±9.45e-02↑	2.83e-07±7.97e-08↑
f_{18}	**1.09e+01±6.87e+00**	1.87e+03±5.79e+02↑	1.54e+03±7.46e+02↑
f_{19}	**2.12e+04±2.21e+03**	2.85e+05±1.74e+04↑	2.91e+04±2.14e+03↑
f_{20}	8.50e+02±2.50e+01	1.05e+03±7.59e+01↑	**3.52e+02±4.43e+02**↓
R^+	—	205.0	167.0
R^-	—	5.0	43.0
p-value	—	1.89e-04	2.06e-02
CEC'2013 Functions			
F	CCFR-IDG2 (CMAES)	MA-SW-Chains	MOS-CEC2013
f_1	3.97e-03±3.15e-04	8.49e-13±1.09e-12↓	**1.27e-22±7.41e-23**↓
f_2	**4.35e+02±3.55e+01**	1.22e+03±1.14e+02↑	8.32e+02±4.48e+01↑
f_3	2.04e+01±5.30e-02	2.14e+01±5.62e-02↑	**9.18e-13±5.12e-14**↓
f_4	**1.98e+05±6.47e+04**	4.58e+09±2.46e+09↑	1.74e+08±7.87e+07↑
f_5	2.19e+06±3.11e+05	**1.87e+06±3.06e+05**↓	6.94e+06±8.85e+05↑
f_6	9.99e+05±1.26e+04	1.01e+06±1.53e+04↑	**1.48e+05±6.43e+04**↓
f_7	**7.55e+01±1.13e+01**	3.45e+06±1.27e+06↑	1.62e+04±9.10e+03↑
f_8	**4.89e+03±1.23e+03**	4.85e+13±1.02e+13↑	8.00e+12±3.07e+12↑
f_9	1.59e+08±3.33e+07	**1.07e+08±1.68e+07**↓	3.83e+08±6.29e+07↑
f_{10}	9.11e+07±1.35e+06	9.18e+07±1.06e+06↑	**9.02e+05±5.07e+05**↓
f_{11}	**5.09e-05±7.89e-05**	2.19e+08±2.98e+07↑	5.22e+07±2.05e+07↑
f_{12}	1.01e+03±5.20e+01	1.25e+03±1.05e+02↑	**2.47e+02±2.54e+02**↓
f_{13}	**2.58e+06±3.00e+05**	1.98e+07±1.82e+06↑	3.40e+06±1.06e+06↑
f_{14}	3.63e+07±3.21e+06	1.36e+08±2.11e+07↑	**2.56e+07±7.94e+06**↓
f_{15}	2.80e+06±2.77e+05	5.71e+06±7.57e+05↑	**2.35e+06±1.94e+05**↓
R^+	—	102.0	77.0
R^-	—	18.0	43.0
p-value	—	1.51e-02	3.59e-01

The symbols ↑ and ↓ have similar meanings as in Table I.

[9] N. Hansen, S. D. Müller, and P. Koumoutsakos, "Reducing the time complexity of the derandomized evolution strategy with covariance matrix adaptation (cma-es)," *Evol. Comput.*, vol. 11, no. 1, pp. 1–18, Mar. 2003.