Supplementary File of 'Efficient Resource Allocation in Cooperative Co-evolution for Large-scale Global Optimization'

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I. THE SENSITIVITY STUDY OF THE PARAMETER U OF CCFR

Table I summarizes the results of CCFR-I with different values of the parameter U (see Eq. (6a) in the paper) on the CEC'2010 and CEC'2013 large-scale functions [1], [2]. D_i is the dimensionality of a subcomponent.

TABLE I
THE AVERAGE FITNESS VALUES ± STANDARD DEVIATIONS ON THE CEC'2010 AND CEC'2013 FUNCTIONS OVER 25 INDEPENDENT RUNS. THE SIGNIFICANT BEST RESULTS ARE IN BOLD FONT (WILCOXON RANK SUM TEST WITH HOLM p-VALUE CORRECTION, α=0.05). R^+, R^- AND p-VALUE ARE OBTAINED THROUGH MULTIPLE-PROBLEM ANALYSIS BY THE WILCOXON TEST BETWEEN CCFR-I (U=D_i) AND ITS COMPETITORS.

CEC'2010 Functions			
F	CCFR-I (U=D_i)	CCFR-I (U=2D_i)	CCFR-I (U=10D_i)
f_1	1.20e-05±4.89e-06	1.31e-05±5.19e-06	1.68e-05±6.54e-06↑
f_2	**2.75e+01±5.25e+00**	5.13e+01±5.04e+00↑	1.52e+02±7.22e+00↑
f_3	**4.56e+00±4.63e-01**	5.56e+00±4.63e-01↑	8.10e+00±4.65e-01↑
f_4	8.33e+10±6.16e+10	8.69e+10±4.68e+10	1.06e+11±4.31e+10↑
f_5	7.23e+07±1.32e+07	7.32e+07±1.22e+07	9.12e+07±1.74e+07↑
f_6	7.74e+05±7.15e+05	7.83e+05±8.28e+05	7.28e+05±8.51e+05
f_7	**1.49e-03±2.47e-04**	1.66e-03±2.78e-04↑	2.14e-03±3.90e-04↑
f_8	3.19e+05±1.08e+06	6.38e+05±1.46e+06	9.57e+05±1.70e+06↑
f_9	9.38e+06±1.18e+06	8.81e+06±1.05e+06	1.05e+07±1.44e+06↑
f_{10}	1.41e+03±1.01e+02	1.42e+03±7.83e+01	1.61e+03±1.10e+02↑
f_{11}	1.03e+01±2.71e+00	9.72e+00±2.11e+00	1.00e+01±2.59e+00
f_{12}	**1.17e+00±4.57e+00**	4.72e+00±1.75e+01↑	7.49e+00±2.30e+01↑
f_{13}	3.18e+02±9.91e+01	3.25e+02±1.01e+02	4.03e+02±9.45e+01↑
f_{14}	2.48e+07±2.85e+06	2.48e+07±2.85e+06	2.48e+07±2.85e+06
f_{15}	2.81e+03±1.31e+02	2.81e+03±1.31e+02	2.81e+03±1.31e+02
f_{16}	2.01e+01±2.62e+00	2.01e+01±2.62e+00	2.01e+01±2.62e+00
f_{17}	9.78e+00±1.09e+01	9.78e+00±1.09e+01	9.78e+00±1.09e+01
f_{18}	1.14e+03±1.82e+02	1.14e+03±1.82e+02	1.14e+03±1.82e+02
f_{19}	1.16e+06±9.47e+04	1.16e+06±9.47e+04	1.16e+06±9.47e+04
f_{20}	1.01e+09±8.96e+08	1.01e+09±8.96e+08	1.01e+09±8.96e+08
R^+	—	168.0	170.0
R^-	—	42.0	40.0
p-value	—	2.66e-02	1.71e-02
CEC'2013 Functions			
F	CCFR-I (U=D_i)	CCFR-I (U=2D_i)	CCFR-I (U=10D_i)
f_1	1.30e-05±3.18e-06	1.40e-05±3.49e-06	1.80e-05±4.65e-06↑
f_2	**5.51e-01±1.47e+00**	5.33e+01±1.70e+01↑	3.14e+02±2.05e+01↑
f_3	2.00e+01±3.06e-07	**2.00e+01±3.23e-07**↓	2.00e+01±3.89e-04↑
f_4	4.50e+07±1.66e+07	5.26e+07±2.22e+07	7.47e+07±2.31e+07↑
f_5	2.53e+06±2.67e+05	2.47e+06±3.75e+05	2.62e+06±3.88e+05
f_6	1.06e+06±1.19e+03	**1.06e+06±1.30e+03**↓	1.07e+06±1.64e+03↑
f_7	8.60e+06±1.90e+07	9.94e+06±2.64e+07	1.04e+07±1.85e+07
f_8	9.61e+09±1.59e+10	9.61e+09±1.59e+10	9.61e+09±1.59e+10
f_9	1.85e+08±2.79e+07	1.84e+08±2.70e+07	1.84e+08±2.73e+07
f_{10}	9.47e+07±1.86e+05	9.46e+07±3.84e+05	**9.43e+07±3.44e+05**↓
f_{11}	3.25e+08±3.24e+08	2.53e+08±3.33e+08	3.28e+08±3.38e+08
f_{12}	6.00e+08±7.09e+08	6.00e+08±7.09e+08	6.00e+08±7.09e+08
f_{13}	9.28e+08±5.33e+08	9.28e+08±5.33e+08	9.28e+08±5.33e+08
f_{14}	2.14e+09±2.11e+09	2.14e+09±2.11e+09	2.14e+09±2.11e+09
f_{15}	8.25e+06±3.28e+06	8.25e+06±3.28e+06	8.25e+06±3.28e+06
R^+	—	49.5	89.5
R^-	—	70.5	30.5
p-value	—	6.25e-01	1.60e-01

The symbols ↑ and ↓ denote that the CCFR-I (U=D_i) algorithm performs significantly better than and worse than this algorithm at a 0.05 significance level by the Wilcoxon rank sum test, respectively..

For the functions with separable variables (i.e., the CEC'2010 functions f_1–f_{13} and the CEC'2013 functions f_1–f_7), the smaller value of U, the the better performance of CCFR in general. This is because CCFR with a small value of U can stop early evolution for the stagnated subpopulations. It can save more computational resources on the separable and stagnated variables than CCFR with a larger value of U. Therefore, we use U=D_i as the default setting of U. For the functions without separable variables, the subpopulations hardly enter the stagnated state, so there is no difference between CCFR-I with different values of U. Overall, CCFR-I with different values of U had similar performances on most of the CEC'2010 and CEC'2013 functions.

TABLE II
THE GROUPING RESULTS ON THE CEC'2010 AND CEC'2013 FUNCTIONS. THE VALUES OF IDG2 AND DG ARE SEPARATED BY "/". THE BOLD FONT DENOTES IDG2 PERFORMED BETTER THAN DG; THE GRAY BACKGROUND DENOTES IDG2 PERFORMED WORSE THAN DG.

CEC'2010 Functions

F	Sep Vars	Non-Sep		IDG2 / DG ($\epsilon = 10^{-3}$)						
				FEs	Sep			Non-sep		
		Vars	Groups		Formed Vars	Captured Vars	Accuracy	Formed Subcomponents	Captured Subcomponents	Accuracy
f_1	1000	0	0	500501 / 1001000	1000 / 1000	1000 / 1000	100.0% / 100.0%	0 / 0	0 / 0	100.0% / 100.0%
f_2	1000	0	0	500501 / 1001000	1000 / 1000	1000 / 1000	100.0% / 100.0%	0 / 0	0 / 0	100.0% / 100.0%
f_3	1000	0	0	500501 / 1001000	0 / 1000	0 / 1000	0.0% / 100.0%	1 / 0	0 / 0	100.0% / 100.0%
f_4	950	50	1	500501 / 14554	950 / 33	950 / 33	**100.0% / 3.5%**	1 / 10	1 / 1	100.0% / 100.0%
f_5	950	50	1	500501 / 905450	950 / 950	950 / 950	100.0% / 100.0%	1 / 1	1 / 1	100.0% / 100.0%
f_6	950	50	1	500501 / 906332	854 / 950	854 / 950	89.9% / 100.0%	2 / 1	1 / 1	100.0% / 100.0%
f_7	950	50	1	500501 / 67742	950 / 248	950 / 248	**100.0% / 26.1%**	1 / 4	1 / 0	**100.0% / 0.0%**
f_8	950	50	1	500501 / 23286	950 / 134	950 / 133	**100.0% / 14.0%**	1 / 5	1 / 0	**100.0% / 0.0%**
f_9	500	500	10	500501 / 270802	500 / 500	500 / 500	100.0% / 100.0%	10 / 10	10 / 10	100.0% / 100.0%
f_{10}	500	500	10	500501 / 272958	500 / 500	500 / 500	100.0% / 100.0%	10 / 10	10 / 10	100.0% / 100.0%
f_{11}	500	500	10	500501 / 270640	0 / 501	0 / 500	0.0% / 100.0%	11 / 10	10 / 9	**100.0% / 90.0%**
f_{12}	500	500	10	500501 / 271390	500 / 500	500 / 500	100.0% / 100.0%	10 / 10	10 / 10	100.0% / 100.0%
f_{13}	500	500	10	500501 / 50328	500 / 131	500 / 107	**100.0% / 21.4%**	10 / 34	10 / 0	**100.0% / 0.0%**
f_{14}	0	1000	20	500501 / 21000	0 / 0	0 / 0	100.0% / 100.0%	20 / 20	20 / 20	100.0% / 100.0%
f_{15}	0	1000	20	500501 / 21000	0 / 0	0 / 0	100.0% / 100.0%	20 / 20	20 / 20	100.0% / 100.0%
f_{16}	0	1000	20	500501 / 21128	0 / 4	0 / 0	100.0% / 100.0%	20 / 20	20 / 16	**100.0% / 80.0%**
f_{17}	0	1000	20	500501 / 21000	0 / 0	0 / 0	100.0% / 100.0%	20 / 20	20 / 20	100.0% / 100.0%
f_{18}	0	1000	20	500501 / 39624	0 / 78	0 / 0	100.0% / 100.0%	20 / 50	20 / 0	**100.0% / 0.0%**
f_{19}	0	1000	1	500501 / 2000	0 / 0	0 / 0	100.0% / 100.0%	1 / 1	1 / 1	100.0% / 100.0%
f_{20}	0	1000	1	500501 / 155430	0 / 33	0 / 0	100.0% / 100.0%	1 / 241	1 / 0	**100.0% / 0.0%**

CEC'2013 Functions

F	Sep Vars	Non-Sep		IDG2 / DG ($\epsilon = 10^{-3}$)						
				FEs	Sep			Non-sep		
		Vars	Groups		Formed Vars	Captured Vars	Accuracy	Formed Subcomponents	Captured Subcomponents	Accuracy
f_1	1000	0	0	500501 / 1001000	1000 / 1000	1000 / 1000	100.0% / 100.0%	0 / 0	0 / 0	100.0% / 100.0%
f_2	1000	0	0	500501 / 1001000	1000 / 1000	1000 / 1000	100.0% / 100.0%	0 / 0	0 / 0	100.0% / 100.0%
f_3	1000	0	0	500501 / 1001000	0 / 1000	0 / 1000	0.0% / 100.0%	1 / 0	0 / 0	100.0% / 100.0%
f_4	700	300	7	500501 / 15792	700 / 40	700 / 40	**100.0% / 5.7%**	7 / 13	7 / 3	**100.0% / 58.3%**
f_5	700	300	7	500501 / 527026	700 / 707	700 / 700	100.0% / 100.0%	7 / 10	7 / 6	**100.0% / 66.7%**
f_6	700	300	7	500501 / 579848	0 / 752	0 / 700	0.0% / 100.0%	8 / 5	7 / 3	**100.0% / 50.0%**
f_7	700	300	7	500501 / 11452	700 / 64	700 / 64	**100.0% / 9.1%**	7 / 10	7 / 0	**100.0% / 0.0%**
f_8	0	1000	20	500501 / 22682	200 / 4	0 / 0	100.0% / 100.0%	18 / 25	18 / 14	**80.0% / 65.0%**
f_9	0	1000	20	500501 / 17650	0 / 0	0 / 0	100.0% / 100.0%	20 / 20	20 / 20	100.0% / 100.0%
f_{10}	0	1000	20	500501 / 48650	0 / 152	0 / 0	100.0% / 100.0%	20 / 18	20 / 14	**100.0% / 65.0%**
f_{11}	0	1000	20	500501 / 9102	0 / 1	0 / 0	100.0% / 100.0%	20 / 18	20 / 0	**100.0% / 0.0%**
f_{12}	0	1000	1	500501 / 149894	0 / 50	0 / 0	100.0% / 100.0%	1 / 222	1 / 0	**100.0% / 0.0%**
f_{13}	0	905	1	409966 / 18786	0 / 0	0 / 0	100.0% / 100.0%	1 / 20	1 / 0	**100.0% / 0.0%**
f_{14}	0	905	1	409966 / 26698	0 / 0	0 / 0	100.0% / 100.0%	1 / 19	1 / 0	**100.0% / 0.0%**
f_{15}	0	1000	1	500501 / 2000	0 / 0	0 / 0	100.0% / 100.0%	1 / 1	1 / 1	100.0% / 100.0%

II. THE PERFORMANCE OF CCFR WITH GROUPINGS

In order to study the effect of decomposition on the performance of CCFR, we test CCFR with two grouping methods (DG [3] and IDG2 [4]). DG is a differential grouping method with a theoretical foundation, which is able to group the interacting variables with a high accuracy. In DG, the parameter ϵ was set to 10^{-3}, which was recommended in [3]. IDG2 is an improved variant of DG, which is able to group the interacting variables better than DG. Table II summarizes the grouping results of IDG2 and DG.

Table III summarizes the optimization results of CCFR, CBCC1 [5], CBCC2 [5] and DECC [3] with IDG2 and DG. Note that, for the algorithms with IDG2 and DG, the function evaluations spent by grouping (see the 'FEs' column in Table II) are counted into the entire function evaluations. The multiple-problem analysis results show that CCFR-IDG2 and CCFR-DG performed better than other peer algorithms on the CEC'2010 and CEC'2013 functions.

CCFR-DG performed significantly better than other peer algorithms with DG on most separable functions f_1–f_3. For the almost partially separable functions (the CEC'2010 functions f_4–f_{18}; the CEC'2013 functions f_4–f_{11}), the difference between results of CCFR-DG and other peer algorithms with DG is not significant. For the CEC'2010 functions f_7, f_8 and f_{13}, DG is not able to identify the interdependence between variables. There is interdependence between the subcomponents formed by DG. CCFR-DG performed worse than CBCC1-DG and DECC-DG by several orders of magnitude. This indicates that if there is interdependence between the subcomponents, it may be a good way to optimize each subcomponent one by one.

CCFR-IDG2 significantly outperformed other peer algorithms on most separable functions f_1–f_3 by several orders of

TABLE III

THE AVERAGE FITNESS VALUES ± STANDARD DEVIATIONS ON THE CEC'2010 AND CEC'2013 FUNCTIONS OVER 25 INDEPENDENT RUNS. THE SIGNIFICANT BEST RESULTS ARE IN BOLD FONT (WILCOXON RANK SUM TEST WITH HOLM p-VALUE CORRECTION, α=0.05). R^+, R^- AND p-VALUE ARE OBTAINED THROUGH MULTIPLE-PROBLEM ANALYSIS BY THE WILCOXON TEST BETWEEN CCFR-IDG2 OR CCFR-DG AND ITS COMPETITOR.

CEC'2010 Functions

F	CCFR-IDG2	CBCC1-IDG2	CBCC2-IDG2	DECC-IDG2	CCFR-DG	CBCC1-DG	CBCC2-DG	DECC-DG
f_1	**1.6e-05±6.5e-06**	1.7e+07±2.1e+07↑	1.7e+07±2.1e+07↑	1.7e+07±2.1e+07↑	4.8e+08±9.8e+07	2.9e+07±3.1e+07↓	2.9e+07±3.1e+07↓	2.9e+07±3.1e+07↓
f_2	**1.7e+02±8.6e+00**	4.7e+03±4.8e+02↑	4.7e+03±4.8e+02↑	4.7e+03±4.8e+02↑	**3.2e+02±1.7e+01**	4.7e+03±4.8e+02↑	4.7e+03±4.8e+02↑	4.7e+03±4.8e+02↑
f_3	1.2e+01±3.7e-01	1.2e+01±3.7e-01	1.2e+01±3.7e-01	1.2e+01±3.7e-01	**1.1e+01±3.8e-01**	1.2e+01±3.7e-01↑	1.2e+01±3.7e-01↑	1.2e+01±3.7e-01↑
f_4	1.3e+11±7.5e+10	7.4e+10±4.8e+10↓	1.1e+11±2.9e+10	8.9e+10±4.6e+10↓	4.3e+10±1.6e+10	3.5e+11±2.0e+11↑	5.1e+10±3.1e+10	7.8e+11±5.5e+11↑
f_5	9.2e+07±1.6e+07	6.8e+07±1.1e+07↓	6.8e+07±9.4e+06↓	6.7e+07±1.0e+07↓	4.9e+08±2.4e+07	6.9e+07±1.0e+07↓	6.9e+07±1.0e+07↓	6.9e+07±1.1e+07↓
f_6	6.8e+05±7.1e+05	1.1e+06±7.9e+05↑	1.1e+06±6.9e+05↑	6.4e+05±6.8e+05	1.1e+07±7.5e+05	1.3e+06±6.4e+05↓	1.3e+06±6.4e+05↓	**8.1e+05±7.2e+05**↓
f_7	**2.0e-03±3.5e-04**	7.9e+04±1.0e+04↑	1.1e+05±1.8e+04↑	4.2e+04±1.2e+04↑	2.7e+07±7.0e+07	1.1e+05±8.5e+04↓	7.6e+09±6.6e+09↑	**6.0e+04±3.3e+04**↓
f_8	**3.2e+05±1.1e+06**	8.8e+05±1.6e+06↑	1.1e+06±1.7e+06↑	5.2e+05±1.3e+06↑	2.6e+08±1.9e+08	**4.6e+06±8.8e+06**↓	6.3e+07±6.0e+07↓	1.5e+07±2.3e+07↓
f_9	1.3e+07±1.7e+06	2.1e+07±2.2e+07	4.4e+09±7.0e+08↑	5.4e+07±6.4e+07↑	1.1e+07±1.4e+06	1.8e+07±2.1e+07	1.8e+07±2.1e+07	3.3e+07±2.0e+07↑
f_{10}	**1.8e+03±1.4e+02**	3.4e+03±1.7e+02↑	4.6e+03±7.7e+02↑	4.3e+03±1.8e+02↑	**1.6e+03±1.2e+02**	3.2e+03±1.7e+02↑	3.2e+03±1.7e+02↑	4.1e+03±1.7e+02↑
f_{11}	**2.0e+01±3.3e+00**	2.4e+01±2.4e+00↑	2.5e+01±2.3e+00↑	2.3e+01±2.1e+00↑	**1.1e+01±2.5e+00**	2.3e+01±2.2e+00↑	2.3e+01±2.1e+00↑	2.3e+01±2.7e+00↑
f_{12}	**2.0e+01±2.2e+01**	2.6e+04±7.4e+03↑	3.7e+04±9.7e+03↑	2.3e+04±8.8e+03↑	**4.6e+00±6.9e+00**	2.2e+04±6.3e+03↑	2.2e+04±6.3e+03↑	1.9e+04±7.3e+03↑
f_{13}	**5.3e+02±1.0e+02**	2.6e+04±7.8e+03↑	3.9e+04±6.2e+03↑	2.5e+04±7.8e+03↑	2.8e+06±9.2e+05	**5.8e+03±4.4e+03**↓	1.6e+04±7.8e+03↓	8.7e+03±3.9e+03↓
f_{14}	**3.1e+07±3.3e+06**	3.5e+07±2.6e+06↑	9.5e+09±5.2e+08↑	3.3e+07±2.7e+06↑	**2.5e+07±2.9e+06**	2.8e+07±2.1e+06↑	2.8e+07±2.1e+06↑	2.7e+07±2.2e+06↑
f_{15}	**3.2e+03±1.5e+02**	4.4e+03±1.5e+02↑	4.6e+03±1.7e+02↑	4.4e+03±1.9e+02↑	**2.8e+03±1.3e+02**	4.0e+03±1.5e+02↑	4.0e+03±1.5e+02↑	4.0e+03±1.6e+02↑
f_{16}	2.0e+01±2.6e+00	1.9e+01±3.2e+00	2.0e+01±3.4e+00	2.0e+01±4.0e+00	2.4e+01±4.3e+00	2.0e+01±3.4e+00↓	2.1e+01±3.1e+00	2.1e+01±3.4e+00
f_{17}	**6.7e+01±8.7e+01**	1.3e+02±8.9e+01↑	7.2e+02±3.4e+02↑	8.0e+01±5.2e+01↑	1.1e+01±1.1e+01	3.6e+01±4.9e+01↑	3.6e+01±4.9e+01↑	2.4e+01±3.7e+01
f_{18}	1.4e+03±1.9e+02	1.3e+03±1.9e+02	1.7e+03±2.4e+02↑	1.2e+03±1.5e+02↓	**1.3e+08±9.9e+07**	6.9e+09±2.3e+09↑	1.4e+10±2.0e+09↑	2.1e+10±3.9e+09↑
f_{19}	1.3e+06±1.0e+05	1.3e+06±1.0e+05	1.3e+06±1.0e+05	1.3e+06±1.0e+05	1.2e+06±9.5e+04	1.2e+06±9.5e+04	1.2e+06±9.5e+04	1.2e+06±9.5e+04
f_{20}	2.0e+09±1.8e+09	2.0e+09±1.8e+09	2.0e+09±1.8e+09	2.0e+09±1.8e+09	**3.1e+07±6.6e+06**	1.4e+10±2.7e+09↑	1.6e+08±1.5e+08↑	3.3e+10±5.9e+09↑
R^+	—	165.0	174.0	153.0	—	123.0	137.0	123.0
R^-	—	45.0	36.0	57.0	—	87.0	73.0	87.0
p-value	—	2.51e-02	1.00e-02	7.31e-02	—	5.02e-01	2.32e-01	5.02e-01

CEC'2013 Functions

F	CCFR-IDG2	CBCC1-IDG2	CBCC2-IDG2	DECC-IDG2	CCFR-DG	CBCC1-DG	CBCC2-DG	DECC-DG
f_1	**1.8e-05±4.5e-06**	4.6e+07±1.3e+08↑	4.6e+07±1.3e+08↑	4.6e+07±1.3e+08↑	4.8e+08±6.9e+07	6.2e+07±1.3e+08↓	6.2e+07±1.3e+08↓	6.2e+07±1.3e+08↓
f_2	**3.6e+02±2.1e+01**	2.1e+04±1.0e+03↑	2.1e+04±1.0e+03↑	2.1e+04±1.0e+03↑	**7.4e+02±4.0e+01**	2.1e+04±1.0e+03↑	2.1e+04±1.0e+03↑	2.1e+04±1.0e+03↑
f_3	2.1e+01±1.2e-02	2.1e+01±1.2e-02	2.1e+01±1.2e-02	2.1e+01±1.2e-02	**2.0e+01±6.0e-07**	2.1e+01±1.1e-02↑	2.1e+01±1.1e-02↑	2.1e+01±1.1e-02↑
f_4	**9.6e+07±4.0e+07**	2.2e+08±6.0e+07↑	6.6e+10±5.6e+09↑	2.9e+08±9.7e+07↑	9.1e+10±5.6e+10	8.7e+10±5.1e+10	4.6e+11±2.8e+11↑	8.3e+10±4.7e+10
f_5	2.8e+06±3.2e+05	2.6e+06±4.3e+05	2.5e+06±4.7e+05↓	3.0e+06±4.7e+05	3.0e+06±5.2e+05	2.8e+06±3.6e+05	2.6e+06±4.4e+05↓	3.3e+06±4.0e+05↑
f_6	1.1e+06±1.0e+03	1.1e+06±1.7e+03↓	1.1e+06±1.8e+03↓	1.1e+06±1.6e+03↓	1.1e+06±1.6e+03	1.1e+06±2.1e+03↓	1.1e+06±1.5e+03↓	1.1e+06±2.3e+03↓
f_7	2.0e+07±2.9e+07	2.2e+07±2.6e+07	9.9e+07±3.7e+08	2.4e+07±3.8e+07	1.4e+08±9.7e+07	1.2e+08±3.9e+07	1.6e+10±1.4e+10↑	1.4e+08±7.1e+07
f_8	**6.6e+10±9.5e+10**	2.3e+13±1.6e+13↑	1.1e+12±1.7e+11↑	7.4e+13±5.8e+13↑	1.6e+15±1.0e+15	2.0e+15±1.5e+15	5.9e+15±4.3e+15↑	2.0e+15±1.4e+15
f_9	**1.9e+08±2.8e+07**	2.6e+08±4.0e+07↑	2.3e+08±3.0e+07↑	3.0e+08±5.7e+07↑	**1.9e+08±2.8e+07**	2.5e+08±3.8e+07↑	2.2e+08±2.9e+07↑	2.9e+08±5.2e+07↑
f_{10}	9.5e+07±1.8e+05	9.4e+07±2.8e+05↓	9.4e+07±2.5e+05↓	9.5e+07±3.0e+05↓	9.5e+07±3.1e+05	9.4e+07±6.1e+05↓	9.4e+07±6.6e+05↓	9.4e+07±2.4e+05↓
f_{11}	4.2e+08±3.4e+08	5.0e+09±1.5e+10	7.3e+10±1.2e+11↑	2.8e+09±1.1e+10	**2.8e+10±6.0e+10**	4.5e+10±6.1e+10↑	5.2e+12±3.7e+12↑	4.7e+10±5.7e+10↑
f_{12}	1.6e+09±1.6e+09	1.6e+09±1.6e+09	1.6e+09±1.6e+09	1.6e+09±1.6e+09	**8.0e+07±8.3e+06**	6.0e+10±8.3e+09↑	6.6e+08±1.3e+08↑	1.2e+11±1.4e+10↑
f_{13}	1.2e+09±6.0e+08	1.2e+09±6.0e+08	1.2e+09±6.0e+08	1.2e+09±6.0e+08	**2.0e+09±1.0e+09**	4.0e+09±1.5e+09↑	4.1e+10±2.7e+10↑	6.3e+09±1.9e+09↑
f_{14}	3.4e+09±3.1e+09	3.5e+09±3.2e+09	3.5e+09±3.2e+09	3.5e+09±3.2e+09	7.4e+09±8.5e+09	1.3e+10±1.2e+10↑	5.0e+11±1.2e+12↑	8.9e+09±6.8e+09
f_{15}	9.8e+06±3.7e+06	9.9e+06±3.7e+06	9.9e+06±3.7e+06	9.9e+06±3.7e+06	8.3e+06±3.3e+06	8.3e+06±3.3e+06	8.3e+06±3.3e+06	8.3e+06±3.3e+06
R^+	—	107.0	107.0	112.0	—	80.0	99.0	91.0
R^-	—	13.0	13.0	8.0	—	40.0	21.0	29.0
p-value	—	5.37e-03	5.37e-03	1.53e-03	—	2.77e-01	2.56e-02	8.33e-02

The symbols ↑ and ↓ denote that the CCFR-IDG2 or CCFR-DG algorithm performs significantly better than and worse than this algorithm at a 0.05 significance level by the Wilcoxon rank sum test, respectively.

magnitude. CCFR-IDG2 outperformed other peer algorithms on most partially separable functions (the CEC'2010 functions f_4–f_{18}; the CEC'2013 functions f_4–f_{11}). For the partially separable functions on which CCFR-IDG2 performed worse, the difference between results of CCFR-I and other peer algorithms is not significant. But for the functions on which CCFR-IDG2 performed better, the difference is significant. For the nonseparable functions (the CEC'2010 functions f_{19}–f_{20}; the CEC'2013 functions f_{12}–f_{15}), all variables are grouped into one subcomponent. There is no significant difference between the algorithms with IDG2.

For most functions, the algorithms with IDG2 performed better than the ones with DG. This is because IDG2 can identify the interdependence between variables with more accuracies than DG. The multiple-problem analysis results show that compared with DG, IDG2 made CCFR performed much better than other peer algorithms. The performances of CCFR-IDG2 and CCFR-DG do not differ greatly on most of the functions where CCFR-IDG2 performed worse than CCFR-DG. But for most of the functions where CCFR-IDG2 performed better than CCFR-DG, CCFR-IDG2 outperformed greatly CCFR-DG by several orders of magnitude due to its higher grouping accuracy for nonseparable variables on these functions (i.e., the CEC'2010 functions f_7, f_8, f_{13} and f_{18}; the CEC'2013 functions f_4, f_7, f_8 and f_{11}). The experimental results show that the performance of CCFR is dependent on the decomposition method. A high grouping accuracy, especially for the nonseparable variables, can improve the performance of CCFR.

REFERENCES

[1] K. Tang, X. Li, P. N. Suganthan, Z. Yang, and T. Weise, "Benchmark functions for the CEC'2010 special session and competition on large-scale global optimization," Nature Inspired Computation and Applications Laboratory, Tech. Rep., 2010.

[2] X. Li, K. Tang, M. N. Omidvar, Z. Yang, and K. Qin, "Benchmark functions for the CEC'2013 special session and competition on large scale global optimization," Evolutionary Computation and Machine Learning Group, RMIT University, Australia, Tech. Rep., 2013.

[3] M. N. Omidvar, X. Li, Y. Mei, and X. Yao, "Cooperative co-evolution with differential grouping for large scale optimization," *Evolutionary Computation, IEEE Transactions on*, vol. 18, no. 3, pp. 378–393, 2014.

[4] M. N. Omidvar, M. Yang, Y. Mei, X. Li, and X. Yao, "IDG: A faster and more accurate differential grouping algorithm," University of Birmingham, School of Computer Science, Tech. Rep. CSR-15-04, September 2015. [Online]. Available: ftp://ftp.cs.bham.ac.uk/pub/tech-reports/2015/CSR-15-04.pdf

[5] M. N. Omidvar, X. Li, and X. Yao, "Smart use of computational resources based on contribution for cooperative co-evolutionary algorithms," in *Proceedings of the 13th annual conference on Genetic and evolutionary computation*. ACM, 2011, pp. 1115–1122.