Operations Research II
Final Report

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Abstract

Space reserved for abstract.

1 Problems Overview

`TODO: Overview of the problem in words.`

2 Mathematical Model

2.1 General Input

- n: Number of students ($n > 0$);
- m: Number of seminars ($m > 0$);
- k: Max number of selections that a student can make ($1 \leq k \leq m$);
- $s_{i,j}$: The j^{th} selection of i^{th} student, where $1 \leq i \leq n$ and $1 \leq j \leq k$. $s_{i,j} = 0$ when the Student i makes no corresponding choice for Rank j;
- q_k: The quota for k^{th} seminar, where $1 \leq k \leq m$.

2.2 Input Constraints

- Positivity: $n, m, k > 0, \forall k \in \{1, \cdots, m\}, q_k > 0$;
- Number of selections for student is bounded by number of available seminars: $k \leq m$;
- **(?)** Student rankings are valid and unique: $\forall(i,j), 1 \leq s_{i,j} \leq m$. And, for each i, all non-zero entries $s_{i,j}$'s take unique values.

2.3 Decision Variables

- $Y_{i,j}$: Indicator variables for whether Student i is assigned to Seminar j, where $1 \leq i \leq n$ and $1 \leq j \leq k$;

2.4 Data Pre-Processing

In order to deal with cases when a student is only willing or allowed to rank $k' < m$ seminars, we automatically set all "unassigned" priorities to $(k + 1)$. Also, we change the representation of students' preference from *(student, ranking)* $\mapsto$ *seminar* to *(student, seminar)* $\mapsto$ *ranking* to make further calculations easier, i.e.

$$X_{i,j} = \begin{cases} l & \text{If Student } i \text{ ranked } j \text{ as } l^{\text{th}} \text{ option, or } s_{i,l} = j \text{ for some } l \in \{1, \cdots, k\} \\ k+1 & \text{If Seminar } k \text{ is not on Student } i\text{'s list, or } s_{i,l} \neq j \text{ for all } l \in \{1, \cdots, k\} \end{cases}.$$

2.5 General Constraints

- $Y_{i,j}$'s are indeed indicator variables: $\forall (i,j), Y_{i,j} \in \mathbb{Z}, Y_{i,j} \geq 0, Y_{i,j} \leq 1$;
- Each student is assigned precisely one seminar: $\forall i$, $\sum_{l=1}^{m} Y_{i,l} = 1$;
- Each seminar is within enrollment quota: $\forall j$, $\sum_{l=1}^{n} Y_{l,j} \leq q_j$;

3 Approach for Various Heuristic Functions

Due to the flexibility of the original problem, we are proposing different objective functions for optimization, including minimizing the total "rank" given by the students, maximizing the number of student getting their top λk choice (where $\lambda \in (0,1)$), etc. In the following sub-sections we present our approach for each heuristic in mathematical terms.

3.1 Minimize Total Rank of Students

In this case, our goal is to minimize the sum of all student rankings for their assigned seminars. To do so, our objective is to minimize $W = \sum_{i=1}^{n} \sum_{j=1}^{m} X_{i,j} Y_{i,j}$.

3.2 Maximize Number of Students Getting Top-Tier Seminars

In this case, we would additionally require the user to input a value $\lambda \in (0,1)$, representing the range of rankings we consider as "top-tier", i.e. $\{1, \cdots, \lfloor \lambda k \rfloor\}$. Given this heuristic, our objective is to maximize $W = \sum_{i=1}^{n} \sum_{j=1}^{m} \mathbb{1}_{X_{i,j} \leq \lfloor \lambda k \rfloor} Y_{i,j}$.

3.3 And Potentially More

4 Summary of Results

5 Further Work & Enhancements

6 References