



Password Guessability with Markov Model

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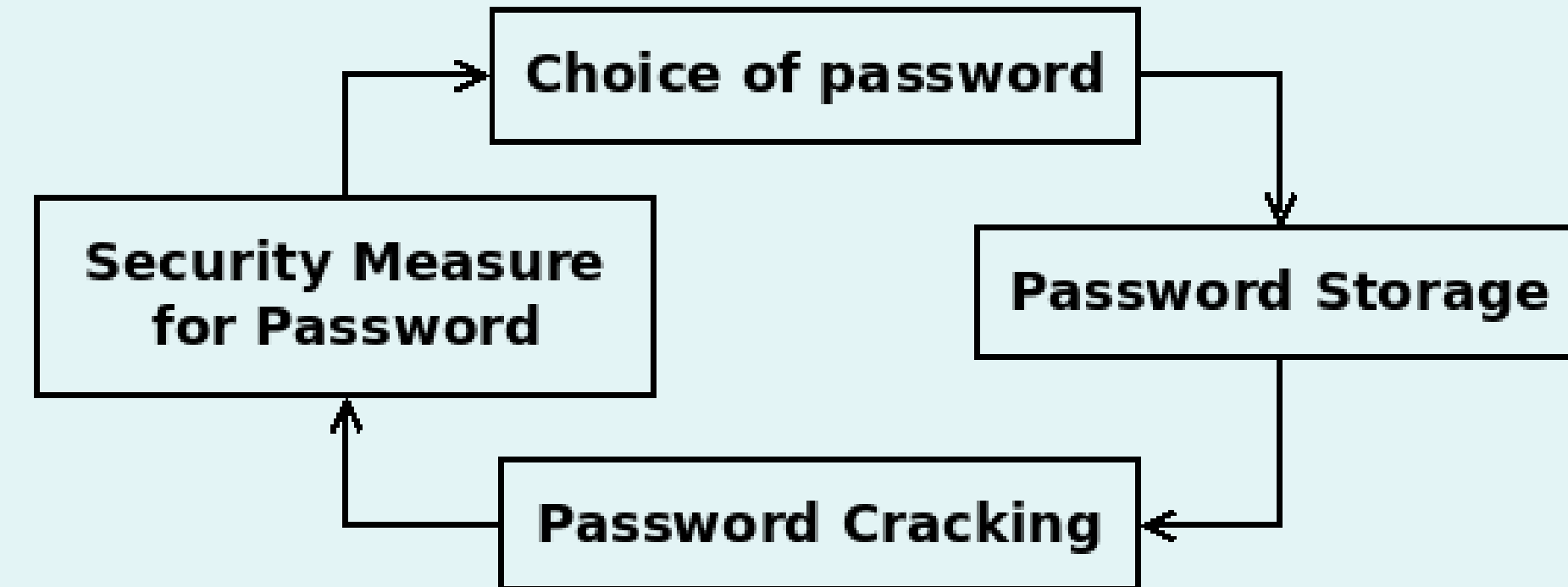
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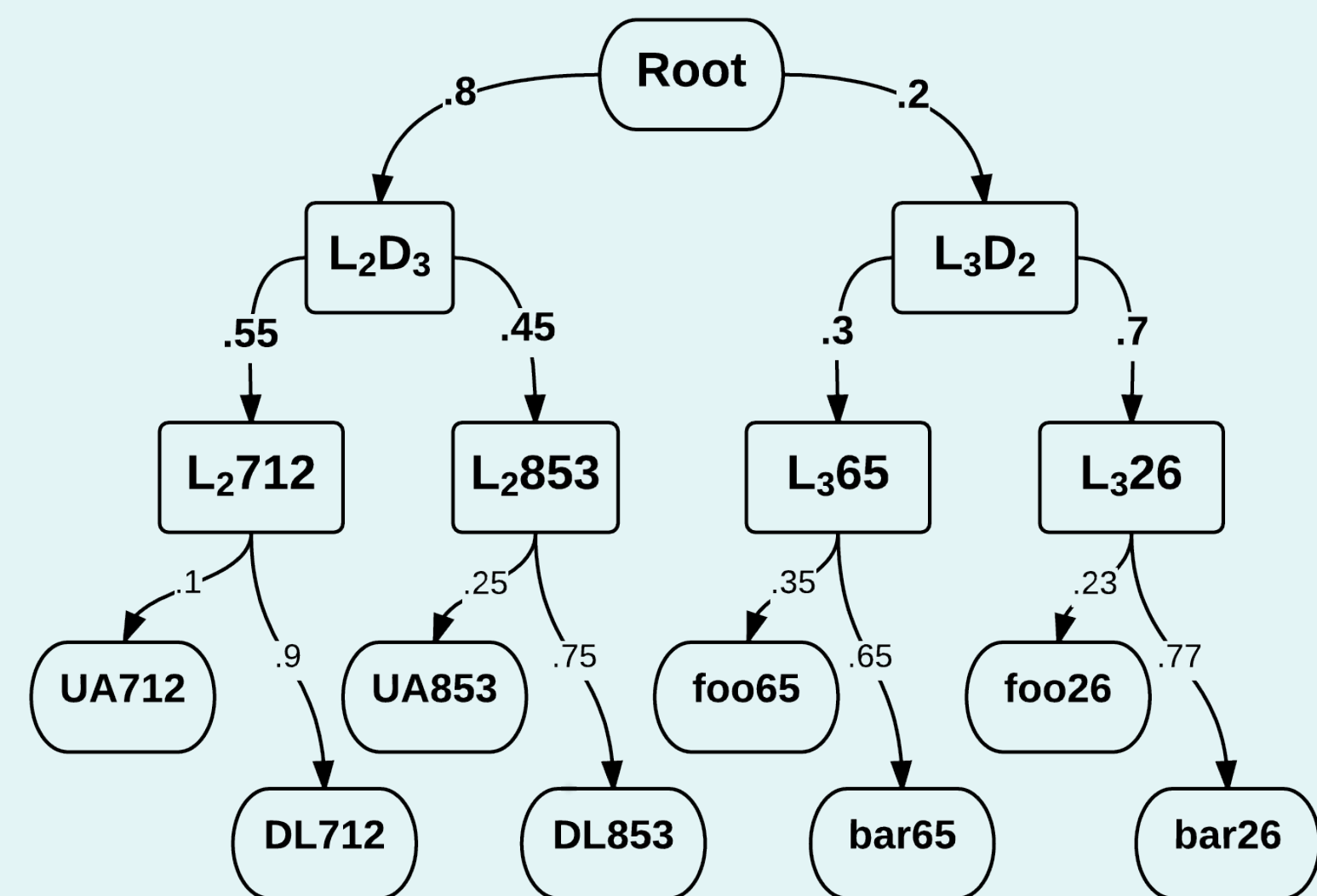
Introduction & Problem Statement

- Passwords has been and will be the most commonly used method of authentication.
- Common question: how many times does an adversary need in order to guess a password?
- Our goal: Study state-of-the-art password cracking techniques, and implement a look-up table to allow efficient guessability analysis with Markov model.

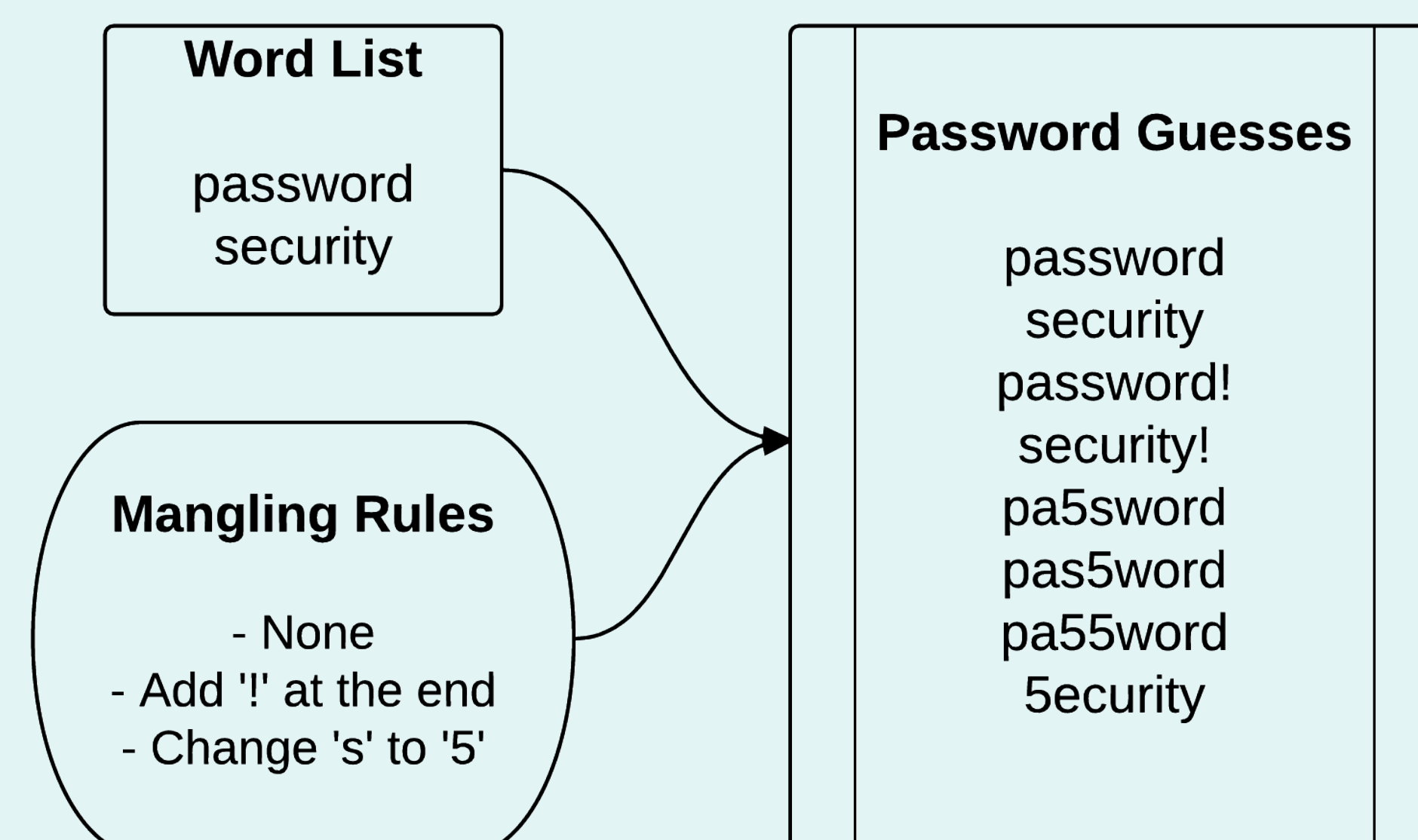


Password Cracking Techniques

- Brute Force / Masked Brute Force:
 - aaa, aab, aac, ..., zzz
 - aaaa, aaab, aaac, ..., zzz
- Probabilistic Context-Free Grammar (PCFG):
 - Splits password into homogeneous “regions”
 - Different regions have different probability
 - Each region is filled with chars independently

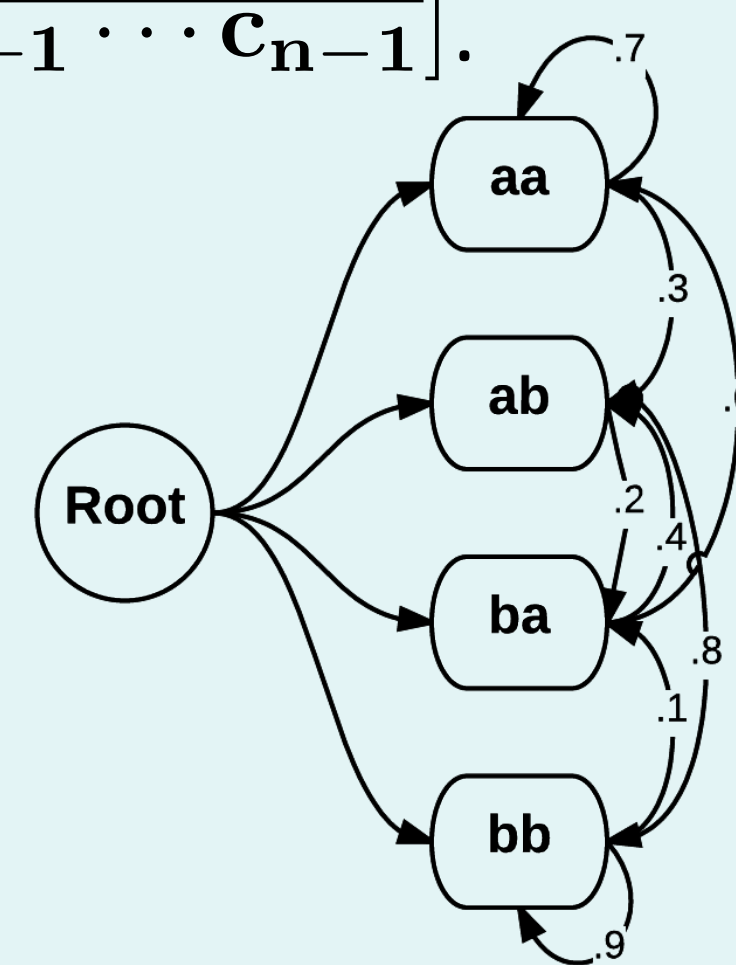


- Word Mangling Rules (used by JtR and HashCat)



Methodology

- Markov Model:
 - Next character depends on its previous (k-1)-gram, i.e. $\Pr[c \mid \overline{x_{n-k+1} \cdots c_{n-1}}]$.

$$\begin{array}{l} \Pr[a \mid aa] = .7 \\ \Pr[b \mid aa] = .3 \\ \Pr[a \mid ab] = .2 \\ \Pr[b \mid ab] = .8 \\ \Pr[a \mid ba] = .6 \\ \Pr[b \mid ba] = .4 \\ \Pr[a \mid bb] = .1 \\ \Pr[b \mid bb] = .9 \end{array}$$


Step One: Learn Probability Parameters

$$\Pr[c \mid \overline{x_{n-k+1} \cdots c_{n-1}}] := \frac{\text{count}(\overline{x_{n-k+1} \cdots c_{n-1}c}) + \delta}{\text{count}(\overline{x_{n-k+1} \cdots c_{n-1}}) + \delta|\Sigma|}$$

$$\Pr[\overline{x_0 \cdots x_{k-2}}] := \frac{\text{count}(\overline{x_0 \cdots x_{k-2}}) + \delta}{|\mathcal{D}| + \delta|\Sigma|^{k-1}}$$

$$\Pr[\overline{x_{n-k+2} \cdots x_n}] := \frac{\text{count}(\overline{x_{n-k+2} \cdots x_n \perp}) + \delta}{|\mathcal{D}| + \delta|\Sigma|^{k-1}}$$

Step Two: Discretize Probabilities

- Initially proposed by Duermuth, et al. In 2015
- Reduce continuous variables to discrete variables: $\hat{\Pr}[\cdot] = -\text{round}(\log(c_1 p + c_2))$.
- Parameters: ℓ_{\max}, c_1, c_2 , satisfying:

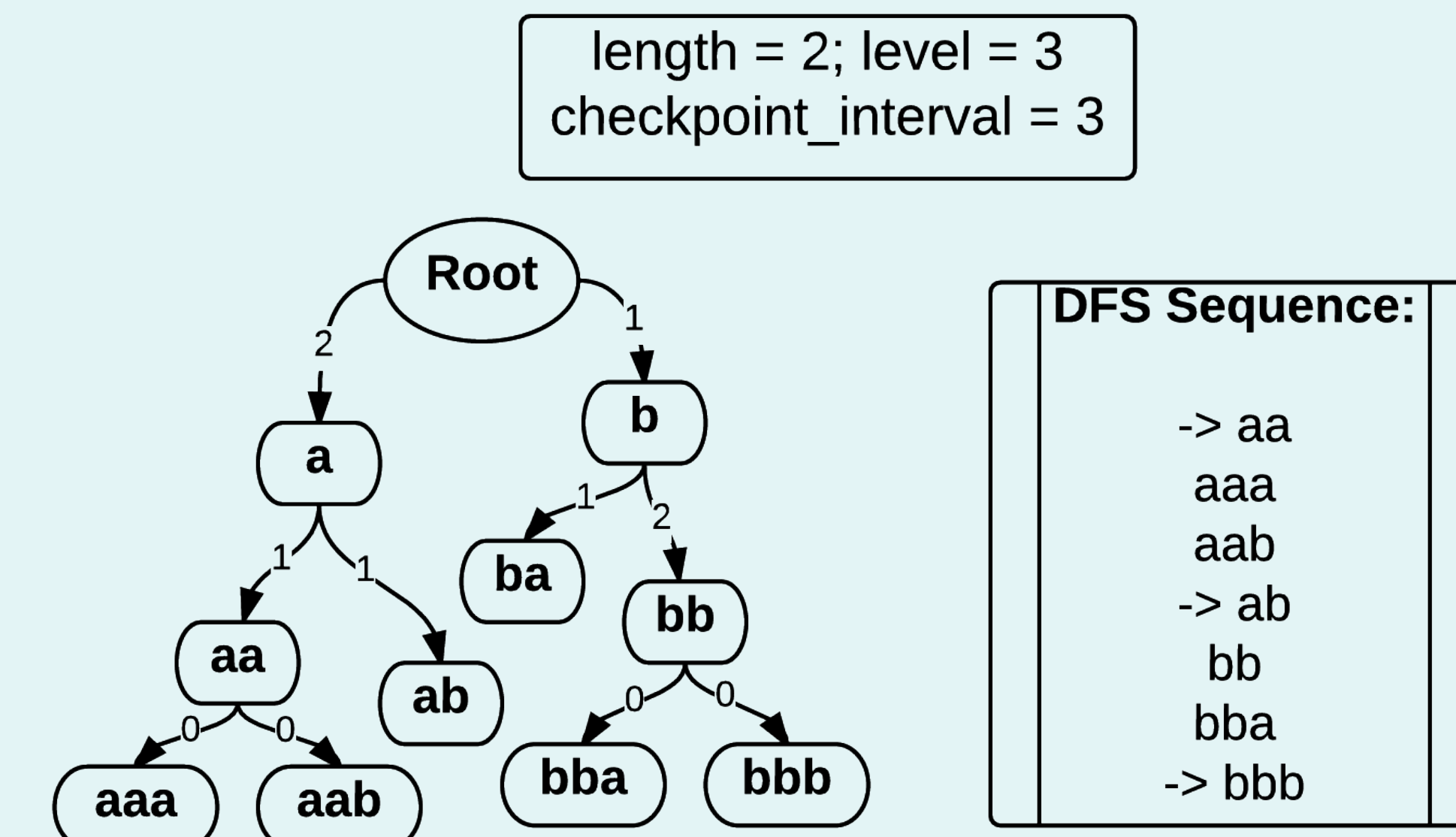
$$\begin{cases} -\log(c_2) = \ell_{\max} \\ \log(c_1 p_{\max} + c_2) = 0 \end{cases}$$

$$\Rightarrow (c_1, c_2) = \left(\frac{1 - \exp(-\ell_{\max})}{p_{\max}}, \exp(-\ell_{\max}) \right).$$

Step Three: Build Enumeration Index

- Level of a password = sum of the levels of its Markov probabilities, e.g:

$$\text{lvl}(\text{"ab"}) := \hat{\Pr}[\text{"a"}] + \hat{\Pr}[\text{"b"} \mid \text{"a"}] + \hat{\Pr}[\text{"a"} \perp].$$
 For each (length, level)-pair, enumerate passwords with certain length and level with DFS and create “checkpoints” along the way for future use:



Methodology (Cont'd)

Step Four: Guess Count Calculation

- Pick up the DFS process from the most recent checkpoint (if exists), and continue until finding the password. Overall, the guess number of password is:

$$g(\mathbf{x}) = \left(\sum_{\text{prior}(\text{len}, \text{lvl})} |\mathcal{I}_{\text{len}, \text{lvl}}| \right) + g_{\text{len}, \text{lvl}}(\mathbf{x}).$$

Results & Evaluation

Index-Builder Efficiency:

Guess Count Calculator Efficiency:

TBD

Further Work & Remarks

- Time-space tradeoff:
 - More storage used for checkpointing gives faster run-time for guess number calculation; vice versa.
 - Alternative model – *layered Markov model*
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- ```

graph LR
 Root((Root)) --> a1((a))
 Root --> b1((b))
 Root --> p1((⊥))
 a1 --> a2((a))
 a1 --> b2((b))
 a1 --> p2((⊥))
 b1 --> a2
 b1 --> b2
 b1 --> p2
 p1 --> a2
 p1 --> b2
 p1 --> p2
 a2 --> a3((a))
 a2 --> b3((b))
 a2 --> p3((⊥))
 b2 --> a3
 b2 --> b3
 b2 --> p3
 p2 --> a3
 p2 --> b3
 p2 --> p3
 a3 --> a3
 a3 --> b3
 a3 --> p3
 b3 --> a3
 b3 --> b3
 b3 --> p3
 p3 --> a3
 p3 --> b3
 p3 --> p3

```
- Other smoothing techniques, e.g. Good-Turing:
 
$$\hat{c}(s) = \begin{cases} 0.22 & \text{for } c(s) = 1 \\ 0.88 & \text{for } c(s) = 2 \\ c(x) - 1 & \text{for } c(s) > 2 \\ \text{previously deducted count} & \text{otherwise} \end{cases}$$
  - Fine-tuned implementation based on our algorithm can be incorporated in a Guessability-as-a-Service module, like in [2].
  - Source code and final report for the project is available in our Github repo [1], and intermediate output data (> 2GB) is available upon request.

## Selected References

- M. Duermuth, F. Angelstorf, C. Castelluccia, D. Perito, and A. Chaabane. OMEN: faster password guessing using an ordered markov enumerator. In *Proc. ESSoS*, 2015.
  - J. Ma, W. Yang, M. Luo, and N. Li. A study of probabilistic password models. In *Proc. IEEE Symp. On Security and Privacy*, 2014.
  - B. Ur, S. M. Segreti, L. Bauer, N. Christin, L. F. Cranor, S. Komanduri, D. Kurilova, M. L. Mazurek, W. Melicher, and R. Shay. Measuring real-world accuracies and biases in modeling password guessability. In *Proc. USENIX Security*, 2015.
- (Full list of references is available in the final report.)

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[1] <https://github.com/ymzong/password-guessability>

[2] <https://pgs.ece.cmu.edu>

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