Bilkent University

EE-202 Circuit Theory

Lab 3

Maximum Power Transfer

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Section 02

Introduction

This lab aims to design two passive linear circuits to transfer maximum power to 180Ω load from a voltage source with 50Ω output impedance. The chosen frequency should be between 5 and 10 MHz. Figure 1 shows the overall illustration of the task.

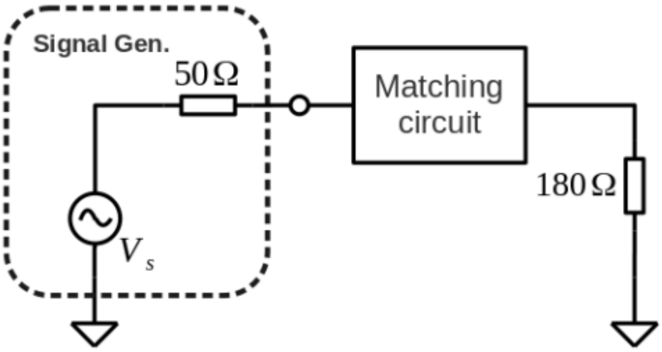

Figure 1: Lab task

Analysis

To achieve maximum power transfer to a desired load, the resistances of the source and the load must match, and their reactance should cancel each other. In other words, the total impedances of the source (Z_S) and the load (Z_L) must be complex conjugates of each other, expressed as: $Z_S = Z_L^*$

Furthermore, it is known that the average power delivered to load is:

$$P_{ave} = |I|^2 \times \frac{R_L}{2} \tag{1}$$

$$|I| = \frac{|V|}{(R_S + R_L)} \tag{2}$$

Inserting for I, the following equation is obtained:

$$P_{ave} = \frac{|V|^2}{2(R_S + R_L)^2} \times R_L \tag{3}$$

When maximum power transfer is achieved, resistive parts of the impedances should be equal such that $R_S = R_L = R$. Inserting again and noting that $|V|$ is the peak value of the input voltage V, the maximum power delivered to the load is:

$$P_{ave} = \frac{|V|^2}{8R} \tag{4}$$

To obtain the maximum power, two methods are proposed.

Method 1: T-Section

The T-Section circuit is shown in Figure 2. Before finding the values of L and C, the source's peak voltage is chosen as 10V, and the frequency is chosen as 8 MHz.

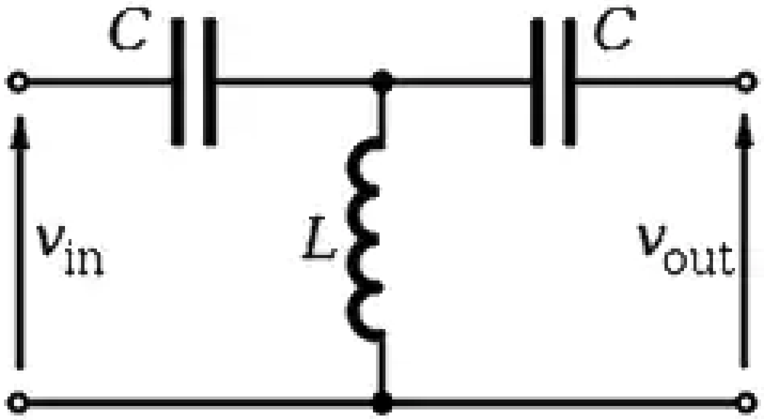

Figure 2: T-Section circuit [1]

It is known that $Z_S = R_S = 50\Omega$, $Z_L = R_L = 180\Omega$ and f = 8 MHz. To calculate the inductance and capacitance values L and C, the following formula, namely impedance inverter, is used:

$$Z_{in} = Z_S = \frac{X^2}{Z_L}$$

$$X^2 = R_S \times R_L = 50 \times 180$$

$$X = \mp 94.87$$

Taking the positive root and using the following equalities:

$$jX = j\omega L\,, -jX = \frac{1}{j\omega C}$$

$$L = \frac{X}{2\pi f} = \frac{94.87}{2\pi \times 8 \times 10^6} \cong 1.89\ \mu H$$

$$C = \frac{1}{2\pi f X} = \frac{1}{2\pi \times 8 \times 10^6 \times 94.87} \cong 210\ pF$$

The calculated values are then used in the circuit.

Method 2: L-Section

The L-Section circuit is shown in Figure 3. Again, the source's peak voltage is chosen as 10V, and the frequency is chosen as 8 MHz.

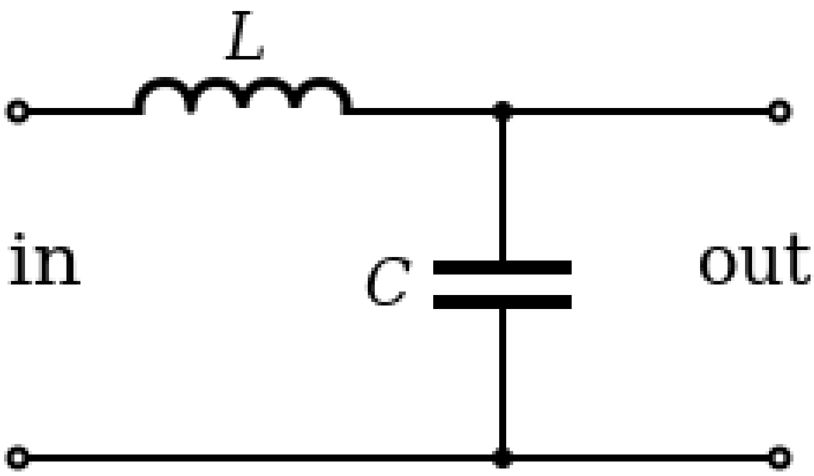

Figure 3: L-Section circuit [2]

Again, $R_S = 50\Omega$, $R_L = 180\Omega$ and f = 8 MHz. This time, to calculate L and C, quality factor Q will be used.

$$R_L = (Q^2+1)R_S \Rightarrow Q = \sqrt{\frac{R_L}{R_S} - 1} = \sqrt{\frac{180}{50} - 1} \cong 1.612$$

$$Q = \frac{2\pi f L}{R_S} \Rightarrow L = \frac{R_S Q}{2\pi f} = \frac{50 \times 1.612}{2\pi \times 8 \times 10^6}$$

$$L \cong 1.6\ \mu H$$

and lastly,

$$\omega = 2\pi f = \sqrt{\frac{1}{LC(1+\frac{1}{Q^2})}} \Rightarrow C = \frac{1}{4\pi^2 f^2 L(1+\frac{1}{Q^2})}$$

$$C \cong 179\ pF$$

The calculated values are then used in the circuit.

Simulations

Firstly, to find the maximum power that can be delivered, equation (4) is used, where $|V|$=10V and R=50Ω:

$$P_{ave,max} = \frac{10^2}{8 \times 50} = 250\ mW$$

To find the power delivered to the 180Ω load without the matching circuit, the circuit in Figure 4 is implemented. The input's peak voltage is 10 V and frequency is 8 MHz. Figure 5 shows the plot of the power delivered to the load.

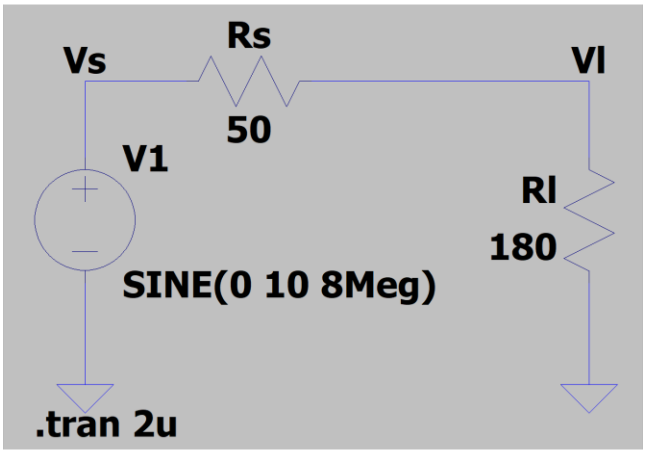

Figure 4: Load without the matching circuit

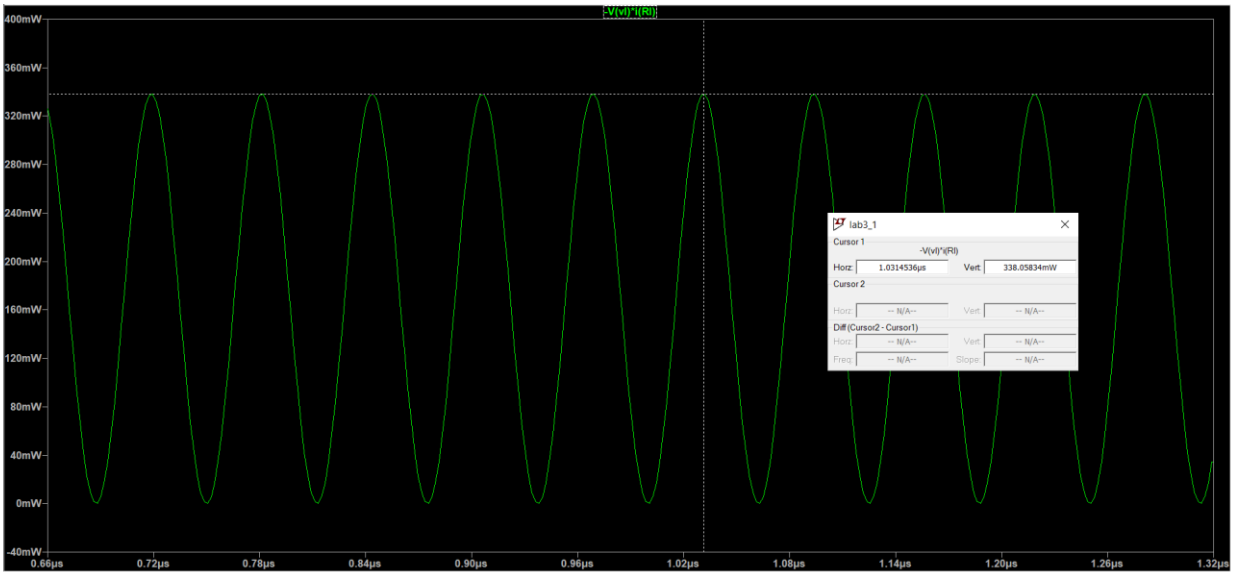

Figure 5: Power plot without the matching circuit

For this case:

$$P_{ave} = \frac{P_{max}}{2} = \frac{338}{2} = 169\ mW \tag{5}$$

Compared to the maximum power that can be delivered, which is 250 mW, it is smaller. The power deliver ratio is 67.6%.

After that, the T-Section and L-Section circuits are implemented on LTSpice. Figures 6-7 show the circuits for T-Section and L-Section methods.

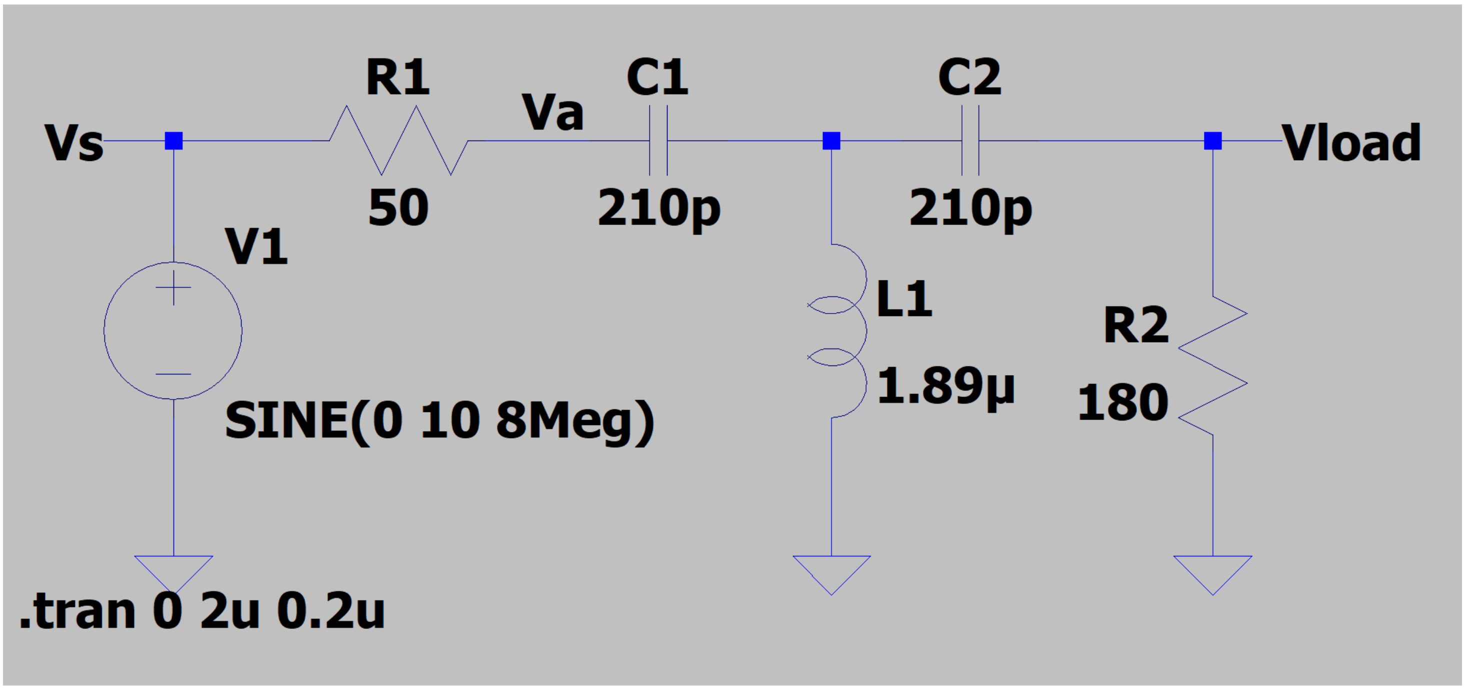

Figure 6: T-Section circuit on software

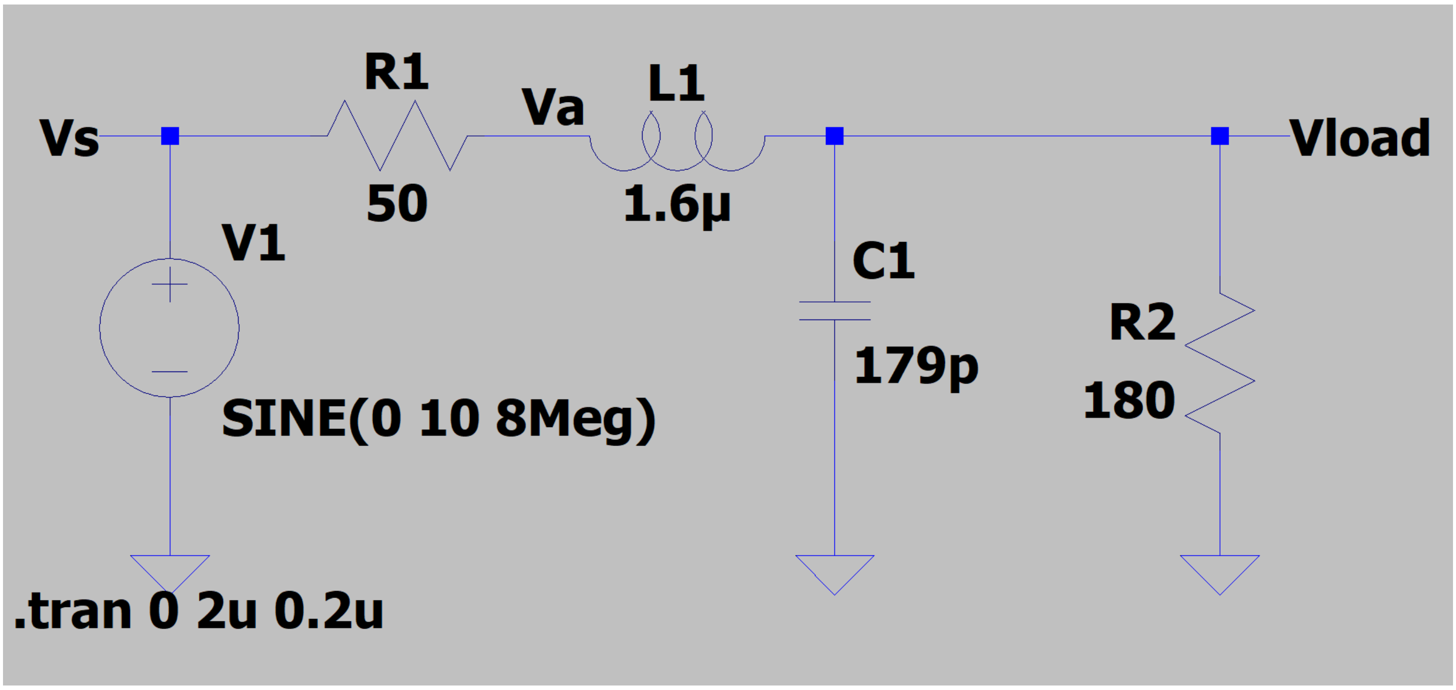

Figure 7: L-Section circuit on software

Figure 8 shows the obtained power plot for the T-Section.

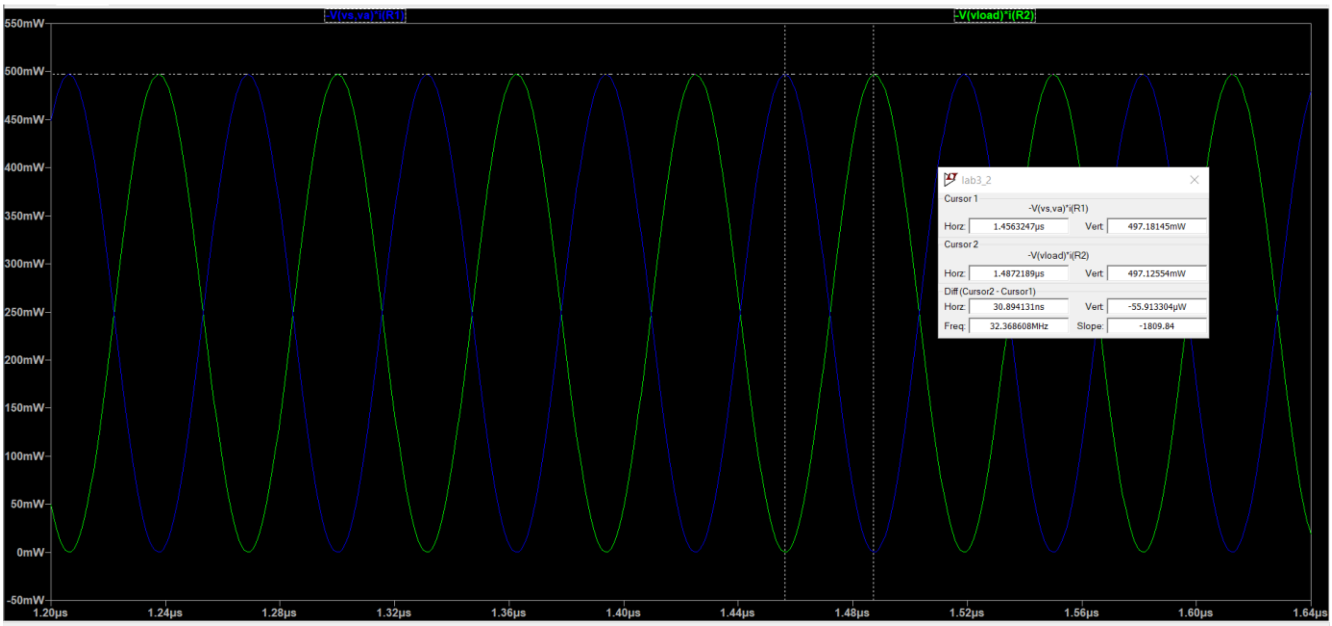

Figure 8: Power plot for the T-Section

Using equation (5), the average power available at the source is 248.59 mW, and the average power delivered to the source is 248.56 mW. The error with respect to the available power is 0.1%. The power deliver ratio with respect to the calculated value is 99.42%, and the power deliver ratio with respect to the measured available power is 99.9%.

Figure 9 shows the obtained power plot for the L-Section.

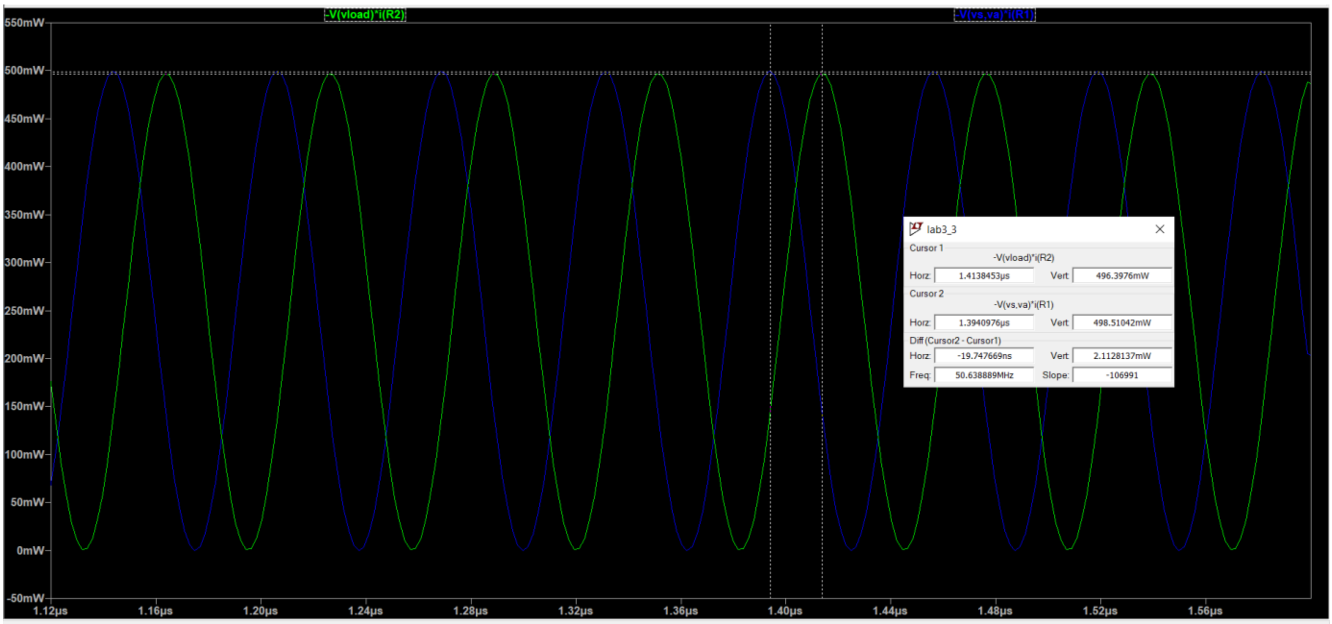

Figure 9: Power plot for the L-Section

Using equation (5), the average power available at the source is 249.25 mW, and the average power delivered to the source is 248.2 mW. The error with respect to the available power is 0.42%. The power deliver ratio with respect to the calculated value is 99.28%, and the power deliver ratio with respect to the measured available power is 99.58%. Overall results are shown in Tables 1 and 2.

	Calculated maximum power	Power delivered to load	Power deliver ratio with respect to the calculated maximum power
Without matching circuit	250 mW	169 mW	67.6%

Table 1: Results for the case without the matching circuit

	Calculated maximum power	Power available at source	Power delivered to load	Power deliver ratio with respect to the calculated maximum power	Power deliver ratio with respect to the measured available power	Error
T-Section	250 mW	248.59 mW	248.56 mW	99.42%	99.9%	0.1%
L-Section	250 mW	249.25 mW	248.2 mW	99.28%	99.58%	0.42%

Table 2: Results for the case with the matching circuits

Hardware Implementation

First, as a reference for the available power, the power transferred to a 47Ω resistor is calculated as indicated in the guidelines. It is connected to a signal generator with a sinusoidal input with 10 V peak voltage and 8 MHz frequency. For this measurement, since the voltage on the resistor is measured directly (as in a voltage divider), the following formula is used:

$$P_{ave} = \frac{|V_R|^2}{2R} \tag{6}$$

In general, for the hardware lab, equation (6) will be used to find the average power. Therefore, measured peak voltages on the resistors ($|V_R|$) and resistances should be considered. Figures 10-11 show the measurements for the 47Ω resistor.

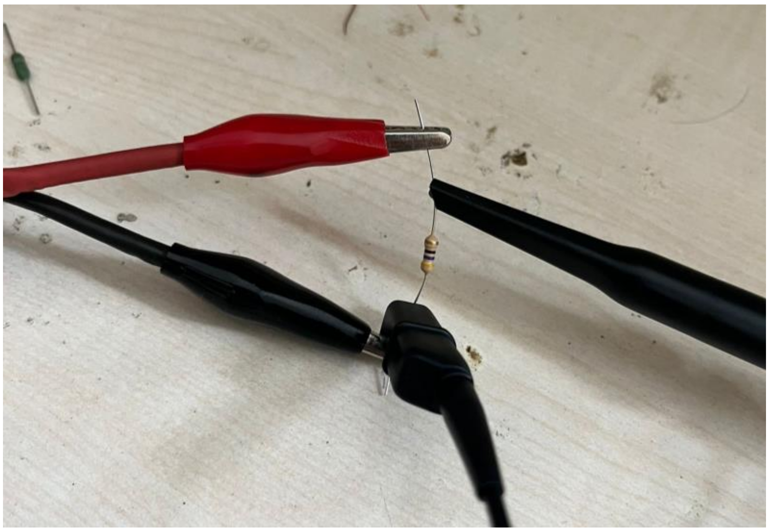

Figure 10: 47Ω resistor connected to the voltage source

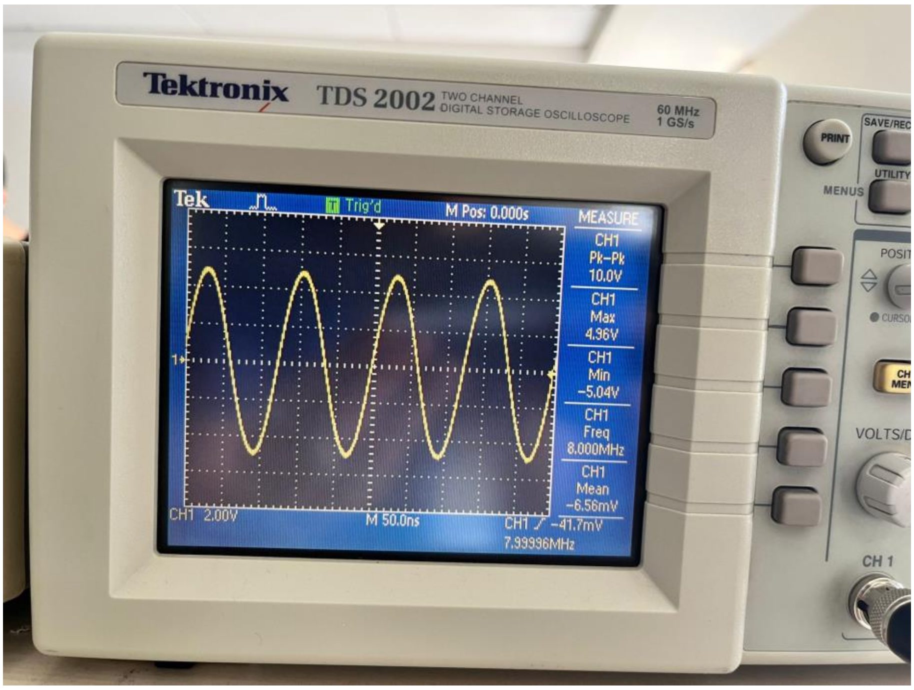

Figure 11: Voltage of the 47Ω resistor, Vmax=4.96 V

Putting $|V_R|$=4.96 V, R=47Ω in (6), the reference power is calculated as 262 mW. The power deliver ratio and the error will be calculated according to this value.

After that, first the T-Section, then the L-Section circuits are implemented. The inductance and capacitance values were obtained by series and parallel combinations of standard values. The voltages on the load resistors are measured with an oscilloscope.

Then the power delivered to the 180Ω load resistance is calculated by inserting the peak voltage values of the load $|V_R|$ and R=180Ω into (6). The power deliver ratios are calculated with respect to both the reference power delivered to the 47Ω resistance and the calculated maximum power which is 250 mW. Figures 12-15 show the implementations and the measurements, and the results are presented in Table 3.

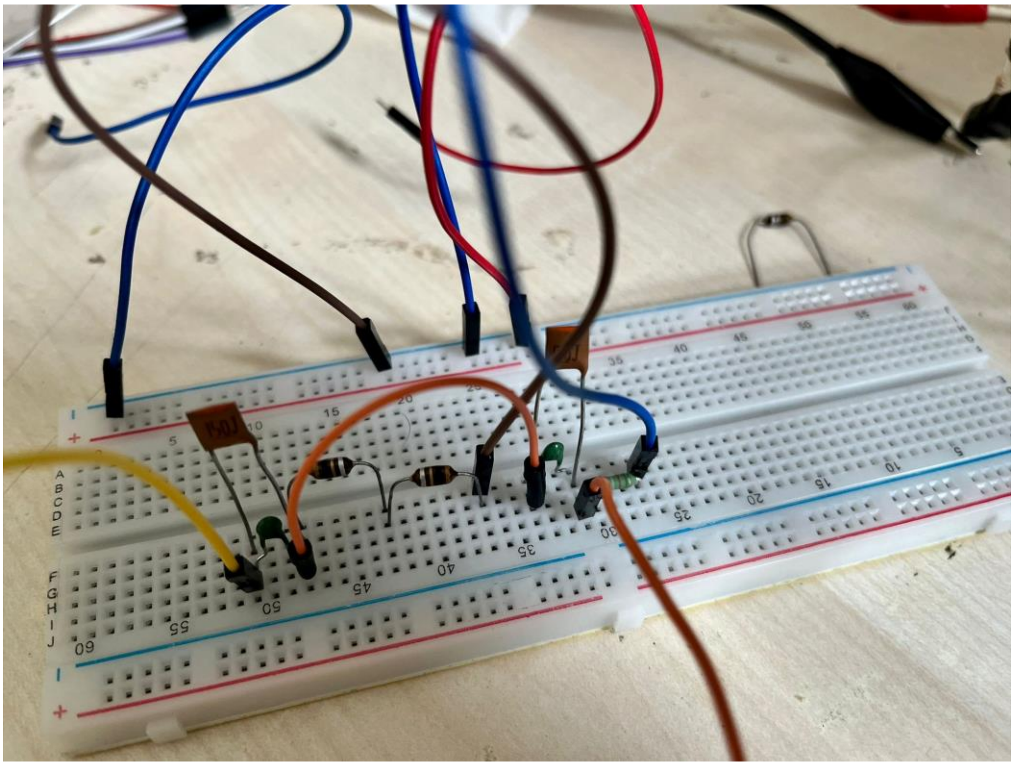

Figure 12: T-Section circuit is implemented on hardware

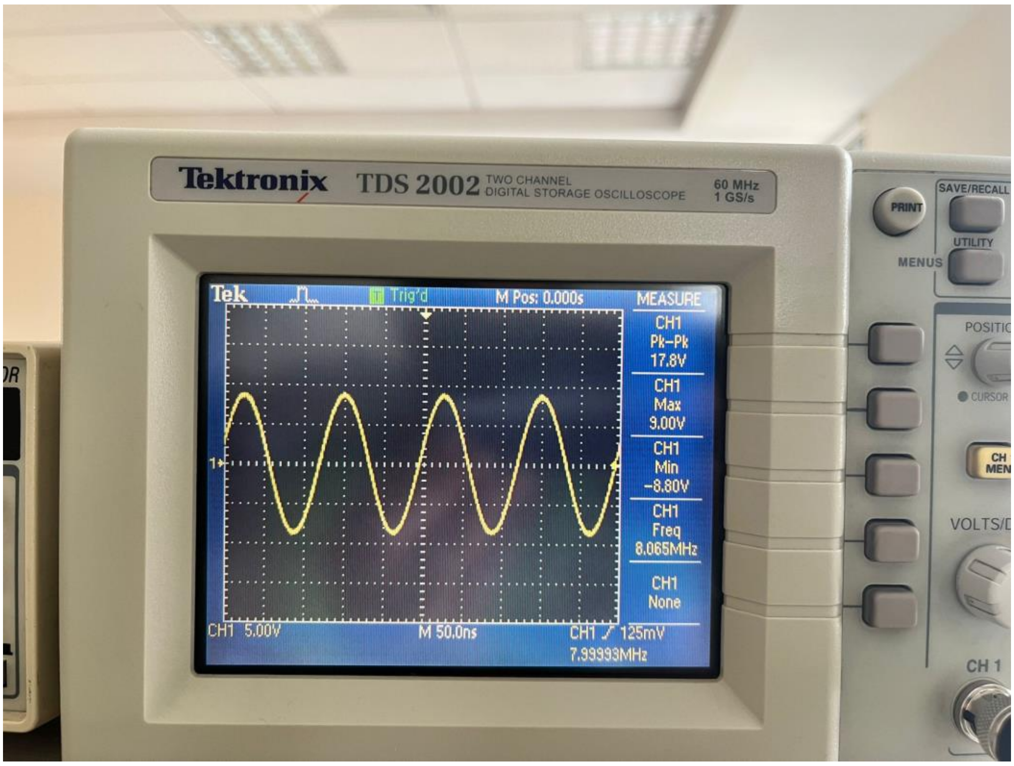

Figure 13: Peak voltage of the load is 9V

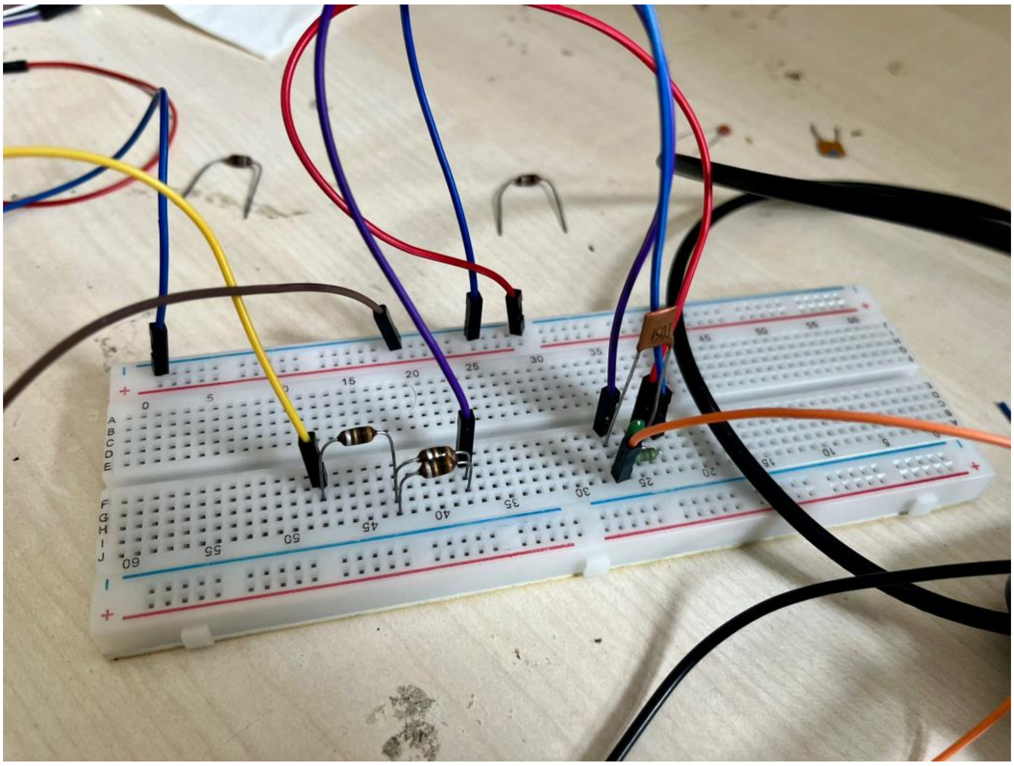

Figure 14: L-Section circuit is implemented on hardware

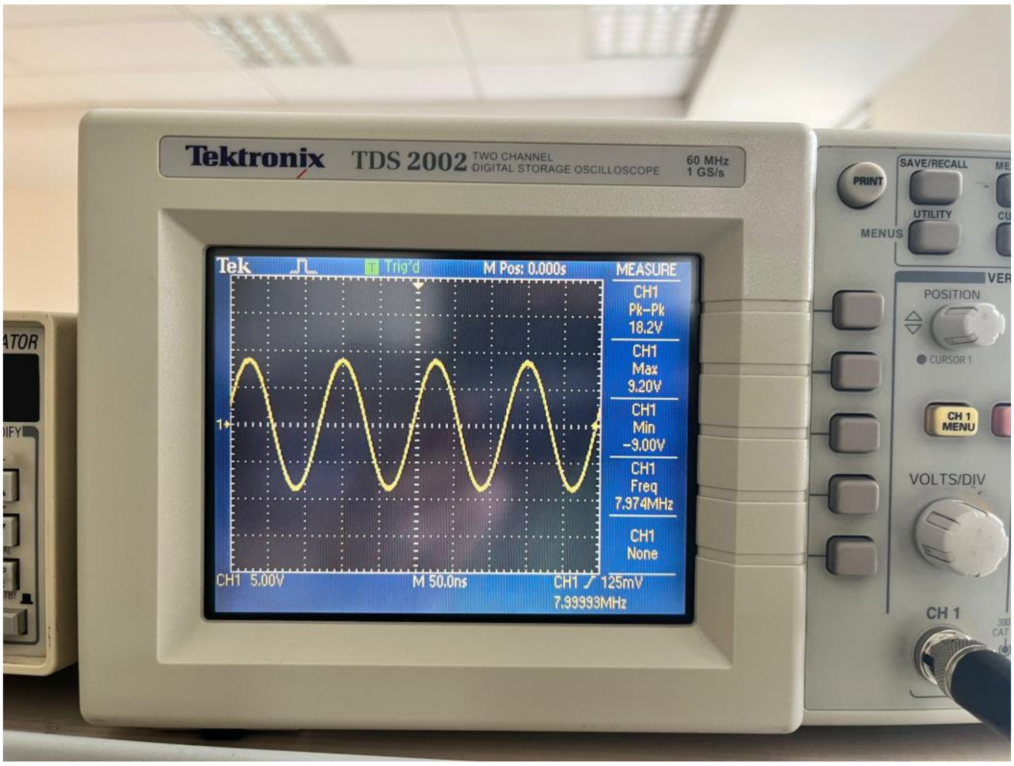

Figure 15: Peak voltage of the load is 9.2V

	Calculated maximum power	Power delivered to 47Ω resistor (as a reference for the avaliable power)	Power delivered to load	Power deliver ratio with respect to the calculated maximum power	Power deliver ratio with respect to the reference available power	Error with respect to the calculated maximum power	Error with respect to the reference available power
T-Section	250 mW	262 mW	225 mW	90%	85.88%	10%	14.12%
L-Section	250 mW	262 mW	235 mW	94%	89.69%	6%	10.31%

Table 3: Results of the hardware lab

Conclusion

For the power delivered to the load, the results of the software and hardware labs match with 9.5% and 5.3% errors for the T-Section and the L-Section respectively. In the software part, the power deliver ratios are high since the components are considered as ideal. Still, despite being very close, they are not 100% because the final values for the inductances are capacitances are rounded up after the calculations. For the software part, there was no significant difference between the ratios of T-Section and L-Section methods, but T-Section method was slightly more efficient with 99.9% ratio than L-Section with 99.58% ratio.

In the hardware lab, however, the errors are relatively high so the power deliver ratios are lower compared to the software lab. The reason for that can be because of the inner resistances of the wires, or the 5-10% tolerance values of the resistors, capacitors and axial inductors. Furthermore, a number of capacitors and inductors were connected in parallel and in series to achieve the calculated values in the software part. Usage of the components in this way might have contributed to the error as well. For the hardware part, considering the power deliver ratios, L-Section method was more successful with 89.69% than T-Section with 85.88%. The efficiency can be improved by choosing the frequency such that the standard values available in the lab match the calculated ones. Despite being relatively high, the errors are still less than 20% as required.

Overall, this lab demonstrated a very important concept in electronics, that is, delivering the maximum power available to a load. It taught how to achieve this both on software and hardware, by building impedance matching circuits and by using necessary components like inductors and capacitors. So, this lab also provided better insight to designing passive, linear circuits.

References

[1] J. Gabay, “Pi, T filters match RF impedances,” Digi, https://www.digikey.com/en/articles/pi-t-filters-match-rf-impedances (accessed May 18, 2023).

[2] Bhakti, “What is choke filter? - L-section filter, working, Advantages & Disadvantages,” Electronics Coach, https://electronicscoach.com/choke-filter.html (accessed May 18, 2023).