ECD – HW 2

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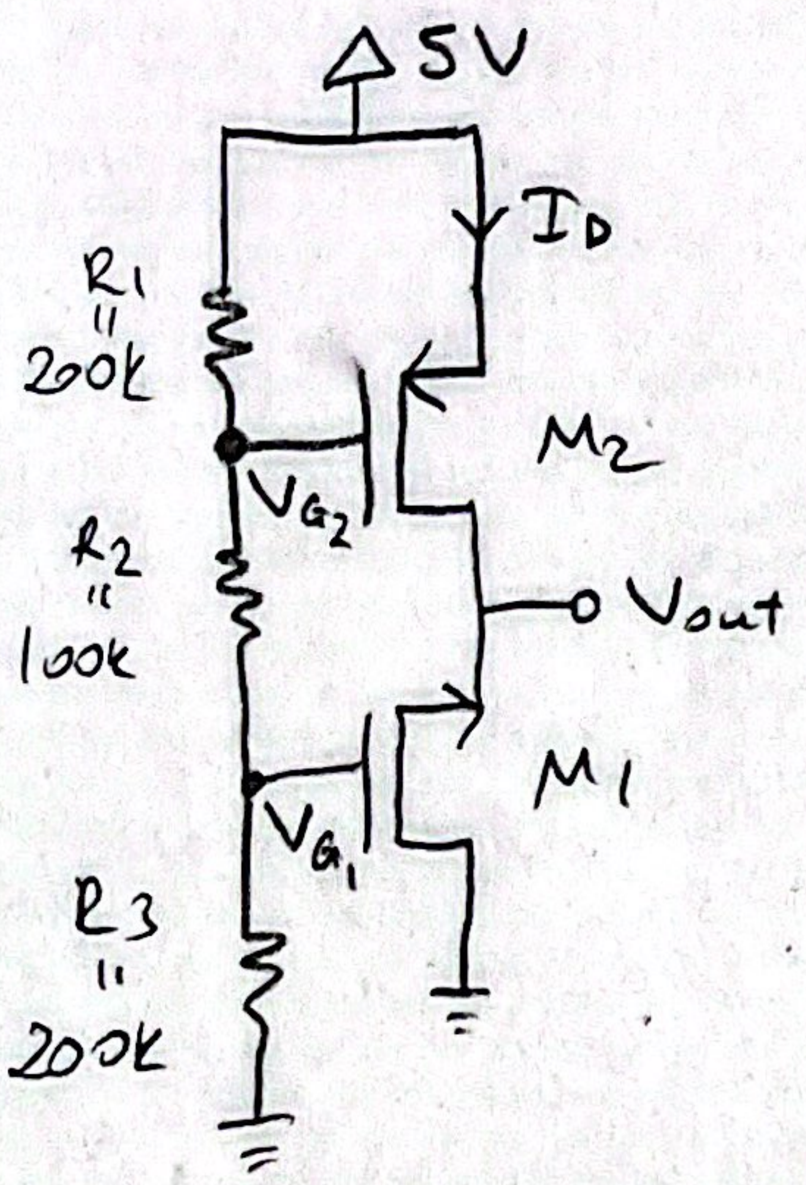
1) a) DC analysis: Open-circuit the capacitors:



By voltage dividers (since gate currents are 0):

$$V_{G_1} = \frac{R_3}{R_1+R_2+R_3} \cdot 5V = 2V$$

$$V_{G_2} = \frac{R_3+R_2}{R_1+R_2+R_3} \cdot 5V = 3V$$

$\Rightarrow V_{GS_1} = 2V,\ V_{DS_1} = V_{out}$
$V_{SG_2} = 2V,\ V_{SD_2} = 5 - V_{out}$ } by KVL.

Assume SAT for both M_1 and M_2:

For M_1: $I_D = \underbrace{K_n}_{0.25}(\underbrace{V_{GS_1}}_{2} - \underbrace{V_{tn}}_{1})^2 = 0.25\ mA$

For M_2: $I_D = \underbrace{K_p}_{0.25}(\underbrace{V_{SG_2}}_{2} - \underbrace{|V_{tp}|}_{1})^2 = 0.25\ mA$

Currents match but how to find V_{DS} (V_{out})?

$\Downarrow$

$\boxed{\text{DON'T ignore } \lambda\text{'s!}}$

Write again the saturation currents:

$I_D = K_n(V_{GS_1} - V_{tn})^2(1+\lambda V_{DS}) = 0.25(1+0.02V_{out}) \rightarrow$ for M_1

and $I_D = K_p(V_{SG_1} - |V_{tp}|)^2(1+\lambda V_{SD}) = 0.25(1+0.01(5-V_{out})) \rightarrow$ for M_2

$\Rightarrow I_D = \cancel{0.25}(\cancel{1}+0.02V_{out}) = \cancel{0.25}(\cancel{1}+0.01(5-V_{out}))$

$\Rightarrow V_{out} = \frac{5}{3}V \cong 1.67V \Rightarrow I_D = 0.25(1+0.02(1.67)) = 0.258\ mA$

Check: $\underbrace{V_{DS_1}}_{1.67V} \ ?> \underbrace{V_{GS_1}}_{2V} - V_{tp}$ $-1V = 1V$ ✓ $\Rightarrow$ M1 in SAT

$\underbrace{V_{SD_2}}_{3.33V} \ ?> \underbrace{V_{SG_2}}_{2V} - V_{tp}$ $-1V = 1V$ ✓ $\Rightarrow$ M2 in SAT

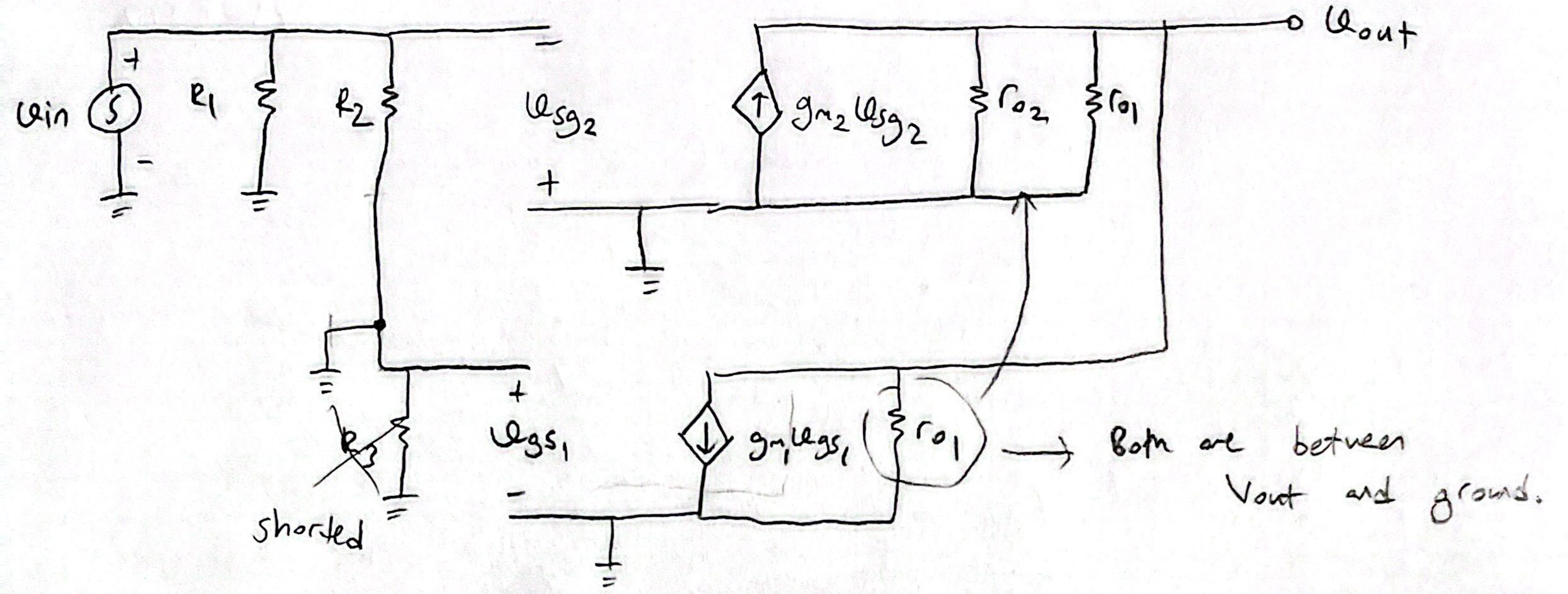
✓

Now, $\boxed{I_D = 0.258\ mA}$, $\boxed{V_{DS} = 1.67V}$

$\boxed{g_{m_1} = 2\sqrt{K_n \cdot I_D} = 0.51\ \frac{mA}{V}}$, $\boxed{r_{o_1} = \frac{1}{\lambda_1 I_{DQ}} = 193.8\ k\Omega}$

$\boxed{g_{m_2} = 2\sqrt{K_p \cdot I_D} = 0.51\ mA/V}$ $\boxed{r_{o_2} = \frac{1}{\lambda_2 I_{DQ}} = 387.6\ k\Omega}$

b) In the small signal analysis, the capacitors will become shorted:



Note that $v_{g_1} = v_{s_1} = 0 \Rightarrow v_{gs_1} = 0,\ g_{m_1} v_{gs_1} = 0 \rightarrow$ Nothing to do with M1.

Now,

$v_{in} = -v_{sg_2}$ and $v_{out} = g_{m_2} v_{sg_2} (r_{o_2} \parallel r_{o_1})$

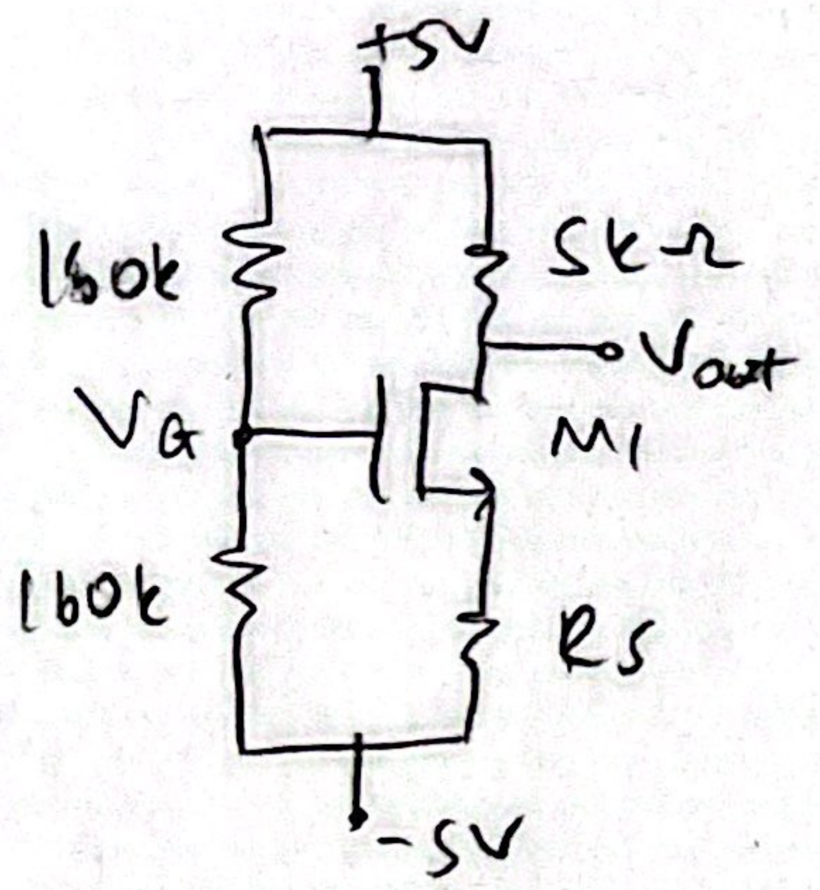
$$\Rightarrow \boxed{A_v = \frac{v_{out}}{v_{in}} = -g_{m_2} (r_{o_1} \parallel r_{o_2})} \quad \Rightarrow \boxed{A_v = -65.892 \ \frac{V}{V}}$$

c) $\boxed{R_{in} = R_1 \parallel R_2 = 200k \parallel 100k = 66.67\ k\Omega}$

To find R_{out}, kill v_{in}, then $v_{sg_2} = 0 \Rightarrow g_{m_2} v_{sg_2} = 0$, open-circuit it

$$\Rightarrow \boxed{R_{out} = r_{o_2} \parallel r_{o_1} = 129.2\ k\Omega}$$

(2) a) DC analysis → open circuit cap's



$V_G = 5V \cdot \frac{160k}{320k} + (-5V) \cdot \frac{160k}{320k} = 0V$ (voltage divider, superposition)

$V_S = I_D R_S - 5$, $V_{out} = V_{DS} + I_D R_S - 5 \Rightarrow \boxed{V_{GS} = -I_D R_S + 5}$

or $\boxed{V_{out} = 5 - 5I_D}$ by KVL

$V_{out} = 5 - 5I_D = 0 \Rightarrow \boxed{I_D = 1\,mA}$ for $V_{out} = 0$

$\Rightarrow$ Assume SAT: $I_D = k_n(V_{GS} - V_{th})^2 = 1mA$

$= 0.25(4 - I_D R_S)^2 = 1\,mA$

$\Rightarrow (4 - R_S)^2 = 4 \Rightarrow \boxed{R_S = 2\,k\Omega}$ we need $V_{GS} > V_{th}$

or $R_S = 6\,k\Omega$

$\Rightarrow \boxed{R_S = 2\,k\Omega}$ for $V_{out} = 0V$ DC.

$g_m = 2\sqrt{k_n I_D} = 2\sqrt{(0.25)(1)} = 1\,mA/V$

$r_o = \frac{1}{\lambda I_D} = \frac{1}{(0.01)1} = \boxed{100\,k\Omega}$ (for later parts)

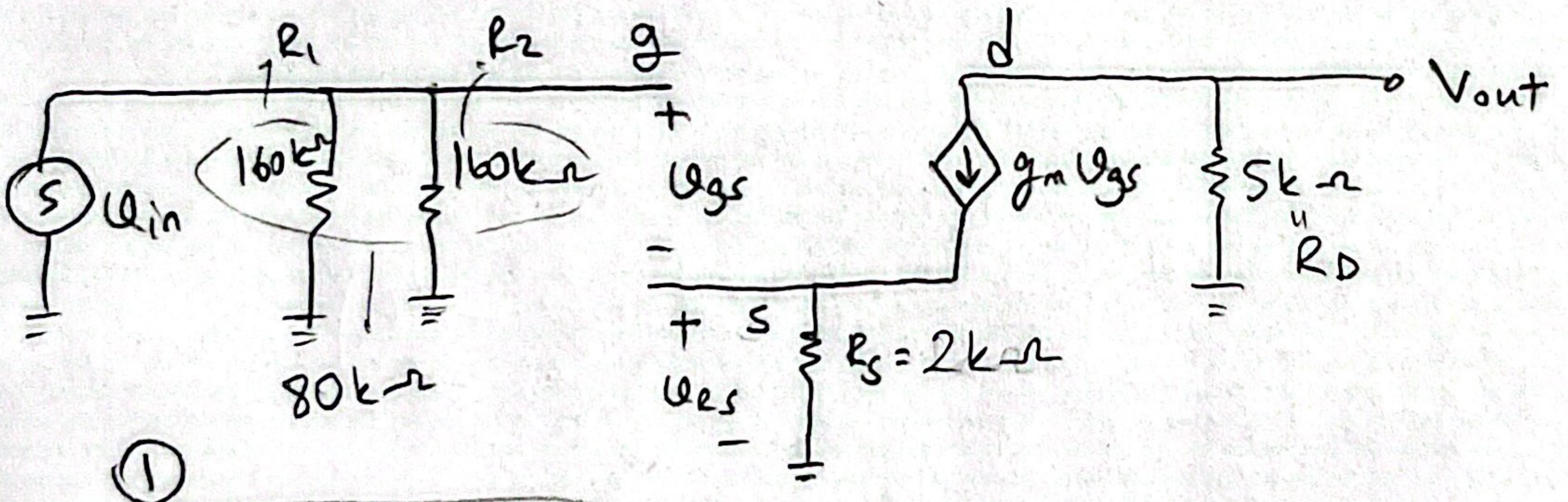
Check:

$V_{DS} = 5 - \underset{1mA}{I_D}\underset{2k}{R_S} = 3V$

$V_{GS} = 5 - I_D R_S = 3V$

$V_{DS} > V_{GS} - V_{th} \Rightarrow$ SAT ✓

b) S.S model:



(1)

$\boxed{v_{out} = -g_m v_{gs} R_D}$, $v_{Rs} = g_m v_{gs} R_s$

by KVL: $v_{in} = v_{gs} + v_{Rs} = v_{gs}(1 + g_m R_s)$

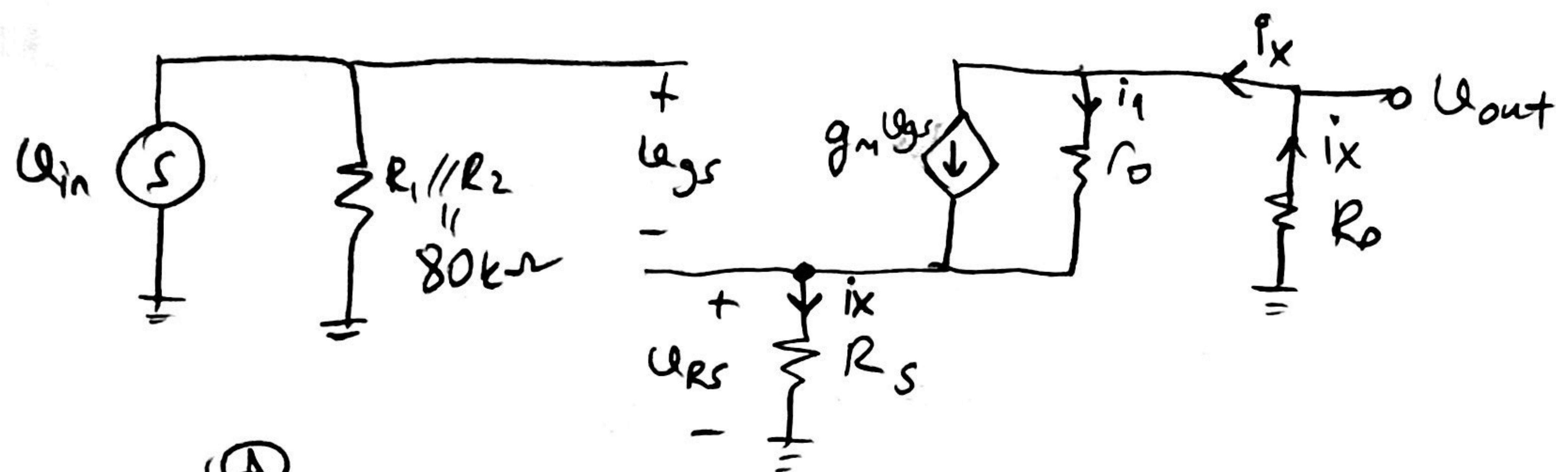
$\Rightarrow v_{gs} = \frac{v_{in}}{1 + g_m R_s}$, Insert into (1): $v_{out} = -g_m R_D \frac{v_{in}}{1 + g_m R_s}$

$$\boxed{A_v = \frac{v_{out}}{v_{in}} = -\frac{g_m R_D}{1 + g_m R_s}}$$

$$\Rightarrow \boxed{A_v = -\frac{1(5k\Omega)}{1 + (1)(2)} = -\frac{5}{3}\ V/V \cong -1.67\ V/V}$$

c) $r_o = 100k\Omega$ from previous part.

S.S model with r_o:



① $\boxed{v_{gs} = v_{in} - v_{RS}}$ and $\boxed{i_x = g_m v_{gs} + \dfrac{v_{out} - v_{RS}}{r_o}}$ --- (1) (KCL at V_{out})

$\Rightarrow$ We also know that $v_{RS} = R_S \cdot i_x$, insert into (1):

$$i_x = g_m \underbrace{v_{gs}}_{\substack{v_{in} - R_s i_x \\ \text{from ①}}} + \frac{v_{out} - R_S i_x}{r_o} \Rightarrow i_x = g_m v_{in} - g_m R_S i_x + \frac{v_{out}}{r_o} - \frac{R_S}{r_o} i_x$$

$$\Rightarrow i_x \left(1 + g_m R_S + \frac{R_S}{r_o}\right) = g_m v_{in} + \frac{v_{out}}{r_o}$$

$$\Rightarrow i_x = \frac{g_m v_{in} + \frac{v_{out}}{r_o}}{1 + g_m R_S + \frac{R_S}{r_o}}$$

Now, $v_{out} = -R_D \cdot i_x$

$$\Rightarrow v_{out} = -R_D \left(\frac{g_m v_{in} + \frac{v_{out}}{r_o}}{1 + g_m R_S + \frac{R_S}{r_o}} \right) \Rightarrow v_{out}\left(1 + g_m R_S + \frac{R_S}{r_o} + \frac{R_D}{r_o}\right) = -v_{in}(g_m R_D)$$

$$\Rightarrow \boxed{A_v = \frac{v_{out}}{v_{in}} = -\frac{g_m R_D}{1 + g_m R_S + \frac{R_D + R_S}{r_o}}}$$

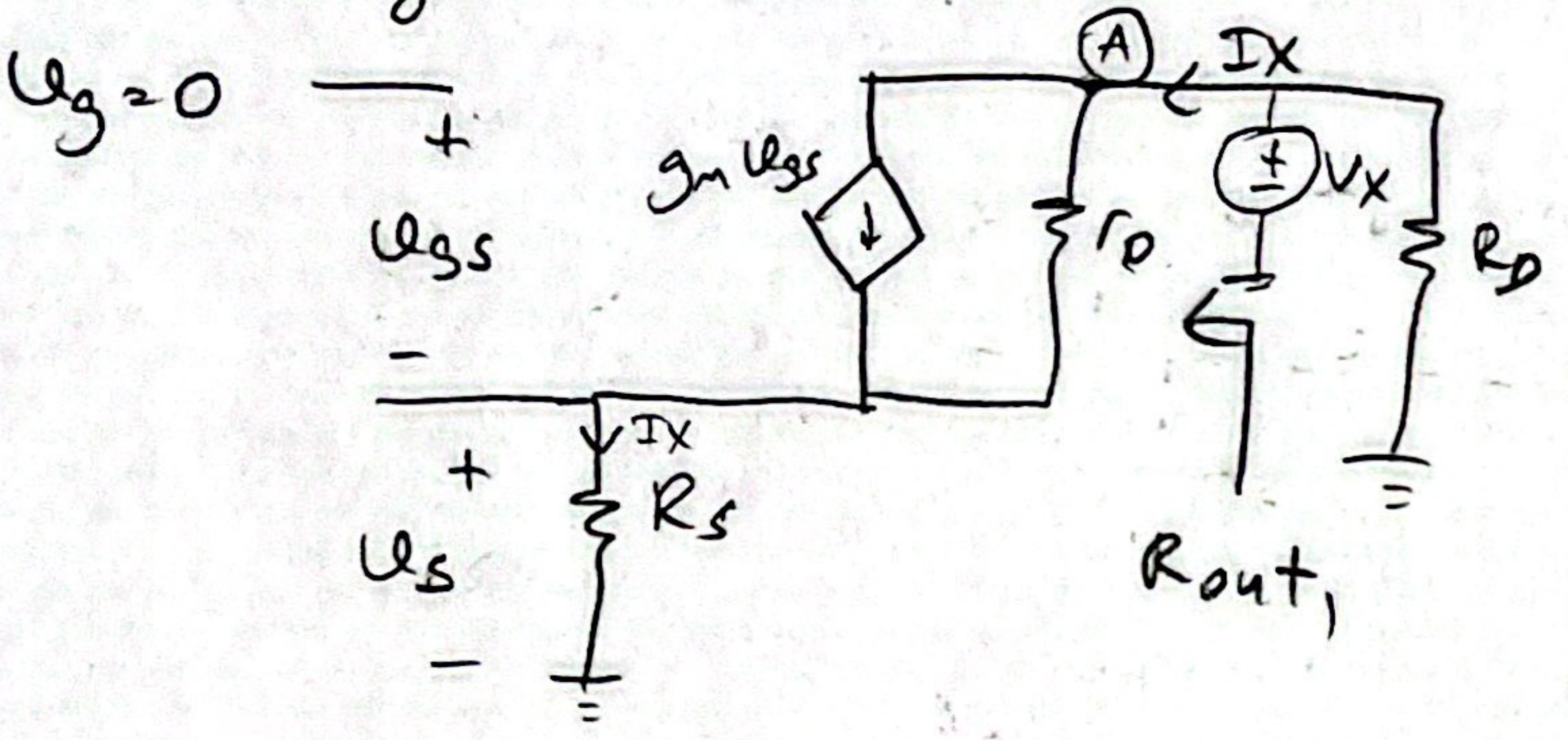
Note that as $\lambda \to 0$, $r_o \to \infty$ $\left(r_o = \frac{1}{\lambda I_D}\right)$

$$\Rightarrow \lim_{r_o \to \infty} A_v = \frac{-g_m R_D}{1 + g_m R_S + \underbrace{\lim_{r_o \to \infty} \frac{R_D + R_S}{r_o}}_{0}} = \frac{-g_m R_D}{1 + g_m R_S} \quad \text{(same in part b)}$$

$$\Rightarrow \boxed{A_v = -\frac{(1)(5k\Omega)}{1 + (1)(2k\Omega) + \frac{7}{100k\Omega}} = \frac{-5}{3 + 0.07} = -1.63\ V/V}$$

d) $\boxed{R_{in} = R_1 // R_2 = 80\,k\Omega}$

To find R_{out}, kill v_{in}:



$v_{gs} = \cancel{v_g} - v_s = -v_s = -v_{RS}$

$-v_s$

KCL at A: $I_X = \dfrac{V_X - v_s}{r_o} + g_m v_{gs}$

$\Rightarrow I_X = \dfrac{V_X - v_s}{r_o} - g_m v_s$

and $\boxed{v_s = I_X R_S}$, insert:

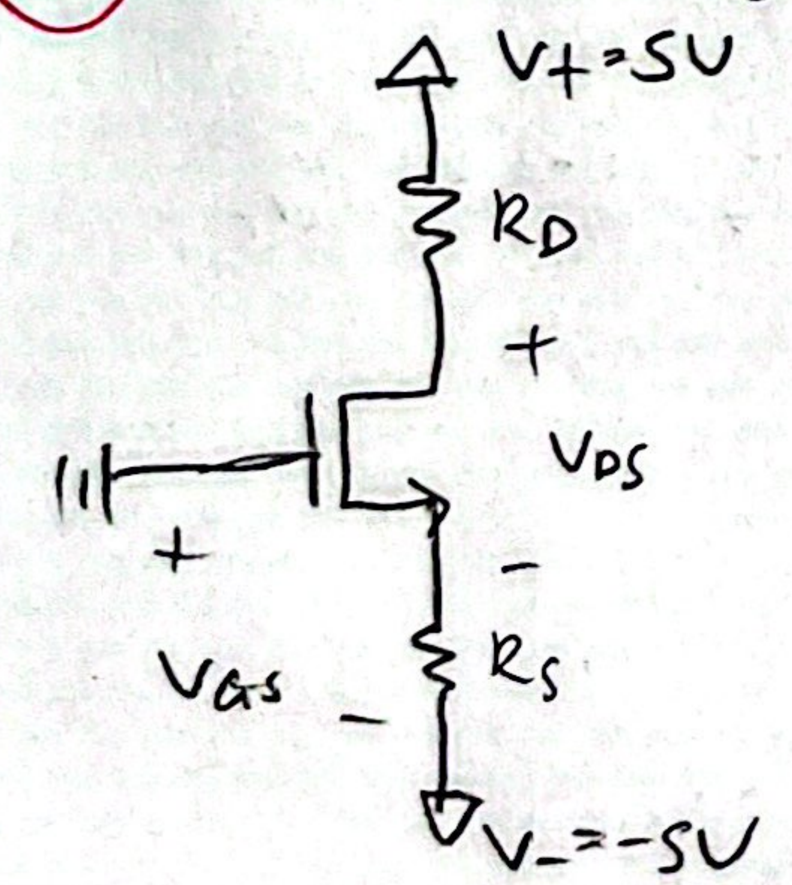
$\Rightarrow I_X = \dfrac{V_X - I_X R_S}{r_o} - g_m R_S I_X$

$\Rightarrow I_X \left(1 + g_m R_S + \dfrac{R_S}{r_o}\right) = \dfrac{V_X}{r_o}$

$\Rightarrow \boxed{R_{out,1} = \dfrac{V_X}{I_X} = r_o(1 + g_m R_S) + R_S}$

finally $\boxed{R_{out} = R_{out,1} // R_D = \left(r_o(1+g_m R_S) + R_S\right) // R_D}$ $\Rightarrow \boxed{R_{out} = 4.89\,k\Omega}$

3 a) DC analysis → Open circuit caps.



Assume SAT, we want $I_{DQ} = 0.2\,mA$ and $V_{DQ} = 1V$

KVL: $10 = \underbrace{I_{DQ}}_{0.2mA}(R_D + R_S) + \underbrace{V_{DQ}}_{1V}$

$\Rightarrow \boxed{R_D + R_S = 45\,k\Omega}$

and $I_D = k_n(V_{GS} - V_{tn})^2$, $V_{GS} = \overset{0}{\cancel{V_G}} - V_S$

$= 5 - I_D R_S$

$\Rightarrow 0.2 = 0.5(V_{GS} - 1)^2$

$\Rightarrow (V_{GS} - 1)^2 = \dfrac{2}{5}$ $\Rightarrow V_{GS} = 1.63\,V$

(to have $V_{GS} > V_{tn}$, take the bigger root)

and $V_{GS} = 5 - I_D R_S \Rightarrow 5 - 0.2 R_S = 1.63$

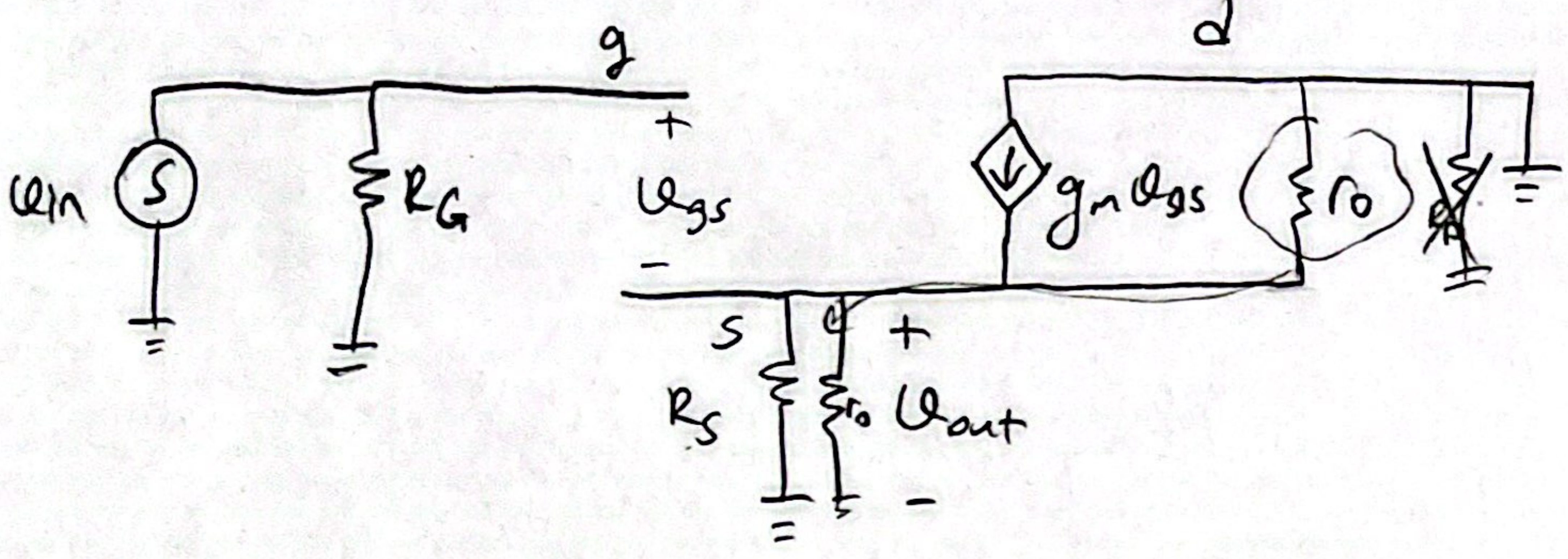
$\Rightarrow \boxed{R_S = 16.85\,k\Omega}$

$\Rightarrow \boxed{R_D = 45 - R_S = 28.15\,k\Omega}$

$\boxed{g_m = 2\sqrt{k_n I_D} = 0.632\,mA/V}$

$\boxed{r_o = \dfrac{1}{\lambda I_D} = 500\,k\Omega}$

b) S.S model:



We can move r_o parallel with R_S since they are both connected between v_{out} and ground.

$$\Rightarrow v_{out} = g_m v_{gs} (R_S // r_o) \quad -- (1)$$

KVL: $v_{in} = v_{gs} + v_{out}$

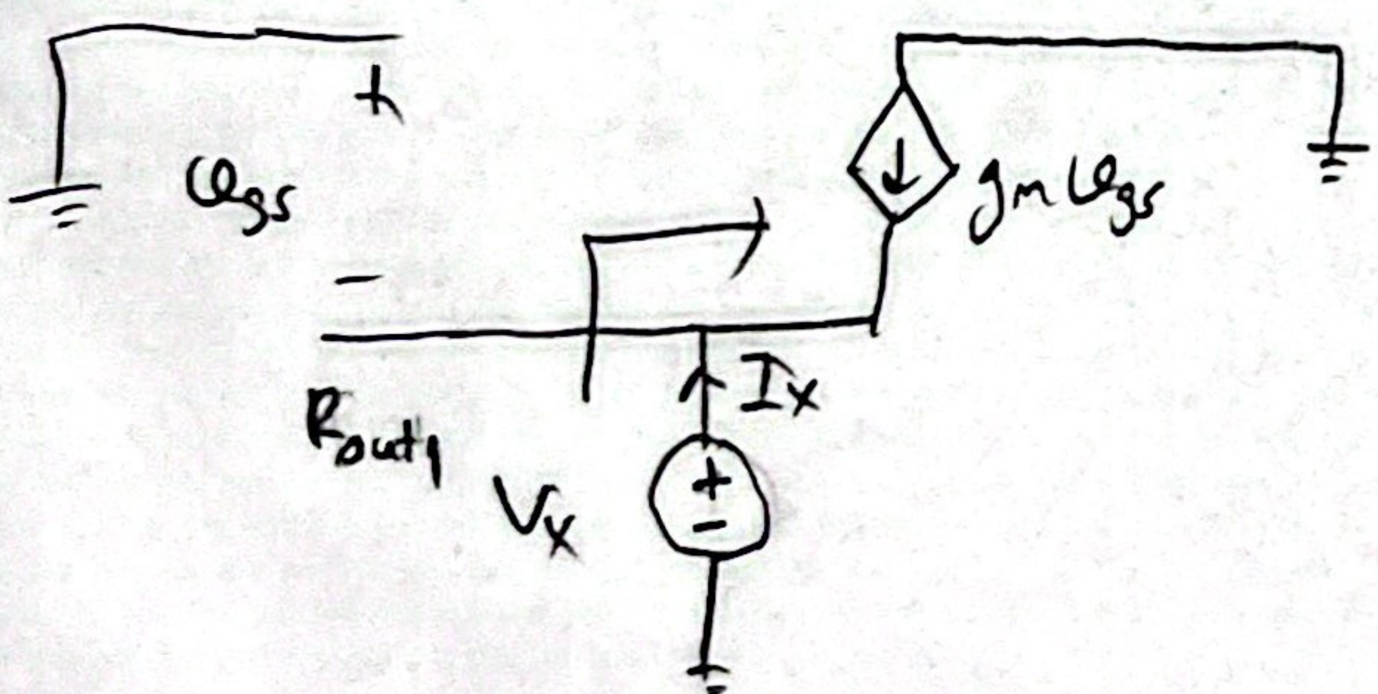
$\Rightarrow v_{gs} = v_{in} - v_{out}$ + insert into (1):

$$\Rightarrow v_{out} = g_m (R_S // r_o) \cdot (v_{in} - v_{out})$$

$$\Rightarrow \boxed{A_v = \frac{v_{out}}{v_{in}} = \frac{g_m (R_S // r_o)}{1 + g_m (R_S // r_o)}} \Rightarrow \boxed{A_v = \frac{(0.632)(500 // 16.85)}{1 + (0.632)(500 // 16.85)} = \frac{10.3}{11.3} = 0.91 \frac{V}{V}}$$

c) $\boxed{R_{in} = R_G = 5M\Omega.}$

To find R_{out}: Kill v_{in}, determine R_{out1} first (look from the source):



We know that $v_{gs} = \cancel{v_g} - v_s = -v_s$

and $v_s = V_X$

$\Rightarrow v_{gs} = -V_X$

$\Rightarrow$ KCL at source:

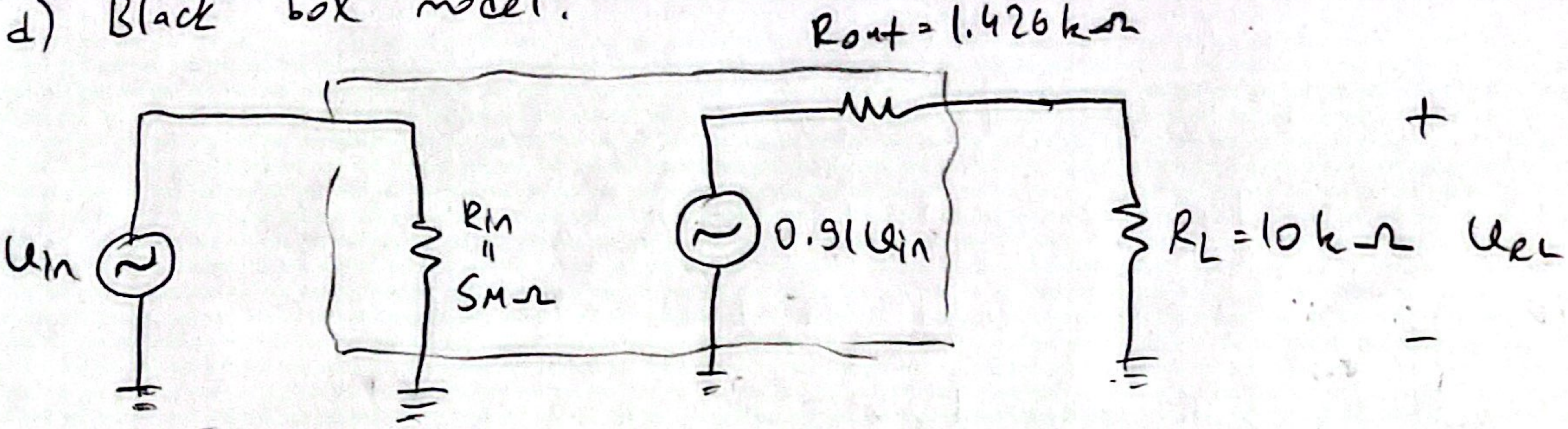
$$\boxed{I_X = -g_m v_{gs} = g_m V_X}$$

$$\Rightarrow \boxed{R_{in1} = \frac{V_X}{I_X} = \frac{1}{g_m}}$$

Now, $R_{out} = R_{out1} // r_o // R_S$

$$\Rightarrow \boxed{R_{out} = \frac{1}{g_m} // r_o // R_S}, \quad \boxed{R_{out} = \left(0.632 + \frac{1}{100} + \frac{1}{16.85}\right)^{-1} = 1.426\ k\Omega}$$

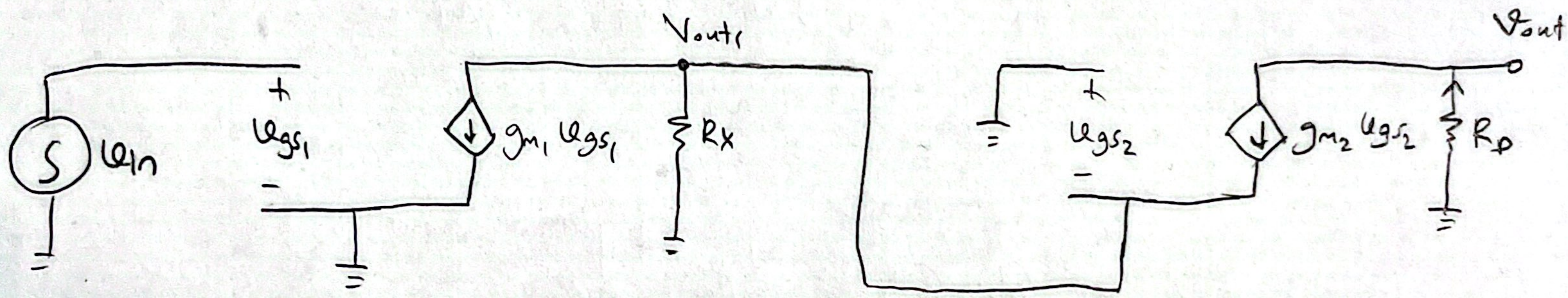
d) Black box model:



Voltage divider: $v_{RL} = \frac{R_L}{R_L + R_{out}} (0.91) v_{in} = 0.796\, v_{in}$

$$\Rightarrow \boxed{A_v = \frac{v_{RL}}{v_{in}} = 0.796 \ \frac{V}{V}}$$

4) Small signal model: (No DC analysis since assumed SAT and g_m's are given)



First, $v_{out} = -g_{m_2} v_{gs_2} R_D$ and $v_{gs_2} = -v_{out_1}$ since $v_{g_2} = 0$

$\Rightarrow v_{out} = g_{m_2} v_{out_1} R_D \Rightarrow \boxed{v_{out_1} = v_{out} \frac{1}{g_{m_2} R_D}}$

Now, KCL at v_{out_1}: $g_{m_1} v_{gs_1} + \frac{v_{out_1}}{R_X} - g_{m_2} v_{gs_2} = 0$

$v_{in} = v_{gs_1}$

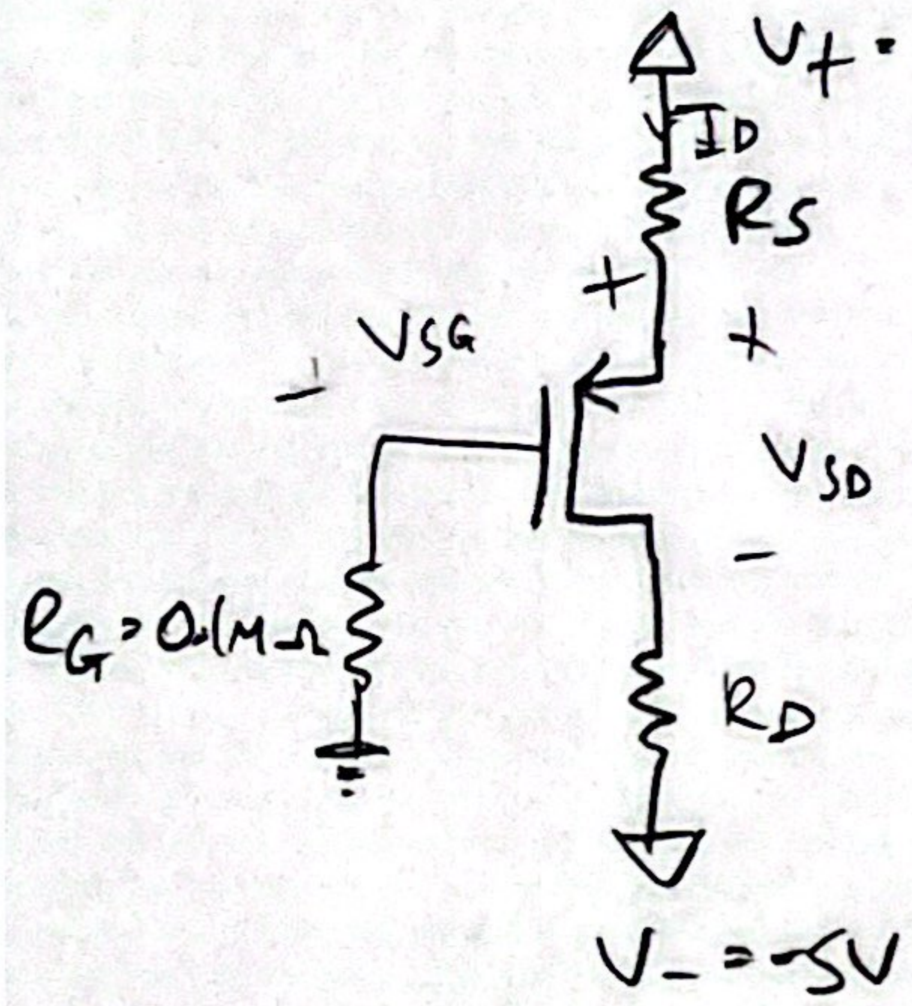
$-v_{out_1} = v_{gs_2}$

$\Rightarrow g_{m_1} v_{in} = -v_{out_1} \left(\frac{1}{R_X} + g_{m_2} \right)$

$g_{m_1} v_{in} = -v_{out} \left(\frac{1}{R_D} \left(\frac{1}{g_{m_2} R_X} + 1 \right) \right)$

$$\Rightarrow \boxed{\frac{v_{out}}{v_{in}} = -\frac{g_{m_1} g_{m_2} R_D R_X}{1 + g_{m_2} R_X}}$$

5) a) In DC, open circuit cap-s:



$\Downarrow$

$V_S = 5 - I_d R_S$

$= 5 - 0.5 R_S$

$\Rightarrow V_{SG} = 5 - 0.5 R_S$

To have $V_{SDQ} = 5V$, apply KVL

$10 = V_{SD} + I_d(R_S + R_D)$

$10 = 5 + 0.5(R_S + R_D)$

$\Rightarrow \boxed{R_S + R_D = 10k\Omega}$

and $I_{DQ} = K_p(V_{SG} - \underbrace{|V_{tp}|}_{1V})^2$, assuming SAT.

$\Rightarrow 0.5 = 0.5(4 - 0.5R_S)^2$

$\Rightarrow \boxed{R_S = 6k\Omega}$ or $\cancel{R_S = 10k\Omega}$

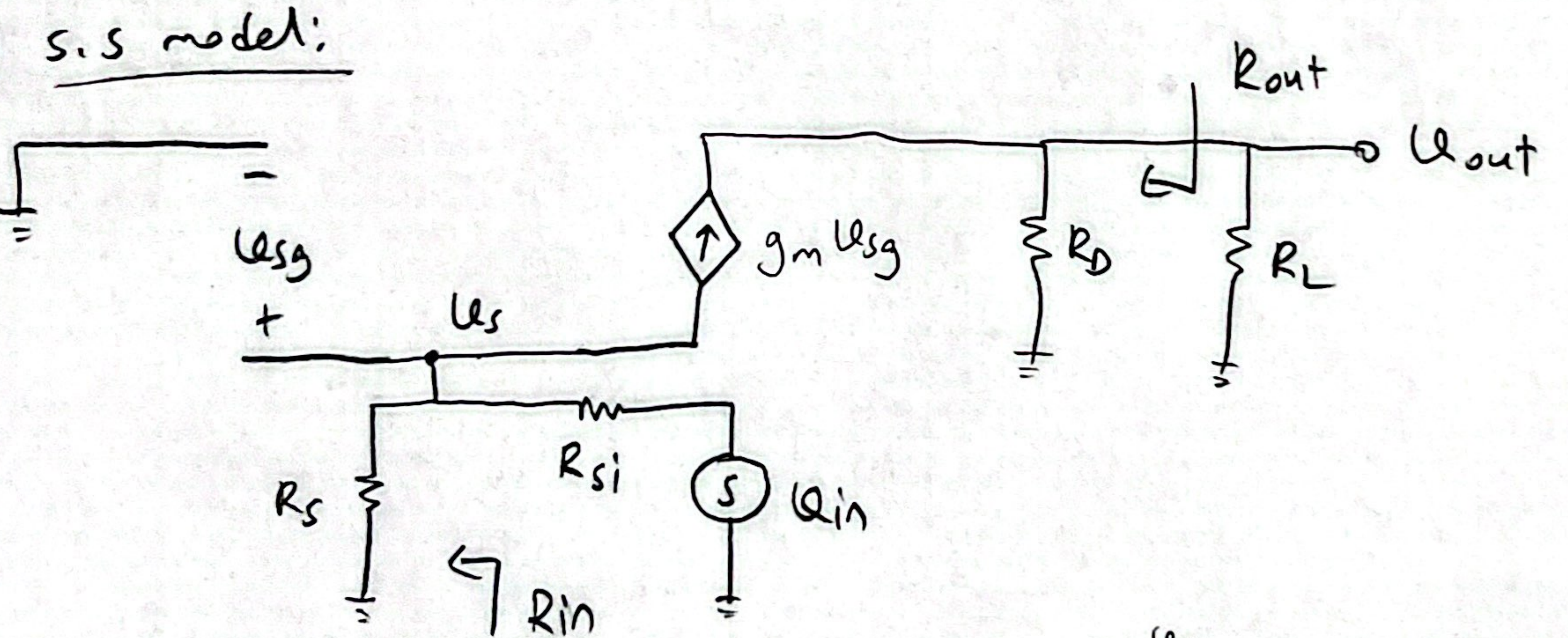
$\Rightarrow \boxed{R_D = 4k\Omega}$

Check!: $V_S = 5 - (0.5)6 = 2V$

$V_{SG} = 2V$, $V_{SD} = 5V$

$\underbrace{V_{SD}}_{5V} > \underbrace{V_{SG}}_{2V} - \underbrace{|V_{tp}|}_{1V} \Rightarrow$ SAT ✓

b) s.s model:



First, KCL at v_s: $\frac{v_s}{R_S} + \frac{v_s - v_{in}}{R_{si}} + g_m v_{sg} = 0$ and $v_{sg} = v_s$ since $v_g = 0$

$\Rightarrow v_s\left(\frac{1}{R_S} + \frac{1}{R_{si}} + g_m\right) \cdot R_{si} = v_{in}$

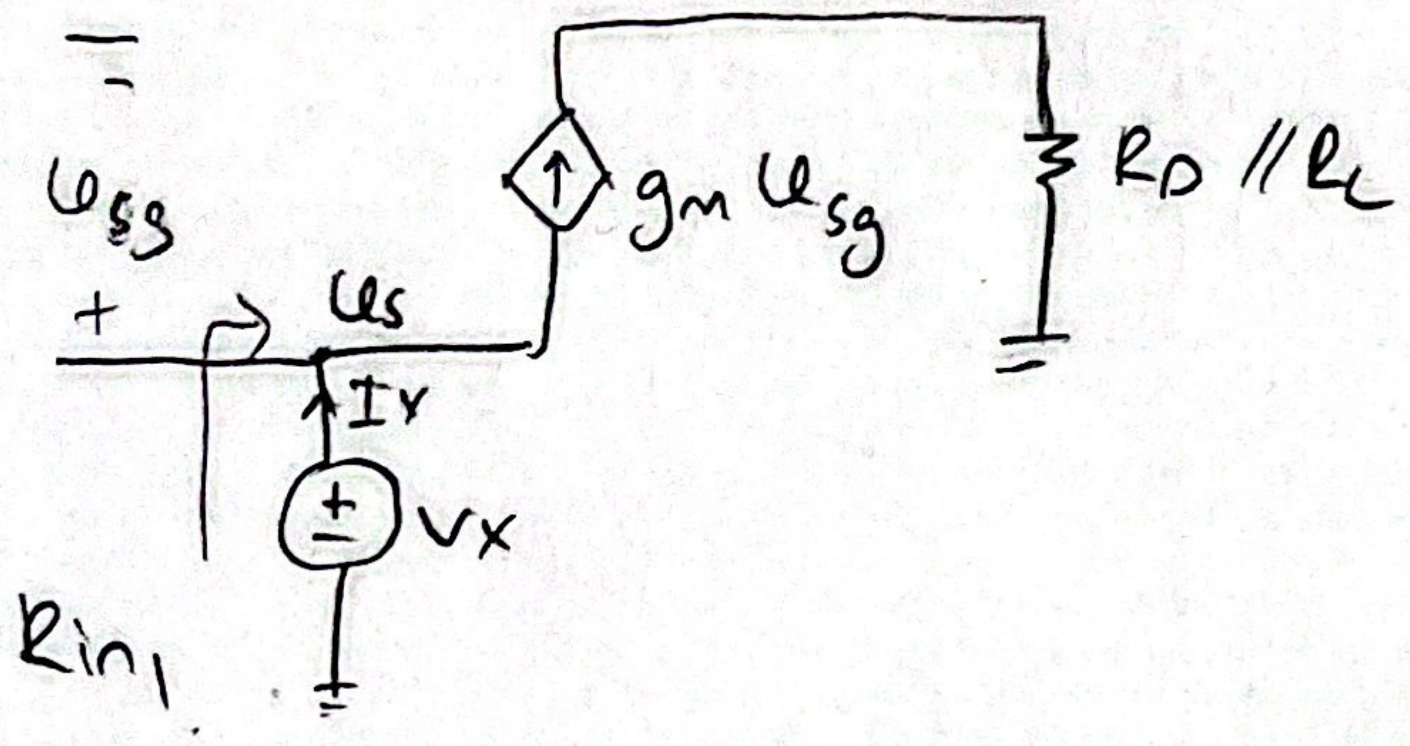
and $v_{out} = g_m v_{sg}(R_D // R_L) = g_m v_s (R_D // R_L)$

$= g_m (R_D // R_L) \cdot \frac{1}{R_{si}\left(\frac{1}{R_S} + \frac{1}{R_{si}} + g_m\right)} \cdot v_{in}$

$\Rightarrow \boxed{A_v = \frac{v_{out}}{v_{in}} = \frac{g_m (R_D // R_L)}{1 + g_m R_{si} + \frac{R_{si}}{R_S}}}$, $g_m = 2\sqrt{\underbrace{K_p}_{0.5} \underbrace{I_D}_{0.5}} = 1\ mA/V$

$\Rightarrow \boxed{A_v = \frac{2k // 4k}{1 + 1\cdot(50k) + \frac{50k}{6k}} = 0.0225\ \frac{V}{V}}$

To find Rin, first ignore R_S :



$v_{sg} = v_s$ and $v_s = V_X \Rightarrow v_{sg} = V_X$

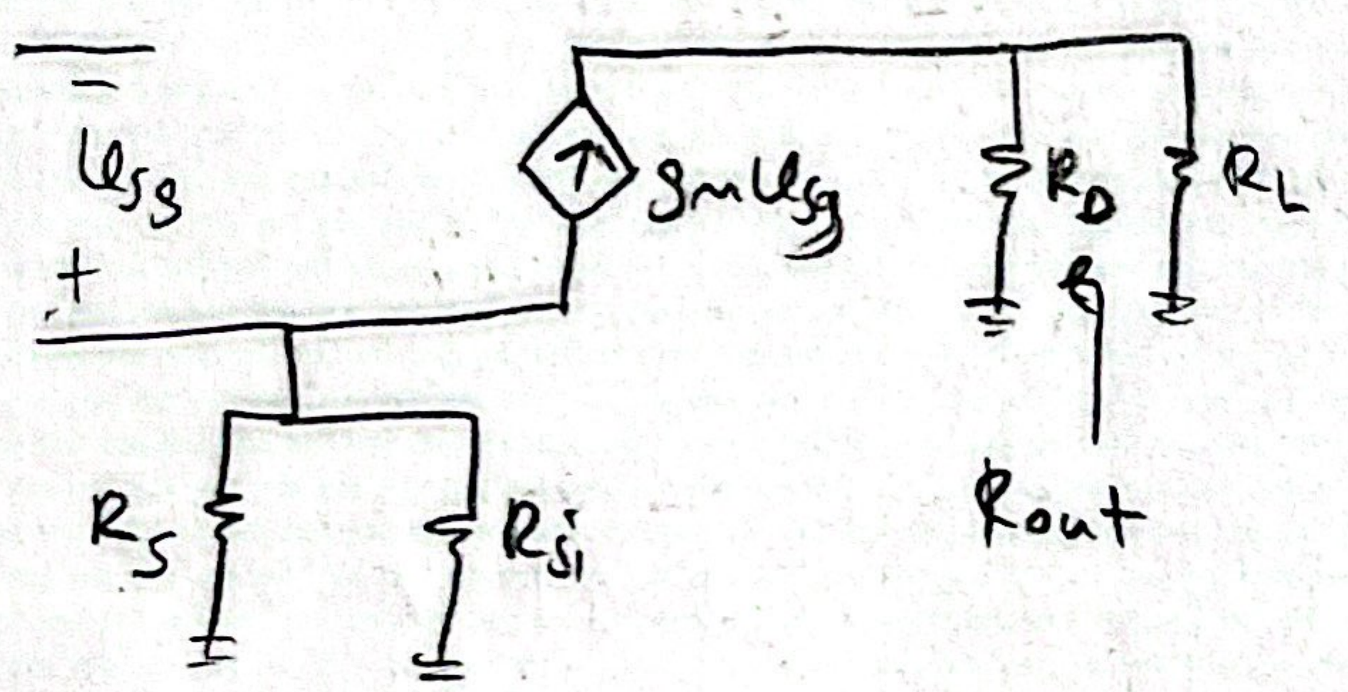
KCL at source: $I_X = g_m v_{sg} = g_m V_X$

$$\Rightarrow Rin_1 = \frac{V_X}{I_X} = \frac{1}{g_m}$$

Now, $\boxed{R_{in} = Rin_1 // R_S = \underbrace{\frac{1}{g_m}}_{1k\Omega} // \underbrace{R_S}_{6k\Omega}}$

$\Rightarrow \boxed{R_{in} = \frac{6}{7} k\Omega \cong 0.86 k\Omega}$

To find R_{out}, kill v_{in}:



$v_{sg} = 0$ since $v_s = 0$

$\Rightarrow g_m v_{sg} = 0$, open-circuit it

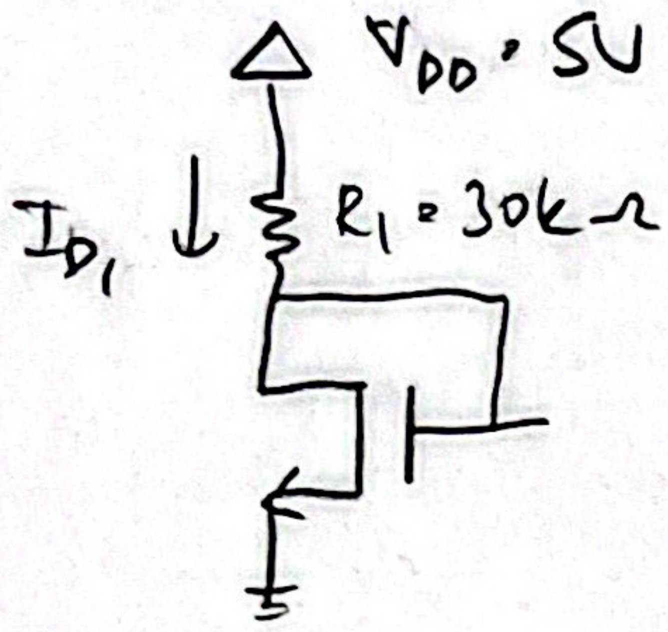
$\Rightarrow \boxed{R_{out} = R_D = 4k\Omega}$

c) Gain was calculated as 0.0225 from prev. part

$\Rightarrow v_{out} = v_{in}(0.0225) = 0.011 \text{ mV} \cos(wt)$

and $\boxed{i_{out} = \frac{v_{out}}{\underbrace{R_L}_{2k\Omega}} = 5.625 \text{ nA} \cos(wt)}$

6) a) For M1:



Assume SAT:

$\Rightarrow I_{D_1} = K_{n_1}(V_{GS_1} - V_{th})^2$

$\Rightarrow I_D = 0.1(4 - 30 I_{D_1})^2$

$I_D = 0.178\,mA$ or $\boxed{I_D = 0.1\,mA}$

since we need $V_{GS} > V_{th}$

$V_{G_1} = V_{D_1} = 5 - I_{D_1}(30k)$
and $V_{S_1} = 0$

$\Rightarrow V_{GS_1} = 5 - (0.1)(30) = 2V$

$\Rightarrow V_{DS_1} = 2V$

Check: $\underbrace{V_{DS_1}}_{2V} > \underbrace{V_{GS_1}}_{2V} - \underbrace{V_{th}}_{1V} \Rightarrow$ M1 in SAT ✓ (Diode connected)

$\Rightarrow \boxed{I_{D_1} = 0.1\,mA}$, $\boxed{V_{G_1} = V_{D_1} = 2V}$

For M2: Assume SAT

$\boxed{V_{G_2} = 2V}$, $V_{S_2} = (1.17) I_{D_2} \Rightarrow V_{GS_2} = 2 - 1.17 I_{D_2}$

$I_{D_2} = K_{n_2}(V_{GS_2} - V_{th})^2 = 0.5(1 - 1.17 I_{D_2})^2$

$\Rightarrow \boxed{I_{D_2} = 0.25\,mA}$ or $\cancel{I_{D_2} = 2.92\,mA}$ $\Rightarrow V_{GS_2} = 1.71V$

we need $V_{GS_2} > V_{th}$

$\boxed{V_{S_2} = 0.29\,V}$

Now, for M3 also assume SAT:

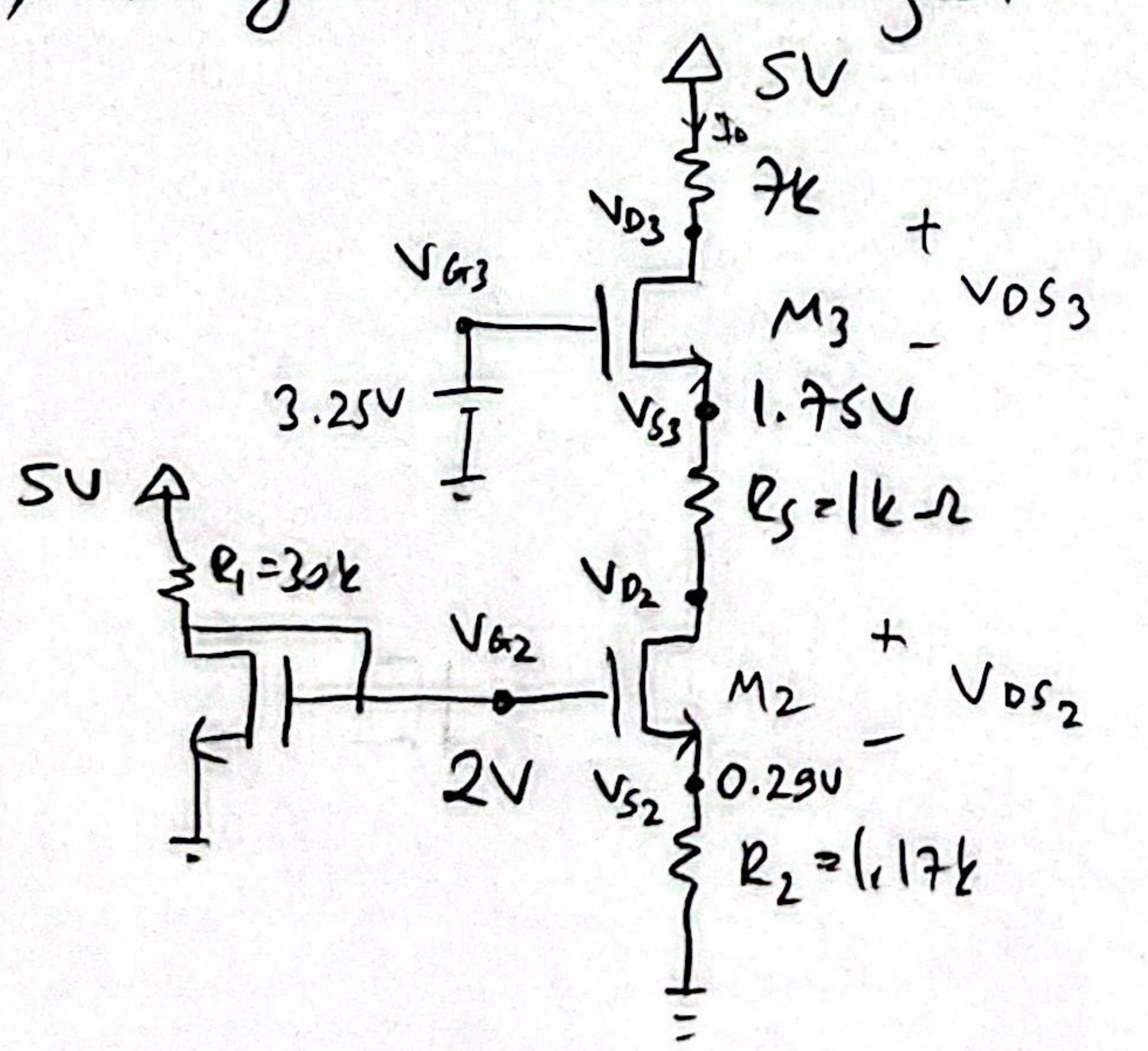
$I_{D_2} = I_{D_3} = K_{n_3}(V_{GS_3} - V_{th})^2 \Rightarrow 0.25 = 1(3.25 - V_{S_3} - 1)^2$

$\boxed{V_{G_3} = 3.25V}$

$\boxed{V_{S_3} = 1.75\,V}$ or $\cancel{V_S = 2.75\,V}$

since we need $V_{GS_3} > V_{th}$ $\Rightarrow V_{GS_3} = 1.5V$

Now, to find node voltages:



$V_{D_3} = 5 - I_D(7k) = 5 - 7(0.25)$

$\Rightarrow \boxed{V_{D_3} = 3.25V}$

$V_{D_2} = V_{S_3} - I_D R_S = 1.75 - (0.25)1$

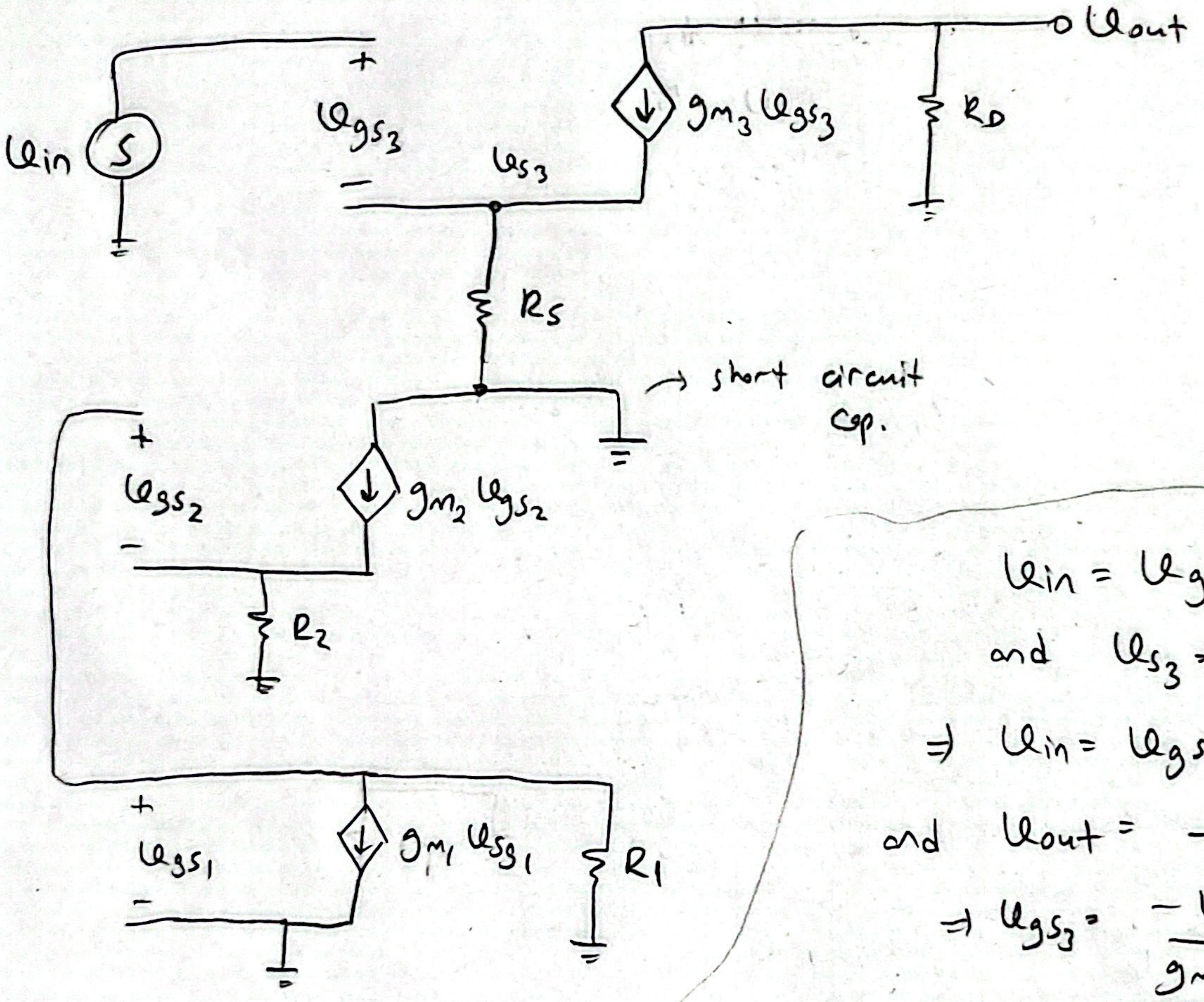
$\Rightarrow \boxed{V_{D_2} = 1.5V}$

Check: $V_{DS_2} = \underbrace{V_{D_2}}_{1.5} - \underbrace{V_{S_2}}_{0.29} = 1.21V$, $V_{DS_3} = \underbrace{V_{D_3}}_{3.25} - \underbrace{V_{S_3}}_{1.75} = 1.5V$

$\Rightarrow \underbrace{V_{DS_2}}_{1.21V} > \underbrace{V_{GS_2}}_{1.71V} - \underbrace{V_{th}}_{1V} \Rightarrow$ M2 in SAT ✓

$\Rightarrow \underbrace{V_{DS_3}}_{1.5V} > \underbrace{V_{GS_3}}_{1.5V} - \underbrace{V_{th}}_{1V} \Rightarrow$ M3 in SAT ✓

b) <u>s.s model:</u>



$\Rightarrow$ M1 and M2 have no effect since R_S is grounded with the capacitor.

$v_{in} = v_{gs_3} + v_{s_3}$

and $v_{s_3} = g_{m_3} v_{gs_3} R_S$

$\Rightarrow v_{in} = v_{gs_3}(1 + g_{m_3} R_S)$

and $v_{out} = -g_{m_3} v_{gs_3} R_D$

$\Rightarrow v_{gs_3} = \dfrac{-v_{out}}{g_{m_3} R_D}$

$$\Rightarrow \boxed{A_v = \frac{v_{out}}{v_{in}} = -\frac{g_{m_3} R_D}{1 + g_{m_3} R_S}}$$

$$g_{m_3} = 2\sqrt{\underbrace{K_{n_3}}_{1} \underbrace{I_{D_2}}_{0.25}} = 1\,mA/V \Rightarrow \boxed{A_v = -\frac{7k\Omega}{1 + 1.(1k\Omega)} = -3.5\,\frac{V}{V}}$$