ECD HW-3

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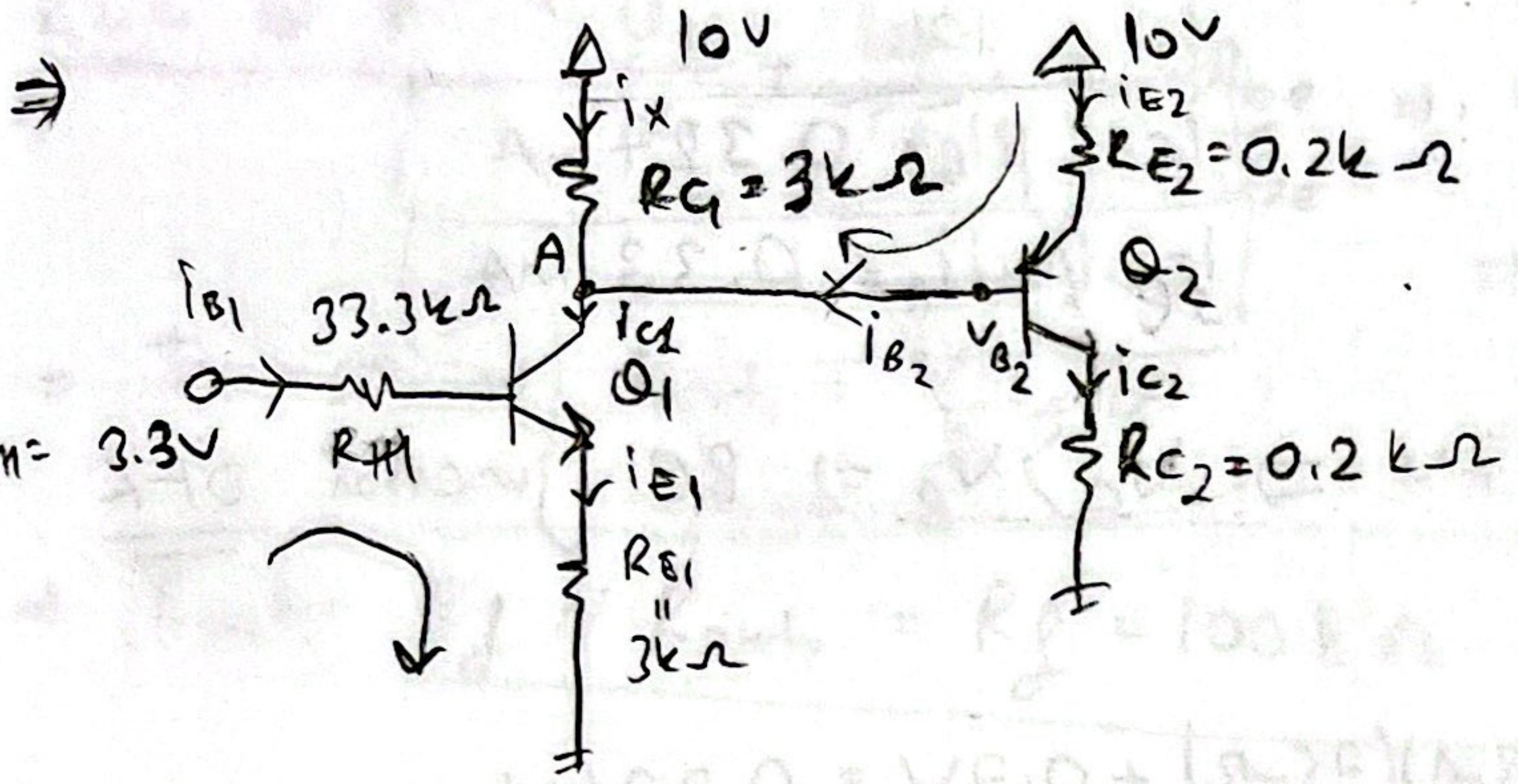
(1) Thévenin conversion for Q_1, $V_{TH} = 10V \cdot \frac{R_1}{R_1+R_2} = 10V \cdot \frac{50k}{150k} = 3.3V$

$$R_{TH} = (100k) // (50k) = 33.3\ k\Omega$$

⇒



Assume Q_1 F.A:

KVL: $V_{TH} = i_{B_1} R_{TH} + V_{BE_1}(ON) + \underbrace{i_{E_1}}_{(\beta+1)i_{B_1}} R_{E_1}$

$\Rightarrow 3.3 = (33.3) i_{B_1} + 0.7 + i_{B_1}(101)(3)$

$\Rightarrow \boxed{i_{B_1} = 7.73\ \mu A}$

$\Rightarrow \boxed{i_{E_1} = (101)(7.73\mu A) = 0.781\ mA}$

$\boxed{i_{C_1} = 100(7.73\mu A) = 0.773\ mA}$

$V_A = V_{B_2} = 10 - 3(\underbrace{i_{C_1}}_{0.773} - i_{B_2}) = 7.681 + 3 i_{B_2}$ ⟶ Note: KCL @ A: $\boxed{i_x = i_{C_1} - i_{B_2}}$

Assume Q_2 F.A.: KVL: $10V = i_{E_2} R_{E_2} + V_{EB_2}(ON) + V_A$

$\Rightarrow 10 = \underbrace{(\beta+1)}_{101} \underbrace{R_{B_2}}_{0.2} i_B + 0.7 + 7.681 + 3 i_{B_2}$

$\Rightarrow \boxed{i_{B_2} = 69.8\ \mu A}$

$\Rightarrow \boxed{i_{E_2} = (101)(69.8\mu A) = 7.05\ mA}$

$\Rightarrow \boxed{i_{C_2} = (100)(69.8\mu A) = 6.98\ mA}$

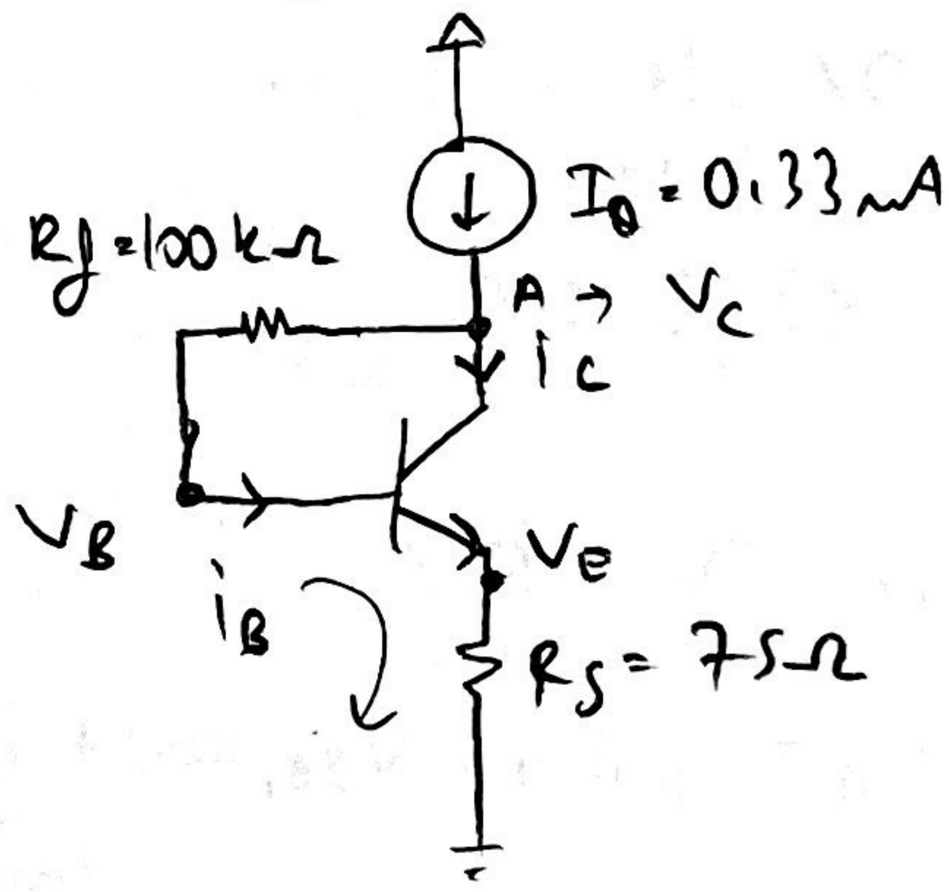
Check: Q_2: $V_{EC} = 10 - \overbrace{(0.2)}^{R_{E2}}\overbrace{(7.05)}^{i_{E2}} - \overbrace{(0.2)}^{R_{C2}}\overbrace{(6.98)}^{i_{C2}} = 7.194V > V_{EC}(SAT) = 0.2V$

$\Rightarrow \boxed{Q_2 \text{ in F.A}}$ ✓

Q_1: $V_{CE} = 10 - \overbrace{3}^{R_{C1}}(\underbrace{\overbrace{0.773}^{i_{C1}} - \overbrace{0.0697}^{i_{B2}}}_{i_x}) - \overbrace{3}^{R_{E1}}(\overbrace{0.781}^{i_{E1}}) = 5.547V > V_{CE}(SAT) = 0.2V$

$\Rightarrow \boxed{Q_1 \text{ in F.A}}$ ✓

(2) a) DC analysis: Kill v_s, open circuit caps.



Assume F.A:

KCL @ A: $I_Q = i_C + i_B = (\beta+1)\, i_B$

$\Rightarrow$ $\boxed{i_B = \frac{0.33}{101} = 3.27\,\mu A}$

$\boxed{i_C = \beta i_B = 0.327\,mA}$

$\boxed{i_E = (\beta+1) i_B = 0.33\,mA}$

Note that $V_C - V_B = i_B R_f = 0.327V \Rightarrow V_C > V_B \Rightarrow$ BC junction OFF

$\Rightarrow Q_1$ in F.A as long as ON.

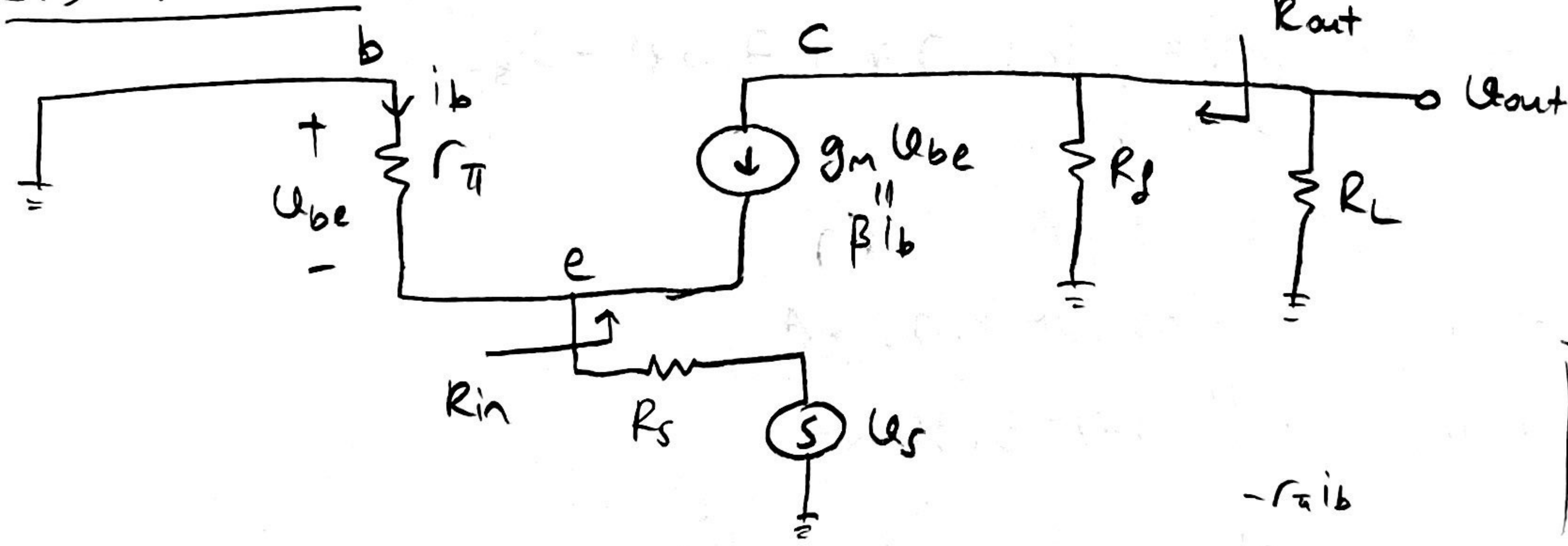
Check: $V_B = i_E R_S + V_{BE}(ON) = (0.33mA)(75\Omega) + 0.7V = 0.724V$

$V_E = 0.0247V \quad (R_S \cdot i_E)$

$V_C = V_B + i_B \cdot R_f = V_B + 0.327V = 1.05V$

$\Rightarrow V_{CE} = V_C - V_E = 1.027V > V_{CE}(SAT) = 0.2V \Rightarrow$ in F.A ✓

b) S.S model:



$\boxed{r_\pi = \frac{26mV}{3.27\,\mu A} = 7.95\,k\Omega}$

$\boxed{g_m = \frac{\beta}{r_\pi} = 12.58\,mA/V}$

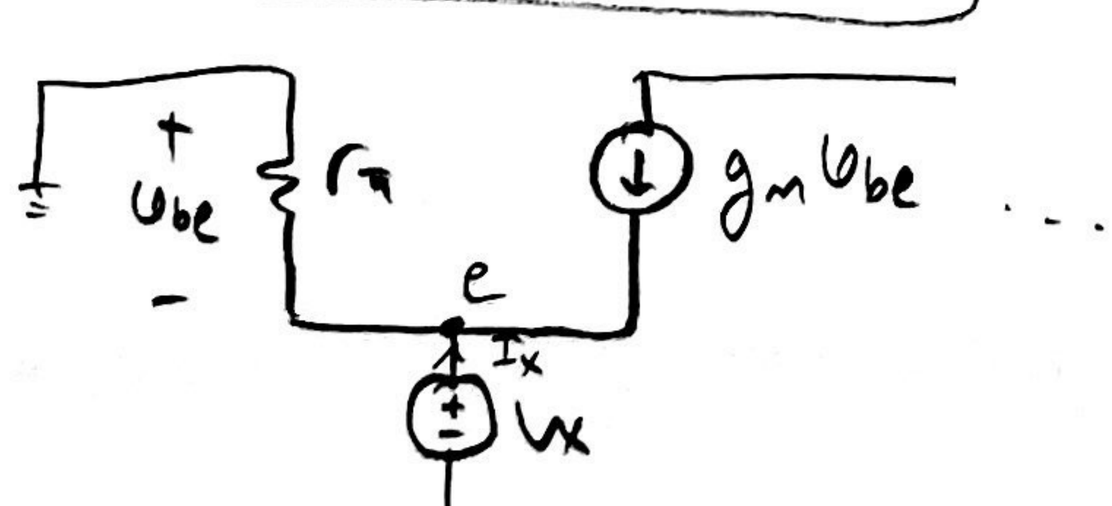
$v_{out} = -\beta i_b (R_f \,//\, R_L)$, KCL @ e: $i_b + \beta i_b = \frac{\overset{-r_\pi i_b}{v_e} - v_s}{R_S}$

and $v_e = -v_{be}$ since $v_b = 0 \Rightarrow v_e = -r_\pi \cdot i_b$

$\Rightarrow v_s = -((\beta+1) R_S i_b + r_\pi i_b)$

$$\Rightarrow \boxed{A_v = \frac{v_{out}}{v_s} = \frac{\beta (R_f \,//\, R_L)}{r_\pi + (\beta+1) R_S}}$$

To find R_{in}:



$V_x = v_e$, $v_{be} = -v_e$ since $v_b = 0$

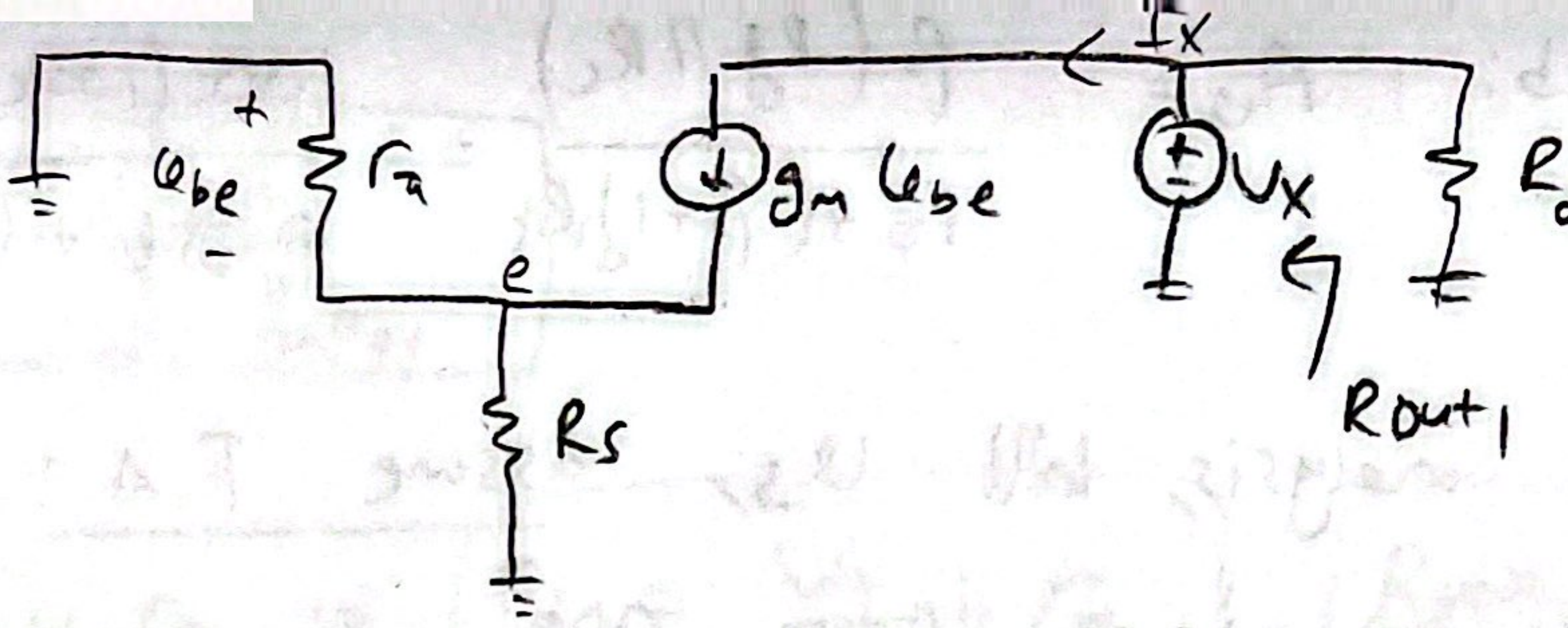
KCL @ e: $I_x = -g_m \underbrace{v_{be}}_{-v_x} + \frac{v_e}{r_\pi}$

$\Rightarrow I_x = v_x \left(g_m + \frac{1}{r_\pi}\right)$

$\Rightarrow \boxed{R_{in} = \frac{V_x}{I_x} = r_\pi \,//\, \frac{1}{g_m}}$

$\boxed{R_{in} = 7.95 \,//\, 0.0795 = 78\,\Omega}$

To find R_{out}: Kill v_{in}!



$v_{be} = -v_e$ since $v_b = 0$

$I_x = g_m v_{be} = -g_m v_e$

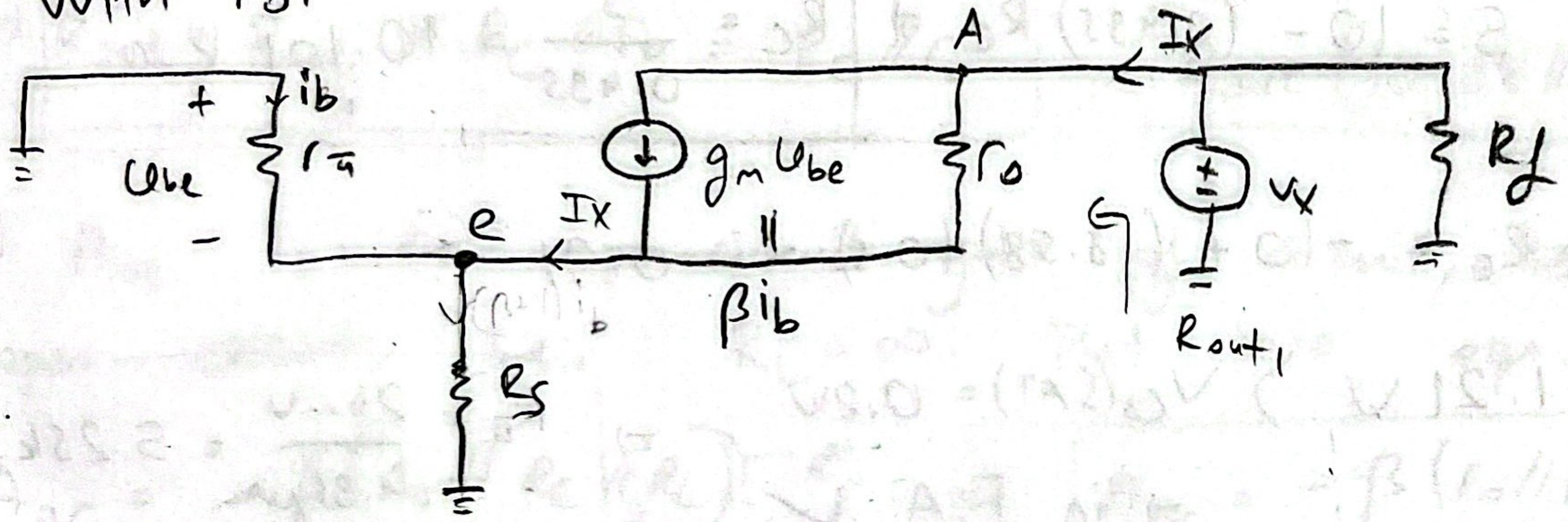
KCL @ e: $\frac{v_e}{R_s} + \frac{v_e}{r_\pi} = g_m \underbrace{v_{be}}_{-v_e} \Rightarrow v_e\left(g_m + \frac{1}{R_s} + \frac{1}{R_E}\right) = 0$

$\Rightarrow v_e = 0$ since $g_m + \frac{1}{R_s} + \frac{1}{R_E} \neq 0$.

$\Rightarrow I_x = \infty \Rightarrow R_{out1} = \frac{V_x}{I_x} = \infty$

$\Rightarrow R_{out} = R_f // R_{out1} = R_f = 100\,k\Omega$

c) With r_o:



KVL: $V_x = (I_x - g_m v_{be}) r_o + I_x (R_s // r_\pi)$

and $v_{be} = -v_e$ since $v_b = 0$, $v_e = I_x (R_s // r_\pi)$

$\Rightarrow V_x = (I_x + g_m I_x (R_s // r_\pi)) r_o + I_x (R_s // r_\pi)$

$\Rightarrow R_{out1} = \frac{V_x}{I_x} = (1 + g_m (R_s // r_\pi)) r_o + (R_s // r_\pi)$

and $R_{out} = R_{out1} // R_f = \left[(1 + g_m (R_s // r_\pi)) r_o + (R_s // r_\pi)\right] // R_f$

$\Rightarrow R_{out} = 65.93\,k\Omega$

d) in part b: $A_v = \dfrac{\beta(R_g // R_L)}{r_\pi + (\beta+1)R_s} = \dfrac{100\,(100k // 1.5k)}{7.95k + (101)(75\Omega)} \cong \boxed{9.52 \frac{V}{V}}$

(3) a) in DC analysis, kill v_s, assume F.A:

$i_E = 0.5mA \Rightarrow i_B = \dfrac{i_E}{\beta+1} = \dfrac{0.5}{101} = 4.95\,\mu A$

KVL: $i_B R_S + V_{BE}(ON) + i_E R_E - 10V = 0$

$\Rightarrow (0.0049)(2.5) + 0.7 + (0.5)R_E = 10 \Rightarrow R_E = \dfrac{10 - 0.7 - i_B R_S}{i_E}$

$\Rightarrow \boxed{R_E = 18.58\,k\Omega}$

b) $i_C = \beta i_b = 0.495\,mA$

$V_C = V_{CC} - i_C R_C \Rightarrow 5 = 10 - (0.495)R_C \Rightarrow \boxed{R_C = \dfrac{5}{0.495} = 10.101\,k\Omega}$

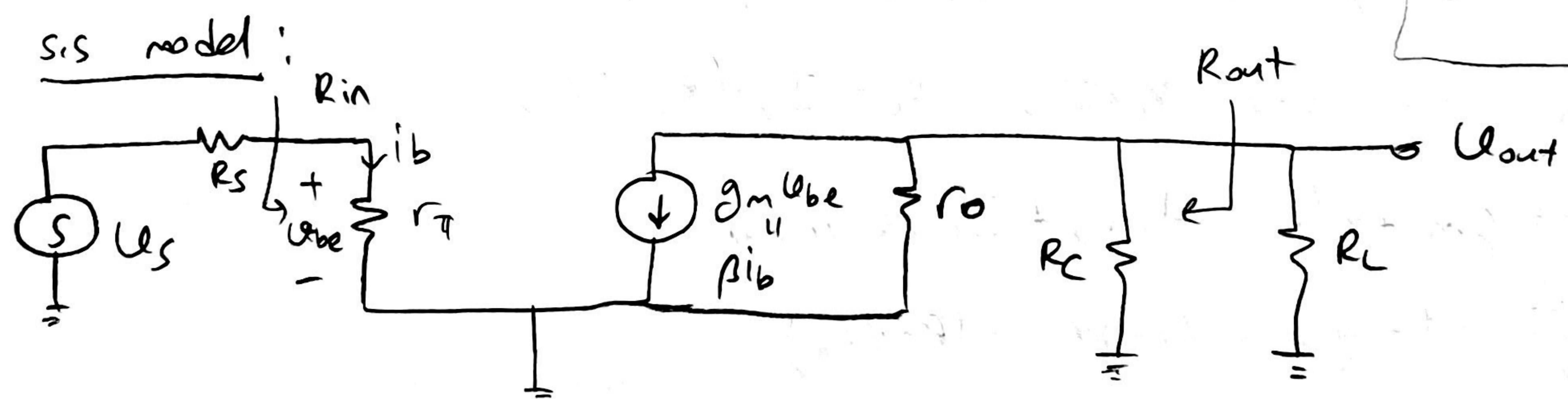
Check: $V_E = -10 + i_E R_E = -10 + (18.58)(0.5) = -0.71$

$V_{CB} = \underbrace{V_C}_{0.5V} - \underbrace{V_E}_{-0.71V} = 1.21\,V > V_{CB}(SAT) = 0.2V$

$\Rightarrow$ in F.A ✓

$\Rightarrow \boxed{r_\pi = \dfrac{26mV}{4.95\mu A} = 5.25k\Omega \quad g_m = \dfrac{\beta}{r_\pi} = 19.04\,mA/V}$

c) s.s model:

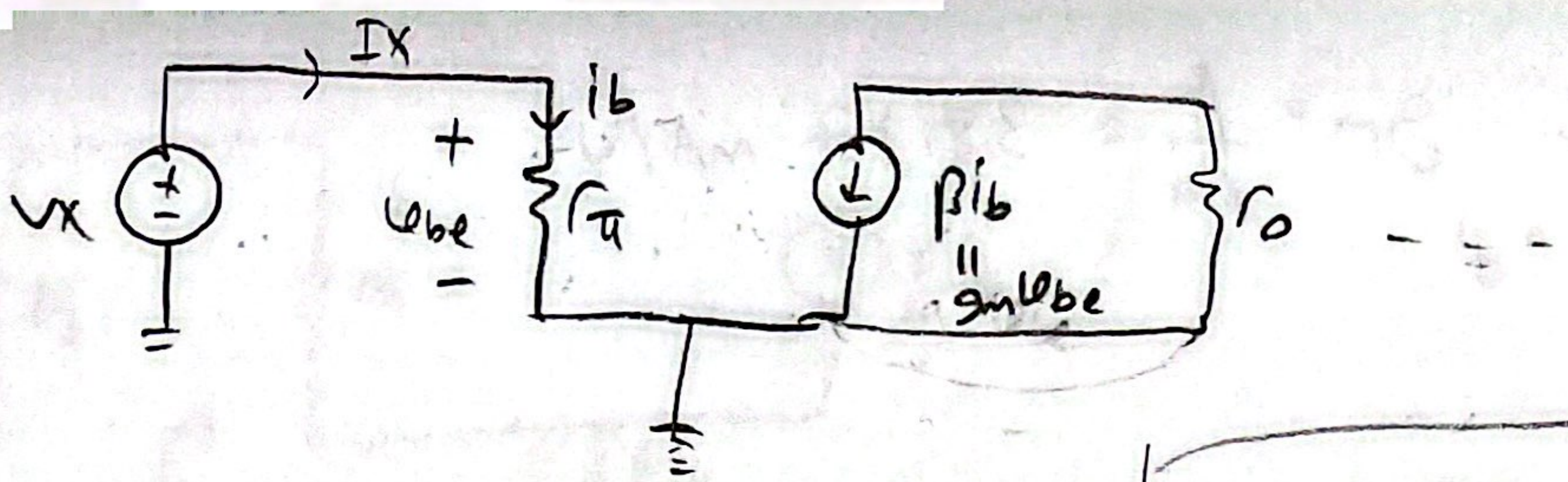


$v_{be} = v_s \cdot \dfrac{r_\pi}{r_\pi + R_s}$ (voltage div.) and $v_{out} = -g_m(r_o // R_C // R_L) \cdot v_{be}$

$\Rightarrow \boxed{A_v = \dfrac{v_{out}}{v_s} = -\dfrac{r_\pi}{r_\pi + R_s} g_m (r_o // R_C // R_L) = \dfrac{-\beta(r_o // R_C // R_L)}{r_\pi + R_s}}$

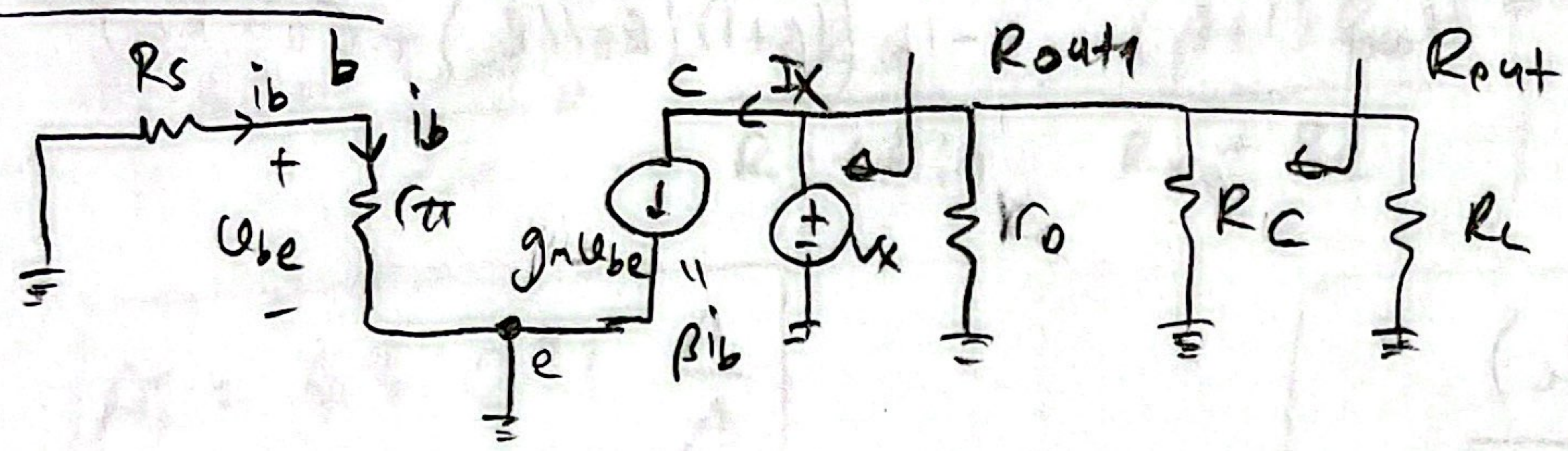
$A_v = \dfrac{-100 \cdot (200k // 10.10k // 10k)}{5.25k + 2.5k} = -63.25 \frac{V}{V}$

d) Find R_{in}:



$I_X = i_b$ and $V_X = v_{be} = i_b \cdot r_\pi \Rightarrow \boxed{R_{in} = \frac{V_X}{I_X} = r_\pi}$ $\boxed{R_{in} = 5.25k\Omega}$

Find R_{out}: kill v_{in}



Note that since $v_e = 0$

$v_b = -R_S i_b = r_\pi i_b$

$\Rightarrow \underline{i_b = 0}$ since $R_S + r_\pi \neq 0$

$\Rightarrow I_X = \beta i_b = 0$, $R_{out_1} = \frac{V_X}{I_X} = \infty$

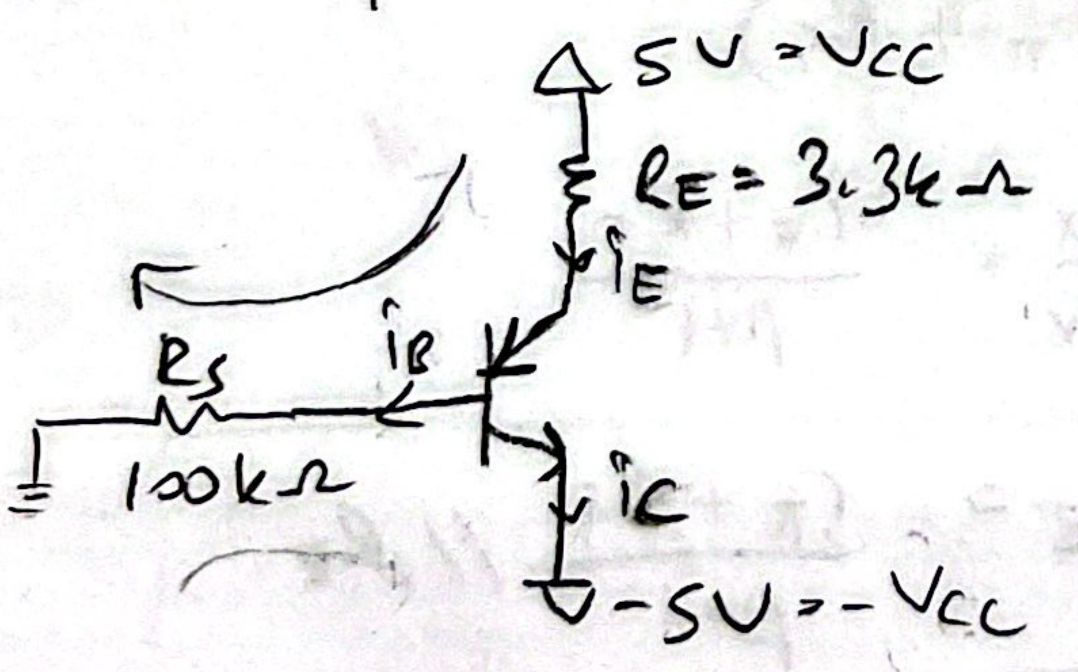
$\Rightarrow \boxed{R_{out} = R_{out_1} \,//\, R_C \,//\, r_o = R_C \,//\, r_o}$ $\boxed{R_{out} = (10.10k \,//\, 200k) = 9.61k\Omega}$

e) A_{oc} is the open-circuit gain of circuit, so we have set R_L to open circuit, i.e. $R_L = \infty$. Put into eqn. found in pt. c:

$$A_{oc} = -\frac{\beta(r_o \,//\, R_C \,//\, \overset{\infty}{R_L})}{r_\pi + R_S} \Rightarrow \boxed{A_{oc} = \frac{-\beta(r_o \,//\, R_C)}{r_\pi + R_S}}$$

$$\Rightarrow \boxed{A_{oc} = -124.058 \ \frac{V}{V}}$$

4) a) DC comp. is zero $\Rightarrow$ kill v_s, open circ. cap.s:



Assume F.A:

KVL: $\underbrace{i_E}_{(\beta+1)i_B} R_E + \underbrace{V_{EB}(ON)}_{0.7V} + i_B R_S = 5V$

$$\boxed{i_B = \frac{5 - 0.7}{101(3.3) + 100} = 9.92\ \mu A}$$

$$\boxed{i_E = (\beta+1)\, i_B = 1.00\ mA}$$

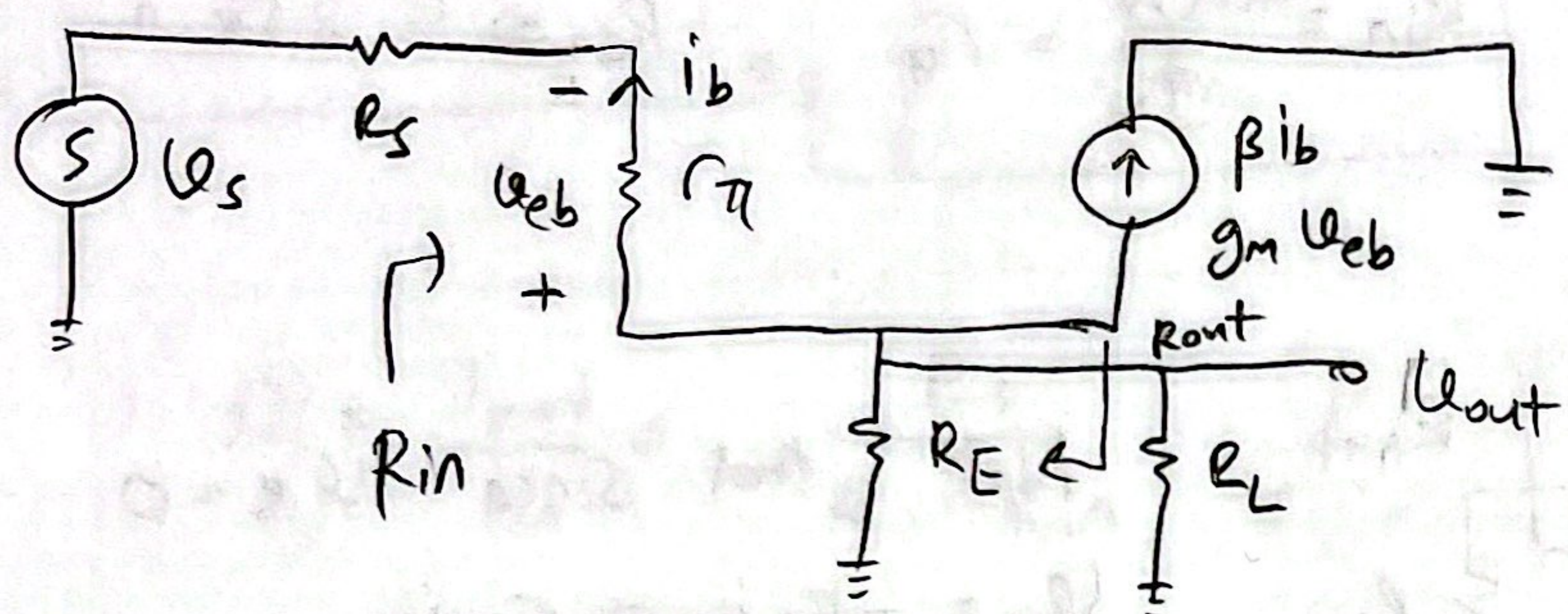
Check: $V_E = 5 - i_E R_E = 5 - 3.3 = 1.7V$

$V_{EC} = V_E - \underbrace{V_C}_{-5V} = 6.7V > V_{EC}(SAT) = 0.2V$

$\Rightarrow$ in F.A ✓

b) $r_\pi = \dfrac{26\,mV}{9.92\,mA} = 2.62\,k\Omega$, $g_m = \dfrac{\beta}{r_\pi} = 38.17\,mA/V$

s.s model:



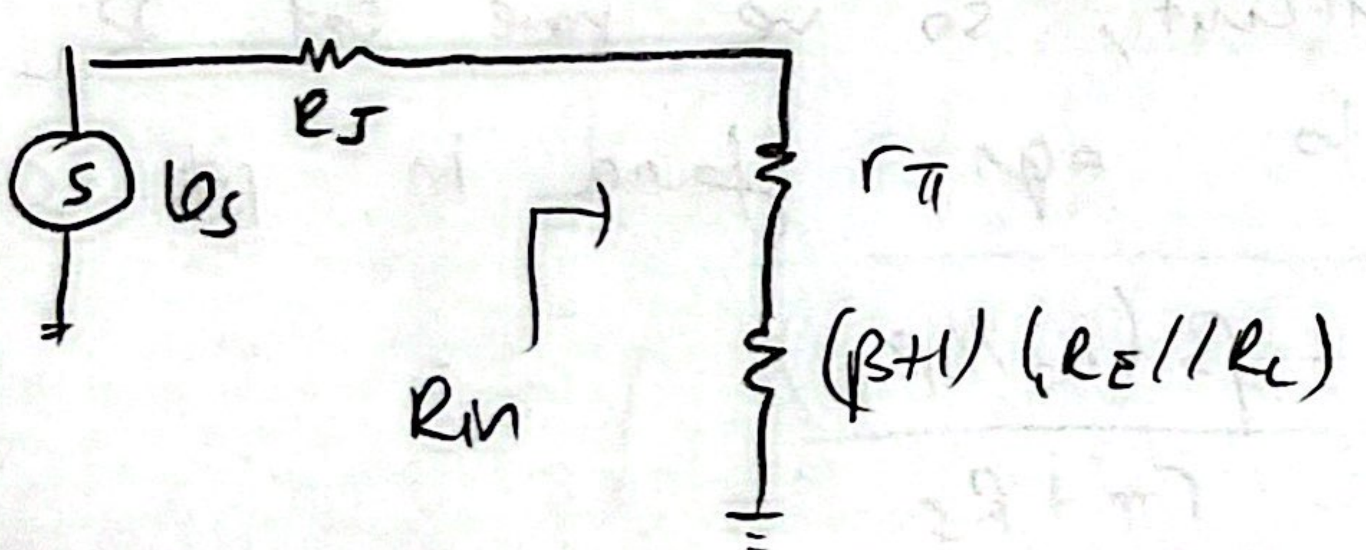
$v_{out} = -(\beta+1)\,i_b\,(R_E // R_L)$

$v_s = -(v_{out} + v_{eb} + i_b R_s)$

$= -i_b\left((\beta+1)(R_E//R_L) + r_\pi + R_s\right)$

$$\Rightarrow \boxed{A_v = \frac{v_{out}}{v_s} = \frac{(\beta+1)(R_E//R_L)}{(\beta+1)(R_E//R_L) + r_\pi + R_s}}$$

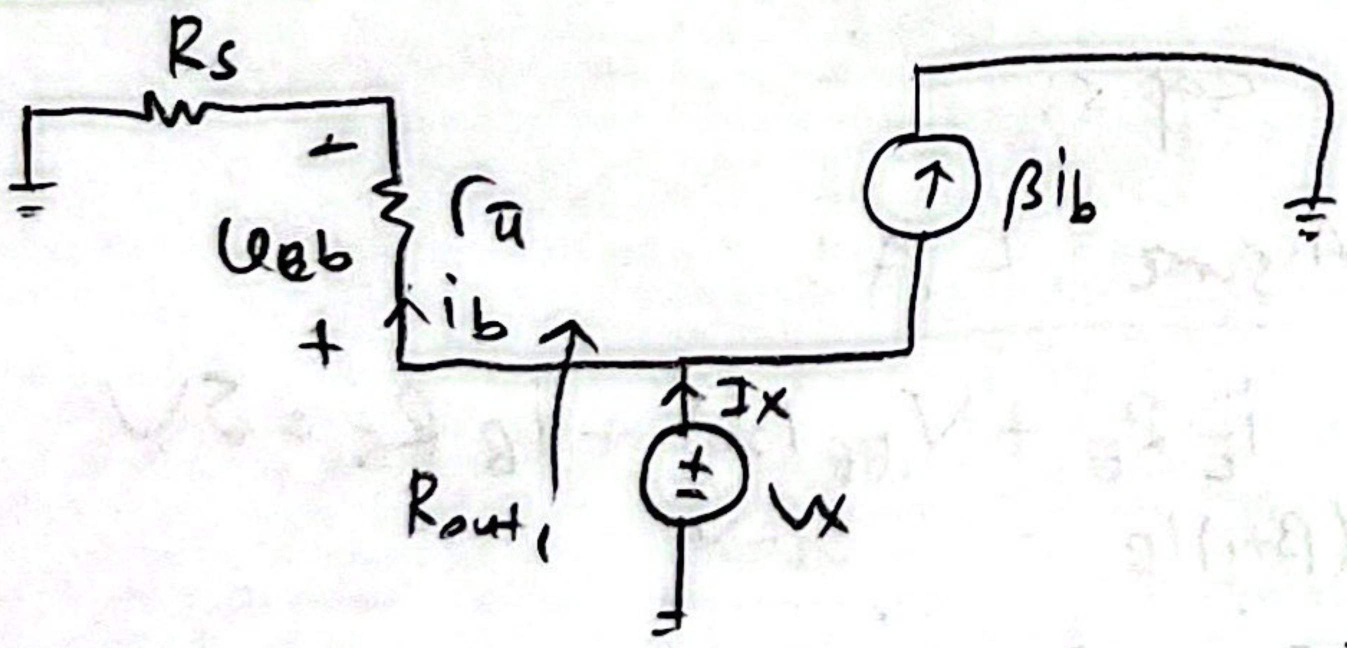
To find R_{in}: we can move R_E and R_L directly to the base side as $(\beta+1)(R_E//R_L)$ since the current over them is $(\beta+1)i_b$!



$$\boxed{R_{in} = r_\pi + (\beta+1)(R_E//R_L)}$$

$$\boxed{R_{in} = 80.13\,k\Omega}$$

To find R_{out}; Kill v_{in}



$I_x = (\beta+1)\,i_b$

$V_x = i_b\,(r_\pi + R_s)$

$\Rightarrow R_{out_1} = \dfrac{V_x}{I_x} = \dfrac{r_\pi + R_s}{\beta+1}$

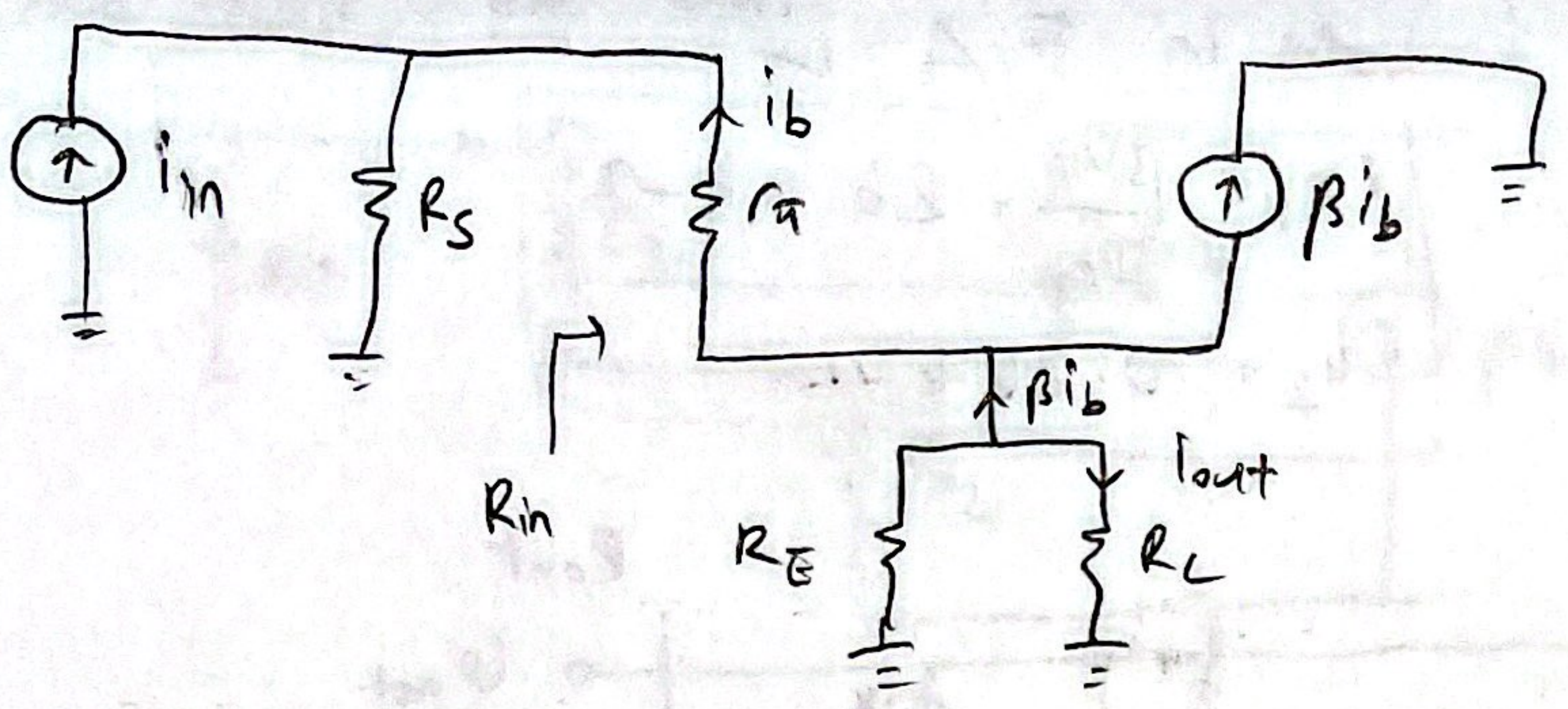
$$\Rightarrow \boxed{R_{out} = R_{out_1} // R_E = \frac{r_\pi + R_s}{\beta+1} // R_E}$$

$$\Rightarrow \boxed{R_{out} = 0.77\,k\Omega}$$

c) from prev. part:

$$\boxed{A_v = \frac{v_{out}}{v_{in}} = \frac{(\beta+1)(R_E//R_L)}{(\beta+1)(R_E//R_L) + r_\pi + R_s} = 0.43\,\frac{V}{V}}$$

d)



$$i_b = -i_{in} \frac{R_S}{R_S + R_{in}} \quad \text{(cur. divider)}$$

$$i_{out} = -(\beta+1) i_b \frac{R_E}{R_E + R_L}$$

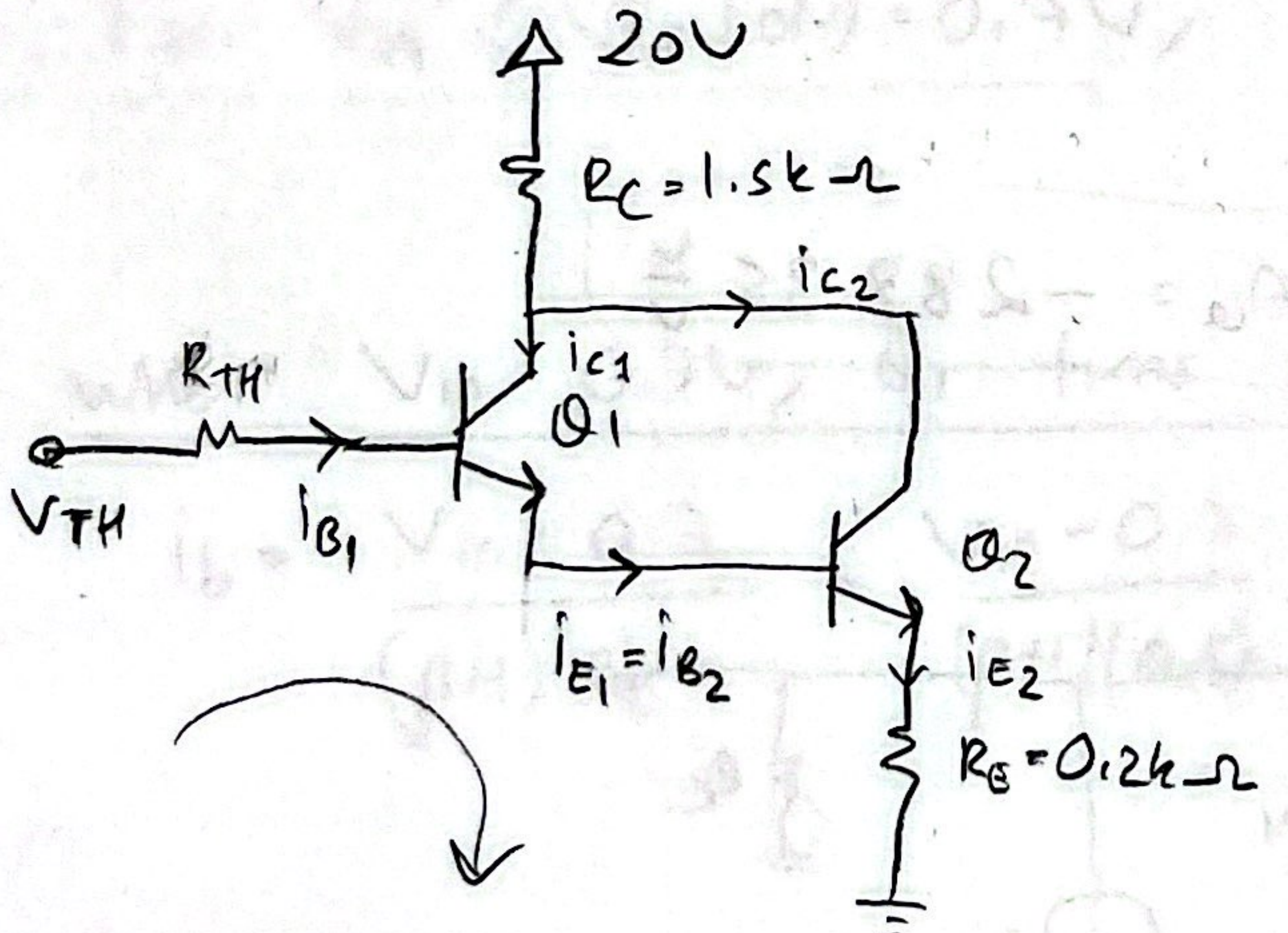
$$\Rightarrow \boxed{A_i = \frac{i_{out}}{i_{in}} = \frac{R_S}{R_S + R_{in}} \cdot \frac{(\beta+1) R_E}{R_E + R_L}} \text{ , } R_{in} \text{ found in pt. b}$$

$$\boxed{A_i = 43.031 \ \frac{A}{A}}$$

5 a) Thev. eq. for Q_1 : $V_{TH} = 20V \cdot \frac{R_2}{R_1 + R_2} = 20 \cdot \frac{17}{17+83} = 3.4V$

$R_{TH} = 83k // 17k = 14.11 \ k\Omega$

$\Rightarrow$



Assume Q_1, Q_2 F.A :

KVL: $V_{TH} = R_{TH} i_{B_1} + V_{BE_1}(ON) + V_{BE_2}(ON) + i_{E_2} \cdot R_E$

and since $i_{E_1} = i_{B_2} = (\beta+1) i_{B_1}$

$\Rightarrow i_{E_2} = (\beta+1) i_{B_2} = (\beta+1)^2 i_{B_1}$

$$\Rightarrow \boxed{i_{B_1} = \frac{3.4 - 2(0.7)}{14.11 + 101^2 (0.2)} = 0.97 \ \mu A}$$

$\Rightarrow \boxed{i_{E_1} = i_{B_2} = 0.098 \ mA}$, $\boxed{i_{C_1} = 0.097 \ mA}$

$\Rightarrow \boxed{i_{E_2} = 9.93 \ mA}$, $\boxed{i_{C_2} = 9.83 \ mA}$

$\Rightarrow V_{C_1} = V_{C_2}$ and $V_{C_1} = 20 - (i_{C_1} + i_{C_2}) R_C = 20 - (9.83 + 0.097) 1.5 = \underline{\underline{5.11V}}$

$V_{E_2} = i_{E_2} R_E = 1.986 V$

$\Rightarrow V_{CE_2} = V_{C_2} - V_{E_2} = 3.124V > V_{CE}(SAT) = 0.2V \Rightarrow$ $\underline{Q_2 \text{ in F.A}}$ ✓

$\Rightarrow V_{E_1} = \underbrace{V_{BE}(ON)}_{0.7 \text{ since } Q_2 \text{ F.A}} + \underbrace{V_{E2}}_{1.986} = 2.686 \Rightarrow V_{CE_1} = V_{C_1} - V_{E_1} = 2.424V > V_{CE}(SAT) = 0.2V$

$\Rightarrow \underline{Q_1 \text{ in F.A}}$ ✓

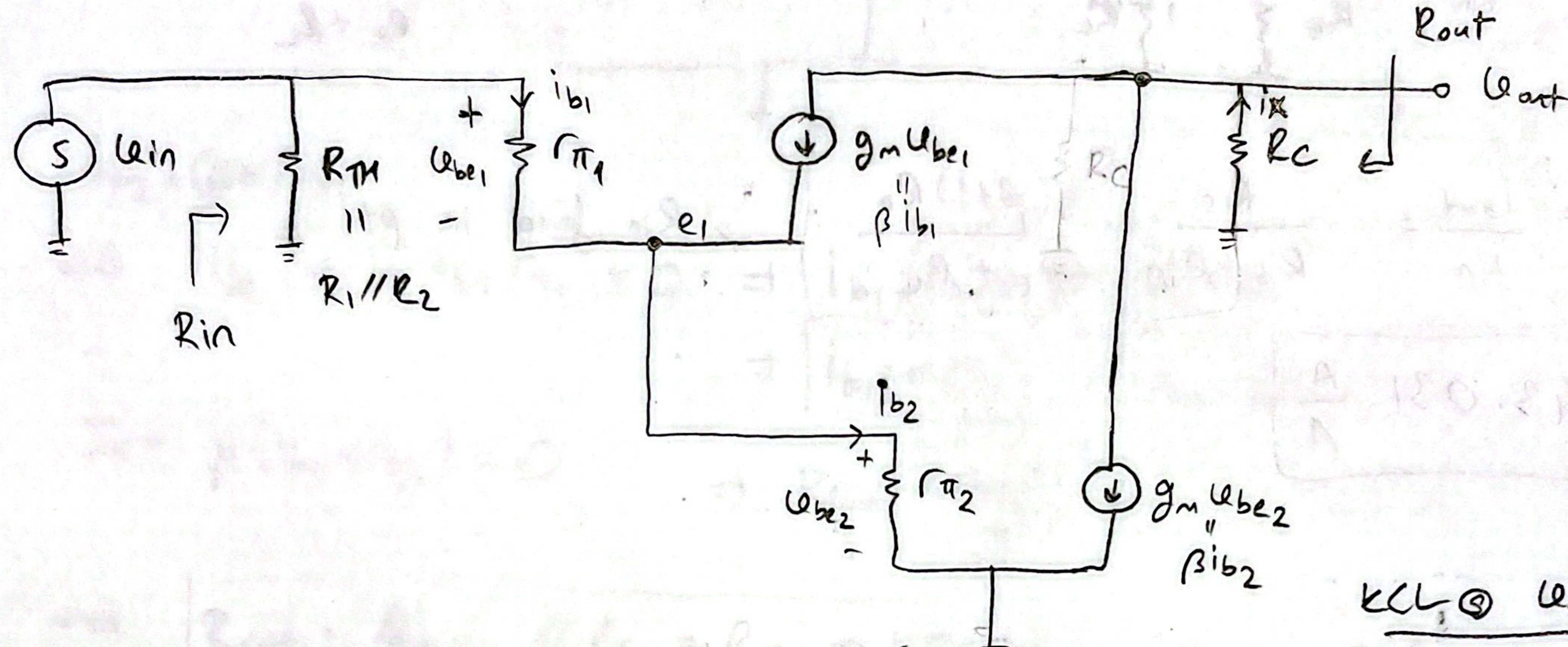
$$\Rightarrow \boxed{\begin{aligned}(I_{C_1}, V_{CE_1}) &= (0.097\,mA,\ 2.424\,V)\\ (I_{C_2}, V_{CE_2}) &= (9.83\,mA,\ 3.124\,V)\end{aligned}}$$

both in F.A ✓

$$\boxed{r_{\pi_1} = \frac{\beta V_T}{I_{C_1}} = 26.8\,k\Omega}$$

$$\boxed{r_{\pi_2} = 0.26\,k\Omega}$$

b) s.s model:



KCL @ vout:

$$i_x = \beta i_{b_1} + \beta i_{b_2}$$
$$= \beta i_{b_1} + \beta(\beta+1) i_{b_1}$$
$$= \beta i_{b_1}(\beta+2)$$

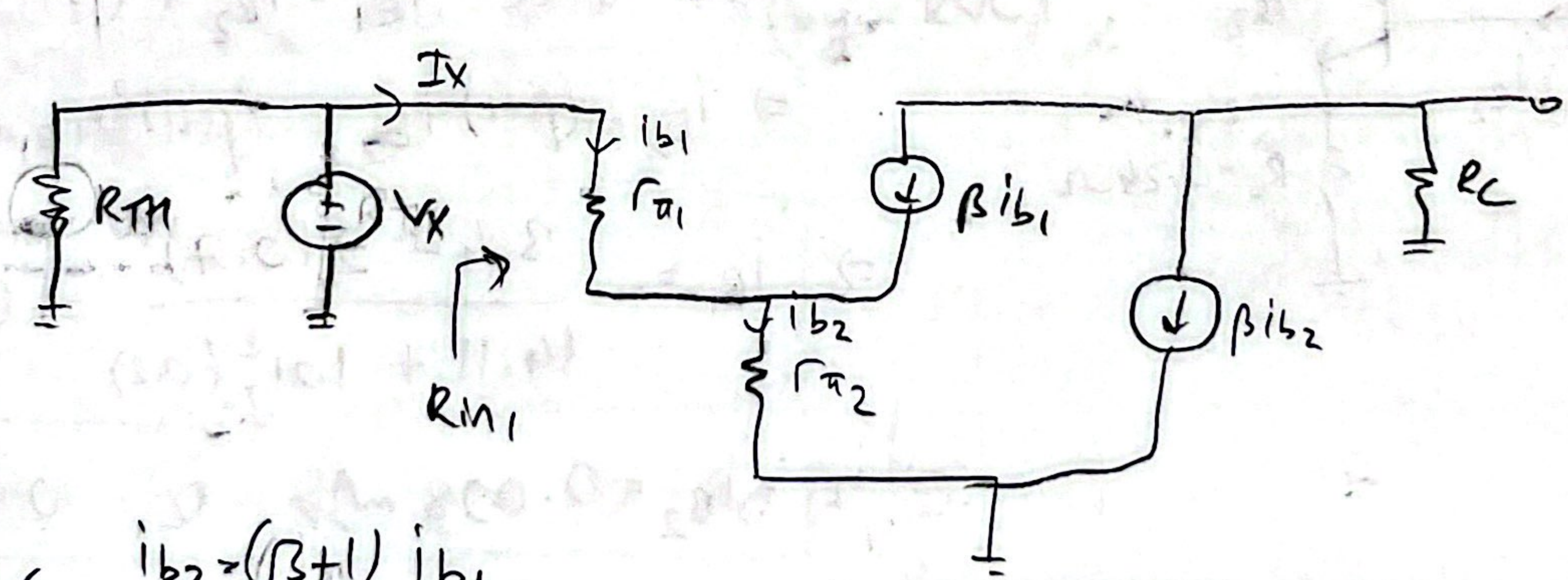
KCL @ e_1: $i_{b_2} = (\beta+1) i_{b_1}$

KVL: $v_{in} = r_{\pi_1} i_{b_1} + r_{\pi_2} i_{b_2} = \left(r_{\pi_1} + (\beta+1) r_{\pi_2}\right) i_{b_1}$

$v_{out} = -R_C i_x = -\beta R_C(\beta+2)\cdot i_{b_1}$

$$\Rightarrow \boxed{A_v = \frac{v_{out}}{v_{in}} = \frac{-\beta(\beta+2) R_C}{r_{\pi_1} + (\beta+1) r_{\pi_2}}} \Rightarrow \boxed{A_v = -288.35\ \tfrac{V}{V}}$$

c) Find Rin:



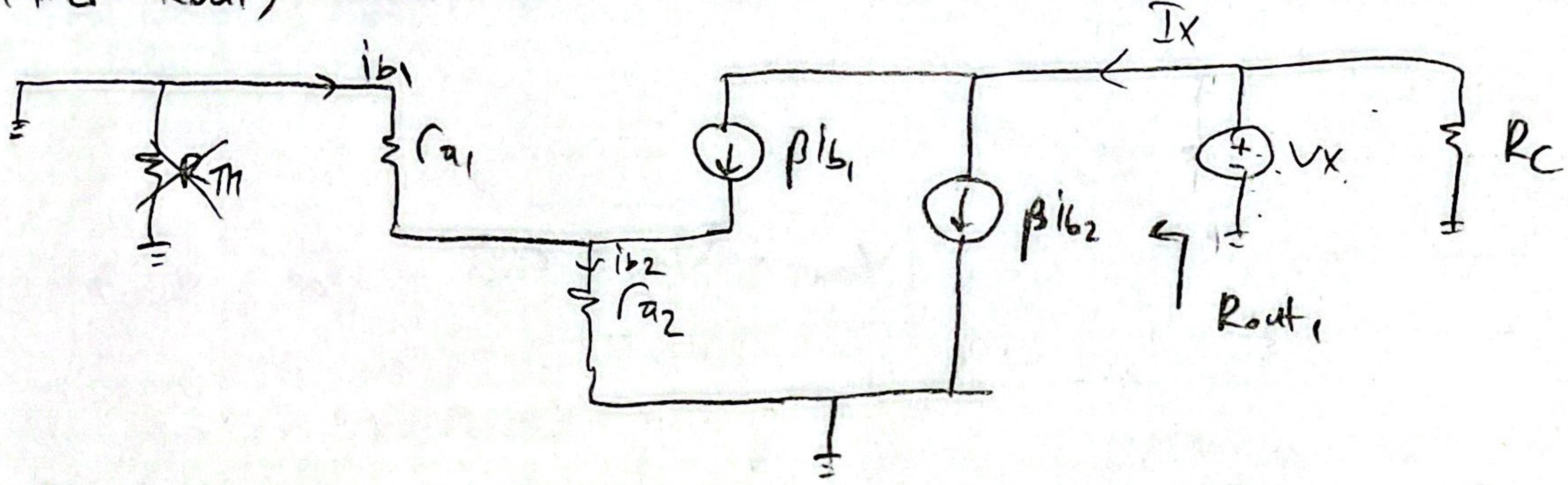
$I_x = i_{b_1}$, $\quad i_{b_2} = (\beta+1) i_{b_1}$
$\qquad\qquad\quad = (\beta+1) I_x$

$\Rightarrow V_x = I_x r_{\pi_1} + (\beta+1) I_x r_{\pi_2}$ from KVL $\Rightarrow R_{in_1} = \dfrac{V_x}{I_x} = r_{\pi_1} + (\beta+1) r_{\pi_2}$

and $\boxed{R_{in} = R_{in_1} \,//\, R_{TH} = \left(r_{\pi_1} + (\beta+1) r_{\pi_2}\right) // R_{TH}}$

$$\Rightarrow \boxed{R_{in} = 11.146\,k\Omega}$$

d) Find R_{out}, kill v_{in}



$i_{b2} = (\beta+1)\, i_{b1}$

and $\quad i_{b1} r_{\pi 1} + i_{b2} r_{\pi 2} = 0 \Rightarrow i_{b1}\big(\overbrace{r_{\pi 1} + (\beta+1) r_{\pi 2}}^{\neq 0}\big) = 0$

$\Rightarrow \boxed{i_{b1} = 0} \Rightarrow i_{b2} = 0$

$I_x = \beta(i_{b1} + i_{b2}) = 0 \qquad \Rightarrow R_{out,} = \dfrac{V_x}{I_x} = \infty$

and $\quad \boxed{R_{out} = \underbrace{R_{out,}}_{\infty} \,//\, R_C = R_C = 1.5\,k\Omega}$

6) For $V_{in} < V_{BE}(ON) = 0.7V$, $\quad Q_1$ is OFF $\Rightarrow i_B = i_C = i_E = 0$

$\Rightarrow V_{out} = 10V$

When $V_{in} = 0.7V$, Q_1 turns ON. With small i_b currents having value:

$i_b = \dfrac{V_{in} - 0.7}{(\beta+1) R_E} = \dfrac{V_{in} - 0.7}{(\beta+1)(0.5)}$ (by KVL), since V_{in} is close to 0.7

$\Rightarrow$ small i_C current

$\Rightarrow$ in F.A mode due to small V_E and large V_C

where $V_C = 10 - R_C \underbrace{i_C}_{small}$

and $\quad V_{out} = 10 - R_C i_C = 10 - R_C \beta i_b$

$= 10 - 2\beta \dfrac{(V_{in} - 0.7)}{(\beta+1)(0.5)}$ while in F.A.

Q_1 is in F.A until BC turns ON, and $V_{BC}(ON) = 0.5V$

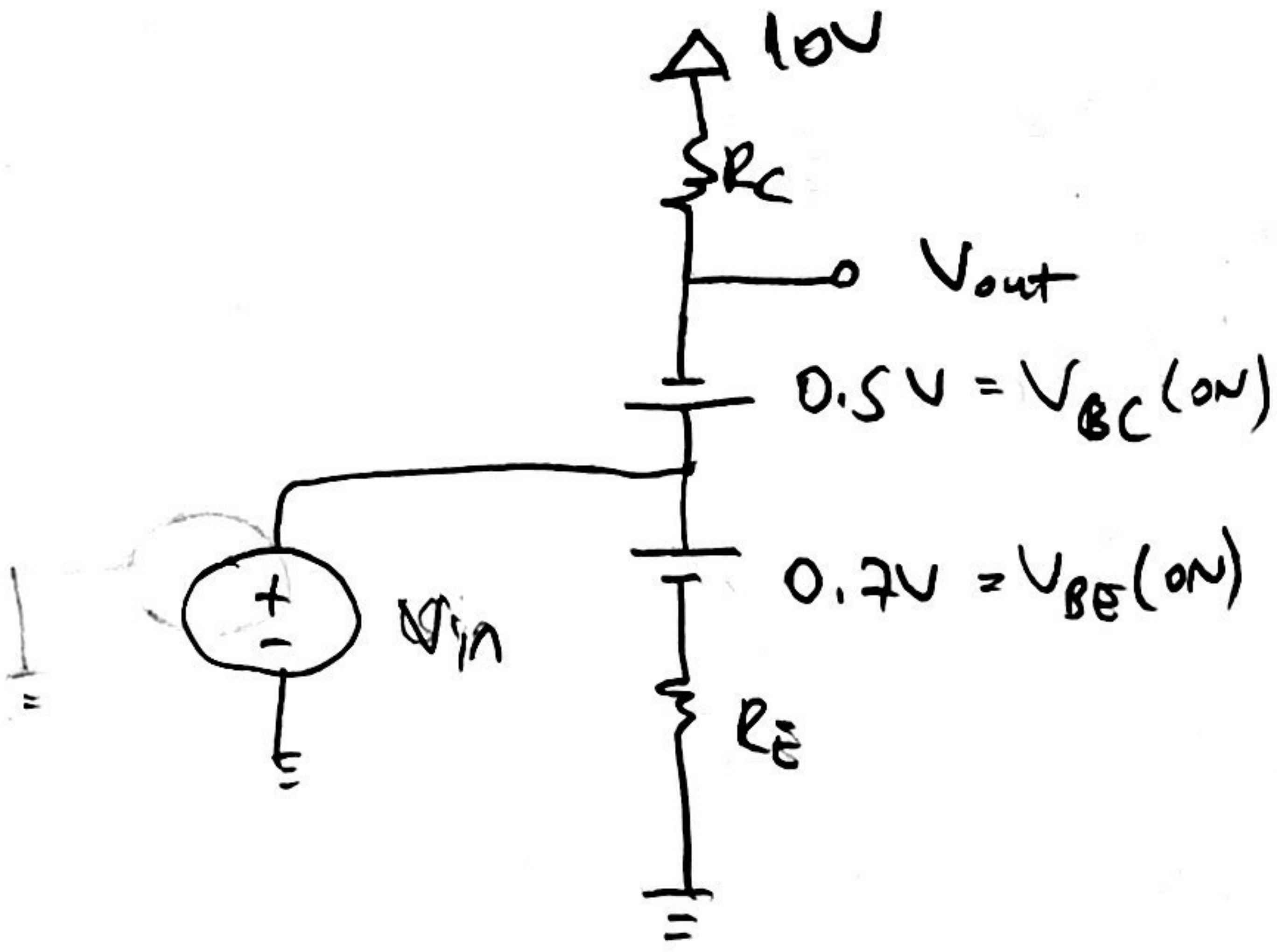
$\Rightarrow$ when $V_{in} - V_{out} = 0.5$, Q_1 goes to SAT mode.

$\Rightarrow V_{in} - V_{out} = V_{in} - \Big(10 - 2\underbrace{\dfrac{\beta}{(\beta+1)}}_{\approx 1 \text{ since } \beta \text{ is large}} \dfrac{V_{in} - 0.7}{0.5}\Big) = V_{in} - (10 - 4V_{in} + 2.8) = 0.5$

$\Rightarrow 5V_{in} = 12.8 + 0.5$

$\Rightarrow \boxed{V_{in} = 2.66\,V}$

After the transistor is in SAT: (both BE, BC ON)



$\Rightarrow \boxed{V_{out} = V_{in} + 0.5}$ for $V_{in} > 2.66V$

$$\Rightarrow V_{out} = \begin{cases} 10V, & V_{in} < 0.7 \\ 10 - 4\frac{\beta}{\beta+1}(V_{in} - 0.7), & 0.7 \leq V_{in} \leq 2.66 \\ V_{in} + 0.5, & V_{in} > 2.66 \end{cases}$$