ECD - HW 4

Yigit Narter
22102718

(1) a) Assume F.A, for Thév. Conversion:

$V_{TH} = V_{CC} \cdot \frac{R_2}{R_1+R_2} = 5 \cdot \frac{22k}{(22+33)k} = 2V$, $R_{TH} = R_1 // R_2 = 22k // 33k = \underline{13.2k}$

$\Rightarrow i_B = \frac{V_{TH} - V_{BE(ON)}}{R_{TH} + (\beta+1)R_E} = \frac{2-0.7}{13.2 + (101)(3)} = 4.11\ \mu A$

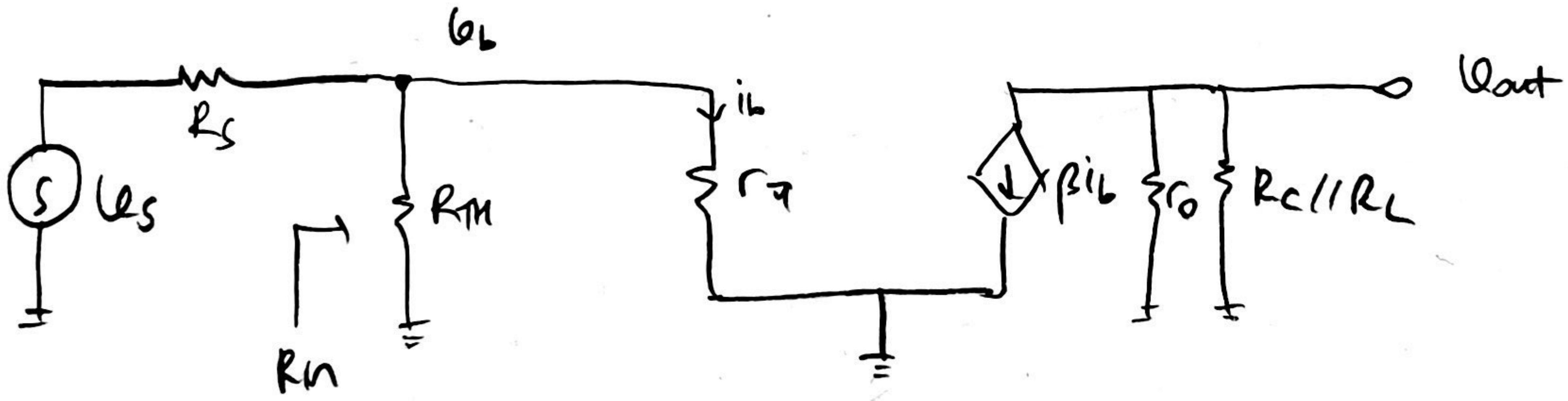
$\Rightarrow \boxed{i_C = \beta i_B = 0.41\ mA}$, $\boxed{i_E = i_C + i_B = 0.415\ mA}$

Check: $V_C = V_{CC} - i_C R_C = 5 - (0.41)(4) = 3.36V$
$V_E = i_E R_E = 1.245V$
$\Rightarrow \boxed{V_{CE} = V_C - V_E = 2.115V} > 0.2V$ in F.A ✓

$\boxed{(i_C, V_{CE}) = (0.41\ mA, 2.115V)}$

$r_\pi = \frac{V_T}{i_B} = \frac{26\ mV}{4.11\ \mu A} = 6.33\ k\Omega$, $g_m = \frac{\beta}{r_\pi} = 15.81\ mA/V$

b) s.s model in midband (short the cap.s):



$\boxed{R_{in} = R_{TH} // r_\pi = 13.2k // 6.33k = 4.28\ k\Omega}$

$v_b = \frac{R_{in}}{R_{in}+R_S} v_s$, $i_b = \frac{v_b}{r_\pi}$, $v_{out} = -\beta i_b (r_o // R_C // R_L)$

$\Rightarrow \boxed{A_o = \frac{v_{out}}{v_s} = \frac{-\beta (r_o // R_C // R_L)}{r_\pi} \cdot \frac{R_{in}}{R_{in}+R_S}}$

$= \frac{-100(200k // 4k // 5k)}{6.33k} \cdot \frac{4.28k}{4.28k + 1k} \cong \boxed{-17.95\ \frac{V}{V}}$

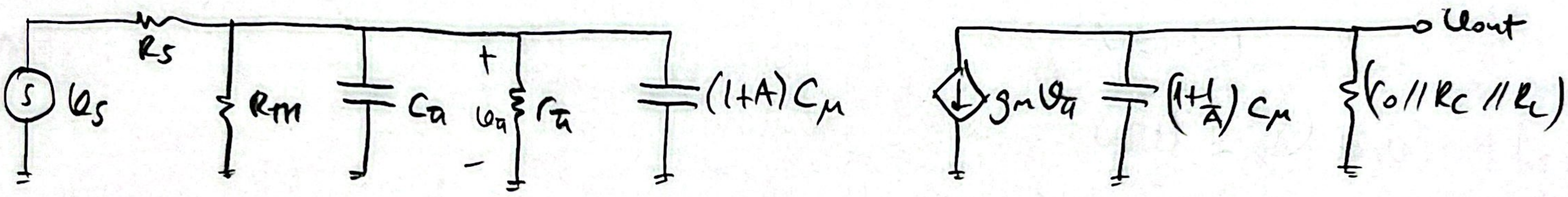
c) $f_T = 700\,MHz$, $C_\mu = 1\,pF$ $\Rightarrow$ $f_T = \dfrac{g_m}{2\pi(C_\mu + C_\pi)}$ $\Rightarrow$ $700\,MHz = \dfrac{15.81\,mA/V}{2\pi(1\,pF + C_\pi)}$

$$\Rightarrow \boxed{C_\pi = 2.59\,pF}$$

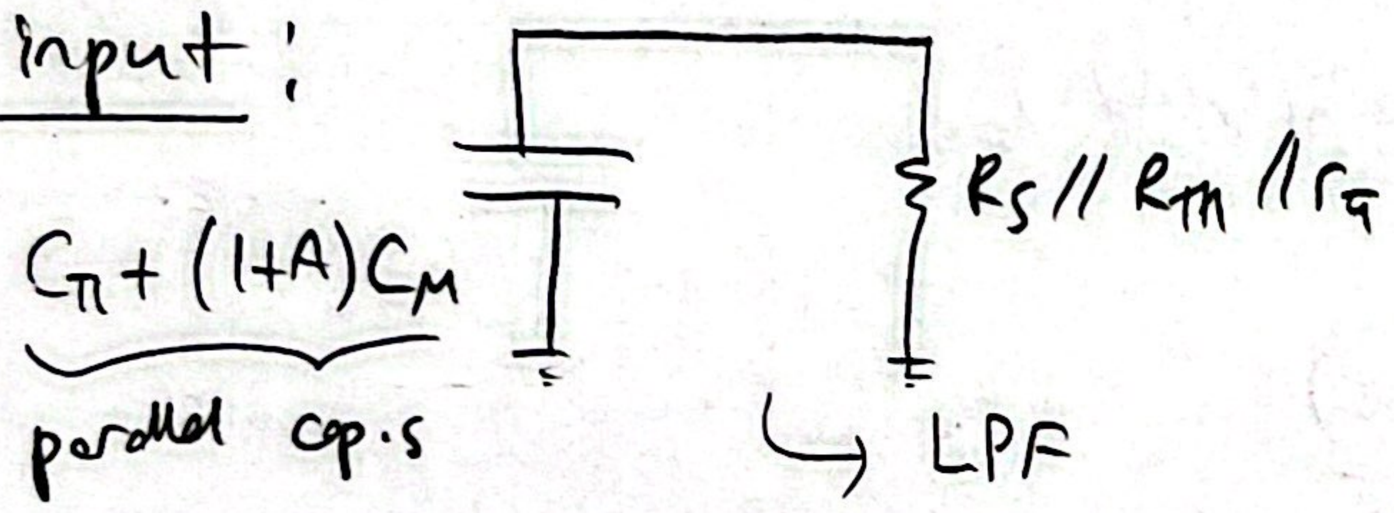
Using Miller effect, split C_μ as $(1+A)C_\mu$, $(1+\frac{1}{A})C_\mu$.
A is the gain between the base and collector $\Rightarrow -A = \dfrac{v_{out}}{v_b}$

$$\boxed{A = -34.72\ \tfrac{V}{V}} \Longleftarrow = -\frac{\beta(r_o // R_C // R_L)}{r_\pi}$$

Redraw the circuit:

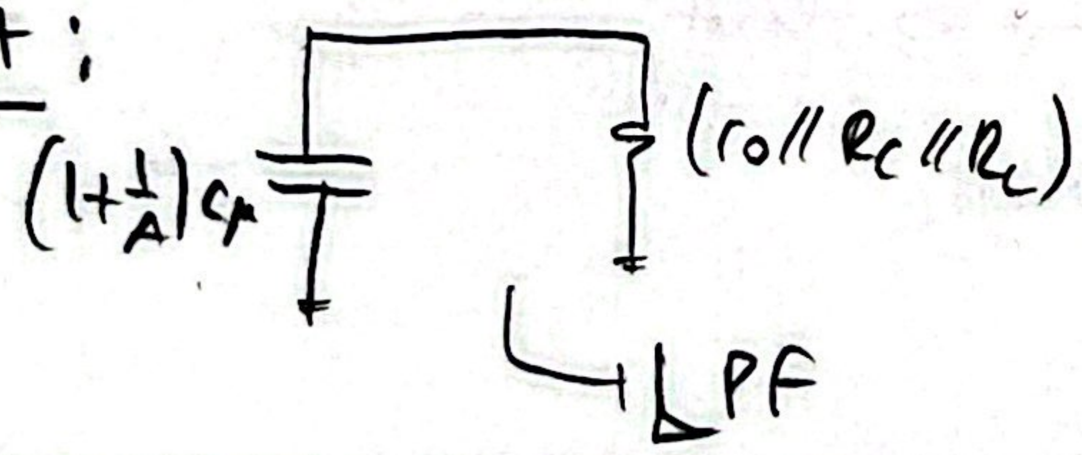


At the input:



$$\tau_1 = (C_\pi + (1+A)C_\mu)(\underbrace{R_S // R_{TH} // r_\pi})$$
$$= (2.59\,pF + 35.72\,pF)(2.067\,k\Omega)$$
$$= 7.92 \times 10^{-8}\ \text{sec.}$$

At the output:



$$\tau_2 = ((1+\tfrac{1}{A})C_\mu)(r_o // R_C // R_L)$$
$$= 2.26 \times 10^{-9}\ \text{sec}$$

$\Rightarrow$ $\boxed{f_1 = \dfrac{1}{2\pi\tau_1} = 2.01\,MHz}$, $f_2 = \dfrac{1}{2\pi\tau_2} = 70.42\,MHz$ → Pick the smaller upper cut-off frequency f_H as it dominates

$$\Rightarrow \boxed{A(j\omega) = -17.95\left(\frac{1}{1+j\frac{f}{f_1}}\right)\left(\frac{1}{1+j\frac{f}{f_2}}\right)}$$ high freq. response

$$= \boxed{-17.95\left(\frac{1}{1+j\frac{\omega}{\omega_1}}\right)\left(\frac{1}{1+j\frac{\omega}{\omega_2}}\right)}$$

Scanned with CamScanner

② a) Assume SAT: $V_{GS}=V_G-V_S=5-10I_D$

$\Rightarrow I_D=K_n(V_{GS}-V_{tn})^2 \Rightarrow I_D=2(4-10I_D)^2$

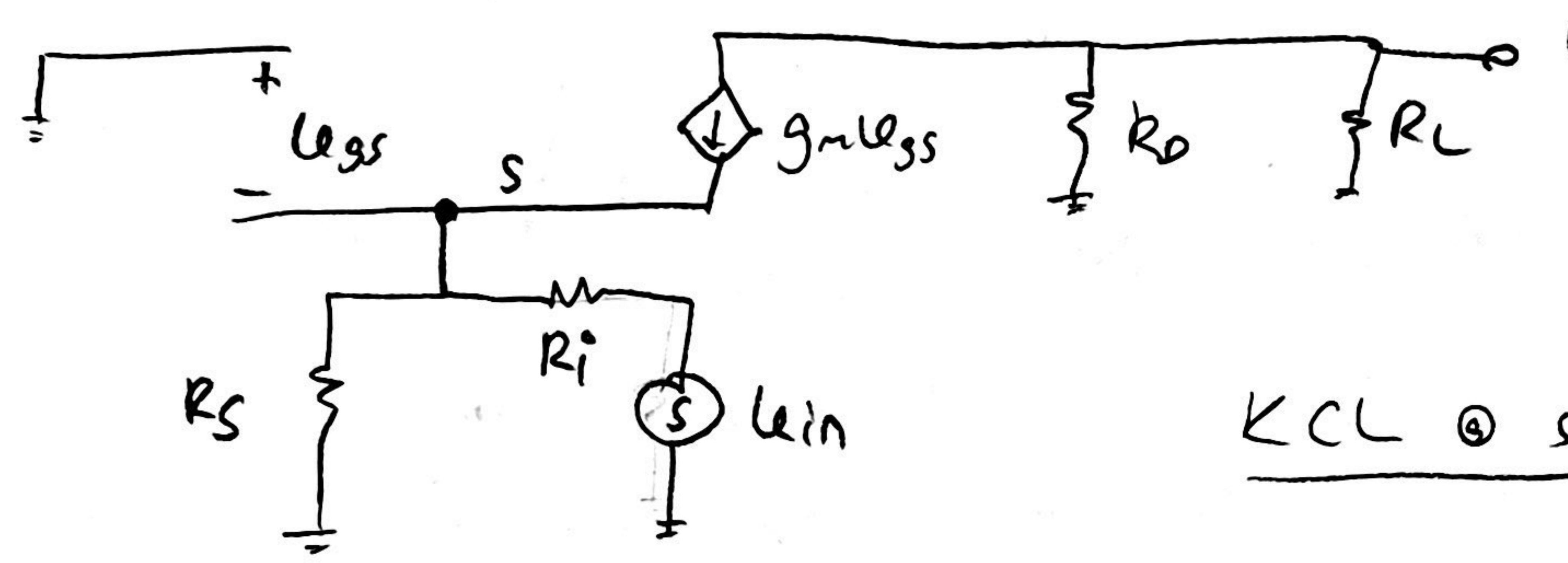
$\boxed{I_D=0.36\,mA}$ or ~~(I_D = 0.45 mA)~~ to have it ON

$V_{DS}=10-I_D(5+10)=10-15(0.36)=4.6V > \underbrace{V_{GS}-V_{TN}}_{0.4V}$ ✓ in SAT

$V_{GS}=5-10I_D=1.4V$

$\Rightarrow (I_D, V_{DS})=(0.36\,mA,\ 4.6V)$

b)



$v_{out}=-g_m v_{gs}(R_D//R_L)$

KCL @ source: $\frac{v_s}{R_S}+\frac{v_s-v_{in}}{R_i}-g_m v_{gs}=0$

and $v_{gs}=-v_s \Rightarrow v_{in}=R_i\cdot v_s\left(g_m+\frac{1}{R_S}+\frac{1}{R_i}\right)$

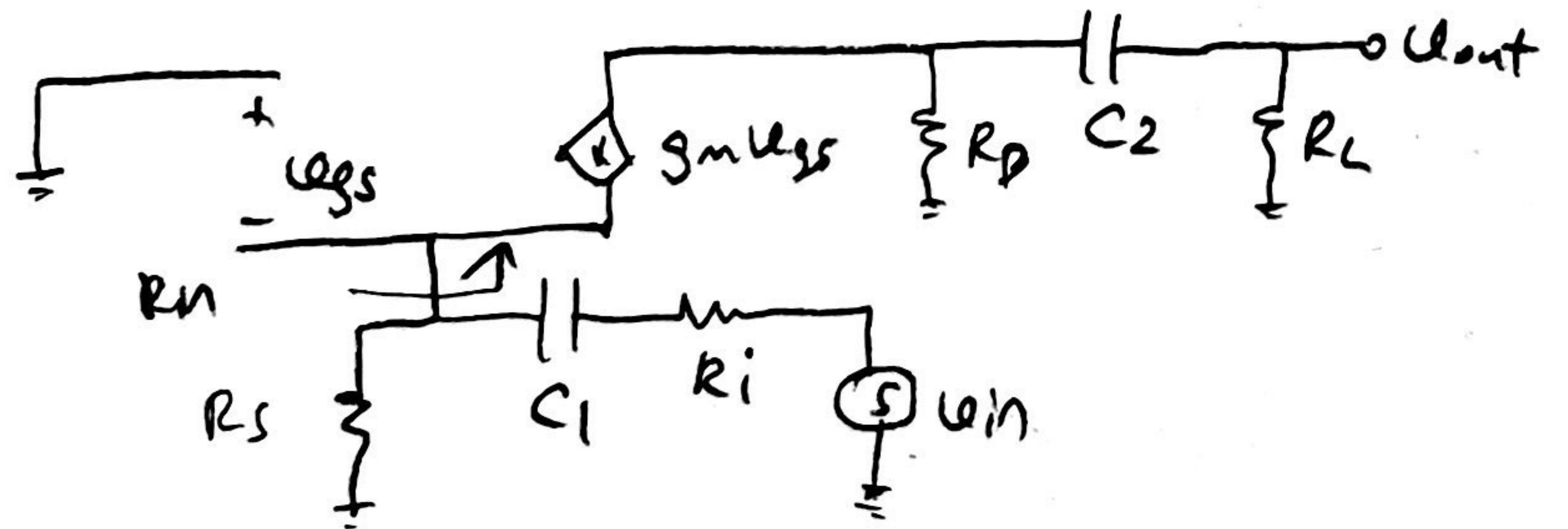
$v_{out}=g_m v_s(R_D//R_L)$

$$\Rightarrow \boxed{A_o=\frac{v_{out}}{v_{in}}=\frac{g_m(R_D//R_L)}{R_i\left(g_m+\frac{1}{R_S}\right)+1}=\frac{g_m(R_D//R_L)\left(R_S//\frac{1}{g_m}\right)}{R_i+\left(R_S//\frac{1}{g_m}\right)}}$$

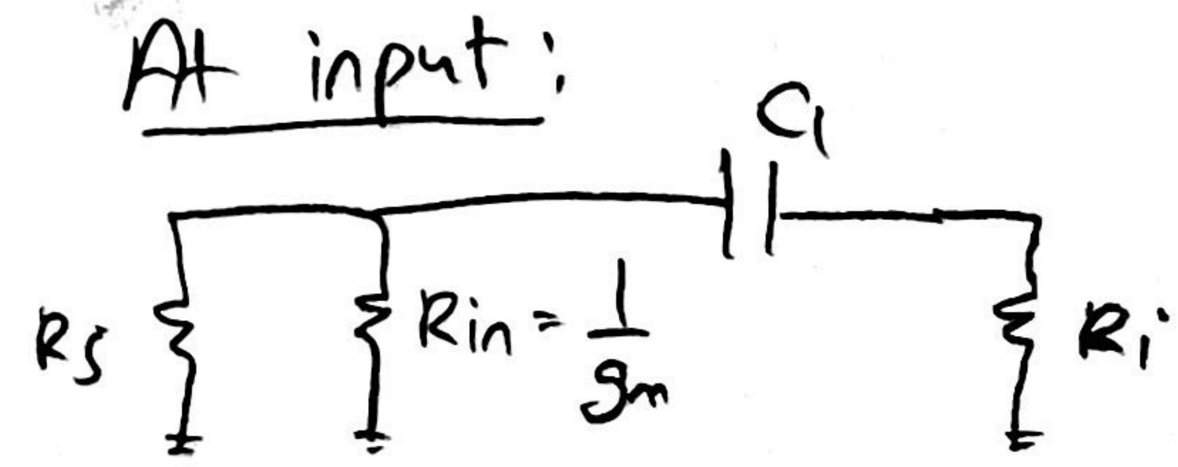
$g_m=2\sqrt{K_n I_D}=2\sqrt{2\cdot(0.36)}=1.7\,mA/V$

$=\boxed{0.82\ \frac{V}{V}}$

c) At lower frequencies, C_{gs} and C_{gd} are open circuited.



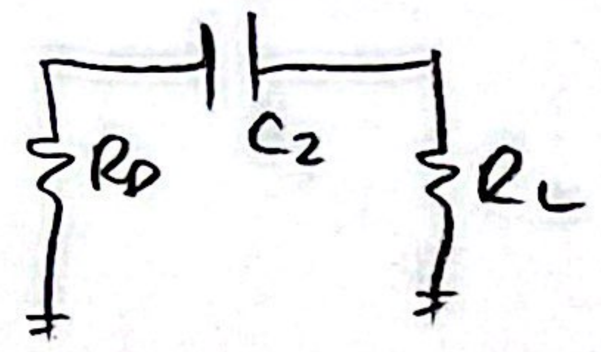
At input:



$\tau_1=C_1\left(\left(R_S//\frac{1}{g_m}\right)+R_i\right)=2.55\,ms$

$\boxed{f_{L1}=\frac{1}{2\pi\tau_1}=62.28\,Hz}$

At output:

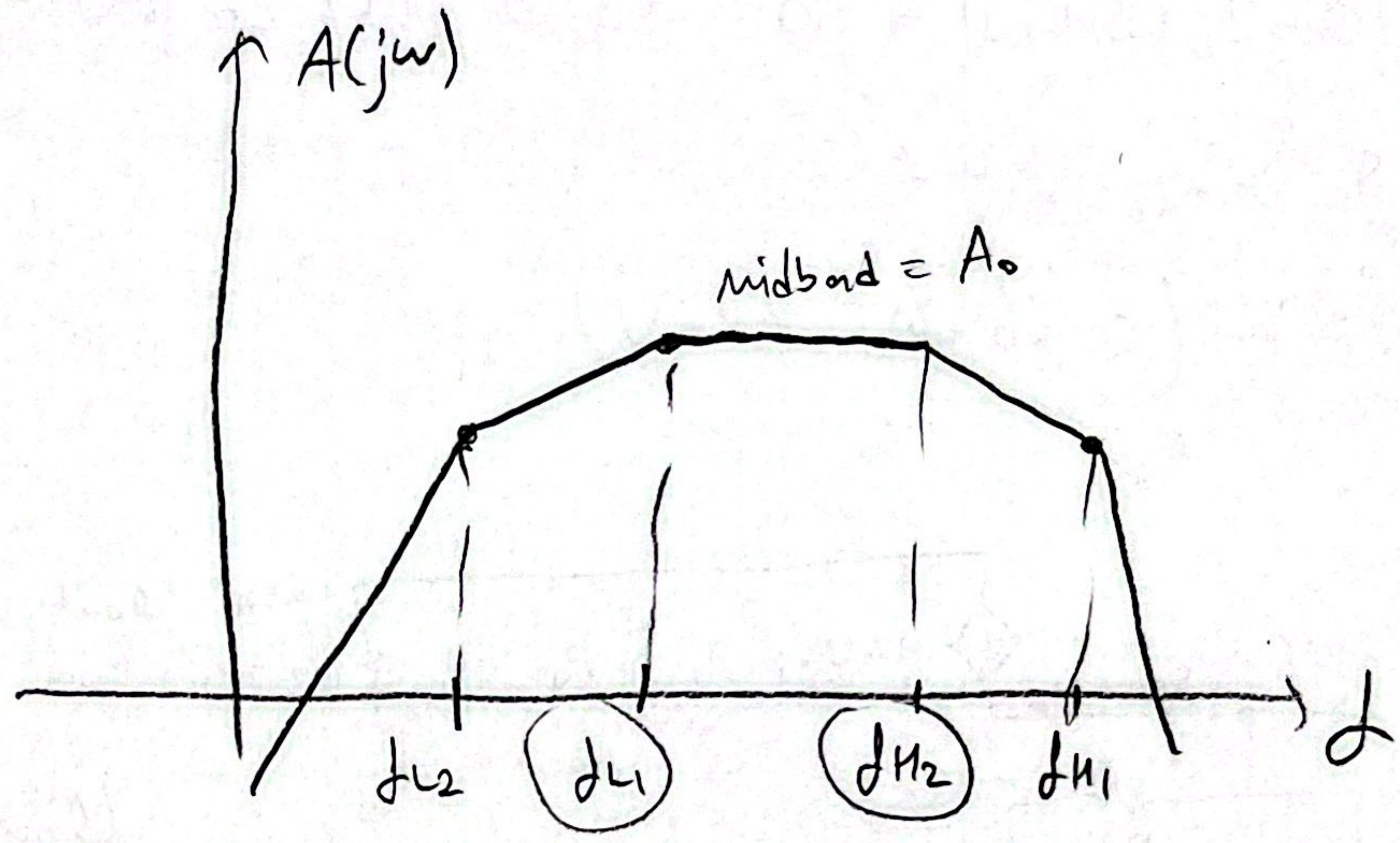


$\tau_2 = C_2(R_D + R_L) = 18\ ms$

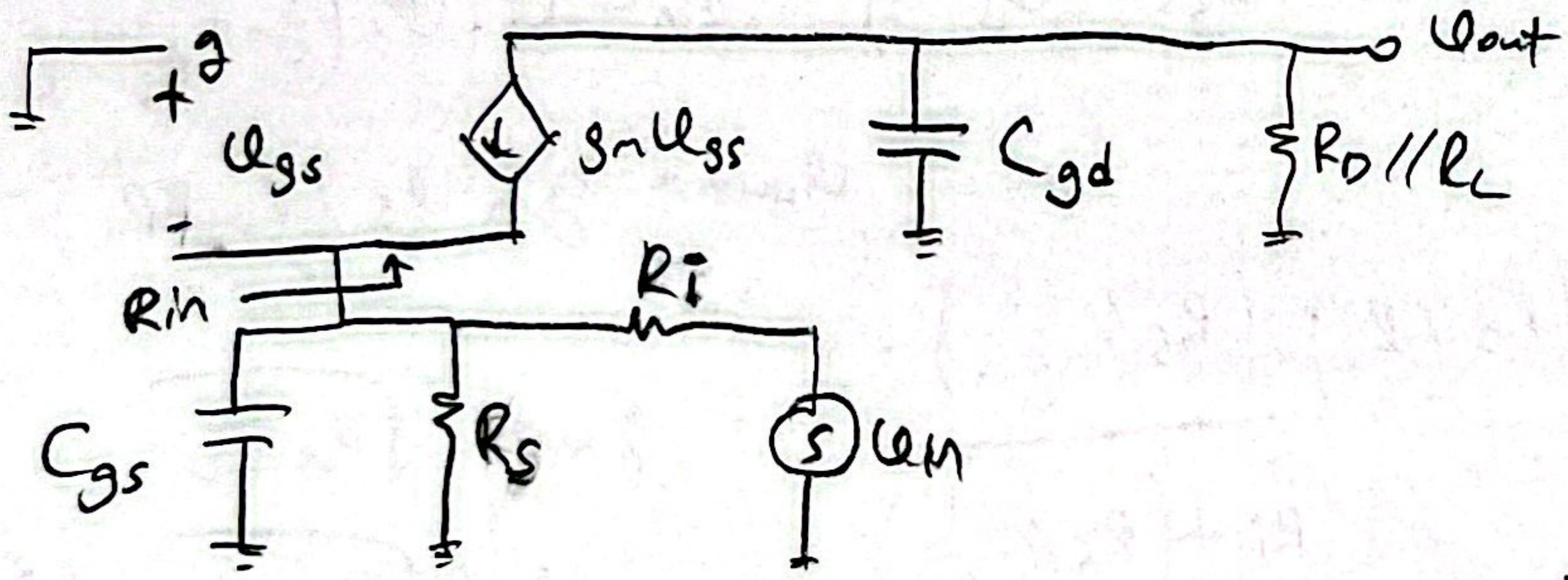
$$\boxed{f_{L_2} = \frac{1}{2\pi \tau_2} = 8.842\ Hz}$$

For f_L, pick the bigger one since it determines the bandwidth:

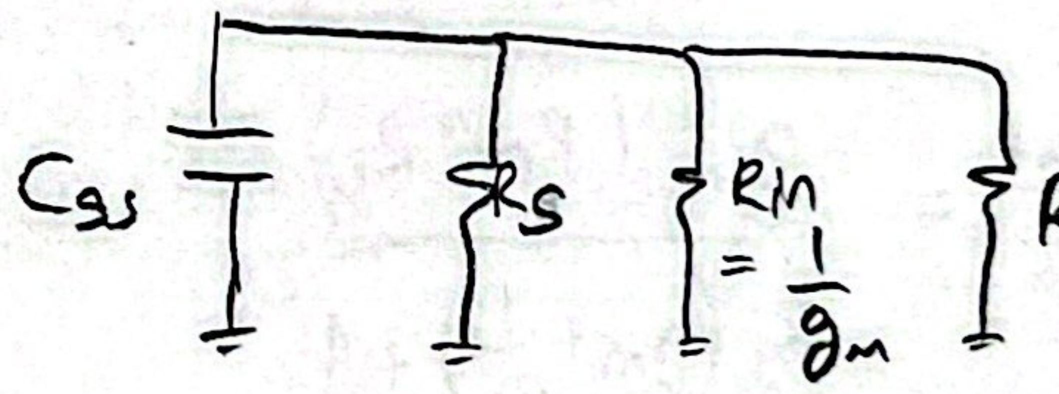
$$\boxed{f_L = 62.28\ Hz}$$



d) At high frequency, assume C_1, C_2 short ckt. Note that since gate is grounded, C_{gs} is in between the source and gnd, and C_{gd} is between drain and gnd:



At input:

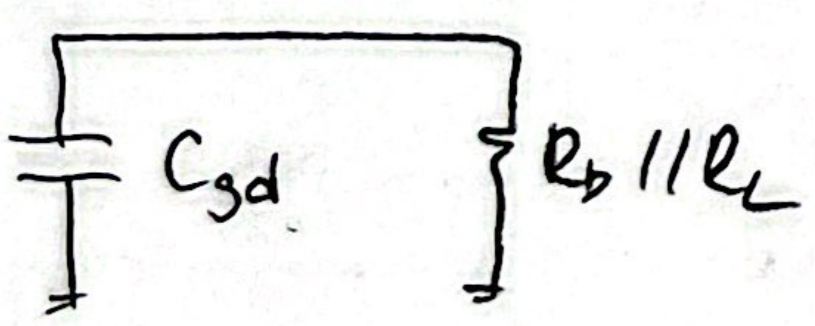


$\Rightarrow \tau_1 = C_{gs}\left(R_S // R_i // \frac{1}{g_m}\right)$

$= 5.22 \times 10^{-9}$ sec

$$\Rightarrow \boxed{f_{H_1} = \frac{1}{2\pi \tau_1} = 30.49\ MHz}$$

At output:

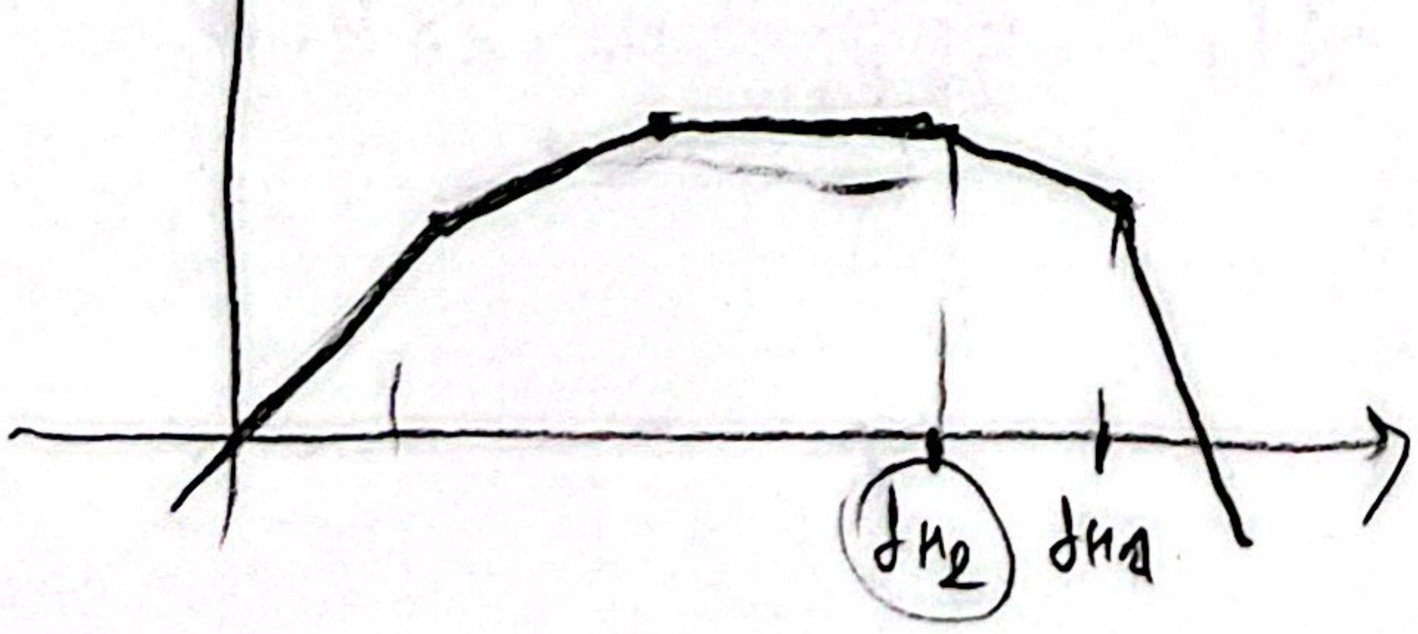


$\tau_2 = C_{gd}(R_D // R_L)$

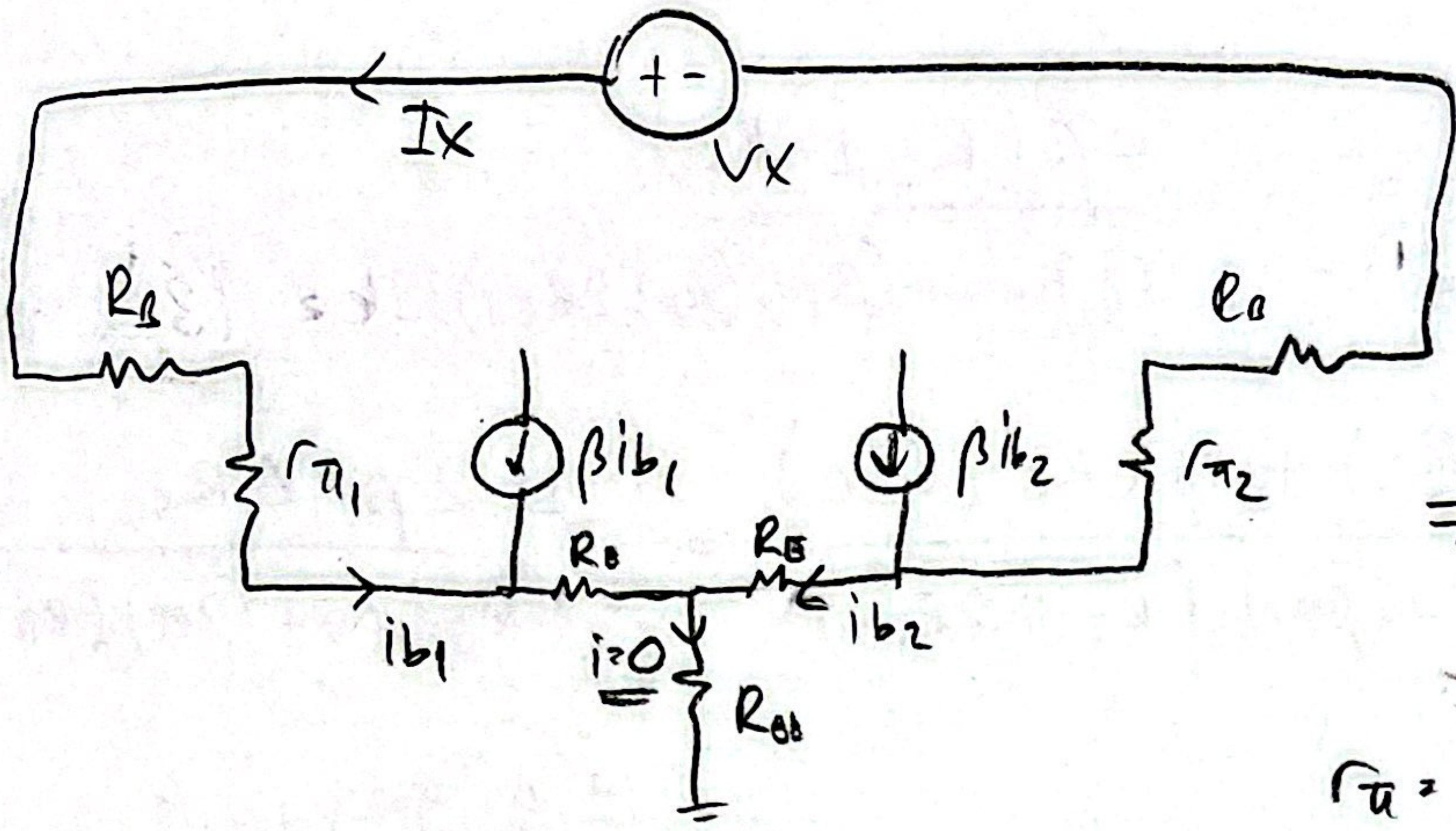
$= 6.66 \times 10^{-9}$ sec

$$\Rightarrow \boxed{f_{H_2} = \frac{1}{2\pi \tau_2} = 23.9\ MHz}$$

Pick the smaller one for f_H: $\boxed{f_H = 23.9\ MHz}$



③ a) In diff. mode, current on R_{EE} is zero since $\boxed{i_{b1} = -i_{b2}}$ due to symmetry



$I_X = i_{b1} = -i_{b2}$

$\Rightarrow V_X = 2\left(i_{b1} r_\pi + (\beta+1) i_{b1} R_E + i_b R_B\right)$

$$\Rightarrow \boxed{R_{id} = \frac{V_X}{I_X} = 2\left(R_B + r_\pi + (\beta+1) R_E\right)}$$

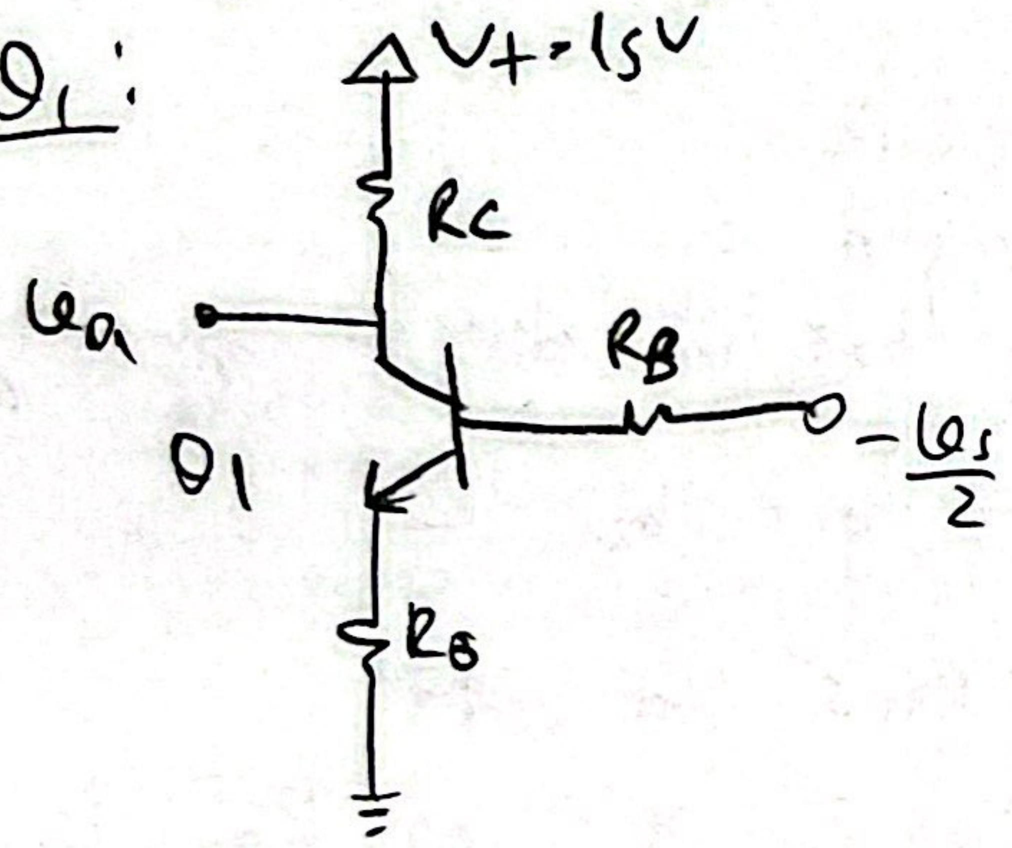
$$r_\pi = \frac{\beta V_T}{I_C} = \frac{(0.026V) \times 100}{0.5mA} = 5.2k\Omega$$

in DC, $\frac{I_Q}{2}$

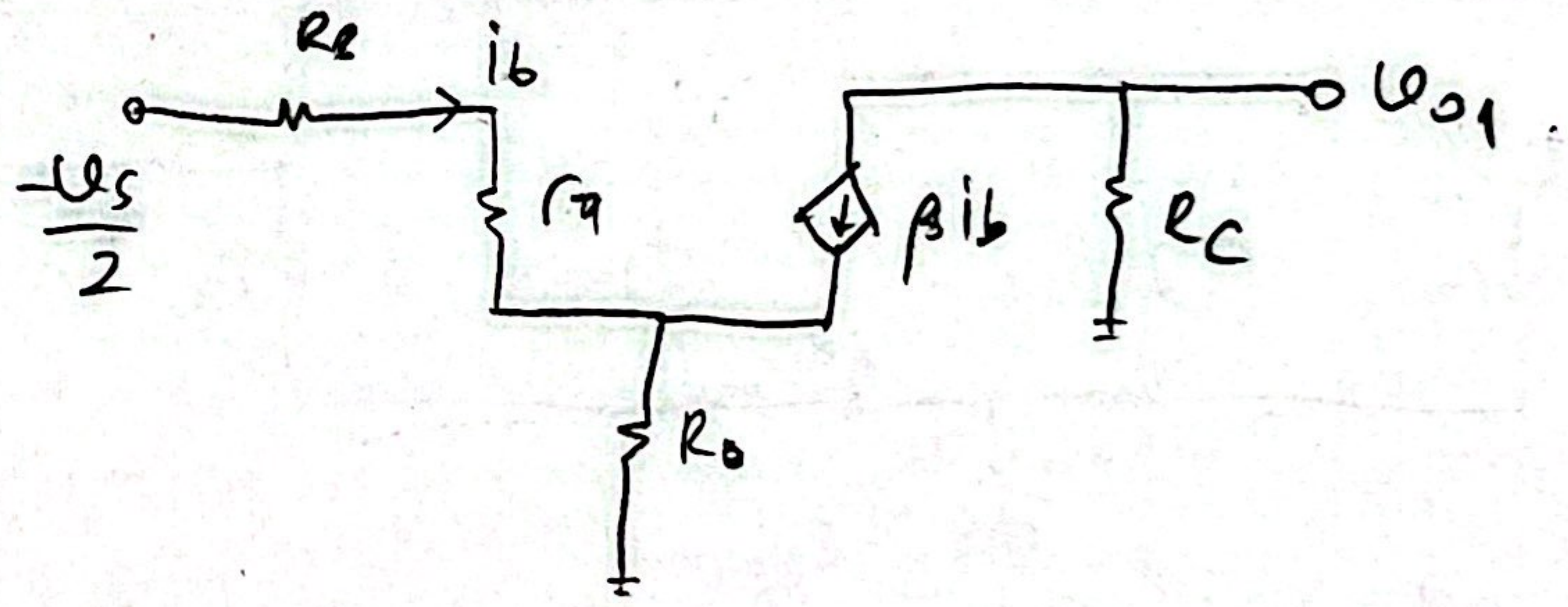
$$\boxed{R_{id_m} = 50.7k\Omega}$$

b) Use half ckt. analysis, omit R_{EE} since no current on it in diff. mode.

For Q_1:



$\equiv$ s.s model



Due to symmetry;

$$A_{dm} = A_{dm1} - \underbrace{A_{dm0}}_{\text{same input but negative of } A_{dm1}}$$

$$\Leftarrow \left\{ \begin{aligned} v_{o1} &= -\beta i_b R_C \\ -\frac{v_s}{2} &= i_b\left(R_B + r_\pi + (\beta+1) R_E\right) \end{aligned} \right\} \Rightarrow A_{dm1} = \frac{v_{o1}}{v_s} = \boxed{\frac{\beta R_C}{2\left(R_B + r_\pi + (\beta+1) R_E\right)}}$$

$$= 2 \frac{\beta R_C}{2\left(R_B + r_\pi + (\beta+1) R_E\right)} \Rightarrow \boxed{A_{dm} = \frac{v_{out}}{v_s} = \frac{\beta R_C}{R_B + r_\pi + (\beta+1) R_E}}$$

$$= \underline{39.45 \frac{V}{V}}$$

If R_C accurate to ±1%
$\Rightarrow \Delta R_C = 0.1k\Omega$

$$\Rightarrow A'_{dm} = A_{dm1} - A_{dm0} = \frac{\beta(R_C \pm \Delta R_C) + \beta(R_C \mp \Delta R_C)}{2\left(R_B + r_\pi + (\beta+1) R_E\right)}$$

$$= \frac{\beta R_C}{R_B + r_\pi + (\beta+1)} = A_{dm} \left(\begin{matrix}\text{still the} \\ \text{same}\end{matrix}\right)$$

c) In common mode half ckt., $2R_{EE}$ will be added to the emitter, hence plugging in:

$$A_{cm,1} = \frac{v_{out}}{v_{cm}} = \frac{-\beta \tilde{R}_C}{R_B + r_\pi + (\beta+1)(R_E + 2R_{EE})} = \frac{-\beta(R_C \pm \Delta R_C)}{R_B + r_\pi + (\beta+1)(R_E + 2R_{EE})}$$

$= 1\% \times 10k = 0.1k$

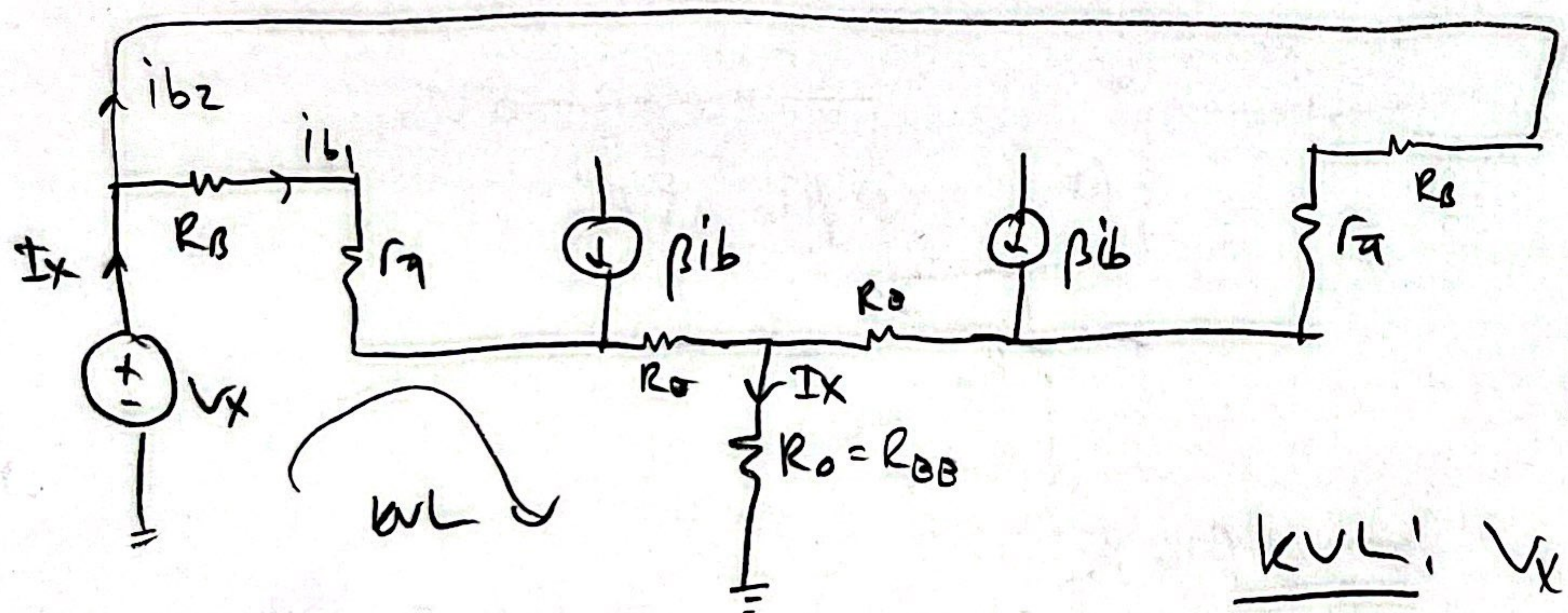
$$A_{cm} = A_{cm,1} - \underbrace{A_{cm,0}}_{} = \frac{-\beta(R_C \pm \Delta R_C) + \beta(R_C \mp \Delta R_C)}{R_B + r_\pi + (\beta+1)(R_E + 2R_{EE})} = \frac{\pm 2\beta \cdot \Delta R_C}{R_B + r_\pi + (\beta+1)(R_E + 2R_{EE})}$$

only $\pm\Delta R_C$ & $\mp\Delta R_C$ due to symmetry, same otherwise

$$= \underline{\underline{\pm 4.95 \times 10^{-4}}}$$

d) $$CMRR = \frac{|A_{dm}|}{|A_{cm}|} = \frac{39.45}{4.95 \times 10^{-4}} = 79696.97 \cong \boxed{98.03 \text{ dB}}$$

e)



$i_{b2} = i_{b1}$ due to symmetry

$\Rightarrow i_{b1} = i_{b2} = \frac{I_X}{2}$

r_π's are equal

by KCL, current on R_{EE} is I_X

KVL: $V_X = \frac{I_X}{2}(R_B + r_\pi) + (\beta+1)\frac{I_X}{2}R_E + (\beta+1)I_X R_{EE}$

$$\Rightarrow \boxed{R_{icm} = \frac{V_X}{I_X} = \frac{R_B + r_\pi + (\beta+1)R_E}{2} + (\beta+1)R_{EE}}$$

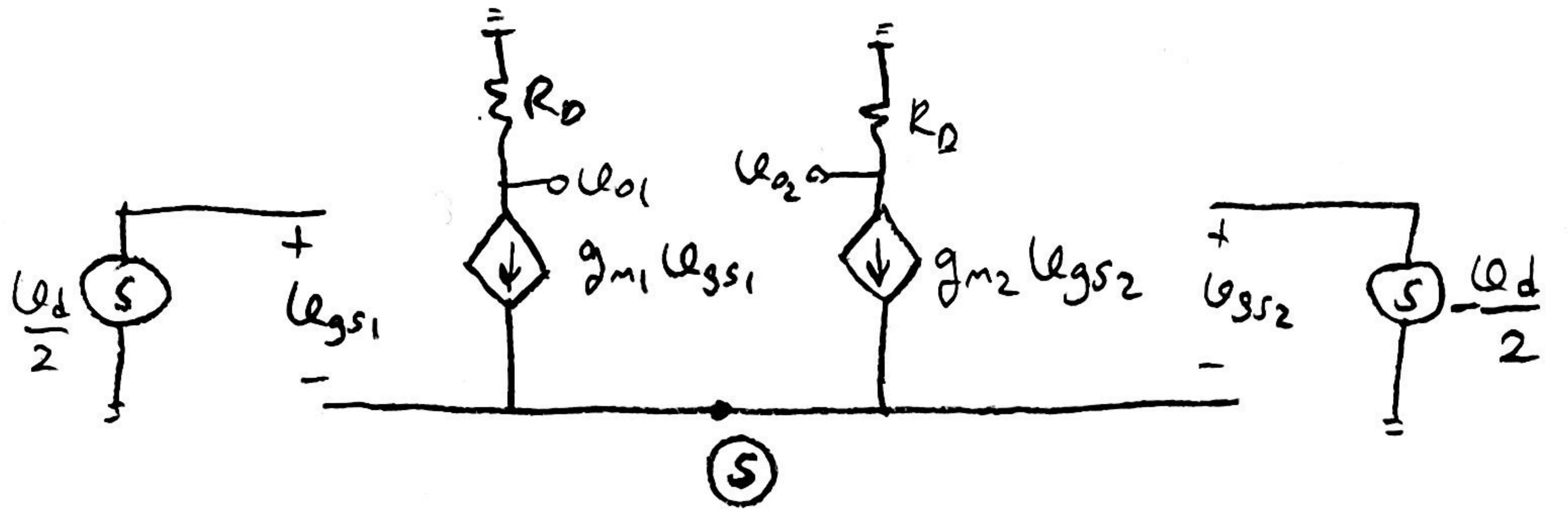
$$= \boxed{20.21 \text{ M}\Omega}$$ very large!

(4) DC biasing is the same → Currents are proportional wrt k_n's

and $k_{n2} = 2k_{n1}$ (since $\left(\frac{W}{L}\right)_2 = 2\left(\frac{W}{L}\right)_1$)

$\Rightarrow I_{D2} = 2I_{D1}$ and $g_{m1} = 2\sqrt{k_{n1} I_{D1}}$ $\Rightarrow$ $\boxed{g_{m2} = 2\sqrt{(2k_{n1})(2I_{D1})} = 2g_{m1}}$

in s.s:



KCL @ S:

$g_{m1} v_{gs1} = -\underbrace{g_{m2}}_{2g_{m1}} v_{gs2}$

$\Rightarrow \boxed{v_{gs1} = -2v_{gs2}}$

and $v_{gs1} - v_{gs2} = v_d$ (by KVL)

$\Rightarrow \boxed{v_{gs1} = \frac{2v_d}{3}, \quad v_{gs2} = -\frac{v_d}{3}}$ $\Rightarrow v_{o2} = -(2g_{m1})\left(-\frac{v_d}{3}\right) R_D$

$v_{o1} = -g_{m1}\left(\frac{2v_d}{3}\right) R_D$

$$\boxed{A_v = \frac{v_{o2} - v_{o1}}{v_d} = \frac{2g_{m1}\frac{v_d}{3}R_D - \left(-g_{m1}\frac{2v_d}{3}R_D\right)}{v_d} = \frac{4}{3} g_{m1} R_D}$$

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(5) a) Assume M_1 SAT: $V_{GS1} = 3V \Rightarrow I_D = k_n(V_{GS} - V_{th})^2 = 0.5(3-1)^2 = 2\,mA$

$\Rightarrow \boxed{I_{C1} = I_{C2} \cong I_{E1} = I_{E2} = 1\,mA}$

As V_{CM} decreases: $V_{E1,2} \downarrow$ to keep $\overbrace{V_{BE}}^{V_{BE}(ON)}$ constant, hence $V_{D1} \downarrow$, $V_{DS1} \downarrow$
so M1 may move out of SAT.

We need $V_{DS1} > V_{GS1} - V_{th} \Rightarrow (V_{CM} - \underbrace{0.7}_{V_{BE}(ON)}) > \underbrace{3-1}_{2V} \Rightarrow \boxed{V_{CM} > 2.7V}$

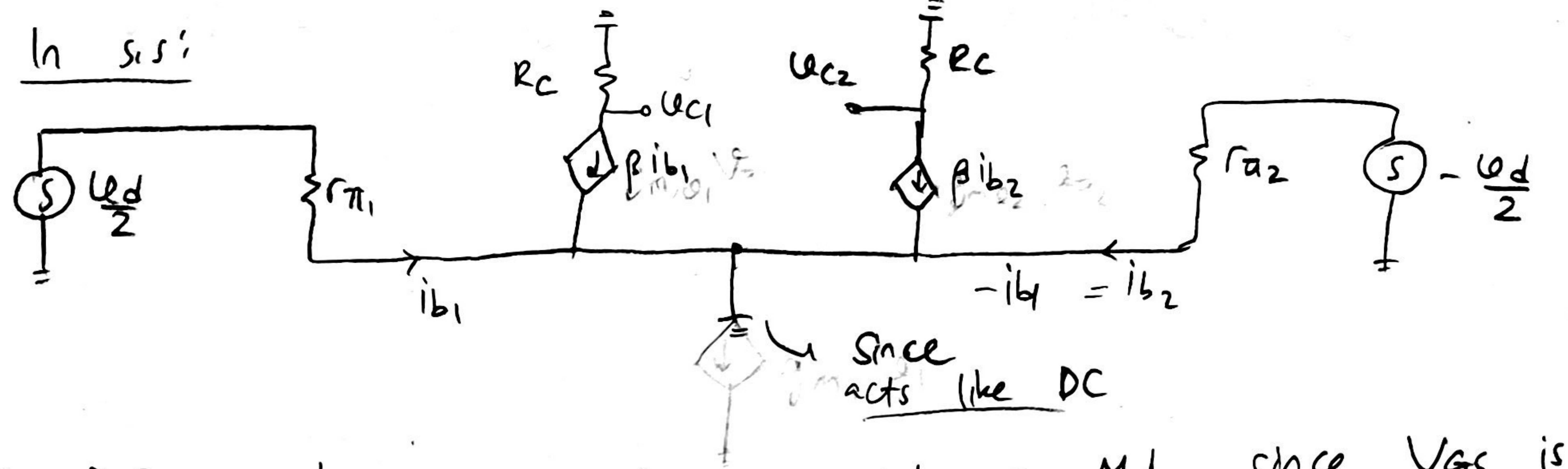
As V_{CM} increases: $V_{B1,2} \uparrow$ so that B-C diode may be turned on,
and Q_1, Q_2 may move into SAT

we need $V_{CE} > V_{CE}(SAT) \Rightarrow (10 - \underbrace{I_C}_{1mA}\underbrace{R_C}_{4}) - (V_{CM} - 0.7) > 0.2$

$\Rightarrow 6.7 - V_{CM} > 0.2 \Rightarrow \boxed{6.5 < V_{CM}}$

Hence, we need $\boxed{6.5V > V_{CM} > 2.7V}$

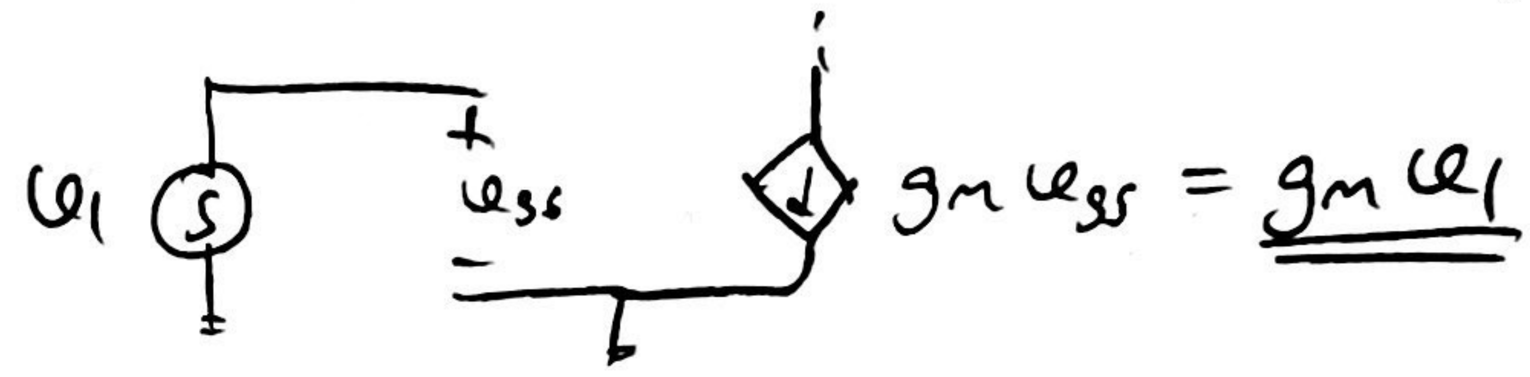
b) In s.s:



In DC, we have a varying current on M1 since V_{GS} is varying. This causes a change in I_C's of the BJT's $\Rightarrow$ change in g_m's.

$$g_m = \underbrace{\frac{I_C}{V_T}}_{\text{coming from 3V bias}} + \underbrace{\frac{\Delta I_C}{V_T}}_{\text{coming from } v_1}$$

Since v_1 is s.s, the model of M1:



This means that the sum of I_C's of Q1 and Q2 should be

$$I_{C1} + I_{C2} = \underbrace{I_D}_{\text{from 3V}} + \underbrace{i_d}_{\text{from } v_1} = 2\,mA + g_m v_1 \Rightarrow$$

Due to symmetry:

$$\boxed{I_{C1} = I_{C2} = 1\,mA + \frac{g_m v_1}{2}}$$

and

$$\boxed{g_{m1} = g_{m2} = \frac{I_{C1/2}}{V_T} = \frac{1\,mA}{V_T} + \frac{g_m v_1}{2V_T}}$$

Now, $v_{C1} = -\beta i_b R_C = \frac{-\beta}{r_\pi} \frac{v_d}{2} R_C = \boxed{-g_{m1} R_C \frac{v_d}{2}}$

$v_{C2} = -\beta i_b R_C = \left(-\frac{\beta}{r_\pi}\right)\left(-\frac{v_d}{2}\right) R_C = \boxed{g_{m2} R_C \frac{v_d}{2}}$

$$\Rightarrow \boxed{v_{out} = g_{m1,2} R_C v_d = \left(\frac{1\,mA}{V_T} + \frac{g_m v_1}{2V_T}\right) R_C v_d}$$

$g_m = 2\sqrt{k_n I_D} = 2\sqrt{(0.5)2}$
$= 2\,mA/V$

v_{out}

$$= \left(\frac{1\,mA}{0.026V} + \frac{(2\,mA/V)\,v_1}{2(0.026)}\right)(4k)\,v_d = \boxed{153.85\,v_d + (153.85)(v_1 \cdot v_d}$$

This is a multiplier circuit whose output is AM modulated. The frequencies present are (using the identity:

$\cos a \cdot \cos b = \frac{1}{2}(\cos(a+b) + \cos(a-b))$)

$$\Rightarrow \boxed{\begin{aligned} w_{01} &= w_1 + w_2 \\ w_{02} &= w_2 - w_1 \end{aligned}}$$

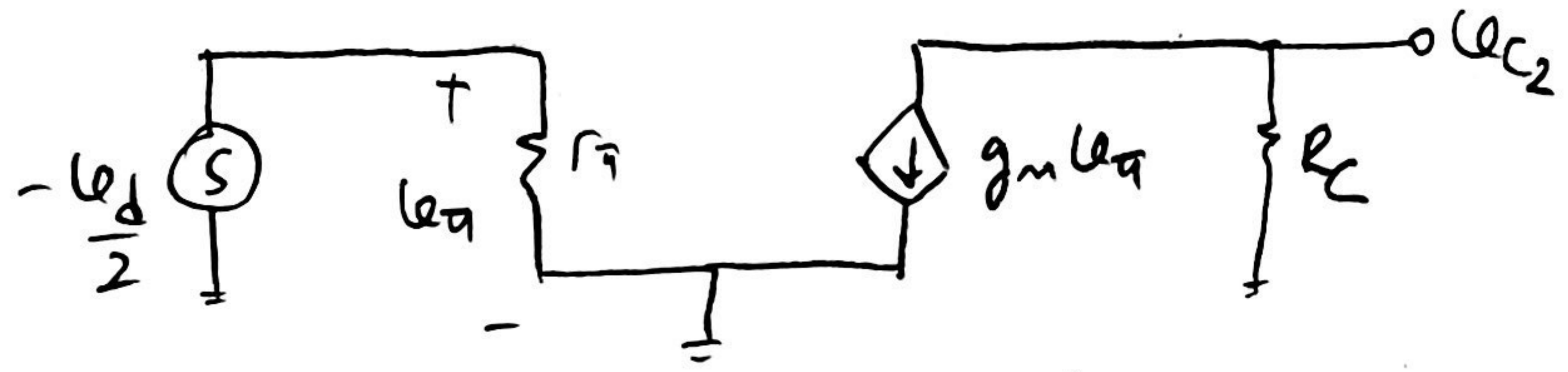
$\Rightarrow$ these frequencies will also be present in the output

$$\Rightarrow \boxed{v_{out} = 0.153\cos(w_1 t) + 0.0153\,\underbrace{\cos(w_1 t)\cos(w_2 t)}_{\text{multiplied!}}}$$

c) Since M1 has no s.s on it, it will act like a resistance with r_o in s.s.

$$r_o = \frac{1}{\lambda I_D} = \frac{1}{(0.02)(2)} = 25k\Omega$$

A_{dm2}: Omit r_o since no current passes on diff. mode



$$\boxed{A_{dm_2} = \frac{v_{C2}}{-v_d} = \frac{g_m R_C}{2}}$$

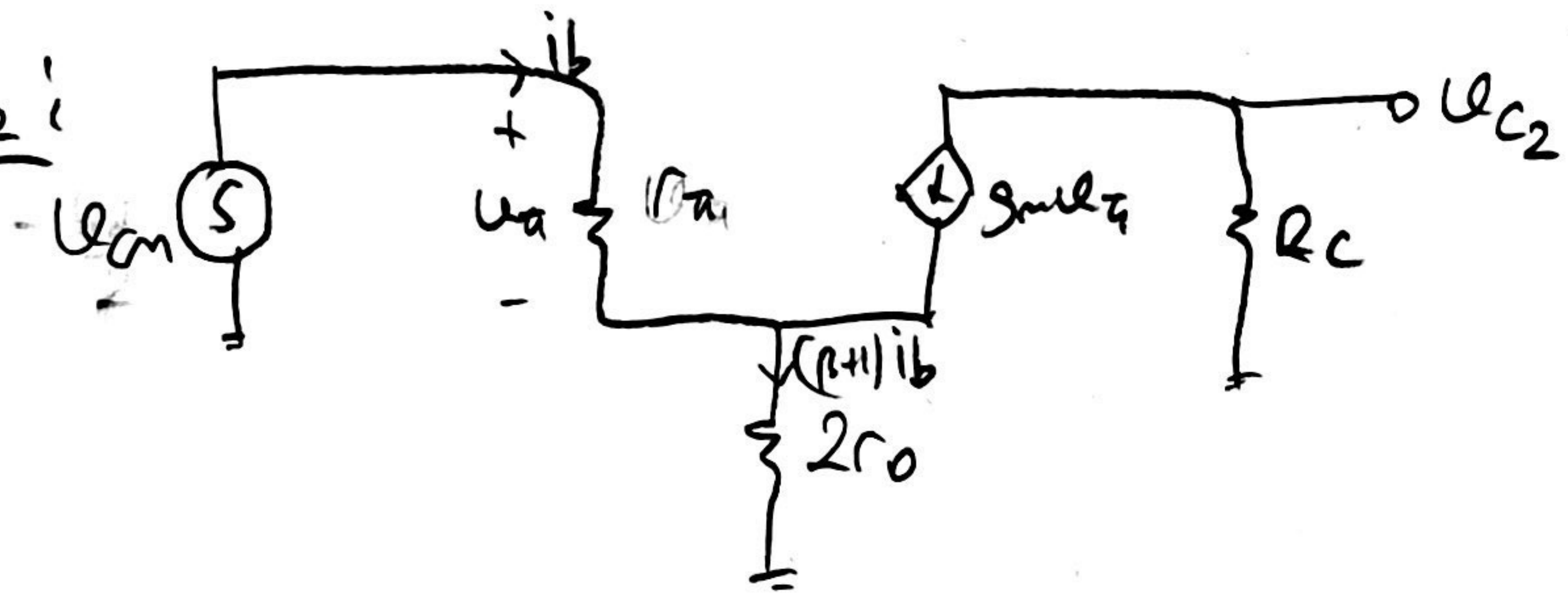
$$v_{C2} = -g_m v_\pi R_C = g_m \frac{v_d}{2} R_C$$

g_m of Q_1, Q_2: $\frac{I_C}{V_T} = \frac{1mA}{0.026V} = 38.46\ mA/V$

$$\Rightarrow \boxed{A_{dm_2} = 76.92\ \frac{V}{V}}$$

$$r_\pi = \frac{\beta}{g_m} = 2.6k\Omega$$

A_{cm2}:



$$v_\pi = v_{cm} \frac{r_\pi}{r_\pi + 2(\beta+1)r_o}$$

$$v_{C2} = -g_m v_\pi R_C = -v_{cm} \frac{\overbrace{g_m r_\pi}^{\beta} R_C}{r_\pi + 2(\beta+1)r_o}$$

$$\Rightarrow \boxed{A_{cm_2} = \frac{v_{C2}}{v_{cm}} = \frac{-\beta R_C}{r_\pi + 2(\beta+1)r_o}}$$

$$\boxed{A_{cm2} = -0.079\ \frac{V}{V}}$$

$$\Rightarrow CMRR = \left|\frac{A_{dm2}}{A_{cm2}}\right| = \frac{76.92}{0.079} = 973.67 \cong \boxed{59.77\ dB}$$