EEE - 321: Signals and Systems

Lab Assignment 2

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Section 02

Part 1

The function SUMCS:

```
function [xs] = SUMCS(t, A, omega)
    xs = zeros(size(t));

    L = length(A);

    for i = 1:L
        xs = xs + A(i) * exp(1j * omega(i) * t);
    end
end
```

In the code, the “rand” function is used. rand(M, N) returns an M-by-N matrix containing “uniformly distributed” pseudorandom values drawn from the standard uniform distribution on the open interval (0,1). The code is presented below.

```
t=[0:0.001:1];
n=mod(22102718, 41);

real_part = 3 * rand(1, n);
imaginary_part = 3 * rand(1, n);
A = real_part + 1j * imaginary_part;

omega = pi * rand(1, n);

xs= SUMCS(t, A, omega);

real_part_xs = real(xs);
imaginary_part_xs = imag(xs);

magnitude_xs = abs(xs);
phase_xs = angle(xs);
```

```
figure
grid("on");
subplot(1, 2, 1);
plot(t, real_part_xs, 'b');
xlabel('Time (t)');
ylabel('Real part of xs(t)');
title('Re(xs) versus t');
grid("on");


subplot(1, 2, 2);
plot(t, imaginary_part_xs, 'r');
xlabel('Time (t)');
ylabel('Imaginary part of xs(t)');
title('Im(xs) versus t');
grid("on");

sgtitle('Real and Imaginary Parts of xs(t)');

figure
subplot(1, 2, 1);
plot(t, magnitude_xs, 'g');
xlabel('Time');
ylabel('Magnitude of xs(t)');
title('Magnitude of xs(t) versus t');
grid("on");

subplot(1, 2, 2);
plot(t, phase_xs, 'm');
xlabel('Time');
ylabel('Phase of xs(t)');
title('Phase of xs(t) versus t');
grid("on");
sgtitle('Magnitude and Phase of xs(t)');
```

The resulting plots are presented below:

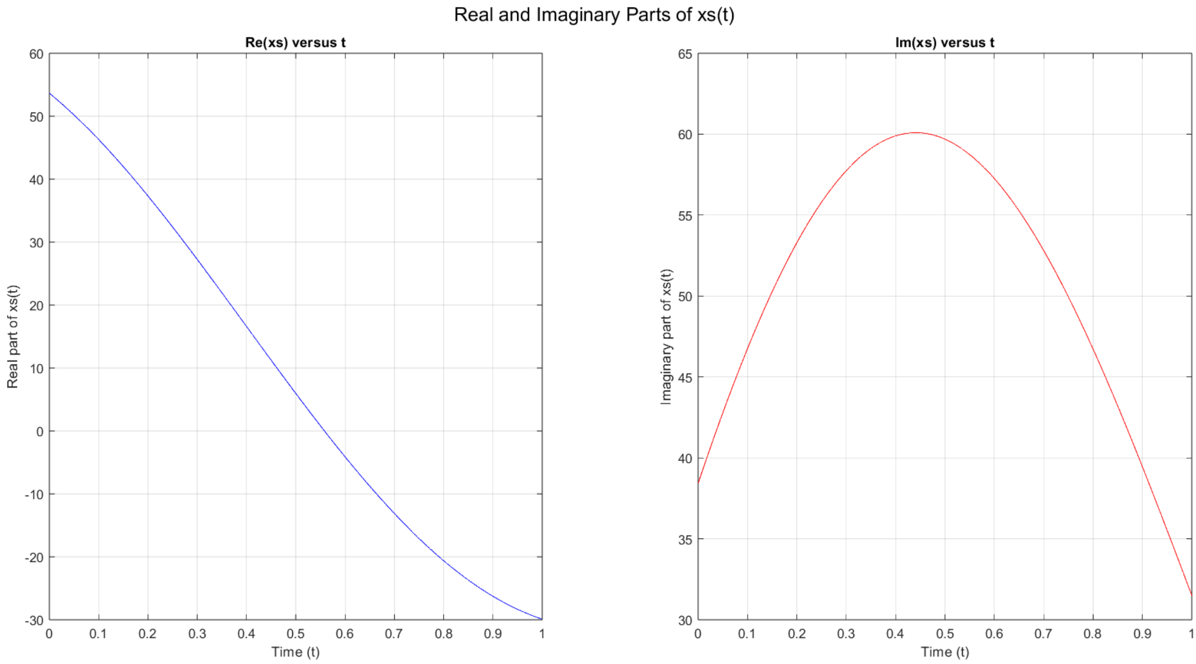


Figure 1: Plot for the real and imaginary parts of xs

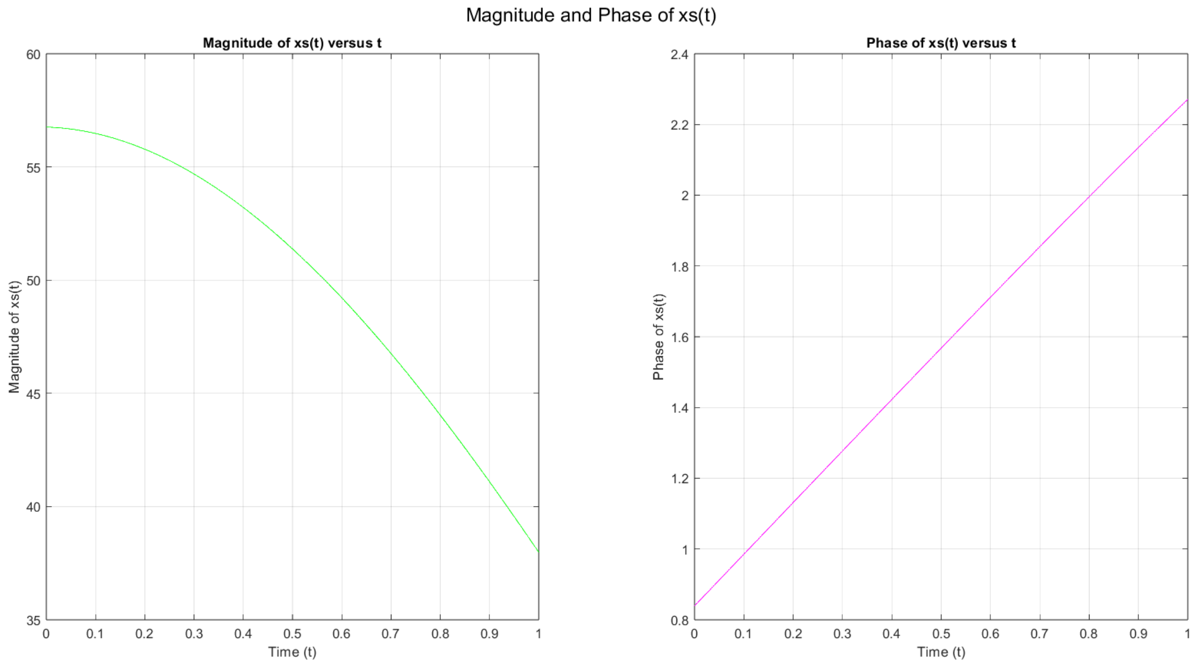


Figure 2: Plot for the magnitude and phase of xs

Part 2

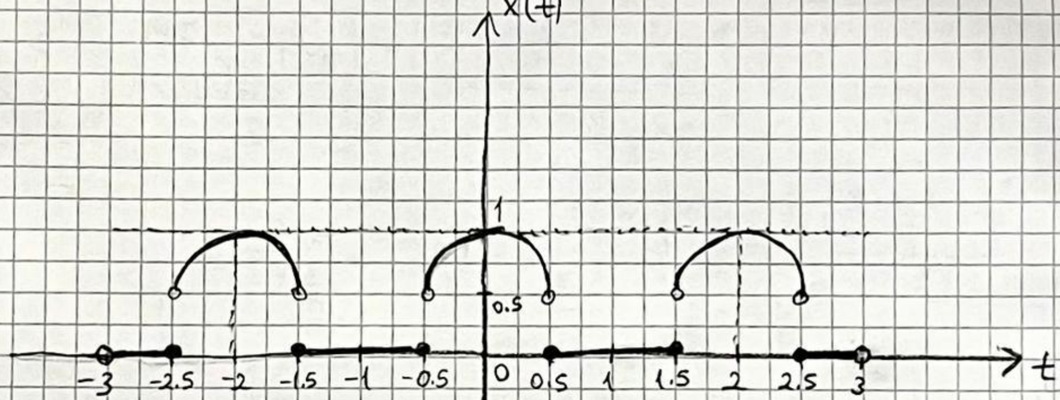
$$x(t)=\sum_{k=-\infty}^{\infty} X_k e^{j\frac{2\pi k t}{T}} \quad , \quad X_k=\frac{1}{T}\int_{-T/2}^{T/2} x(t)\, e^{-j\frac{2\pi k t}{T}}\, dt$$

$$\tilde{x}(t)=\sum_{k=-K}^{K} X_k e^{j\frac{2\pi k t}{T}}$$

$\boxed{W=1,\ T=2,\ \text{sketch } x(t) \text{ over } \underline{-3<t<3}:}$

$$\Rightarrow \boxed{x(t)=\begin{cases} 1-2t^2 & , \ -0.5<t<0.5 \\ 0 & , \ \text{otherwise} \end{cases}}$$

and $x(t)$ is periodic with $T=2$.



$$X_k=\frac{1}{T}\int_{-T/2}^{T/2} x(t)\, e^{-j\frac{2\pi k t}{T}}\, dt$$

$$\Rightarrow X_k=\frac{1}{T}\left(\underbrace{\int_{-T/2}^{-W/2} 0\cdot e^{-j\frac{2\pi k t}{T}}\, dt}_{0}+\int_{-W/2}^{W/2} (1-2t^2)\, e^{-j\frac{2\pi k t}{T}}\, dt+\underbrace{\int_{W/2}^{T/2} 0\cdot e^{-j\frac{2\pi k t}{T}}\, dt}_{0}\right)$$

$$\Rightarrow X_k=\frac{1}{T}\int_{-W/2}^{W/2} (1-2t^2)\, e^{-j\frac{2\pi k t}{T}}\, dt=\frac{1}{T}\left(\int_{-W/2}^{W/2} e^{-j\frac{2\pi k t}{T}}\, dt-2\underbrace{\int \underbrace{t^2}_{u}\, \underbrace{e^{-j\frac{2\pi k t}{T}}\, dt}_{dv}}_{\text{use IBP}}\right)$$

$$=\frac{1}{T}\left[\left.\frac{e^{j\frac{2\pi k}{T}t}}{j\frac{2\pi k}{T}}\right|_{\frac{W}{2}}^{-\frac{W}{2}}-2\left(\left.\frac{t^2 e^{-j\frac{2\pi k}{T}t}}{j\frac{2\pi k}{T}}\right|_{\frac{W}{2}}^{-\frac{W}{2}}-\underbrace{\int_{-W/2}^{W/2}\frac{2t\, e^{-j\frac{2\pi k}{T}t}}{-j\frac{2\pi k}{T}}\, dt}_{\substack{\text{use IBP again} \\ u=t,\ dv=\frac{e^{-j\frac{2\pi k}{T}t}}{-j\frac{\pi k}{T}}}}\right)\right]$$

$$= \frac{1}{T}\left[\left.\frac{e^{-j\frac{2\pi k}{T}t}}{j\frac{2\pi k}{T}}\right|_{w/2}^{-w/2} - 2\left(\left.\frac{t^2 e^{-j\frac{2\pi k}{T}t}}{j\frac{2\pi k}{T}}\right|_{w/2}^{-w/2} - \left(\left.\frac{t e^{-j\frac{2\pi k}{T}t}}{\frac{\pi^2 k^2}{T^2}\cdot 2}\right|_{w/2}^{-w/2} - \int_{-w/2}^{w/2} \frac{e^{-j\frac{2\pi k}{T}t}}{-2\frac{\pi^2 k^2}{T^2}}\, dt \right)\right)\right]$$

$$= \frac{1}{T}\left[\left.\frac{e^{-j\frac{2\pi k}{T}t}}{j\frac{2\pi k}{T}}\right|_{w/2}^{-w/2} - 2\left(\left.\frac{t^2 e^{-j\frac{2\pi k}{T}t}}{j\frac{2\pi k}{T}}\right|_{w/2}^{-w/2} - \left(\left.\frac{t e^{-j\frac{2\pi k}{T}t}}{\frac{2\pi^2 k^2}{T^2}}\right|_{w/2}^{-w/2} - \left.\frac{e^{-j\frac{2\pi k}{T}t}}{4j\frac{\pi^3 k^3}{T^3}}\right|_{-w/2}^{w/2} \right)\right)\right]$$

$$\frac{e^{j\frac{2\pi k}{T}\frac{w}{2}} - e^{-j\frac{2\pi k}{T}\frac{w}{2}}}{j\frac{2\pi k}{T}} = \frac{\sin\left(\frac{2\pi k}{T}\cdot\frac{w}{2}\right)}{\frac{\pi k}{T}}$$

$$\frac{\frac{w^2}{4}\left(e^{j\frac{2\pi k}{T}\frac{w}{2}} - e^{-j\frac{2\pi k}{T}\frac{w}{2}}\right)}{2j\frac{\pi k}{T}} = \frac{\frac{w^2}{4}\sin\left(\frac{2\pi k}{T}\frac{w}{2}\right)}{\frac{\pi k}{T}}$$

$$\frac{-\frac{w}{2}\left(e^{j\frac{2\pi k}{T}\frac{w}{2}} + e^{-j\frac{2\pi k}{T}\frac{w}{2}}\right)}{\frac{\pi^2 k^2}{T^2}\cdot 2} = \frac{-\frac{w}{2}\cos\left(\frac{2\pi k}{T}\frac{w}{2}\right)}{\frac{\pi^2 k^2}{T^2}}$$

$$\frac{\sin\left(\frac{2\pi k}{T}\frac{w}{2}\right)}{2\frac{\pi^3 k^3}{T^3}}$$

$$= \frac{1}{T}\left[\frac{\sin\left(\frac{\pi k w}{T}\right)\cdot T}{\pi k} - 2\left(\frac{w^2 T}{4\pi k}\sin\left(\frac{\pi k w}{T}\right) + \frac{wT^2}{2\pi^2 k^2}\cos\left(\frac{\pi k w}{T}\right) + \frac{T^3}{2\pi^3 k^3}\sin\left(\frac{\pi k w}{T}\right)\right)\right]$$

$$= \frac{1}{\pi k}\sin\left(\frac{\pi k w}{T}\right) - \frac{w^2}{2\pi k}\sin\left(\frac{\pi k w}{T}\right) - \frac{wT}{\pi^2 k^2}\cos\left(\frac{\pi k w}{T}\right) - \frac{T^2}{\pi^3 k^3}\sin\left(\frac{\pi k w}{T}\right)$$

$$= \boxed{\frac{(2-w^2)\pi^2 k^2 - 2T^2}{2\pi^3 k^3}\sin\left(\frac{\pi k w}{T}\right) - \frac{wT}{\pi^2 k^2}\cos\left(\frac{\pi k w}{T}\right)}$$

Part 3

The FSWave function is presented below.

```
function [xt] = FSWave(t,K,T,W)

    Xk = zeros(1,2*K+1);
    omega = zeros(1,2*K+1);

    for k = -K:K
        omega(k+K+1) = 2*pi*k/T;
        syms x
        fun = (1 - 2* x^2) *exp(-1j * omega(k+K+1) * x);
        Xk(k+K+1) = (1/T)*int(fun, -W/2, W/2);
    end

    xt = SUMCS(t, Xk, omega);

end
```

To obtain the plots:

```
T=2;
W=1;
t=[-5:0.001:5];
D11 = mod(22102718, 11);
D5  = mod(22102718, 5);
K=20+D11;
K1=2+D5;
K2=7+D5;
K3=15+D5;
K4=50+D5;
K5=100+D5;

xt = FSWave(t, K, T, W);

re_xt = real(xt);
im_xt = imag(xt);
```

```
figure
grid("on");
subplot(1, 2, 1);
plot(t, re_xt, 'b');
xlabel('Time (t)');
ylabel('Real part of xt(t)');
title('Re(xt) versus t');
grid("on");


subplot(1, 2, 2);
plot(t, im_xt, 'r');
xlabel('Time (t)');
ylabel('Imaginary part of xt(t)');
title('Im(xt) versus t');
grid("on");


sgtitle('Real and Imaginary Parts of xt(t)');
```

The real and the imaginary parts are plotted as:

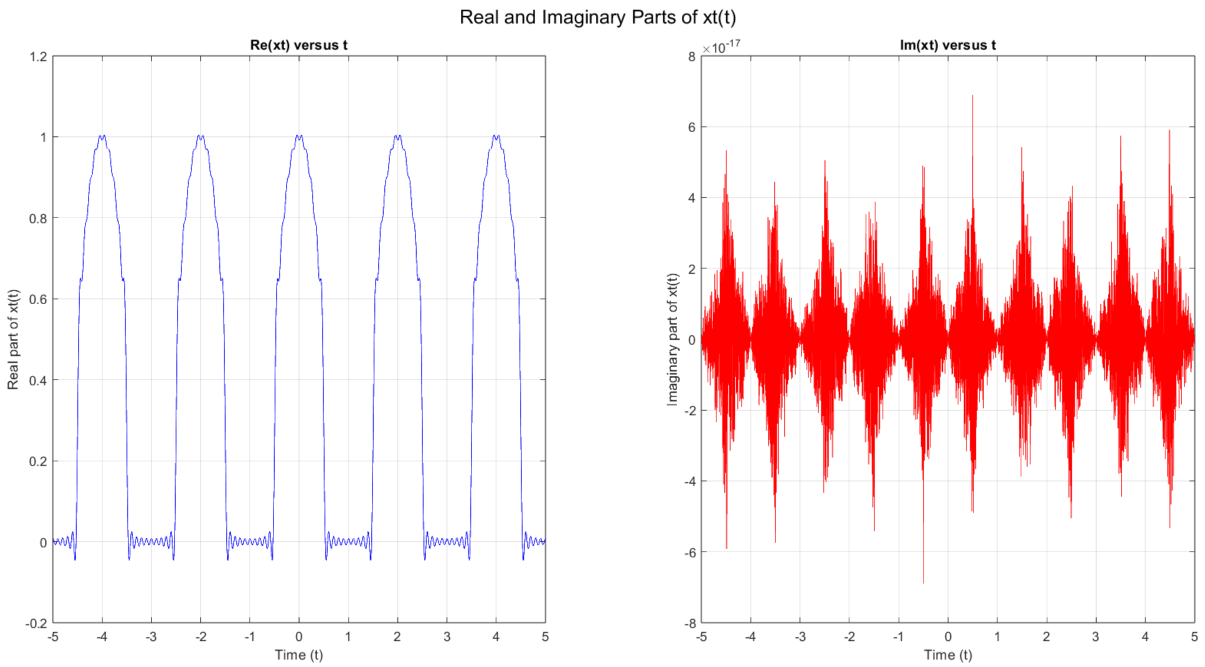


Figure 3: Plot for the real and imaginary parts of xt

The values are given as D_{11} = mod(your ID number, 11) and D_5 = mod(your ID number, 5), T=2, W=1, K=20+D_{11} and t=[-5:0.001:5]. It can be observed that on the real part plot, the minimum value oscillates around 0, while the maximum value is very close to 1. The plot also resembles the graph of the function defined in Part 2. For the imaginary part, the minimum value is approximately $-7*10^{-17}$ and the maximum value is approximately $7*10^{-17}$.

The values for the imaginary parts are nearly 10^{17} times smaller compared to those of the real parts. Since they are much smaller than the real parts, they can be ignored. In fact, the imaginary parts of xt should be 0 since x(t) is a real-valued signal. The reason why they are not perfectly zero is because computers don't have infinite precision to calculate numbers, for example π has infinitely many decimal points. Instead, the numbers are approximated as precisely as possible. For this reason, when sin(π/6) − 0.5 is computed the MATLAB, the result is given as -5.5511e-17 ($-5.551 * 10^{-17}$), again a very small number that is not equal to zero.

Then, for five different values of K, the real parts of xt are plotted:

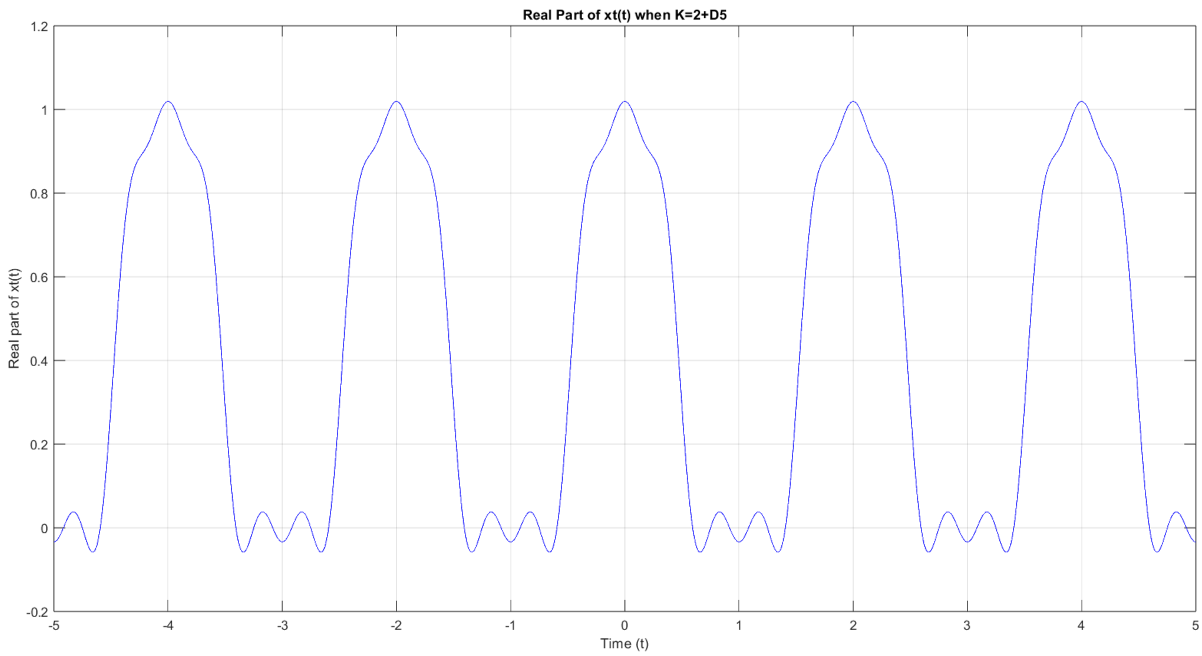


Figure 4: Real part of xt when K=2+D5

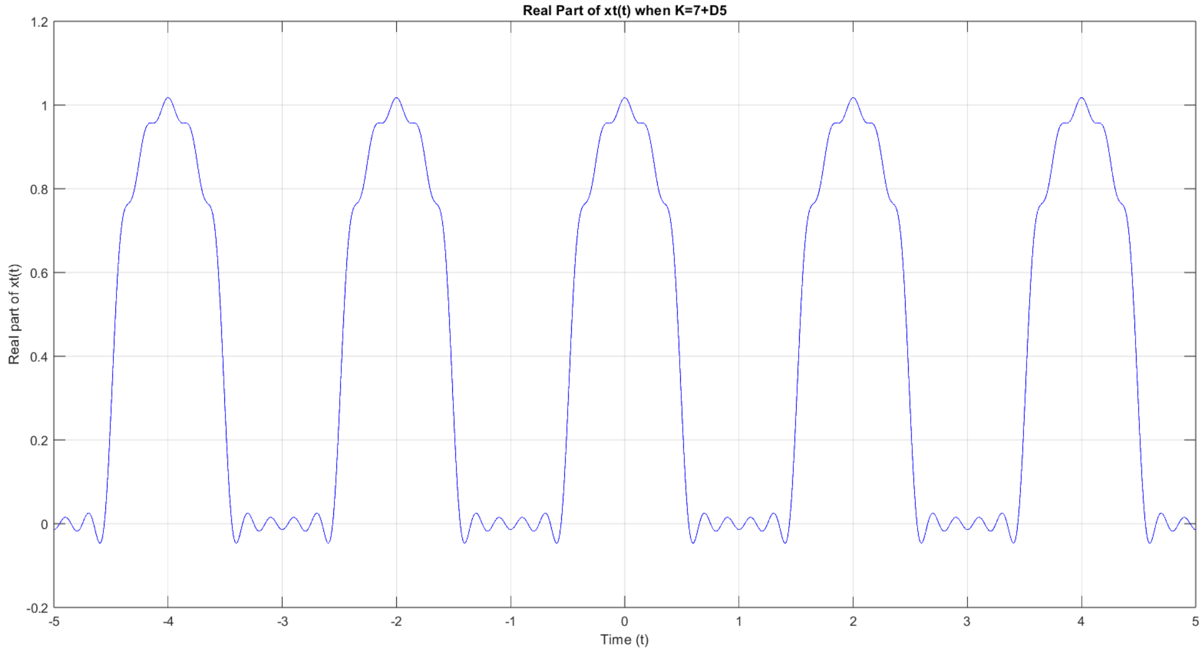


Figure 5: Real part of xt when K=7+D5

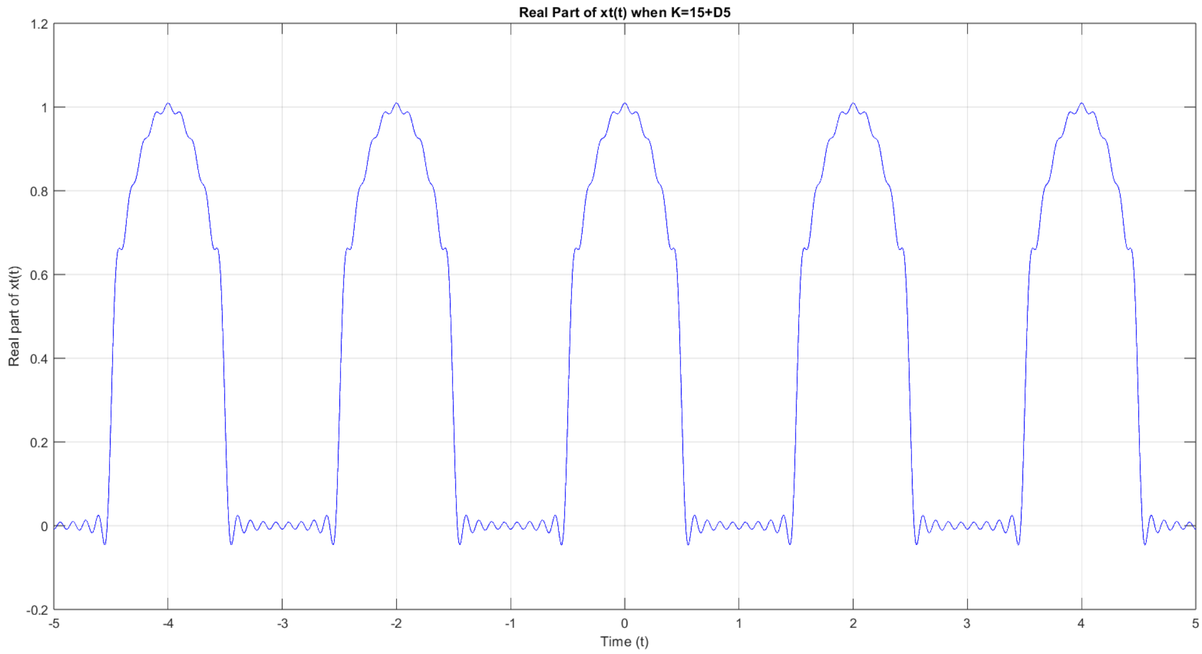


Figure 6: Real part of xt when K=15+D5

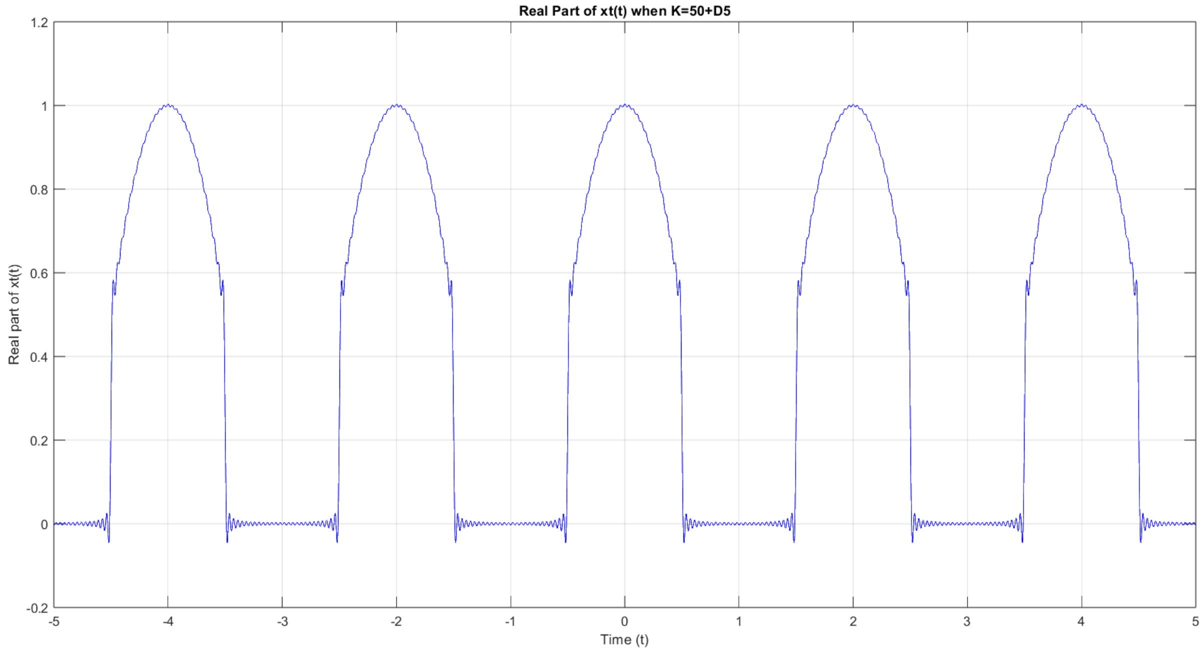


Figure 7: Real part of xt when K=50+D5

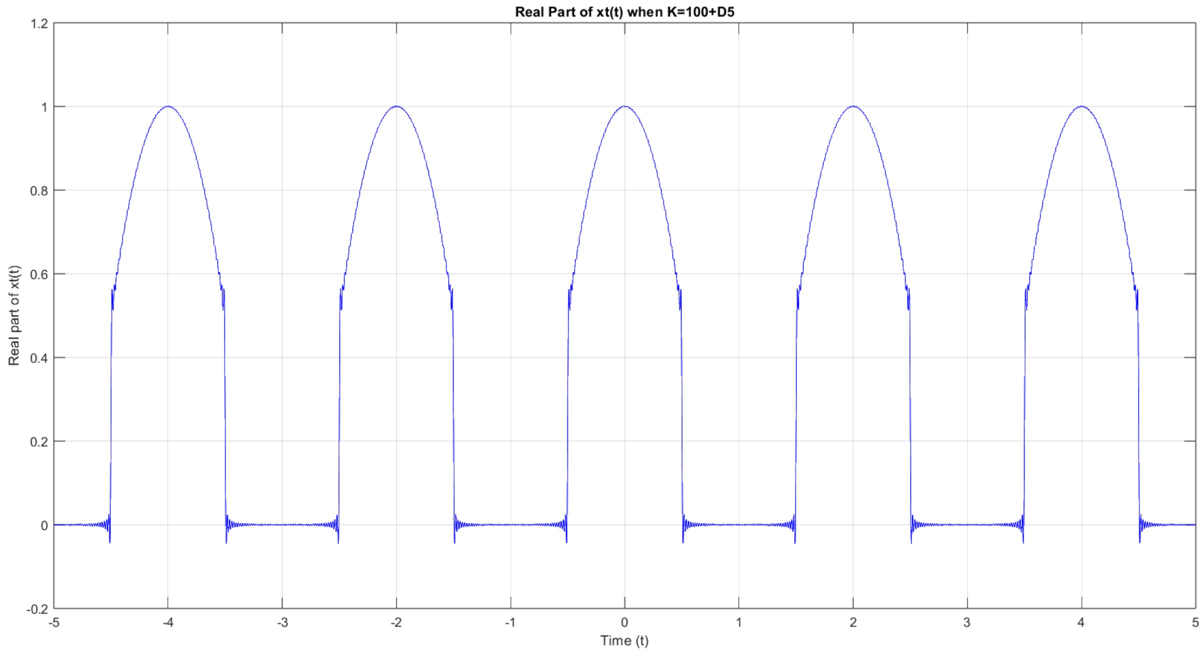


Figure 8: Real part of xt when K=100+D5

It can be observed that as the value of K is increased, the oscillations around the discontinuities are increased and are getting smaller, this way the plots are getting smoother. The plot, therefore, resembles the graph of x(t) (which was drawn in Part 2) more and the error in the approximation is reduced. The success of x˜(t) in approximating x(t) increases as K is increased. This is because K keeps approaching to infinity as it is increased, hence the Fourier series of x(t) is approximated better.

Part 4

a) The FSWave function becomes:

```
function [xt] = FSWave(t,K,T,W)

    Xk = zeros(1,2*K+1);
    omega = zeros(1,2*K+1);

    for k = -K:K
        omega(k+K+1) = 2*pi*k/T;
        syms x
        fun = (1 - 2* x^2) *exp(-1j * omega(k+K+1) * x);
        Xk(k+K+1) = (1/T)*int(fun, -W/2, W/2);
    end

    %for Part 4a
    Yk = Xk(2*K+1 : -1 : 1);

    xt = SUMCS(t, Yk, omega);

end
```

This corresponds to the coefficients, and therefore x˜(t), being mirrored with respect to the line t=0 (or the y-axis). However, since the function is an even function, there is no difference between the resulting plot and the initial plot.

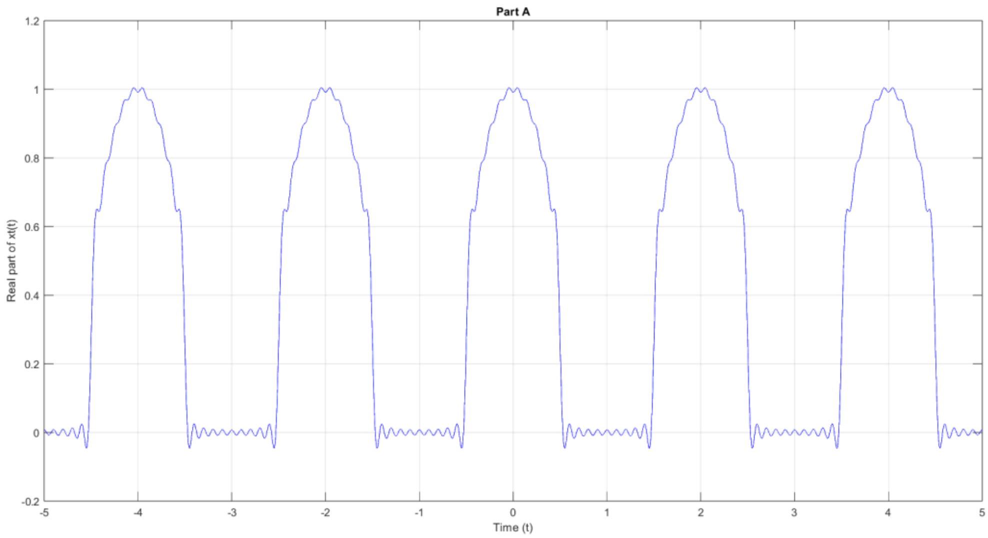


Figure 9: Plot for part A, function remains the same

b) The FSWave function becomes:

```
function [xt] = FSWave(t,K,T,W)

    Xk = zeros(1,2*K+1);
    omega = zeros(1,2*K+1);

    for k = -K:K
        omega(k+K+1) = 2*pi*k/T;
        syms x
        fun = (1 - 2* x^2) *exp(-1j * omega(k+K+1) * x);
        Xk(k+K+1) = (1/T)*int(fun, -W/2, W/2);
    end

    %for Part 4b
    t_0 = 0.6;
    arr = [-K:1:K];
    phase = exp(-1j*2*pi*t_0*arr/T);
    Yk = Xk .* phase;

    xt = SUMCS(t, Yk, omega);

end
```

This multiplication actually corresponds to a phase in time domain, in other words, a time domain translation or shifting. The function is shifted to the right by t_0, which is 0.6 in this case.

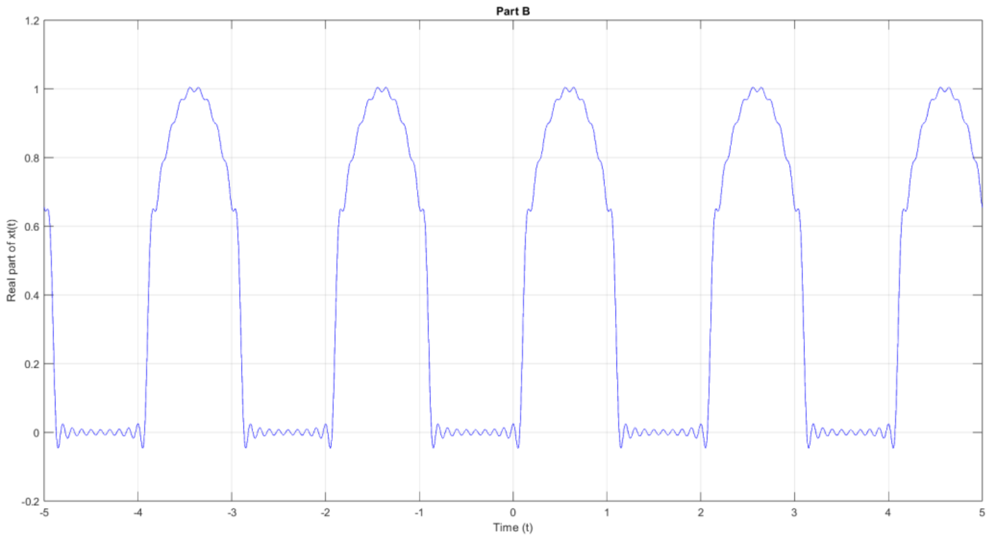


Figure 10: Plot for part B, function is shifted by 0.6

c) The FSWave function becomes:

```
function [xt] = FSWave(t,K,T,W)

    Xk = zeros(1,2*K+1);
    omega = zeros(1,2*K+1);

    for k = -K:K
        omega(k+K+1) = 2*pi*k/T;
        syms x
        fun = (1 - 2* x^2) *exp(-1j * omega(k+K+1) * x);
        Xk(k+K+1) = (1/T)*int(fun, -W/2, W/2);
    end

    %for Part 4c
    arr = [-K:1:K];
    Yk=1j*2*pi*arr.*Xk / T;

    xt = SUMCS(t, Yk, omega);

end
```

This operation corresponds to taking the derivative of x˜(t). The plot, therefore, indicates the rate of change of x˜(t).

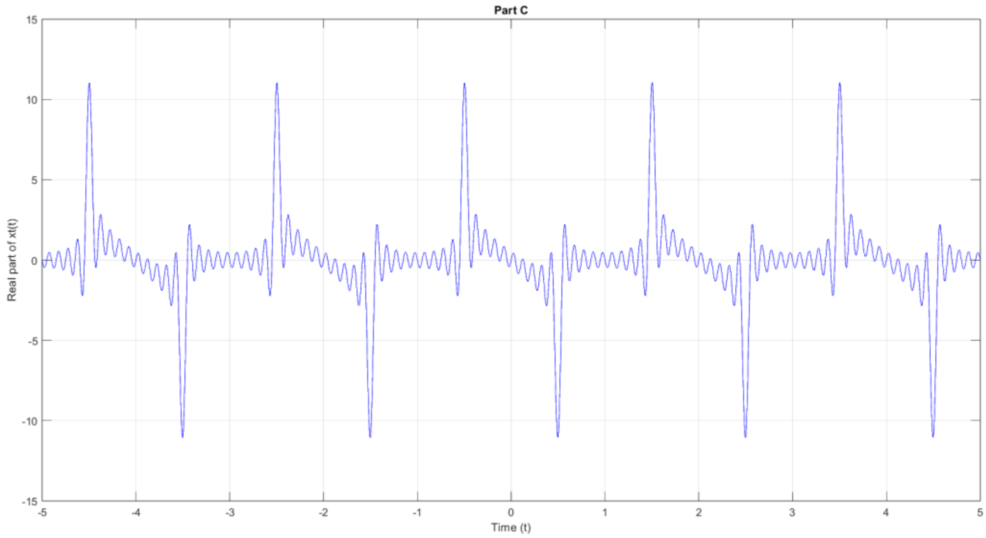


Figure 11: Plot for part C

d) The FSWave function becomes:

```
function [xt] = FSWave(t,K,T,W)

    Xk = zeros(1,2*K+1);
    omega = zeros(1,2*K+1);

    for k = -K:K
        omega(k+K+1) = 2*pi*k/T;
        syms x
        fun = (1 - 2* x^2) *exp(-1j * omega(k+K+1) * x);
        Xk(k+K+1) = (1/T)*int(fun, -W/2, W/2);
    end

    %for Part 4d
    Yk = [Xk(K:-1:1)  Xk(K+1)  Xk(2*K+1:-1:K+2)];

    xt = SUMCS(t, Yk, omega);

end
```

The operation rearranges the Xk coefficients with respect to the given condition. The values are interchanged for three different intervals.

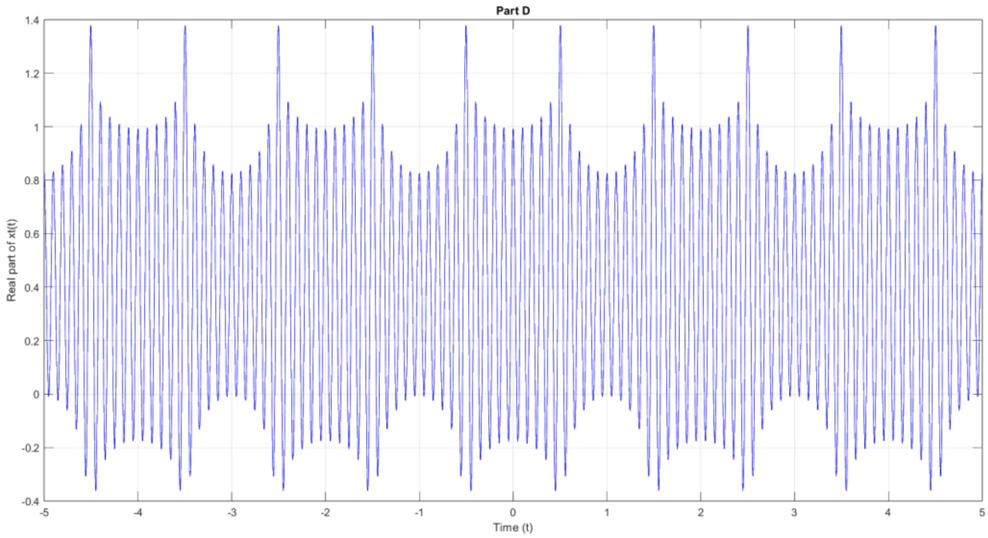


Figure 12: Plot for part D