EEE - 321: Signals and Systems

Lab Assignment 3

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Section 02

Part 1.1: DTMF Transmitter

The function DTMFTRA:

```
Number = [5 3 8 7 3 4 5 2 5 4];
x = DTMFTRA(Number);
soundsc(x,8192);

function [x]=DTMFTRA(Number)
t = 0:1/8192:0.25;
t = t(1:end-1);
len_t = length(t);
len_N = length(Number);
x = zeros(1,len_N*len_t);
R = [941 697 697 697 770 770 770 852 852 852];
C = [1336 1209 1336 1477 1209 1366 1477 1209 1336 1477];
for k = 1:len_N
    fr = R(Number(k)+1);
    fc = C(Number(k)+1);
    x((k-1)*len_t+1 : k*len_t) = cos(2*pi*fr*t)+cos(2*pi*fc*t);
end


end
```

The sound resembles the typing sounds on old phones when a number is dialed. I observed that every digit generates a different sound since the frequencies are different for each digit.

Part 1.2: DTMF Receiver

This time, Number is modified such that it includes the last 4 digits of the student number in reverse order and the number corresponding to the section. Hence Number = [8 1 7 2 2].

Using the definition of Fourier transform and its properties, the Fourier transforms of the below functions are computed.

a) $x(t) = e^{j2\pi f_0 t} \Rightarrow X(w) = \int_{-\infty}^{\infty} e^{j2\pi f_0 t} \cdot e^{-jwt} \, dt$, let $\boxed{2\pi f_0 = w_0}$ for all parts!

$$\Rightarrow X(w) = \int_{-\infty}^{\infty} e^{-j(w-w_0)t} \, dt = \lim_{a \to \infty} \frac{-1}{j(w-w_0)} \underbrace{\left[e^{-j(w-w_0)t} \right]_{-a}^{a}}_{e^{-j(w-w_0)a} - e^{j(w-w_0)a} = -2j \cdot \sin((w-w_0)a)}$$

$$\Rightarrow X(w) = \lim_{a \to \infty} \frac{\cancel{-}1}{\cancel{j(w-w_0)}} \cdot \cancel{-2j} \sin((w-w_0)a) = \lim_{a \to \infty} 2a \underbrace{\frac{\sin((w-w_0)a)}{(w-w_0)a}}_{\text{sinc}((w-w_0)a)}$$

$$= 2\pi \underbrace{\lim_{a \to \infty} \frac{a}{\pi} \text{sinc}((w-w_0)a)}_{\delta(w-w_0)} = \boxed{2\pi \delta(w-w_0)}$$

b) $x(t) = \cos(2\pi f_0 t)$. Let $2\pi f_0 = w_0$, use Euler's identity:

$$x(t) = \cos(w_0 t)$$

$$= \left(\frac{e^{jw_0 t} + e^{-jw_0 t}}{2} \right) = \frac{1}{2} e^{jw_0 t} + \frac{1}{2} e^{-jw_0 t} \rightarrow \text{linearity of F.T}$$

from part a $\xleftrightarrow{F} \frac{1}{2} \left(2\pi \delta(w-w_0) + 2\pi \delta(w+w_0) \right)$

$$\Rightarrow \boxed{X(w) = \pi \left(\delta(w-w_0) + \delta(w+w_0) \right)}$$

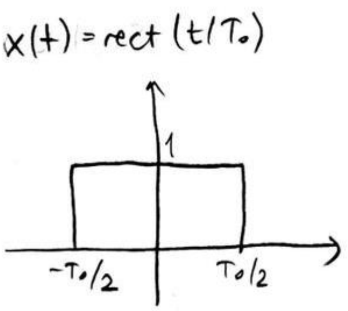
c) $x(t) = \sin(2\pi f_0 t) = \sin(w_0 t) = \frac{e^{jw_0 t} - e^{-jw_0 t}}{2j} = \frac{1}{2j} e^{jw_0 t} - \frac{1}{2j} e^{-jw_0 t}$

linearity $\leftarrow$ $\updownarrow F$ (part a)

$$\frac{1}{2j} \left(2\pi \delta(w-w_0) + 2\pi \delta(w+w_0) \right)$$

$$\Rightarrow \boxed{X(w) = j\pi \left(\delta(w+w_0) - \delta(w+w_0) \right)}$$

d) $x(t) = \text{rect}(t/T_0)$



$$X(w) = \int_{-\infty}^{\infty} \underbrace{x(t)}_{1 \text{ between } T_0/2,\, -T_0/2} e^{-jwt} \, dt = \int_{-T_0/2}^{T_0/2} e^{-jwt} \, dt$$

$$= \frac{1}{jw} \left[e^{-jwt} \right]_{T_0/2}^{-T_0/2} = \frac{2}{w} \underbrace{\frac{e^{jwT_0/2} - e^{-jwT_0/2}}{2j}}_{\sin(w\frac{T_0}{2})}$$

$$= \boxed{\frac{2}{w} \sin\left(\frac{wT_0}{2} \right)}$$

e) $x(t) = \underbrace{e^{j2\pi f_0 t}}_{k(t)} \underbrace{\text{rect}(t/T_0)}_{m(t)}$. <u>Multiplication property</u> : $k(t) \cdot m(t) \longleftrightarrow \dfrac{K(\omega) * M(\omega)}{2\pi}$

From previous parts: $e^{j2\pi f_0 t} = e^{j\omega_0 t} \longleftrightarrow 2\pi\,\delta(\omega-\omega_0) = K(\omega)$

and $\text{rect}(t/T_0) \longleftrightarrow \dfrac{2}{\omega}\sin\left(\dfrac{\omega T_0}{2}\right) = M(\omega)$

we know that $K(\omega) * \delta(\omega-\omega_0) = K(\omega-\omega_0)$

$$\Rightarrow X(\omega) = \frac{K(\omega) * M(\omega)}{2\pi} = \frac{1}{\cancel{2\pi}}\cdot\cancel{2\pi}\left(\underbrace{\frac{2}{\omega}\sin\left(\frac{\omega T_0}{2}\right) * \delta(\omega-\omega_0)}\right)$$

$$\Rightarrow \boxed{X(\omega) = \frac{2}{\omega-\omega_0}\sin\left(\frac{(\omega-\omega_0)T_0}{2}\right)}$$ where $\omega_0 = 2\pi f_0$

f) $x(t) = \underbrace{\cos(2\pi f_0 t)}_{k(t)} \underbrace{\text{rect}(t/T_0)}_{m(t)}$. <u>Mult. property</u>: $k(t)\cdot m(t) \longleftrightarrow \dfrac{K(\omega) * M(\omega)}{2\pi}$

and $\cos(\omega_0 t) \longleftrightarrow \pi\left(\delta(\omega-\omega_0) + \delta(\omega+\omega_0)\right) = K(\omega)$

$\text{rect}(t/T_0) \longleftrightarrow \dfrac{2}{\omega}\sin\left(\dfrac{\omega T_0}{2}\right) = M(\omega)$

$$\Rightarrow X(\omega) = \frac{K(\omega) * M(\omega)}{2\pi} = \frac{1}{2\cancel{\pi}}\cdot\cancel{\pi}\left(\frac{2}{\omega}\sin\left(\frac{\omega T_0}{2}\right) * \left(\delta(\omega-\omega_0) + \delta(\omega+\omega_0)\right)\right)$$

$$= \frac{1}{2}\left(\left(\frac{2}{\omega}\sin\left(\frac{\omega T_0}{2}\right) * \delta(\omega-\omega_0)\right) + \left(\frac{2}{\omega}\sin\left(\frac{\omega T_0}{2}\right) * \delta(\omega+\omega_0)\right)\right)$$ → <u>distributive prop. of convolution</u>

$$= \frac{1}{\cancel{2}}\left(\frac{\cancel{2}}{\omega-\omega_0}\sin\left(\frac{(\omega-\omega_0)T_0}{2}\right) + \frac{\cancel{2}}{\omega+\omega_0}\sin\left(\frac{(\omega+\omega_0)T_0}{2}\right)\right)$$

$$\Rightarrow \boxed{X(\omega) = \frac{\sin\left(\frac{(\omega-\omega_0)T_0}{2}\right)}{\omega-\omega_0} + \frac{\sin\left(\frac{(\omega+\omega_0)T_0}{2}\right)}{\omega+\omega_0}}$$

g) $x(t) = \text{rect}\left(\dfrac{t-t_0}{T_0}\right)$. <u>Time shifting property</u>: $x(t) \longleftrightarrow X(\omega)$

$x(t-t_0) \longleftrightarrow e^{-j\omega t_0} X(\omega)$

From prev. part $\text{rect}\left(\dfrac{t}{T_0}\right) \longleftrightarrow \dfrac{2}{\omega}\sin\left(\dfrac{\omega T_0}{2}\right)$

$$\Rightarrow \boxed{X(\omega) = e^{-j\omega t_0}\,\frac{2}{\omega}\sin\left(\frac{\omega T_0}{2}\right)}$$

h) $x(t) = \underbrace{e^{j2\pi f_0 t}}_{k(t)} \underbrace{\mathrm{rect}\left(\frac{t-t_0}{T_0}\right)}_{m(t)}$. Mult. prop: $k(t)\cdot m(t) \longleftrightarrow \frac{K(\omega) * M(\omega)}{2\pi}$

$$e^{j\omega_0 t} \longleftrightarrow 2\pi\,\delta(\omega-\omega_0) = K(\omega)$$

$$\mathrm{rect}\left(\frac{t-t_0}{T_0}\right) \longleftrightarrow e^{-j\omega t_0}\,\frac{2}{\omega}\sin\left(\frac{\omega T_0}{2}\right) = M(\omega)$$

from prev. parts

$$\Rightarrow X(\omega) = \frac{1}{2\pi}\left(K(\omega) * M(\omega)\right) = \frac{1}{\cancel{2\pi}}\cancel{2\pi}\underbrace{\left(\delta(\omega-\omega_0) * e^{-j\omega t_0}\,\frac{2}{\omega}\sin\left(\frac{\omega T_0}{2}\right)\right)}$$

$$\Rightarrow \boxed{X(\omega) = e^{-j(\omega-\omega_0)t_0}\,\frac{2}{\omega-\omega_0}\sin\left(\frac{(\omega-\omega_0)T_0}{2}\right)}$$

i) $x(t) = \underbrace{\cos(2\pi f_0 t)}_{k(t)} \underbrace{\mathrm{rect}\left(\frac{t-t_0}{T_0}\right)}_{m(t)}$. Use mult. property.

$$\cos(\omega_0 t) \longleftrightarrow \pi\left(\delta(\omega-\omega_0) + \delta(\omega+\omega_0)\right) = K(\omega)$$

$$\mathrm{rect}\left(\frac{t-t_0}{T_0}\right) \longleftrightarrow e^{-j\omega t_0}\,\frac{2}{\omega}\sin\left(\frac{\omega T_0}{2}\right) = M(\omega)$$

Dist. property of conv.

$$\Rightarrow X(\omega) = \frac{1}{2\pi}\left(K(\omega) * M(\omega)\right) = \frac{1}{2\cancel{\pi}}\cancel{\pi}\left(\left(\delta(\omega-\omega_0)+\delta(\omega+\omega_0)\right) * e^{-j\omega t_0}\,\frac{2}{\omega}\sin\left(\frac{\omega T_0}{2}\right)\right)$$

$$= \frac{1}{2}\left(\underbrace{\left(e^{-j\omega t_0}\,\frac{2}{\omega}\sin\left(\frac{\omega T_0}{2}\right) * \delta(\omega-\omega_0)\right)} + \underbrace{\left(e^{-j\omega t_0}\,\frac{2}{\omega}\sin\left(\frac{\omega T_0}{2}\right) * \delta(\omega+\omega_0)\right)}\right)$$

$$\Rightarrow \boxed{X(\omega) = \frac{1}{2}\left(e^{-j(\omega-\omega_0)t_0}\,\frac{2}{\omega-\omega_0}\sin\left(\frac{(\omega-\omega_0)T_0}{2}\right) + e^{-j(\omega+\omega_0)t_0}\,\frac{2}{\omega+\omega_0}\sin\left(\frac{(\omega+\omega_0)T_0}{2}\right)\right)}$$

The code to produce the plot, also to examine the sound:

```
Number = [8 1 7 2 2];
x = DTMFTRA(Number);
soundsc(x,8192);

X=FT(x);
omega=linspace(-8192*pi,8192*pi,10241);
omega=omega(1:10240);
plot(omega,abs(X));
grid("on");
xlabel("Omega (W)");
ylabel("Magnitude of X(W)");
title("Magnitude of X(W) vs W");
```

The resulting plot is presented in Figure 1.

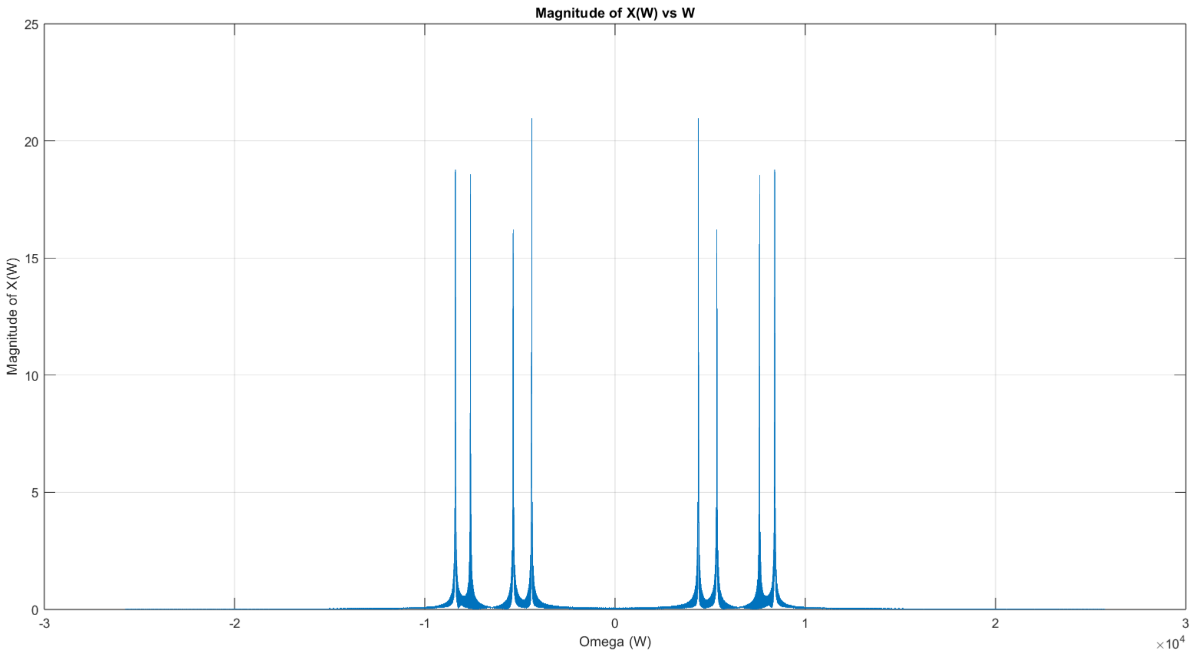


Figure 1: Magnitude of X(ω) versus ω

When the magnitude of the Fourier transform of x(t) is plotted, it can be seen that there are peaks in different frequencies. Considering the Number in this part, the corresponding frequencies first need to be converted into angular frequency. Hence, they need to be multiplied by 2π ($\omega = 2\pi f$).

Number	Corresponding frequency (fr, fc) (in Hz)	Angular frequency (multiplied by 2π) (in rad/sec)
8	852, 1336	5353, 8394
1	697, 1209	4379, 7596
7	852, 1209	5353, 7596
2	697, 1336	4379, 8394
2	697, 1336	4379, 8394

Table 1: Conversion to angular frequency

The problem is that there are common frequencies for different numbers, so the plot is not enough to determine the digits of the number. To find the frequencies at which the peaks occur, the peak points on the plot can be marked:

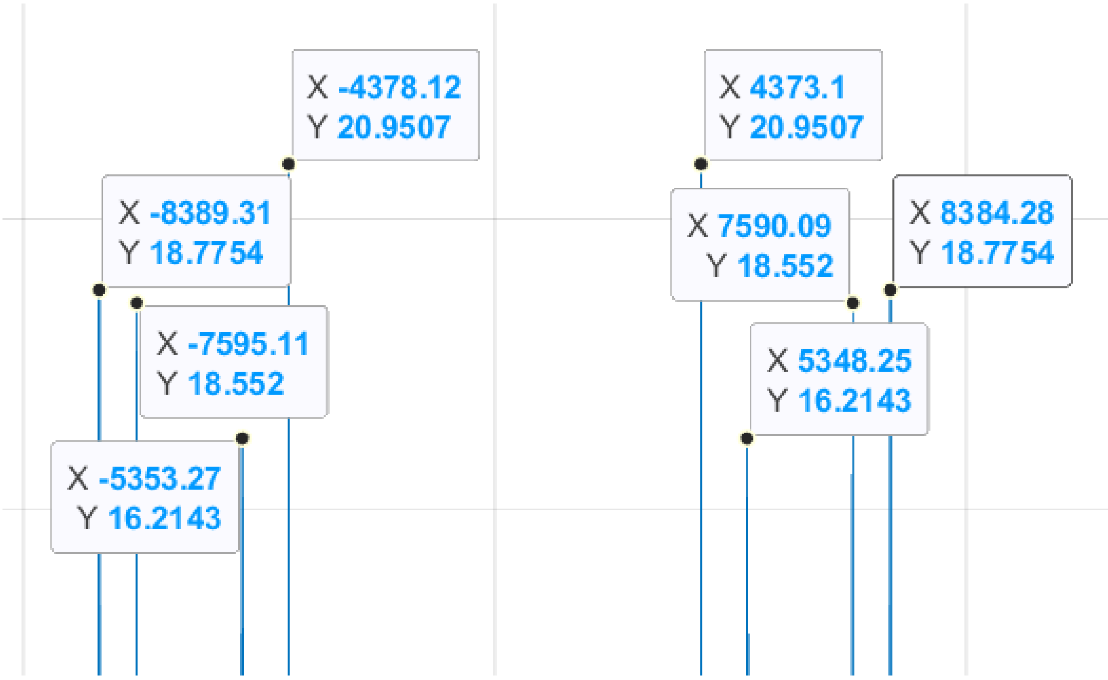


Figure 2: Peak frequencies

As expected, the values match the four different angular frequencies in Table 1 with minor differences. However, the digits can't be determined only by this plot.

For the new defined signal $x_1(t)$, $x(t)$ needs to be multiplied by a rectangular signal. This signal can be defined as:

$$rect(t) = \begin{cases} 1, & for\ 0 \le t \le 0.25 \\ 0, & otherwise \end{cases}$$

Now, to create this function in MATLAB:

```
Number = [8 1 7 2 2];
x = DTMFTRA(Number);
omega=linspace(-8192*pi,8192*pi,10241);
omega=omega(1:10240);
len_t = 2048;
len_n = length(Number);

rec= zeros(1, length(omega));
rec(1, 0*len_t +1 : 1*len_t) = ones(1, len_t);
x_1 = rec .* x;
X = FT(x_1);

plot(omega, abs(X));
grid("on");
xlabel("Omega (W)");
ylabel("Magnitude of X(W)");
title("First digit magnitude");
```

The resulting plot is shown in Figure 3.

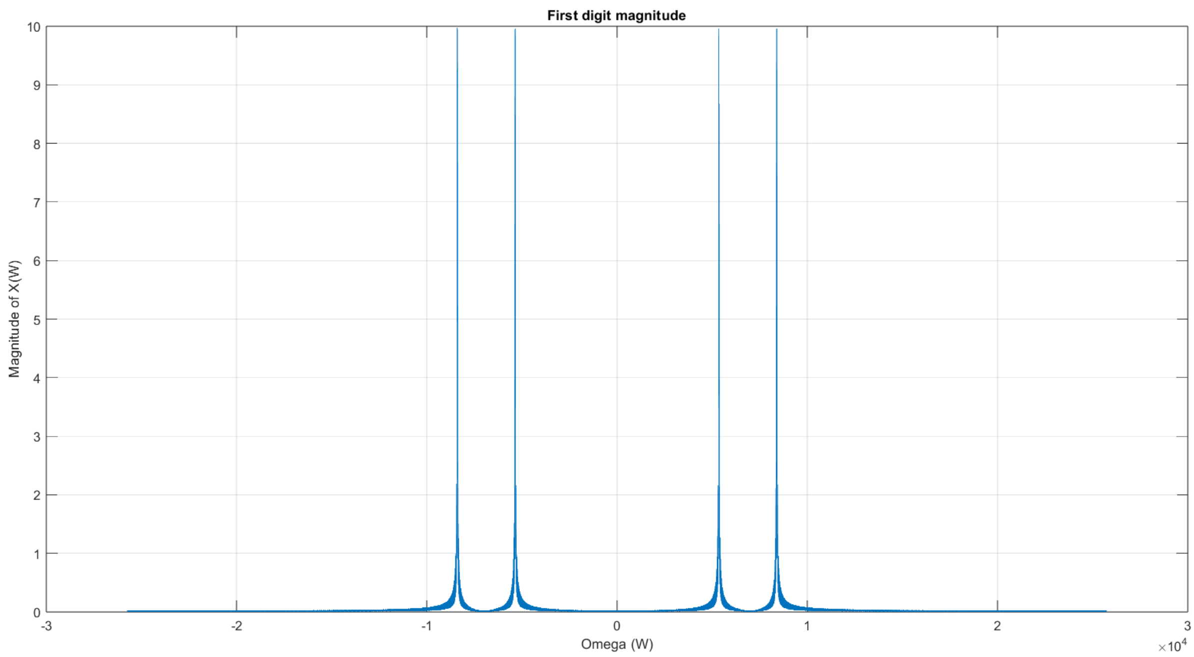


Figure 3: Magnitude plot for first digit

By multiplying with a rectangular signal, the received signal is sampled for the first digit only, acting like a filter. This is because the signal is eliminated after t>0.25, before which only the first digit is received. Hence, the resulting transform corresponds to only the first digit. Now, to locate the point where the peaks occur:

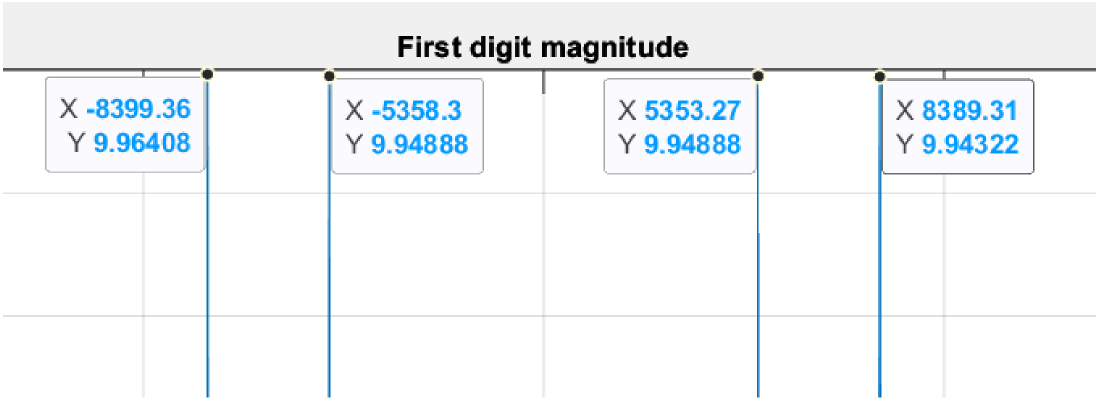


Figure 4: Peak frequencies of the first digit

Since these are the angular frequencies, they need to be divided by 2π. After that, the frequencies can be looked up from the DTMF frequencies table. This way, the digit can be determined. For the first digit, the peak frequencies are approximately at the positive and negative values of 5353 and 8394, which correspond to 8 as seen in Table 1.

To find the other digits, the rectangle needs to be shifted by 0.25 per digit. As the rectangle is shifted, every 0.25 second will be sampled one-by-one, corresponding to each digit separately. For example, for the second digit we need:

$$rect2(t) = \begin{cases} 1, & for\ 0.25 \leq t \leq 0.5 \\ 0, & otherwise \end{cases}$$

since the second digit corresponds to $0.25 \leq t \leq 0.5$. To modify the rectangle accordingly, the line

```
rec(1, 0*len_t +1 : 1*len_t) = ones(1, len_t)
```

should be modified as:

`rec(1, 1*len_t +1 : 2*len_t) = ones(1, len_t)` for the second digit

`rec(1, 2*len_t +1 : 3*len_t) = ones(1, len_t)` for the third digit

`rec(1, 3*len_t +1 : 4*len_t) = ones(1, len_t)` for the fourth digit

`rec(1, 4*len_t +1 : 5*len_t) = ones(1, len_t)` for the fifth digit

This is a way of changing the time interval by shifting every 0.25 seconds. By using the "ones()" function, the corresponding interval is set to 1, while the remaining array is 0. The len_t corresponds to the length of the omega array (in which the element number is 10240) divided by 5, since there are five digits to be sampled in total. Every five interval is plotted separately to determine the other digits.

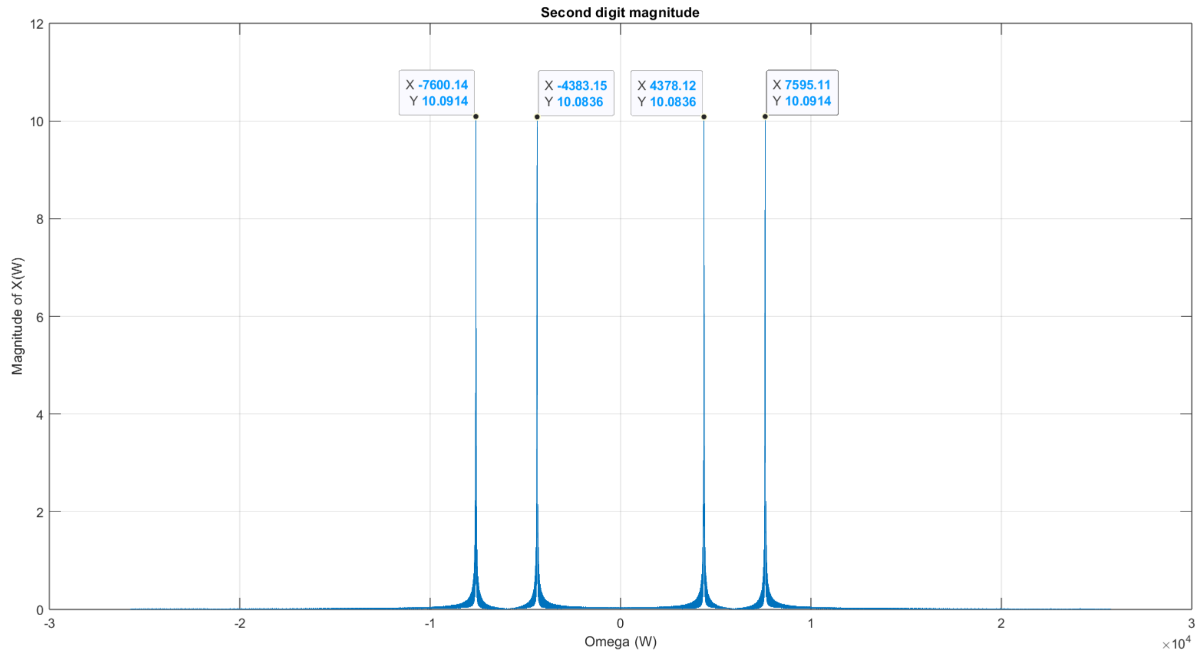


Figure 5: Plot for the second digit

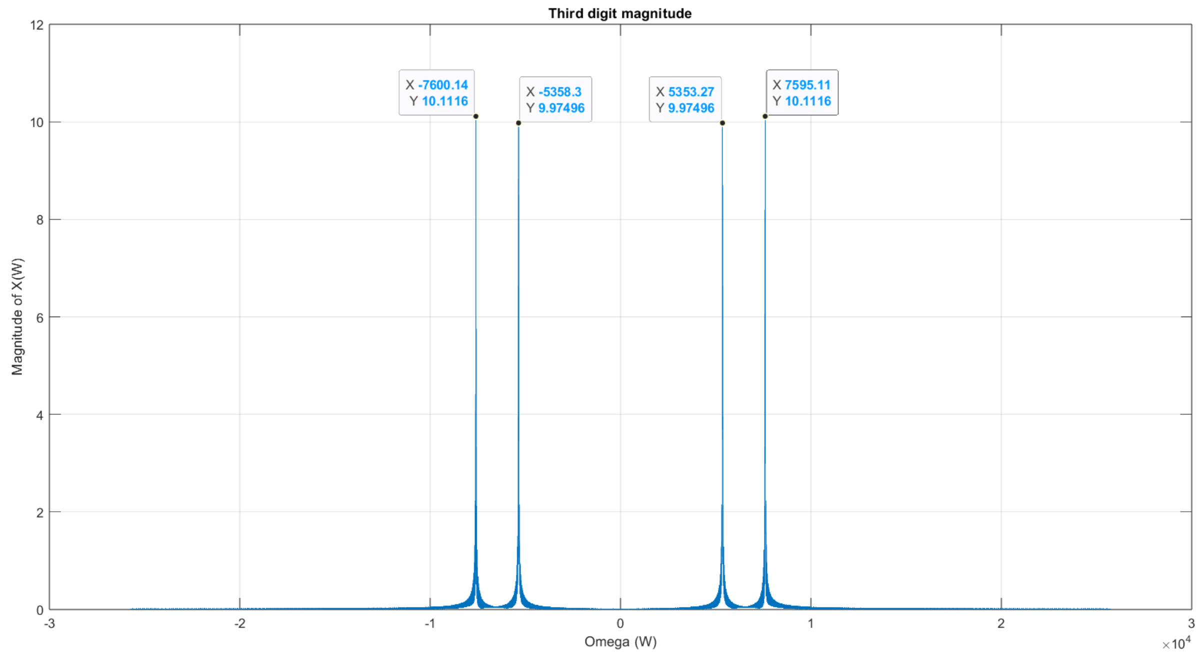


Figure 6: Plot for the third digit

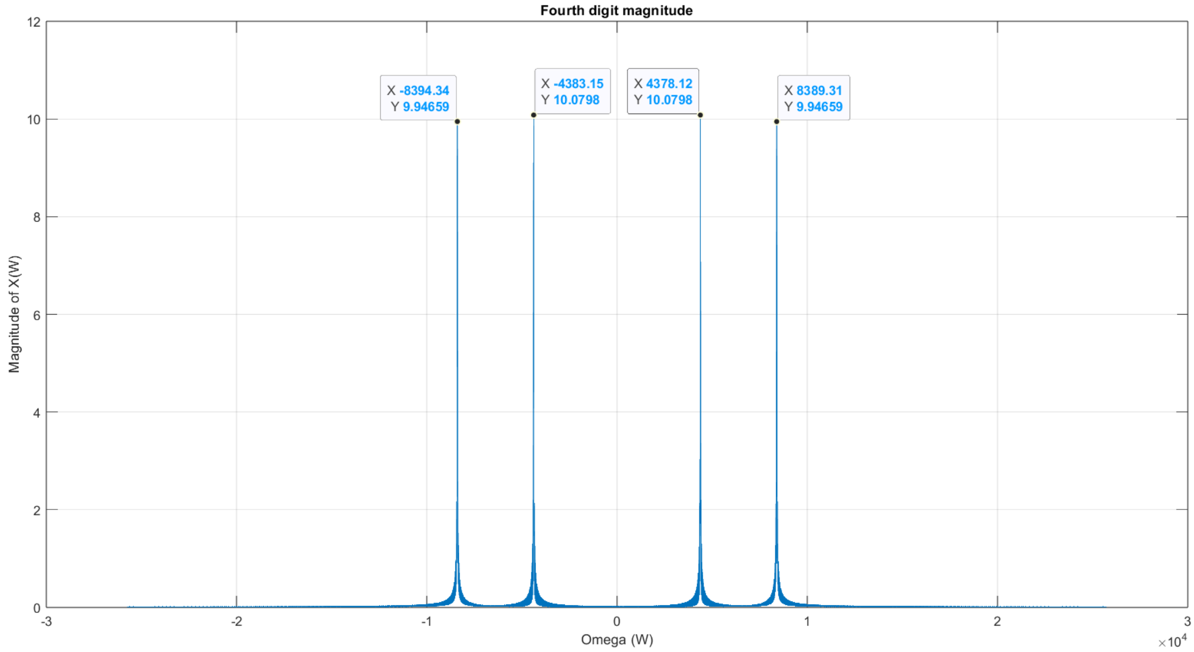


Figure 7: Plot for the fourth digit

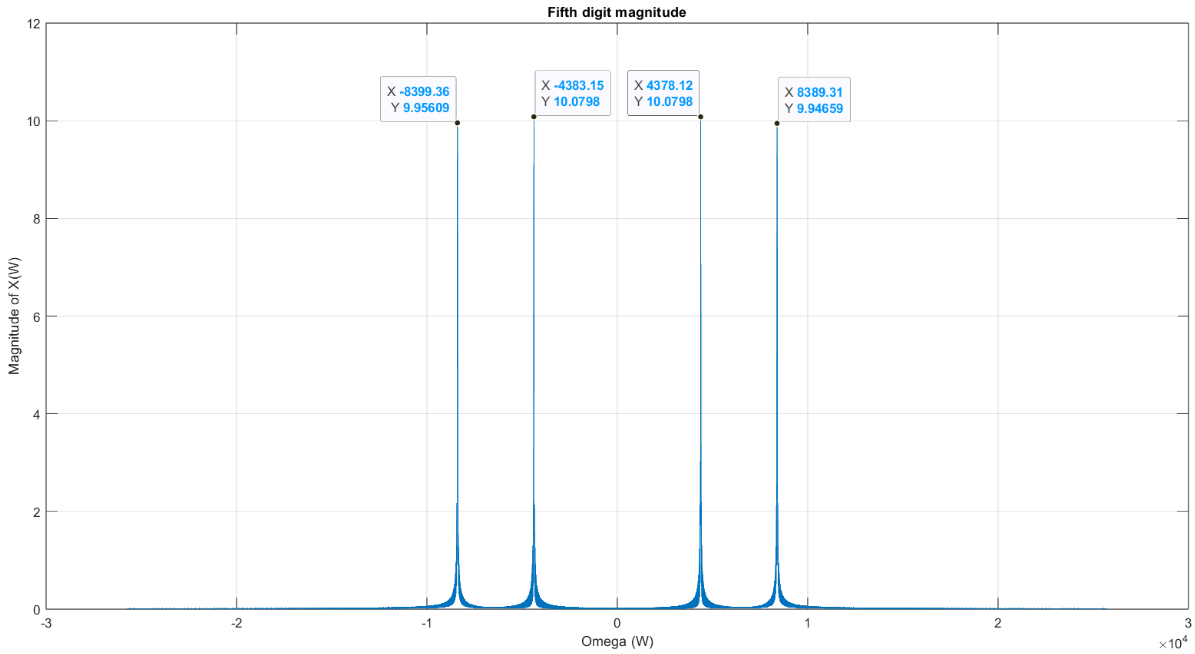


Figure 8: Plot for the fifth digit

All the digit values can be found and the peak frequencies match with the angular frequencies in Table 1.

- For the first digit, the peak frequencies are very close to 5353 and 8394, corresponding to 8.
- For the second digit, the peak frequencies are very close to 4379 and 7596, corresponding to 1.
- For the third digit, the peak frequencies are very close to 5353 and 7596, corresponding to 7.
- For the fourth and fifth digits, the peak frequencies are very close to 4379 and 8394, corresponding to 2.

It should be noted that the frequencies don't match exactly since the multiplication with 2π in Table 1 is approximated. MATLAB also doesn't have infinite precision required for 2π, hence it also approximates. This results in differences, although minimal.

As mentioned earlier, the reason why checking all digits at the same time is not enough to determine the digits is because the digits share common frequencies. For example, 7596 on the plot may correspond to 1, 4 or 7. However, when the digits are checked separately, there are only two peak frequencies. This is enough to determine the digits since the (fr, fc) of all digits are distinct, while fr and fc separately belong to multiple digits at the same time.

Part 2

I recorded my speech from my phone, then I opened the voice recording in MATLAB using the "audioread()" function. This function returns x as an array of samples and fs as the sampling rate when used with the syntax below. I cropped x in case its length exceeded 8192*12, since the recording is supposed to be 12 seconds. Also, to test the recording, I also played x with the "soundsc()" function. The recording played successfully, and my speech was heard clearly.

My speech was: "I am Yiğit Narter and my number is 22102718. I am doing my signals and systems lab right now." The code used:

```
sample_rate = 8192;
[x, fs] = audioread("part2record.wav");
x = x(1:sample_rate*12)'; %crop the recording
len_x = length(x);

soundsc(x);
```

$$y(t) = x(t) + \sum_{i=1}^{M} A_i\, x(t-t_i)$$

a) $h(t)$ is the impulse response of the system, and $y(t) = h(t) * x(t)$

So put $x(t) = \delta(t) \Rightarrow y(t) = h(t) \Rightarrow$ impulse response (response to $\delta(t)$)

$$\Rightarrow \boxed{h(t) = \delta(t) + \sum_{i=1}^{M} A_i\, \delta(t-t_i)}$$

b) Fourier transform of $h(t)$: $H(\omega) = \int_{-\infty}^{\infty} \left[\delta(t) + \sum_{i=1}^{M} A_i\, \delta(t-t_i) \right] e^{-j\omega t}\, dt$

$$\Rightarrow H(\omega) = \underbrace{\int_{-\infty}^{\infty} \delta(t) e^{-j\omega t}\, dt}_{\substack{\text{non-zero when} \\ t=0 \text{ only}}} + \sum_{i=1}^{M} A_i \underbrace{\int_{-\infty}^{\infty} \delta(t-t_i) e^{-j\omega t}\, dt}_{\substack{\text{non-zero for} \\ t=t_i \text{ only}}}$$

$$= e^{-j\omega \cdot 0} \underbrace{\int_{-\infty}^{\infty} \delta(t)\, dt}_{1} + \sum_{i=1}^{M} A_i \cdot e^{-j\omega t_i} \underbrace{\int_{-\infty}^{\infty} \delta(t-t_i)\, dt}_{1}$$

$$\Rightarrow \boxed{H(\omega) = 1 + \sum_{i=1}^{M} A_i\, e^{-j\omega t_i}}$$

c) $x(t) = \delta(t) \Rightarrow X(\omega) = \int_{-\infty}^{\infty} \delta(t) e^{-j\omega t} dt = 1$ and $Y(\omega) = H(\omega)$

$\Rightarrow \boxed{Y(\omega) = H(\omega) \cdot X(\omega)}$ This can be shown by multiplication property of FT;

$$\left.\begin{aligned} a(t) &\leftrightarrow A(\omega) \\ b(t) &\leftrightarrow B(\omega) \end{aligned}\right\} \Rightarrow \underbrace{a(t) * b(t)}_{y(t)} \longleftrightarrow \underbrace{A(\omega)\, B(\omega)}_{Y(\omega)}$$

When the impulse response ($h(t)$) is known for LTI systems, its F.T $H(\omega)$ can be used to find $Y(\omega)$, by finding $X(\omega)$.
In previous step, $H(\omega)$ was found. Since $x(t) = \delta(t)$ for this case, $y(t) = h(t)$, meaning $Y(\omega) = H(\omega)$, and $X(\omega) = F(\delta(t)) = 1$. The relation between them is therefore a multiplication. For any $x(t)$, this can be generalized:

$y(t) = h(t) * x(t)$ for LTI systems:

$$\Rightarrow Y(\omega) = \int_{-\infty}^{\infty} y(t) e^{-j\omega t} dt = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x(\tau) h(t-\tau)\, d\tau\; e^{-j\omega t} dt = \int_{-\infty}^{\infty} x(\tau) \underbrace{\left(\int_{-\infty}^{\infty} h(t-\tau) e^{-j\omega t} dt \right)}_{\text{shifted by } \tau \;\leftarrow\; H(\omega) \cdot e^{-j\omega \tau}} d\tau$$

$$= H(\omega) \cdot \underbrace{\int_{-\infty}^{\infty} x(\tau) e^{-j\omega \tau} d\tau}_{X(\omega)} = H(\omega) \cdot X(\omega).$$

d) From part c, $\boxed{X(\omega) = \dfrac{Y(\omega)}{H(\omega)}}$

For our case, since $y(t) = h(t) \Rightarrow Y(\omega) = H(\omega) \Rightarrow X(\omega) = 1$

$\Rightarrow$ find $x(t)$ whose F.T is 1 ($X(\omega) = 1$)

$\Rightarrow x(t) = \delta(t)$, check: $X(\omega) = \int_{-\infty}^{\infty} \delta(t) e^{-j\omega t} dt = \underbrace{e^{-j\omega \cdot 0}}_{1} \underbrace{\int_{-\infty}^{\infty} \delta(t)\, dt}_{1} = \underline{\underline{1}}$

The code used to generate y from x:

```
t=0:1/8192:12-1/8192;

Ai = [0.65 0.50 0.30 0.22 0.15 0.1];
ti = [0.25 0.75 1 1.25 2 3.25];

x_1 = [zeros(1, sample_rate*ti(1)) Ai(1)*x(1: len_x - sample_rate*ti(1))];
x_2 = [zeros(1, sample_rate*ti(2)) Ai(2)*x(1: len_x - sample_rate*ti(2))];
x_3 = [zeros(1, sample_rate*ti(3)) Ai(3)*x(1: len_x - sample_rate*ti(3))];
x_4 = [zeros(1, sample_rate*ti(4)) Ai(4)*x(1: len_x - sample_rate*ti(4))];
x_5 = [zeros(1, sample_rate*ti(5)) Ai(5)*x(1: len_x - sample_rate*ti(5))];
x_6 = [zeros(1, sample_rate*ti(6)) Ai(6)*x(1: len_x - sample_rate*ti(6))];

x_1 = x_1(1:sample_rate*12); %crop the delayed signals
x_2 = x_2(1:sample_rate*12);
x_3 = x_3(1:sample_rate*12);
x_4 = x_4(1:sample_rate*12);
x_5 = x_5(1:sample_rate*12);
x_6 = x_6(1:sample_rate*12);

y = x + x_1 + x_2 + x_3 + x_4 + x_5 + x_6;
```

The plots are given in Figures 9-10.

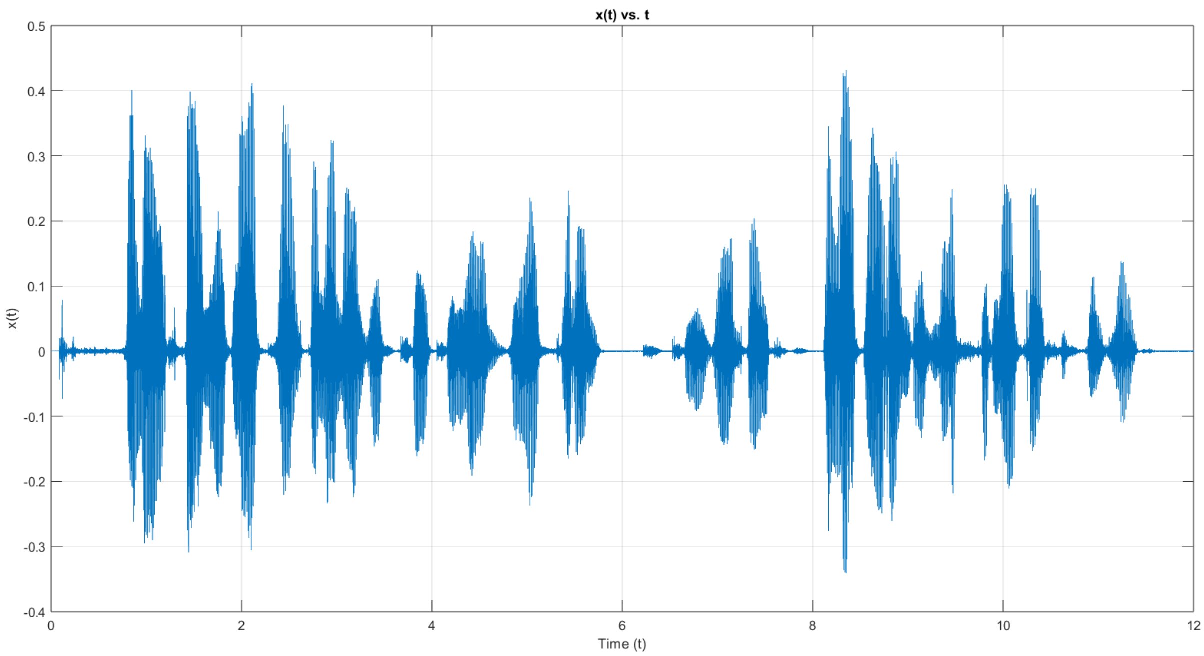


Figure 9: Plot of x(t) vs. t

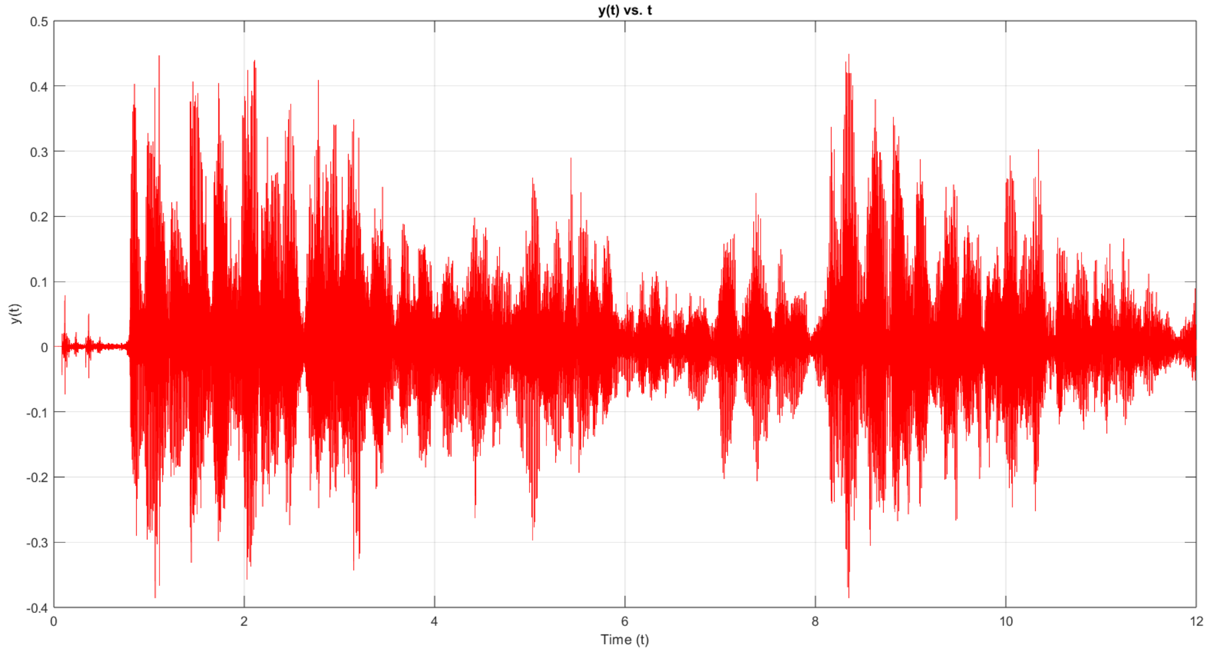


Figure 10: Plot of y(t) versus t

When I listened to y using the soundsc() function, the recording had echoes and the words were hardly comprehensible. Hence, y is actually an echo-added version of x. The sound resembled the echoes in caves, where the sound is reflected back.

Now, to generate the non-echo version of y, the impulse response will be used. The previously computed equation (item d) states that the Fourier transform of x(t) can be computed using the Fourier transforms of y(t) and h(t). Since I found H(ω) in part b, in MATLAB H(ω) can be computed as:

```
Y=FT(y);

omega=linspace(-8192*pi,8192*pi,98305);
omega=omega(1:98304);

H = ones(1, length(omega));
for i=1:6
    H = H+Ai(i)*exp(-1j * omega * ti(i));
end
h = IFT(H);

X_e = Y./H;
x_e = IFT(X_e);

soundsc(x_e);
```

Lastly, the estimated version of x(t), named $x_e(t)$, is computed. The relation found in item d was used to compute it. The plots for h(t) vs. t, |H(ω)| vs. ω, and $x_e(t)$ vs. t are given in Figures 11-13.

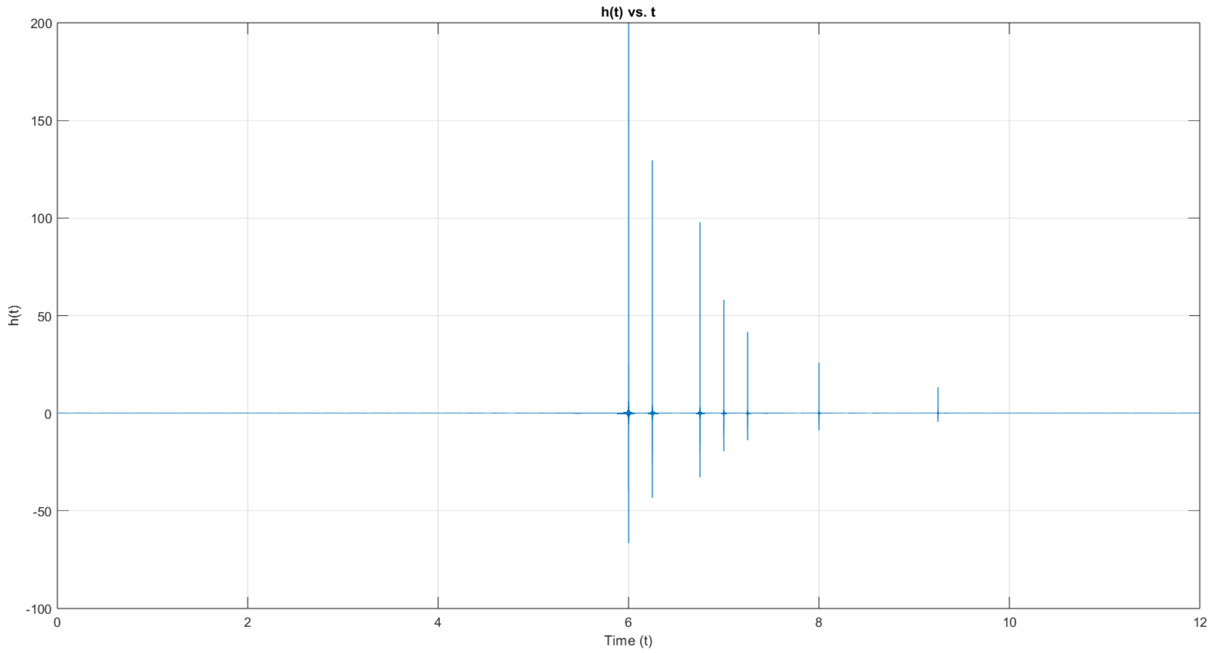


Figure 11: Plot for h(t) vs. t

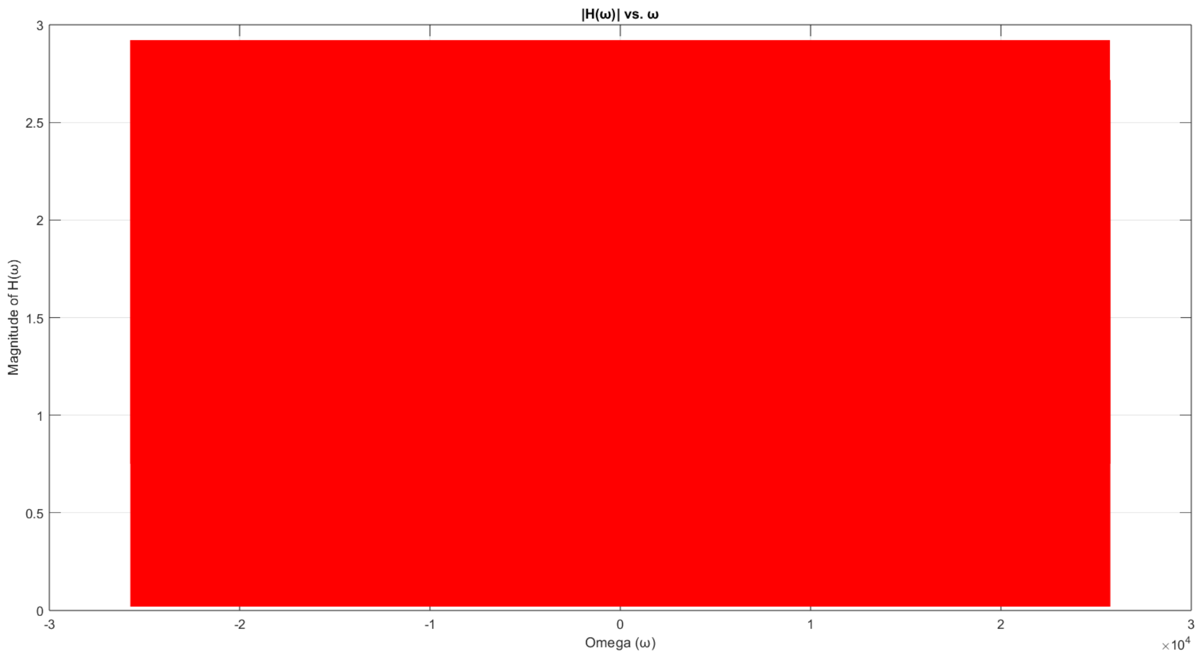


Figure 12: Plot for |H(ω)| vs. ω

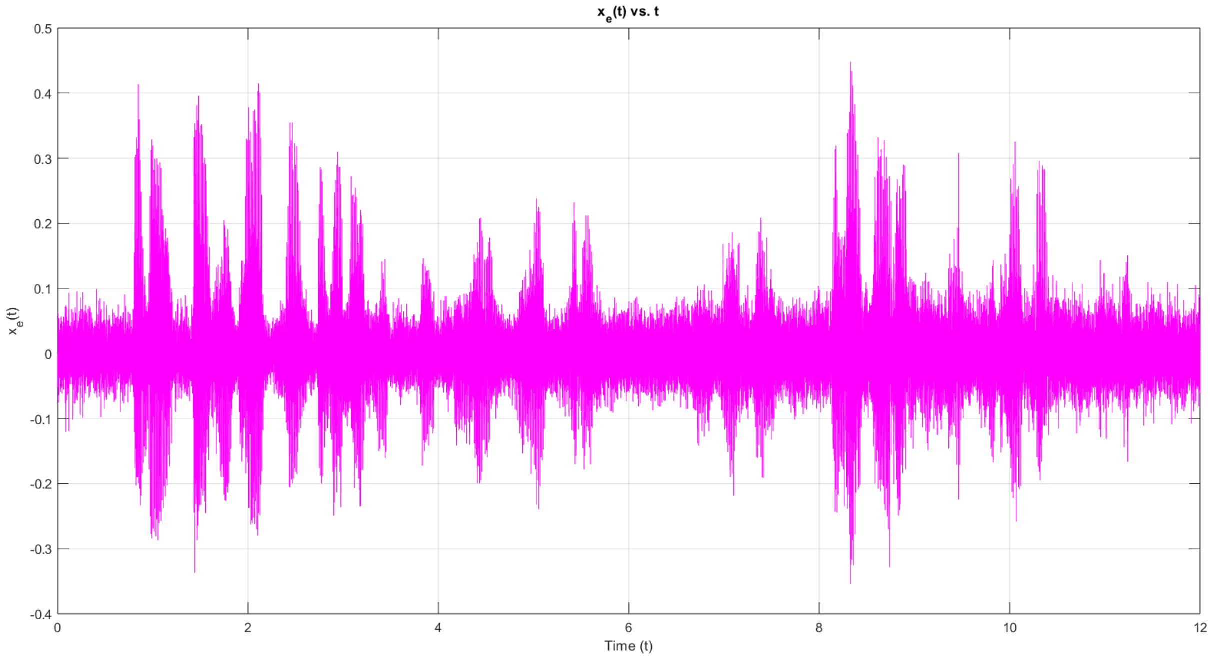


Figure 13: Plot for $x_e(t)$ vs. t

When I listened to the estimated speech x_e, the echoes were all gone and the speech was heard clearly, just like in the original recording x. However, there was some noise in the background. This may have occurred because of the background noise as well as the sound system and the recording setup of my computer. When the plots of x(t) and $x_e(t)$ are also compared, the plots are nearly identical with the only difference being the noise added to the parts where I wasn't speaking (in these parts x(t) values are nearly zero while $x_e(t)$ values are larger).

The voice recordings are presented below. They are saved as .wav files using the "audiowrite()" function in MATLAB.

First speech (x):

https://drive.google.com/file/d/1Iry2j0J0M9pVi53Pb3ub2nfYnnUKbTw7/view?usp=sharing

Speech with echo (y):

https://drive.google.com/file/d/1ESUl6-rZOhklX9dorq2bJsrtVIxou2jb/view?usp=sharing

Estimated speech (x_e):

https://drive.google.com/file/d/17RJlzQLPi7-JqZMEWLwgtW43r_wU9ead/view?usp=sharing