

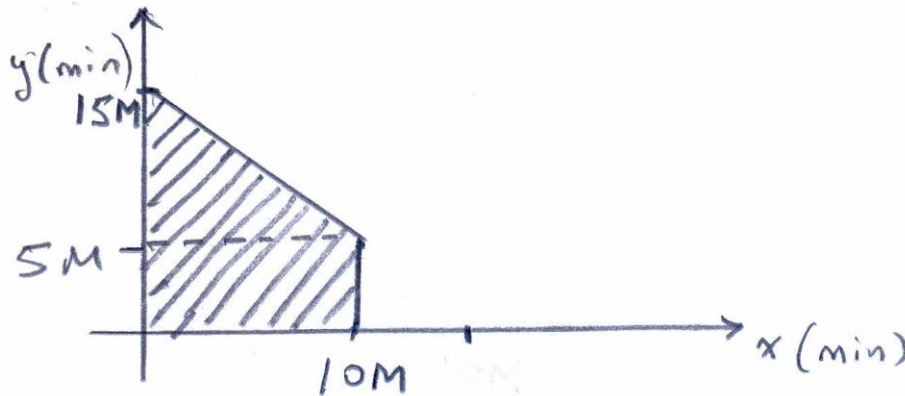
## MATH 255 Homework 3

(Clearly justify all answers)

(Due 14 November 2023)

Write your Bilkent student number, which is an eight-digit number. Label the digits as  $x_0, x_1 \dots x_7$ , where  $x_0$  represents the least significant digit, and  $x_7$  represent the most significant digit. Compute  $M = [(x_4 + x_3 + x_2) \bmod 6] + 3$ . Example: for student number 21003141,  $M = [(0 + 3 + 1) \bmod 6] + 3 = 7$ .

Q1- Let  $X$  and  $Y$  be two random variables. The joint pdf of these two random variables is uniformly distributed over the shaded area given in the graph below.



a) Find the marginal probability density functions for  $X$  and  $Y$ .

b) Find the conditional probability density functions  $f_{X|Y=45}(x)$  and  $f_{Y|X=45}(y)$ .

c) Find the expected values  $E\{X\}$ ,  $E\{Y\}$ ,  $E\{X|Y = 45\}$ .

d) Find the probabilities i)  $X < 40$ , ii)  $Y \geq 40$ , iii)  $X + Y < 40$ , iv)  $|X - Y| \leq 5$

e) Are  $X$  and  $Y$  independent?

f) Let  $X$  and  $Y$  represent the arrival times (in minutes, from a given instant) of trains to city A, and B, respectively. What is the probability that the train to A arrives its destination earlier than train to B arrives its destination? What is the probability that the difference of the arrival times of the trains is less than 15 minutes?

} All numbers  
are in  
minutes.