MATH255 Homework 2
(Clearly justiyfy all answers.)

(Due 22 Oct 2023. Upload the solutions to MOODLE.)

Write your Bilkent student number. Take its three least signifcant digits as a three digit number; call it N. Compute $M = [(N^3 - 2) \mod 5] + 2$. Example: for student number 21003141, $M = [(141^3 - 2) \mod 5] + 2 = 6$.

Q1- A traffic police directs incoming traffic from a highway into two outgoing routes. The police sends a vehicle to Route 1 with probability $1/M$, and therefore, to Route 2 with probability $(1 - 1/M)$. The police makes the decision for each vehicle, independently. Both routes go to the same destination, lets say A. If a vehicle enters Route 1, it takes either 2 hours to reach A, with probability 0.7; or 4 hours. If, however, a vehicle enters Route 2, it takes either 1 hour to reach A, with probability 0.4; or 2 hours. The distance from the police to the destination A is 100km via Route 1 and 70km via Route 2.

a) An observer at the destination A finds out that it took 2 hours for a vehicle to arrive. Find the probability that this vehicle came from Route 1. Also find the probability that this vehicle came from Route 2.

b) If it is known that it took a vehicle 2 hours to arrive, what is the probability that the travel time of the next vehicle is also 2 hours?

c) What is the expected value of the travel time for a vehicle?

d) Bob and Alice are at point A, and they play a game by betting on the arriving vehicles: If it takes 2 hours for a vehicle to arrive, Bob pays Alice 10TL; otherwise, Alice pays Bob 40TL. What is the expected gain, per bet, for Bob? Do you recommend him to play this game, if his goal is to make (and not to lose) money? In other words, is this a "good" bet?

Q2- There are M females and M males in an animal population at instant t_0, forming M adult couples. At time t_1, where $t_1 > t_0$,

- Each such formed couple may have one baby with probability p_1 or no baby at all, independent of other couples.

- Each adult (either male of female) may die with probability p_2 independent of all other adults.

- To have a baby and to die are also independent events.

Find the expected number of babies, expected number of adults, expected number of female adults, expected number of male adults and the expected total number of animals in this population at instant t_1.

Q3- Cars arrive to a parking lot in a Poisson distributed manner with an average arrival rate of 10 cars/hour.

i- What is the probability that exactly 5 cars arrive between 10:00 and 10:30? ii- What is the probability that no cars arrive between 11:00 and 13:00? iii- Given that the number of cars arrived between 15:00 and 16:00 is in the interval $[3, 6]$, find the probability that 3 cars arrived in that same interval and the expected number of cars.