Bilkent University **Spring 2017-2018**

Math255 Probability and Statistics
Midterm 1
5 March 2017

Name and Lastname:

Bilkent ID No:

Math 255 Section No:

Score (for instructor use)

(P1)	(P2)	(P3)	(P4)	(P5)	Sum

Instructions

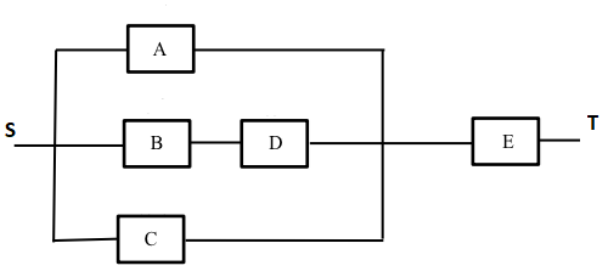
- The exam is 120 minutes. There are five problems and each problem is six points. This is a closed book exam. You may use one one-sided A4-size sheet of notes.
- Before the exam starts:
  - Make sure you are taking the exam in the classroom your are assigned to. Your exam may not be graded if you are in a wrong room.
  - All electronic devices must be switched off and stored away.
  - Remove all objects from your desk, including bags, coats, books, calculators, cell phones, wallets. The only items allowed on your desk are one cheat-sheet, writing utensils, and one bottle of water. No snacks no food!
  - Once the exam starts, you will not be allowed to leave the room and come back for any reason. But you may leave any time if you finish the exam early.
- During the exam:
  - Place your ID visibly on your desk and keep it that way throughout the exam.
  - Keep your exam sheets in front of you at all times. Exposing your answer sheets to others shall be treated as cheating.
  - Write legibly. If your answer cannot be read you will receive no credit.
  - In each problem, show your work in the space provided for that problem and write your final answer in the designated box.
  - Double check all your results and pay attention to the following! You will receive no credit if
    - there is not sufficient justification for a correct answer,
    - the answer does not appear in the designated box,
    - numerical answers are not given as a real number of a simple fraction of integers,
    - the answer is wrong for one reason or another, including trivial mistakes.

Problem 1. [6 pts] Consider the set all distinct 10-letter words that can be created by rearranging the letters of the word KAUNAKAKAI. Suppose a word is chosen at random from this set, what is the probability (call it p) that no K in the word is adjacent to another K? Show your reasoning. (Compute the result as a real number or simplified fraction of two integers.)

$p =$	**Do not write in this space.**

Problem 2. [6 pts]

An electrical system (shown on the right) consists of 5 components, each of which is operational with probability $p = \frac{1}{2}$. Assume that the states of the components (whether operational or not) are jointly independent. The system is operational (call this event F) if there is a path from S to T such that all components on the path are operational. Find the probability $P(F)$ that the system is operational. Show your work. Give the result as a real number or a simple fraction.



$P(F) =$	**Do not write in this space.**

Problem 3. [6 pts] Suppose there are three coins that look identical, except the first coin has heads on both faces, the second has tails on both faces, and the third is a regular coin with heads on one face and tails on the other. Assume that each coin is fair in the sense that when flipped each face is equally likely to come up. The coins are put in a bag, a coin is picked at random without revealing the identity of the coin, the coin is flipped and allowed to land on the ground. Given that the up-face of the coin on the ground is heads (call this event B), what is the conditional probability that the coin on the ground is the third coin (call this event A)? (Write your answer as a real number or a simple fraction.)

$P(A\|B) =$	**Do not write in this space.**

Problem 4. [6 pts] A (six-faced) fair die is rolled twice. Let X and Y be the minimum and the maximum of the two rolls, respectively. Compute $\mathbf{E}[X + Y|Y = 3]$ and $\text{var}(X + Y)$. The answer must be simplified to numbers.

$\mathbf{E}[X + Y\|Y = 3] =$	$\text{var}(X + Y) =$	**Do not write in this space.**

Problem 5. [6 pts] Consider an experiment that consists of tossing a fair four-faced die until all four faces appear at least once. Let X denote the duration of the experiment and compute $\mathbf{E}[X]$. Show your reasoning. Compute your answer as a real number or a simple fraction of integers. Do not leave any uncomputed sums.

$\mathbf{E}[X] =$	**Do not write in this space.**

Solution of Problem 1 **only**.

Solution of Problem 2 **only**.

Solution of Problem 3 **only**.

Solution of Problem 4 **only**.

Solution of Problem 5 **only**.

Extra space. Your work on this page will not be evaluated unless you indicate otherwise.