Math 255, Probability and Statistics
Spring 2019-2020, Midterm 2
16 April 2020

10 points per question. Show all your work. Answers must be numeric.

P1. Let X, Y, Z be independent identically-distributed (IID) random variables, each uniformly distributed on $\{1, 2, 3, 4, 5, 6\}$. Compute $P(A)$ and $P(A|B)$ where $A = \{X < Y < Z\}$ and $B = \{X < Z\}$.

P2. Let X and Y be independent random variables with f_X and f_Y uniform on $[0, 1]$. Let $Z = 2X + Y$. Compute $f_Z(1)$ and $\text{var}(Z)$.

P3. Let X, Y be conditionally independent given Q with

$$f_{X,Y,Q}(x, y, q) = f_{X|Q}(x|q) f_{Y|Q}(y|q) f_Q(q)$$

where $f_{X|Q}(x|q) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(x-q)^2}{2}}$, $f_{Y|Q}(y|q) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(y-q)^2}{2}}$, and $f_Q(q) = 3q^2$ for $0 \le q \le 1$. Let $Z = X + Y$. Compute $\mathbf{E}[Z]$ and $\text{var}(Z)$.