MATH255 Homework 1 Part A
(Clearly justiyfy all answers.)

(Due 9 Oct 2023. Upload your solutions to MOODLE.)

Write your last name using capital letters. Associate the integer numbers 1 through 29 to each letter in your last name according to the ordering of letters in the Turkish alphabet: $A = 1, \cdots, Z = 29$. For those letters in the English alphabet that are not in Turkish, use the following numbers: $Q = 30$, $W = 31$; associate English capital letter I, with Turkish I,ı. Add the numbers corresponding to the letters in your last name; call the result N. (Example: for BAL $N = 2{+}1{+}15 = 18$, for WILLIAM $N = 31{+}11{+}15{+}15{+}11{+}1{+}16 = 100$) Find $M = \big[(N) \bmod 5\big] + 1$. (Example: for BAL, $M = 4$, for WILLIAM, $M = 1$)

Q1- A fair six-sided die is rolled twice, independently. Let us denote the results of the two rolls as (i, j), where i and j are the results of the first and second rolls, respectively; $i, j \in \{1, 2, 3, 4, 5, 6\}$.

a) Find the probability of the event A which is defined as $i = M + 1$.

b) Find the probability of the event B which is defined as $i + j < 2M - 1$.

c) Find the probability of the event C which is defined as $M - 2 < j \leq M + 1$

d) Find the probability of the joint event $A \cap B$

e) Find the probability of the joint event $A \cap C^c$

f) Are A and B independent?

g) Are A and C independent?

h) Are B and C independent?

i) Are A, B, C independent?

Q2- There are nine balls; each ball has a single color; a number is written on each ball. There are three distinct colors, denoted by letters R, G and B. The balls are identified with their colors and numbers, and the elements of the set of these balls are: R1, R2, R3, G2, G3, G4, B5, B6, B7. A ball is drawn randomly (fair).

a) Find the probability of the event "the number on the drawn ball is larger than $M + 1$".

b) Find the probability of the event "the number on the drawn ball is larger than $M + 1$, given that the ball has the color B".

c) Find the probability of the event "the ball has color R, given that the number on the ball is larger than M".