MATH255 Homework 5
(Clearly justiyfy all answers.)

(Due 14 Dec 2023. Upload the solutions to MOODLE.)

1- An FIR filter is discribed by $y[n] = 0.5x[n] + x[n-1] + 0.5x[n-2]$. The input $x[n]$ is independent identically distributed random variables which are uniformly distributed in $[0, 1]$.

a) Find $\text{Cov}(y[n], x[n])$ and $\text{Cov}(y[n], y[n-1])$.

b) Find $\text{Var}(x[n])$ and $\text{Var}(y[n])$.

c) Find the linear least mean square estimator $\hat{y}[n+1]$ given $y[n]$: $\hat{y}[n+1] = ay[n] + b$. (Find a and b.) (Hint: Are the results of (a) and (b) depend on n?

2-a) Go to www.random.org and download an array (size=2000) of i.i.d. random numbers uniformly distributed in $[0, 1]$. Write a computer program which takes this array as an input to the given FIR filter, and find its output. Then implement your estimator found in (1-c) by writing a computer program and generate the array $\hat{y}[n]$. Also compute the error array $e[n] = y[n] - \hat{y}[n]$.

b) Plot three graphs: i) $x[n]$, ii) $y[n]$ and $\hat{y}[n]$ on top of each other using different colors, iii) $e[n]$. For all graphs the range of n is $[256, 511]$. Include you computer code as an appendix to your report.

3- Repeat questions 1 and 2, by replacing your FIR filter by $y[n] = -0.5x[n] + x[n-1] - 0.5x[n-2]$.

4- Comment on the results: i) Is $x[n+1]$ predictable from $x[n]$? ii) Why $y[n+1]$ is predictable from $y[n]$. iii) Describe the nature of correlation between two consecuitive elements of the output array for the given two different FIR filters.