MATH 255 Final Exam 21 December 2023

150 min. CLEARLY JUSTIFY ALL ANSWERS

1-) There are two boxes: Box 1 and Box 2. Box 1 has 112 balls in it; 42 of them are blue and 70 of them are red. Box 2 has 58 balls in it; 40 of them are blue and 18 of them are red. First, a box is selected: selecting Box 1 has probability $2/3$. Then a ball is randomly selected (selecting each ball has the same probability). The selected ball is then put back into its box.

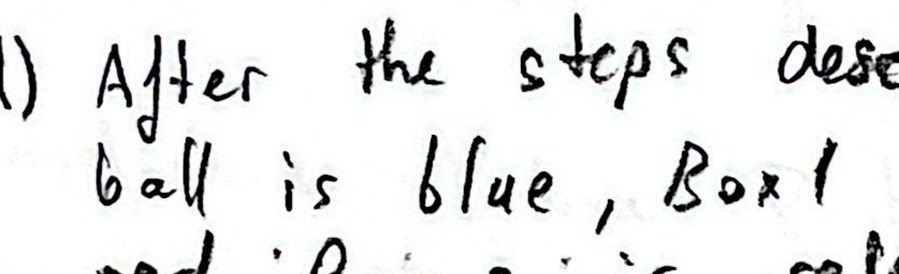
5pts a) Find the probability of getting a red ball

5pts b) Find the probability that a selected red ball comes from Box 1.

5pts c) A person plays a game based on the description given above: If the selected ball is blue he wins 100TL; otherwise he loses 200 TL. Find the expected winning per game.

5pts d) After the steps described above, if the resultant ball is blue, Box 1 is selected, if the ball is red Box 2 is selected, and then a ball is drawn randomly. Find the probability that the final ball is blue.

2-) A random variable X has the pdf:



2pts a) Find a

5pts b) Find $P\{-2 < X \leq 1\}$

7pts c) Find $P\{-3 < X < 0.5 \mid X < 0\}$

3-) A student takes a course until he/she passes. At k^{th} attempt, the probability of success is $p(k)$. The $p(k)$ are given as:

$$p(k) = \begin{cases} 0.6 & \text{if } k=1 \\ 0.7 & \text{if } k=2 \\ 0.8 & \text{if } k=3 \\ (0.8)^{k-2} & \text{if } k \geq 4 \end{cases}$$

a) Let $\underset{\sim}{K}$ denote the random variable which indicates the number of attempts for an eventual success.

5pts i) Find the pmf $p_{\underset{\sim}{K}}(k)$

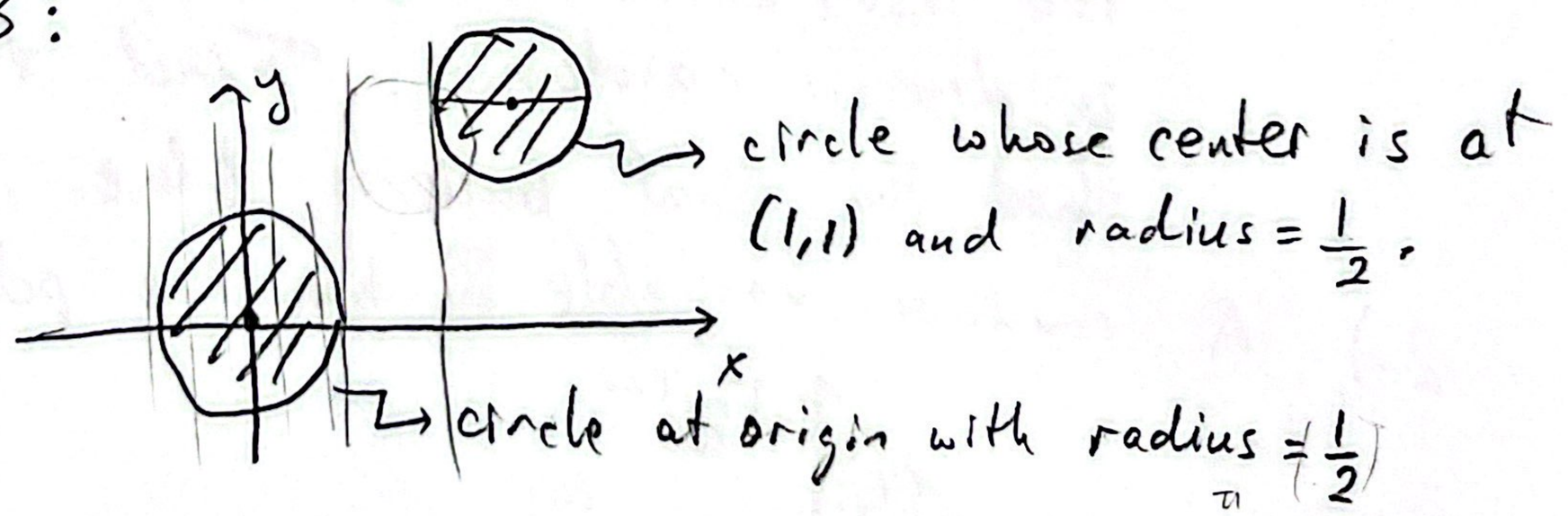
7pts ii) Find $E\{\underset{\sim}{K}\}$

4-) There is a biased six-faced die.

5pts a) Design an experiment to find out the unknown probabilities of each face.

5pts b) Convince me that your experiment gives "good" results.

5-) The joint pdf of $\underset{\sim}{X}$ and Y is uniform in shaded area B:



10pts a) Find $f_{\underset{\sim}{X}}(x)$.

4pts b) Find $f_{\underset{\sim}{X}|Y=1}(x)$

4pts c) Are $\underset{\sim}{X}$ and Y independent? Are they correlated?

6-) Let $\underline{X}$ be uniformly distributed over $[6, 12]$. A random error obscures $\underline{X}$: We observe $Y = \underline{X} + N$ where N is random error uniformly distributed over $[-1/2, 1/2]$. $\underline{X}$ and N are independent.

8pts a) Find and plot the joint pdf of $\underline{X}$ and Y.

4pts b) Find $E\{\underline{X} | Y=y\}$; plot the result.

4pts c) Find an estimate $\hat{\underline{X}}$ of $\underline{X}$, given $Y=y$, such that $E\{(X - \hat{\underline{X}})^2\}$ is minimized. Plot $\hat{\underline{X}}$ as a function of y.