Bilkent University
Spring 2007-08
Math 250 Probability
Midterm I - 18/3/2008

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100 minutes. Three problems. 30 points. Closed book. No calculators. Switch off cell phones. One A4-size one-sided page of notes allowed (no photocopies). You may receive no credit on correct answers if not fully justified. Write your name, student no and **Math 250 section number** on the first sheet. Write your name on all pages. Number the pages. Good Luck.

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1. [10 pts] Let A be the event that a given person has Chagas' disease, a human tropical parasitic disease which occurs in the Americas. Let B be the event that the ELISA test indicates the presence of the Chagas' disease. Suppose the following data are given $P(A) = 0.1$ (the probability that an individual chosen at random from the population has the disease). Let $P(B|A) = 0.80$ (the test detects the disease), $P(B|A^c) = 0.01$ (false detection). Suppose that when the test result is positive (indicating disease), the doctors order a second test to reduce the risk of false alarm. Thus, the possible test results for an individual are B_1B_2 (indicating two positives), $B_1B_2^c$ (first test positive second test negative), and B_1^c (first test negative). Assume that B_1 and B_2 are conditionally independent given A.

(a) [3] Sketch a tree diagram that describes all possible scenarios in such a test, with the first level branching indicating whether A or A^c occurs and later branchings covering all possible test sequences that may follow. Mark each branch of the tree in the usual way with the corresponding conditional probability and each node with the corresponding event.

(b) [3] Compute the probabilities $P(B_1)$ and $P(B_1B_2)$.

(c) [4] Compute the probabilities $P(A|B_1)$, $P(A|B_1B_2)$, and $P(A|B_1^c)$. State what these probabilities mean.

2. [10 pts] Let (X, Y) be a pair of random variables with a joint p.m.f. as given in the figure below.

Y values					
4			1/10	1/10	
3		3/10			
2		1/10			1/10
1	2/10			1/10	
	1	2	3	4	5
			X values		

(a) [2] Compute the marginal p.m.f.'s $p_X(x)$ and $p_Y(y)$ for all possible values of x and y.

(b) [2] Compute $E[XY]$.

(c) [3] Compute the conditional p.m.f $p_{X|Y}(x|2)$ for each x and conditional expectation $E[X|Y = 2]$.

(d) [3] Let $Z = X + Y$. Determine the p.m.f. $p_Z(z)$ for all possible z.

3. [10 pts] There are 10 persons in a room. Assume that their birthdays are independent random variables drawn from the uniform distribution over the days of the calendar. To simplify calculations, assume that we are using a 100-day calendar (instead of 365 days). Let Y denote the number of pairs of persons in the room who have the same birthday. (If A and B have the same birthday, count them only once.)

(a) [4] Compute $E[Y]$. Show your reasoning.

(b) [6] Compute $\mathrm{Var}(Y)$. Show your reasoning. (This part is trickier than it may appear.)