CSE 015: Discrete Mathematics
Fall 2020
Homework #6
Solution

Tony Doan
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1. **Question 1: Recursively defined functions**
   f(0) = 3 for n = 1, 2, 3, ...

   (a) f(n + 1) = -2f(n)
   f(1) = f(0 + 1) = -2f(0) = -2 * 3 = -6
   f(2) = f(1 + 1) = -2f(1) = -2 * -6 = 12
   f(3) = f(2 + 1) = -2f(2) = -2 * 12 = -24
   f(4) = f(3 + 1) = -2f(3) = -2 * -24 = 48
   f(5) = f(4 + 1) = -2f(4) = -2 * 42 = -96

   (b) f(n + 1) = 3f(n) + 7
   f(1) = f(0 + 1) = 3f(0) + 7 = (3 * 3) + 7 = 16
   f(2) = f(1 + 1) = 3f(1) + 7 = (3 * 16) + 7 = 55
   f(3) = f(2 + 1) = 3f(2) + 7 = (3 * 55) + 7 = 172
   f(4) = f(3 + 1) = 3f(3) + 7 = (3 * 172) + 7 = 523
   f(5) = f(4 + 1) = 3f(4) + 7 = (3 * 523) + 7 = 1576

   (c) f(n + 1) = $f(n)^2$ - 2f(n) - 2
   f(1) = f(0 + 1) = $f(0)^2$ - 2f(0) - 2 = 3^2 - (2 * 3) - 2 = 9 - 6 - 2 = 1
   f(2) = f(1 + 1) = $f(1)^2$ - 2f(1) - 2 = 1^2 - (2 * 1) - 2 = 1 - 2 - 2 = -3
   f(3) = f(2 + 1) = $f(2)^2$ - 2f(2) - 2 = -3^2 - (2 * -3) - 2 = 9 - (-6) - 2 = 13
   f(4) = f(3 + 1) = $f(3)^2$ - 2f(3) - 2 = 13^2 - (2 * 13) - 2 = 169 - 26 - 2 = 141
   f(5) = f(4 + 1) = $f(4)^2$ - 2f(4) - 2 = 141^2 - (2 * 141) - 2 = 19881 - 282 - 2 = 19597

   (d) f(n + 1) = $3^{(\frac{f(n)}{3})}$
   f(1) = f(0 + 1) = $3^{(\frac{f(0)}{3})} = 3^{(\frac{3}{3})} = 3^{(1)} = 3$
   f(2) = f(1 + 1) = $3^{(\frac{f(1)}{3})} = 3^{(\frac{3}{3})} = 3^{(1)} = 3$
   Since f(1) = 3 and f(2) = 3, plugging in 3 for f(3), f(4), f(5) will also equal to 3 since they use the same equation recursively...
   f(3) = 3
   f(4) = 3
   f(5) = 3

2. **Question 2: Recursively Defined Sequences**

   (a) a_n = 4n - 2

   a_0 = 4(0) - 2 = -2
   a_1 = 4(1) - 2 = 2
   a_2 = 4(2) - 2 = 6
   a_3 = 4(3) - 2 = 10

   Basis:
   a_0 = 4(0) - 2 = -2

   Induction:
   Assume n = k... a_k = 4k - 2
   Let n = k + 1... a_{k+1} = 4(k + 1) - 2
   a_{k+1} = 4(k + 1) - 2
   a_{k+1} = 4k + 4 - 2
   a_{k+1} = 4k + 2, True

   (b) a_n = 1 + $(-1)^n$

   a_0 = 1 + $(-1)^0$ = 1 + 1 = 2
   a_1 = 1 + $(-1)^1$ = 1 + (-1) = 0
   a_2 = 1 + $(-1)^2$ = 1 + 1 = 2
   a_3 = 1 + $(-1)^3$ = 1 + (-1) = 0

   Basis:
   a_0 = 1 + $(-1)^0$ = 1 + 1 = 2

   Induction:
   Assume n = k... a_k = 1 + $(-1)^k$
   Let n = k + 1... a_{k+1} = 1 + $(-1)^{k+1}$
   a_{k+1} = 1 + $(-1)^{k+1}$
   a_{k+1} = 1 + $(-1)^{k+1}$, True

   (c) a_n = n(n - 1)

   a_0 = 0(0 - 1) = 0 * -1 = 0
   a_1 = 1(1 - 1) = 1 * 0 = 0
   a_2 = 2(2 - 1) = 2 * 1 = 2
   a_3 = 3(3 - 1) = 3 * 2 = 6

   Basis:
   a_0 = 0(0 - 1) = 0 * -1 = 0

   Induction:
   Assume n = k... a_k = k(k - 1)
   Let n = k + 1... a_{k+1} = (k + 1) ((k + 1) - 1)

a_{k+1} = (k + 1) ((k + 1) - 1)
a_{k+1} = (k + 1) (k)
a_{k+1} = k^2 + k, True

(d) $a_n = n^2$

$a_0 = 0^2 = 0$
$a_1 = 1^2 = 1$
$a_2 = 2^2 = 4$
$a_3 = 3^2 = 9$

Basis:
$a_0 = 0^2 = 0$

Induction:
Assume n = k... $a_k = k^2$
Let n = K + 1... $a_{k+1} = (k+1)^2$
$a_{k+1} = (k+1)^2$
a_{k+1} = k^2 + 2k + 1, True

3. **Question 3: Recursively Defined Sets**
Σ = 0, 1
S = {(0, 1), (00, 11), (000, 111), (0000, 1111), (00000, 11111), ... }
In set S for whatever number of "0" there is, it combined with the equal amount of "1".