CSE 015: Discrete Mathematics
Fall 2020
Homework #5
Solution

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1. **Question 1: Mathematical Induction 1**

   P(n) = $1^3 + 2^3 + 3^3 + ... + n^3 = (\frac{n(n+1)}{2})^2$

   (a) **P(1):**
   $1^3 = (\frac{1(1+1)}{2})^2$

   (b) $1^3 = (\frac{1(1+1)}{2})^2$
   $1 = (\frac{1(2)}{2})^2$
   $1 = (\frac{1\cancel{(2)}}{\cancel{2}})^2$
   $1 = 1^2$
   1 = 1, True

   (c) Assume n = k is true.
   P(k):
   $k^3 = (\frac{k(k+1)}{2})^2$

   (d) Add $(k + 1)^3$ to both sides
   P(k + 1):
   $k^3 + (k + 1)^3 = (\frac{k(k+1)}{2})^2 + (k + 1)^3$
   $k^3 + (k + 1)^3 = (\frac{k^2(k+1)^2}{4}) + (k + 1)^3$
   $k^3 + (k + 1)^3 = (\frac{k^2(k+1)^2}{4}) + (\frac{4(k+1)^3}{4})$
   $k^3 + (k + 1)^3 = (\frac{k^2(k+1)^2+4(k+1)^3}{4})$
   $k^3 + (k + 1)^3 = (\frac{(k+1)^2(k^2+4(k+1))}{4})$
   $k^3 + (k + 1)^3 = (\frac{(k+1)^2(k^2+4k+4)}{4})$
   $k^3 + (k + 1)^3 = (\frac{(k+1)^2(k+2)^2}{4})$
   $k^3 + (k + 1)^3 = (\frac{(k+1)(k+2)}{2})^2$, True

2. **Question 2: Mathematical Induction 2**

   (a) P(1) = 0 = 0
   P(2) = 0 + 2 = 2
   P(3) = 0 + 2 + 4 = 6
   P(4) = 0 + 2 + 4 + 6 = 12

   formula: n^2 - n
   P(1) = 1^2 - 1 = 0, P(1) = 0
   P(2) = 2^2 - 2 = 2, P(2) = 2
   P(3) = 3^2 - 3 = 6, P(3) = 6
   P(4) = 4^2 - 4 = 12, P(4) = 12

   (b) P(n) = n^2 - n
   P(1) = 1^2 - 1 = 0, True

   Induction: Let n = k be true:
   P(k) = k^2 - k

   Use P(k + 1) and add (k + 1) to both sides:
   P(k + 1) = $(k+1)^2$ - (k + 1)
   P(k + 1) = (k^2 + 2k + 1) + (-k - 1)
   P(k + 1) = (k^2 + k), True

3. **Question 3: Mathematical Induction 3**

   P(n) = n! < n^n

   (a) The statement P(2) states that two factorial is less than two squared.

   (b) P(2) = 2! < 2^2
   P(2) = (2 * 1) < 4
   P(2) = 2 < 4, True

   (c) Induction:
   Let n = k be true:
   P(k) = k! < k^k

   Use P(k + 1):
   P(k + 1) = (k + 1)! < $(k+1)^{(k+1)}$, True