# CSE 015: Discrete Mathematics
# Fall 2020
# Homework #4
# Solution

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1. **Question 1: Set Operations**

   (a) A $\cup$ B
   UCM students are either registered in CSE015 or live in Merced county.

   (b) A $\cap$ C
   UCM students are both registered in CSE015 and freshmen.

   (c) C $\setminus$ B
   UCM students who live in Merced county but who are not freshmen.

   (d) $\overline{A}$
   UCM students who are not registered in CSE015.

   (e) A $\cap$ B $\cap$ C
   UCM students who are registered in CSE015, live in Merced county, and are freshmen.

2. **Question 2: Cartesian Product**

   (a) C $\times$ A
   $\{$*(”True”, 1), (”True”, 2), (”True”, 3), (”True”, 4), (”False”, 1), (”False”, 2), (”False”, 3), (”False”, 4)*$\}$

   (b) B $\times$ B
   $\{$*(”a”, ”a”), (”a”, ”b”), (”a”, ”c”), (”b”, ”a”), (”b”, ”b”), (”b”, ”c”), (”c”, ”a”), (”c”, ”b”), (”c”, ”c”)*$\}$

   (c) B $\times$ A $\times$ C
   $\{$*(”a”, 1, ”True”), (”a”, 1, ”False”), (”a”, 2, ”True”), (”a”, 2, ”False”), (”a”, 3, ”True”), (”a”, 3, ”False”), (”a”, 4, ”True”), (”a”, 4, ”False”), (”b”, 1, ”True”), (”b”, 1, ”True”), (”b”, 2, ”True”), (”b”, 2, ”False”), (”b”, 3, ”True”), (”b”, 3, ”False”), (”b”, 4, ”True”), (”b”, 4, ”False”), (”c”, 1, ”True”), (”c”, 1, ”True”), (”c”, 2, ”True”), (”c”, 2, ”False”), (”c”, 3, ”True”), (”c”, 3, ”False”), (”c”, 4, ”True”), (”c”, 4, ”False”)*$\}$

3. **Question 3: Composite Cartesian Products**
   A $\times$ (B $\cup$ C) $\equiv$ (A $\times$ B) $\cup$ (A $\times$ C)
   <u>Proof:</u>

$= A \times (B \cup C) = \{(x, y) \mid x \in A \text{ and } y \in B \cup C\}$
$= \{(x, y) \mid ((x, y) \in A \times B) \cup ((x, y) \in A \times C)\}$
$= (A \times B) \cup (A \times C)$

True, because if $x \in A$ and $y \in B \cup C$, there will be (x, y) for either or both $A \times B$ and $A \times C$.

4. **Question 4: Relations**

(a) $R_1 = \{(a ,b), (a, c), (a, a), (b, a), (c, a)\}$

| **1** | **2** |
|---|---|
| a | b |
| a | c |
| a | a |
| b | a |
| c | a |

**Reflexive:**
*(a, a) but no (b, b), (c, c), or (d, d)*

**Symmetric:**
*(a, b), (b, a)*
*(a, c), (c, a)*
*(a, a)*

**Anti-Symmetric:**
*(a, b), (b, a)*
*(a, c), (c, a)*
*(a, a)*
*If (a, b) were to be swapped to (b, a), it would cause there to be two pairs (b, a), so it can't be anti-symmetric.*

**Transitive:**
*(a, b) to (b, a) to (a, a)*
*(b, a) to (a, b) but no (b, b)*
*(c, a) to (a, c)*

It would be symmetric.

(b) $R_2 = \{(a, b), (b, b), (b, c), (c, c), (a, c)\}$

| **1** | **2** |
|---|---|
| a | b |
| b | b |
| b | c |
| c | c |
| a | c |

**Reflexive:**
*There is a (b, b), (c, c) but not (a, a) or (d, d).*

**Symmetric:**
*(a, b) but no (b, a)*
*(b, c) but no (c, b)*
*(a, c) but no (c, a)*

**Anti-Symmetric:**
*The above explanation for symmetric shows that it can be anti-symmetric because if we flip the elements, there are no pairs that match.*

**Transitive:**
*(a, b) to (b, b) to (a, b)*
*(a, b) to (b, c) to (a, c)*
*(b, c) to (c, c) to (b, c)*

It would be both transitive and anti-symmetric.

(c) $R_3 = \{(a,\ b),\ (d,\ c),\ (c,\ a),\ (c,\ d),\ (a,\ b)\}$

| **1** | **2** |
|---|---|
| a | b |
| d | c |
| c | a |
| c | d |
| a | b |

**Reflexive:**
*No (a, a), (b, b), (c, c), (d, d)*
*There are no reflexive pairs.*

**Symmetric:**
*(a, b) but no (b, a)*
*(d, c) and (c, d)*
*(c, a) but no (a, c)*
*(a, b) repeats*

**Anti-Symmetric:**
*(a, b) repeats*

**Transitive:**
*(a, b) but no other elements with b*
*(d, c) to (c, d)*
*(c, a) to (a, b) but no (b, c)*

It would be none of the above because it doesn't fit the criteria.

(d) $R_4 = \{(a,\ a),\ (b,\ b),\ (c,\ c)\}$

| 1 | 2 |
|---|---|
| a | a |
| b | b |
| c | c |

**Reflexive:**
*There is (a, a), (b, b), (c, c) but no (d, d)*

**Symmetric:**
*All the elements could be swapped and still be the same with no repeats.*

**Anti-Symmetric:**
*All the elements could be swapped and still be the same with no repeats.*

**Transitive:**
*None of them equal each other and transitive can not have an element point to itself, like (a, a).*

It would be both symmetric and anti-symmetric.

5. **Question 5: Functions**

(a) $f(m, n) = 2m - n$
Surjective, because you set m to 0 and n to any integer, you get (0, n). If you plug in any integer for n, you will get an output no matter what you choose.

(b) $f(m, n) = m^2 - n^2$
Not surjective, because it is a perfect square so (m, n) can not equal 2. It can only equal to 1 or greater than 2.
$f(1, 0) = 1^2 - 0^2 = 1 - 0 = 1$
$f(2, 0) = 2^2 - 0^2 = 4 - 0 = 4$

(c) $f(m, n) = |m| - |n|$
Surjective, because any integer that is plugged in will have an output of all integers.

(d) $f(m, n) = m^2 - 4$
Not surjective, because you can not get all the integers. The lowest integer that you can get is -4.
$f(0, n) = 0^2 - 4 = 0 - 4 = -4$
$f(-1, n) = -1^2 - 4 = 1 - 4 = -3$
$f(1, n) = 1^2 - 4 = 1 - 4 = -3$