CSE 015: Discrete Mathematics
Fall 2020
Homework #02
Solution

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1. **Question 1:**

   (a) $P(2),\ x < x^3$
   $(2 < 2^3) = (2 < 8)$
   True, because 2 is less than 8.

   (b) $P(-1),\ x < x^3$
   $(-1 < -1^3) = (-1 < -1)$
   False, because -1 is not less than -1. They are equal.

   (c) $\forall x P(x),\ x < x^3$
   $P(0)$: $(0 < 0^3) = (0 < 0)$
   $P(-4)$: $(-4 < -4^3) = (-4 < -64)$
   False, because $P(x)$ where x is equal to 1 or less is false, so it can't all be true.

   (d) $\exists x P(x),\ x < x^3$
   $P(4)$: $(4 < 4^3) = (4 < 64)$
   True because $P(4)$ is one example of it being true.

   (e) $\exists! x P(x),\ x < x^3$
   $P(4)$: $(4 < 4^3) = (4 < 64)$
   $P(2)$: $(2 < 2^3) = (2 < 8)$
   False, because $P(x)$ where x is equal to 2 or greater is true, so there is more than one outcome where it will be true.

2. **Question 2:**

   (a) $\neg\forall x(S(m) \wedge M(x))$

   (b) $\forall x(S(m) \vee M(x))$

   (c) $\exists x(S(x) \wedge \neg M(x))$

3. **Question 3:**

   (a) $\forall x(A(x) \wedge B(x)) \equiv \forall(A(x) \rightarrow B(x))$

$\forall x(A(x) \wedge B(x))$:

A(x)	**B(x)**	**A(x) $\wedge$ B(x)**	**$\forall$x(A(x) $\wedge$ B(x))**
F	F	F	F
F	T	F	F
T	F	F	F
T	T	T	T

$\forall$x(A(x) $\wedge$ B(x)) is false

$\forall(A(x) \rightarrow B(x))$:

A(x)	**B(x)**	**A(x) $\rightarrow$ B(x)**	**$\forall$(A(x) $\rightarrow$ B(x))**
F	F	T	T
F	T	T	T
T	F	F	F
T	T	T	T

$\forall$(A(x) $\rightarrow$ B(x) is false

Both statements are logically equivalent because all the outcomes contain one or more false statements, therefore making them both false.

4. **Question 4:**

   (a) $\exists x \forall y A(x,y)$
   There exists a real number x for every real number y that xy = 0.
   (1)(0) = 0
   (1)(1) = 1
   False

   (b) $\exists x \exists y B(x,y)$
   There exists a real number x for a real number y that x + y = 0.
   -4 + 4 = 0
   True

   (c) $\forall x \exists y A(x,y)$
   For every real number x, there exists a real number y that xy = 0.
   (1)(0) = 0
   (2)(0) = 0
   True

   (d) $\exists x \forall y(A(x,y) \wedge B(x,y))$
   There exists a real number x for all real numbers y that xy = 0 and x + y = 0.
   (0)(1) = 0 and 0 + 1 = 1
   False

   (e) $\exists x \exists y(A(x,y) \wedge \neg B(x,y))$
   There exists a real number x for a real number y that xy = 0 and x + y $\neq$ 0.
   (4)(0) = 0 and 4 + 0 $\neq$ 0
   True

5. **Question 5:**

   (a) $\neg \exists x \exists y(P(x) \rightarrow Q(y))$
   $P(y) = \exists y(P(x) \rightarrow Q(y))$, *so* $\neg \exists x P(y)$
   $\neg \exists x P(y) \equiv \forall x \neg P(y)$
   $\neg P(y) = \neg \exists y(P(x) \rightarrow Q(y))$, *so* $\forall y \neg(P(x) \rightarrow Q(y))$
   Answer: $\forall x \forall y(\neg P(x) \rightarrow \neg Q(y))$

(b) $\neg\exists y(\exists x A(x,y) \vee \forall x B(x,y))$

$N(x) = \exists x A(x,y)$; $S(x) = \forall x B(x,y)$, so $\neg\exists y(N(x) \vee S(x))$

$\neg\exists y(N(x) \vee S(x) \equiv \forall y \neg(N(x) \wedge S(x))$

$\neg(N(x) \wedge S(x)) \equiv (\neg\exists x A(x,y) \wedge \neg\forall x B(x,y))$

$(\neg\exists x A(x,y) \wedge \neg\forall x B(x,y)) \equiv (\forall x \neg A(x,y) \wedge \exists x \neg B(x,y))$

Answer: $\forall y(\forall x \neg A(x,y) \wedge \exists x \neg B(x,y))$