

Q1)

a)

$$P(S=3 | D2) * P(D2) / P(S=3) = ?$$

$$\begin{aligned} P(S=3) &= P(S=3 | D1) * P(D1) + P(S=3 | D2) * P(D2) + P(S=3 | D3) * P(D3) \\ &= 1/6 + 2 * ((1/6)^2) * ((1/2)^2) + (1/6)^3 * (1/2)^3 \\ &= 169 / (12^3) \end{aligned}$$

$$P(S=3 | D2) * P(D2) = 2 * ((1/6)^2) * ((1/2)^2) = 1/72$$

$$\Rightarrow ? = 24 / 169$$

b)

$$P(S=5 | D2) * P(D2) + P(S=5 | D4) * P(D4) / P(S=5) = ?$$

$$\begin{aligned} P(S=5) &= P(S=5 | D1) * P(D1) + P(S=5 | D2) * P(D2) + P(S=5 | D3) * P(D3) + P(S=5 | D4) * P(D4) \\ &\quad + P(S=5 | D2) * P(D2) \end{aligned}$$

$$= 1/12 + (1/72) + (1/(72*6)) + (1/(72*72)) + (1/(72*72*96))$$

$$P(S=5 | D2) * P(D2) + P(S=5 | D4) * P(D4) = (1/72) + (1/(72*72))$$

$$\Rightarrow ? = 7008 / 49633$$

c)

$$P(S=5 | D2 | \text{Dice1} = 2) * P(D2) / P(S = 5) = ?$$

$$P(S = 5) = P(S=5 | D1) * P(D1) + P(S=5 | D2) * P(D2) + P(S=5 | D3) * P(D3) + P(S=5 | D4) * P(D4) \\ + P(S=5 | D2) * P(D2)$$

$$= 1/12 + (1/72) + (1/(72*6)) + (1/(72*72)) + (1/(72*72*96))$$

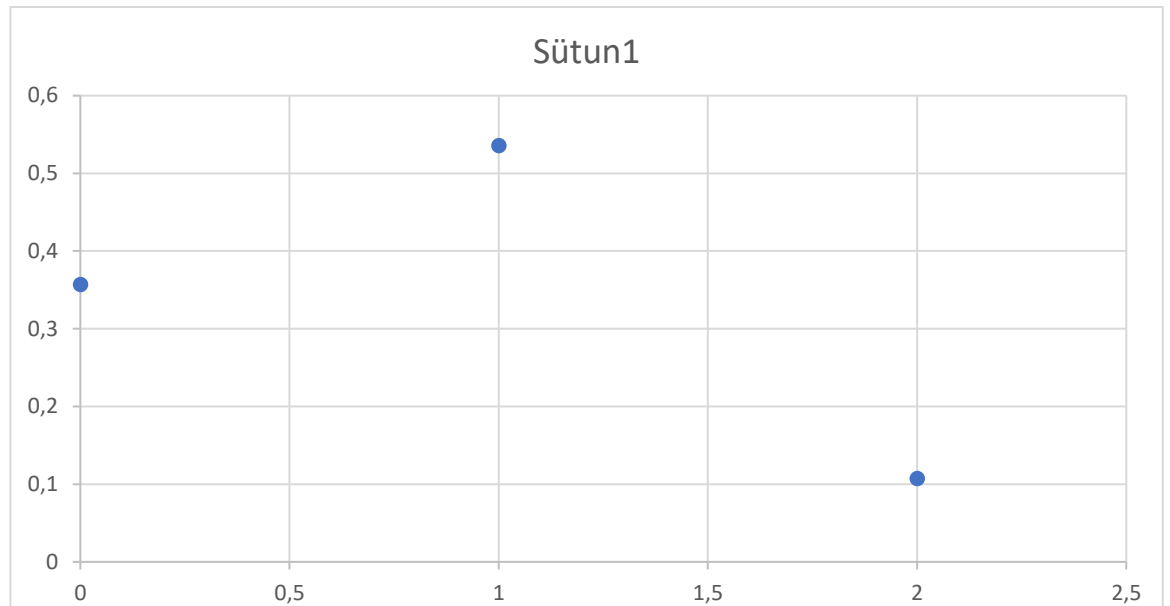
$$P(S=5 | D2 | \text{Dice1} = 2) * P(D2) = 1/144$$

$$\Rightarrow ? = 3456 / 49633$$

d)

Q2)

1)



2)

$$F(a) = f(0) + f(1) + \dots + f(a)$$

$$P(a < x < b) = F(b) - F(a)$$

a)

$$P(X = 0) = P(0 \leq x \leq 0) = F(0) - F(0) = 0$$

b)

$$P(0 < x < 2) = F(2) - F(0)$$

$$F(2) = 1, \quad F(0) = 20/56 \quad \Rightarrow \quad P = 36 / 56$$

Q3)

$$[-\infty, +\infty] \int f(x) dx = 1 \text{ and } [a, b] \int f(x) dx = P(a < x < b)$$

a)

$$[1, +\infty] \int f(x) dx = [-c*(x^4-4)/4] [1, +\infty] = 1$$

$$\Rightarrow c = 4$$

b)

$$F(x) = [1, +\infty] \int x*f(x) dx = [-c*(x^3-3)/3] [1, +\infty]$$

$$\Rightarrow F(x) = \begin{cases} 0, & x < 1 \\ -c/(3*x^3), & x \geq 1 \end{cases}$$

c)

$$P(x > 2) = F(+\infty) - F(2)$$

$$= 0 - (-c/24) = c/24$$

Q4)

a)

$$g(x) = [0,1] \int (x-y) dy = [xy - y^2/2] [0,1] \Rightarrow g(x) = x - 1/2$$

$$h(y) = [2,1] \int (x-y) dx = (x^2/2 - xy) [2,1] \Rightarrow h(y) = 3/2 - y$$

b)

$$f(x,y) =? g(x)*h(y) \Rightarrow x-y \neq (x - 1/2)*(3/2 - y)$$

$$\Rightarrow f(x,y) \neq g(x)*h(y) \Rightarrow \text{it is not independent !!!}$$

c)

$$f(x|y) = f(x,y) / h(y) = (x-y) / (3/2 - y)$$

$$P(x > 1.5 | y = 0.5) = \int_{1.5}^2 (x-y) / (3/2 - y) dx$$

$$= \int_{1.5}^2 (x-0.5) / (3/2 - 0.5) dx = [(x^2/2) - (x/2)]_{1.5}^2$$

$$\Rightarrow P = 5/8$$