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1-)

μ_X = E(X) and n = 45

$= \sum_x x . f(x); x = 2, 4, 6, 8$

$= 2\ x\ \frac{1}{4} +\ 4\ x\ \frac{1}{4} + 6\ x\ \frac{1}{4} + 8\ x\ \frac{1}{4}$

$= \frac{20}{4} = 5$

=>E(X) = 5

$E(X^2) = \sum_x x^2 . f(x); x = 2, 4, 6, 8$

$= 4\ x\ \frac{1}{4} +\ 16\ x\ \frac{1}{4} + 36\ x\ \frac{1}{4} + 64\ x\ \frac{1}{4}$

$= \frac{120}{4} = 30$

=>$E(X^2) = 30$

Variance of X = $\sigma^2_x = E(X^2) - [E(X)]^2$

$= 30 - 5^2 = 30 - 25$

$= \sigma^2_x = 5$

Variance of $\overline{X}$ = $\sigma^2_{\overline{x}}$

$= \frac{\sigma_x^2}{n}$

$=\frac{5}{45}$

$\sigma^2_{\bar{x}} = \frac{1}{9}$

Standard Deviation of $\bar{X} = \sigma_{\bar{x}} = \sqrt{\frac{1}{9}} = \frac{1}{3}$

$$Z_1 \frac{\bar{X} - \mu}{\sigma_{\bar{x}}} = \frac{5.1-5}{\frac{1}{3}} = 0.3$$

$$Z_2 \frac{\bar{X} - \mu}{\sigma_{\bar{x}}} = \frac{5.4-5}{\frac{1}{3}} = 1.2$$

$P[5.1 < \bar{X} < 5.4] = P[0.3 < Z < 1.2]$

$= P(Z < 1.2) - P(Z < 0.3)$

$= 0.88493 - 0.61791 = 0.26702$

2-)

Since p-value > α, H_0 is accepted.

The difference between the sample standard deviation (s) of the Group1 and Group2 populations is not big enough to be statistically significant.

The p-value equals 0.07571. ($p(x \leq F) = 0.9621$). It means that the chance of type error, rejecting a correct Ho is too high: 0.07571 (7.57%).

The larger the p-value the more it supports H_0

The test statistic F equals 5.6984, which is in the 95% region of acceptance:

[0.167:6.9777].

S1/S2=2.39, is in the 95% region of acceptance:
[0.4087: 2.6415].

The 95% confidence interval of σ_1^2/σ_2^2 is:

[0.8167, 34.1196].

3-)

Number of degrees of freedom are

$n_1 + n_2 - 2 = 15 + 17 - 2 = 30$

$t_c = t_{1-\alpha/2;\, n-1}$ and $\alpha = 0.01$ and number of degrees of freedom(ndf) = 30

$\Rightarrow t_c = 2.75$

$$s_p = \sqrt{\frac{(n_1 - 1)^2\, s_1^2 + (n_2 - 1)^2\, s_2^2}{\mathrm{n1} + \mathrm{n2} - 2}}$$

$$= \sqrt{\frac{(15-1)^2\, x\, (1.4)^2 + (17-1)^2\, x\, (1.7)^2}{15 + 17 - 2}}$$

$= 1.567$

$$\text{Standard error(se)} = s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$= 1.567 \text{ x } \sqrt{\frac{1}{15} + \frac{1}{17}}$$

$= 0.556$

Confidence Interval (CI) =

$(\overline{X}_1 - \overline{X}_2 - t_c \text{ x se},\ \overline{X}_1 - \overline{X}_2 + t_c \text{ x se})$

$(16 - 18 - 2.75 \text{ x } 0.556,\ 16 - 18 + 2.75 \text{ x } 0.556)$

$(-3.529, -0.471)$

$-3.529 < \mu_1 - \mu_2 < -0.471$

4-)

$\overline{X} - t_{(1-\alpha)/2} * \frac{s}{\sqrt{n}} < \mu < \overline{X} + t_{\alpha} * \frac{s}{\sqrt{n}}$

Number of Degrees of Freedom(ndf) = n – 1 = 19

$\alpha = 0.98$

$t_{0.01} = 2.5395$

Standard Error of the Mean (SEM) = $\frac{\sigma}{\sqrt{n}}$

$= \frac{8.3}{\sqrt{20}}$

= 1.8559

$\overline{X} - t_{(1-\alpha)/2} * \frac{s}{\sqrt{n}} < \mu < \overline{X} + t_{\alpha} * \frac{s}{\sqrt{n}}$

=> 83.4 – 2.5395 * 1.8559 < μ < 83.4 + 2.5395 * 1.8559

=>78.6868 < μ < 88.1132

5-)

a-)

$H_0 : \mu = 40$

$H_1 : \mu \neq 40$

$\overline{X} = 45.22$

$S^2 = 196 \Rightarrow s = 14$

n = 30

$\Rightarrow \frac{\left|\overline{X} - \mu\right| * \sqrt{n}}{s} = \frac{|45.22 - 40| * \sqrt{30}}{14}$

= 2.042

Number of Degrees of Freedom (ndf) = n – 1

= 29

From the t table we see that ndf = 29 and α = 0.01 for two tailed, the critical value is 2.756

2.042 < 2.756

=>We may accept the null hypothesis at 0.01 level of sig

=> We can conclude that the company is not wrong with its claim.

b-)

To reject H_0 , at α=0.01, the critical value(2.756) is smaller than test statistic value.

Thus,

$$\frac{|45.22-40| * \sqrt{n}}{14} > 2.756$$

$$=> \sqrt{n} > \frac{38.584}{5.22}$$

$$=> n > (\frac{38.584}{5.22})^2$$

=> n > 54.635 = 55(approximately)

=> n = 55 to reject the null hypothesis

$$\text{so } t = \frac{\left|\overline{X} - \mu\right| * \sqrt{n}}{s} = \frac{|45.22-40| * \sqrt{55}}{14} = 2.765 \text{ at ndf} = 29$$

new test statistic value = 2.765 is greater than 2.756 and much smaller than 3.08, so p value between 0.01 and 0.005, but 0.01 is much closer than 0.005.

Thus, 0.01 – 2 * (0.01 - 0.005) / 5 = 0.008

=> p-value can be obtained 0.008.

6-)

H_0 : Die is fair

H_a : Die is not fair

Face	1	2	3	4	5	6
Observed(O)	4	12	3	9	13	7
Expectation(E)	8	8	8	8	8	8
$(O-E)^2$ / E	2	2	3.125	0.125	3.125	0.125

=>$\chi^2 = \sum((\mathrm{O} - \mathrm{E})2 \,/\, \mathrm{E})$ = 10.5

Number of degrees of freedom (ndf) = 6 – 1 = 5

p-value = 0.0622

So, p-value = 0.0622 > 0.05

This is enough to fail to reject the null hypothesis, so the die is fair.