Instructor: Fatih Temiz

Name: Yunus Emre Topçu

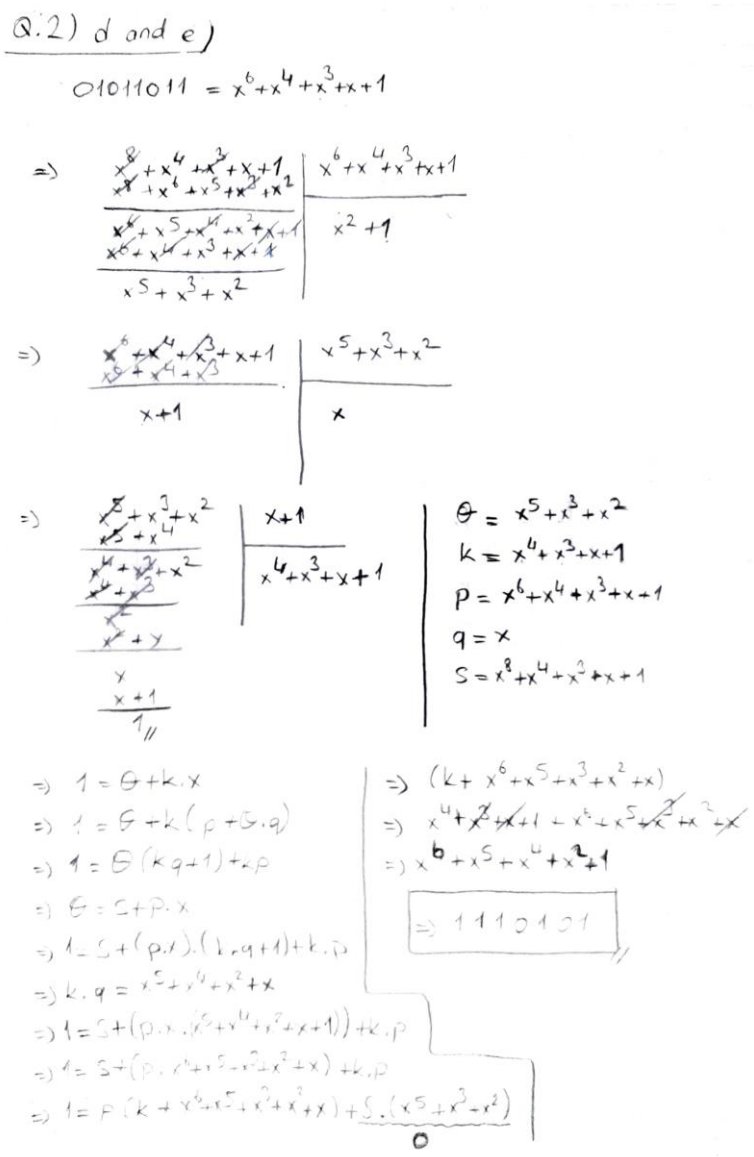
Student No: 180209801

Department: Software Engineering

ANSWERS

Q2)

a) 80209801 (mod2) = 00001001
b) Row number = 82 = 01010010
c) 00001001 (XOR) 01010010 = 01011011
d)
and
e)



Q3)

a)

- ⇨ p = 557
- ⇨ 557 is prime

for 558;

- ⇨ $2 * 3^2 * 31 = 558$
- ⇨ **A** = 558 / 2 = 279
- ⇨ **B** = 558 / 3 = 186
- ⇨ **C** = 558 / 31 = 18

```
import math
arr = []
i = 2
count = 0

while (i<557):
    if (pow(i,279)%557!=1):
        if (pow(i,186)%557!=1):
            if (pow(i,18)%557!=1):
                arr.append(i)
                count+=1
    i += 1

print(arr)
print("557 has", count, "primitive roots")
```

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[2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38,
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527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554,
555]
557 has 554 primitive roots
```

for 556;

- ⇨ $2^2 * 139 = 556$
- ⇨ **A** = 556 / 2 = 278
- ⇨ **B** = 556 / 139 = 4

```
import math
arr = []
i = 2
count = 0

while (i<557):
    if (pow(i,278)%557!=1):
        if (pow(i,4)%557!=1):
                arr.append(i)
                count+=1
    i += 1

print(arr)
print("557 has", count, "primitive roots")
```

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[2, 3, 5, 8, 11, 12, 13, 14, 18, 20, 21, 23, 27, 30, 31, 32, 34, 35, 37, 38, 41, 44, 45, 47, 48, 50, 51, 52, 53, 56,
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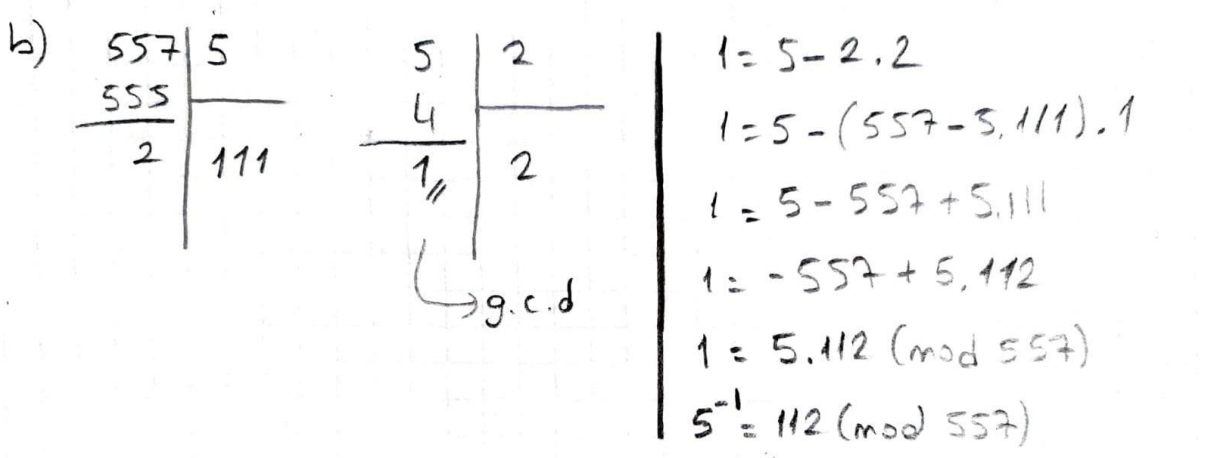
 543, 544, 545, 546, 549, 552, 554, 555]
557 has 276 primitive roots
```

b)



Q4)

a)

⇨ q = 571

⇨ 571 is prime

for 570;

⇨ 2 * 3 * 5 * 19 = 570

⇨ **A** = 570 / 2 = 285

⇨ **B** = 570 / 3 = 190

⇨ **C** = 570 / 5 = 114

⇨ **D** = 570 / 19 = 30

```
import math
arr = []
i = 2
count = 0

while (i<571):
    if (pow(i,285)%557!=1):
         if (pow(i,190)%557!=1):
            if (pow(i,114)%557!=1):
                   if (pow(i,30)%557!=1):
                        arr.append(i)
                        count+=1
    i += 1

print(arr)
print("570 has", count, "primitive roots")
```

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[2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32,
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63, 564, 565, 566, 567, 568, 569, 570]
570 has 567 primitive roots
```

for 572;

⇨ 2^2 * 11 * 13 = 572

⇨ **A** = 572 / 2 = 286

⇨ **B** = 572 / 11 = 52

⇨ **C** = 572 / 13 = 44

```python
import math
arr = []
i = 2
count = 0

while (i<571):
    if (pow(i,286)%557!=1):
        if (pow(i,52)%557!=1):
            if (pow(i,44)%557!=1):
                arr.append(i)
                count+=1
    i += 1

print(arr)
print("570 has", count, "primitive roots")
```

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[2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32,
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65, 566, 567, 568, 569, 570]
572 has 565 primitive roots
```

b)

Step 1 → Alice and Bob get public numbers q=571, G = 7

Step 2 → Alice selected private key a = 32 and Bob selected private key b = 64

Step 3 → Compute the values

- ⇨ Alice: $x = (7^{32} \bmod 571) = 377$
- ⇨ Bob: $y = (7^{64} \bmod 571) = 521$

Step 4 → Alice and Bob Exchange the numbers

Step 5 → Alice receives public key y = 521 and bob receives public key x=377

Step 5 → Compute symmetric keys

keyAlice = $(521^{32} \bmod 571)$ = 182

keyBob = $(377^{64} \bmod 571)$ = 182

Step 7 → 182 is the shared secret.

My code:

```
import math

x = int(input("number 1: "))
y = int(input("number 2: "))

sol = pow(x,y)
z = int(input("mod: "))

final = sol %  z

print(final)
```

Alice

```
number 1: 7
number 2: 32
mod: 571
377
```

keyAlice

```
number 1: 521
number 2: 32
mod: 571
182
```

Bob

```
number 1: 7
number 2: 64
mod: 571
521
```

keyBob

```
number 1: 377
number 2: 64
mod: 571
182
```

c) I used the code I gave in option b.

a = 7, q = 571

- ⇨ $a^{-1} = a^{q-2} \pmod q$
- ⇨ $7^{-1} = 7^{569} \pmod{571}$
- ⇨ 408

```
number 1: 7
number 2: 569
mod: 571
408
```