

# Computational Experiments of AE

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## 1 Letchford's test instances

**Number of facilities:** In his test instances, Letchford has  $n$  clients and  $m$  facilities and sets  $m = n$ .

**Assignment (transaction) costs** Each facility and customer location is set to a random point on the unit square, therefore setting each assignment cost to the euclidian distance between their locations on the unit square.

### 1.1 Description

There are four different test cases depending on how large the facility costs are and whether they are randomized or constant. Small and constant facility costs, medium and constant, large and constant, and entirely varied.

$m, n$  are the number of facilities and the right columns are the time both algorithms take in seconds (according to what Letchford reports in his paper).

#### Small facility costs

All facility (fixed) costs are set to  $\sqrt{n}/1000$  Letchford:

m=n	Reduction time 1 (s)	Eliminated	Reduction time 2 (s)	Eliminated
500	0.01	97	0.03	97
1000	0.03	99	0.52	99
2000	0.23	98	2.56	98

AE:

m=n	Time (s)	%Eliminated
500	0.128	29
1000	0.445	12
2000	1.016	0.6

**Lemma 1.1.** *The reason why AE's reduction rates are so low is the following: since SPLP is sub-modular, in the first step of the reduction procedure, if*

$$\Pi(0, \dots, 1, \dots, 0) - \Pi(0, \dots, 0)$$

is less than zero, we could set the entry of 1 to 0 permanently. However, since in SPLP there is no capacity on how many clients a facility can serve, opening a new facility while all of the others are closed leads to a situation where every client is served by this new facility. Considering the assignment profits are random numbers between 0 and 1, the total gain from opening the new facility is on average greater than  $\frac{n}{2}$ , which becomes greater than  $\sqrt{n}/1000$  as  $n$  gets large. So, AE's infimum test cannot reduce well in SPLP.

## Medium facility costs

All facility costs are set to  $\sqrt{n}/100$

Letchford:

m=n	Reduction 1 (s)	Eliminated	Reduction 2 (s)	Eliminated
500	0.02	89	0.04	96
1000	0.42	76	0.67	96
2000	0.92	71	1.45	97

AE:

m=n	Time (s)	%Eliminated
500	0.097	2.400
1000	0.875	2.700
2000	1.767	0.450

## Large facility costs

All facility costs are set to  $\sqrt{n}/10$  AE:

m=n	Time (s)	Eliminated
500	0.036	56.200
1000	0.156	22.300
2000	0.787	1.9

## 2 Other test cases

### Quadratic cases

In the cases where the fixed cost is  $k\sqrt{n}$ , where  $k$  is a constant less than 0.0001, AE can reduce due to the initial supremum check.

The following case: the fixed costs are all constant and equal to  $\frac{\sqrt{n}}{100,000}$ . AE:

m=n	Time (s)	Eliminated
1000	0.170s	99
5000	10 s	92

### Linear-fixed cost cases

When the fixed cost is equal to  $\frac{n}{2}$ , we get the following AE:

m=n	Time (s)	Eliminated
4000	26 s	76

Fixed cost =  $0.96n$  gives even better results but is a very unrealistic case.

m=n	Time (s)	Eliminated
500	0.032 s	87
1000	0.098 s	91
2000	0.558 s	79
4000	2.188 s	79