I. APPENDIX

A. A simple example of ordered submodularity functions

Assuming that there are 2 POIs, and $k_1 = 1, k_2 = 2$, which means that POI 1 must be covered once, and POI 2 must be covered twice. There are two users u_1, u_2, assuming that the contribution values for POI are $v_{11} = 10, v_{12} = 5, v_{21} = 5, v_{22} = 10$. It can be known from formula (2) that if we calculate the total revenue according to the order of (u_1, u_2), it can be obtained that

$$\begin{aligned} V_1(u_1, u_2) &= \sum_{i=1}^{\min\{k_1, dim(u_1,u_2)\}} v_{i1} \\ &= v_{11} = 10 \end{aligned}$$

$$\begin{aligned} V_2(u_1, u_2) &= \sum_{i=1}^{\min\{k_2, dim(u_1,u_2)\}} v_{i2} \\ &= v_{12} + v_{22} = 5 + 10 = 15 \end{aligned}$$

Thus, $V(u_1, u_2) = V_1(u_1, u_2) + V_2(u_1, u_2) = 25$. If we calculate the total revenue according to the order of (u_2, u_1), we can obtain

$$\begin{aligned} V_1(u_2, u_1) &= \sum_{i=1}^{\min\{k_1, dim(u_2,u_1)\}} v_{i1} \\ &= v_{21} = 5 \end{aligned}$$

$$\begin{aligned} V_2(u_2, u_1) &= \sum_{i=1}^{\min\{k_2, dim(u_2,u_1)\}} v_{i2} \\ &= v_{22} + v_{12} = 10 + 5 = 15 \end{aligned}$$

Thus, $V(u_2, u_1) = V_1(u_2, u_1) + V_2(u_2, u_1) = 20$. The user's order effects the function value.

B. Property of RL-OMC

***Lemma** 1 ($V_m(\cdot)$ **is an ordered submodularity function**):*
Proof. According to definition 1, $V_m(\cdot)$ is an ordered submodularity function, which means that for any sequence A'_m, B'_m and any element i, j in the sequence A_m, we must prove

$$V_m(A'_m||i) - V_m(A'_m) \geq V_m(A'_m||i||B'_m) - V_m(A'_m||j||B'_m).$$

According to the definition of $V_m(\cdot)$ in formula (2)

$$\begin{aligned} & V_m(A'_m||i) - V_m(A'_m) \\ &= \sum_{n=1}^{\min\left\{k_m, dim\left(A'_m||i\right)\right\}} v_{nm} - \sum_{n=1}^{\min\left\{k_m, dim\left(A'_m\right)\right\}} v_{nm} \\ &= \begin{cases} v_{im}, & k_m > dim(A'_m) \\ 0, & otherwise \end{cases} \end{aligned}$$

$$\begin{aligned} & V_m(A'_m||i||B'_m) - V_m(A'_m||j||B'_m) \\ &= \sum_{n=1}^{\min\left\{k_m, dim\left(A'_m||i||B'_m\right)\right\}} v_{nm} - \sum_{n=1}^{\min\left\{k_m, dim\left(A'_m||j||B'_m\right)\right\}} v_{nm} \\ &= \begin{cases} v_{im} - v_{jm}, & k_m > dim(A'_m) \\ 0, & otherwise \end{cases} \end{aligned}$$

We can obtain

$$V_m(A'_m||i) - V_m(A'_m) \geq V_m(A'_m||i||B'_m) - V_m(A'_m||j||B'_m);$$

Thus, $V_m(\cdot)$ is an ordered submodularity function.

***Theorem** 1 ($V(\cdot)$ **is an ordered submodularity function**):*
Proof. According to definition 1, $V_m(\cdot)$ is an ordered submodularity function, which means that for any sequence A', B' and any element i, j in the sequence A, we must prove

$$V(A'||i) - V(A') \geq V(A'||i||B') - V(A'||j||B').$$

According to formula (3), we can obtain $V(A) = \sum_{m \in \mathcal{M}} V_m(A_m)$, and Lemma 1 proves that

$$V_m(A'_m||i) - V_m(A'_m) \geq V_m(A'_m||i||B'_m) - V_m(A'_m||j||B'_m).$$

After accumulating the above inequality, we can obtain

$$\begin{aligned} & \sum_{m \in \mathcal{M}} V_m(A'_m||i) - \sum_{m \in \mathcal{M}} V_m(A'_m) \\ & \geq \sum_{m \in \mathcal{M}} V_m(A'_m||i||B'_m) - \sum_{m \in \mathcal{M}} V_m(A'_m||j||B'_m). \end{aligned}$$

A_m is a subsequence of A; that is,

$$V(A'||i) - V(A') \geq V(A'||i||B') - V(A'||j||B')$$

Therefore, $V(\cdot)$ is an ordered submodular function.

***Lemma** 2 (**The RL-OMC is monotonicity**):*
Proof. Because users cannot lie about their marginal value, only untruthful bids and arrival times can be submitted.

1. If the algorithm is monotonic, it must be proven that if user i is accepted, then user i can still be accepted after reducing the bid $b'_i < b_i$.
According to algorithms 1 and 2, user i is accepted, which means:

$$q_1 = \sum_{j=1}^{d} w_{1j} s_i(j) \geq q_0 = \sum_{j=1}^{d} w_{0j} s_i(j)$$

According to definition 8 and formula (8), $b_i \in \mathcal{D}$, when user i is accepted, $w_{1b_i} = \min\{w_{0b_i}, w_{1b_i}\} \leq w_{0b_i}$; thus, if user i submits $b'_i < b_i$, we can obtain:

$$q'_1 = \sum_{j=1}^{d} w_{1j} s(j) - w_{1b_i}(b_i - b'_i)$$

and

$$q'_0 = \sum_{j=1}^{d} w_{0j} s(j) - w_{0b_i}(b_i - b'_i)$$

It can be seen

$$\begin{aligned} q'_1 - q'_0 &= \sum_{j=1}^{d} w_{1j} s(j) - \sum_{j=1}^{d} w_{0j} s(j) + (w_{0b_i} - w_{1b_i})(b_i - b'_i) \\ &= q_1 - q_0 + (w_{0b_i} - w_{1b_i})(b_i - b'_i) \end{aligned}$$

Because $q_1 \geq q_0, w_{0b_i} \geq w_{1b_i}, b_i > b'_i$, we can obtain $q'_1 \geq q'_0$. According to the algorithm, user i can still be accepted.

2. If the algorithm is monotonic, it must be proven that if user i is rejected, then user i is still rejected by arriving later $t_i' > t_i$ with the other state features.
According to algorithms 1 and 2, user i is rejected, which means:

$$q_1 = \sum_{j=1}^{d} w_{1j} s_i(j) < q_0 = \sum_{j=1}^{d} w_{0j} s_i(j)$$

According to definition 8 and formula (8), $t_i \in \mathcal{D}$, when user i is rejected, $w_{0t} = \max\{w_{1t}, w_{0t}\} \geq w_{1t}$; thus, if user i submits $t_i' > t_i$, we can obtain:

$$q_1' = \sum_{j=1}^{d} w_{1j} s(j) - w_{1t_i}(t_i - t_i')$$

and

$$q_0' = \sum_{j=1}^{d} w_{0j} s(j) - w_{0t_i}(t_i - t_i')$$

It can be seen

$$\begin{aligned} q_1' - q_0' &= \sum_{j=1}^{d} w_{1j} s(j) - \sum_{j=1}^{d} w_{0j} s(j) + (w_{0t_i} - w_{1t_i})(t_i - t_i') \\ &= q_1 - q_0 + (w_{0t_i} - w_{1t_i})(t_i - t_i') \end{aligned}$$

Because $q_1 < q_0, w_{0t_i} \geq w_{1t_i}, t_i > t_i'$, we can obtain $q_1' < q_0'$. According to the algorithm, user i is still rejected.

Thus, the allocation algorithm can guarantee monotonicity.

Lemma 3 (RL-OMC satisfies the critical value theory):
Proof. According to critical value theory, if the user is accepted, then if the user's bid is higher than the critical value, it will be rejected; however, if the user's bid is lower than the critical value, the user will still be accepted
According to algorithms 1 and 2, we know that $p_i = \min\left\{B\frac{V_i}{V_{max}}, ep_i\right\}$, and there are two cases as follows.

1. If $B\frac{V_i}{V_{max}} > ep_i$, then,

$$p_i = ep_i = \frac{\sum\limits_{j=1, s_i(j)\neq b_i}^{d} w_{1j} s_i(j) - \sum\limits_{j=1, s_i(j)\neq b_i}^{d} w_{0j} s_i(j)}{(w_{0b_i} - w_{1b_i})}$$

a) If the user bids $b_i' > p_i = ep_i$, according to the definition 9 of ep_i and $w_{0b_i} > w_{1b_i}$, we can obtain:

$$\sum_{j=1, s_i(j)\neq b_i}^{d} w_{1j} s_i(j) + w_{1b_i} b_i' < \sum_{j=1, s_i(j)\neq b_i}^{d} w_{0j} s_i(j) + w_{0b_i} b_i'$$

Thus, the user will be rejected at $q_1 < q_0$.

b) If the user bids $b_i' < p_i = ep_i$, according to the definition 9 of ep_i and $w_{0b_i} > w_{1b_i}$, we can obtain:

$$\sum_{j=1, s_i(j)\neq b_i}^{d} w_{1j} s_i(j) + w_{1b_i} b_i' > \sum_{j=1, s_i(j)\neq b_i}^{d} w_{0j} s_i(j) + w_{0b_i} b_i'$$

Thus, $q_1 > q_0$, and the user will still be accepted.

2. If $B\frac{V_i}{V_{max}} \leq ep_i$, then $p_i = B\frac{V_i}{V_{max}}$.

a) If the user bids $b_i' > p_i = B\frac{V_i}{V_{max}}$, according to algorithms 1 and 2, the necessary condition for the user to be accepted is $b_i \leq B\frac{V_i}{V_{max}}$. Therefore, the user will be rejected.

b) If the user bids $b_i' < p_i = B\frac{V_i}{V_{max}} \leq ep_i$, according to the definition of ep_i and $w_{0b_i} > w_{1b_i}$, we obtain:

$$\sum_{j=1, s_i(j)\neq b_i}^{d} w_{1j} s_i(j) + w_{1b_i} b_i' > \sum_{j=1, s_i(j)\neq b_i}^{d} w_{0j} s_i(j) + w_{0b_i} b_i'$$

Thus, $q_1 > q_0$, and the user will still be accepted. Thus, RL-OMC satisfies the critical value theory.

Theorem 2 (The RL-OMC is truthfulness):
Proof. According to lemmas 2 and 3, we know that the RL-OMC satisfies the monotonicity of allocation, and the payment satisfies the critical value theory; thus, RL-OMC is truthful.

Lemma 4 (When the user is accepted, $p_i \geq b_i$):
Proof. According to algorithms 1 and 2, we know that $p_i = \min\left\{B\frac{V_i}{V_{max}}, ep_i\right\}$, and there are two cases as follows.

1. If $B\frac{V_i}{V_{max}} > ep_i$, then $p_i = ep_i$. When the user is accepted, we obtain $q_1 \geq q_0$; that is,

$$\sum_{j=1, s(j)\neq b_i}^{d} w_{1j} s_i(j) + w_{1b_i} b_i \geq \sum_{j=1, s_i(j)\neq b_i}^{d} w_{0j} s_i(j) + w_{0b_i} b_i$$

According to formula (8), we know that $w_{0b_i} \geq w_{1b_i}$; thus, $w_{0b_i} b_i \geq w_{1b_i} b_i$, and we obtain

$$p_i = \frac{\sum\limits_{j=1, s_i(j)\neq b_i}^{d} w_{1j} s_i(j) - \sum\limits_{j=1, s_i(j)\neq b_i}^{d} w_{0j} s_i(j)}{(w_{0b_i} - w_{1b_i})} \geq b_i$$

2. If $B\frac{V_i}{V_{max}} \leq ep_i$, then $p_i = B\frac{V_i}{V_{max}}$, and the user should be paid according to the marginal value ratio at this time because a necessary condition for user acceptance is $b_i \leq B\frac{V_i}{V_{max}}$; thus, $p_i = B\frac{V_i}{V_{max}} \geq b_i$.
In summary, when the user is accepted, $p_i \geq b_i$.

Theorem 3 (RL-OMC satisfies individual rationality):
Proof. According to formula (5) and algorithms 1 and 2, if the user is rejected, the utility value is 0. If the user is accepted, according to lemma 4, $p_i \geq b_i$; thus, the utility value is $u_i = p_i - b_i \geq 0$. In summary, $u_i \geq 0$, so the RL-OMC satisfies individual rationality.

Theorem 4 (RL-OMC satisfies budget feasibility):
Proof. According to algorithms 1 and 2, we know $p_i = \min\left\{B\frac{V_i}{V_{max}}, ep_i\right\} \leq B\frac{V_i}{V_{max}}$, where $V_{max} = \sum\limits_{m\in\mathcal{M}} k_m r_m$ is the maximum revenue that the

service provider can theoretically obtain. It is obvious that $V_{max} \geq OPT \geq V(A) = \sum\limits_{i \in A} V_i$, and $p_i \leq B\frac{V_i}{V_{max}}$, so $\sum\limits_{i \in A} p_i \leq B\frac{\sum\limits_{i \in A} V_i}{V_{max}} = B\frac{V(A)}{V_{max}} \leq B$; thus, RL-OMC satisfies budget feasibility.

***Theorem** 5 (**RL-OMC satisfies computational efficiency**):*
Proof. In Algorithm 1, the time complexity of allocation and payment for each user is only $O(1)$, so the complexity of the RL-OMC mechanism is $O(N)$. Thus, RL-OMC satisfies computational efficiency.