I. APPENDIX

A. A simple example of ordered submodular functions

Suppose that there are 2 POIs and that $k_1 = 1$ and $k_2 = 2$, meaning that POI 1 must be covered once and POI 2 must be covered twice. Additionally suppose that there are two users, u_1 and u_2, with contribution values for the POIs of $v_{11} = 10$, $v_{12} = 5$, $v_{21} = 5$, and $v_{22} = 10$. According to formula (2), if we calculate the total revenue for the user order (u_1, u_2), we obtain

$$\begin{aligned} V_1(u_1, u_2) &= \sum_{i=1}^{\min\{k_1, dim(u_1,u_2)\}} v_{i1} \\ &= v_{11} = 10 \end{aligned}$$

$$\begin{aligned} V_2(u_1, u_2) &= \sum_{i=1}^{\min\{k_2, dim(u_1,u_2)\}} v_{i2} \\ &= v_{12} + v_{22} = 5 + 10 = 15 \end{aligned}$$

Thus, $V(u_1, u_2) = V_1(u_1, u_2) + V_2(u_1, u_2) = 25$. In contrast, if we calculate the total revenue for the user order (u_2, u_1), we obtain

$$\begin{aligned} V_1(u_2, u_1) &= \sum_{i=1}^{\min\{k_1, dim(u_2,u_1)\}} v_{i1} \\ &= v_{21} = 5 \end{aligned}$$

$$\begin{aligned} V_2(u_2, u_1) &= \sum_{i=1}^{\min\{k_2, dim(u_2,u_1)\}} v_{i2} \\ &= v_{22} + v_{12} = 10 + 5 = 15 \end{aligned}$$

Thus, $V(u_2, u_1) = V_1(u_2, u_1) + V_2(u_2, u_1) = 20$. The order of the users affects the function value.

B. Properties of RL-OMC

***Lemma 1** ($V_m(\cdot)$ **is an ordered submodular function**):*
Proof. According to definition 1, to prove that $V_m(\cdot)$ is an ordered submodular function, we must prove that for any sequences A'_m and B'_m and any elements i and j in sequence A_m,

$$V_m(A'_m||i) - V_m(A'_m) \geq V_m(A'_m||i||B'_m) - V_m(A'_m||j||B'_m)$$

According to the definition of $V_m(\cdot)$ in formula (2),

$$\begin{aligned} &V_m(A'_m||i) - V_m(A'_m) \\ &= \sum_{n=1}^{\min\left\{k_m, dim\left(A'_m||i\right)\right\}} v_{nm} - \sum_{n=1}^{\min\left\{k_m, dim\left(A'_m\right)\right\}} v_{nm} \\ &= \begin{cases} v_{im}, & k_m > dim(A'_m) \\ 0, & otherwise \end{cases} \end{aligned}$$

$$\begin{aligned} &V_m(A'_m||i||B'_m) - V_m(A'_m||j||B'_m) \\ &= \sum_{n=1}^{\min\left\{k_m, dim\left(A'_m||i||B'_m\right)\right\}} v_{nm} - \sum_{n=1}^{\min\left\{k_m, dim\left(A'_m||j||B'_m\right)\right\}} v_{nm} \\ &= \begin{cases} v_{im} - v_{jm}, & k_m > dim(A'_m) \\ 0, & otherwise \end{cases} \end{aligned}$$

We can obtain

$$V_m(A'_m||i) - V_m(A'_m) \geq V_m(A'_m||i||B'_m) - V_m(A'_m||j||B'_m)$$

Thus, $V_m(\cdot)$ is an ordered submodular function.

***Theorem 1** ($V(\cdot)$ **is an ordered submodular function**):*
Proof. According to definition 1, to prove that $V_m(\cdot)$ is an ordered submodular function, we must prove that for any sequences A' and B' and any elements i and j in sequence A,

$$V(A'||i) - V(A') \geq V(A'||i||B') - V(A'||j||B').$$

From formula (3), we can obtain $V(A) = \sum_{m\in\mathcal{M}} V_m(A_m)$, and lemma 1 proves that

$$V_m(A'_m||i) - V_m(A'_m) \geq V_m(A'_m||i||B'_m) - V_m(A'_m||j||B'_m)$$

By summing the above inequality over all POIs, we obtain

$$\begin{aligned} &\sum_{m\in\mathcal{M}} V_m(A'_m||i) - \sum_{m\in\mathcal{M}} V_m(A'_m) \\ &\geq \sum_{m\in\mathcal{M}} V_m(A'_m||i||B'_m) - \sum_{m\in\mathcal{M}} V_m(A'_m||j||B'_m) \end{aligned}$$

A_m is a subsequence of A; that is,

$$V(A'||i) - V(A') \geq V(A'||i||B') - V(A'||j||B')$$

Therefore, $V(\cdot)$ is an ordered submodular function.

***Lemma 2** (**RL-OMC is monotonic**):*
Proof. Because users cannot lie about their marginal values, only untruthful bids and untruthful arrival times can be submitted.

1. To prove that the algorithm is monotonic, we must prove that if user i is accepted, then user i will still be accepted if she or he reduces her or his bid to $b'_i < b_i$.
If user i is accepted according to Algorithms 1 and 2, this means that

$$q_1 = \sum_{j=1}^{d} w_{1j}\boldsymbol{s}_i(j) \geq q_0 = \sum_{j=1}^{d} w_{0j}\boldsymbol{s}_i(j)$$

According to definition 8 and formula (8), $b_i \in \mathcal{D}$, and when user i is accepted, $w_{1b_i} = \min\{w_{0b_i}, w_{1b_i}\} \leq w_{0b_i}$; thus, for the case in which user i submits $b'_i < b_i$, we can obtain

$$q'_1 = \sum_{j=1}^{d} w_{1j}\boldsymbol{s}_i(j) - w_{1b_i}(b_i - b'_i)$$

and

$$q'_0 = \sum_{j=1}^{d} w_{0j}\boldsymbol{s}_i(j) - w_{0b_i}(b_i - b'_i)$$

It can be seen that

$$\begin{aligned} q'_1 - q'_0 &= \sum_{j=1}^{d} w_{1j}\boldsymbol{s}_i(j) - \sum_{j=1}^{d} w_{0j}\boldsymbol{s}_i(j) + (w_{0b_i} - w_{1b_i})(b_i - b'_i) \\ &= q_1 - q_0 + (w_{0b_i} - w_{1b_i})(b_i - b'_i) \end{aligned}$$

Because $q_1 \geq q_0$, $w_{0b_i} \geq w_{1b_i}$, and $b_i > b_i'$, we can obtain $q_1' \geq q_0'$. According to the algorithm, user i will still be accepted.

2. To prove that the algorithm is monotonic, we must prove that if user i is rejected, then user i will still be rejected if she or he arrives later (at $t_i' > t_i$) with all other state features remaining the same.
If user i is rejected according to Algorithms 1 and 2, this means that

$$q_1 = \sum_{j=1}^{d} w_{1j}\boldsymbol{s}_i(j) < q_0 = \sum_{j=1}^{d} w_{0j}\boldsymbol{s}_i(j)$$

According to definition 8 and formula (8), $t_i \in \mathcal{D}$, and when user i is rejected, $w_{0t} = \max\{w_{1t}, w_{0t}\} \geq w_{1t}$; thus, for the case in which user i submits $t_i' > t_i$, we can obtain

$$q_1' = \sum_{j=1}^{d} w_{1j}\boldsymbol{s}_i(j) - w_{1t_i}(t_i - t_i')$$

and

$$q_0' = \sum_{j=1}^{d} w_{0j}\boldsymbol{s}_i(j) - w_{0t_i}(t_i - t_i')$$

It can be seen that

$$\begin{aligned} q_1' - q_0' &= \sum_{j=1}^{d} w_{1j}\boldsymbol{s}_i(j) - \sum_{j=1}^{d} w_{0j}\boldsymbol{s}_i(j) + (w_{0t_i} - w_{1t_i})(t_i - t_i') \\ &= q_1 - q_0 + (w_{0t_i} - w_{1t_i})(t_i - t_i') \end{aligned}$$

Because $q_1 < q_0$, $w_{0t_i} \geq w_{1t_i}$, and $t_i > t_i'$, we can obtain $q_1' < q_0'$. According to the algorithm, user i is still rejected.

Thus, the allocation algorithm can guarantee monotonicity.

Lemma 3 (RL-OMC satisfies critical value theory): **Proof.** According to critical value theory, for an accepted user, there exists a critical value such that if the user's bid were higher than this critical value, then the user would be rejected, but as long as the user's bid is lower than the critical value, the user will always be accepted.
According to Algorithms 1 and 2, $p_i = \min\left\{B\frac{V_i}{V_{max}}, ep_i\right\}$, and there are two corresponding cases, as follows.

1. If $B\frac{V_i}{V_{max}} > ep_i$, then

$$p_i = ep_i = \frac{\sum\limits_{j=1,\boldsymbol{s}_i(j)\neq b_i}^{d} w_{1j}\boldsymbol{s}_i(j) - \sum\limits_{j=1,\boldsymbol{s}_i(j)\neq b_i}^{d} w_{0j}\boldsymbol{s}_i(j)}{(w_{0b_i} - w_{1b_i})}$$

a) For the case in which the user bids $b_i' > p_i = ep_i$, according to definition 9 (the definition of ep_i) and $w_{0b_i} > w_{1b_i}$, we can obtain

$$\sum_{j=1,\boldsymbol{s}_i(j)\neq b_i}^{d} w_{1j}\boldsymbol{s}_i(j) + w_{1b_i}b_i' < \sum_{j=1,\boldsymbol{s}_i(j)\neq b_i}^{d} w_{0j}\boldsymbol{s}_i(j) + w_{0b_i}b_i'$$

Thus, the user will be rejected because $q_1 < q_0$.

b) For the case in which the user bids $b_i' < p_i = ep_i$, according to definition 9 and $w_{0b_i} > w_{1b_i}$, we can obtain

$$\sum_{j=1,\boldsymbol{s}_i(j)\neq b_i}^{d} w_{1j}\boldsymbol{s}_i(j) + w_{1b_i}b_i' > \sum_{j=1,\boldsymbol{s}_i(j)\neq b_i}^{d} w_{0j}\boldsymbol{s}_i(j) + w_{0b_i}b_i'$$

Thus, $q_1 > q_0$, and the user will still be accepted.

2. If $B\frac{V_i}{V_{max}} \leq ep_i$, then $p_i = B\frac{V_i}{V_{max}}$.

a) If the user bids $b_i' > p_i = B\frac{V_i}{V_{max}}$, because the necessary condition for the user to be accepted according to Algorithms 1 and 2 is $b_i \leq B\frac{V_i}{V_{max}}$, the user will be rejected.

b) For the case in which the user bids $b_i' < p_i = B\frac{V_i}{V_{max}} \leq ep_i$, according to the definition of ep_i and $w_{0b_i} > w_{1b_i}$, we can obtain

$$\sum_{j=1,\boldsymbol{s}_i(j)\neq b_i}^{d} w_{1j}\boldsymbol{s}_i(j) + w_{1b_i}b_i' > \sum_{j=1,\boldsymbol{s}_i(j)\neq b_i}^{d} w_{0j}\boldsymbol{s}_i(j) + w_{0b_i}b_i'$$

Thus, $q_1 > q_0$, and the user will still be accepted. Therefore, RL-OMC satisfies critical value theory.

Theorem 2 (RL-OMC is truthful):
Proof. According to lemmas 2 and 3, the RL-OMC mechanism satisfies monotonicity of allocation, and the payment satisfies critical value theory; thus, RL-OMC is truthful.

Lemma 4 (When user i is accepted, $p_i \geq b_i$): **Proof.** According to Algorithms 1 and 2, $p_i = \min\left\{B\frac{V_i}{V_{max}}, ep_i\right\}$, and there are two corresponding cases, as follows.

1. If $B\frac{V_i}{V_{max}} > ep_i$, then $p_i = ep_i$. When the user is accepted, $q_1 \geq q_0$, that is,

$$\sum_{j=1,\boldsymbol{s}_i(j)\neq b_i}^{d} w_{1j}\boldsymbol{s}_i(j) + w_{1b_i}b_i \geq \sum_{j=1,\boldsymbol{s}_i(j)\neq b_i}^{d} w_{0j}\boldsymbol{s}_i(j) + w_{0b_i}b_i$$

From formula (8), we know that $w_{0b_i} \geq w_{1b_i}$; thus, $w_{0b_i}b_i \geq w_{1b_i}b_i$, and we obtain

$$p_i = \frac{\sum\limits_{j=1,\boldsymbol{s}_i(j)\neq b_i}^{d} w_{1j}\boldsymbol{s}_i(j) - \sum\limits_{j=1,\boldsymbol{s}_i(j)\neq b_i}^{d} w_{0j}\boldsymbol{s}_i(j)}{(w_{0b_i} - w_{1b_i})} \geq b_i$$

2. If $B\frac{V_i}{V_{max}} \leq ep_i$, then $p_i = B\frac{V_i}{V_{max}}$, and the user should be paid in accordance with the marginal value ratio because a necessary condition for user acceptance is $b_i \leq B\frac{V_i}{V_{max}}$; thus, $p_i = B\frac{V_i}{V_{max}} \geq b_i$.
In summary, when user i is accepted, $p_i \geq b_i$.

Theorem 3 (RL-OMC exhibits individual rationality):
Proof. According to formula (5) and Algorithms 1 and 2, if a user is rejected, the utility value for that user is 0. If the user is accepted, according to lemma 4, $p_i \geq b_i$; thus, the utility value is $u_i = p_i - b_i \geq 0$. In summary, $u_i \geq 0$; therefore, RL-OMC guarantees individual rationality.

***Theorem** 4 (**RL-OMC exhibits budget feasibility**):*
Proof. According to Algorithms 1 and 2, $p_i = \min\left\{B\frac{V_i}{V_{max}}, ep_i\right\} \le B\frac{V_i}{V_{max}}$, where $V_{max} = \sum\limits_{m\in\mathcal{M}} k_m r_m$ is the maximum revenue that the service provider can theoretically obtain. It is evident that $V_{max} \ge OPT \ge V(A) = \sum\limits_{i\in A} V_i$, and $p_i \le B\frac{V_i}{V_{max}}$; thus, $\sum\limits_{i\in A} p_i \le B\frac{\sum\limits_{i\in A} V_i}{V_{max}} = B\frac{V(A)}{V_{max}} \le B$. Therefore, RL-OMC guarantees budget feasibility.

***Theorem** 5 (**RL-OMC exhibits computational efficiency**):*
Proof. In Algorithm 1, the time complexity of allocation and payment for each user is only $O(1)$; thus, the complexity of the RL-OMC mechanism is $O(N)$. Therefore, RL-OMC exhibits computational efficiency.