PROBLEM 1:RELATIONS

Name / Surname : Yunus Emre Geyik
No: 1801042635
Department: Computer Engineering

Problem 1: Relations

$B = \{0, 1, 2, 3, 4, 5\}$

we have to find the poset for greater than or equal.
"$\geq$" says "a relation (a,b) where $a, b \in B$, a greater than or equal to b"

Let the set is A. Then,

$A = \{(0 \to 0), (1 \to 1), (2 \to 2), (3 \to 3), (4 \to 4), (5 \to 5), (1 \to 0), (2 \to 1), (2 \to 0), (3 \to 2), (3 \to 1), (3 \to 0), (4 \to 3), (4 \to 2), (4 \to 1), (4 \to 0), (5 \to 4), (5 \to 3), (5 \to 2), (5 \to 1), (5 \to 0)\}$

Let's draw the directed grap the relation above in figure 1:

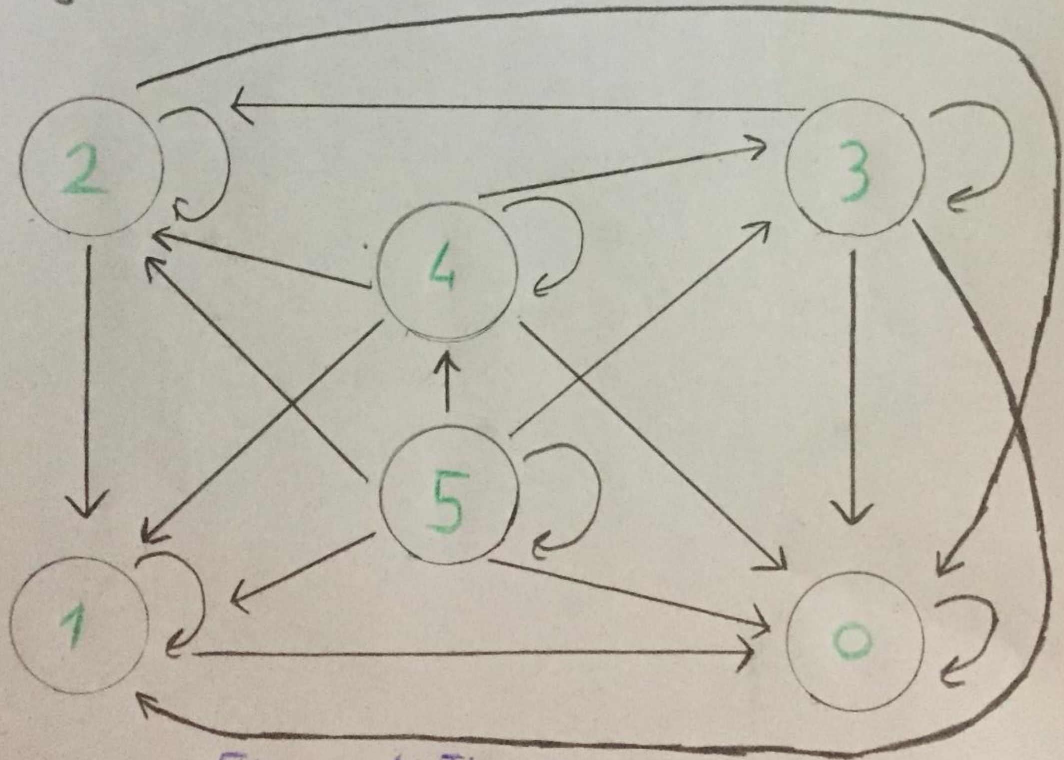


Figure 1: The orginal graph

PROBLEM 1 CONTINUE...

Problem 1 Continue...

Since we know that a poset MUST provide reflexivity, we also do not need the reflexive relations in A. Hence A can be updated as:

$A = \{(1 \to 0), (2 \to 1), (2 \to 0), (3 \to 2), (3 \to 1), (3 \to 0), (4 \to 3), (4 \to 2), (4 \to 1), (4 \to 0), (5 \to 4), (5 \to 3), (5 \to 2), (5 \to 1), (5 \to 0)\}$

In the next step, remove the self-loops in Figure 2:

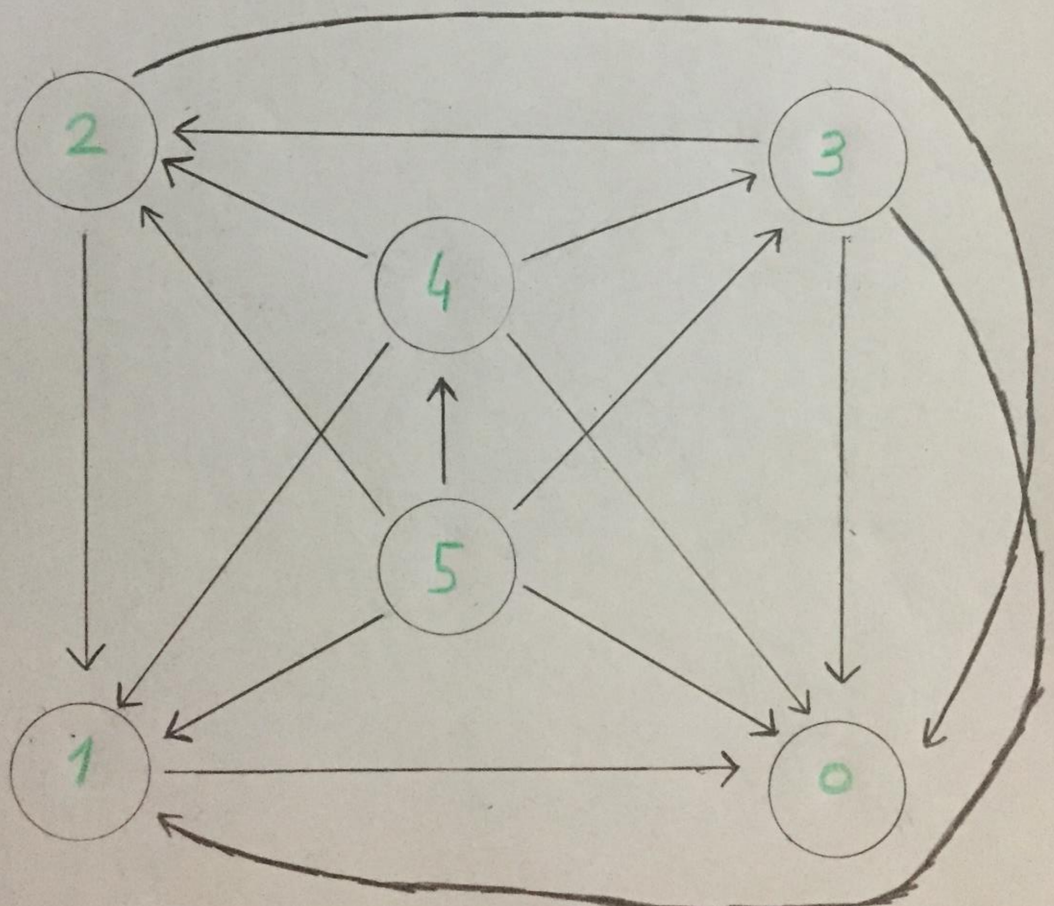


Figure 2: The graph without self-loops

PROBLEM 1 CONTINUE...

Problem 1 Continue...

Remove the transivite edges and the hasse diagram is obtained in Figure 3:

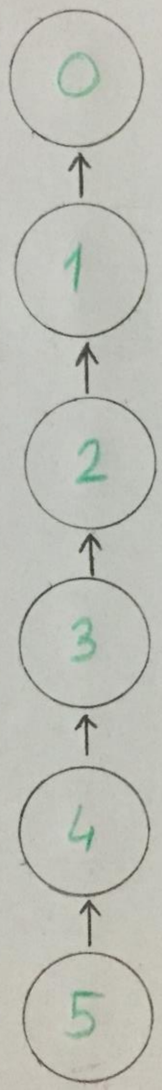
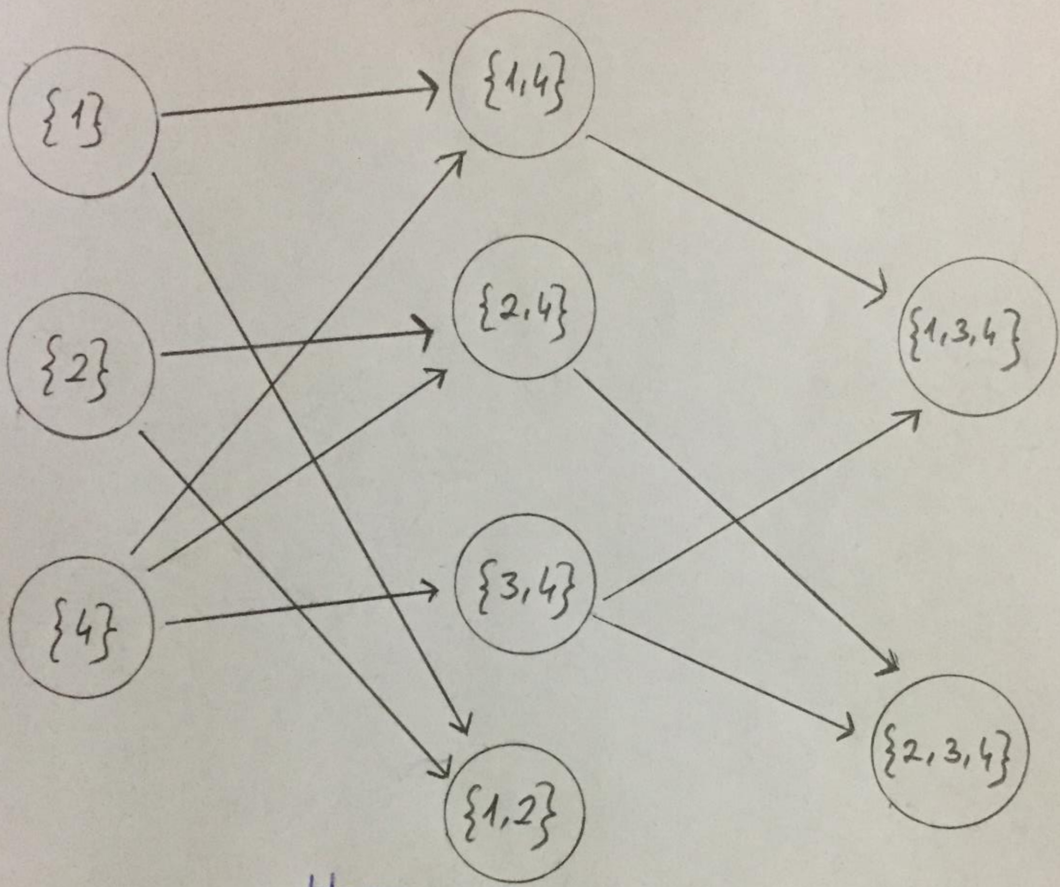


Figure 3: The hasse diagram of $(\{0,1,2,3,4,5\}, \geq)$

PROBLEM 2:RELATIONS

Name / Surname: Yunus Emre Geyik
No: 1801042635
Department: Computer Engineering



Hasse Diagram

a) In a Hasse diagram, a vertex corresponds to a maximal element if there is no edge leaving the vertex.

In Hasse Diagram, we can see that {1,3,4} and {2,3,4} are maximal elements of the poset.

PROBLEM 2 CONTINUE...

b) In a Hasse diagram, a vertex corresponds to a minimal element if there is no edge entering the vertex.
In the Hasse diagram, we can see that $\{1\}, \{2\}$ and $\{4\}$ are minimal elements of the poset.

c) In the Hasse Diagram, the least element does not exist since there is no any one element that precedes all the elements.

d) Upper Bounds;
→ Does not have to belong to set
→ Must be greater than all elements in set $\{\{2\},\{4\}\}$

$\begin{pmatrix} \{2\} \subseteq \{2,4\}, \\ \{4\} \subseteq \{2,4\} \end{pmatrix}, \begin{pmatrix} \{2\} \subseteq \{2,3,4\}, \\ \{4\} \subseteq \{2,3,4\} \end{pmatrix}$

The upper Bounds of $\{\{2\},\{4\}\}$ is $\Rightarrow \{\{2,4\},\{2,3,4\}\}$

e) Yes it is exist. $\{2,4\}$ is the smallest among the Upper Bounds.

Upper Bounds $\Rightarrow \{\{2,4\},\{2,3,4\}\}$

$(\{2,4\} \subseteq \{2,3,4\})$, $(\{2,4\} \to \{2,3,4\})$

the least upper bound of $\{\{2\},\{4\}\}$ is $\Rightarrow \{2,4\}$

PROBLEM 2 CONTINUE...

f) a lower bound;

→ Does not have to belong to a set $\{\{1,3,4\},\{2,3,4\}\}$

→ Must be smaller than all element in set $\{\{1,3,4\},\{2,3,4\}\}$

$\begin{pmatrix} \{3,4\} \subseteq \{1,3,4\}, \\ \{3,4\} \subseteq \{2,3,4\} \end{pmatrix}, \begin{pmatrix} \{4\} \subseteq \{1,3,4\}, \\ \{4\} \subseteq \{2,3,4\} \end{pmatrix}$

The Lower Bounds of of $\{\{1,3,4\},\{2,3,4\}\}$ is $\{\{4\},\{3,4\}\}$

e) Yes it is exist. $\{3,4\}$ is the greatest among the lower bounds.

The Lower Bounds $\Rightarrow \{\{4\},\{3,4\}\}$

$(\{4\} \subseteq \{3,4\})$, $(\{4\} \rightarrow \{3,4\})$

the greatest lower bound of $\{\{1,3,4\},\{2,3,4\}\}$ is

$\{3,4\}$