Overview of inequality measures

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Outline

1. Inequality of what?

   Some choices for the statistician/researcher.

2. Data sources and caveats.

3. Measures of inequality.

   Most widely used indicators to measure inequality and understand the sensitivity to different parts of the distribution.

1. Inequality of what?

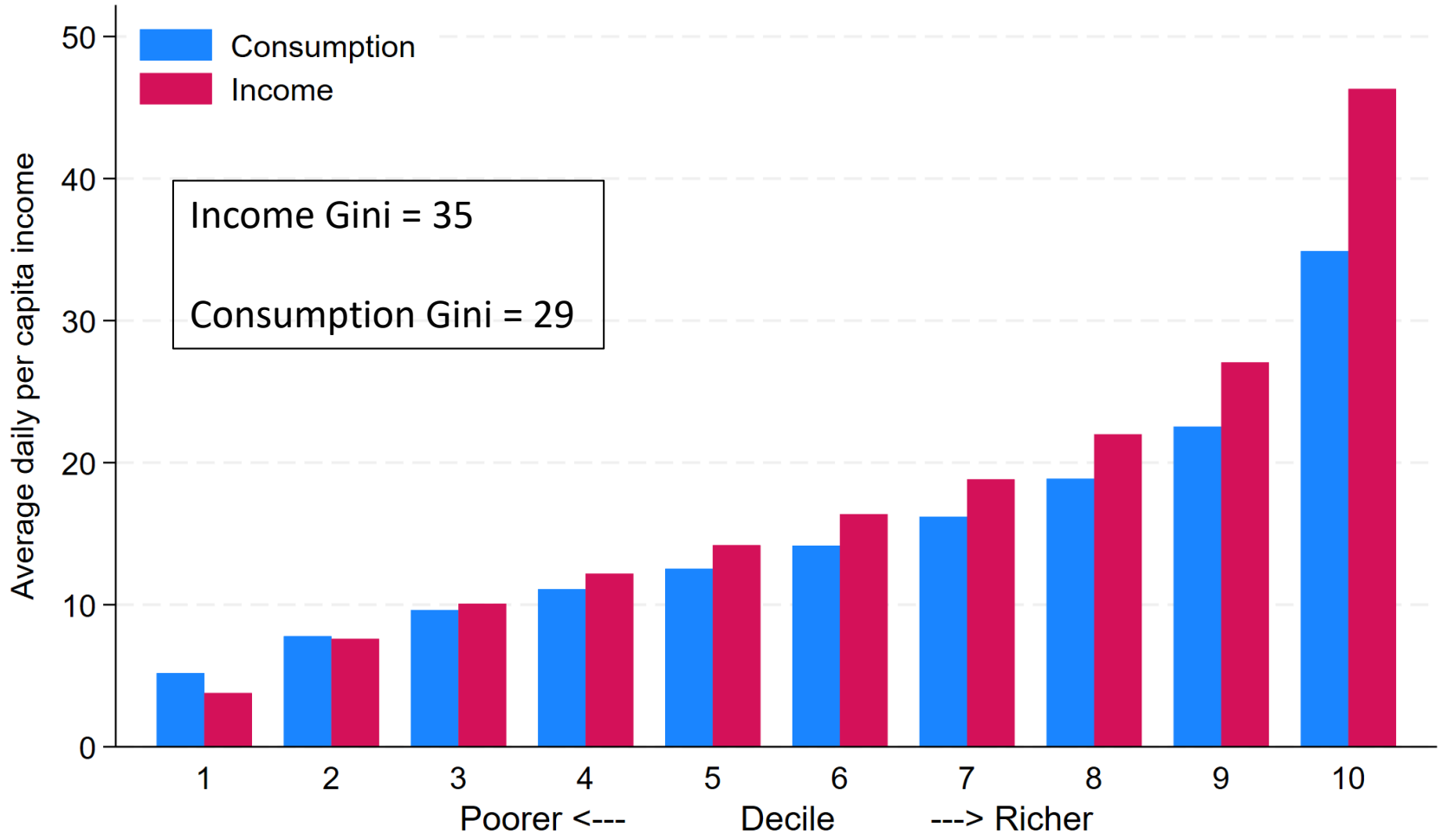
Some decisions to make…

A. Income or consumption (expenditures or wealth)
B. Absolute or relative inequality
C. Recipient: household or individual
D. Ranking: per capita or equivalence scales
E. Type: Market or Disposable or other definitions
F. Price adjustments across time and space

A. Income vs. consumption?

- Income: gives actual *economic power*
- Expenditures or consumption: give actual *standard of living*
- Savings (as % of income) generally larger for higher income households
  => inequality of income greater than inequality of consumption
- Income can be negative; Consumption cannot be
  => inequality of income greater than inequality of consumption
- So at both ends, income gives higher inequality (would also give greater poverty)

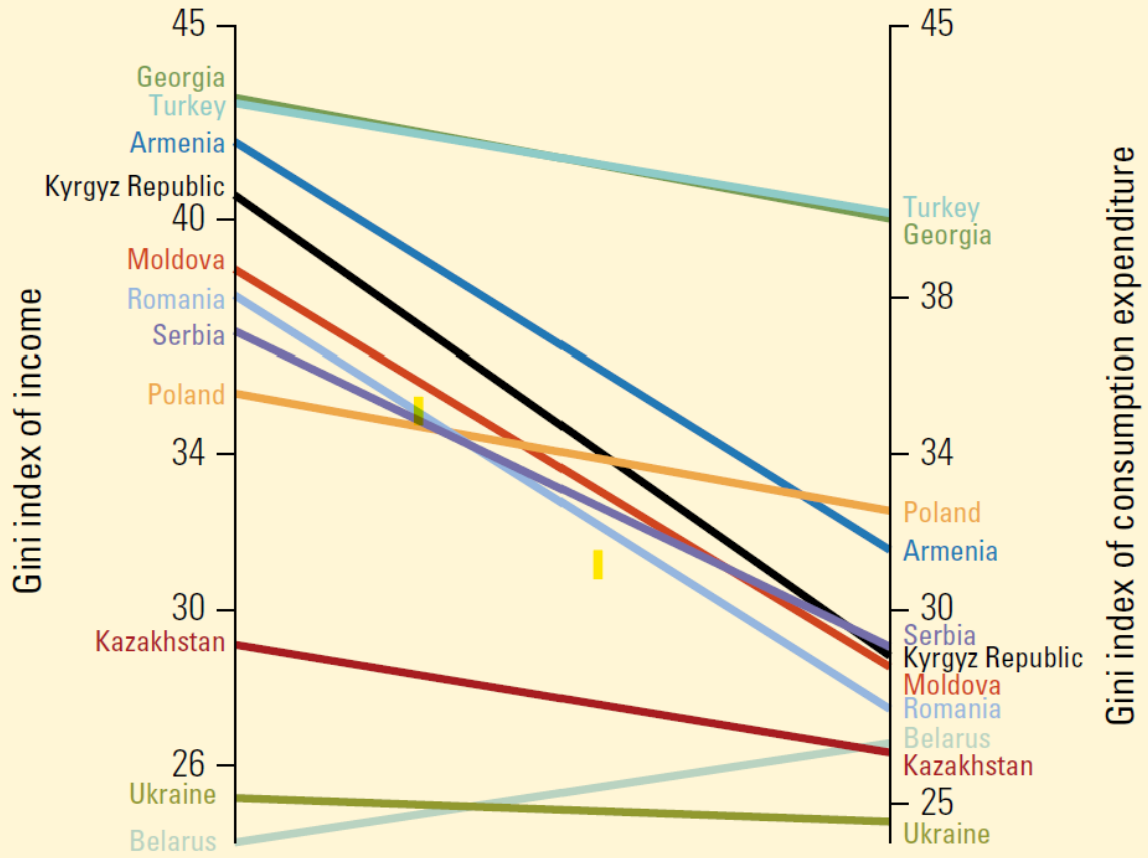
Average income or consumption across deciles, Serbia 2019



Source: Poverty and Inequality Platform, World Bank

Comparison of income and consumption Gini coefficients across various countries in PIP

FIGURE B4.4.1 Levels of Income and Consumption, Gini Indexes, 2013



Source: Poverty and Shared Prosperity 2016: Taking on inequality, World Bank

B. Relative and absolute inequality

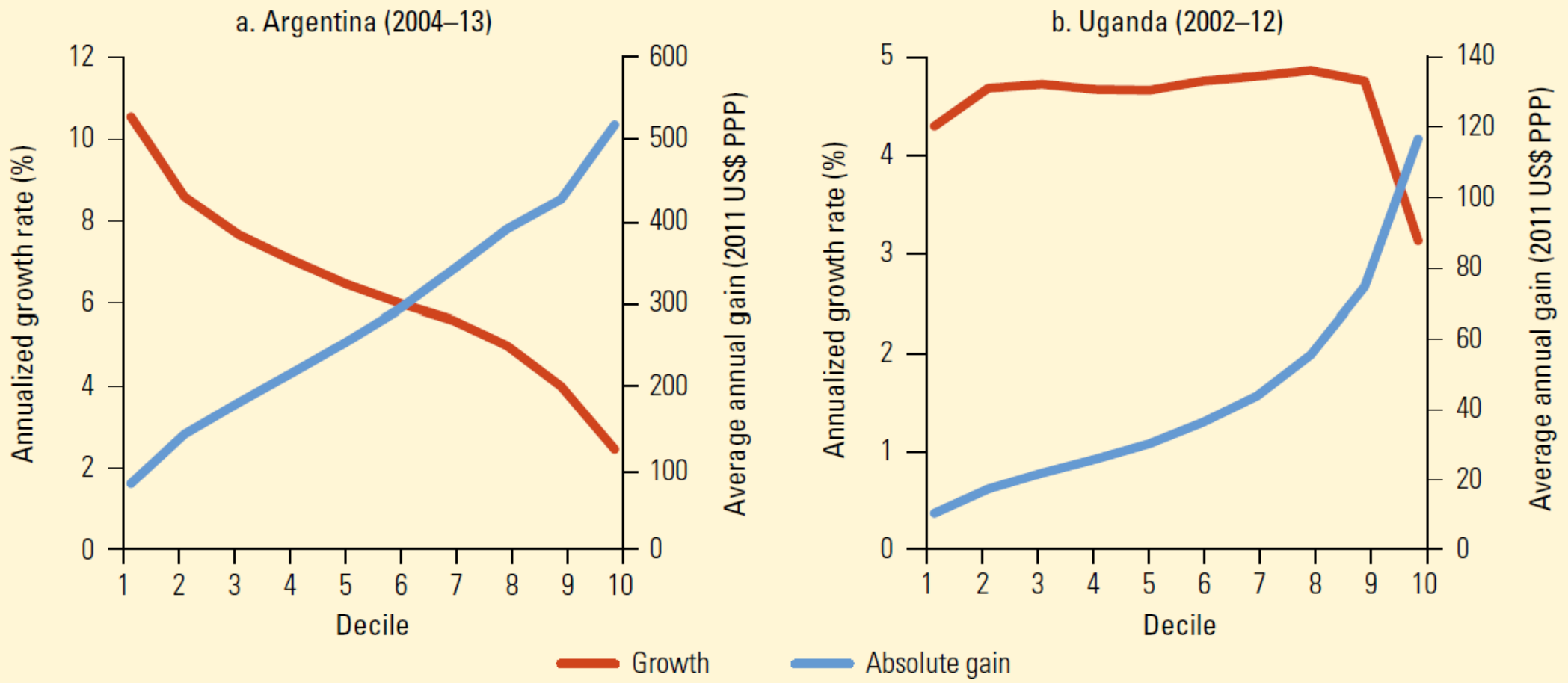
- Relative inequality is about ratios; absolute inequality is about differences.
  - *State A: two incomes $1,000 and $10,000 per year*
  - *State B: these rise to $2,000 and $20,000*
  - *Ratio is unchanged but the absolute gain to the rich is twice as large in state B*
- Ravallion (2016) found that in experimental studies, university students in Germany, Israel, the United Kingdom, and the United States are approximately evenly split between caring about relative measures and caring about absolute measures.

Disadvantages of absolute inequality measures

- Penalizes growth and does not make it easy to distinguish between periods of differently rising inequality
- Growth rate (a relative measure) should be related to inequality measure (relative). They are changes in the 1^{st} and 2^{nd} moment of the distribution.
- Relative measure correlated with changes in utility (if U=log(y))

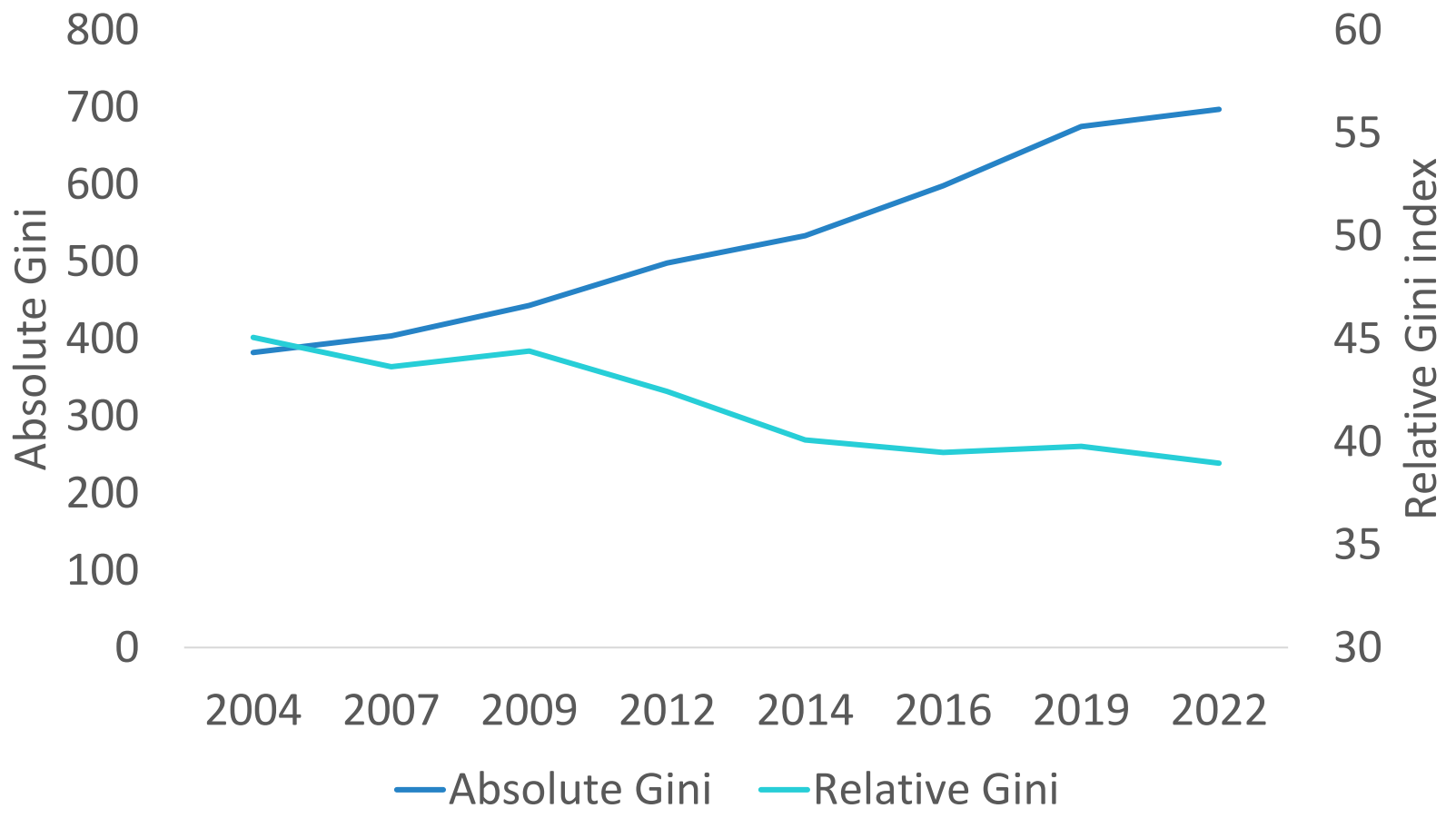
Absolute and relative gains can differ

FIGURE B4.3.1 Comparing Absolute and Relative Gains across the Distribution



Source: Poverty and Prosperity Report 2016: Taking on Inequality, World Bank

Absolute and relative Gini for Malaysia



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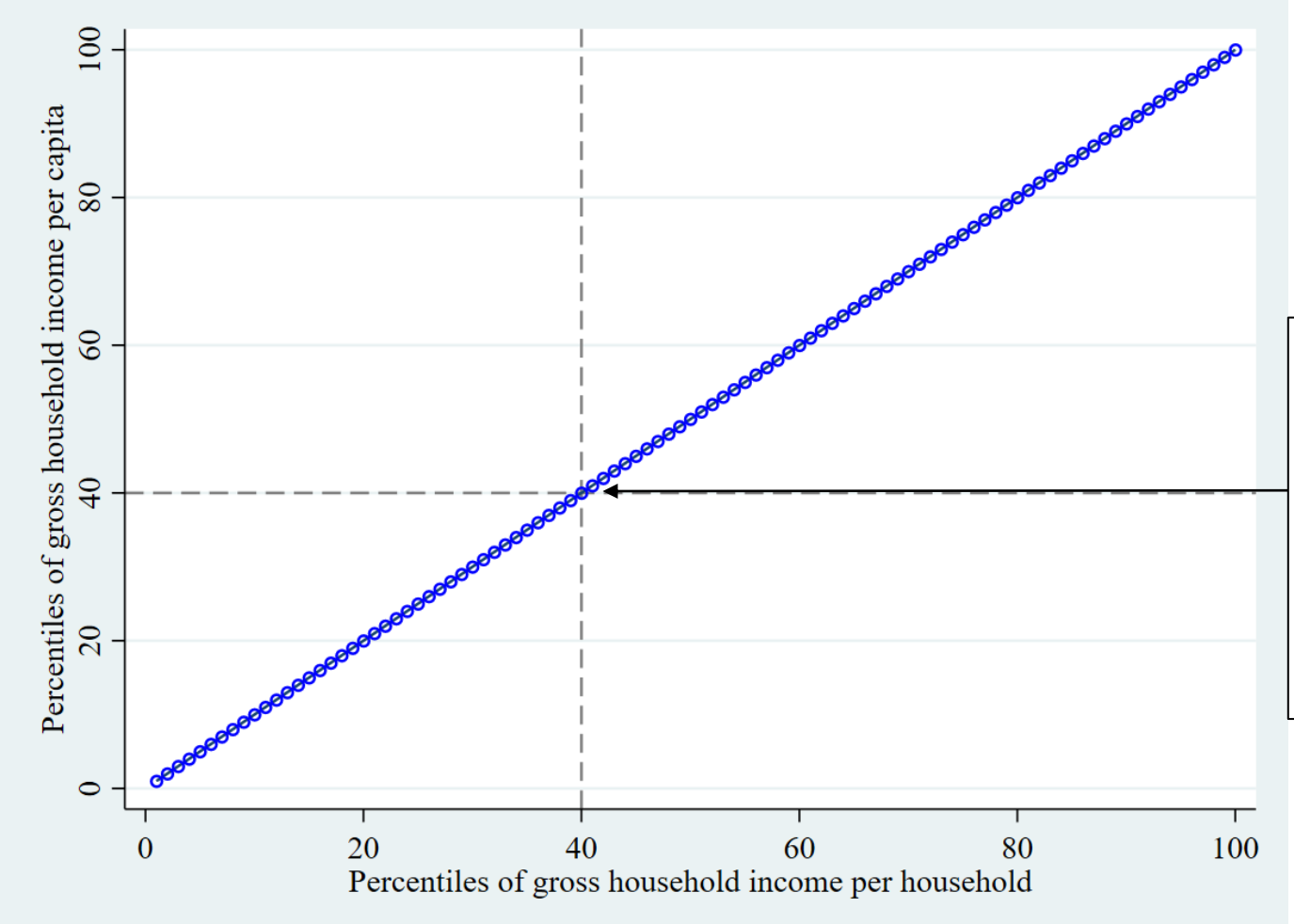
Source: Measuring and Addressing Inequality in Malaysia (2024), World Bank;
Note: Gini of household per capita income

C. Recipient of income/consumption

Recipient	Income	Inequality
Household income	D(H|Yh)	No inequality within households; household size doesn't matter
Household income per capita	D(H|Yp)	No inequality within households; household size matters
Personal income	D(p|Yp)	Inequality within households included

Same household size and income ranking



If all households had the same number of people:

The 40th ranked individual would be ranked 40th in the household distribution (true for all individuals)

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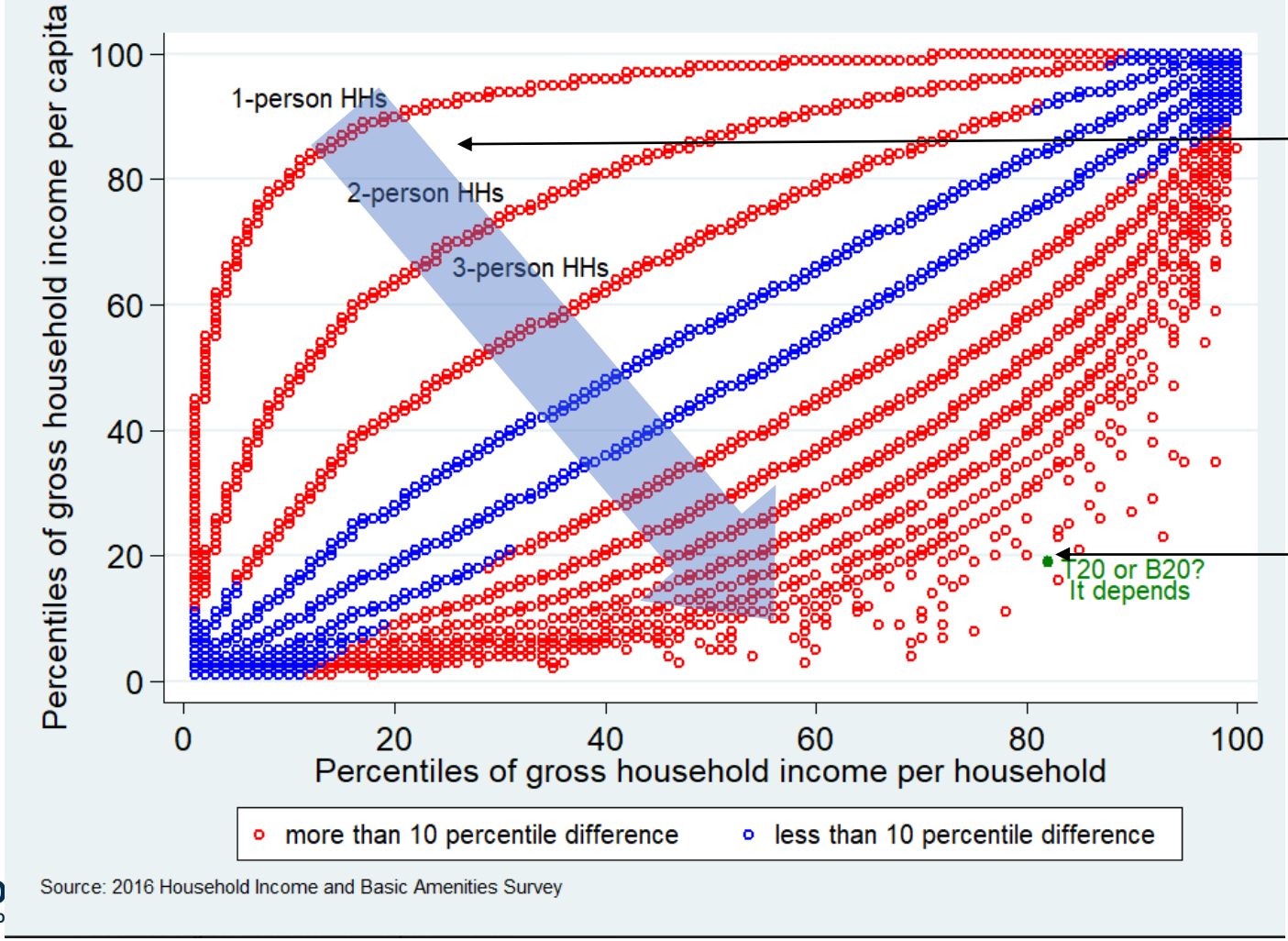
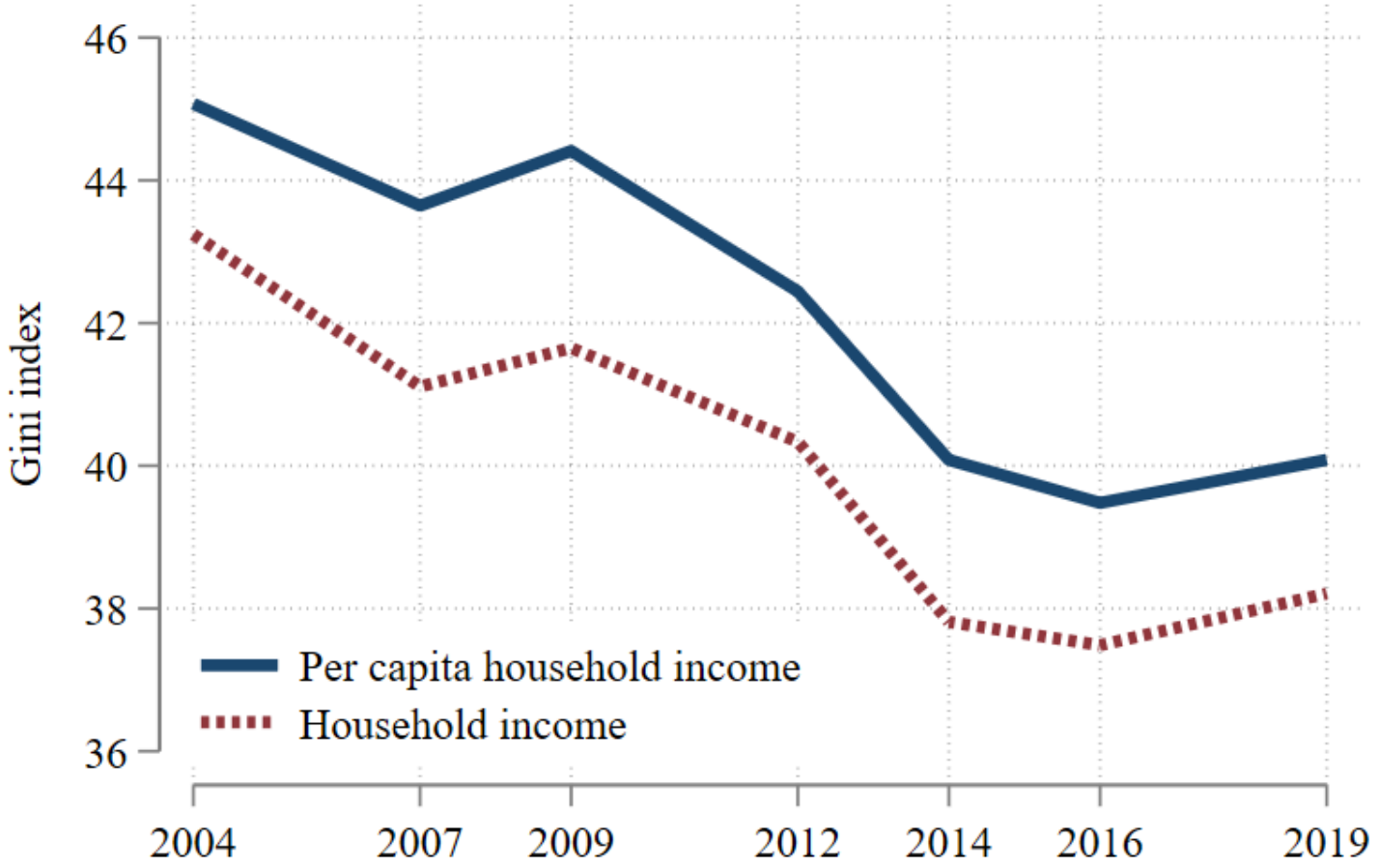
Unit of measurement makes a difference: Re-rankings between per capita or household incomes



Increasing size of households.

This entry is the 80th percentile in the distribution of household income, but only the 20th percentile in the distribution per capita household income

Malaysia inequality levels between households and individuals



Source: Measuring and Addressing Inequality in Malaysia (2024), World Bank

D. Equivalence scales

- The basic idea: to reach the same degree of utility, people may not need the same amount of income
- But we know nothing about how individuals “convert” income into utility (no inter-personal comparisons)
- What we know (or suppose):
  i. cost of food is less for children than for adults;
  ii. people who live together share public goods (“it’s cheaper in per capita terms for two people to live together than individually”; think of heating costs) => what we refer to as “economies of scale”

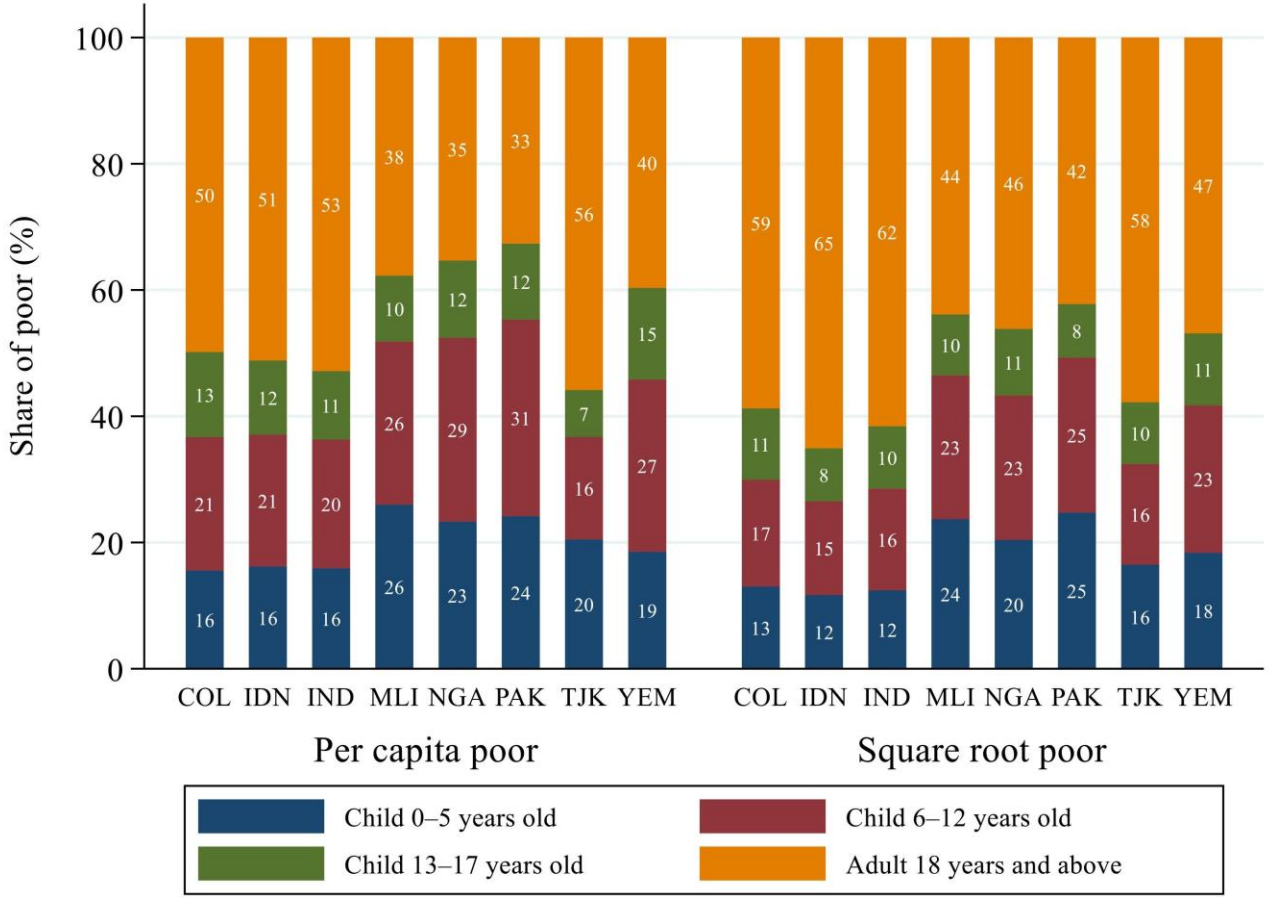
Some equivalence scales in practice

Household size	Equivalence scale				
	per-capita income	"Oxford" scale ("Old OECD scale")	"OECD-modified" scale	Square root scale	Household income
1 adult	1	1	1	1	1
2 adults	2	1.7	1.5	1.4	1
2 adults, 1 child	3	2.2	1.8	1.7	1
2 adults, 2 children	4	2.7	2.1	2.0	1
2 adults, 3 children	5	3.2	2.4	2.2	1
Elasticity[1]	1	0.73	0.53	0.50	0

[1] Using household size as the determinant, equivalence scales can be expressed through an "equivalence elasticity", i.e. the power by which economic needs change with household size. The equivalence elasticity can range from 0 (when unadjusted household disposable income is taken as the income measure) to 1 (when per capita household income is used). The smaller the value for this elasticity, the higher the economies of scale in consumption.

Source: What are equivalence scales? OECD. https://www.oecd.org/els/soc/OECD-Note-EquivalenceScales.pdf

How consumption/income is distributed within household matters for welfare.

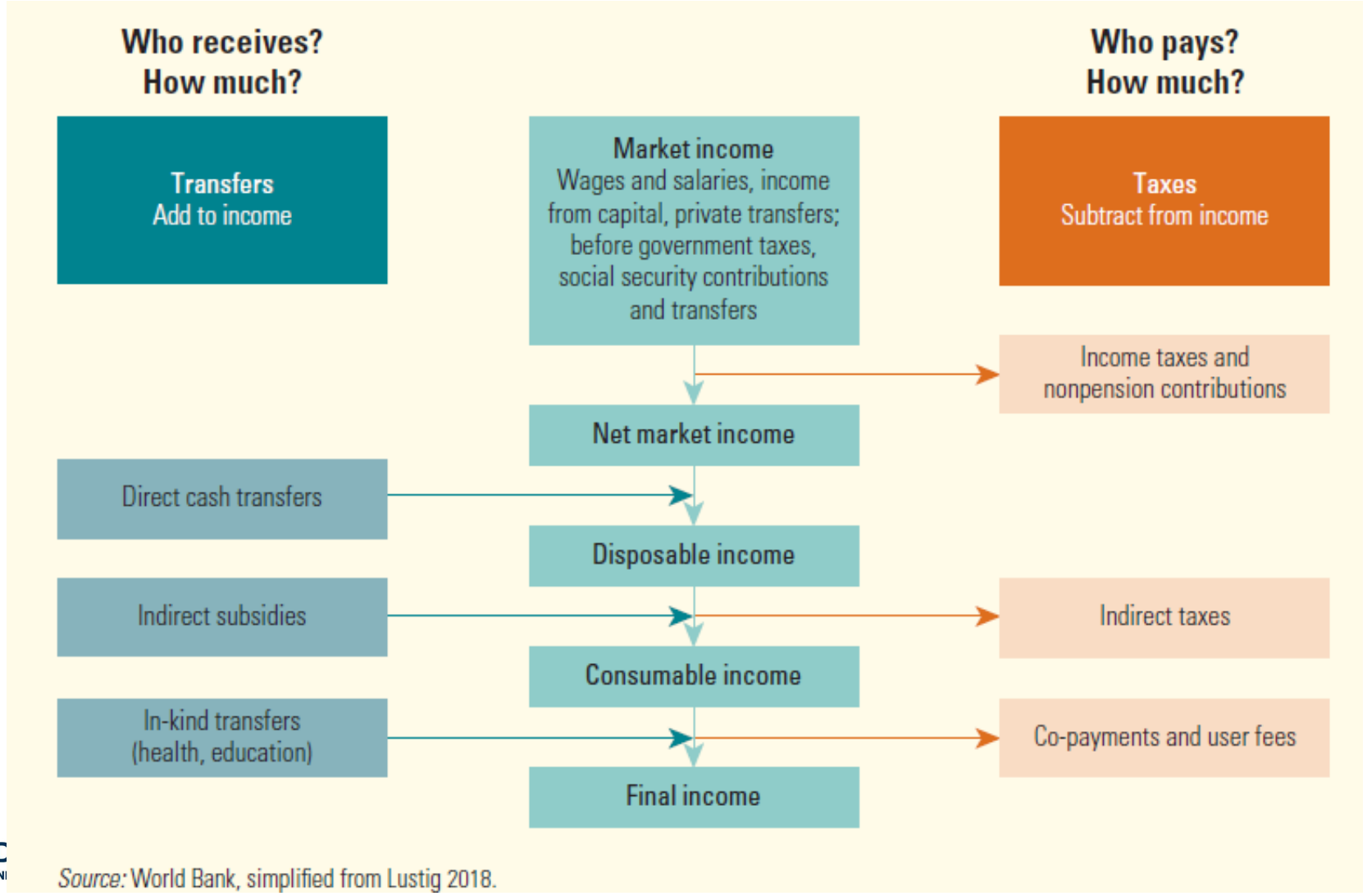


Source: Jolliffe and Tetteh-Baah (2024)

Sources: Global Monitoring Data (GMD), Luxembourg Income Study (LIS)

Notes: This chart shows the share of the per-capita and square-root poor by age cohort. Four age cohorts are identified, namely children between up to 5 years, between 6 and 12 years, between 13 and 17 years, and adults.

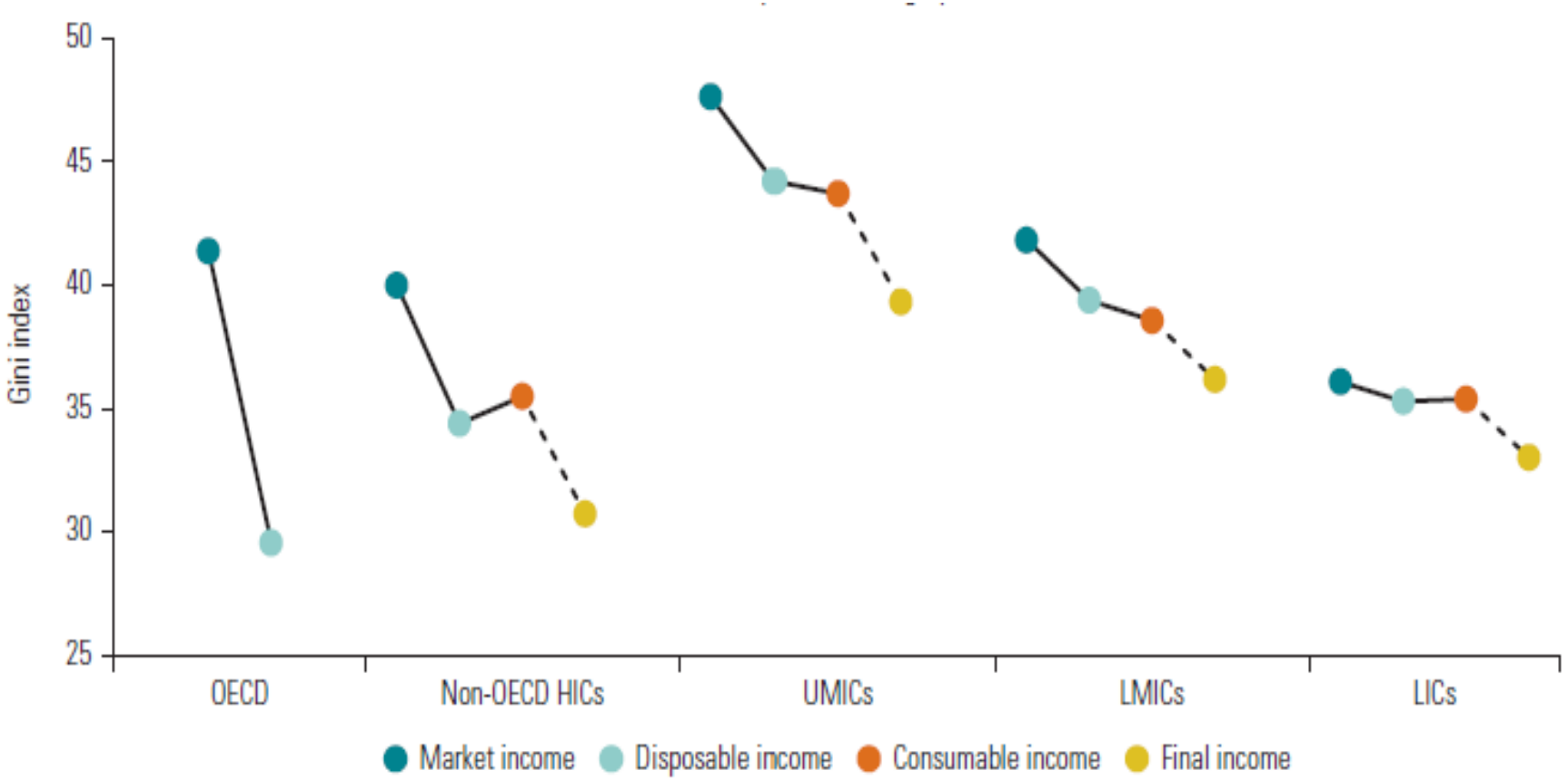
E. Market v disposable income



Source: World Bank, simplified from Lustig 2018.

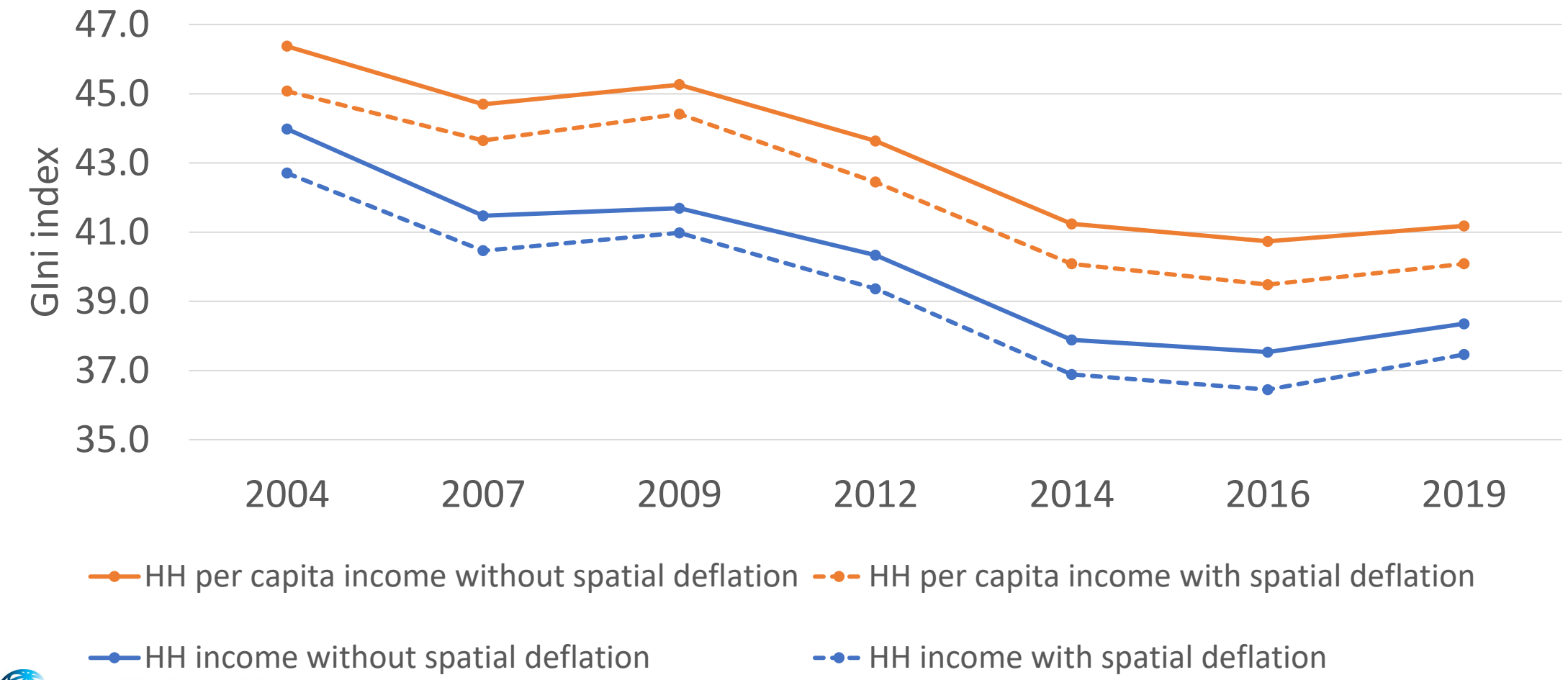
Source: Poverty and Shared Prosperity 2022: Correcting Course

Inequality level with various income definition



Source: Poverty and Shared Prosperity 2022: Correcting Course

F. Spatial price adjustments matter



Outcomes, Opportunity, Mobility, Perception

- Thinking beyond just outcomes (income or consumption or wealth) today and the opportunities for tomorrow can be important.
- Economic mobility provides a way to think about inequality more dynamically.
- Perceptions of inequality may or may not match objective measures, but they still matter:
    i. people's attitudes and behaviours are based on perceptions;
    ii. perceptions of high inequality or low opportunities for mobility can lead to unhappiness and discontent (Gupta 1990; Keefer and Knack 2002); and
    iii. they can also influence support for policies to address inequality.
- Note that these concepts interact and affect each other.

Perceptions and Inequality of Opportunity

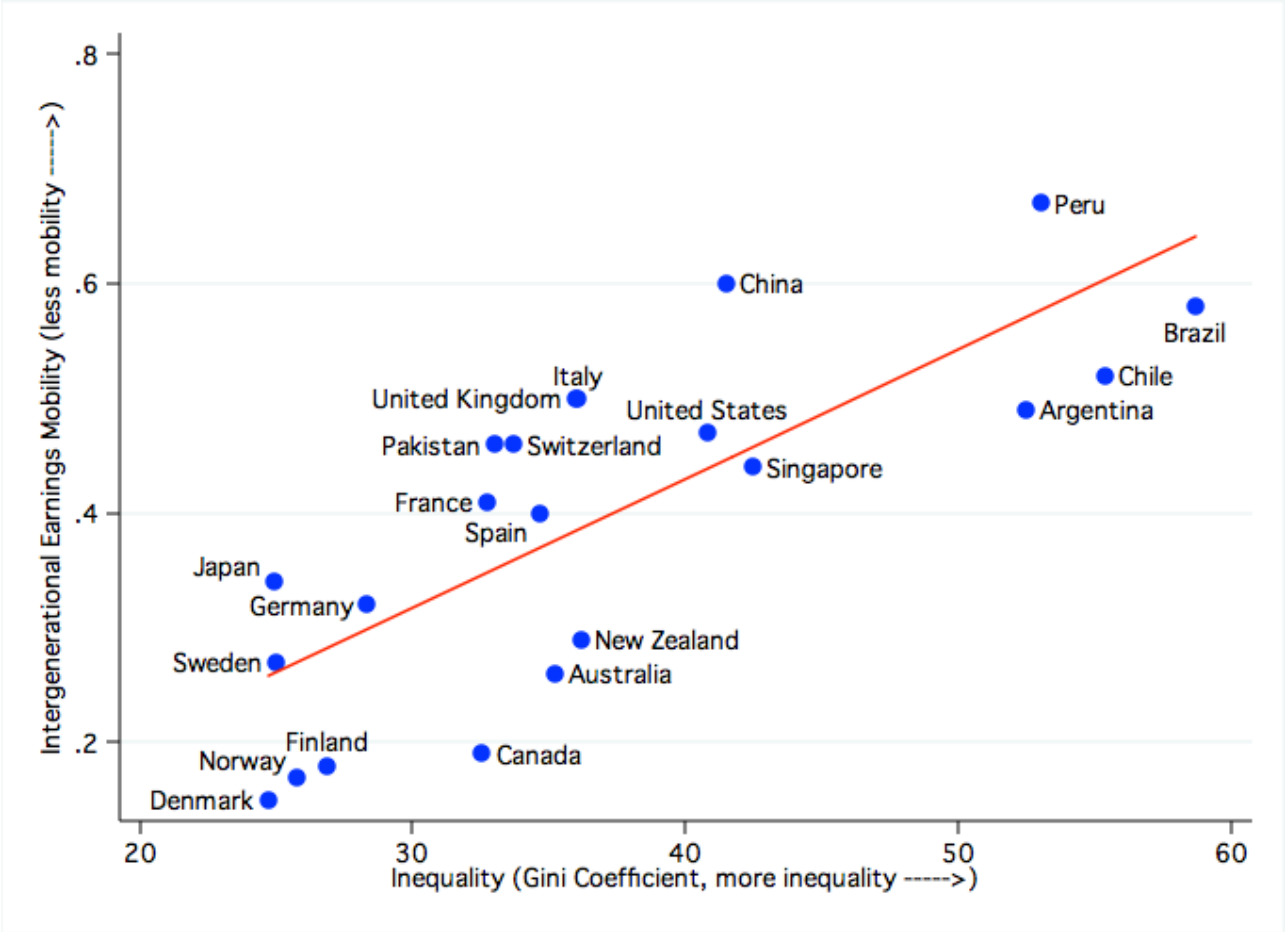
- Perceptions of inequality are often strongly linked to notions of *fairness*
  - Link to Nozick's influential theory of justice
  - Inequality arising from different application of effort is acceptable
  - But inequality due to inequality of *opportunity* is unjust
    - Outcome differences due to factors (circumstances) beyond an individual's control are unfair
- How to assess the extent of inequality of opportunity?
  - We can decompose inequality between groups defined in terms of "morally irrelevant" circumstances
    - Calculate the share of "between-group" inequality to total inequality
    - Anand (1983) examines how much of Malaysian inequality is attributable to racial differences (between Malay, Chinese, Indian Malay, and "other"races)
    - Suggests that between 8-10% of total inequality in Malaysia is due to differences in average incomes of these four groups
    - This provides a *lower bound* estimate of inequality of opportunity in Malaysia in 1970.
  - Decomposition analyses like this are widely conducted

Inequality and Mobility

- Mobility analysis enquires into *dynamics* of distributional change
  - Follow individuals over time
    - This typically requires panel data
    - Some headway is possible with “synthetic” panels and/or retrospective data
- Intragenerational mobility: tracking the same individuals over time
  - Provides a window on chronic and transitory poverty
  - Can help to identify “poverty traps” and “inequality traps”
  - Helps policy makers choose between structural intervention or safety nets
- Intergenerational mobility: tracking individuals across generations
  - Provides an alternative window on inequality of opportunity:
    - With equal opportunity there would likely be considerable intergenerational mobility

Outcomes today are linked to economic mobility tomorrow

- Figure shows the relationship between inequality today and intergenerational earnings mobility
- Referred to as “the Gatsby Curve” (Corak, 2012)



2. Data sources and concerns

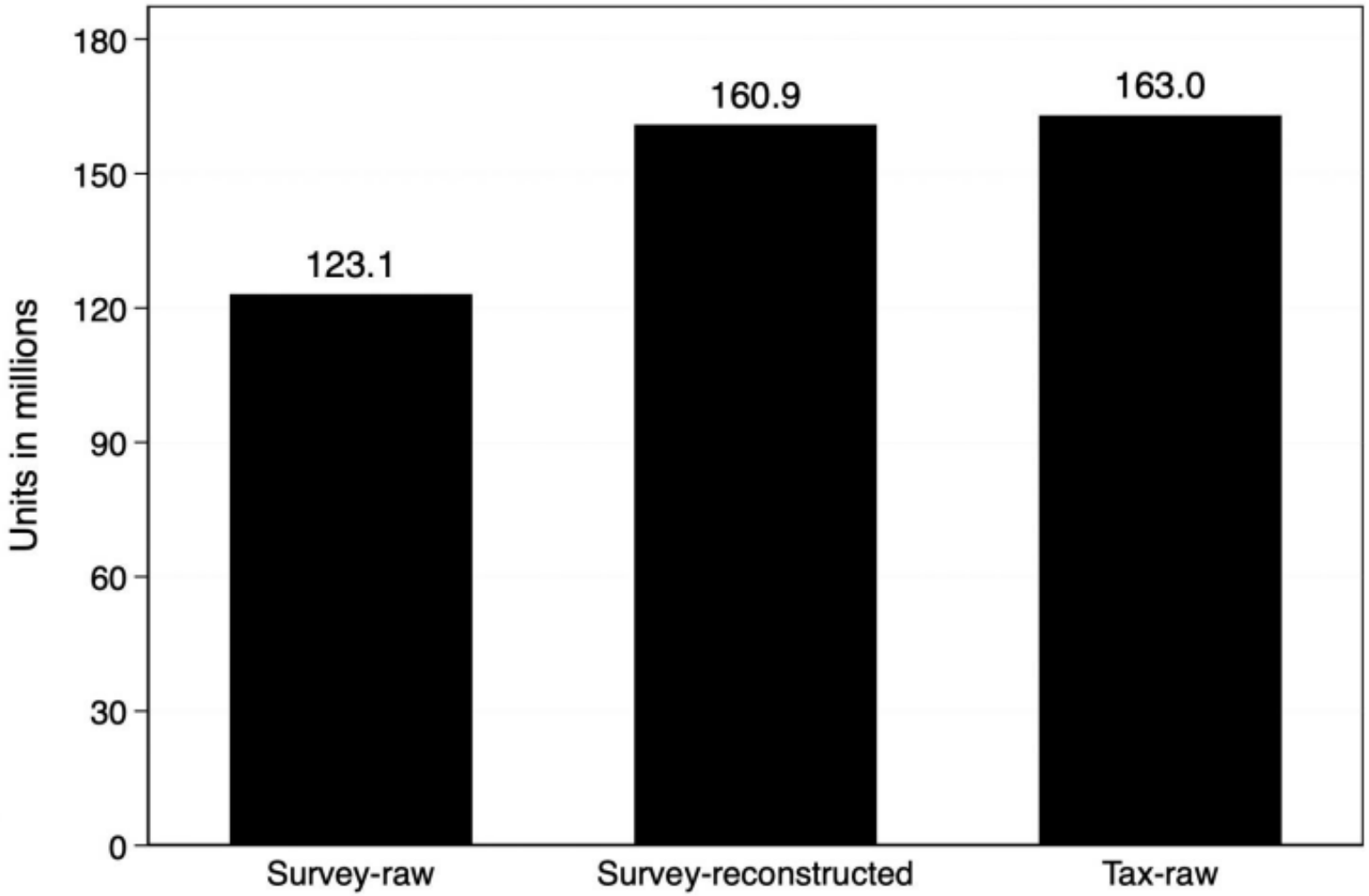
Measurement concerns with Survey data

- Non-compliance (refusal to participate), underreporting, top-coding.
- Income: valuation of home consumption, imputed rent, self-employment income, property income; net or gross income.
- Coverage and classification of consumption
- High probability that non-compliance increases in income because (mean) richer areas generally show higher % of refusal to participate (Lustig 2020).
- US inequality may be underestimated by as much as 4 Gini points or 10% (Korinek, Mistiaen, Ravallion, 2006)

Measurement concerns with tax data

- Tax data suffers from tax exemptions, evasion, avoidance, and misreporting (e.g. Piketty et al. 2011)
- Definitions of income varies as a function of the types of income that are taxed in a given country/region and across time
- The nature of tax units also depends on overarching rules, such as whether couples can file jointly, separately, or both.
- Tax data does not cover all individuals in society.
- Other administrative data sources?

Example of differences in units between survey data and tax data, USA 2013



Source: Yonzan et al. (2023)

3. Inequality Measures

What makes a “good” inequality measure?

1. **Symmetry or anonymity**: if two people swap positions, inequality does not change.
2. **Population invariance**: If number of recipients is multiplied (at each income level) by a constant, inequality does not change
3. **Scale invariance**: If all incomes are multiplied by a constant (Y1=Y*C), inequality does not change.
   - Increase of all incomes by a constant (Y1=Y+C), reduces inequality.
4. **Pigou-Dalton transfer sensitive**: Progressive transfer (which does not change the rankings of individuals) reduces inequality (Dalton’s axiom). (Dalton improvement = income of the poor ↑ by at least as much as income of the rich goes down. “Leakages” may prevent Dalton improvement.)

Measures of inequality

Desirable properties and how different measures satisfy them.

	Gini	GE(0) or mean log deviation (not entropy)	GE(1) or entropy index	Relative mean deviation
Formulas	$\frac{1}{2m}\frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}\lvert y_i - y_j \rvert$	$\frac{1}{n}\sum \log \frac{1/n}{y_i/mn}$	$T*(1) = \frac{1}{n}\sum_i \frac{yi}{m}\ln\frac{yi}{m}$	$\frac{1}{2m}\frac{1}{n}\sum_{i=1}^{n}\lvert y_i - m \rvert$
	$\mathbf{Kow}\left[\mathbf{u} - \frac{1}{\bar{y}}\mathbf{y}\right]$	$\log m - \frac{1}{n}\sum_i \log(y_i)$		
	$\frac{2}{m} cov(y, r_y)$			
Compares person's income to:	Other persons' income (*absolute differences*)	Individual's share in population total income	Mean	Mean
Features	Mean-normalized measure. Shows percentage difference between incomes of two randomly selected individuals	Compares one's share in population (1/n) to one's share in total income (yi/mn).	Both weight and income comparison the same (yi/m)	Mean-normalized measure
Intuitive explanation	Gini of 30 means that the expected difference in income between 2 randomly selected persons is 60 percent of overall mean income; 30% of people have 0 income, and remaining 70% share all income equally			Shows percentage of total income that should be transferred so that all incomes are the same.
Income-scale independence (if all incomes increase by the same %, measure does not change)	Yes	Yes	Yes	Yes

	Gini	Theil 0 or mean log deviation	Theil 1 or entopy index	Relative mean deviation
Absolute increase of all incomes reduces inequality	Yes	Yes	Yes	Yes
Size independence (population size does not affect the measure)	Yes	Yes	Yes	Yes
Progressive transfer reduces inequality (The Pigou-Dalton transfer principle)	Yes	Yes	Yes	Not if both individuals have incomes greater (or lower) than the mean.
Symmetrical (if two people change their places, measure is not affected)	Yes	Yes	Yes	Yes
Measure varies between 0 and 1	Yes	Not bounded from above;	Ln n is maximum	Yes
Decomposability (between recipients and between income sources)	Yes, between income sources No, between recipients	Yes (both)	Yes (both)	No
Sensitivity to transfers	Greatest at the mode (varies as density function of the distribution)	Greatest at low income levels		Insensitive if transfers take place between two individuals with income greater (or lower) than the mean.

“Partial” inequality measures

- *Kuznets ratios*: income share of the top *x* percent over the share of bottom *y* percent
- *Top income shares*: e.g., income share of top 10%

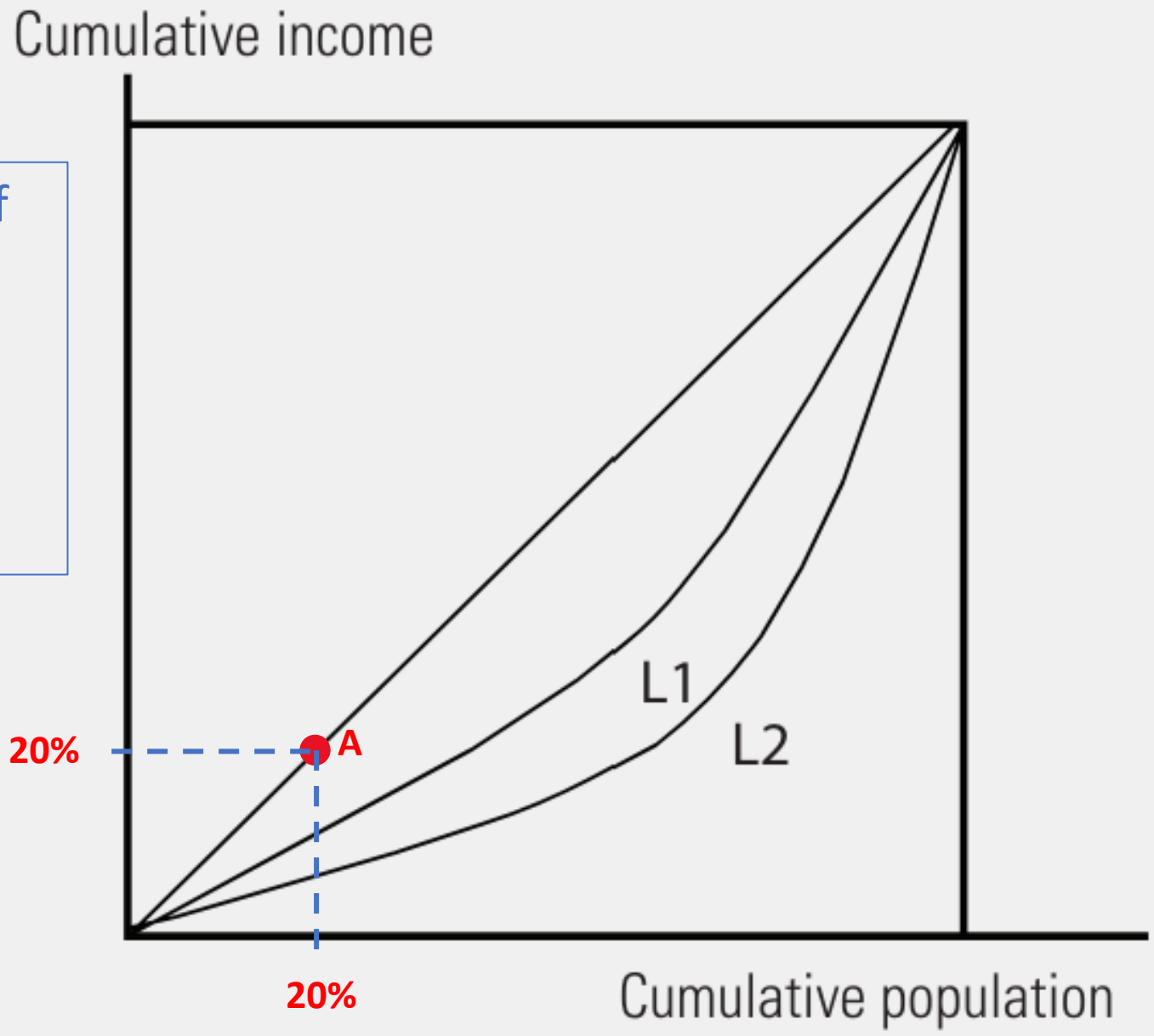
- These measures satisfy principles 1-3 and are *weakly* transfer sensitive. These measures focus on limited ranges of income.
- *Weak transfer principle*: if there is a regressive transfer, the inequality index can never decline, but it may remain unchanged.
- One important reason that these measures are continually used is their ease of communication (this will come up again with Prosperity Gap).

Lorenz curve

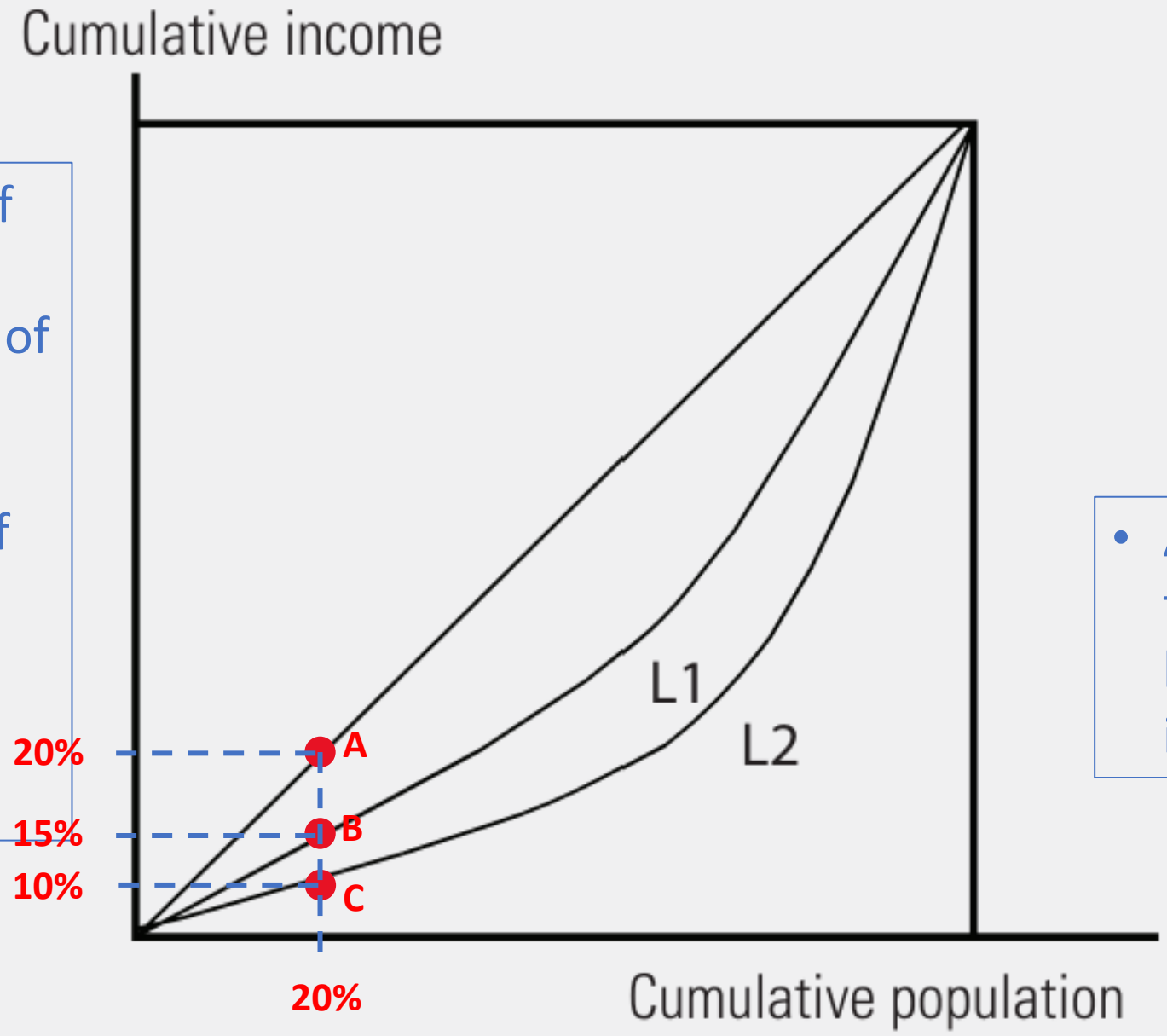
1. Rank the population according to income from lowest to highest
2. Plot cumulative shares of individuals in the population against their respective cumulative share in total income.
3. Draw the curve known as the Lorenz curve.

- The horizontal axis of the Lorenz curve shows the *cumulative percentages of the population* arranged in increasing order of income.
- The vertical axis shows the *percentage of total income* received by a fraction of the population.

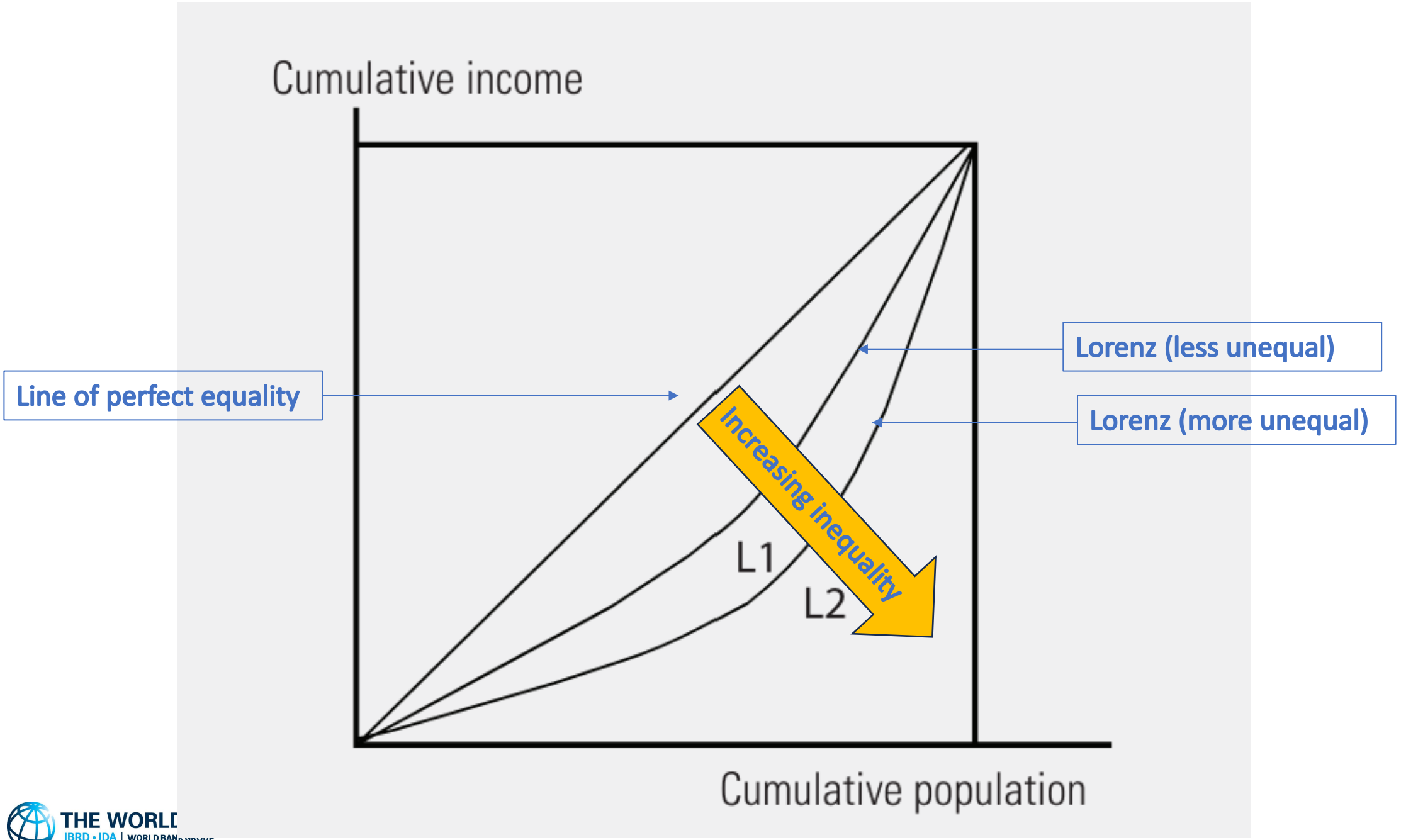
- At point A, 20% of the population have 20% of the income



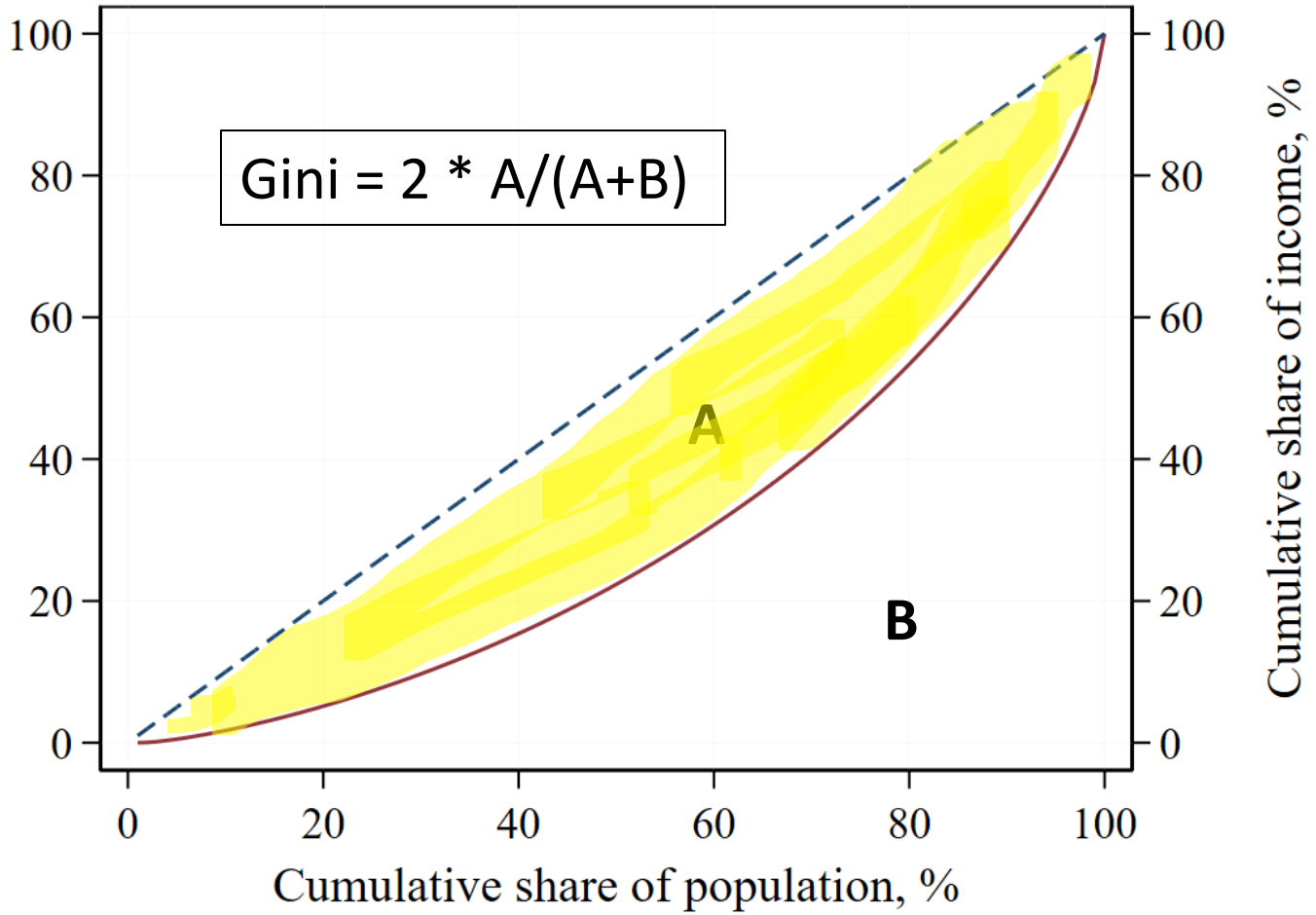
- At point A, 20% of the population have exactly 20% of the income.
- At point B, 20% of the population have 15% of the income.



- At point C, 20% of the population have 10% of the income.


Cumulative income
Line of perfect equality
Lorenz (less unequal)
Lorenz (more unequal)
Increasing inequality
L1
L2
Cumulative population

Gini index

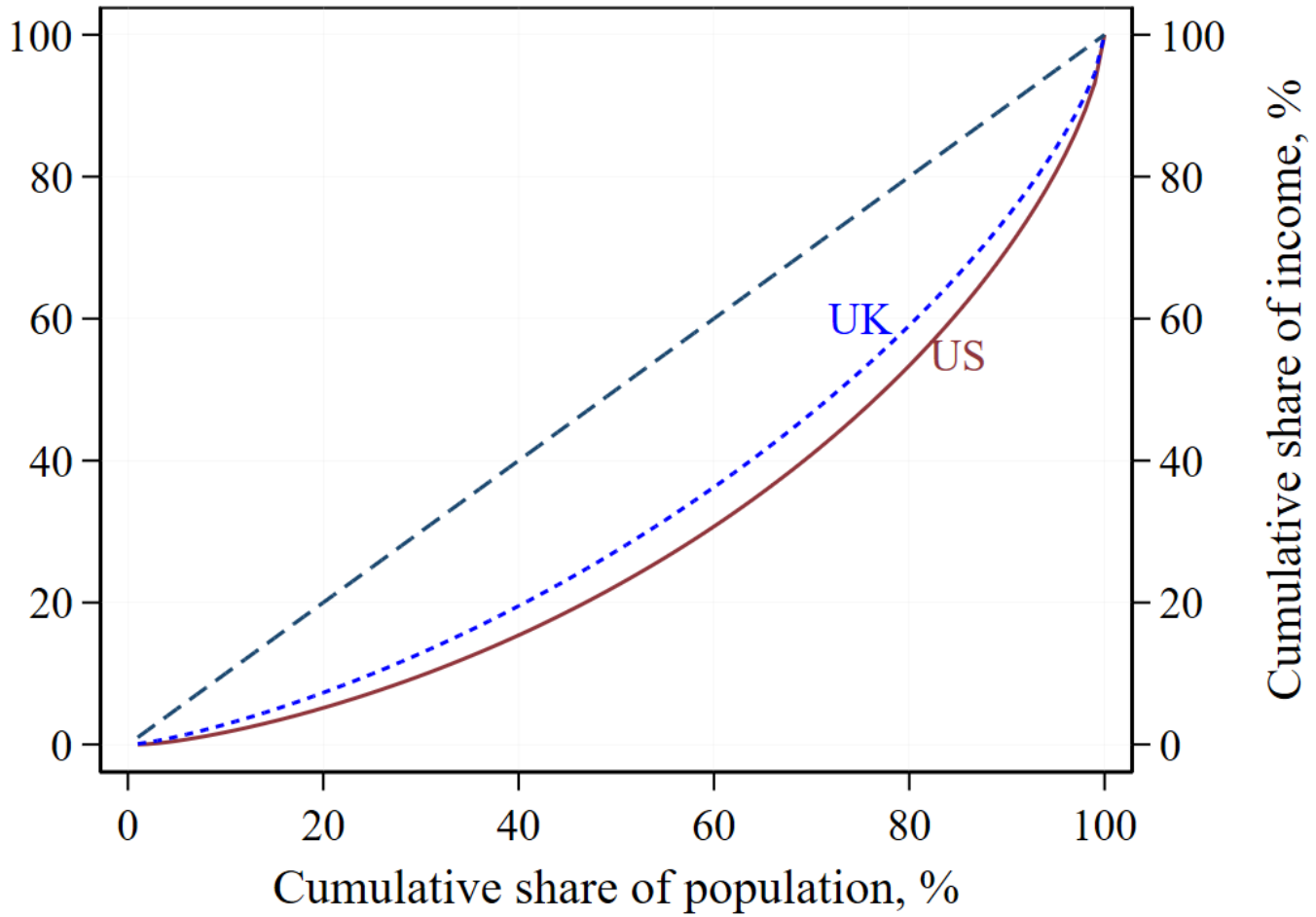


- Gini for the US in 2017 is 0.41

Source: Poverty and Inequality Platform (version 20230328_2017_01_02_PROD) [data set]. pip.worldbank.org.

Lorenz/Gini: US compared to UK in 2017

- Gini for the US in 2017 is **0.41**
- Gini for the UK in 2017 is **0.34**
- Area between the Lorenz for the UK and the 45-degree line is less than the area for the US.



- Gini ranges between 0 and 1 (or 1 to 100).

Source: Poverty and Inequality Platform (version 20230328_2017_01_02_PROD) [data set]. pip.worldbank.org.

Gini index in a 3-person society

- Three individuals A, B, and C with incomes $20, $40, and $60.

Absolute income difference ($\lvert y_i - y_j \rvert$)	**A ($20)**	**B ($40)**	**C ($60)**	**Sum of differences**
A ($20)	0	20	40	
B ($40)	20	0	20	
C ($60)	40	20	0	
Total	**60**	**40**	**60**	**160**

- Gini = $\frac{1}{2\mu} \times \frac{1}{n^2} \sum_{j=1}^{m} \sum_{i=1}^{n} \lvert y_i - y_j \rvert$
- Mean income, $\mu = 40$
- Total number of individuals, n = 3
- Gini = $\frac{1}{2 \times 40} \times \frac{1}{3^2} \times 160$
- Gini = 0.22
- The **expected difference in income between any 2 randomly selected persons is 44 percent (i.e., 0.22 x 2) of the mean income**.

Mean Log Deviation (MLD)

- Class of generalized entropy measures, the GE(0). Also known as the Theil *L*.

$$MLD = \frac{1}{N}\sum_{i=1}^{N} \ln \frac{\bar{x}}{x_i}$$

- It is the average distance from mean income in logs.
- It is not bounded, i.e. doesn't have an upper threshold.
- Useful property: it can be additively decomposed into within and between group inequality.

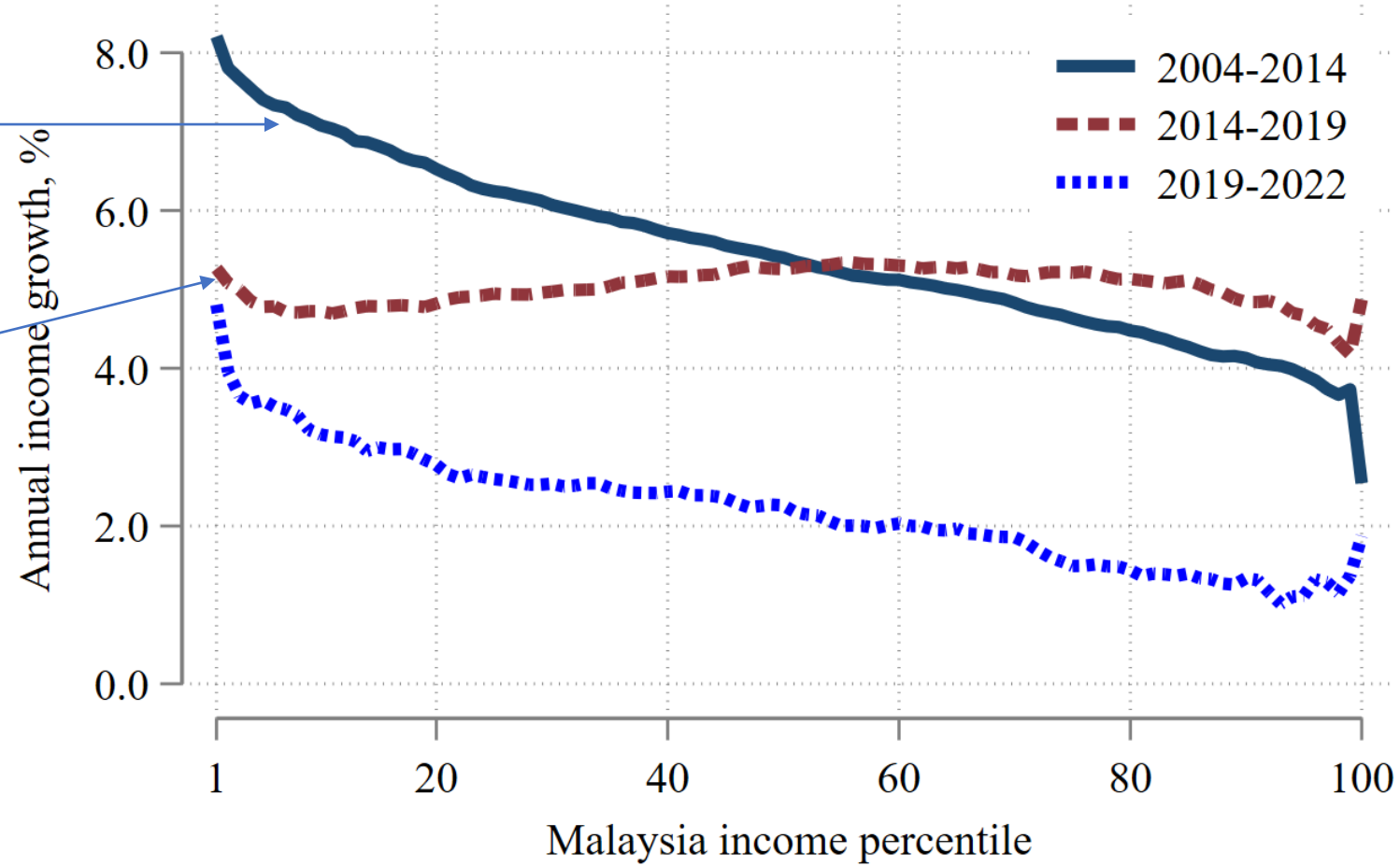
Growth incidence curve

- The Growth Incidence Curve (GIC) captures income growth rate of every person ranked from the poorest to the richest. So, it is a useful tool to visualize inequality changes.
- Note that when using cross-sectional data, one cannot identify individuals across time and hence the income growth of percentiles is used for GIC. This is referred to as the *anonymous* GIC.
- Shape of GIC:
  - Horizontal (flat) => everyone's income grew at the same rate, no change in inequality
  - downward sloping => pro-poor growth
  - upward sloping => pro-rich growth

Growth incidence curve of Malaysia

Pro-poor GIC

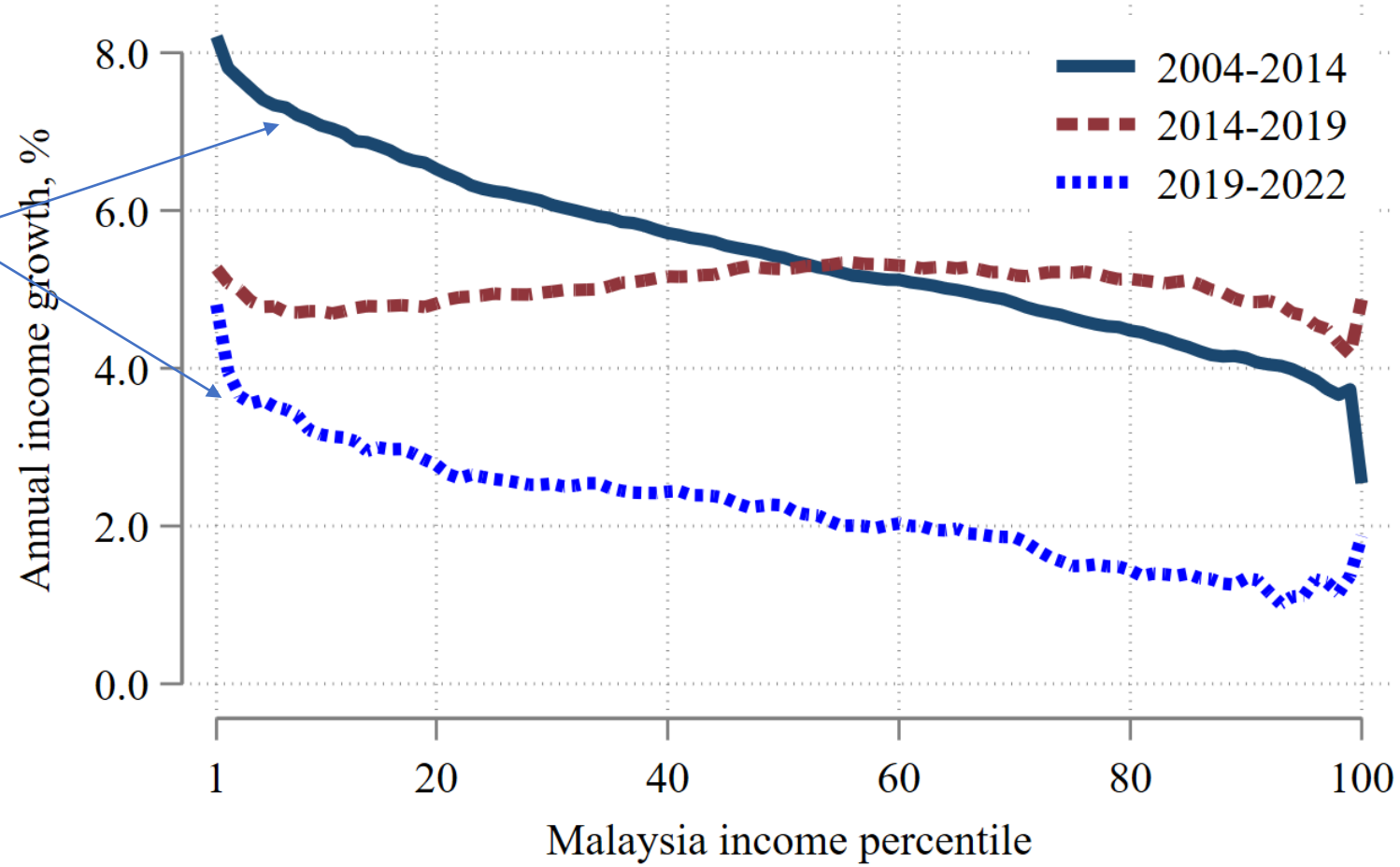
Fairly flat GIC, implying inequality didn't change much



Source: Measuring and Addressing Inequality in Malaysia (2024), World Bank

Growth incidence curve of Malaysia

Note that both are pro-poor GIC, however, the levels of growth are different



Source: Measuring and Addressing Inequality in Malaysia (2024), World Bank

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