8.1 for this problem, refer to the prior ones and their results.

$H_0: X = N \qquad z = \frac{1}{\sigma_n^2}\sum_i\sum_j x_{ij}S_{ij} \qquad D(z_1) = \frac{1}{\sigma_n^2}\sum_i\sum_j S_{ij}^2.$

$H_1: X = S+N.$

$f(z|H_0) \sim N(0, \sum_i\sum_j \frac{S_{ij}^2}{\sigma_n^2})$

$f(z|H_1) \sim N(\sum_i\sum_j \frac{S_{ij}^2}{\sigma_n^2}, \sum_i\sum_j \frac{S_{ij}^2}{\sigma_n^2}).$

since by definition.

$$\lambda(z) = \frac{f(z|H_1)}{f(z|H_0)} = \frac{\cancel{\frac{1}{\sqrt{2\pi D(z)}}}\exp\left(-\frac{(z-D(z))^2}{2D(z)}\right)}{\cancel{\frac{1}{\sqrt{2\pi D(z_1)}}}\exp\left(-\frac{z^2}{2D(z)}\right)}.$$

$$= \exp\left(\frac{\cancel{z^2}-\cancel{z^2}+2zD(z)-D^2(z)}{2D(z_1)}\right)$$

$$= \exp\left(z - \tfrac{1}{2}D(z_1)\right)$$

$$= \exp\left(z - \sum_i\sum_j \frac{S_{ij}^2}{2\sigma_n^2}\right)$$

$\therefore \ln\lambda = z - \frac{1}{2\sigma_n^2}\sum_i\sum_j S_{ij}^2$

then $\ln\lambda$ is the ~~like~~ natural log of likelihood ratio for this specific problem.

8.2 b) $f(\ln\lambda|H_0) \Leftrightarrow f(g(z)|H_0) \quad g(z) = \ln\lambda = z - \frac{1}{\sigma_n^2}\sum_i\sum_j S_{ij}^2$

$\therefore f(\ln\lambda|H_0) = \frac{1}{\sqrt{2\pi D(z_1)}}\exp\left[-\frac{(z-D(z_1))^2}{2D(z_1)}\right] \qquad \int_{-\infty}^{\beta} f(\ln\lambda|H_0)\,dz = P_F.$

$f(\ln\lambda|H_1) = \frac{1}{\sqrt{2\pi D(z_1)}}\exp\left[-\frac{(z-2D(z_1))^2}{2D(z_1)}\right]. \qquad \int_{-\infty}^{\beta} f(\ln\lambda(z)|H_1)\,dz = P_D.$