Metrics of the degree of innovation in music notes

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Summary

In this project, I focused on the following points

- First, I use English plain text to test my programs concerning mutual information, entropy and entropy rate and obtained similar results comparing with the lecture notes.
- Second, I calculated the entropy and first order conditional entropy for source music pieces, to get a general understanding of them.
- Third, I used four different kinds of Hidden Markov Models proposed in the article to generate the Algorithm Composed Music, and obtained the data to process.
- Fourth, I calculated the mutual information between input sequence and output sequence, and the first order conditional entropy of the hidden Markov Chain(which should equal to the entropy rate of it)
- Finally, I analyzed the results and give further discussion.

Background Theory

- Hidden Markov Model(HMM)
- Entropy rate: for Markov Chain or for general stationary random processes
- Mutual information, entropy and other metrics of the degree of innovation(RMSE of entropy, Musicality, etc.)
- Baum-Welch Algorithm(which I only used but not delved into)
- Manipulations of text, MIDI format music and symbol sequences with Python…

$$H(\chi) = \lim_{n\to\infty} H(X_{n+1} \mid X_1, \ldots, X_n)$$

$$H_M(\chi) = H(X_2 \mid X_1)$$

$$I(X;Y) = \sum_{y\in Y}\sum_{x\in X} p(x,y)\log\left(\frac{p(x,y)}{p(x)\,p(y)}\right)$$

$$p\left(x_{1:n}, z_{1:n}\right) = p\left(z_1\right)p\left(x_1 \middle| z_1\right)\prod_{t=2}^{n} p\left(z_t \middle| z_{t-1}\right)p\left(x_t \middle| z_t\right)$$

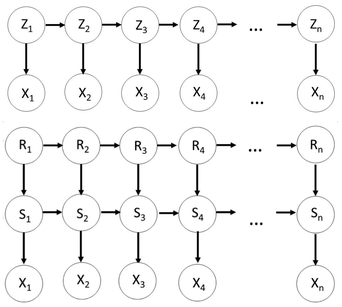
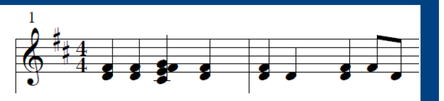
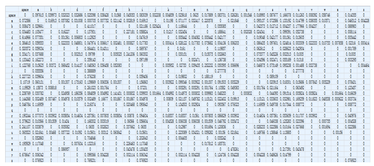


Figure 1. Background theories: HMM, 2-states HMM, formulae

1. Text analysis



After programming the codes to calculate entropy, mutual information and entropy rate, I tested my code with English text. As it was referred in the lecture, the approximated entropy rate of English corpus is 1.3bits. I used the whole novel of *The Lord of the Rings* to get a overall estimation of around 1bit. Also, the entropy of English letters is around 4.2bits, which is lower than the uniform distribution of all 26 letters and a space.



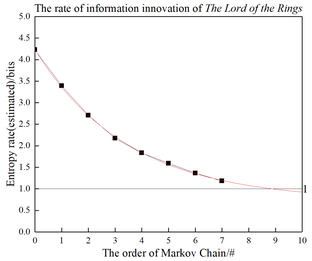


Figure 2. Entropy rate approximation of English text

2. Source music analysis

According to the data acquired from the original pieces of music, it could be found that the overall length and complexity can be reflected by the entropy and first order conditional entropy. The easier and less wavy sounds are lower in both metrics, while the complex and changeful pieces scored higher.

3. Algorithm composition

The generated pieces are not as natural as the original pieces to human audition, although several recognizable bars are among the new pieces when more hidden states and lower convergence are allowed. Also, according to the results in the article, 4 major models are used to generate new pieces.

4. Metrics of innovation

1. Entropy of new pieces

For all the new pieces with all models, the entropy follow the same trend with corresponding original pieces. Also, shorter pieces generally have lowest entropy

We can get a feeling of being simple of a certain music by just hearing it.

2. Mutual Information between original and new piece

The more complex the model is, the less MI remains. That's might be why the higher order generated pieces are not euphoric. A simple piece of music do not need too heavy hidden states modeling.

Also, lower MI means the learnt model is not successful. The models for game music are less successful than for traditional music.

3. Entropy rate of HMM pieces

While the simplest *Ode to Joy* have lowest entropy rate, the highest was obtained with *Bach Cello Suite 1b*. Surprisingly we found that generated game music is not as "meaningful" as Bach ones, either because of the worse learnt model or a less meaningful original creation.

Also, according to the data, music pieces are more likely to be improvised in a note-by-note continuous fashion, rather than a notes-jumping fashion.

	single entropy	entropy rate 1st order	length(lines)
bach cello 1a	3.9938	2.5889	594
bach cello 1b	3.9018	2.6799	1010
beethoven ode to joy	3.0371	1.8014	97
FC contra	5.1441	3.0161	5983
FC supermario	4.5048	2.2367	897
pachelbel canon	4.0010	2.2205	155
FC slms	5.2106	2.6550	18977
GBA ruby	4.9997	4.1586	4741

Table 1. Information metrics for original pieces

5. Conclusion

The project mainly focused on how to gain a insight of the "meaning" and degree of innovation of a given music piece. Hidden Markov Models are used to investigate the underlying essence of music. It was shown that music pieces are tend to be continuous and non-random. Music with higher entropy sounds more complex, while which with higher entropy rate for it's hidden Markov Chain are more meaningful and innovative.

References

[1] Yanchenko, Anna K., and Sayan Mukherjee. "Classical Music Composition Using State Space Models." arXiv preprint arXiv:1708.03822 (2017).

[2] Coffman, Don D. "Measuring musical originality using information theory." Psychology of Music 20.2 (1992): 154-161.

[3] Walker, J. (2008, January). Midi-csv