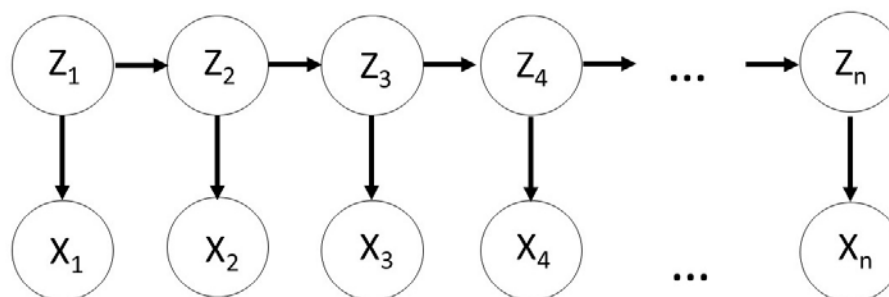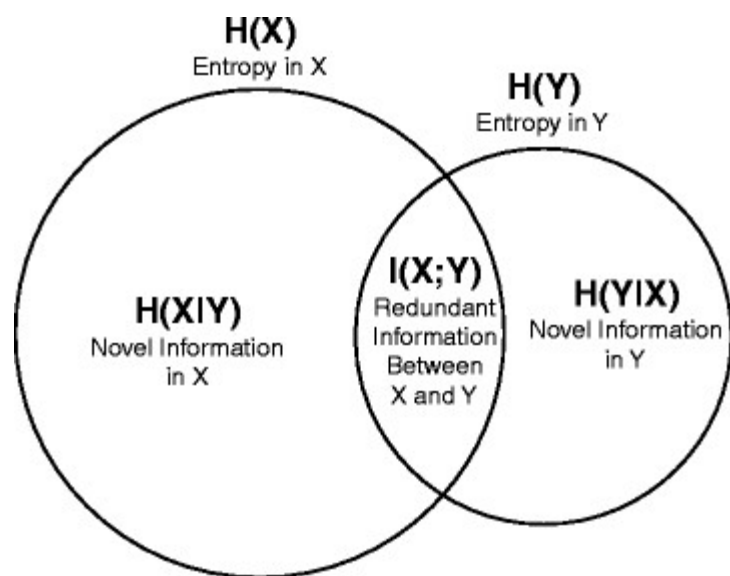# *Metrics of the degree of innovation in music notes*

Background theories

Information theory

- Entropy

- Mutual Information

- Entropy rate

- Entropy rate for Markov Chain

$$H(\chi) = \lim_{n\to\infty} H(X_{n+1} \mid X_1, \ldots, X_n)$$

$$H_M(\chi) = H(X_2 \mid X_1)$$

$$I(X;Y) = \sum_{y\in Y}\sum_{x\in X} p(x,y)\log\left(\frac{p(x,y)}{p(x)p(y)}\right)$$

Music theory

- Harmonic

- Melodic

- Dissonance

Background theories

Hidden Markov Chain(HMM)

- First-order, higher-order, two-states, layered, etc.

Baum-Welch Algorithm

- Given an observation sequence x, learn the model λ's parameters that maximize p(x|λ)

Algorithm Composition

- With HMM

- With Recurrent Neural Networks(RNN)

Python programming

1. Text analysis

The Lord of the Rings

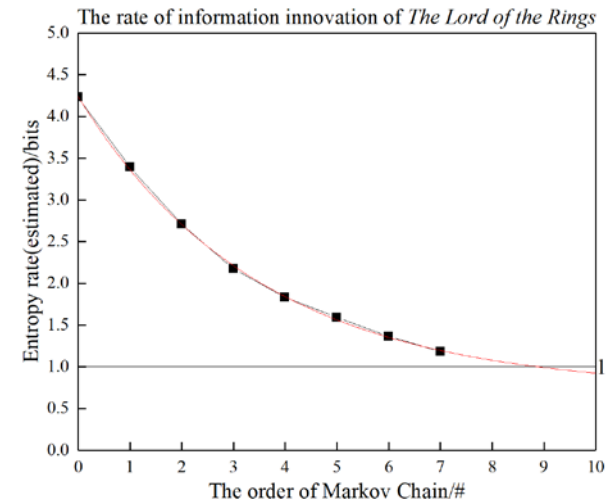- Entropy 4.2bit, Entropy rate -> 1bit

Declaration of Independence

- H = 4.08, H1 = 3.19

Harry Potter

- H = 4.10, H1 = 3.33

The rate of information innovation of *The Lord of the Rings*

The error of estimation: length of the text:

$27^5 \approx 10M$ and a lot prob. will be zero!

2. Source music analysis

Classical music pieces

- Not too changeful, lower zero & first order conditional entropy

Ode to Joy is definitely easiest piece! Continuous and repetitive.

FC game music pieces

- Very complex, with a variety of notes included

- ***Super Mario*** is more predictable(first order) than other FC games! Maybe that's why it was famous?

*(some inconsistent numeric with article, might be caused by different source version)

	single entropy	entropy rate 1st order	length(lines)
bach cello 1a	3.9938	2.5889	594
bach cello 1b	3.9018	2.6799	1010
beethoven ode to joy	3.0371	1.8014	97
FC contra	5.1441	3.0161	5983
FC supermario	4.5048	2.2367	897
pachelbel canon	4.0010	2.2205	155
FC slms	5.2106	2.6550	18977
GBA ruby	4.9997	4.1586	4741
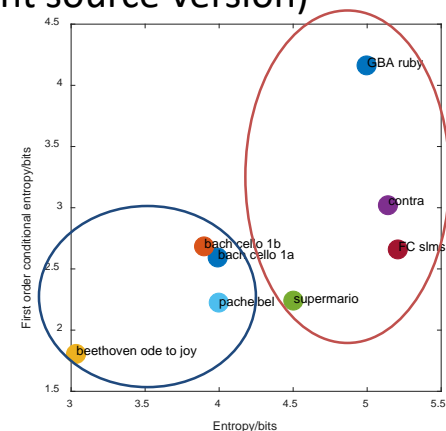
3. Algorithm composition

1st order HMM

2nd order HMM

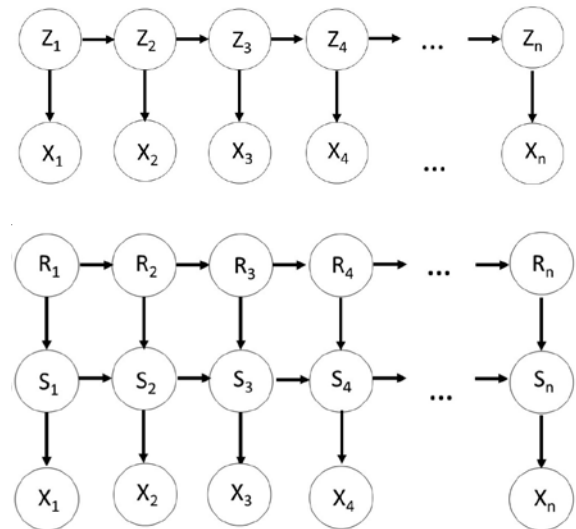- Computation starts to become slower

Layered HMM

- More acceptable result

2 hidden states HMM

- Even slower

Other thoughts: the original model in the article could be improved, with the processing of notes and MIDI file.

- Leak Notes for harmonic notes

4. Metrics of innovation

Entropy of new pieces

- Easier pieces have lower entropy/entropy rate.

- Obey the same trend with original pieces

Mutual Information between the original and the new

- 2nd order is lower than 1st order and layered model, which might be why we think higher order model not euphoric

- For every model, game music are less than traditional music, still might be because too changeful, should add more hidden states to learn the model

- Lower MI means the model learnt is not so successful.

4. Metrics of innovation

Entropy rate of the HMC modeled by the input sequence

- Overall, *Ode to Joy* is the least innovative music in the test, while the Bach Cello Suite 1b is the most innovative!

- Surprisingly we found that *Bach Cello 1b* is actually more complex than *1a* despite a less usage of notes

- 2nd order model is already enough?

- Contra could have been more innovative!

    (How to select the parameters...)

	average 1st order single entropy	average mutual information	average entropy rate
bach cello 1a	4.040	0.620	1.402
bach cello 1b	3.869	0.348	1.588
beethoven ode to joy	2.967	0.790	0.814
contra	4.870	0.429	1.460
supermario	3.344	0.476	1.378
pachelbel canon	3.992	1.378	1.188

Conclusion

- How could we interpret a piece of music with the insight of information theory

- Is it proper to analyze the "feelings" or "meanings" of a music with a Markov Chain? – Possibly

- The real "meanings" can be somehow reflected with the entropy rate of the hidden Markov model derived from certain music.(Thinking about the entropy rate of text)

- Game music are more complex, but contains less "meanings"

- Practice pieces are really meant to be easiest!

- Other interesting topics related:

Improving the model/algorithm composition/pure music to MIDI, ML for de-vocal/