Worksheet answers

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A.
$x_1 = \pi$ $\quad a_1 = \frac{x_1}{x_2} = \frac{\pi}{2}$

$x_2 = 2$ $\quad a_2 = a_1^{x_3} = \frac{\pi}{2}$

$x_3 = 1$

$$\frac{dy}{dx_1} = \frac{dy}{da_2}\cdot\frac{da_2}{da_1}\cdot\frac{da_1}{dx_1} = -\sin(a_2)\cdot x_3\cdot a_1^{x_3-1}\cdot\frac{1}{x_2} = \boxed{-\frac{1}{2}}$$

$$\frac{dy}{dx_2} = \frac{dy}{da_2}\cdot\frac{da_2}{da_1}\cdot\frac{da_1}{dx_2} = -\sin(a_2)\cdot x_3\cdot a_1^{x_3-1}\cdot -\frac{x_1}{x_2^2} = \boxed{\frac{\pi}{4}}$$

$$\frac{dy}{dx_3} = \frac{dy}{da_2}\cdot\frac{da_2}{dx_3} = -\sin(a_2)\cdot a_2^{x_3}\cdot \ln(a_2) = \boxed{\ln\left(\frac{\pi}{2}\right)\cdot -\frac{\pi}{2}}$$

B. $y = \cos\left(\left(\frac{x_1}{x_2}\right)^{x_3}\right)$

$$\frac{dy}{dx_1} = -\sin\left(\left(\frac{x_1}{x_2}\right)^{x_3}\right)\cdot\left(\frac{x_1}{x_2}\right)^{x_3-1}\cdot\frac{1}{x_2}$$

$$\frac{dy}{dx_2} = -\sin\left(\left(\frac{x_1}{x_2}\right)^{x_3}\right)\cdot\left(\frac{x_1}{x_2}\right)^{x_3-1}\cdot -\frac{x_1}{x_2^2}$$

$$\frac{dy}{dx_3} = -\sin\left(\left(\frac{x_1}{x_2}\right)^{x_3}\right)\cdot\left(\frac{x_1}{x_2}\right)^{x_3}\cdot \ln\left(\left(\frac{x_1}{x_2}\right)^{x_3}\right)$$

C.
$$f = \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} -2 \\ 3 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 10 \end{bmatrix}$$

$$\sigma[f] = \begin{bmatrix} \max(0,0) \\ \max(10,0) \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 10 \end{bmatrix}$$

next layer =>

$$\begin{bmatrix} a_1 \\ a_2 \end{bmatrix}[v_1 \ v_2] = \begin{bmatrix} 0 \\ 10 \end{bmatrix}[3 \ 2] = 20$$

output = $\boxed{y = 20}$

D.

$y = a_1 v_1 + a_2 \ 2$

$\mathrm{Relu}(z_2) = \begin{cases} z_2 & z_2 > 0 \\ 0 & z_2 \le 0 \end{cases}$

$z_2 = w_{2,1}x_1 + w_{2,2}x_2 + b_2$

$y = 20$

$v_2 = 2$

$x_2 = 3$

$$\frac{dL}{dw_{2,2}} = \frac{dL}{dy}\cdot\frac{dy}{da_2}\cdot\frac{da_2}{dz_2}\cdot\frac{dz_2}{dw_{2,2}}$$

$$\frac{dz_2}{dw_{2,2}} = x_2$$

$$\frac{da_2}{dz_2} = \begin{cases} 1 & z_2 > 0 \\ 0 & z_2 \le 0 \end{cases}$$

$$\frac{dy}{da_2} = v_2$$

$$\frac{dL}{dy} = 2(y-22)$$

final gradient: $\frac{dL}{dw_{2,2}} = 2(y-22)\cdot v_2\cdot \mathbb{1}(z_2 > 0)\cdot x_2$

$= 2(-2)\cdot 2\cdot 1\cdot 3$

$= -24$