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Problem 1.10
Indicating for each distinct pair i and j whether $f_i(n)$ is $O(f_j(n))$ and whether $f_i(n)$ is $\Omega(f_j(n))$
Case 1 – pair i = 1, j =2
$f_2(n)$ is $O\big(f_1(n)\big)$ or $O(n^2)$ since there exists a constant c and n_0 such that $f_2(n) \leq cf_1(n)$ whenever n > n_0

$$f_2(n) = n^2 + 1000n \;\; and \; f_1(n) = \;\; n^2$$

Proof

Assuming c is some arbitrary constant, $cn^2 \geq \; n^2 + 1000n \rightarrow c(\frac{n^2}{n}) \geq \; (\frac{n^2}{n}) + 1000(\frac{n}{n})$

$$\rightarrow cn \geq n + 1000 \rightarrow (c-1)n \geq 1000$$

$$\rightarrow n = \; \frac{1000}{c-1} \; and \; c = \; \frac{1000}{n} + 1 \approx c = 6$$

assuming c is equal to 6,

$$\rightarrow 6n^2 \geq \; n^2 + 1000n \rightarrow 6\left(\frac{n^2}{n}\right) \geq \; \left(\frac{n^2}{n}\right) + 1000\left(\frac{n}{n}\right) \rightarrow 6n - n \; \geq \; 1000$$

$$\rightarrow 5n \geq 1000 \rightarrow n \geq 200$$

thus when $n \geq \; \frac{1000}{c-1}$ or in this case $n \geq 200$, $f_2(n)$ is $O(f_1(n))$. The inverse relationship is present when $n \leq \; \frac{1000}{c-1}$ or $n < \; n_0 \approx n \leq 200$ when c=6 in this case, thus defining the lower bound of $f_2(n)$ modelled as $f_2(n)$ is $\Omega(f_1(n))$

Case 2 – pair I =1 and j =3
$f_1(n)$ is equally dominated by $f_3(n)$ and dominates $f_3(n)$ over all ranges of operation or infinity due to $f_3(n)$'s piecewise behavior.

$$f_3(n) = \begin{cases} n \;\; if \; n \; is \; odd \\ n^3 \; if \; n \; is \; even \end{cases} \; and \; f_1(n) = \;\; n^2$$

Proof

Case A – even values in the range [0, inf).

$f_1(n)$ is $O(f_{(3,odd)}(n))$ or $O(n^3)$ since there exists a constant c and n_0 such that $f_1(n) \leq cf_3(n)$ whenever n > n_0. $f_3(n)$ dominates $f_1(n)$.

Case A Proof

Assuming c is some arbitrary constant c, $cn^3 \geq \; n^2$

$\rightarrow c\frac{n^3}{n^3} \geq \; \frac{n^2}{n^3} \rightarrow c \geq \frac{1}{n}$

$\rightarrow$ if $c \geq \frac{1}{n} \approx (\frac{1}{4})$, $\left(\frac{1}{4}\right)n^3 \geq \; n^2 \rightarrow \; n^3 \geq \; 4n^2 \rightarrow n \geq 4$

$\rightarrow$ thus, n must be greater than or equal to some constant c and there exists a constant c and n_0 such that $f_1(n) \; is \; O(n^3)$ whenever $n >$

n_0 *which in this specific case is* $n \geq 1/c$ which is always true as c is the inverse of the magnitude of n.

Case B – odd values in the range [0, inf).

$f_{(3,even)}(n)$ is $O(f_1(n))$ or $O(n^2)$ since there exists a constant c and n_0 such that $f_3(n) \leq cf_1(n)$ whenever n > n_0. Thus $f_1(n)$ dominates $f_3(n)$.

Case B Proof

Assuming c is some arbitrary constant c, $cn^2 \geq n$

→ $c\frac{n^2}{n^2} \geq \frac{n}{n^2} \rightarrow c \geq \frac{1}{n}$

→ if $c \geq \frac{1}{n} \approx (\frac{1}{4})$, $\left(\frac{1}{4}\right)n^2 \geq n \rightarrow n^2 \geq 4n \rightarrow n \geq 4$

→ thus, n must be greater than or equal to some constant c whose magnitude must be greater than a maximal magnitude of 1 obtained from the inverse of n and there exists a constant c and n_0 such that $f_3(n)$ *is* $O(n^2)$ whenever $n > n_0$ *which in this specific case is* $n > 1/c$ which is always true as c is the inverse of the magnitude of n.

Now because both piecewise behaviors displayed above fluctuate equally over all values residing in the range [0, inf), $f_1(n)$ is equally dominated by $f_3(n)$ and dominates $f_3(n)$ over all ranges of operation.

Case 3 – pair i = 1, j =4

$f_1(n)$ is $O\big(f_4(n)\big)$ or $O(n^3)$ since there exists a constant c and n_0 such that $f_1(n) \leq cf_4(n)$ whenever n >100.

$$f_4(n) = \begin{cases} n \ \ if\ n \leq 100 \\ n^3\ if\ n > 100 \end{cases} \ and\ f_1(n) = \ n^2$$

Proof

Assuming c is some arbitrary constant, $cn^3 \geq n^2 \rightarrow c(\frac{n^3}{n^3}) \geq (\frac{n^2}{n^3})$

→ $c \geq (\frac{1}{n}) \approx (\frac{1}{101})$

assuming c is equal to $\frac{1}{101}$,

→ $\frac{1}{101}n^3 \geq n^2 \rightarrow n^3 \geq 101n^2 \rightarrow n \geq 101$

thus when $n \geq \frac{1*n^2}{c*n^2}$ or in this case $n \geq 101\ or\ n > 100,\ f_1(n)$ is $O(f_4(n))$.

The inverse relationship is present when $n \leq \frac{1*n^2}{c*n^2}$ or $n < n_0 \approx n < 100$, thus defining the lower bound of $f_1(n)$ modelled as $f_1(n)$ is $\Omega(f_4(n))$ or $\Omega(n)$ since there exists a constant c and n_0 such that $f_1(n) \geq cf_4(n)$

Proof

Assuming c is some arbitrary constant, $\mathrm{cn} \leq \mathrm{n}^2 \rightarrow \mathrm{c}\left(\frac{\mathrm{n}}{\mathrm{n}}\right) \leq (\frac{\mathrm{n}^2}{\mathrm{n}})$

$\rightarrow \mathrm{c} \leq \mathrm{n} \approx 50$

assuming c is equal to 50,

$\rightarrow 50\mathrm{n} \leq \mathrm{n}^2 \rightarrow 50 \leq \mathrm{n}$

thus when $\mathrm{n} \geq 50$ and $\mathrm{c} \leq \mathrm{n}$, $\mathrm{f_1(n)}$ is $\Omega(\mathrm{f_4(n)})$ or $\Omega(\mathrm{n})$.

$f_4(n)$ is $O(f_1(n))$ or $O(n^2)$ since there exists a constant c and n_0 such that $f_4(n) \leq cf_1(n)$ whenever $\mathrm{n} \leq 100$.

Proof

Assuming c is some arbitrary constant $cn^2 \geq n \rightarrow c\left(\frac{n^2}{n}\right) \geq \left(\frac{n}{n}\right) \rightarrow cn \geq 1$

$$\rightarrow c \geq \frac{1}{n} \approx \frac{1}{5}$$

assuming c is equal to $\frac{1}{5}, \rightarrow \frac{1}{5}n^2 \geq n \rightarrow n \geq 5$

thus, when $n \geq 5$ and $c \geq \frac{1}{n} \approx \frac{1}{5}$, $f_4(n)$ is $O(f_1(n))$ or $O(n^2)$

Case 4 – pair I = 2 and j =3

$$f_3(n) = \begin{cases} n \ \ if\ n\ is\ odd \\ n^3\ if\ n\ is\ even \end{cases}\ and\ f_2(n) = n^2 + 1000n$$

This case is similar to case 2 in that the piecewise behavior of $f_3(n)$ allows $f_2(n)$ to dominate $f_3(n)$ for odd values residing in the range [0, inf) and $f_2(n)$ to be dominated by $f_3(n)$ for even values residing in the range [0, inf). Hence, there is an equal level of domination between both function as n approaches infinity. The proof follows the exact form of case 2's.

Case 5 – pair i = 2 and j =4

$f_2(n)$ is $O(f_4(n))$ or $O(n^3)$ since there exists a constant c and n_0 such that $f_2(n) \leq cf_4(n)$ whenever $\mathrm{n} > 100$.

$$f_4(n) = \begin{cases} n\ \ if\ n \leq 100 \\ n^3\ if\ n > 100 \end{cases}\ and\ f_2(n) = n^2 + 1000n$$

Proof

Assuming c is some arbitrary constant, $cn^3 \geq n^2 + 100n \rightarrow c\left(\frac{n^3}{n^3}\right) \geq \left(\frac{n^2}{n^3}\right) + \frac{1000n}{n^3}$

$\rightarrow c \geq (\frac{1}{n} + \frac{1000}{n^2}) \approx (\frac{1}{101})$

assuming c is equal to $\frac{1}{101}$,

$\rightarrow \frac{1}{101}n^3 \geq n^2 + 1000n \rightarrow n^3 \geq 101n^2 + 1000 * 101n \rightarrow n \geq 101 + \frac{101*1000}{n} \rightarrow n \geq 101$

thus when $n \geq \frac{1}{c} + \frac{1}{cn} * 1000$ or in this case $n \geq 101\ or\ n > 100$, $f_2(n)$ is $O(f_4(n))$.

The inverse relationship is present when $n < \frac{1}{c} + \frac{1}{cn}$ or $n < n_0 \approx n \leq 100$, thus defining the lower bound of $f_1(n)$ modelled as $f_2(n)$ is $\Omega(f_4(n))$ or $\Omega(n)$ since there exists a constant c and n_0 such that $f_2(n) \geq cf_4(n) \rightarrow$ by lower bound I am insinuating loosely lower bounded by n.

Proof

Assuming c is some arbitrary constant, $cn \leq n^2 + 1000n \rightarrow c\left(\frac{n}{n}\right) \leq \left(\frac{n^2}{n}\right) + 1000$

$\rightarrow c \leq n + 1000 \approx 100$

assuming c is equal to 100,

$\rightarrow 100n \leq n^2 + 1000n \rightarrow n \geq -900$

thus when $n \geq -900$ and $c \leq n + 1000$ $or\ c = 100\ in\ this\ case,$ $f_2(n)$ is $\Omega(f_4(n))$ $or\ \Omega(n)$ or loosely lower bounded by n.

$f_4(n)$ is $O(f_2(n))$ or $O(n^2)$ since there exists a constant c and n_0 such that $f_4(n) \leq cf_2(n)$ whenever n $\leq$ 100.

Proof

Assuming c is some arbitrary constant $cn^2 \geq n \rightarrow c\left(\frac{n^2}{n}\right) \geq \left(\frac{n}{n}\right) \rightarrow cn \geq 1$

$$\rightarrow c \geq \frac{1}{n} \approx \frac{1}{5}$$

assuming c is equal to $\frac{1}{5}$, $\rightarrow \frac{1}{5}n^2 \geq n \rightarrow n \geq 5$

thus, when $n \geq 5$ and $c \geq \frac{1}{n} \approx \frac{1}{5}$

$f_4(n)$ is $O(f_2(n))$ or $O(n^2)$

Case 6 – pair I = 3 and j = 4

$$f_4(n) = \begin{cases} n\ \ if\ n \leq 100 \\ n^3\ if\ n > 100 \end{cases}\ and\ f_3(n) = \begin{cases} n\ \ if\ n\ is\ odd \\ n^3\ if\ n\ is\ even \end{cases}$$

$f_3(n)$ is significantly dominated by $f_4(n)$ due to the fact that for values greater than n_0 = 100, $f_3(n)$ grows significantly slower than $f_4(n)$ resulting in an upper bound of $O(f_4(n))$ or $O(n^3)$. This can be seen in the function definitions as the functions dominate each other equally until the point when n reaches 100 at which point the function have the same complexity for even values but $f_4(n)$ defines an upper bound of $O(n^3)$ for odd values.

Problem 1.12

- **a.** Procedure matmy - 3 loops with each loop embedded in the other results in a worst case complexity of $O(n^3)$ as for each iteration in loop 1, loop 2 runs n times and for each iteration in loop 2, loop 3 runs n times resulting in n*n*n which is (n^3).

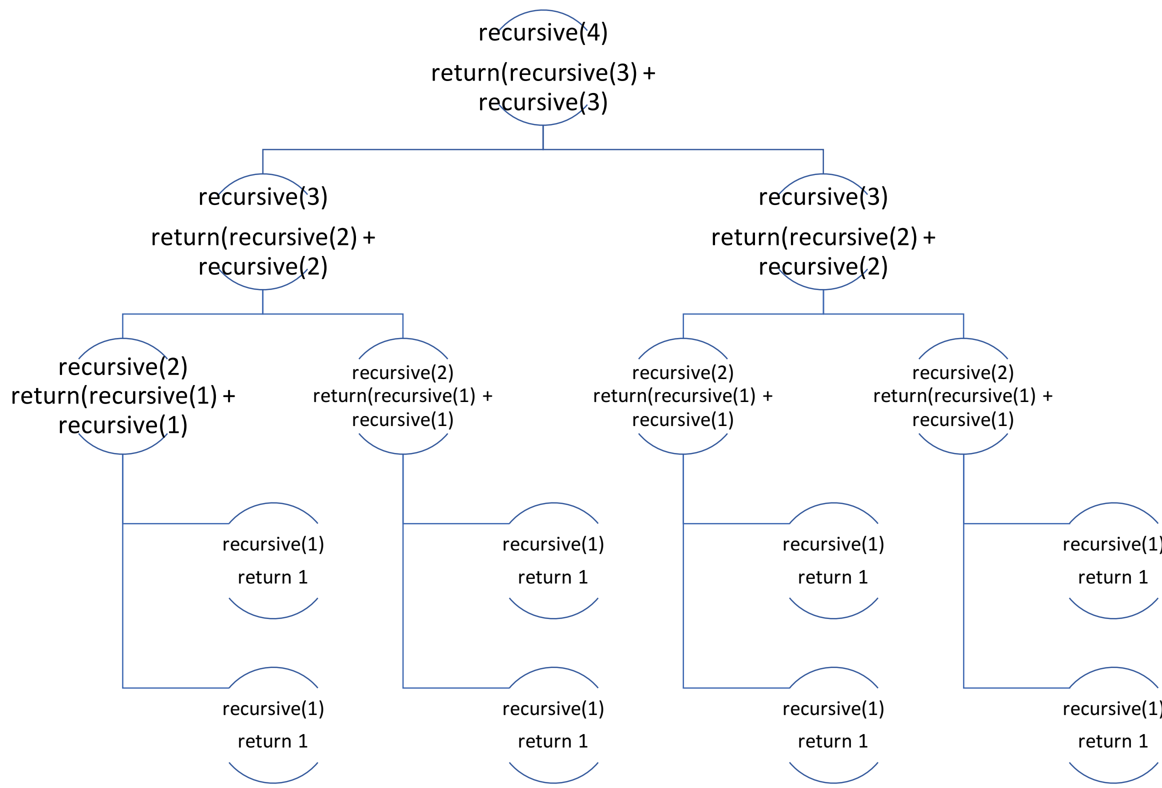
- **b.** Procedure mystery- loop 1 runs n-1 times, loop 2 run n-1 times, and loop 3 runs in proportion to the magnitude of the point of iteration of loop 2 which is n, thus the algorithmic run time complexity is $(n-1)^2 * n \rightarrow O(n^3)$

c. Procedure very odd- The first loop runs n times, if an odd is conditioned upon, 2 inner loops execute resulting in an inner worst case complexity of $O(cn)$. The total worst case complexity in the case where an odd value is operated on is $O(cn^2)$. While, for an even value, it is $O(n)$.

$$F(n) = \begin{cases} even\ O(n) \\ odd\ \ O(n^2) \end{cases}$$

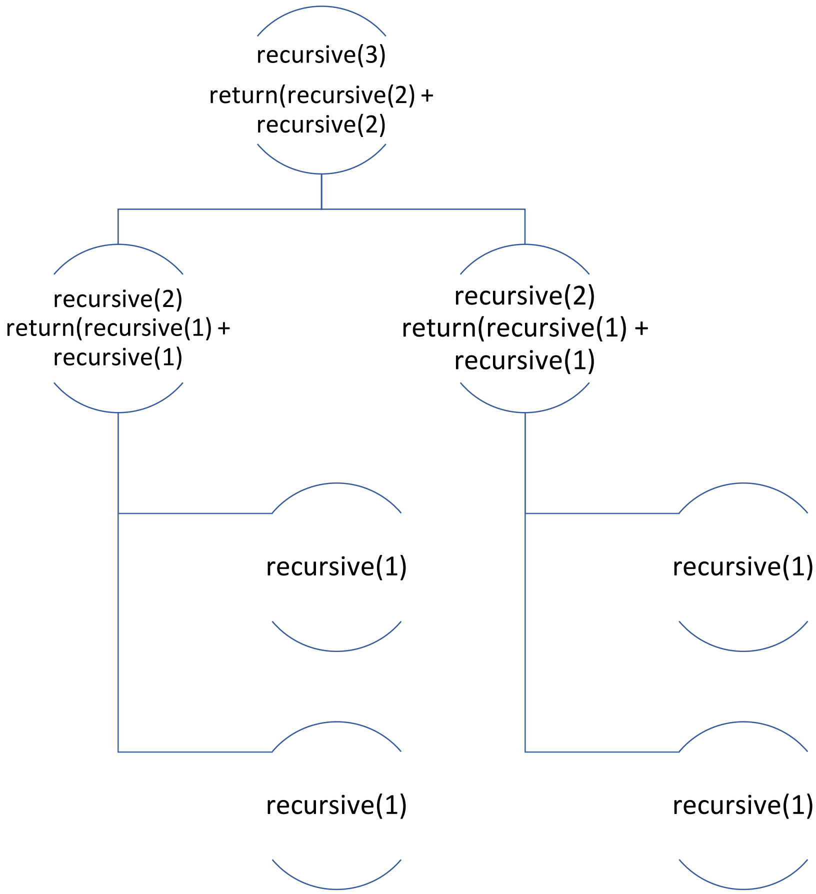
d. Procedure recursive

Case N = 4



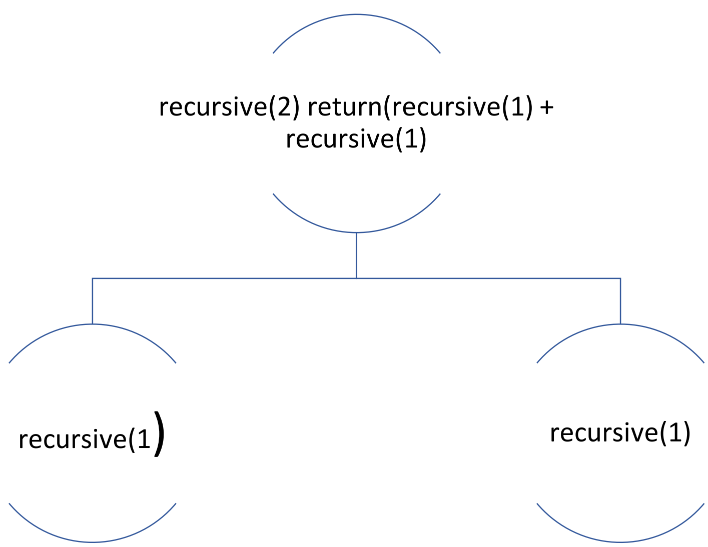
15 function calls performed.

Case N = 3



7 function calls performed.

Case N = 2



3 function calls performed

15 function calls for input n = 4
7 function calls for input n =3
3 function calls for input n =2

From the above expressions, we can see that the number of function calls performed is dependent on the size of the input n and can be modelled by the following expression $2^n - 1$ therefore, this recursive procedure is is upper bound by 2^n function calls