Predictors that Specialize

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Outline

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The specialists setup

bounding cumulative loss using relative entropy

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- Generalizes the switching experts setup.

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On each iteration $t = 1, 2, 3, \ldots$

- Adversary chooses a set $E^t \subseteq \{1, \ldots, N\}$ of awake specialists.

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- Algorithm suffers loss. Specialists in E^t suffer loss. Sleeping specialists suffer no loss.

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- $\mathbf{u}$: comparator distribution, $u_i \geq 0, \ \sum_i u_i = 1$.
- Average loss w.r.t. $\mathbf{u}$: $\ell_{\mathbf{u}}^t \doteq \frac{\sum_{i \in E^t} u_i \ell_i^t}{\sum_{i \in E^t} u_i}$

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- Average loss w.r.t. $\mathbf{u}$: $\ell^t_{\mathbf{u}} \doteq \frac{\sum_{i \in E^t} u_i \ell^t_i}{\sum_{i \in E^t} u_i}$
- Goal: $L_A \leq \min_{\mathbf{u}} \sum_{t=1}^{T} \ell^t_{\mathbf{u}} + \text{something small}$

Ideas

- We focus on normalized weights:

$$v_i^t = \frac{w_i^t}{\sum_{j=1}^{N} w_i^t}, \quad \mathbf{v}^t = \frac{\mathbf{w}^t}{W^t}$$

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- In particular: total weight is always 1.

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$$\text{if } i \notin E_t : \quad p_{t+1,i} = p_{t,i}$$

Bound for SBayes

- For any sequence of awake specialists $E_1, \ldots, E_T$, specialist predictions and outcomes, and for any comparator $\mathbf{u}$:

$$\sum_{t=1} u(E^t)\ell_A^t \leq \sum_{t=1}^{T} \sum_{i \in E^t} u_i \ell_i^t + \mathrm{RE}\left(\mathbf{u} \parallel \mathbf{v}^1\right)$$

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- If we assume that $u(E^t) = U$ is constant, we get

$$L_A \leq \sum_{t=1}^{T} \ell_{\mathbf{u}}^t + \frac{\mathrm{RE}\left(\mathbf{u} \parallel \mathbf{v}^1\right)}{U}$$

Proof of Bound (1)

Lemma:

$$\mathrm{RE}\left(\mathbf{u} \parallel p_t\right) - \mathrm{RE}\left(\mathbf{u} \parallel p_{t+1}\right) = u(E_t)L(\hat{y}_t, y_t) - \sum_{i \in E_t} u_i L(x_{t,i}, y_t)$$

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From definition of $\mathrm{RE}\left(\;\|\; \right)$:

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If $y_t = 1$ the RHS is equal to

$$\begin{aligned} \sum_{i \in E_t} u_i \ln \frac{x_{t,i}}{\hat{y}_t} &= \sum_{i \in E_t} u_i \ln x_{t,i} - u(E_t) \ln \hat{y}_t \\ &= -\sum_{i \in E_t} u_i L(X_{t,i}, y_t) + u(E_t) L(\hat{y}_t, y_t) \end{aligned}$$

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Similarly for $y_t = 0$

Visual intuition

$$\mathrm{RE}\left(\mathbf{u} \;\|\; \mathbf{v}^t\right) - \mathrm{RE}\left(\mathbf{u} \;\|\; \mathbf{v}^{t+1}\right) = \ell_A^t - \mathbf{u} \cdot \boldsymbol{\ell}^t$$

Visual intuition

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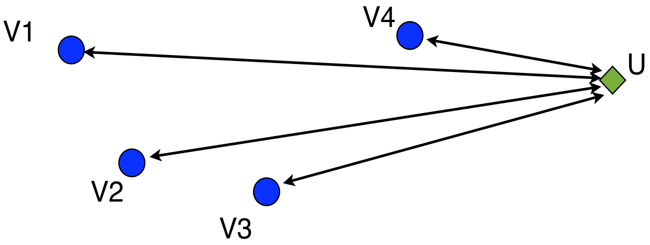

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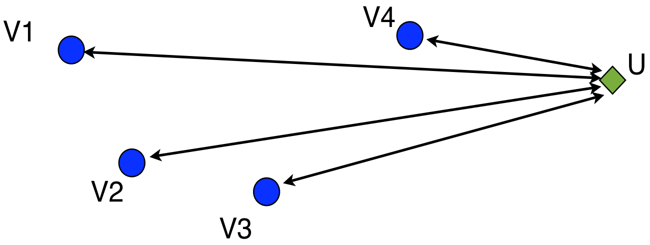

$\mathbf{v}^{t+1}$ is chosen to minimize $\mathrm{RE}\left(\mathbf{v}^{t+1} \;\|\; \mathbf{v}^t\right) + \mathbf{v}^{t+1} \cdot \boldsymbol{\ell}^t$

Proof of Bound (2)

Summing over $t = 1, \ldots, T$:

$$\mathrm{RE}\left(\mathbf{u} \parallel p_t\right) - \mathrm{RE}\left(\mathbf{u} \parallel p_{t+1}\right) = u(E_t)L(\hat{y}_t, y_t) - \sum_{i \in E_t} u_i L(x_{t,i}, y_t)$$

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We get

$$\begin{aligned}
\mathrm{RE}\left(\mathbf{u} \parallel p_1\right) &\geq \mathrm{RE}\left(\mathbf{u} \parallel p_1\right) - \mathrm{RE}\left(\mathbf{u} \parallel p_{T+1}\right) \\
&= \sum_{t=1}^{T} u(E_t)L(\hat{y}_t, y_t) - \sum_{t=1}^{T} \sum_{i \in E_t} u_i L(x_{t,i}, y_t)
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- $L_A \leq \min_{\mathbf{u}} \left(a\mathbf{u} \cdot \sum_{t=1}^T \boldsymbol{\ell}^t + c\mathrm{RE}\left(\mathbf{u} \parallel \mathbf{v}^1\right)\right)$

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- For any mixable loss, $a = 1$, using $\mathbf{u} = \langle 0, \ldots, 0, 1, 0, \ldots, 0 \rangle$ and $\mathbf{v}^1 = \langle 1/N, \ldots, 1/N \rangle$ we get the old bound: $L_A \leq \min_i L_i + c \log N$

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- But much easier to generalize.

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- I don't know, could not find in the literature.