Laplacian coordinates for a graph

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1 Definitions

Let $G = (V, E)$ be a graph with edges E and vertice V.

Let $v_1, \ldots, v_k \in V$ be a set of vertices we call landmarks. Denote $B = \{v_1, \ldots, v_k\}$

For $i = 1, \ldots, k$ define the diffusion function $f_i : V \to R$ as follows.

- $f(v_i) = 1$
- For $1 \leq j \leq k, j \neq i$, $f(v_j) = 0$
- $f_i(v)$ for $v \notin B$ satisfy the diffusion operator $L - I$

2 Coordinate System

We define a coordinate system $C : v \to (f_1(v), \ldots, f_k(v))$.

We say that the coordinate system is complete if for any $u, v\ inV$, if $\|C(u) - C(v)\| \leq \epsilon$ then $d(u, v) \leq a\epsilon$ for some appropriately defined measure of distance d.

One can use diffusion metric or maybe there is a way to consider adding u or v to the landmarks and checking whether the result is over-complete.

3 Claim

I believe that if the vertices are points on a manifold of dimension d then there is a set of $d + 1$ landmarks that create a complete coordinate system.

On the other hand I don't think that having doubling dimension d is sufficient. For example, if the graph is disconnected then we need at least one landmark on each connected component. I also think that having one connected component + small doubling dimension is enough.