



# A Projected Lloyd's Algorithm for Coverage Control Problems

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M.Sc. Seminar

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Introduction

Mathematical Background

Problem Solution

Simulations



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Introduction

Mathematical Background

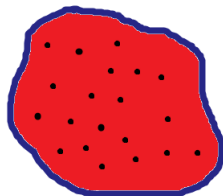
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# Motivation

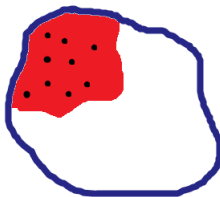
Covering an area - (relatively) easy



# Motivation

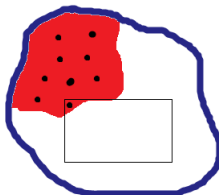
Covering an area with not sufficient amount of sensors - not so easy

- *Requires better definition of behaviour*



# Motivation

Maintain contact with home base (at least in steady state) - hard



# Problem Formulation

- ▶ There is some area  $A \in \mathbb{R}^2$  That we aim to cover
- ▶ We have set of sensors  $S = \{s_1 \dots s_n\}$  located in positions  $p_i \in \mathbb{R}^2$  (for  $i = 1, \dots, n$ ) at time  $t$ 
  - ▶ Each sensor has coverage radius  $R$  (assuming all sensors are identical)
  - ▶ Each sensor can cover a disk  $D(p_i(t), R) \subset \mathbb{R}^2$ , centred at  $p_i(t)$
- ▶ Thus, the coverage:

$$D(p_i(t), R) = D_i(t) = \{x \in \mathbb{R}^2 \mid \|x - p_i(t)\|_2 \leq R\}. \quad (1)$$

- ▶ We also assume  $D_i(t) < A$





## Problem Formulation

Coverage Constraint:

- ▶ A given area inside the area  $A_m \subset A$  must be covered always (e.g. ground station).

*A configuration:* A configuration  $c$  at time  $t$  is the stack of the sensor positions at time  $t$ ,

$$c(t) = \begin{bmatrix} p_1^T(t) & \cdots & p_n^T(t) \end{bmatrix}^T \in \mathbb{R}^{2n}. \quad (2)$$

### Notice

Since  $D_i(t) < A$ , there is no one configuration that can cover the entire area  $A$  at once



## Problem Formulation

Our goal is to find the set of configuration  $C$  which contains configurations that all together provide full coverage of the area  $A$ , and yet maintains the coverage of  $A_m$ .

### Problem

Find the set  $C = \left[ c_1^T(t_1) \quad \cdots \quad c_n^T(t_n) \right]^T$  that provide coverage at time  $i$  to some area  $A_c(t_i)$ , such that:

1. after time  $n$ , each point of  $A$  was visited at least once,
2. At each time there was coverage to some area  $A_s m \subset A_m$ .



# Literature Review

## ► Covering an area<sup>1,2,3</sup>:

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<sup>1</sup>Nigam, N., Bieniawski, S., Kroo, I., & Vian, J. (2012). Control of multiple UAVs for persistent surveillance: Algorithm and flight test results. *IEEE Transactions on Control Systems Technology*, 20(5), 1236–1251.

<sup>2</sup>Montijano, E., Sagues, C., & Llorente, S. (2016). Multi-Robot Persistent Coverage with Optimal Times, (Cdc), 3511–3517.

<sup>3</sup>Loizou, S. G., & Constantinou, C. C. (2016). Multi-Robot Coverage on Dendritic Topologies Under Communication Constraints, (Cdc).

<sup>4</sup>Cassandras, C. G., & Li, W. (2005). Sensor Networks and Cooperative Control. *Decision and Control, 2005 and 2005 European Control Conference. CDC-ECC '05. 44th IEEE Conference On*, 4237–4238.



# Literature Review

- ▶ Covering an area<sup>1,2,3</sup>:
  - ▶ Photographing
  - ▶ Tracking
  - ▶ iRobot...

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# Literature Review

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  - ▶ iRobot...
- ▶ *Coverage Control*<sup>4</sup>

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# Literature Review

- Widely used concept - set of trajectories<sup>1,2,3</sup>

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<sup>1</sup>Atinç, G. M., Stipanović, D. M., Voulgaris, P. G., & Karkoub, M. (2013). Supervised coverage control with guaranteed collision avoidance and proximity maintenance. Proceedings of the IEEE Conference on Decision and Control, 3463–3468. <https://doi.org/10.1109/CDC.2013.6760414>

<sup>2</sup>Hussein, I. I., & Stipanovic, D. M. (2007). Effective Coverage Control for Mobile Sensor Networks With Guaranteed Collision Avoidance. IEEE Transactions on Control Systems Technology, 15(4), 642–657.

<sup>3</sup>Du, Q., Faber, V., & Gunzburger, M., “Centroidal Voronoi Tessellations: Applications and Algorithms,” SIAM Review, Vol. 41, No. 4, 1999, pp. 637–676

<sup>4</sup>Cortes, J., & Martinez, S. (2004). Coverage control for mobile sensing networks. Robotics and Automation, ..., 20(2), 13.



## Literature Review

- ▶ Widely used concept - set of trajectories<sup>1,2,3</sup>
- ▶ Another concept - *Voronoi Partitioning*

According to [Cortes2004]<sup>4</sup>, It is possible to use partitioning for full coverage.

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# Lyapunov Stability

- ▶ Lyapunov stable - if we are near the equilibrium point  $x_{eq}$ , then the controller will stay near  $x_{eq}$  forever.



# Lyapunov Stability

- ▶ Lyapunov stable - if we are near the equilibrium point  $x_{eq}$ , then the controller will stay near  $x_{eq}$  forever.
- ▶ Asymptotically stable - Lyapunov stable + converge to  $x_{eq}$ .



# Lyapunov Stability

How to prove Lyapunov stable and asymptotically stable? Using Lyapunov direct method!

1. Define a *Candidate Lyapunov Function*  $V(x) : \mathbb{R}^n \rightarrow \mathbb{R}$



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1. Define a *Candidate Lyapunov Function*  $V(x) : \mathbb{R}^n \rightarrow \mathbb{R}$
2. To show that we're Lyapunov stable, make sure that:
  - 2.1  $V(0) = 0$
  - 2.2  $V(\zeta) > 0 \Leftrightarrow \zeta \neq 0$
  - 2.3  $\dot{V}(\zeta) \leq 0 \Leftrightarrow \zeta \neq 0$



# Lyapunov Stability

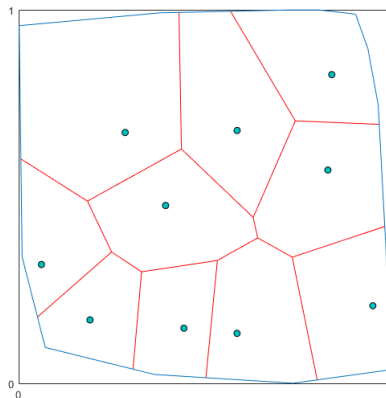
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  - 2.3  $\dot{V}(\zeta) \leq 0 \Leftrightarrow \zeta \neq 0$
3. To show that we are asymptotically stable, show that condition 2.3 is:  $\dot{V}(\zeta) < 0 \Leftrightarrow \zeta \neq 0$



# Voronoi Partitioning

Let's start with a simple intuitive explanation...



# Voronoi Partitioning

While being a method to partition an area with some cost function, the is a widely-used representation in the coverage problem ([Cortes2004][Hussein2007][Du1999]).

The Voronoi Diagram of a region  $\Omega \subset \mathbb{R}^2$  is the set of partitions  $\mathcal{V} = \{V_i \mid \cup V_i = \Omega\}$ , generated by the generators  $\mathcal{Z} = \{z_1, \dots, z_n \mid z_i \in \Omega\}$ , such that

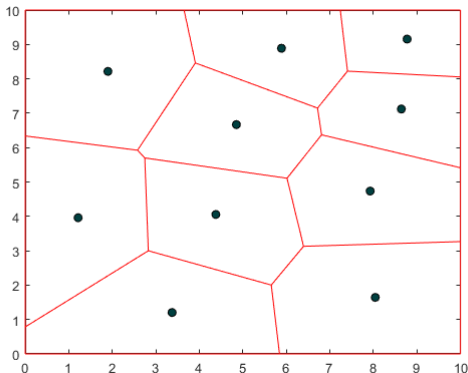
$$V_i = \{q \in \Omega \mid \|q - z_i\| \leq \|q - z_j\| \forall z_i, z_j \in \mathcal{Z}\}, \quad (3)$$

where  $V_i$  corresponds to the  $i$ -th element of  $\mathcal{Z}$ , and  $\|\cdot\|$  denotes the Euclidean distance.



## Central Voronoi Tessellations

And yet again, let's have an intuitive explanation...





## Central Voronoi Tessellations

Let us define a density function,  $\rho_i$ , for each Voronoi partition  $V_i$ . Then, we can define the center of mass for each partition as

$$z_i^* = \frac{\int_{V_i} y \rho(y) dy}{\int_{V_i} \rho(y) dy}. \quad (4)$$

If a generator  $z_i = z_i^* \forall V_i$ , we call this partitioning a *centroidal Voronoi tessellation* (CVT).



# Lloyd's Algorithm

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**Algorithm 1** Lloyd's Algorithm <sup>1</sup>.

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- 1: Calculate the Voronoi diagram for the current agents positions.
  - 2: Calculate the center of mass for every cell.
  - 3: Move the agents to the center of mass.
- 

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<sup>1</sup>Lloyd, S., "Least squares quantization in PCM," IEEE Transactions on Information Theory, Information Theory, IEEE Transactions on, IEEE Trans. Inform. Theory, Vol. 28, No. 2, 1982, pp. 129–137, doi:10.1109/TIT.1982.1056489

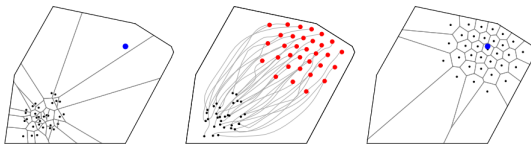


## Lloyd's Algorithm

According to [Cortes2004], if we define agent  $i$  position as  $p_i$  and the  $i$ 's partition centroid as  $C_{V_i}$ , then for some proportional constant  $k_{prop}$ , the controller can be defined as:

$$u_i = -k_p (p_i - C_{V_i}) \quad (5)$$

Moreover, this controller is locally asymptotically stable.



**Figure:** A simulation from [Cortes2004] with 32 agents



## Formation Control

For a complete background, some prior knowledge on algebraic graph theory is needed. Let's simplify:

- ▶ We have agents  $1 \dots n$  on positions  $p_i$



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## Formation Control

For a complete background, some prior knowledge on algebraic graph theory is needed. Let's simplify:

- ▶ We have agents  $1 \dots n$  on positions  $p_i$
- ▶ We want that the distance between agents  $i, j (i \neq j)$  will be  $d_{ij}$
- ▶ Lets assume that there isn't connection between all the agents. Only  $\varepsilon$  agents can share information.



## Formation Control

Then, for a single agent  $p_i$ , the controller will have the following form:

$$\dot{p}_{f_i} = - \sum_{i \sim j} (\|p_i - p_j\|^2 - d_{ij}^2) (p_i - p_j) \quad (6)$$

This controller is locally asymptotically stable.



## Projection Operator

The projection linear operator is defined as a linear transformation  $P$  from a vector space to itself such as  $P^2 = P$ . In other words, the transformation  $P$  is idempotent.





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## Projected Lloyd's Algorithm

We should supply a solution for the "Coverage Constraint"



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## Reminder

A given area inside the area  $A_m \subset A$  must be covered always



## Projected Lloyd's Algorithm

We should supply a solution for the "Coverage Constraint"

### Reminder

A given area inside the area  $A_m \subset A$  must be covered always

We came up with a rather simple solution for this problem.



# Projected Lloyd's Algorithm

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**Algorithm 2** Projected Lloyd's Algorithm (PLA)

---

- 1: Calculate the Voronoi diagram for the current agents positions.
  - 2: Calculate the center of mass for every cell.
  - 3: Project the center of mass of every cell to the area constraint limiting polygon.
  - 4: Move the agents the projected center of mass.
  - 5: Repeat until converge.
- 

Writing this algorithm as a controller:

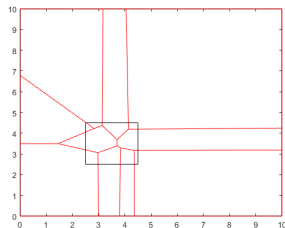
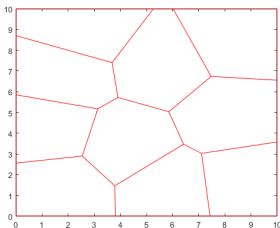
$$u_i = -k_p (p_i - \text{proj}(C_{V_i})) \quad (7)$$



# Projected Lloyd's Algorithm

In the following example:

- ▶ Left - regular CVT, build with the original Lloyd's Algorithm.
- ▶ Right - A partitioning built with the PLA.



# Projected Lloyd's Algorithm

## Theorem

*The projected Lloyd's Algorithm is locally asymptotically stable*

As the projection is a linear operator, the controller is also locally asymptotically stable, and the proof is virtually the same as was given by Cortes et al. The proof is based on a proposal of Lyapunov function, and then using the direct Lyapunov method to prove the stability



## Proof of stability

As the projection is a linear operator, the controller is also locally asymptotically stable, and the proof is virtually the same as was given by Cortes et al.





## Problem Solution Algorithm

So far, we've given solution for:

- ▶ Covering a given area using Voronoi partitioning
- ▶ Partition and area such that the coverage constraint is fulfilled.

Therefore, we are ready for the problem solution algorithm...



# Problem Solution Algorithm

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## Algorithm 3 Problem Solution Algorithm

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- 1: Using some random initial guess, partition the whole area using PLA.
  - 2: For each partition (assuming that the agents can actually cover each partition with their coverage radius), calculate the CVT. The initial positions for the CVT calculation is the previous partition CVT.
- 



# Lloyd's Algorithm and Formation Control

- ▶ Problem solved.



# Lloyd's Algorithm and Formation Control

- ▶ Problem solved.
- ▶ Make it more interesting...



# Lloyd's Algorithm and Formation Control

- ▶ Problem solved.
- ▶ Make it more interesting...
- ▶ Combine Lloyd's Algorithm with distance-based formation control!



# Lloyd's Algorithm and Formation Control

- ▶ Problem solved.
- ▶ Make it more interesting...
- ▶ Combine Lloyd's Algorithm with distance-based formation control!
  - ▶ create and maintain spatial properties partially or fully (We do not provide a condition where this combination meets the requirements).



## Lloyd's Algorithm and Formation Control

As both of the controllers are convex, we propose to simply combine them with some coefficient:

$$u_i = \alpha (-k_p (p_i - C_{V_i})) + (1 - \alpha) \left[ - \sum_{i \sim j} (\|p_i - p_j\|^2 - d_{ij}^2) (p_i - p_j) \right] \quad (8)$$



## Lloyd's Algorithm and Formation Control

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### Theorem

*The combined controller is Locally Asymptotically Stable*





# Lloyd's Algorithm and Formation Control

How to prove:

- ▶ Pretty long and technical, based on Lyapunov function.
- ▶ We know the Lyapunov function of each controller - Let's combine!
- ▶ After long calculations, we can show using the Lyapunov direct method that this controller is locally asymptotically stable.
- ▶ In the same way, we can show it works with the PLA.



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## Some Simulation

List of simulations to create:

- ▶ 3 agents, 5 big partitions, no formation, no PLA
- ▶ 3 agents, 5 big partitions, no formation, PLA
- ▶ 10 agents, 5 big partitions, no formation, PLA
- ▶ 6 agents, 5 big partitions, some formation, PLA

