

Non-Vertical Cultural Transmission, Assortment, and the Evolution of Cooperation

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Abstract

8 Cultural evolution of cooperation under vertical and non-vertical cultural transmission is studied,
and conditions are found for fixation and coexistence of cooperation and defection. The evolution
10 of cooperation is facilitated by its horizontal transmission and by an association between social
interactions and horizontal transmission. The effect of oblique transmission depends on the horizontal
12 transmission bias. Stable polymorphism of cooperation and defection can occur, and when it does,
reduced association between social interactions and horizontal transmission evolves, which leads to a
14 decreased frequency of cooperation and lower population mean fitness. The deterministic conditions
are compared to outcomes of stochastic simulations of structured populations. Parallels are drawn
16 with Hamilton's rule incorporating relatedness and assortment.

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30 1 Introduction

Cooperative behavior can reduce an individual's fitness and increase the fitness of its conspecifics or
32 competitors [1]. Nevertheless, cooperative behavior appears to occur in many animals [2], including
humans, primates [3], rats [4], birds [5, 6], and lizards [7]. Evolution of cooperative behavior has
34 been an important focus of research in evolutionary theory since at least the 1930s [8]. Since the work
of Hamilton [9] and Axelrod and Hamilton [1], theories for the evolution of cooperative and altruistic
36 behaviors have been intertwined often under the rubric of *kin selection*. Kin selection theory posits
that natural selection is more likely to favor cooperation between more closely related individuals. The
38 importance of *relatedness* to the evolution of cooperation and altruism was demonstrated by Hamilton
[9], who showed that an allele that determines cooperative behavior will increase in frequency if the
40 reproductive cost to the actor that cooperates, c , is less than the benefit to the recipient, b , times the
relatedness, r , between the recipient and the actor. This condition is known as *Hamilton's rule*:

$$42 \quad c < b \cdot r, \quad (1)$$

where the relatedness coefficient r measures the probability that an allele sampled from the cooperator
44 is identical by descent to one at the same locus in the recipient.

There is an ongoing debate about to what extent kin selection explains evolution of cooperation and
46 altruism. It has been suggested that kin selection to explain the cooperative behaviour of eusocial
insects like the honey bee. The most significant argument against kin selection is that cooperation can
48 evolve with zero relatedness [10]. This makes Hamilton's rule incomplete according to Wilson [10].
Foster et al. [11] reject this claim. They argue that altruism without relatedness can not evolve. They
50 refer us to Hamilton who claimed that relatedness can arise without recent common ancestry. Wilson
also criticises kin selection on the grounds that environmental or ecological factors probably be more
52 important than relatedness in determining social actions. On the other hand, Foster et al. [11] argue
that kin selection does not ignore ecology. Hamilton's rule shows that environmental factors causing
54 a high benefit: cost ratio will favour cooperation.

Beside kin selection, two more major theories were suggested to explain to evolution of coopera-
56 tion.

Reciprocity suggests repeating interactions or individual recognition as key factors for explaining the
58 evolution of cooperation. In *direct reciprocity* there are a repeated encounters between the same two
individuals. In every encounter, each player has a choice between cooperation and defection. If I
60 cooperate now, you may cooperate later. Hence, it may pay off to cooperate. This game-theoretic
framework is known as the repeated Prisoner's Dilemma. Direct reciprocity can only lead to the
62 evolution of cooperation if the cost is smaller than w the probability for another encounter between
the same two individuals multiplied by the benefit.

$$64 \quad c < bw \quad (2)$$

Direct reciprocity assumes that both players are in a position to cooperate. Direct reciprocity can
66 not explain cooperation in asymmetric interactions. In humans, such interactions happen often, for

example humans often donate money. *Indirect reciprocity* has been suggested to explain this behavior.

68 Nowak [12] claims that direct reciprocity is like a barter economy based on the immediate exchange of
goods, while indirect reciprocity resembles the invention of money. The money that "fuels the engines"
70 of indirect reciprocity is reputation. However, Reciprocity assume repeating interactions and therefore,
has difficulty explaining evolution of cooperation if the no repeating interactions occurs.

72 **Group Selection** theory posits that cooperation is favoured because of the advantage to the whole
group, if selection acts at the group level in addition to the individual level. A common model for group
74 selection work as is: the population is divided into groups. In each groups there are cooperators, which
help to other group members and defectors which do not help. Individuals reproduce proportional to
76 their fitness. Offspring are added to the same group. If a group reaches a certain size it can split to two
groups. A group that grow faster will split more often. Groups of cooperators are growing faster than
78 group of defectors. Therefore, cooperation can evolve in this model when the ratio between benefit
b and cost c is more than one plus the ratio between the maximum group size n and the number of
80 groups m:

$$\frac{b}{c} > 1 + \frac{n}{m} \quad (3)$$

82 All three theories mentioned above assume that cooperation is genetically determined. This raise
the question, is it possible that cooperation is determined by environmental or social influences.
84 Cooperative behavior can be subject to *cultural transmission*, which allows an individual to acquire
attitudes or behavioral traits from other individuals in its social group through imitation, learning, or
86 other modes of communication [13, 14]. Cultural transmission may be modeled as vertical, horizontal,
or oblique: vertical transmission occurs between parents and offspring, horizontal transmission occurs
88 between individuals from the same generation, and oblique transmission occurs to offspring from the
generation to which their parents belong (i.e. from non-parental adults). Evolution under either of these
90 transmission models can be be more rapid than under pure vertical transmission [13, 15, 16].

Here, we study models for the cultural evolution of cooperation that include both vertical and non-
92 vertical transmission. In our models behavioral changes are mediated by cultural transmission that
can occur specifically during social interactions. For instance, there may be an association between
94 the choice of partner for social interaction and the choice of partner for cultural transmission, or when
an individual interacts with an individual of a different phenotype, exposure to the latter may lead the
96 former to convert its phenotype. Our results demonstrate that cultural transmission, when associated
with social interactions, can enhance the evolution of cooperation even when genetic transmission
98 cannot, partly because it facilitates the generation of assortment [17], and partly because it diminishes
the effect of selection (due to non-vertical transmission from non-reproducing individuals [16]).

100 2 Related Work

102 Eshel and Cavalli-Sforza [18] studied a related model for the evolution of cooperative behavior. Their
 104 model included *assortative meeting*, or non-random encounters, where a fraction m of individuals in
 106 the population each interact specifically with an individual of the same phenotype, and a fraction $1 - m$
 interacts with a randomly chosen individual. Such assortative meeting may be due, for example, to
 population structure or active partner choice. In their model, cooperative behavior can evolve if [18,
 eq. 3.2]

$$108 c < b \cdot m, \quad (4)$$

where b and c are the benefit and cost of cooperation¹.

The role of assortment in the evolution of altruism was emphasized by Fletcher and Doebeli [17].
 110 They found that in a *public-goods* game, altruism will evolve if cooperative individuals experience
 more cooperation, on average, than defecting individuals, and “thus, the evolution of altruism requires
 112 (positive) assortment between focal *cooperative* players and cooperative acts in their interaction
 environment.” With some change in parameters, this condition is summarized by [17, eq. 2.3]

$$114 c < b \cdot (p_C - p_D), \quad (5)$$

where p_C is the probability that a cooperator receives help, and p_D is the probability that a defector
 116 receives help². Bijma and Aanen [19] obtained a result related to inequality 5 for other games.

Cooperation can also evolve when interactions are determined by population structure. For example,
 118 Ohtsuki et al. [20] studied populations on graphs with average degree k , that is, the average individual
 has k potential interaction partners. Assuming that selection is weak and that the population size is
 120 much larger than k (i.e. sparse structure), they found that cooperative behaviour can evolve if [20]

$$c < b \cdot \frac{1}{k}. \quad (6)$$

122 They thus interpret $1/k$ as *social relatedness* or *social viscosity* [20].

Feldman et al. [21] introduced the first model for the evolution of altruism by cultural transmission
 124 with kin selection and demonstrated that if the fidelity of cultural transmission of altruism is φ , then
 the condition for evolution of altruism in the case of sib-to-sib altruism is [21, Eq. 16]

$$126 c < b \cdot \varphi - \frac{1 - \varphi}{\varphi}. \quad (7)$$

In inequality 7, φ replaces relatedness (r in inequality 1) or assortment (m in inequality 4), but the ef-
 128 fective benefit $b \cdot \varphi$ is reduced by $(1 - \varphi)/\varphi$. This shows that under a cultural transmission, the condition
 for the evolutionary success of altruism entails a modification of Hamilton’s rule (inequality 1).

¹In an extended model, which allows an individual to encounter N individuals before choosing a partner, the right hand side is multiplied by $E[N]$, the expected number of encounters [18, eq. 4.6].

²Inequality 5 generalizes inequalities 1 and 4 by substituting $p_C = r + p$, $p_D = p$ and $p_C = m + (1 - m)p$, $p_D = (1 - m)p$, respectively, where p is the frequency of cooperators.

130 Both Woodcock [22] and Lewin-Epstein et al. [23] demonstrated that non-vertical transmission can help
explain the evolution of cooperative behavior, the former using simulations with cultural transmission,
132 the latter using a model where cooperation is mediated by host-associated microbes. Indeed, models
in which microbes affect their host's behavior [23, 24, 25] are mathematically similar to models of
134 cultural transmission, and they also emphasize the role of non-vertical transmission [13].

3 Models

- 136 Consider a large population whose members can be one of two phenotypes: $\phi = A$ for cooperators or $\phi = B$ for defectors. An offspring inherits its phenotype from its parent via vertical transmission
 138 with probability v or from a random individual in the parental population via oblique transmission with probability $(1 - v)$ (Figure 1a). Following Ram et al. [16], given that the parent's phenotype is
 140 ϕ and assuming uni-parental inheritance [26], the conditional probability that the phenotype ϕ' of the offspring is A is

$$142 \quad P(\phi' = A | \phi) = \begin{cases} v + (1 - v)p, & \text{if } \phi = A \\ (1 - v)p, & \text{if } \phi = B \end{cases}, \quad (8)$$

where $p = P(\phi = A)$ is the frequency of A among all adults in the parental generation.

- 144 Not all adults become parents, and we denote the frequency of phenotype A among parents by \dot{p} . Therefore, the frequency \hat{p} of phenotype A among juveniles (after selection and vertical and oblique
 146 transmission) is

$$\hat{p} = \dot{p}[v + (1 - v)p] + (1 - \dot{p})[(1 - v)p] = v\dot{p} + (1 - v)p. \quad (9)$$

- 148 Individuals are assumed to interact according to a *prisoner's dilemma*. Specifically, individuals interact in pairs; a cooperator suffers a fitness cost $0 < c < 1$, and its partner gains a fitness benefit
 150 b , where we assume $c < b$. Figure 1a shows the payoff matrix, i.e. the fitness of an individual with phenotype ϕ_1 when interacting with a partner of phenotype ϕ_2 .

- 152 Social interactions occur randomly: two juvenile individuals with phenotype A interact with probability \hat{p}^2 , two juveniles with phenotype B interact with probability $(1 - \hat{p})^2$, and two juveniles with
 154 different phenotypes interact with probability $2\hat{p}(1 - \hat{p})$. Horizontal cultural transmission occurs between pairs of individuals from the same generation. It occurs between socially interacting partners
 156 with probability α , or between a random pair with probability $1 - \alpha$ (see Figure 1b). However, horizontal transmission is not always successful, as one partner may reject the other's phenotype. The
 158 probability of successful horizontal transmission of phenotypes A and B are T_A and T_B , respectively (Table 1, Figure 1d). Thus, the frequency p' of phenotype A among adults in the next generation, after
 160 horizontal transmission, is

$$\begin{aligned} p' &= \hat{p}^2[\alpha + (1 - \alpha)(\hat{p} + (1 - \hat{p})(1 - T_B))] + \\ &\quad \hat{p}(1 - \hat{p})[\alpha(1 - T_B) + (1 - \alpha)(\hat{p} + (1 - \hat{p})(1 - T_B))] + \\ &\quad (1 - \hat{p})\hat{p}[\alpha T_A + (1 - \alpha)\hat{p}T_A] + (1 - \hat{p})^2[(1 - \alpha)\hat{p}T_A] \\ &= \hat{p}^2(T_B - T_A) + \hat{p}(1 + T_A - T_B). \end{aligned} \quad (10)$$

- 162 The frequency of A among parents (i.e. after selection) follows a similar dynamic, but also includes the effect of natural selection, and is therefore

$$\begin{aligned} 164 \quad \bar{w}\dot{p}' &= \hat{p}^2(1 + b - c)[\alpha + (1 - \alpha)(\hat{p} + (1 - \hat{p})(1 - T_B))] + \\ &\quad \hat{p}(1 - \hat{p})(1 - c)[\alpha(1 - T_B) + (1 - \alpha)(\hat{p} + (1 - \hat{p})(1 - T_B))] + \\ &\quad (1 - \hat{p})\hat{p}(1 + b)[\alpha T_A + (1 - \alpha)\hat{p}T_A] + (1 - \hat{p})^2[(1 - \alpha)\hat{p}T_A], \end{aligned} \quad (11)$$

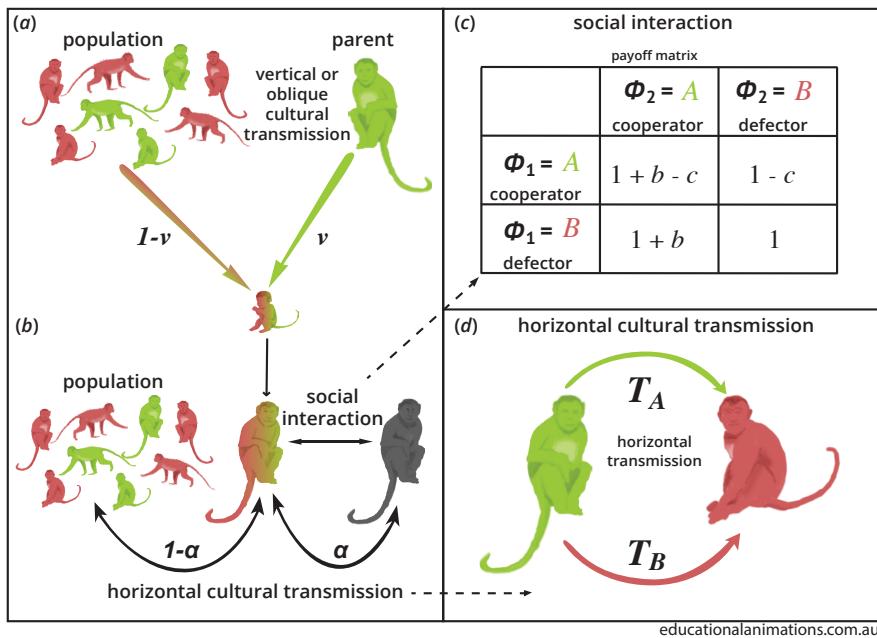


Figure 1: Model illustration. **(a)** First, offspring inherit their parent's phenotype via vertical cultural transmission with probability v , or the phenotype of a random non-parental adult via oblique cultural transmission with probability $1 - v$. **(b)** Second, adults socially interact in pairs in a prisoner's dilemma game. Horizontal cultural transmission occurs from a random individual in the population, with probability $1 - \alpha$, or from the social partner, with probability α , where α is the interaction-transmission association parameter. **(c)** The prisoner's dilemma payoff matrix shows the fitness of phenotype ϕ_1 when interacting with phenotype ϕ_2 . **(d)** The probabilities of successful horizontal cultural transmission of phenotypes A (cooperator) and B (defector) are T_A and T_B , respectively.

where fitness values are taken from Figure 1c and Table 1, and the population mean fitness is
166 $\bar{w} = 1 + \hat{p}(b - c)$. Starting from Eq. 9 with $\hat{p}' = v\hat{p}' + (1 - v)p'$, we substitute p' from Eq. 10 and \hat{p}'
from Eq. 11 and obtain

$$\begin{aligned}
\hat{p}' = & \frac{v}{\bar{w}} \left[\hat{p}^2(1 + b - c) \left(1 - (1 - \hat{p})(1 - \alpha)T_B \right) \right] + \\
& \frac{v}{\bar{w}} \left[\hat{p}(1 - \hat{p})(1 - c) (\hat{p}(1 - \alpha)T_B + 1 - T_B) \right] + \\
& \frac{v}{\bar{w}} \left[\hat{p}(1 - \hat{p})(1 + b) (\hat{p}(1 - \alpha) + \alpha)T_A \right] + \\
& \frac{v}{\bar{w}} (1 - \hat{p})^2 \hat{p}(1 - \alpha)T_A + (1 - v)\hat{p}^2(T_B - T_A) + (1 - v)\hat{p}(1 + T_A - T_B).
\end{aligned} \tag{12}$$

168

Table 2 lists the model variables and parameters.

Table 1: Interaction frequency, fitness, and transmission probabilities.

Phenotype ϕ_1	Phenotype ϕ_2	Frequency	Fitness of ϕ_1	$P(\phi_1 = A)$ via horizontal transmission:	
				from partner, α	from population, $(1 - \alpha)$
A	A	\hat{p}^2	$1 + b - c$	1	$\hat{p} + (1 - \hat{p})(1 - T_B)$
A	B	$\hat{p}(1 - \hat{p})$	$1 - c$	$1 - T_B$	$\hat{p} + (1 - \hat{p})(1 - T_B)$
B	A	$\hat{p}(1 - \hat{p})$	$1 + b$	T_A	$\hat{p}T_A$
B	B	$(1 - \hat{p})^2$	1	0	$\hat{p}T_A$

Table 2: Model variables and parameters.

Symbol	Description	Values
A	Cooperator phenotype	
B	Defector phenotype	
p	Frequency of phenotype A among adults	$[0, 1]$
\dot{p}	Frequency of phenotype A among parents	$[0, 1]$
\hat{p}	Frequency of phenotype A among juveniles	$[0, 1]$
v	Vertical transmission rate	$[0, 1]$
c	Cost of cooperation	$(0, 1)$
b	Benefit of cooperation	$c < b$
α	Probability of interaction-transmission association	$[0, 1]$
T_A, T_B	Horizontal transmission rates of phenotype A and B	$(0, 1)$

170 4 Results

We determine the equilibria of the model in Eq. 12 and analyze their local stability. We then analyze
172 the evolution of a modifier of interaction-transmission association, α . Finally, we compare derived
conditions to outcomes of stochastic simulations with a structured population.

174 4.1 Evolution of cooperation

To learn about the evolution of cooperation we investigate local stability of the equilibria of the model
176 in Eq. 12. The equilibria are the solutions of $\hat{p}' - \hat{p} = 0$. Note that Eq. 12 is not simple polynomial
but a polynomial fraction, therefore the solution is not trivial. The fixed points (equilibria) of the
178 recursion (Eq. 12) are $\hat{p} = 0$, $\hat{p} = 1$, and (see Appendix B Eq. B5)

$$\hat{p}^* = \frac{\alpha bvT_A - cv(1 - T_B) + (T_A - T_B)}{[c(1 - v) - b(1 - \alpha v)](T_A - T_B)}. \quad (13)$$

180 Define the following cost thresholds, γ_1 and γ_2 , and the vertical transmission threshold, \hat{v} ,

$$\gamma_1 = \frac{bvaT_A + (T_A - T_B)}{v(1 - T_B)}, \quad \gamma_2 = \frac{bvaT_B + (1 + b)(T_A - T_B)}{v(1 - T_B) + (1 - v)(T_A - T_B)}, \quad \hat{v} = \frac{T_B - T_A}{1 - T_A}. \quad (14)$$

182 Then we have the following result.

184 **Result 1.** *With vertical, horizontal, and oblique transmission, the cultural evolution of a cooperation
follows one of the following scenarios in terms of the cost thresholds γ_1 and γ_2 and the vertical
186 transmission threshold \hat{v} (Eq. 14) :*

1. Fixation of cooperation: if (i) $T_A \geq T_B$ and $c < \gamma_1$; or if (ii) $T_A < T_B$ and $v > \hat{v}$ and $c < \gamma_2$.
- 188 2. Fixation of defection: if (iii) $T_A \geq T_B$ and $\gamma_2 < c$; or if (iv) $T_A < T_B$ and $\gamma_1 < c$.
3. Stable polymorphism: if (v) $T_A < T_B$ and $v < \hat{v}$ and $c < \gamma_1$; or if (vi) $T_A < T_B$ and $v > \hat{v}$ and
190 $\gamma_2 < c < \gamma_1$.
4. Unstable polymorphism: if (vii) $T_A > T_B$ and $\gamma_1 < c < \gamma_2$.

192 Thus, cooperation can take over the population if it has either a horizontal transmission advantage, or
if it has a horizontal transmission disadvantage, but the vertical transmission rate is high enough. In
194 either case, the cost of cooperation must be small enough. A stable polymorphism can exist between
cooperation and defection only if defection has a horizontal transmission advantage. In this case,
196 the existence of a stable polymorphism depends on the interplay between the benefit and cost of
cooperation and the vertical transmission rate. These conditions are illustrated in Figures 2a, 2b, 3a,
198 and 3b, and the analysis is in Appendix B. Note that *stable* and *unstable* polymorphism are also called,
respectively, *coexistence* and *bistable competition*.

200 Much of the literature on evolution of cooperation focuses on conditions for an initially rare cooperative phenotype to invade a population of defectors. The following remarks address this condition.

Remark 1. If the initial frequency of cooperation is very close to zero, then its frequency will increase

204 if the cost of cooperation is low enough,

$$c < \gamma_1 = \frac{bvaT_A + (T_A - T_B)}{v(1 - T_B)} . \quad (15)$$

206 This unites the conditions for fixation of cooperation and for stable polymorphism, both of which entail instability of the state where defection is fixed, $\hat{p} = 0$.

208 Importantly, increasing interaction-transmission association α increases the cost threshold ($\partial\gamma_1/\partial\alpha > 0$), making it easier for cooperation to increase in frequency when initially rare. Similarly, increasing 210 the horizontal transmission of cooperation, T_A , increases the threshold ($\partial\gamma_1/\partial T_A > 0$), facilitating 212 the evolution of cooperation ((Figure 3a and 3b)). However, increasing the horizontal transmission of 214 defection, T_B , can increase or decrease the cost threshold, but it increases the cost threshold when the threshold is already above one ($c < 1 < \gamma_1$): $\partial\gamma_1/\partial T_B$ is positive when $T_A > \frac{1}{1+\alpha bv}$, which 216 gives $\gamma_1 > 1/v$. Therefore, increasing T_B decreases the cost threshold and limits the evolution of 218 cooperation, but only if $T_A < \frac{1}{1+\alpha bv}$.

220 Increasing the vertical transmission rate, v , can either increase or decrease the cost threshold, depending 222 on the horizontal transmission bias, $T_A - T_B$, because $\text{sign}(\partial\gamma_1/\partial v) = -\text{sign}(T_A - T_B)$. When $T_A < T_B$ we have $\partial\gamma_1/\partial v > 0$, and as the vertical transmission rate increases, the cost threshold increases, 224 making it easier for cooperation to increase when rare (Figure 2b). In contrast, when $T_A > T_B$ we get $\partial\gamma_1/\partial v < 0$, and therefore as the vertical transmission rate increases, the cost threshold decreases, 226 making it harder for cooperation to increase when rare (Figure 2a).

228 In general, this condition cannot be formulated in the form of Hamilton's rule due to the bias in horizontal transmission, represented by $T_A - T_B$. If $T_A = T_B$, then, from Result 1 and inequality 15, 230 cooperation will take over the population from any initial frequency if the cost is low enough,

$$c < b \cdot \frac{\alpha T}{1 - T} , \quad (16)$$

232 and regardless of the vertical transmission rate, v . This condition can be interpreted as a version of 234 Hamilton's rule ($c < b \cdot r$, inequality 1) or as a version of inequality 5, where $\alpha T/(1 - T)$ can be 236 regarded as the *effective relatedness* or *effective assortment*, respectively. Note that the right-hand side of inequality 16 equals γ_1 when $T = T_A = T_B$.

238 From inequality 15, without interaction-transmission association ($\alpha = 0$), cooperation will increase 240 when it is rare if there is horizontal transmission bias for cooperation, $T_A > T_B$, and

$$c < \frac{T_A - T_B}{v(1 - T_B)} . \quad (17)$$

242 Figure 3a illustrates this condition (for $v = 1$), which is obtained by setting $\alpha = 0$ in inequality 15.

244 In this case, the benefit of cooperation, b , does not affect the evolution of cooperation, and the 246 outcome is determined only by cultural transmission. Further, inequality 15 shows that with perfect 248 interaction-transmission association ($\alpha = 1$), cooperation will increase when rare if

$$c < \frac{bvT_A + (T_A - T_B)}{v(1 - T_B)} . \quad (18)$$

238 In the absence of oblique transmission, $v = 1$, the only equilibria are the fixation states, $\dot{p} = 0$ and
 $\dot{p} = 1$, and cooperation will evolve from any initial frequency (i.e. $\dot{p}' > \dot{p}$) if inequality 18 applies
240 (Figure 3). This is similar to case of microbe-induced cooperation studied by Lewin-Epstein et al.
[23]; therefore when $v = 1$, this remark is equivalent to their eq. 1.

242 It is interesting to examine the general effect of interaction-transmission association α on the evolution
of cooperation. Define the interaction-transmission association thresholds, a_1 and a_2 , as

$$244 \quad a_1 = \frac{c \cdot v(1 - T_A) - (T_A - T_B)(1 + b - c)}{b \cdot v \cdot T_B}, \quad a_2 = \frac{c \cdot v(1 - T_B) - (T_A - T_B)}{b \cdot v \cdot T_A}. \quad (19)$$

Remark 2. *Cooperation will increase when rare if interaction-transmission association is high
246 enough, specifically if $a_2 < \alpha$.*

Figures 2c and 2d illustrate this condition. With horizontal transmission bias for cooperation, $T_A > T_B$,
248 cooperation can fix from any initial frequency if $a_2 < \alpha$ (green area in the figures). With horizontal
bias favoring defection, $T_A < T_B$, cooperation can fix from any frequency if α is large enough, $a_1 < \alpha$
250 (green area with $T_A < T_B$), and can reach stable polymorphism if α is intermediate, $a_2 < \alpha < a_1$
(yellow area). Without horizontal bias, $T_A = T_B$, fixation of cooperation occurs if α is high enough,
252 $\frac{c}{b} \cdot \frac{1-T}{T} < \alpha$ (inequality 16; in this case $a_1 = a_2$).

Interestingly, because $\text{sign}(\partial a_2 / \partial v) = \text{sign}(T_A - T_B)$, the effect of the vertical transmission rate v
254 on a_1 and a_2 depends on the horizontal transmission bias. That is, if $T_A > T_B$, then evolution of
cooperation is facilitated by oblique transmission, whereas if $T_A < T_B$, then evolution of cooperation
256 is facilitated by vertical transmission (Figures 2c and 2d).

258 Next, we examine the roles of vertical and oblique transmission in the evolution of cooperation.
Fixation of cooperation is possible only if the vertical transmission rate is high enough,

$$260 \quad v > \hat{v} = \frac{T_B - T_A}{1 - T_A}. \quad (20)$$

This condition is necessary for fixation of cooperation, but it is not sufficient. If horizontal transmission
262 is biased for cooperation, $T_A > T_B$, cooperation can fix with any vertical transmission rate (because
 $\hat{v} < 0$). In contrast, if horizontal transmission is biased for defection, $T_A < T_B$, cooperation can fix
264 only if the vertical transmission rate is high enough: in this case oblique transmission can prevent
fixation of cooperation (see Figures 2b and 2d).

266 With only vertical transmission ($v = 1$), from inequality 15, cooperation increases when rare if

$$c < \frac{b\alpha T_A + (T_A - T_B)}{1 - T_B}, \quad (21)$$

268 which can also be written as

$$\frac{c(1 - T_B) - (T_A - T_B)}{bT_A} < \alpha. \quad (22)$$

270 In the absence of vertical transmission ($v = 0$), from recursion 12 we see that the frequency of the
cooperator phenotype among adults increases every generation, i.e. $p' > p$, if there is a horizontal

272 transmission bias in favor of cooperation, namely $T_A > T_B$. That is, if $v = 0$, then selection plays no
273 role in the evolution of cooperation (i.e. b and c do not affect \hat{p}'). The dynamics are determined solely
274 by differential horizontal transmission of the two phenotypes. With no bias in horizontal transmission,
 $T_A = T_B$, phenotype frequencies do not change, $\hat{p}' = \hat{p}$.

276 Cooperation and defection can coexist at frequencies \hat{p}^* and $1 - \hat{p}^*$ (Eq. 13). When it is feasible, this
277 equilibrium is stable or unstable under the conditions of Result 1, parts 3 and 4, respectively. The
278 yellow and blue areas in Figures 3 and 2 show cases of stable and unstable polymorphism, respectively.
279 When \hat{p}^* is unstable, cooperation will fix if its initial frequency is $\hat{p} > \hat{p}^*$, and defection will fix if
280 $\hat{p} < \hat{p}^*$. \hat{p}^* is unstable when there is horizontal transmission bias for cooperation, $T_A > T_B$, and the
cost is intermediate, $\gamma_1 < c < \gamma_2$. Figure 3d shows $\hat{p}' - \hat{p}$ as a function of \hat{p} .

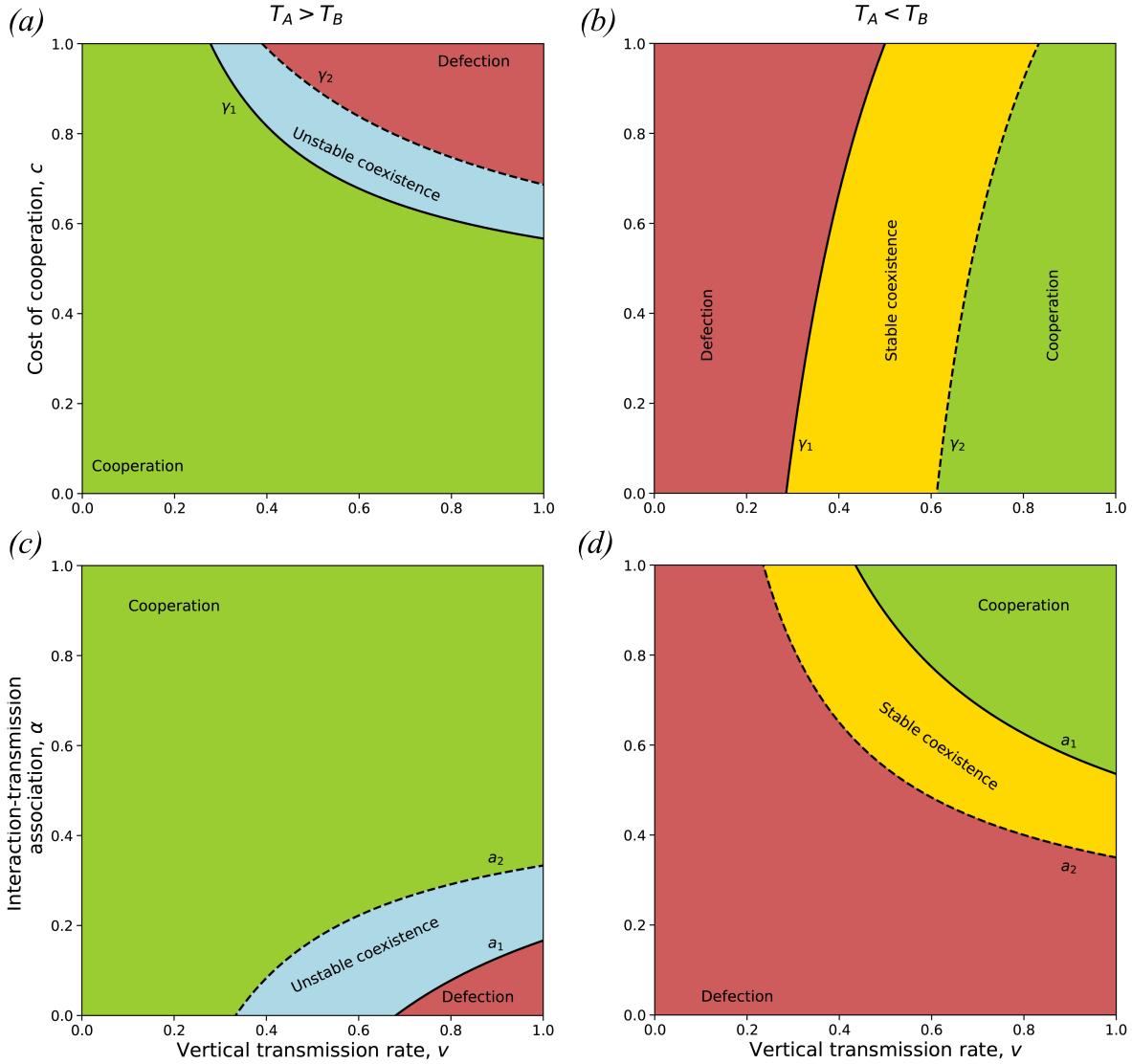


Figure 2: Evolution of cooperation under vertical, oblique, and horizontal cultural transmission.

The figure shows parameter ranges for global fixation of cooperation (green), global fixation of defection (red), fixation of either cooperation or defection depending on the initial conditions, i.e. unstable polymorphism (blue), and stable polymorphism of cooperation and defection (yellow). In all cases the vertical transmission rate v is on the x-axis. **(a-b)** Cost of cooperation c is on the y-axis and the cost thresholds γ_1 and γ_2 (Eqs. 14) are represented by the solid and dashed lines, respectively. **(c-d)** Interaction-transmission association α is on the y-axis and the interaction-transmission association thresholds a_1 and a_2 (Eqs. 19) are represented by the solid and dashed lines, respectively. Horizontal transmission is biased in favor of cooperation, $T_A > T_B$, in **(a)** and **(c)**, or defection, $T_A < T_B$, in **(b)** and **(d)**. Here, $T_A = 0.5$, and **(a)** $b = 1.2$, $T_B = 0.4$, $\alpha = 0.4$; **(b)** $b = 2$, $T_B = 0.7$, $\alpha = 0.7$; **(c)** $b = 1.2$, $T_B = 0.4$, $c = 0.5$; **(d)** $b = 2$, $T_B = 0.7$, $c = 0.5$.

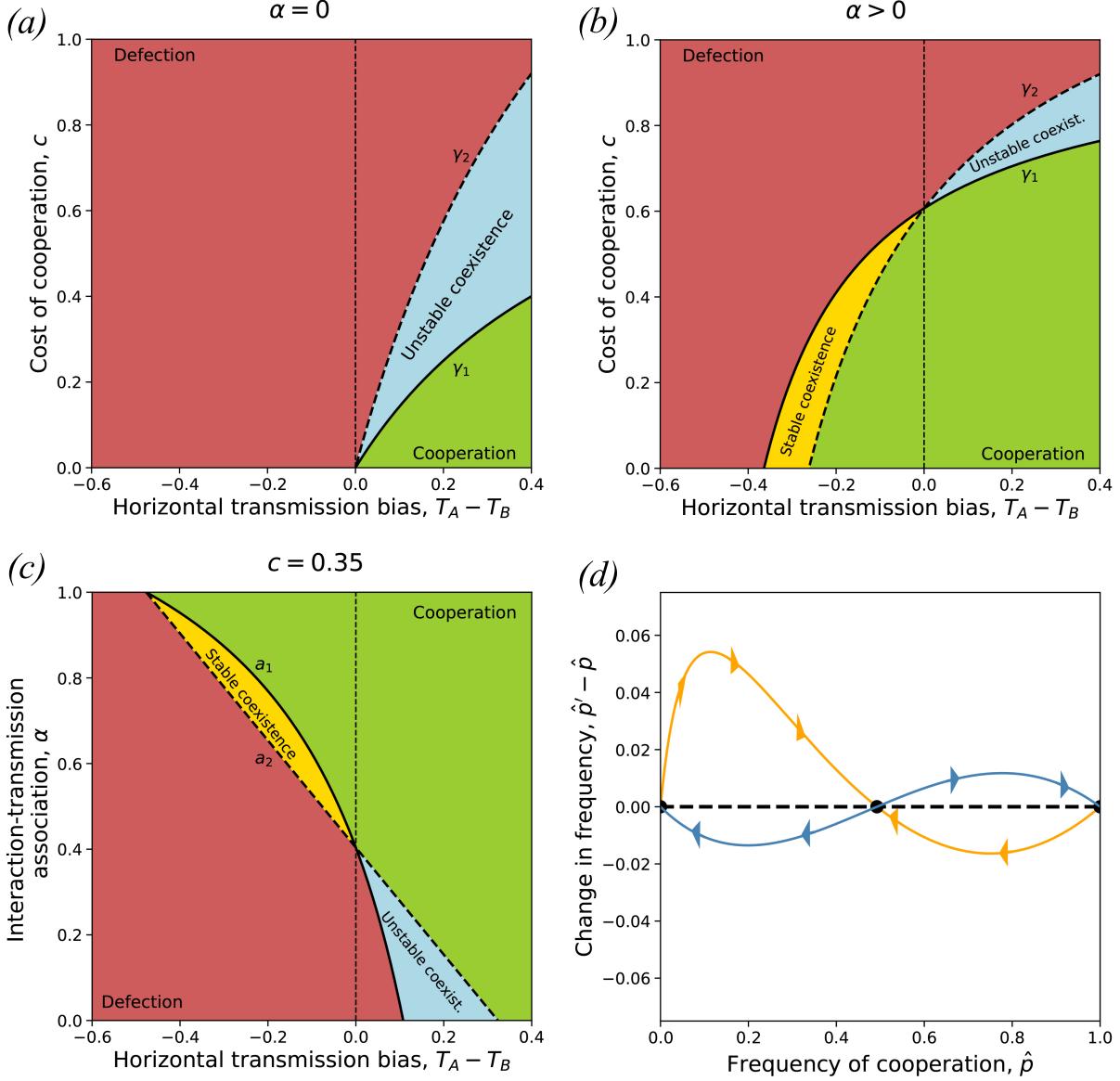


Figure 3: Evolution of cooperation under vertical and horizontal cultural transmission ($v=1$).

The figure shows parameter ranges for global fixation of cooperation (green), global fixation of defection (red), fixation of either cooperation or defection depending on the initial conditions, i.e. unstable polymorphism (blue), and stable polymorphism of cooperation and defection (yellow). **(a-c)** The horizontal transmission bias ($T_A - T_B$) is on the x-axis. In panels **(a)** and **(b)**, the cost of cooperation c is on the y-axis and the cost thresholds γ_1 and γ_2 (Eq. 14) are the solid and dashed lines, respectively. In panel **(c)**, interaction-transmission association α is on the y-axis and the interaction-transmission association thresholds a_1 and a_2 (Eqs. 19) are the solid and dashed lines, respectively. Here, $b = 1.3$, $T_A = 0.4$, $v = 1$, (a) $\alpha = 0$, (b) $\alpha = 0.7$, (c) $c = 0.35$. **(d)** Change in frequency of cooperation among juveniles ($\hat{p}' - \hat{p}$) as a function of the frequency (\hat{p}), see Eq. 12. The orange curve shows convergence to a stable polymorphism ($T_A = 0.4$, $T_B = 0.9$, $b = 12$, $c = 0.35$, $v = 1$, and $\alpha = 0.45$). The blue curve shows fixation of either cooperation or defection, depending on the initial frequency ($T_A = 0.5$, $T_B = 0.1$, $b = 1.3$, $c = 0.904$, $v = 1$, and $\alpha = 0.4$). Black circles show the three equilibria.

282 **4.2 Evolution of interaction-transmission association**

We now focus on the evolution of interaction-transmission association under perfect vertical transmission, $v = 1$, assuming that the population is initially at a stable polymorphism of the two phenotypes, cooperation A and defection B , where the frequency of A among juveniles is \hat{p}^* (Eq. 13). Note that for a stable polymorphism, there must be horizontal bias for defection, $T_A < T_B$, and an intermediate cost of cooperation, $\gamma_2 < c < \gamma_1$ (Eq. 14), see Figure 3b. The equilibrium population mean fitness is $\bar{w}^* = 1 + \hat{p}^*(b - c)$, which is increasing in \hat{p}^* , and \hat{p}^* is increasing in α (Appendix C). Therefore, \bar{w}^* increases as α increases. But can this population-level advantage lead to the evolution of α ?

To answer this question, we add a “modifier locus” [27, 28, 29, 30] that determines the value of α but has no direct effect on fitness. This locus has two alleles, M and m , which induce interaction-transmission associations α_1 and α_2 , respectively. Suppose that the population has evolved to a stable equilibrium \hat{p}^* when only allele M is present. We study the local stability of this equilibrium to invasion by the modifier allele m ; this is called “external stability” [29, 31] and obtain the following result.

Result 2. *From a stable polymorphism between cooperation and defection, a modifier allele can successfully invade the population if it decreases the interaction-transmission association α .*

The analysis is in Appendix D. This reduction principle entails that successful invasions will reduce the frequency of cooperation, as well as the population mean fitness (Figure 4). Furthermore, if we a modifier allele that decreases α appears and invades the population from time to time, then the value of α will continue to decrease, further reducing the frequency of cooperation and the population mean fitness. This evolution will proceed as long as there is a stable polymorphism, that is, as long as $a_2 < \alpha < a_1$ (Remark 2, Figure 3c). Thus, we can expect the value of α to approach a_2 , the frequency of cooperation to fall to zero, and the population mean fitness to decrease to one (Figure 4).

We can do similiar analysis for the general case where $0 < v < 1$ as we did in Appendix D. First we found the value of the characteristic polynomial at 1, $R(1)$ using SymPy [32], a Python library for symbolic mathematics.

$$R(1) = \frac{cv\hat{p}^*[T_A b\hat{p}^{*2}(v\alpha_2 - 1) - 2T_A b\hat{p}^*v\alpha_2 + T_A b\hat{p}^*(1 + v\alpha_2)]}{b\hat{p}(b\hat{p}^* - 2\hat{p}^* + 2) + c\hat{p}^*(c\hat{p}^* - 2) + 1} \\ + \frac{cv\hat{p}^*[T_A c\hat{p}^{*2}(1 - v) + T_A c\hat{p}^*(v - 1) - T_A \hat{p}^*(1 - c) + T_A]}{b\hat{p}(b\hat{p}^* - 2\hat{p}^* + 2) + c\hat{p}^*(c\hat{p}^* - 2) + 1} \\ + \frac{cv\hat{p}^*[T_B b\hat{p}^{*2}(1 - v\alpha_2) + T_B b\hat{p}^2(v\alpha_2 - 1) + T_B c\hat{p}^{*2}(v - 1)]}{b\hat{p}(b\hat{p}^* - 2\hat{p}^* + 2) + c\hat{p}^*(c\hat{p}^* - 2) + 1} \\ + \frac{cv\hat{p}^*[T_B c\hat{p}^*(1 - v) + T_B cv(1 - \hat{p}^* + T_B(\hat{p}^* - 1) + cv(\hat{p}^* - 1))]}{b\hat{p}(b\hat{p}^* - 2\hat{p}^* + 2) + c\hat{p}^*(c\hat{p}^* - 2) + 1} \quad (23)$$

Similarly to what we did in Appendix D, we should find when $R(1) < 0$. However, when adding the vertical transmission into the system the math is more complicated. Therefore, we will use a different technique. We know that when $\alpha_1 = \alpha_2$, $R(1) = 0$. Now, let’s assume that $\alpha_2 = \alpha_1 + \epsilon$. The derivative

312 of $R(1)$ by ϵ will give us better understand about the sign of $R(1)$ when $\alpha_2 < \alpha_1$.

$$\frac{\partial R(1)}{\partial \epsilon} = \frac{cbv^2\hat{p}^*[T_A\hat{p}^{*2} - 2T_A\hat{p}^* + T_A - T_B\hat{p}^{*2} + T_B\hat{p}^*]}{b\hat{p}(b\hat{p}^* - 2\hat{p}^* + 2) + c\hat{p}^*(c\hat{p}^* - 2) + 1} \quad (24)$$

314 We can simplify Eq. 24

$$\frac{\partial R(1)}{\partial \epsilon} = \frac{cbv^2\hat{p}^*[(T_A - T_B)\hat{p}^{*2} + (T_B - 2T_A)\hat{p}^* + T_A]}{(b\hat{p}^* - c\hat{p}^*)^2 + 2\hat{p}^*(b - c) + 1} \quad (25)$$

316 Since $b > c$ the denominator is always positive. The numerator is quadratic polynomial of \hat{p}^* with the following roots:

318 $\hat{p}_1^* = \frac{T_A}{T_A - T_B}\hat{p}_2^* = 1$ (26)

$\hat{p}_1^* < 0$ since $T_B > T_A$ and since the quadratic polynomial has negative leading coefficient then the 320 numerator is positive for every $\hat{p}_1^* < \hat{p}^* < 1$ which is always true. We found that derivative of $R(1)$ by ϵ is positive for every ϵ . Therefore, $R(1)$ grows as ϵ grows, or in other words as $\alpha_2 - \alpha_1$ grows.

322 Thus, $R(1) < 0$ if and only if $\alpha_2 < \alpha_1$. Therefore, different value of vertical transmission does not change the evolution of social-interaction association and invasion is possible when $\alpha_2 < \alpha_1$.

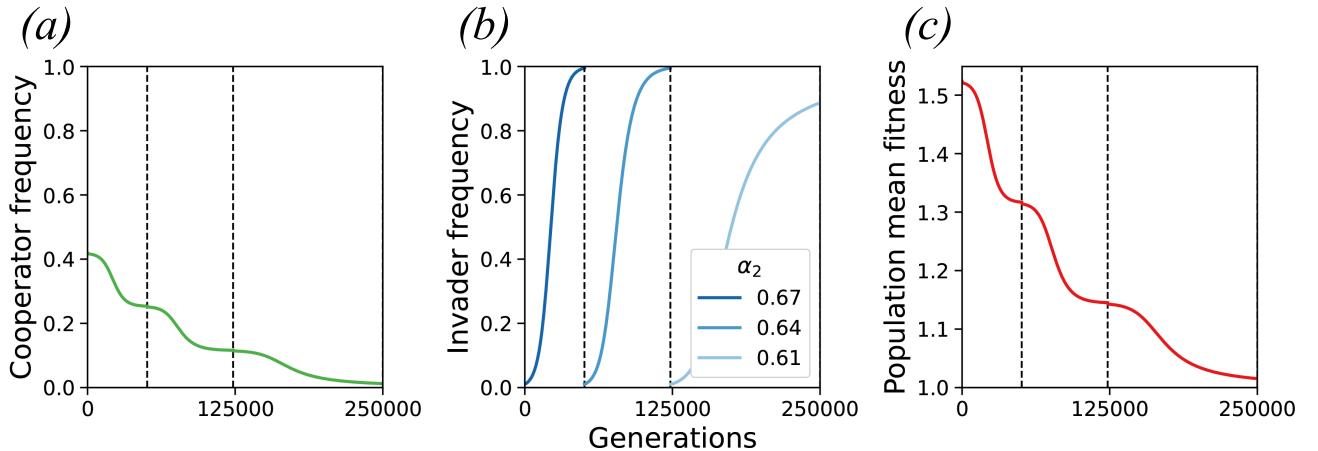


Figure 4: Reduction principle for interaction-transmission association. Consecutive fixation of modifier alleles that reduce interaction-transmission association α in numerical simulations of evolution with two modifier alleles (Eq. D1). When an invading modifier allele is established in the population (frequency > 99.95%), a new modifier allele that reduces interaction-transmission association by 5% is introduced (at initial frequency 0.5%). **(a)** The frequency of the cooperative phenotype A over time. **(b)** The frequency of the invading modifier allele m over time. **(c)** The population mean fitness (\bar{w}) over time. Here, $v = 1$, $c = 0.05$, $b = 1.3$, $T_A = 0.4 < T_B = 0.7$, initial interaction-transmission association $\alpha_1 = 0.7$, lower interaction-transmission association threshold $\alpha_2 = 0.605$.

324 **4.3 Population structure**

All the simulations in this section were made by Ohad Lewin-Epstein from Tel Aviv University.
326 Interaction-transmission association may also emerge from population structure. Consider a pop-
ulation colonizing a two-dimensional grid of size 100-by-100, where each site is inhabited by one
328 individual, similarly to the model of Lewin-Epstein and Hadany [24]. Each individual is characterized
by its phenotype: either cooperator, A , or defector, B . Initially, each site in the grid is randomly
330 colonized by either a cooperator or a defector, with equal probability. In each generation, half of the
individuals are randomly chosen to "initiate" interactions, and these initiators interact with a random
332 neighbor (i.e. individual in a neighboring site) in a prisoners' dilemma game (Figure 1c) and a random
neighbor (with replacement) for horizontal cultural transmission (Figure 1b). The expected number of
334 each of these interactions per individual per generation is one, but the realized number of interactions
can be zero, one, or even more than one, and in every interaction both individuals are affected, not just
336 the initiator. The effective interaction-transmission association α in this model is the probability that
the same neighbor is picked for both interactions, or $\alpha = 1/M$, where M is the number of neighbors.
338 On an infinite grid, $M = 8$ (i.e. Moore neighbourhood [33]), but on a finite grid M can be lower
in neighbourhoods close to the grid border. As before, T_A and T_B are the probabilities of successful
340 horizontal transmission of phenotypes A and B , respectively.

The order of the interactions across the grid at each generation is random. After all interactions take
342 place, an individual's fitness is determined by $w = 1 + b \cdot n_b - c \cdot n_c$, where n_b is the number of interactions
that individual had with cooperative neighbors, and n_c is the number of interactions in which that
344 individual cooperated (note that the phenotype may change between consecutive interactions due to
horizontal transmission). Then, a new generation is produced, and the sites can be settled by offspring
346 of any parent, not just the neighboring parents. Selection is global, rather than local, in accordance
with our deterministic model: The parent is randomly drawn with probability proportional to its
348 fitness, divided by the sum of the fitness values of all potential parents. Offspring are assumed to have
the same phenotype as their parents (i.e. $v = 1$).

350 The outcomes of stochastic simulations with such a structured population are shown in Figure 5, which
demonstrates that the highest cost of cooperation c that permits the evolution of cooperation agrees
352 with the conditions derived above for our model without population structure or stochasticity. An
example of stochastic stable polymorphism is shown in Figure 5c. Changing the simulation so that
354 selection is local (i.e. sites can only be settled by offspring of neighboring parents) had only a minor
effect on the agreement with the derived conditions (Figure 6).
356 These comparisons between the deterministic unstructured model and the stochastic structured model
show that the conditions derived for the deterministic model can be useful for predicting the dynamics
358 under complex scenarios. Moreover, this structured population model demonstrates that our parameter
for interaction-transmission association, α , can represent local interactions between individuals.

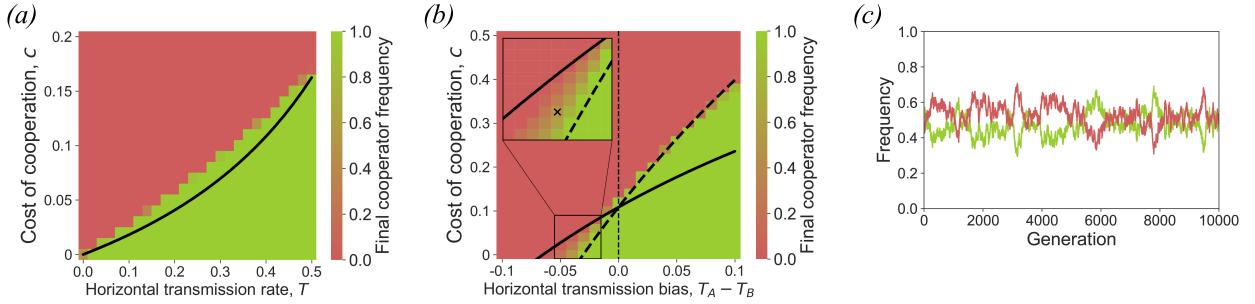


Figure 5: Evolution of cooperation in a structured population. (a-b) The expected frequency of cooperators in a structured population after 10,000 generations is shown (red for 0%, green for 100%) as a function of both the cost of cooperation, c , on the y-axis, and either the symmetric horizontal transmission rate, $T = T_A = T_B$, on the x-axis of panel (a), or the transmission bias, $T_A - T_B$, on the x-axis of panel (b). Black curves represent the cost thresholds for the evolution of cooperation in a well-mixed population with interaction-transmission association, where $\alpha = 1/8$ in inequality 16 for panel (a) and in Eqs. 14 for panel (b). The inset in panel (b) focuses on an area of the parameter range in which neither phenotype is fixed throughout the simulation, maintaining a stochastic locally stable polymorphism [34]. This stochastic polymorphism is illustrated in panel (c), which shows the frequency of cooperators (green) and defectors (red) over time for the parameter set marked by an x in panel (b). In all cases, the population evolves on a 100-by-100 grid. Cooperation and horizontal transmission are both local between neighbouring sites, and each site has 8 neighbours. Selection operates globally (see Figure S2 for results from a model with local selection). Simulations were stopped at generation 10,000 or if one of the phenotypes fixed. 50 simulations were executed for each parameter set. Benefit of cooperation, $b = 1.3$; perfect vertical transmission $v = 1$. (a) Symmetric horizontal transmission, $T = T_A = T_B$; (b) Horizontal transmission rate T_A is fixed at 0.4, and T_B varies, $0.3 < T_B < 0.5$. (c) Horizontal transmission rates $T_A = 0.4 < T_B = 0.435$ and cost of cooperation $c = 0.02$.

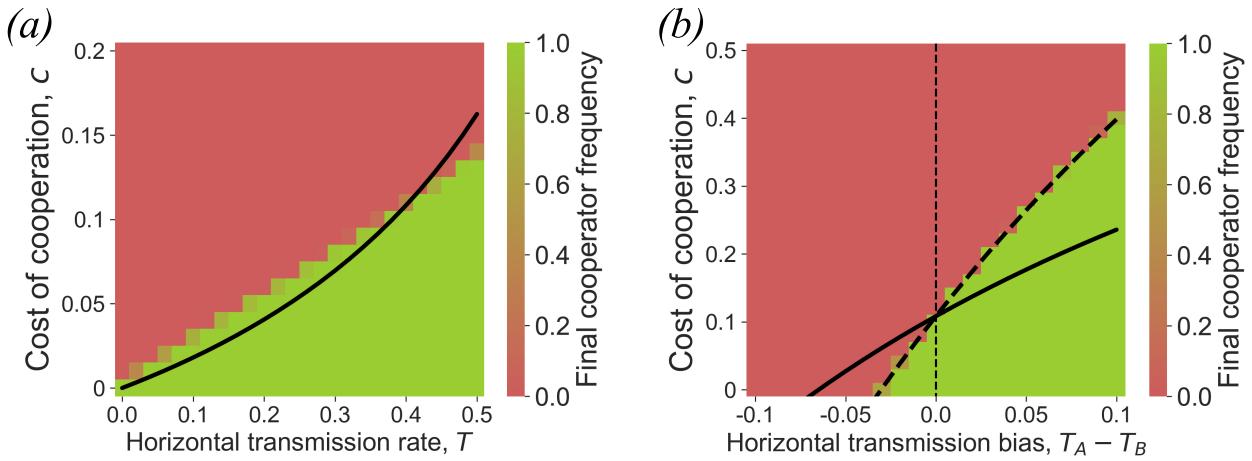


Figure 6: Evolution of cooperation in a structured population with local selection. The expected frequency of cooperators in a structured population after 10,000 generations is shown (red for 0%, green for 100%) as a function of both the cost of cooperation (c) on the y-axis, and the symmetric horizontal transmission rate ($T = T_A = T_B$) on the x-axis of panel **(a)**, or the transmission bias $T_A - T_B$ on the x-axis of panel **(b)**. Cooperation and horizontal transmission are both local between neighbouring sites, and each site had 8 neighbours. Selection operates locally (see Figure 4 for results from a model with global selection). The black curves represent the cost thresholds for the evolution of cooperation in a well-mixed population with interaction-transmission association, where $\alpha = 1/8$ in inequality 14 for panel **(a)** and in Eqs. 12 for panel **(b)**. The population evolves on a 100-by-100 grid. Simulations were stopped at generation 10,000 or if one of the phenotypes fixed. 50 simulations were executed for each parameter set. Here, benefit of cooperation, $b = 1.3$; perfect vertical transmission $v = 1$. **(a)** Symmetric horizontal transmission, $T = T_A = T_B$. **(b)** Horizontal transmission rate T_A is fixed at 0.4, and T_B varies, $0.3 < T_B < 0.5$.

360 **5 Discussion**

Under a combination of vertical, oblique, and horizontal transmission with payoffs in the form
362 of a prisoner’s dilemma game, cooperation or defection can either fix or coexist, depending on
the relationship between the cost and benefit of cooperation, the horizontal transmission bias, and
364 the association between social interaction and horizontal transmission (Result 1, Figures 2 and 3).
Importantly, cooperation can increase when initially rare (i.e. invade a population of defectors) if and
366 only if, rewriting inequality 15, $c \cdot v(1 - T_B) < b \cdot v\alpha T_A + (T_A - T_B)$, namely, the effective cost of
cooperation (left-hand side) is smaller than the effective benefit plus the horizontal transmission bias
368 (right-hand side). This condition cannot be formulated in the form of Hamilton’s rule, $c < b \cdot r$, due to
the effect of biased horizontal transmission, represented by $(T_A - T_B)$. Remarkably, a polymorphism
370 of cooperation and defection can be stable if horizontal transmission is biased in favor of defection
 $(T_A < T_B)$ and both c and α are intermediate (yellow areas in Figures 2 and 3).

372 We find that stronger interaction-transmission association α leads to evolution of higher frequency
of cooperation and increased population mean fitness. Nevertheless, when cooperation and defection
374 coexist, α is expected to be reduced by natural selection, leading to extinction of cooperation and
decreased population mean fitness (Result 2, Figure 4). With $\alpha = 0$, the benefit of cooperation cannot
376 facilitate its evolution; it can only succeed if horizontal transmission is biased in its favor.

Indeed, in our model, horizontal transmission plays a major role in the evolution of cooperation:
378 increasing the transmission of cooperation, T_A , or decreasing the transmission of defection, T_B , facilitates
the evolution of cooperation. However, the effect of oblique transmission is more complicated.
380 When there is horizontal transmission bias in favor of cooperation, $T_A > T_B$, increasing the rate of
oblique transmission, $1 - v$, will facilitate the evolution of cooperation. In contrast, when the bias is
382 in favor of defection, $T_A < T_B$, higher rates of vertical transmission, v , are advantageous for cooperation,
and the rate of vertical transmission must be high enough ($v > \hat{v}$) for cooperation to fix in the
384 population.

Our deterministic model provides a good approximation to outcomes of simulations of a complex
386 stochastic model with population structure in which individuals can only interact with and transmit
to their neighbors. In these structured populations interaction-transmission association arises due to
388 both social interactions and horizontal cultural transmission being local (Figure 5).

Feldman et al. [21] studied the dynamics of an altruistic phenotype with vertical cultural transmission
390 and a gene that modifies the transmission of the phenotype. Their results are very sensitive to
this genetic modification: without it, the conditions for invasion of the altruistic phenotype reduce
392 to Hamilton’s rule. Further work is needed to incorporate such genetic modification of cultural
transmission into our model. Woodcock [22] stressed the significance of non-vertical transmission for
394 the evolution of cooperation and carried out simulations with prisoner’s dilemma payoffs but without
horizontal transmission or interaction-transmission association ($\alpha = 0$). Nevertheless, his results
396 demonstrated that it is possible to sustain altruistic behavior via cultural transmission for a substantial
length of time. He further hypothesized that horizontal transmission can play an important role in the

398 evolution of cooperation, and our results provide strong evidence for this hypothesis.

To understand the role of horizontal transmission, we first review the role of *assortment*. Eshel and
400 Cavalli-Sforza [18] showed that altruism can evolve when the tendency for *assortative meeting*, i.e.
402 for individuals to interact with others of their own phenotype, is strong enough. Fletcher and Doebeli
404 [17] further argued that a general explanation for the evolution of altruism is given by *assortment*: the
406 correlation between individuals that carry an altruistic trait and the amount of altruistic behavior in
408 their interaction group (see also Bijma and Aanen [19]). They suggested that to explain the evolution of
410 altruism, we should seek mechanisms that generate assortment, such as population structure, repeated
412 interactions, and individual recognition. Our results highlight another mechanism for generating
assortment: an association between social interactions and horizontal transmission that creates a
408 correlation between one's partner for interaction and the partner for transmission. This mechanism
does not require repeated interactions, population structure, or individual recognition. We show that
410 high levels of such interaction-transmission association greatly increase the potential for evolution of
cooperation. With enough interaction-transmission association, cooperation can increase in frequency
412 when initially rare even when there is horizontal transmission bias against it ($T_A < T_B$).

How does non-vertical transmission generate assortment? Lewin-Epstein et al. [23] and Lewin-
414 Epstein and Hadany [24] suggested that microbes that induce their hosts to act altruistically can
be favored by selection, which may help to explain the evolution of cooperation. From the kin
416 selection point-of-view, if microbes can be transmitted *horizontally* from one host to another during
host interactions, then following horizontal transmission the recipient host will carry microbes that
418 are closely related to those of the donor host, even when the two hosts are (genetically) unrelated.
From the assortment point-of-view, infection by behavior-determining microbes during interactions
420 effectively generates assortment because a recipient of help may be infected by a behavior-determining
microbe and consequently become a helper. Cultural horizontal transmission can similarly generate
422 assortment between cooperators and enhance the benefit of cooperation if cultural transmission and
helping interactions occur between the same individuals, i.e. when there is interaction-transmission
424 association, so that the recipient of help may also be the recipient of the cultural trait for cooperation.
Thus, with horizontal transmission, “assortment between focal cooperative players and cooperative
426 acts in their interaction environment” [17] is generated not because the helper is likely to be helped,
but rather because the helped is likely to become a helper.

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Appendices

504 Appendix A Local stability criterion

Let $f(p) = \lambda \cdot (p' - p)$, where $\lambda > 0$, and 0 and 1 are equilibria, that is, $f(0) = 0$ and $f(1) = 0$.

506 Set $p > p^* = 0$. Using a linear approximation for $f(p)$ near 0, we have

$$p' < p \Leftrightarrow f(p)/p < 0 \Leftrightarrow \frac{f'(0) \cdot p + O(p^2)}{p} < 0 \Leftrightarrow f'(0) + O(p) < 0. \quad (\text{A1})$$

508 Therefore, by definition of big-O notation, if $f'(0) < 0$ then there exists $\epsilon > 0$ such that for any local perturbation $0 < p < \epsilon$, it is guaranteed that $0 < p' < p$; that is, p' is closer to zero than p .

510 Set $p < p^* = 1$ Using a linear approximation for $f(p)$ near 1, we have

$$1 - p' < 1 - p \Leftrightarrow -\frac{f(p)}{1 - p} < 0 \Leftrightarrow \frac{f'(1)(p - 1) + O((p - 1)^2)}{p - 1} < 0 \Leftrightarrow f'(1) - O(1 - p) < 0. \quad (\text{A2})$$

512 Therefore, if $f'(1) < 0$ then there exists $\epsilon > 0$ such that for any $1 - \epsilon < 1 - p < 1$ we have $1 - p' < 1 - p$; that is, p' is closer to one than p .

514 Appendix B Equilibria and stability

Let $f(\hat{p}) = \bar{w}(\hat{p}' - \hat{p})$. Then, using SymPy [32], a Python library for symbolic mathematics, this
516 simplifies to

$$f(\hat{p}) = \bar{w}(\hat{p}' - \hat{p}) = \beta_1 \hat{p}^3 + \beta_2 \hat{p}^2 + \beta_3 \hat{p} , \quad (\text{B1})$$

518 where

$$\begin{aligned} \beta_1 &= [c(1 - v) - b(1 - \alpha v)](T_A - T_B) , \\ \beta_2 &= -\beta_1 - \beta_3 , \\ \beta_3 &= \alpha b v T_A - c v (1 - T_B) + (T_A - T_B) . \end{aligned} \quad (\text{B2})$$

520 If $T = T_A = T_B$ then $\beta_1 = 0$ and $\beta_3 = -\beta_2 = \alpha b v T - c v (1 - T)$, and $f(\hat{p})$ becomes a quadratic
polynomial,

$$522 \quad f(\hat{p}) = \hat{p}(1 - \hat{p})[\alpha b v T - c v (1 - T)] . \quad (\text{B3})$$

Clearly the only two equilibria are the fixations $\hat{p} = 0$ and $\hat{p} = 1$, which are locally stable if
524 $f'(\hat{p}) < 0$ near the equilibrium (see Appendix A), where $f'(\hat{p}) = (1 - 2\hat{p})[\alpha b v T - c v (1 - T)]$, so
that

$$\begin{aligned} 526 \quad f'(0) &= \alpha b v T - c v (1 - T) , \\ f'(1) &= -\alpha b v T + c v (1 - T) . \end{aligned} \quad (\text{B4})$$

In the general case where $T_A \neq T_B$, the coefficient β_1 is not necessarily zero, and $f(\hat{p})$ is a cubic
528 polynomial. Therefore, three equilibria may exist, two of which are $\hat{p} = 0$ and $\hat{p} = 1$, and the third
is

$$530 \quad \hat{p}^* = \frac{\beta_3}{\beta_1} = \frac{\alpha b v T_A - c v (1 - T_B) + (T_A - T_B)}{[c(1 - v) - b(1 - \alpha v)](T_A - T_B)} . \quad (\text{B5})$$

Note that the sign of the cubic (Eq. B1) at positive (negative) infinity is equal (opposite) to the sign of
532 β_1 . If $T_A > T_B$, then

$$\beta_1 < [c(1 - \alpha v) - b(1 - \alpha v)](T_A - T_B) = (1 - \alpha v)(c - b)(T_A - T_B) < 0 , \quad (\text{B6})$$

534 since $c < b$ and $\alpha v < 1$. Hence the signs of the cubic at positive and negative infinity are negative and
positive, respectively. First, if $\beta_3 < \beta_1$ then $1 < \hat{p}^*$. Also, $f'(0) < 0$ and $f'(1) > 0$; that is, fixation
536 of the defector phenotype B is the only locally stable feasible equilibrium. Second, if $\beta_1 < \beta_3 < 0$
then $0 < \hat{p}^* < 1$ and therefore $f'(0) < 0$ and $f'(1) < 0$ so that both fixations are locally stable
538 and \hat{p}^* separates the domains of attraction. Third, if $0 < \beta_3$ then $\hat{p}^* < 0$ and therefore $f'(0) > 0$
and $f'(1) < 0$; that is, fixation of the cooperator phenotype A is the only locally stable legitimate
540 equilibrium.

Similarly, if $T_A < T_B$, then

$$542 \quad \beta_1 > [c(1 - \alpha v) - b(1 - \alpha v)](T_A - T_B) = (1 - \alpha v)(c - b)(T_A - T_B) > 0 , \quad (\text{B7})$$

since $c < b$ and $\alpha v < 1$, and the signs of the cubic at positive and negative infinity are positive and
544 negative, respectively. First, if $\beta_3 < 0$ then $\hat{p}^* < 0$ and therefore $f'(0) < 0$ and $f'(1) > 0$; that

is, fixation of the defector phenotype A is the only locally stable legitimate equilibrium. Second, if
546 $0 < \beta_3 < \beta_1$ then $0 < \hat{p}^* < 1$ and therefore $f'(0) > 0$ and $f'(1) > 0$; that is, both fixations are locally
548 unstable and \hat{p}^* is a stable polymorphic equilibrium. Third, if $\beta_1 < \beta_3$ then $\hat{p}^* > 1$ and therefore
 $f'(0) > 0$ and $f'(1) < 0$, and fixation of the cooperator phenotype A is the only locally stable feasible
equilibrium.

550 This analysis can be summarized as follows:

1. *Fixation of cooperation*: if (i) $T = T_A = T_B$ and $c < b \cdot \frac{\alpha T}{1-T}$; or if (ii) $T_A > T_B$ and $0 < \beta_3$; or if
552 (iii) $T_A < T_B$ and $\beta_1 < \beta_3$.
2. *Fixation of the defection*: if (iv) $T = T_A = T_B$ and $c > b \cdot \frac{\alpha T}{1-T}$; or if (v) $T_A > T_B$ and $\beta_3 < \beta_1 < 0$;
554 or if (vi) $T_A < T_B$ and $\beta_3 < 0$.
3. *polymorphism of both phenotypes at \hat{p}^** : if (vii) $T_A < T_B$ and $0 < \beta_3 < \beta_1$.
- 556 4. *Fixation of either phenotype depending on initial frequency*: if (viii) $T_A > T_B$ and $\beta_1 < \beta_3 < 0$.

We now proceed to use the cost thresholds, γ_1 and γ_2 , and the vertical transmission threshold, \hat{v} (Eq. 14).

558 First, assume $T_A < T_B$. $\beta_3 < 0$ requires $\gamma_1 < c$. For $\beta_3 < \beta_1$ we need $c[v(1-T_B)+(1-v)(T_A-T_B)] > b\alpha T_B + (1+b)(T_A - T_B)$. Note that the expression in the square brackets is positive if and only if
560 $v > \hat{v}$. Thus, for $\beta_3 < \beta_1$ we need $v > \hat{v}$ and $\gamma_2 < c$ or $v < \hat{v}$ and $c < \gamma_2$, and for $0 < \beta_3 < \beta_1$ we need
562 $v > \hat{v}$ and $\gamma_2 < c < \gamma_1$, or $v < \hat{v}$ and $c < \min(\gamma_1, \gamma_2)$. For $\beta_1 < \beta_3$ we need $v > \hat{v}$ and $c < \gamma_2$ or $v < \hat{v}$
and $\gamma_2 < c$. However, some of these conditions cannot be met, since $v < \hat{v}$ implies $c < 1 < \gamma_2$.

Second, assume $T_A > T_B$. $\beta_3 > 0$ requires $\gamma_1 > c$. For $\beta_1 < \beta_3$ we need $c[v(1-T_B)+(1-v)(T_A-T_B)] < b\alpha T_B + (1+b)(T_A - T_B)$. Thus for $\beta_1 < \beta_3$ we need $v > \hat{v}$ and $c < \gamma_2$ or $v < \hat{v}$ and $c > \gamma_2$. But $\hat{v} < 0$ when $T_A > T_B$, and therefore we have $\beta_1 < \beta_3$ if $c < \gamma_2$. Similarly, we have $\beta_3 < \beta_1$ if $c > \hat{\gamma}_2$.

566 This analysis is summarized in Result 1.

Appendix C Effect of interaction-transmission association on mean fitness

568

To determine the effect of increasing α on the stable population mean fitness, $\bar{w}^* = 1 + (b - c)\hat{p}^*$, we
570 must analyze its effect on \hat{p}^* ,

$$\frac{\partial \hat{p}^*}{\partial \alpha} = \frac{bT_A - c(1 - T_B) + (T_A - T_B)}{b(1 - \alpha)^2(T_B - T_A)}. \quad (\text{C1})$$

572 Note that stable polymorphism implies $c < \gamma_1$, and because $\alpha < 1$, we have

$$c < \gamma_1 = \frac{b\alpha T_A + (T_A - T_B)}{1 - T_B} < \frac{bT_A + (T_A - T_B)}{1 - T_B}. \quad (\text{C2})$$

574 Therefore, the numerator in Eq. C1 is positive. Since $T_A < T_B$, the denominator in Eq. C1 is also
positive, and hence the derivative $\partial \hat{p}^*/\partial \alpha$ is positive. Thus, the population mean fitness increases as
576 interaction-transmission association α increases.

Appendix D Reduction principle

578 We assume here that $v = 1$, i.e. no oblique transmission, and therefore $\hat{p} = \dot{p}$. Denote the frequencies of the pheno-genotypes AM , BM , Am , and Bm by $\mathbf{p} = (\dot{p}_1, \dot{p}_2, \dot{p}_3, \dot{p}_4)$. The frequencies of the
580 pheno-genotypes in the next generation are defined by the recursion system,

$$\begin{aligned}
\bar{w}\dot{p}'_1 &= \dot{p}_1x(1+b-c)(1-(1-\alpha_1)(1-x)T_B) + \\
&\quad \dot{p}_1(1-x)(1-c)(1-\alpha_1T_Bx-T_B(1-x)) + \\
&\quad \dot{p}_2x(1+b)T_A(x+\alpha_1(1-x)) + \\
&\quad \dot{p}_2(1-x)x(1-\alpha_1)T_A, \\
\bar{w}\dot{p}'_2 &= \dot{p}_1x(1+b-c)(1-\alpha_1)(1-x)T_B + \\
&\quad \dot{p}_1(1-x)(1-c)(\alpha_1T_B+(1-\alpha_1)(1-x)T_B) + \\
&\quad \dot{p}_2x(1+b)(1-\alpha_1T_A(1-x)-T_AX) + \\
&\quad \dot{p}_2(1-x)(1-(1-\alpha_1)xT_A), \\
\bar{w}\dot{p}'_3 &= \dot{p}_3x(1+b-c)(1-(1-\alpha_2)(1-x)T_B) + \\
&\quad \dot{p}_3(1-x)(1-c)(1-\alpha_2T_Bx-T_B(1-x)) + \\
&\quad \dot{p}_4x(1+b)T_A(x+\alpha_2(1-x)) + \\
&\quad \dot{p}_4(1-x)x(1-\alpha_2)T_A, \\
\bar{w}\dot{p}'_4 &= \dot{p}_3x(1+b-c)(1-\alpha_2)(1-x)T_B + \\
&\quad \dot{p}_3(1-x)(1-c)(\alpha_2T_B+(1-\alpha_2)(1-x)T_B) + \\
&\quad \dot{p}_4x(1+b)(1-\alpha_2T_A(1-x)-T_AX) + \\
&\quad \dot{p}_4(1-x)(1-(1-\alpha_2)xT_A),
\end{aligned} \tag{D1}$$

582 where $x = \dot{p}_1 + \dot{p}_3$ is the total frequency of the cooperative phenotype A , and $\bar{w} = 1 + (b - c)x$ is the population mean fitness.

584 The equilibrium where only allele M is present is $\mathbf{p}^* = (\dot{p}^*, 1 - \dot{p}^*, 0, 0)$, where

$$\dot{p}^* = \frac{c(1 - T_B) - b\alpha_1 T_A - (T_A - T_B)}{b(1 - \alpha_1)(T_A - T_B)}, \tag{D2}$$

586 setting $\alpha = \alpha_1$ and $v = 1$ in Eq. 13. When $v = 1$, \dot{p}^* is a feasible polymorphism ($0 < \dot{p}^* < 1$) if $T_A < T_B$ and $\gamma_2 < c < \gamma_1$ (Result 1).

588 The local stability of \mathbf{p}^* to the introduction of allele m is determined by the linear approximation \mathbf{L}^* of the transformation in Eq. D1 near \mathbf{p}^* (i.e. the Jacobian of the transformation at the equilibrium).

590 \mathbf{L}^* is known to have a block structure, with the diagonal blocks occupied by the matrices \mathbf{L}_{in}^* and \mathbf{L}_{ex}^* [29, 31]. The latter is the external stability matrix: the linear approximation to the transformation near
592 \mathbf{p}^* involving only the pheno-genotypes Am and Bm , derived from Eq. D1, with $\bar{w}^* = 1 + (b - c)\dot{p}^*$ as

the stable population mean fitness,

$$\mathbf{L}_{ex}^* = \frac{1}{\bar{w}^*} \begin{bmatrix} l_{11} & l_{12} \\ l_{21} & l_{22} \end{bmatrix} = \frac{1}{\bar{w}^*} \begin{bmatrix} \frac{\partial \bar{w} \dot{p}_3'}{\partial \dot{p}_3}(\mathbf{p}^*) & \frac{\partial \bar{w} \dot{p}_3'}{\partial \dot{p}_4}(\mathbf{p}^*) \\ \frac{\partial \bar{w} \dot{p}_4'}{\partial \dot{p}_3}(\mathbf{p}^*) & \frac{\partial \bar{w} \dot{p}_4'}{\partial \dot{p}_4}(\mathbf{p}^*) \end{bmatrix} =$$

$$\frac{1}{\bar{w}^*} \begin{bmatrix} (1 + b\dot{p}^* - c)(1 - T_B(1 - \dot{p}^*)) + b\dot{p}^*\alpha_2 T_B(1 - \dot{p}^*) & (1 + b\dot{p}^*)T_A\dot{p}^* + b\dot{p}^*\alpha_2 T_A(1 - \dot{p}^*) \\ (1 + b\dot{p}^* - c)T_B(1 - \dot{p}^*) - b\dot{p}^*\alpha_2 T_B(1 - \dot{p}^*) & (1 + b\dot{p}^*)(1 - T_A\dot{p}^*) - b\dot{p}^*\alpha_2 T_A(1 - \dot{p}^*) \end{bmatrix}. \quad (D3)$$

594

Because we assume that \mathbf{p}^* is internally stable (i.e. locally stable to small perturbations in the frequencies of AM and BM), the stability of \mathbf{p}^* is determined by the eigenvalues of the external stability matrix \mathbf{L}_{ex}^* . This is a positive matrix, and due to the Perron-Frobenius theorem, the leading eigenvalue of \mathbf{L}_{ex}^* is real and positive. Thus, if the leading eigenvalue is less (greater) than one, then the equilibrium \mathbf{p}^* is externally stable (unstable) and allele m cannot (can) invade the population of allele M . The eigenvalues of \mathbf{L}_{ex}^* are the roots of the characteristic polynomial, $R(\lambda)$, which is a quadratic with a positive leading coefficient. Therefore, $\lim_{\lambda \rightarrow \pm\infty} R(\lambda) = \infty$, and the leading eigenvalue is less than one (implying stability) if and only if $R(1) > 0$ and $R'(1) > 0$. Thus, a sufficient condition for external instability of \mathbf{p}^* is $R(1) < 0$.

604 $R(\lambda)$ is defined as a determinant, $R(\lambda) = \det(\mathbf{L}_{ex}^* - \lambda\mathbf{I})$, where \mathbf{I} is the 2-by-2 identity matrix. Since multiplication by a positive factor doesn't change the sign, and using the properties of the determinant, 606 we have

$$\begin{aligned} \text{sign } R(1) &= \text{sign } \det(\mathbf{L}_{ex}^* - \mathbf{I}) = \text{sign}(\bar{w}^*)^2 \det(\mathbf{L}_{ex}^* - \mathbf{I}) = \\ \text{sign } \det(\bar{w}^* \mathbf{L}_{ex}^* - \bar{w}^* \mathbf{I}) &= \text{sign } \det \begin{bmatrix} l_{11} - \bar{w}^* & l_{12} \\ l_{21} & l_{22} - \bar{w}^* \end{bmatrix}, \end{aligned} \quad (D4)$$

608 where l_{ij} are defined in Eq. D3. Adding the second row in Eq. D4 to the first row, which does not change the determinant, and substituting $\bar{w}^* = 1 + (b - c)\dot{p}^*$, we get

$$\begin{aligned} \text{sign } R(1) &= \text{sign } \det \begin{bmatrix} -c(1 - \dot{p}^*) & c\dot{p}^* \\ (1 - \dot{p}^*)[(1 + b\dot{p}^* - c)T_B - b\alpha_2 T_B \dot{p}^*] & \dot{p}^*[-(1 + b\dot{p}^*)T_A - b\alpha_2 T_A(1 - \dot{p}^*) + c] \end{bmatrix} = \\ &= \text{sign} \left[c\dot{p}^*(1 - \dot{p}^*) \cdot \det \begin{bmatrix} -1 & 1 \\ (1 + b\dot{p}^* - c)T_B - b\alpha_2 T_B \dot{p}^* & -(1 + b\dot{p}^*)T_A - b\alpha_2 T_A(1 - \dot{p}^*) + c \end{bmatrix} \right] = \\ &= \text{sign } \det \begin{bmatrix} -1 & 1 \\ (1 + b\dot{p}^* - c)T_B - b\alpha_2 T_B \dot{p}^* & -(1 + b\dot{p}^*)T_A - b\alpha_2 T_A(1 - \dot{p}^*) + c \end{bmatrix}, \end{aligned} \quad (D5)$$

610

since $c > 0$, $0 < \dot{p}^* < 1$. That is,

$$\begin{aligned} \text{sign } R(1) &= \text{sign} \left[(1 + b\dot{p}^*)T_A + b\alpha_2 T_A(1 - \dot{p}^*) - c - (1 + b\dot{p}^* - c)T_B + b\dot{p}^*\alpha_2 T_B \right] = \\ &= \text{sign} \left[(1 + b(1 - \alpha_2)\dot{p}^*)(T_A - T_B) + b\alpha_2 T_A - c(1 - T_B) \right]. \end{aligned} \quad (D6)$$

Substituting \dot{p}^* from Eq. D2, we get

$$\begin{aligned}
R(1) < 0 \Leftrightarrow & [c(1 - T_B) - b\alpha_1 T_A - (T_A - T_B)] \frac{1 - \alpha_2}{1 - \alpha_1} - c(1 - T_B) + b\alpha_2 T_A + (T_A - T_B) < 0 \Leftrightarrow \\
& (1 - \alpha_2)[c(1 - T_B) - b\alpha_1 T_A - (T_A - T_B)] < (1 - \alpha_1)[c(1 - T_B) - b\alpha_2 T_A - (T_A - T_B)] \Leftrightarrow \\
& -b\alpha_1 T_A - \alpha_2 c(1 - T_B) + \alpha_2 (T_A - T_B) < -b\alpha_2 T_A - \alpha_1 c(1 - T_B) + \alpha_1 (T_A - T_B) \Leftrightarrow \\
& \alpha_1[c(1 - T_B) - bT_A - (T_A - T_B)] < \alpha_2[c(1 - T_B) - bT_A - (T_A - T_B)] \Leftrightarrow \\
& \alpha_1[bT_A + (T_A - T_B) - c(1 - T_B)] > \alpha_2[bT_A + (T_A - T_B) - c(1 - T_B)] .
\end{aligned}
\tag{D7}$$

614

We assumed $c < \gamma_1$, and since $0 \leq \alpha_1 \leq 1$,

616

$$\begin{aligned}
c < \gamma_1 = \frac{b\alpha_1 T_A + (T_A - T_B)}{1 - T_B} \Leftrightarrow \\
0 < b\alpha_1 T_A + (T_A - T_B) - c(1 - T_B) \Rightarrow \\
0 < bT_A + (T_A - T_B) - c(1 - T_B) .
\end{aligned}
\tag{D8}$$

Combining inequalities D7 and D8, we find that $R(1) < 0$ if and only if $\alpha_1 > \alpha_2$, which is a sufficient condition for external instability. Therefore, if α_2 , the interaction-transmission association of the invading modifier allele m , is less than α_1 , the interaction-transmission association of the resident allele M , then invasion will be successful.

618

Determining a necessary and sufficient condition for successful invasion is more complicated, requiring analysis of the sign of $R'(1)$. However, we have numerically validated that the leading eigenvalue is greater than one if and only if $\alpha_1 > \alpha_2$.

620