

# Non-Vertical Cultural Transmission, Assortment, and the Evolution of Cooperation

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## Abstract

We construct models for the cultural evolution of cooperation in a well-mixed population with vertical, horizontal, and oblique transmission. Conditions are found for fixation and coexistence of cooperation and defection. Comparisons are drawn with Hamilton's rule and the concepts of relatedness and assortment. We show that assortment between cooperation and horizontal transmission facilitates the evolution of cooperation.

## 16 Introduction

Cooperative behavior can reduce an individual's fitness and increase the fitness of its conspecifics or competitors (Axelrod and Hamilton, 1981). Nevertheless, cooperative behavior appears to occur in many non-human animals (Dugatkin, 1997), including primates (Jaeggi and Gurven, 2013), rats (Rice and Gainer, 1962), birds (Stacey and Koenig, 1990; Krams et al., 2008), and lizards (Sinervo et al., 2006). Evolution of cooperative behavior remains an important conundrum in evolutionary biology. Since the work of Hamilton (1964) and Axelrod and Hamilton (1981), theories for the evolution of cooperative and altruistic behaviors have been intertwined often under the rubric of *kin selection*.

Kin selection theory posits that natural selection is more likely to favor cooperation between more closely related individuals. The importance of relatedness to the evolution of cooperation and altruism was demonstrated by Hamilton (1964), who showed that an allele that determines cooperative behavior will increase in frequency if the reproductive cost to the actor that cooperates,  $c$ , is less than the benefit to the recipient,  $b$ , times the relatedness,  $r$ , between the recipient and the actor. This relatedness coefficient  $r$  measures a probability that an allele sampled from the cooperator is identical by descent to one at the same locus in the recipient. This condition is known as *Hamilton's rule*:

$$c < b \cdot r. \quad (1)$$

Eshel and Cavalli-Sforza (1982) studied a related model for the evolution of cooperative behavior. Their model included *assortative meeting*, or non-random encounters, where a fraction  $m$  of individuals in the population each interact with an individual of the same phenotype, and a fraction  $1 - m$  interacts with a randomly chosen individual. Such assortative meeting may be due, for example, to population structure or active partner choice. In their model, cooperative behavior can evolve if<sup>1</sup> (Eshel and Cavalli-Sforza, 1982, eq. 3.2)

$$c < b \cdot m, \quad (2)$$

where  $b$  and  $c$  are the benefit and cost of cooperation. Here  $m$  in inequality 2 takes the role of the relatedness  $r$  in inequality 1.

The role of assortment in the evolution of altruism was studied by Fletcher and Doebeli (2009) in a *public goods* game. They found that for altruism to evolve, cooperative individuals must experience more cooperation, on average, than defecting individuals, and “thus, the evolution of altruism requires (positive) assortment between focal *cooperative* players and cooperative acts in their interaction environment.” This condition can be summarized by (Fletcher and Doebeli, 2009, eq. 2.3)

$$c < b \cdot (e_C - e_D), \quad (3)$$

where  $e_C$  is the probability that a cooperator receives help, and  $e_D$  is the probability that a defector receives help.<sup>2</sup>

Here we study the evolution of a cooperative behavior that is subject to *cultural transmission*, which allows an individual to acquire attitudes or behavioral traits from other individuals in its social group through imitation, learning, or other modes of communication (Cavalli-Sforza and Feldman, 1981; Richerson and Boyd, 2008). Feldman et al. (1985) introduced the first model for the evolution of altruism by cultural transmission. They demonstrated that if the fidelity of cultural transmission of

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<sup>1</sup>In an extended model, which allows an individual to encounter  $N$  individuals before choosing a partner, the righthand side is multiplied by  $E[N]$ , the expected number of encounters (Eshel and Cavalli-Sforza, 1982, eq. 4.6).

<sup>2</sup>inequality 3 generalizes inequality 1 and inequality 2 by substituting  $e_C = r + p$ ,  $e_D = p$  and  $e_C = m + (1 - m)p$ ,  $e_D = (1 - m)p$ , respectively, where  $p$  is the frequency of cooperators.

54 altruism is  $\varphi$ , then the condition for evolution of altruism in the case of sib-to-sib altruism is (Feldman  
et al., 1985, Eq. 16)

$$56 \quad c < b \cdot \varphi - \frac{1 - \varphi}{\varphi}. \quad (4)$$

Here  $\varphi$  takes the role of ‘relatedness’ ( $r$  in Eq. 1) or ‘assortment’ ( $m$  in Eq. 2), but the effective benefit  
58  $b \cdot \varphi$  is further reduced by  $(1 - \varphi)/\varphi$ . This shows that under a combination of genetic and vertical  
(parent-to-offspring) cultural transmission, the condition for the evolutionary success of altruism  
60 entails a modification of Hamilton’s rule (Eq. 1).

Cultural transmission may be either viewed as vertical, horizontal or oblique: vertical transmission  
62 occurs between parents and offspring, horizontal transmission occurs between individuals from the  
same generation, and oblique transmission occurs to offspring from the generation to which their  
64 parents belong (i.e. from non-parental adults). Evolution under either of these transmission models  
can be more rapid than under pure vertical transmission (Cavalli-Sforza and Feldman, 1981; Lycett  
66 and Gowlett, 2008; Ram et al., 2018). Both Woodcock (2006) and Lewin-Epstein et al. (2017)  
demonstrated that non-vertical transmission can help explain the evolution of cooperative behavior  
68 (the former using simulations with cultural transmission, the latter using a model where cooperation is  
mediated by microbes that manipulate their host’s behavior.) Some of the analyses by Lewin-Epstein  
70 et al. (2017) can be applied to cultural transmission, because models of cultural transmission are  
mathematically similar to those for transmission of infectious diseases (Cavalli-Sforza and Feldman,  
72 1981). Importantly, Woodcock (2006) showed the significance of non-vertical transmission for the  
evolution of cooperation. In their model individuals play a prisoner’s dilemma game. They do not  
74 link between interaction and transmission (i.e. no assortment), but their model demonstrates that it is  
possible to sustain altruistic behaviour via cultural transmission for a significant duration of time in  
76 the absence of any direct biological propensity favouring this behaviour.

We therefore hypothesize that non-vertical cultural transmission can enhance the evolution of coop-  
78 eration. To test this hypothesis we suggest a model in which behavioral changes are mediated by  
cultural transmission that can occur during social interactions. That is, if there exists assortative  
80 meeting in the choice of the social interaction partner, or assortative learning in choice of the cul-  
tural transmission partner. For example, if an individual interacts with a cooperative individual, it  
82 might learn that cooperation is an advantageous behavior and will be cooperative in the future. Our  
cultural evolution models include both vertical and non-vertical transmission of cooperation, and we  
84 investigate these models using mathematical analysis and simulations. Our results demonstrate that  
cultural transmission can facilitate the evolution of cooperation even when genetic transmission can  
86 not because it facilitates the generation of assortment, and that treatment of cooperation as a cultural  
trait, rather than a genetic one, can lead to a better understanding of its evolutionary dynamics.

## 88 **Models**

Consider a very large population whose members are characterized by their phenotype  $\phi$ , which can  
90 be of two types,  $\phi = A$  for cooperators or  $\phi = B$  for defectors. An offspring inherits its phenotype  
from its parent via vertical transmission with probability  $v$  or from a random individual in the parental  
92 population via oblique transmission with probability  $(1 - v)$ . Following Ram et al. (2018), given that  
the parent phenotype is  $\phi$  and assuming uni-parental inheritance, the conditional probability that the  
94 phenotype  $\phi'$  of the offspring is  $A$  is

$$P(\phi' = A \mid \phi) = \begin{cases} v + (1 - v)p, & \text{if } \phi = A \\ (1 - v)p, & \text{if } \phi = B \end{cases}, \quad (5)$$

96 where  $p = P(\phi = A)$  is the frequency of  $A$  among all adults in the parental generation.

Not all adults become parents due to natural selection, and we denote the frequency of phenotype  $A$   
 98 among parents by  $\tilde{p}$ . Therefore, the frequency  $\hat{p}$  of phenotype  $A$  among juveniles (after selection and vertical and oblique transmission) is

$$\begin{aligned}\hat{p} &= \tilde{p}[v + (1 - v)p] + (1 - \tilde{p})[(1 - v)p] \\ &= v\tilde{p} + (1 - v)p.\end{aligned}\tag{6}$$

Individuals are assumed to interact according to a *prisoner's dilemma*. Specifically, individuals  
 102 interact in pairs; a cooperator suffers a fitness cost  $0 < c < 1$ , and its partner gains a fitness benefit  $b$ , where we assume  $c < b$ . Table 1 shows the payoff matrix, i.e. the fitness of an individual with  
 104 phenotype  $\phi_1$  when interacting with a partner of phenotype  $\phi_2$ .

	$\phi_2 = A$	$\phi_2 = B$
$\phi_1 = A$	$1 + b - c$	$1 - c$
$\phi_1 = B$	$1 + b$	$1$

**Table 1: Payoff matrix for prisoner's dilemma.** The fitness of phenotype  $\phi_1$  when interacting with phenotype  $\phi_2$ .  $A$  is a cooperative phenotype,  $B$  is a defector phenotype,  $b$  is the benefit gained by an individual interacting with a cooperator, and  $c$  is the cost of cooperation.  $b > c > 0$ .

Social interactions occur randomly: two individuals with phenotype  $A$  interact with probability  $\hat{p}^2$ ,  
 106 two individuals with phenotype  $B$  interact with probability  $(1 - \hat{p})^2$ , and two individuals with different phenotypes interact with probability  $2\hat{p}(1 - \hat{p})$ .

Horizontal cultural transmission occurs between pairs of individuals from the same generation. It  
 108 occurs between social partners with probability  $\alpha$ , or between a random pair with probability  $1 - \alpha$   
 110 (see Figure 1). The assortment rate  $\alpha$  is therefore the fraction of population that receives (horizontal transmission) from the social interaction partner, and  $1 - \alpha$  receives randomly. Horizontal transmission  
 112 is not always successful, as one partner may reject the other's phenotype. The probability for successful horizontal transmission of phenotypes  $A$  and  $B$  are  $T_A$  and  $T_B$ , respectively (Table 2).

114 Therefore, the frequency  $p'$  of phenotype  $A$  among adults in the next generation, after horizontal

Phenotype $\phi_1$	Phenotype $\phi_2$	Frequency	Fitness of $\phi_1$	$P(\phi_1 = A)$ via horizontal transmission:	
				from partner, $\alpha$	from population, $(1 - \alpha)$
$A$	$A$	$\hat{p}^2$	$1 + b - c$	1	$\hat{p} + (1 - \hat{p})(1 - T_B)$
$A$	$B$	$\hat{p}(1 - \hat{p})$	$1 - c$	$1 - T_B$	$\hat{p} + (1 - \hat{p})(1 - T_B)$
$B$	$A$	$\hat{p}(1 - \hat{p})$	$1 + b$	$T_A$	$\hat{p}T_A$
$B$	$B$	$(1 - \hat{p})^2$	1	0	$\hat{p}T_A$

**Table 2: Interaction frequency, fitness, and transmission probabilities.**

transmission, is

$$\begin{aligned}
 p' = & \hat{p}^2[\alpha + (1 - \alpha)(\hat{p} + (1 - \hat{p})(1 - T_B))] \\
 & + \hat{p}(1 - \hat{p})[\alpha(1 - T_B) + (1 - \alpha)(\hat{p} + (1 - \hat{p})(1 - T_B))] \\
 & + (1 - \hat{p})\hat{p}[\alpha T_A + (1 - \alpha)\hat{p}T_A] \\
 & + (1 - \hat{p})^2[(1 - \alpha)\hat{p}T_A],
 \end{aligned} \tag{7}$$

which simplifies to

$$p' = \hat{p}^2(T_B - T_A) + \hat{p}(1 + T_A - T_B). \tag{8}$$

The frequency of  $A$  among parents (i.e. after selection) follows a similar dynamic, but also includes the effect of natural selection, and is therefore

$$\begin{aligned}
 \bar{w}\tilde{p}' = & \hat{p}^2(1 + b - c)[\alpha + (1 - \alpha)(\hat{p} + (1 - \hat{p})(1 - T_B))] \\
 & + \hat{p}(1 - \hat{p})(1 - c)[\alpha(1 - T_B) + (1 - \alpha)(\hat{p} + (1 - \hat{p})(1 - T_B))] \\
 & + (1 - \hat{p})\hat{p}(1 + b)[\alpha T_A + (1 - \alpha)\hat{p}T_A] \\
 & + (1 - \hat{p})^2[(1 - \alpha)\hat{p}T_A],
 \end{aligned} \tag{9}$$

where fitness values are taken from Table 1 and Table 2, and the population mean fitness is

$$\bar{w} = 1 + \hat{p}(b - c). \tag{10}$$

Eq. 9 can be simplified to

$$\begin{aligned}
 \bar{w}\tilde{p}' = & \hat{p}^2(1 + b - c)(1 - (1 - \hat{p})(1 - \alpha)T_B) \\
 & + \hat{p}(1 - \hat{p})(1 - c)(\hat{p}(1 - \alpha)T_B + 1 - T_B) \\
 & + (1 - \hat{p})\hat{p}(1 + b)(\hat{p}(1 - \alpha) + \alpha)T_A \\
 & + (1 - \hat{p})^2\hat{p}(1 - \alpha)T_A.
 \end{aligned} \tag{11}$$

where  $\hat{p} = v\tilde{p} + (1 - v)p$ .

## Results

### Oblique and Horizontal Transmission

With only oblique and horizontal transmission, i.e.  $v = 0$ , Eq. 6 becomes  $\hat{p} = p$  and Eq. 8 becomes

$$p' = p^2(T_B - T_A) + p(1 + T_A - T_B), \tag{12}$$

which gives the following result.

**Result 1** (Oblique and horizontal transmission of cooperation). *Without vertical transmission ( $v = 0$ ), if there is a horizontal transmission bias in favor of cooperation, namely*

$$T_A > T_B, \tag{13}$$

*then  $p' > p$ , and the frequency of the cooperator phenotype among adults increases every generation.*

If  $T_A = T_B$ , then  $p' = p$  and the population is static. Therefore, in the absence of vertical transmission, selection plays no role in the evolution of cooperation. Hence, cooperation will evolve if the cooperator phenotype has a horizontal transmission bias (see Figure 5c).

## Vertical and Horizontal Transmission

142 With only vertical and horizontal transmission, i.e.  $v = 1$ , Eq. 6 becomes  $\hat{p} = \tilde{p}$ , and Eq. 11 for  
 144 the frequency of the cooperative phenotype among parents in the next generation  $\tilde{p}'$  can be written  
 as

$$\begin{aligned}\bar{w}\tilde{p}' &= \tilde{p}^2(1+b-c)[1-(1-\tilde{p})(1-\alpha)T_B] \\ &+ \tilde{p}(1-\tilde{p})(1-c)[\tilde{p}(1-\alpha)T_B+1-T_B] \\ &+ \tilde{p}(1-\tilde{p})(1+b)[\tilde{p}(1-\alpha)+\alpha]T_A \\ &+ (1-\tilde{p})^2\tilde{p}(1-\alpha)T_A.\end{aligned}\tag{14}$$

146 Fixation of either cooperation,  $\tilde{p} = 1$ , or defection,  $\tilde{p} = 0$ , are equilibria of Eq. 14, that is, they solve  
 $\tilde{p}' = \tilde{p}$ . We therefore assume for the remainder of the analysis that  $0 < \tilde{p} < 1$ .

148 If  $\alpha = 1$ , then  $\tilde{p}' = \tilde{p}$  is reduced to

$$\tilde{p}(1-\tilde{p})[(1+b)T_A + (1-c)(1-T_B) - 1] = 0,\tag{15}$$

150 and there are no additional equilibria. For cooperation to take over the population (for  $\tilde{p} = 1$  to be  
 globally stable) we require  $\tilde{p}' > \tilde{p}$ ; that is,

$$\tilde{p}^2(1+b-c) + \tilde{p}(1-\tilde{p})[(1-c)(1-T_B) + (1+b)T_A] > \bar{w}\tilde{p}.\tag{16}$$

Factoring out  $\tilde{p}(1-\tilde{p})$  and setting  $\bar{w} = 1 + \tilde{p}(b-c)$ , we find that  $\tilde{p}' > \tilde{p}$  if

$$(1-c)(1-T_B) + (1+b)T_A > 1\tag{17}$$

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If  $\alpha < 1$ , divide both sides of Eq. 14 by  $\tilde{p}$  and set  $\bar{w} = 1 + \tilde{p}(b-c)$ . Then  $\tilde{p}' > \tilde{p}$  if

$$\begin{aligned}1 + \tilde{p}(b-c) &< \tilde{p}(1+b-c)(1-(1-\tilde{p})(1-\alpha)T_B) \\ &+ (1-\tilde{p})(1-c)(\tilde{p}(1-\alpha)T_B+1-T_B) \\ &+ (1-\tilde{p})(1+b)(\tilde{p}(1-\alpha)+\alpha)T_A \\ &+ (1-\tilde{p})^2(1-\alpha)T_A.\end{aligned}\tag{18}$$

Simplifying, we find that  $\tilde{p}' > \tilde{p}$  if and only if

$$c(1-T_B) - b\alpha T_A - (T_A - T_B) < \tilde{p} \cdot b(1-\alpha)(T_A - T_B).\tag{19}$$

Besides the fixation states  $\tilde{p} = 0$  and  $\tilde{p} = 1$ , there may be an actual polymorphic equilibrium of  $\tilde{p}' = \tilde{p}$   
 162 in Eq. 14, namely

$$\tilde{p}^* = \frac{c(1-T_B) - b\alpha T_A - (T_A - T_B)}{b(1-\alpha)(T_A - T_B)},\tag{20}$$

164 which is legitimate if  $0 < \tilde{p}^* < 1$ .

Since all parameters are positive, we can apply inequality 19 and see that for  $\tilde{p}' > \tilde{p}$  we require that  
 either

$$T_A > T_B \quad \text{and} \quad \tilde{p} > \tilde{p}^*, \quad \text{or}\tag{21}$$

$$T_A < T_B \quad \text{and} \quad \tilde{p} < \tilde{p}^*.\tag{22}$$

We define the initial frequency of the cooperator phenotype among parents,  $\tilde{p}_0$ , and the *cost boundaries*,

$$\gamma_1 = \frac{b\alpha T_A + (T_A - T_B)}{1 - T_B}, \quad \gamma_2 = \frac{b\alpha T_B + (1 + b)(T_A - T_B)}{1 - T_B}. \quad (23)$$

Then, applying Eqs. 20, 21, and 22, we summarize the possible outcomes in following result and corollaries.

**Result 2** (Vertical and horizontal transmission of cooperation). *With vertical and horizontal but without oblique transmission ( $v = 1$ ), the cultural evolution of a cooperator phenotype will follow one of the following scenarios, depending on the cost boundaries  $\gamma_1$  and  $\gamma_2$  (Eq. 23):*

1. Fixation of cooperation: if (i)  $T = T_A = T_B$  and  $c < b \cdot \alpha \frac{T}{1-T}$ ; or if (ii)  $T_A > T_B$  and  $c < \gamma_1$ ; or if (iii)  $T_A < T_B$  and  $c < \gamma_2$ .
2. Fixation of defection: if (iv)  $T = T_A = T_B$  and  $c > b \cdot \alpha \frac{T}{1-T}$ ; or if (vi)  $T_A > T_B$  and  $\gamma_2 < c$ ; or if (vi)  $T_A < T_B$  and  $\gamma_1 < c$ .
3. Fixation of either phenotype depending on initial frequency: if (vii)  $T_A > T_B$  and  $\gamma_1 < c < \gamma_2$ .
4. Coexistence of both phenotypes at  $\tilde{p}^*$ : if (viii)  $T_A < T_B$  and  $\gamma_2 < c < \gamma_1$ .

These conditions are illustrated in Figure 3. Note that cooperation and defection can coexist stably if there is horizontal bias for defection and the cost of cooperation is large but not too large. The recursion dynamic for this case is illustrated in Figure 2.

Much of the literature on evolution of cooperation focuses on conditions for an initially rare cooperative phenotype to invade a population of defectors. The next corollary deals with such a condition, followed by a corollary that deals with symmetric horizontal transmission, i.e.  $T_A = T_B$ .

**Corollary 1** (Condition for cooperation to increase when initially rare). *If the initial frequency of the cooperative phenotype is very close to zero,  $\tilde{p}_0 \approx 0$ , then this frequency will increase if*

$$T_A > T_B \text{ and } c < \gamma_1, \quad \text{or} \quad T_A < T_B \text{ and } \gamma_2 < c < \gamma_1. \quad (24)$$

In general, these conditions cannot be formulated in the form of Hamilton's rule ( $c < b \cdot r$ ) due to the horizontal transmission bias  $T_A - T_B$ . Without horizontal transmission bias, i.e., with  $T = T_A = T_B$ , these conditions reduce to a form of Hamilton's rule.

**Corollary 2** (Symmetric horizontal transmission). *If  $T = T_A = T_B$ , then cooperation will take over the population if*

$$c < b \cdot \alpha \frac{T}{1-T}. \quad (25)$$

Inequality 25 is obtained by setting  $T = T_A = T_B$  in inequality 19 and can be interpreted as a version of Hamilton's rule (inequality 1) or as a version of inequality 3, where  $\alpha T/(1 - T)$  can be regarded as the 'effective relatedness' or 'effective assortment', respectively. Figure 5a demonstrates this condition.

**Corollary 3** (No assortment of transmission and cooperation). *Without assortment ( $\alpha = 0$ ) and with horizontal bias for cooperation ( $T_A > T_B$ ), then (1) if the cost is low compared to the bias ( $c < (T_A - T_B)/(1 - T_B)$ ), then cooperation will fix from any positive frequency; (2) if the cost is low compared to the benefit ( $c < (1 + b)(T_A - T_B)(1 - T_B)$ ), then cooperation will fix if the initial frequency is high enough ( $\tilde{p}_0 > \tilde{p}^*$ ).*

Figure 3a illustrates these conditions, where the third equilibrium given by Eq. 20 becomes

$$\tilde{p}^*(\alpha = 0) = \frac{c(1 - T_B) - (T_A - T_B)}{b(T_A - T_B)}, \quad (26)$$

and the cost boundaries are

$$\gamma_1(\alpha = 0) = \frac{T_A - T_B}{1 - T_B}, \quad \gamma_2(\alpha = 0) = (1 + b)\frac{T_A - T_B}{1 - T_B}. \quad (27)$$

If  $T_A > T_B$  then  $0 < \gamma_1(\alpha = 0) < \gamma_2(\alpha = 0)$ . So either  $c < \gamma_1(\alpha = 0)$  or  $\gamma_1(\alpha = 0) < c < \gamma_2(\alpha = 0)$  will allow fixation of cooperation, the latter only if the initial frequency is high enough. If  $T_A < T_B$  then  $\gamma_2(\alpha = 0) < \gamma_1(\alpha = 0) < 0 < c$ , and defection will fix.

**Corollary 4** (Perfect assortment of transmission and cooperation). *With perfect assortment ( $\alpha = 1$ ), there are only two equilibria,  $\tilde{p} = 0$  and  $\tilde{p} = 1$ . The condition for evolution of cooperation (i.e. global stability of  $\tilde{p} = 1$ ) is found from inequality 17, namely*

$$c < \frac{b \cdot T_A + (T_A - T_B)}{1 - T_B}. \quad (28)$$

With perfect assortment horizontal transmission always co-occurs with the cooperative interaction. The same occurs in Lewin-Epstein et al. (2017), and therefore this corollary is equivalent to their result (see their eq. 1).

In terms of the cost boundaries, inequality 28 is equivalent to  $c < \gamma_1$ , and if  $T_A > T_B$  then that suffices for fixation of cooperation. If  $T_B > T_A$  then  $\gamma_2(\alpha = 1) < 0$  and again, inequality 28 is sufficient for increase in the frequency of A. Inequality 28 can be written as

$$1 - (1 - c)(1 - T_B) < (1 + b)T_A, \quad (29)$$

which provides an interesting interpretation for the success of cooperation. In the interaction between a cooperator and a defector,  $(1 - c)(1 - T_B)$  is the probability that the cooperator remains cooperative and also reproduces. Therefore,  $1 - (1 - c)(1 - T_B)$  is the probability that either the cooperator becomes a defector, or that it fails to reproduce. This is the *effective cost of cooperation* from this interaction, while  $(1 + b)T_A$  is the probability that the defector becomes cooperative and reproduces, which is the *effective benefit of cooperation* from this interaction. Thus, inequality 28 entails that cooperation can evolve if the effective cost of cooperation is less than the effective benefit.

Given the previous two corollaries for no assortment and perfect assortment ( $\alpha = 0$  and  $\alpha = 1$ ), it is interesting to examine the general effect of assortment on the evolution of cooperation. We denote the assortment boundaries by

$$a_1 = \frac{c(1 - T_B) - (1 + b)(T_A - T_B)}{b \cdot T_B}, \quad a_2 = \frac{c(1 - T_B) - (T_A - T_B)}{b \cdot T_A}. \quad (30)$$

Applying Eqs. 20, 21, and 22, we have the following corollary.

**Corollary 5** (Intermediate assortment of transmission and cooperation). *The cooperative phenotype will increase from rarity if the assortment rate is high enough, or specifically if*

$$\begin{aligned} T_A > T_B \text{ and } a_2 < \alpha, \quad \text{or} \\ T_A < T_B \text{ and } a_1 < \alpha. \end{aligned} \quad (31)$$



Figure 3c demonstrates these conditions. With horizontal bias for cooperation ( $T_A > T_B$ ; positive x-axis) cooperation fixes in the population from any initial positive frequency (green) if  $\alpha$  (y-axis) is above the orange line ( $a_1$ ), or if initially present at a high enough frequency (blue) if  $\alpha$  is above the blue line ( $a_2$ ). With horizontal bias for defection ( $T_B > T_A$ ; negative x-axis) cooperation fixes if  $\alpha$  is above the blue line ( $a_2$ ), but can be maintained in coexistence with defection (yellow) if  $\alpha$  is above the orange line ( $a_1$ ). Without horizontal bias ( $T_A = T_B$ ) fixation occurs if the assortment rate is high enough,  $\alpha > \frac{c}{b} \cdot \frac{1-T}{T}$  (inequality 25).

## With Vertical and Oblique Transmission

With both vertical and oblique transmission,  $0 < v < 1$ , the recursion system is more complex, and we focus on local rather than on global stability. To proceed, we note that Eq. 6 can give  $\hat{p}'$  as a function of both  $p'$  and  $\tilde{p}'$ . Eq. 8 gives  $p'$  as a function of  $\tilde{p}$ , since  $\hat{p}$  is given in Eq. 6 as a function of  $\tilde{p}$  and Eq. 11 gives  $\tilde{p}'$  as a function of  $\hat{p}$ . Combining these equations, we find an equation for  $\hat{p}'$  as a function of  $\hat{p}$  (shown in Appendix A). We then determine the equilibria, namely, solutions of  $\hat{p}' = \hat{p}$ , and analyse their local stability.

We apply Eqs. 6, 8, and 11 to obtain the function  $f(\hat{p})$  (see Appendix A):

$$f(\hat{p}) = \bar{w}(\hat{p}' - \hat{p}) = \beta_1 \hat{p}^3 + \beta_2 \hat{p}^2 + \beta_3 \hat{p}, \quad (32)$$

where

$$\begin{aligned} \beta_1 &= [c(1-v) - b(1-\alpha v)](T_A - T_B), \\ \beta_2 &= -\beta_1 - \beta_3, \\ \beta_3 &= \alpha b v T_A - c v(1 - T_B) + (T_A - T_B). \end{aligned} \quad (33)$$

If  $T = T_A = T_B$  then  $\beta_1 = 0$  and  $\beta_3 = -\beta_2 = \alpha b v T - c v(1 - T)$ , and  $f(\hat{p})$  becomes a quadratic polynomial:

$$f(\hat{p}) = \hat{p}(1 - \hat{p})[\alpha b v T - c v(1 - T)]. \quad (34)$$

Clearly the only two equilibria are the fixations  $\hat{p} = 0$  and  $\hat{p} = 1$ . These equilibria are locally stable if  $f'(\hat{p}) < 0$  near the equilibrium (Appendix B), and

$$f'(\hat{p}) = (1 - 2\hat{p})[\alpha b v T - c v(1 - T)], \quad (35)$$

with

$$\begin{aligned} f'(0) &= \alpha b v T - c v(1 - T), \\ f'(1) &= -\alpha b v T + c v(1 - T). \end{aligned} \quad (36)$$

Therefore with symmetric horizontal transmission, fixation of the cooperative phenotype ( $\hat{p} = 1$ ) occurs under the same condition as Corollary 1.1, namely Eq. 25.

In the general case where  $T_A \neq T_B$ , the coefficient  $\beta_1$  is not necessarily zero, and  $f(\hat{p})$  is a cubic polynomial. Therefore, three equilibria may exist, two of which are  $\hat{p} = 0$  and  $\hat{p} = 1$ , and the third is

$$\hat{p}^* = \frac{\beta_3}{\beta_1}. \quad (37)$$

Note that the sign of the cubic (Eq. 32) at positive (negative) infinity is equal (opposite) to the sign of  $\beta_1$ . If  $T_A > T_B$ , then

$$\beta_1 < [c(1 - \alpha v) - b(1 - \alpha v)](T_A - T_B) = (1 - \alpha v)(c - b)(T_A - T_B) < 0, \quad (38)$$

276 since  $c < b$  and  $1 > \alpha v$ . Hence the signs of the cubic at positive and negative infinity are negative  
 and positive, respectively. First, if  $\beta_3 < \beta_1$  then  $1 < \hat{p}^*$  and therefore  $f'(0) < 0$  and  $f'(1) > 0$ ; that  
 278 is, fixation of the defector phenotype  $B$  is the only locally stable legitimate (i.e. between 0 and 1)  
 equilibrium. Second, if  $\beta_1 < \beta_3 < 0$  then  $0 < \hat{p}^* < 1$  and therefore  $f'(0) < 0$  and  $f'(1) < 0$ , that  
 280 is, both fixations are locally stable and  $\hat{p}^*$  separates the domains of attraction. Third, if  $0 < \beta_3$  then  
 $\hat{p}^* < 0$  and therefore  $f'(0) > 0$  and  $f'(1) < 0$ ; that is, fixation of the cooperorator phenotype  $A$  is the  
 282 only locally stable legitimate equilibrium.

Similarly, if  $T_B > T_A$ , then

$$284 \quad \beta_1 > [c(1 - \alpha v) - b(1 - \alpha v)](T_A - T_B) = (1 - \alpha v)(c - b)(T_A - T_B) > 0, \quad (39)$$

since  $c < b$  and  $1 > \alpha v$ , and the signs of the cubic at positive and negative infinity are positive and  
 286 negative, respectively. First, if  $\beta_3 < 0$  then  $\hat{p}^* < 0$  and therefore  $f'(0) < 0$  and  $f'(1) > 0$ ; that is,  
 fixation of the defector phenotype  $A = B$  is the only locally stable legitimate equilibrium. Second,  
 288 if  $0 < \beta_3 < \beta_1$  then  $0 < \hat{p}^* < 1$  and therefore  $f'(0) > 0$  and  $f'(1) > 0$ ; that is, both fixations  
 are locally unstable and  $\hat{p}^*$  is a stable polymorphic equilibrium. Third, if  $\beta_1 < \beta_3$  then  $\hat{p}^* > 1$  and  
 290 therefore  $f'(0) > 0$  and  $f'(1) < 0$ , and fixation of the cooperorator phenotype  $A$  is the only locally stable  
 legitimate equilibrium.

292 We define new cost boundaries,  $\hat{\gamma}_1$  and  $\hat{\gamma}_2$ ,

$$\hat{\gamma}_1 = \frac{bv\alpha T_A + (T_A - T_B)}{v(1 - T_B)}, \quad \hat{\gamma}_2 = \frac{bv\alpha T_B + (1 + b)(T_A - T_B)}{v(1 - T_B) + (1 - v)(T_A - T_B)}, \quad (40)$$

294 and define a vertical transmission threshold,

$$\hat{v} = \frac{T_B - T_A}{1 - T_A}. \quad (41)$$

296 The following result summarizes the possible outcomes.

298 **Result 3** (Vertical, oblique, and horizontal transmission of cooperation). *With vertical, horizontal,*  
*and oblique transmission, the cultural evolution of a cooperorator phenotype will follow one of the*  
 300 *following scenarios depending on the cost boundaries  $\hat{\gamma}_1$  and  $\hat{\gamma}_3$  (Eq. 40) and the vertical threshold  $\hat{v}$*   
*(Eq. 41) :*

- 302 1. Fixation of cooperation: if (i)  $T = T_A = T_B$  and  $c < b \cdot \alpha \frac{T}{1-T}$ ; or if (ii)  $T_A > T_B$  and  $c < \hat{\gamma}_1$ ; or  
 if (iiia)  $T_A < T_B$  and  $v > \hat{v}$  and  $c < \hat{\gamma}_2$ ; or if (iiib)  $T_A < T_B$  and  $v < \hat{v}$  and  $\hat{\gamma}_2 < c$ .
- 304 2. Fixation of the defection: if (iv)  $T = T_A = T_B$  and  $c > b \cdot \alpha \frac{T}{1-T}$ ; or if (v)  $T_A > T_B$  and  
 $\hat{\gamma}_1 < c < \hat{\gamma}_2$ ; or if (vi)  $T_A < T_B$  and  $\hat{\gamma}_1 < c$ .
- 306 3. Coexistence of both phenotypes at  $\hat{p}^*$ : if (viiia)  $T_A < T_B$  and  $v < \hat{v}$  and  $c < \hat{\gamma}_1$  and  $c < \hat{\gamma}_2$ ; or if  
 (viiib)  $T_A < T_B$  and  $v > \hat{v}$  and  $c < \hat{\gamma}_1$  and  $\hat{\gamma}_2 < c$ .
- 308 4. Fixation of either phenotype depending on initial frequency: if (viii)  $T_A > T_B$  and  $\hat{\gamma}_2 < c$  and  
 $\hat{\gamma}_1 < c$  and  $v > \hat{v}$ .

## 310 Discussion

We hypothesized that non-vertical transmission can explain the evolution of cooperation and evaluated  
 312 this hypothesis using a deterministic discrete-time evolutionary model with fitnesses in the form of  
 payoffs from a prisoner's dilemma game.

314 Under oblique and horizontal cultural transmission, a horizontal transmission bias for the cooperative  
 316 phenotype was found to be sufficient and necessary for evolution of cooperation (Result 1). Under  
 horizontal and vertical cultural transmission, cooperation or defection can either fix or coexist at a  
 stable polymorphism, depending on the relationship between the cost and benefit of cooperation,  
 318 the horizontal bias, and the assortment between cooperation and transmission (Result 2). Under a  
 combination of vertical, oblique, and horizontal transmission the dynamics are more complicated,  
 320 but all outcomes can still occur, depending on parameter values (Result 3). Remarkably, stable  
 coexistence between cooperation and defection can be maintained if horizontal transmission is biased  
 322 for defection ( $T_B > T_A$ ) and the cost of cooperation and assortment ratio are intermediate (yellow areas  
 Figure 3).

324 Eshel and Cavalli-Sforza (1982) showed that with assortative meeting, namely, a probability  $m$  that  
 individuals interact within their phenotypic group, cooperation can evolve if  $c < b \cdot m$ . Fletcher and  
 326 Doebeli (2009) have further argued that a simple and general explanation for the evolution of altruism  
 is given by assortment between individuals that carry an altruistic trait, and the altruistic behavior of  
 328 others. They therefore suggested that to explain the evolution of altruism, we should seek mechanisms  
 that generate such assortment, and that one such major mechanism is population structure. Our results  
 330 highlight another possibility for assortment, namely, individuals socially interacting at rate  $\alpha$  with  
 their horizontal transmission partners. This mechanism does not require population structure. Rather,  
 332 it depends on the co-occurrence of two social processes: cultural transmission and cooperation. We  
 show that high levels of assortment significantly increase the potential for evolution of cooperation  
 334 (Figure 3). With enough assortment ( $\alpha > (c(1 - T_B) + (T_B - T_A))/bT_A$ ), cooperation can increase in  
 frequency when initially rare even when there is horizontal bias against it ( $T_B > T_A$ , see Result 2).

336 This study was partially inspired by the work of Lewin-Epstein et al. (2017) and Lewin-Epstein and  
 Hadany (2020), who hypothesised that microbes that manipulate their hosts to act altruistically can  
 338 be favored by selection, and may play a role in the evolution of cooperation. Indeed, it has been  
 shown that microbes can mediate behavioral changes in their hosts (Dobson, 1988; Poulin, 2010).  
 340 Therefore, natural selection on microbes may favor manipulation of the host so that it cooperates with  
 others. Microbes can be transmitted *horizontally* from one host to another during host interactions,  
 342 and following horizontal transfer, the recipient host may carry microbes that are closely related to  
 the microbes of the donor host, even when the two hosts are (genetically) unrelated (Lewin-Epstein  
 344 et al., 2017). Although this result can be described from a kin selection point-of-view, we are  
 more interested in the assortment point-of-view: infection by behavior-determining microbes during  
 346 interactions effectively generates assortment, as a cooperating donor can transmit to the recipient  
 both the benefit of the cooperative behavior and behavior-determining microbe. Horizontal cultural  
 348 transmission can similarly generate assortment between the cooperative phenotype and the benefit  
 of cooperation if cultural transmission and helping interactions occur between the same individuals,  
 350 which in our model occurs with rate  $\alpha$ . So, because non-vertical transmission can effectively generate  
 assortment, we suggest a revision of the conclusion made by Fletcher and Doebeli (2009)<sup>3</sup>: what is  
 352 necessary for the evolution of altruism is assortment between focal *pheno-genotype* and phenotypic  
 help.

354 Feldman et al. (1985) studied the dynamics of an altruistic phenotype with vertical cultural transmission  
 and a gene that modifies the transmission of the phenotype. Their results are very sensitive to this  
 356 genetic modification: without it, the conditions for invasion of the altruistic phenotype reduce to  
 Hamilton’s rule. Further work is needed to include such genetic modification of cultural transmission  
 358 to our model.

An important implication of our results is that cooperation can evolve even without population struc-

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<sup>3</sup>“what is necessary for the evolution of altruism is assortment between focal genotype and phenotypic help, rather than the assortment among genetic types often emphasized in kin selection theory.”

360 ture, repeated interactions, or individual recognition. This highlights the potential importance of  
non-vertical cultural transmission for explaining complex evolutionary phenomena, and furthers our  
362 understating of the cultural evolution of cooperation.

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## 366 Appendices

### Appendix A

368 We want to find the frequency of juveniles with phenotype  $A$  in next generation  $\hat{p}'$  as a function of frequency of juveniles with phenotype  $A$  in the current generation  $\hat{p}$ . Starting from Eq. 6,

$$370 \quad \hat{p}' = v\tilde{p}' + (1-v)p', \quad (\text{A1})$$

we substitute  $p'$  using Eq. 8 and  $\tilde{p}'$  using Eq. 11, we have

$$\begin{aligned} \hat{p}' = & \frac{v}{\bar{w}} \left\{ \hat{p}^2(1+b-c) \left[ 1 - (1-\hat{p})(1-\alpha)T_B \right] \right\} \\ & + \frac{v}{\bar{w}} \left\{ \hat{p}(1-\hat{p})(1-c) \left[ \hat{p}(1-\alpha)T_B + 1 - T_B \right] \right\} \\ 372 \quad & + \frac{v}{\bar{w}} \left\{ \hat{p}(1-\hat{p})(1+b) \left[ \hat{p}(1-\alpha) + \alpha \right] T_A \right\} \\ & + \frac{v}{\bar{w}} (1-\hat{p})^2 \hat{p}(1-\alpha)T_A \\ & + (1-v)\hat{p}^2(T_B - T_A) + (1-v)\hat{p}(1+T_A - T_B), \end{aligned} \quad (\text{A2})$$

where  $\bar{w} = 1 + \hat{p}(b-c)$ . We define  $f(\hat{p})$  as

$$374 \quad f(\hat{p}) = \bar{w}(\hat{p}' - \hat{p}) \quad (\text{A3})$$

Using *SymPy* (Meurer et al., 2017), a Python library for symbolic mathematics, we simplify Eq. A3 to eqs. 32-33.

### Appendix B

378 Denote  $f(p) = \lambda(p' - p)$ , where  $\lambda > 0$ , and assume  $f(p^*) = 0$ ; i.e.,  $p^*$  is an equilibrium. We want a condition for  $|p' - p^*| < |p - p^*|$ .

380 If  $p > p^* = 0$ , we want a condition for  $p' < p$ , or  $\frac{p'}{p} < 1$ , or  $\lambda \frac{p'-p}{p} < 0$ , or  $\frac{f(p)}{p} < 0$ . Using a linear approximation for  $f(p)$  near 0, we have

$$\begin{aligned} & p' < p \Leftrightarrow \\ 382 \quad & \frac{f'(0) \cdot p + O(p^2)}{p} < 0 \Leftrightarrow \\ & f'(0) + O(p) < 0. \end{aligned} \quad (\text{B1})$$

Therefore, by definition of big-O notation, if  $f'(0) < 0$  then there exists  $\epsilon > 0$  such that for any  
384  $0 < p < \epsilon$ , it is guaranteed that  $0 < p' < p$ , that is,  $p'$  is closer than  $p$  to zero.

If  $p < p^* = 1$ , we want a condition for  $1 - p' < 1 - p$ , or  $\frac{1-p'}{1-p} < 1$ , or  $\lambda \frac{-(p'-p)}{1-p} < 0$ , or  $-\frac{f(p)}{1-p} < 0$ .  
 386 Using a linear approximation for  $f(p)$  near 1, we have

$$\begin{aligned} 1 - p' < 1 - p &\Leftrightarrow \\ \frac{f'(1)(p-1) + O((p-1)^2)}{p-1} < 0 &\Leftrightarrow \\ f'(1) - O(1-p) < 0. \end{aligned} \tag{B2}$$

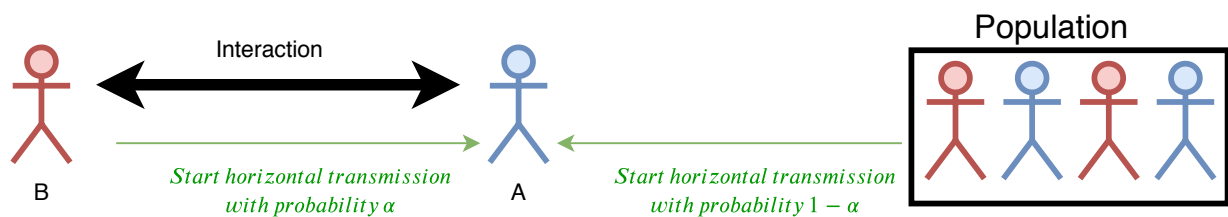
388 Therefore, if  $f'(1) < 0$  then there exists  $\epsilon > 0$  such that for any  $1 - \epsilon < 1 - p < 1$  it is guaranteed that  
 $1 - p' < 1 - p$ , that is,  $p'$  is closer than  $p$  to one.

## 390 References

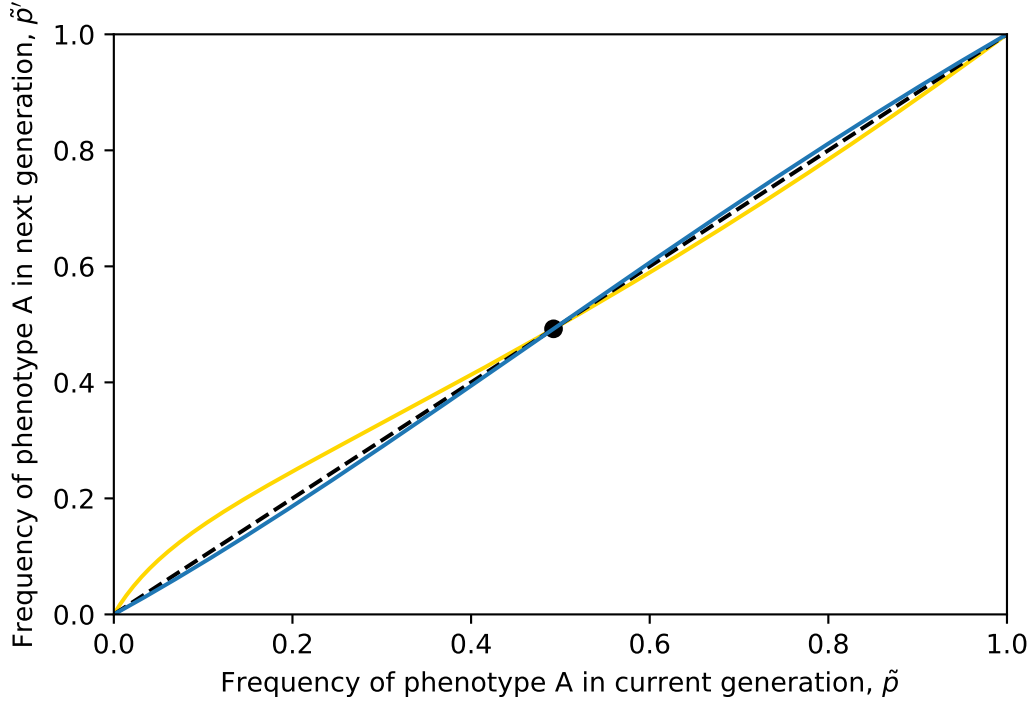
- Robert Axelrod and William D Hamilton. The evolution of cooperation. *Science*, 211(4489):1390–  
 392 1396, 1981.
- Luigi Luca Cavalli-Sforza and Marcus W Feldman. *Cultural transmission and evolution: A quantitative approach*. Number 16. Princeton University Press, 1981.
- Andrew P Dobson. The population biology of parasite-induced changes in host behavior. *The Quarterly Review of Biology*, 63(2):139–165, 1988.
- Lee Alan Dugatkin. *Cooperation among Animals: An Evolutionary Perspective*. Oxford University Press on Demand, 1997.
- Ilan Eshel and Luigi Luca Cavalli-Sforza. Assortment of encounters and evolution of cooperativeness. *Proceedings of the National Academy of Sciences*, 79(4):1331–1335, 1982.
- Marcus W Feldman, Luca L Cavalli-Sforza, and Joel R Peck. Gene-culture coevolution: models  
 402 for the evolution of altruism with cultural transmission. *Proceedings of the National Academy of Sciences*, 82(17):5814–5818, 1985.
- 404 Jeffrey A. Fletcher and Michael Doebeli. A simple and general explanation for the evolution of altruism. *Proc. R. Soc. B Biol. Sci.*, 276(1654):13–19, 2009. ISSN 14712970. doi: 10.1098/rspb.2008.0829.
- 406 William D Hamilton. The genetical evolution of social behaviour. ii. *Journal of Theoretical Biology*, 7(1):17–52, 1964.
- 408 Adrian V Jaeggi and Michael Gurven. Natural cooperators: food sharing in humans and other primates. *Evolutionary Anthropology: Issues, News, and Reviews*, 22(4):186–195, 2013.
- 410 Indrikis Krams, Tatjana Krama, Kristine Igaune, and Raivo Mänd. Experimental evidence of reciprocal altruism in the pied flycatcher. *Behavioral Ecology and Sociobiology*, 62(4):599–605, 2008.
- 412 Ohad Lewin-Epstein and Lilach Hadany. Host-microbiome coevolution can promote cooperation in a rock-paper-scissors dynamics. *Proc. R. Soc. B Biol. Sci.*, 287(1920):20192754, feb 2020. ISSN 0962-8452. doi: 10.1098/rspb.2019.2754.
- 414 Ohad Lewin-Epstein, Ranit Aharonov, and Lilach Hadany. Microbes can help explain the evolution of host altruism. *Nature Communications*, 8:14040, 2017.
- Stephen J Lycett and John AJ Gowlett. On questions surrounding the acheulean ‘tradition’. *World Archaeology*, 40(3):295–315, 2008.
- 418 Aaron Meurer, Christopher P Smith, Mateusz Paprocki, Ondřej Čertík, Sergey B Kirpichev, Matthew Rocklin, AMiT Kumar, Sergiu Ivanov, Jason K Moore, Sartaj Singh, et al. Sympy: symbolic computing in python. *PeerJ Computer Science*, 3:e103, 2017.

- 422 Robert Poulin. Parasite manipulation of host behavior: an update and frequently asked questions. In  
*Advances in the Study of Behavior*, volume 41, pages 151–186. Elsevier, 2010.
- 424 Yoav Ram, Uri Liberman, and Marcus W Feldman. Evolution of vertical and oblique transmission  
under fluctuating selection. *Proceedings of the National Academy of Sciences*, 115(6):E1174–  
426 E1183, 2018.
- George E Rice and Priscilla Gainer. “Altruism” in the albino rat. *Journal of Comparative and*  
428 *Physiological Psychology*, 55(1):123, 1962.
- Peter J Richerson and Robert Boyd. *Not by Genes Alone: How Culture Transformed Human Evolution*.  
430 University of Chicago Press, 2008.
- Barry Sinervo, Alexis Chaine, Jean Clobert, Ryan Calsbeek, Lisa Hazard, Lesley Lancaster, Andrew G  
432 McAdam, Suzanne Alonzo, Gwynne Corrigan, and Michael E Hochberg. Self-recognition, color  
signals, and cycles of greenbeard mutualism and altruism. *Proceedings of the National Academy of*  
434 *Sciences*, 103(19):7372–7377, 2006.
- Peter B Stacey and Walter D Koenig, editors. *Cooperative breeding in birds: long term studies of*  
436 *ecology and behaviour*. Cambridge University Press, 1990.
- Scott Woodcock. The significance of non-vertical transmission of phenotype for the evolution of  
438 altruism. *Biology and Philosophy*, 21(2):213–234, 2006.

## Figures

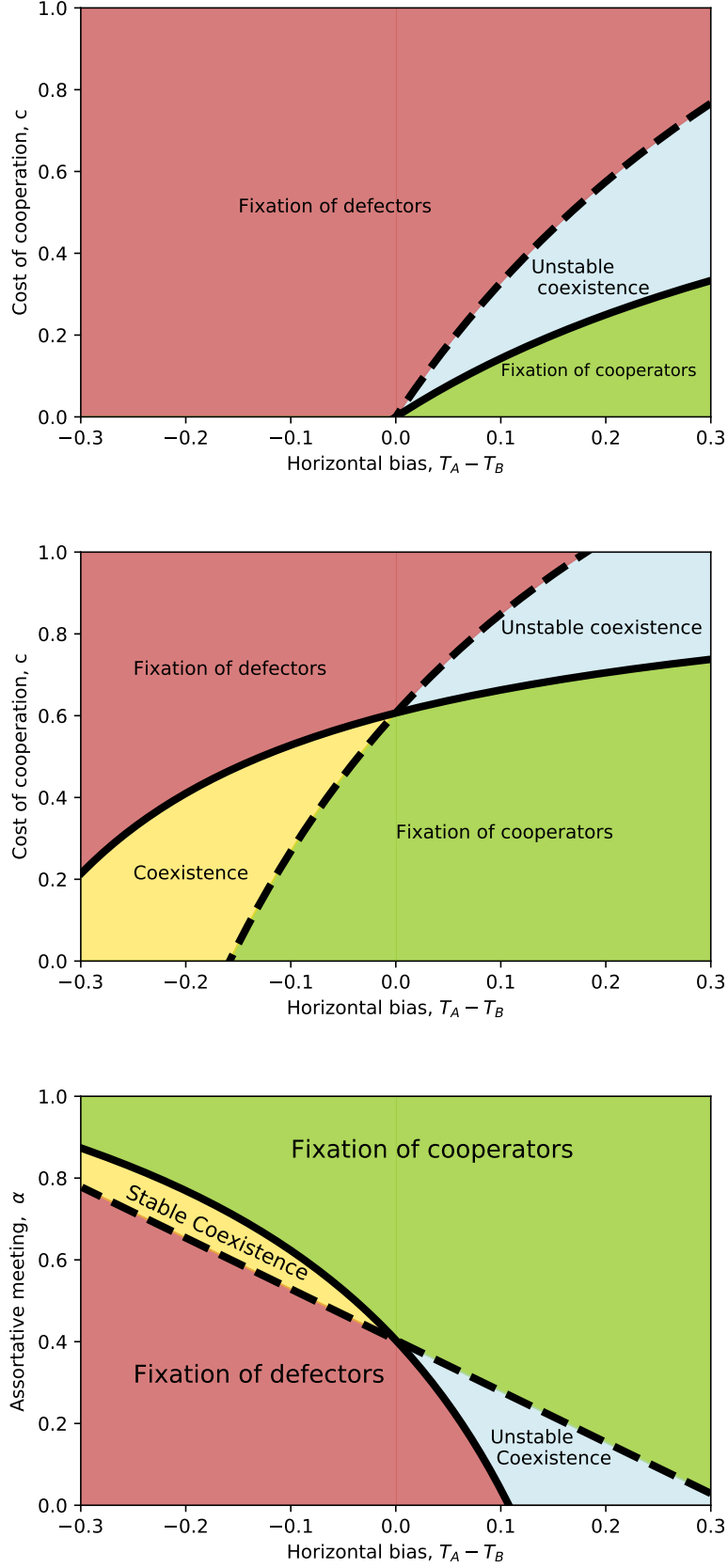


**Figure 1: Cultural horizontal transmission with assortment.** Transmission occurs between interacting partners with probability  $\alpha$  (left) or between two random peers with probability  $1 - \alpha$ .

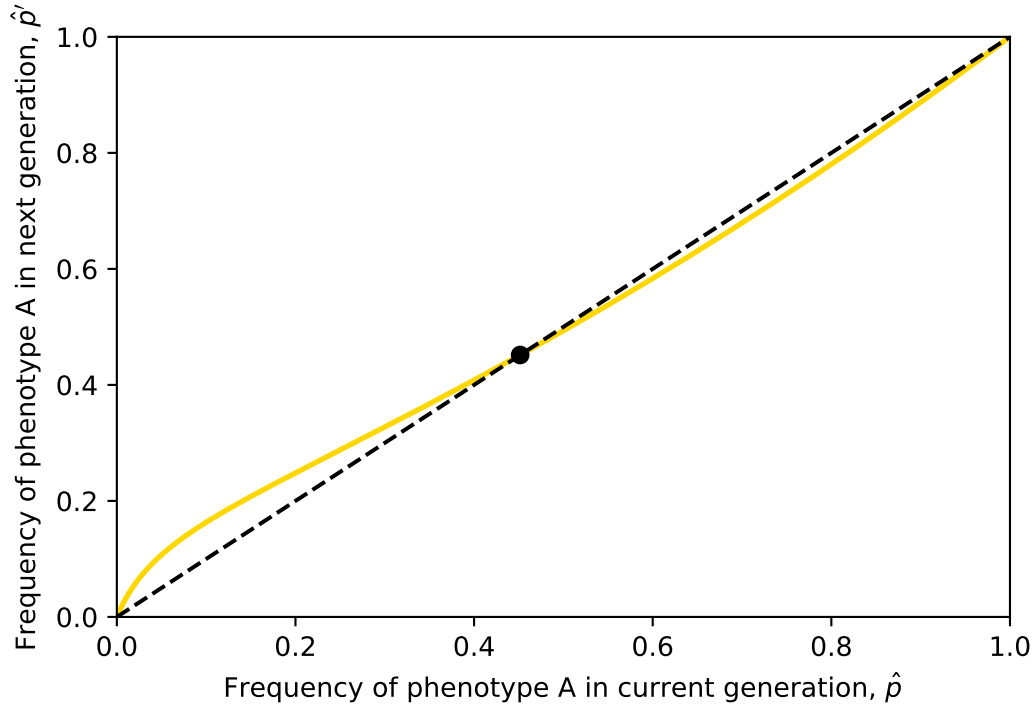


**Figure 2: Stable and unstable coexistence between cooperation and defection without oblique transmission.** The curves show the frequency of the cooperative phenotype A among parents in the next generation  $\tilde{p}'$  vs. the current generation  $\tilde{p}$  (Eq. 14). The dashed black line is  $\tilde{p}' = \tilde{p}$ . The curves and the dashed line intersect at the polymorphic equilibrium  $\tilde{p}^*$  (black circle). When the curves are above the dashed line,  $\tilde{p}' > \tilde{p}$ , then  $\tilde{p}$  increase. When the curves are below the dashed line,  $\tilde{p}' < \tilde{p}$ , then  $\tilde{p}$  decreases. The yellow curve, for which the polymorphic equilibrium is stable, is given by  $T_A = 0.4$ ,  $T_B = 0.9$ ,  $b = 12$ ,  $c = 0.35$ , and  $\alpha = 0.45$ , which give  $\gamma_2 < c < \gamma_1$  (Eq. 23) The blue curve, for which the equilibrium is unstable, is given by  $T_A = 0.5$ ,  $T_B = 0.1$ ,  $b = 1.3$ ,  $c = 0.904$ , and  $\alpha = 0.4$ , which give  $\gamma_1 < c < \gamma_2$ . In both cases there is no oblique transmission,  $\nu = 1$ ; see Figure 4 for  $\nu < 1$ .

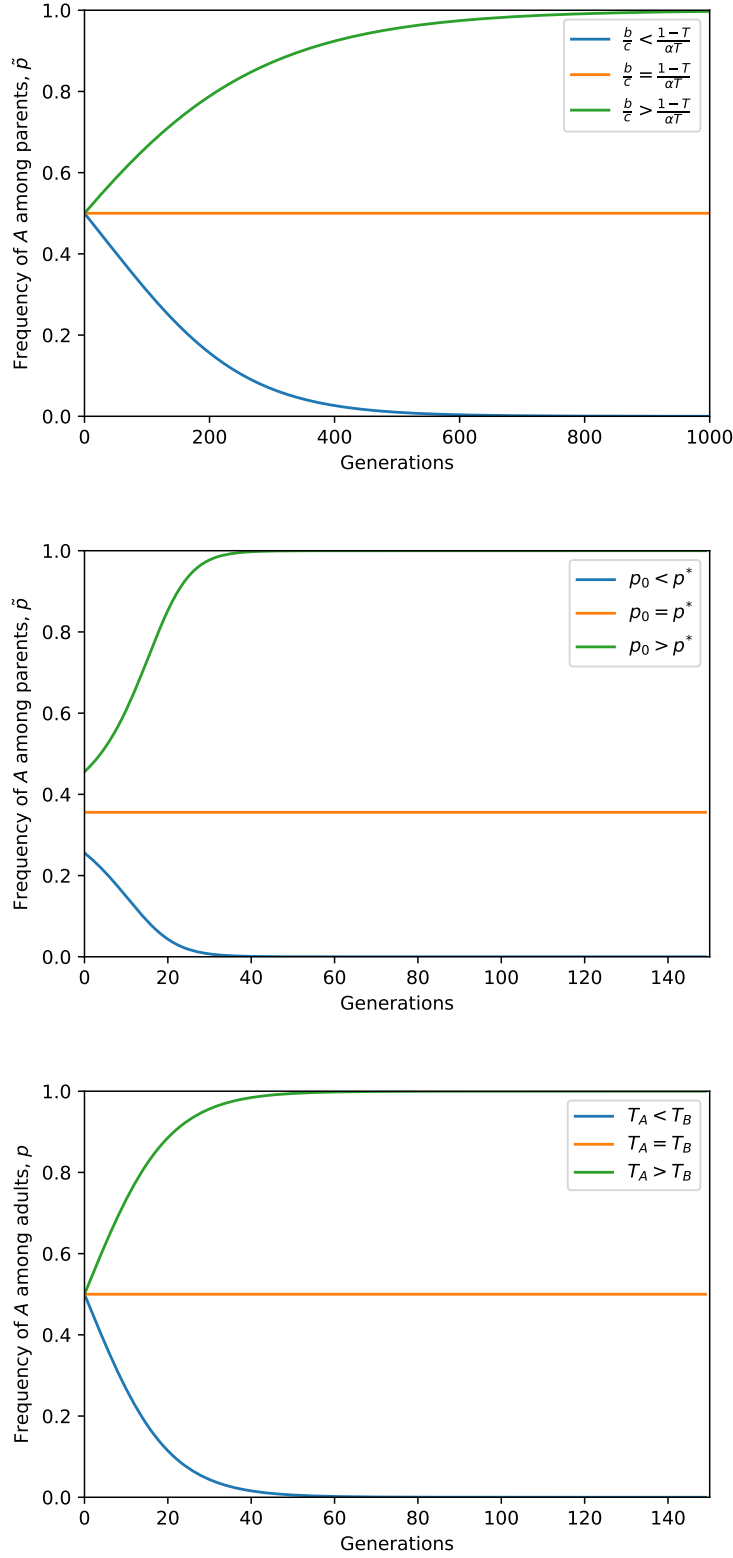




**Figure 3: Evolution of cooperation under vertical and horizontal cultural transmission.** The figure shows the global fixation of cooperation (green), global fixation of defection (red), fixation of either cooperation or defection depending on the initial conditions, i.e. unstable coexistence (blue), and stable coexistence of cooperation and defection (yellow). In all cases the horizontal bias ( $T_A - T_B$ ) is on the x-axis. **(a-b)** the cost of cooperation  $c$  is on the y-axis; the cost boundaries  $\gamma_1$  and  $\gamma_2$  (Eq. 23) are the solid and dashed lines. **(c)** the assortment  $\alpha$  is on the y-axis; the assortment boundaries  $a_1$  and  $a_2$  (Eq. 30) are the solid and dashed lines. Here,  $b = 1.3$ ,  $T_A = 0.4$ . **(a)**  $\alpha = 0$ . **(b)**  $\alpha = 0.7$ . **(c)**  $c = 0.35$ .



**Figure 4: Stable coexistence between cooperation and defection with oblique transmission.** The curve shows the frequency of the cooperative phenotype  $A$  among juveniles in the next generation  $\hat{p}'$  vs. the current generation  $\hat{p}$  (Eq. 6). The dashed black line is  $\hat{p}' = \hat{p}$ . The curve and the dashed line intersect at the stable equilibrium  $\hat{p}^*$  (black circle). When  $\hat{p} < \hat{p}^*$  then the curve is above the dashed line,  $\hat{p}' > \hat{p}$ , and  $\hat{p}$  increases towards  $\hat{p}^*$ . When  $\hat{p} > \hat{p}^*$  then the curve is below the dashed line,  $\hat{p}' < \hat{p}$ , and  $\hat{p}$  decreases towards  $\hat{p}^*$ . Here,  $T_A = 0.4$ ,  $T_B = 0.9$ ,  $b = 20$ ,  $c = 0.1$ ,  $\alpha = 1$ , and  $v = 0.4$ , which give  $0 < \beta_3 < \beta_1$  (Eq. 33).



**Figure 5: Dynamics of the frequency of cooperation.** The frequency  $\tilde{p}$  of parents with cooperative phenotype A in (a-b) and the frequency  $p$  of adults with cooperative phenotype A in (c). The different lines correspond to parameter values that lead to fixation of cooperation (green), extinction of cooperation (red), or stable coexistence of cooperators and defectors (yellow). (a)  $v = 1$ ,  $T_A = T_B = T = 0.2$ ,  $\alpha = 0.5 \neq 0$ ,  $\tilde{p}_0 = 0.5$  and  $c = 0.1$ ; (b)  $v = 1$ ,  $\alpha = 0$ ,  $\tilde{p}^* \approx 0.35$ ,  $T_A = 0.65$ ,  $T_B = 0.1$ ,  $b = 1.3$  and  $c = 0.65$ ; (c)  $v = 0$ ,  $\alpha = 0.5$ ,  $p_0 = 0.5$ ,  $T_A = 0.5$ ,  $b = 1.3$  and  $c = 0.5$ .