

Non-Vertical Cultural Transmission, Assortment, and the Evolution of Cooperation

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Abstract

We study the cultural evolution of cooperation under vertical and non-vertical cultural transmission. Conditions are found for fixation and coexistence of cooperation and defection. The evolution of cooperation is facilitated by horizontal transmission and by an association between social interactions and horizontal transmission. The effect of oblique transmission depends on the horizontal transmission bias. A stable coexistence between cooperation and defection can occur. When it does, selection favors the evolution of higher association between social interactions and horizontal transmission, which leads to an increased frequency of cooperation and higher population mean fitness. The derived conditions are compared to outcomes of stochastic simulations of structured populations. Comparisons are drawn with Hamilton's rule and the concepts of relatedness and assortment.

22 Introduction

Cooperative behavior can reduce an individual's fitness and increase the fitness of its conspecifics or competitors (Axelrod and Hamilton, 1981). Nevertheless, cooperative behavior appears to occur in many non-human animals (Dugatkin, 1997), including primates (Jaeggi and Gurven, 2013), rats (Rice and Gainer, 1962), birds (Stacey and Koenig, 1990; Krams et al., 2008), and lizards (Sinervo et al., 2006). Evolution of cooperative behavior remains an important conundrum in evolutionary biology (Haldane, 1932, Appendix).

Since the work of Hamilton (1964) and Axelrod and Hamilton (1981), theories for the evolution of cooperative and altruistic behaviors have been intertwined often under the rubric of *kin selection*. Kin selection theory posits that natural selection is more likely to favor cooperation between more closely related individuals. The importance of *relatedness* to the evolution of cooperation and altruism was demonstrated by Hamilton (1964), who showed that an allele that determines cooperative behavior will increase in frequency if the reproductive cost to the actor that cooperates, c , is less than the benefit to the recipient, b , times the relatedness, r , between the recipient and the actor. This condition is known as *Hamilton's rule*:

$$c < b \cdot r, \quad (1)$$

where the relatedness coefficient r measures the probability that an allele sampled from the cooperator is identical by descent to one at the same locus in the recipient.

Eshel and Cavalli-Sforza (1982) studied a related model for the evolution of cooperative behavior. Their model included *assortative meeting*, or non-random encounters, where a fraction m of individuals in the population each interact with an individual of the same phenotype, and a fraction $1 - m$ interacts with a randomly chosen individual. Such assortative meeting may be due, for example, to population structure or active partner choice. In their model, cooperative behavior can evolve if¹ (Eshel and Cavalli-Sforza, 1982, eq. 3.2)

$$c < b \cdot m, \quad (2)$$

where b and c are the benefit and cost of cooperation. Here m in inequality 2 takes the role of the relatedness coefficient r in inequality 1.

The role of assortment in the evolution of altruism was emphasized by Fletcher and Doebeli (2009). They found that in a *public-goods* game, altruism will evolve if cooperative individuals experience more cooperation, on average, than defecting individuals, and “thus, the evolution of altruism requires (positive) assortment between focal *cooperative* players and cooperative acts in their interaction environment.” With some change in parameters, this condition is summarized by (Fletcher and Doebeli, 2009, eq. 2.3)

$$c < b \cdot (p_C - p_D), \quad (3)$$

where p_C is the probability that a cooperator receives help, and p_D is the probability that a defector receives help.² See Bijma and Aanen (2010) for treatment of non-public-goods games.

In this paper we study the evolution of a cooperative behavior that is subject to *cultural transmission*, which allows an individual to acquire attitudes or behavioral traits from other individuals in its social group through imitation, learning, or other modes of communication (Cavalli-Sforza and Feldman, 1981; Richerson and Boyd, 2008). Feldman et al. (1985) introduced the first model for the evolution of altruism by cultural transmission. They demonstrated that if the fidelity of cultural transmission of

¹In an extended model, which allows an individual to encounter N individuals before choosing a partner, the righthand side is multiplied by $E[N]$, the expected number of encounters (Eshel and Cavalli-Sforza, 1982, eq. 4.6).

²Inequality 3 generalizes inequality 1 and inequality 2 by substituting $p_C = r + p$, $p_D = p$ and $p_C = m + (1 - m)p$, $p_D = (1 - m)p$, respectively, where p is the frequency of cooperators.

altruism is φ , then the condition for evolution of altruism in the case of sib-to-sib altruism is (Feldman et al., 1985, Eq. 16)

$$c < b \cdot \varphi - \frac{1 - \varphi}{\varphi}. \quad (4)$$

In inequality 4, φ takes the role of relatedness (r in inequality 1) or assortment (m in inequality 2), but the effective benefit $b \cdot \varphi$ is reduced by $(1 - \varphi)/\varphi$. This shows that under a combination of genetic and cultural transmission, the condition for the evolutionary success of altruism entails a modification of Hamilton's rule (1).

Cultural transmission may be viewed as vertical, horizontal or oblique: vertical transmission occurs between parents and offspring, horizontal transmission occurs between individuals from the same generation, and oblique transmission occurs to offspring from the generation to which their parents belong (i.e. from non-parental adults). Evolution under either of these transmission models can be more rapid than under pure vertical transmission (Cavalli-Sforza and Feldman, 1981; Lycett and Gowlett, 2008; Ram et al., 2018). Both Woodcock (2006) and Lewin-Epstein et al. (2017) demonstrated that non-vertical transmission can help explain the evolution of cooperative behavior (the former using simulations with cultural transmission, the latter using a model where cooperation is mediated by microbes that manipulate their host's behavior.) Some of the analyses by Lewin-Epstein et al. (2017) can be applied to cultural transmission, because models of cultural transmission are mathematically similar to those for transmission of infectious diseases (Cavalli-Sforza and Feldman, 1981).

Here, we study cultural-evolution models of cooperation that include both vertical and non-vertical transmission. We investigate these models using mathematical analysis and simulations. In our models behavioral changes are mediated by cultural transmission that can occur specifically during social interactions. For instance, there may be an association between the choice of partner for social interaction and the choice of partner for cultural transmission. As another example, when an individual interacts with an individual of a different phenotype, exposure to the latter may lead the former to convert its phenotype. Our results demonstrate that cultural transmission can enhance the evolution of cooperation even when genetic transmission cannot, partly because it facilitates the generation of assortment (Fletcher and Doebeli, 2009), and partly because non-vertical transmission can diminish the effect of natural selection (Ram et al., 2018). This further emphasizes that treatment of cooperation as a cultural trait, rather than a genetic one, can lead to a broader understanding of its evolutionary dynamics.

Models

Consider a large population whose members can be one of two phenotypes: $\phi = A$ for cooperators or $\phi = B$ for defectors. An offspring inherits its phenotype from its parent via vertical transmission with probability v or from a random individual in the parental population via oblique transmission with probability $(1 - v)$. Following Ram et al. (2018), given that the parent phenotype is ϕ and assuming uni-parental inheritance, the conditional probability that the phenotype ϕ' of the offspring is A is

$$P(\phi' = A \mid \phi) = \begin{cases} v + (1 - v)p, & \text{if } \phi = A \\ (1 - v)p, & \text{if } \phi = B \end{cases}, \quad (5)$$

where $p = P(\phi = A)$ is the frequency of A among all adults in the parental generation.

Not all adults become parents due to natural selection, and we denote the frequency of phenotype A among parents by \tilde{p} . Therefore, the frequency \hat{p} of phenotype A among juveniles (after selection and vertical and oblique transmission) is

$$\hat{p} = \tilde{p}[v + (1 - v)p] + (1 - \tilde{p})[(1 - v)p] = v\tilde{p} + (1 - v)p . \quad (6)$$

106 Individuals are assumed to interact according to a *prisoner's dilemma*. Specifically, individuals
 108 interact in pairs; a cooperator suffers a fitness cost $0 < c < 1$, and its partner gains a fitness benefit
 b , where we assume $c < b$. Table 1 shows the payoff matrix, i.e. the fitness of an individual with
 phenotype ϕ_1 when interacting with a partner of phenotype ϕ_2 .

110 Social interactions occur randomly: two juvenile individuals with phenotype A interact with probability
 \hat{p}^2 , two juveniles with phenotype B interact with probability $(1 - \hat{p})^2$, and two juveniles with different
 112 phenotypes interact with probability $2\hat{p}(1 - \hat{p})$.

Horizontal cultural transmission occurs between pairs of individuals from the same generation. It
 114 occurs between socially interacting partners with probability α , or between a random pair with
 probability $1 - \alpha$ (see Figure 1). However, horizontal transmission is not always successful, as one
 116 partner may reject the other's phenotype. The probability for successful horizontal transmission of
 phenotypes A and B are T_A and T_B , respectively (Table 2).

118 Therefore, the frequency p' of phenotype A among adults in the next generation, after horizontal
 transmission, is

$$\begin{aligned} p' = & \hat{p}^2 [\alpha + (1 - \alpha)(\hat{p} + (1 - \hat{p})(1 - T_B))] + \\ & \hat{p}(1 - \hat{p}) [\alpha(1 - T_B) + (1 - \alpha)(\hat{p} + (1 - \hat{p})(1 - T_B))] + \\ & (1 - \hat{p})\hat{p} [\alpha T_A + (1 - \alpha)\hat{p} T_A] + \\ & (1 - \hat{p})^2 [(1 - \alpha)\hat{p} T_A] , \end{aligned} \quad (7)$$

which simplifies to

$$p' = \hat{p}^2(T_B - T_A) + \hat{p}(1 + T_A - T_B) . \quad (8)$$

The frequency of A among parents (i.e. after selection) follows a similar dynamic, but also includes
 124 the effect of natural selection, and is therefore

$$\begin{aligned} \bar{w}\tilde{p}' = & \hat{p}^2(1 + b - c) [\alpha + (1 - \alpha)(\hat{p} + (1 - \hat{p})(1 - T_B))] + \\ & \hat{p}(1 - \hat{p})(1 - c) [\alpha(1 - T_B) + (1 - \alpha)(\hat{p} + (1 - \hat{p})(1 - T_B))] + \\ & (1 - \hat{p})\hat{p}(1 + b) [\alpha T_A + (1 - \alpha)\hat{p} T_A] + \\ & (1 - \hat{p})^2 [(1 - \alpha)\hat{p} T_A] , \end{aligned} \quad (9)$$

126 where fitness values are taken from Table 1 and Table 2, and the population mean fitness is

$$\bar{w} = 1 + \hat{p}(b - c). \quad (10)$$

128 Eq. 9 can be simplified to

$$\begin{aligned} \bar{w}\tilde{p}' = & \hat{p}^2(1 + b - c) [1 - (1 - \hat{p})(1 - \alpha)T_B] + \\ & \hat{p}(1 - \hat{p})(1 - c) [\hat{p}(1 - \alpha)T_B + 1 - T_B] + \\ & (1 - \hat{p})\hat{p}(1 + b) [\hat{p}(1 - \alpha) + \alpha] T_A + \\ & (1 - \hat{p})^2 \hat{p}(1 - \alpha) T_A . \end{aligned} \quad (11)$$

130 Finally, we find an equation for the frequency of phenotype A among juveniles in the next generation
 \hat{p}' as a function of the frequency in the current generation. Starting from Eq. 6, we substitute Eq. 8

132 for p' and Eq. 11 for \hat{p}' . We therefore have

$$\begin{aligned}\hat{p}' = & \frac{v}{\bar{w}} \left[\hat{p}^2(1+b-c) \left[1 - (1-\hat{p})(1-\alpha)T_B \right] \right] + \\ & \frac{v}{\bar{w}} \left[\hat{p}(1-\hat{p})(1-c)(\hat{p}(1-\alpha)T_B + 1 - T_B) \right] + \\ & \frac{v}{\bar{w}} \left[\hat{p}(1-\hat{p})(1+b)(\hat{p}(1-\alpha) + \alpha)T_A \right] + \\ & \frac{v}{\bar{w}} (1-\hat{p})^2 \hat{p}(1-\alpha)T_A + \\ & (1-v)\hat{p}^2(T_B - T_A) + (1-v)\hat{p}(1+T_A - T_B),\end{aligned}\tag{12}$$

134 where $\bar{w} = 1 + \hat{p}(b-c)$.

Table 3 summarizes the model variables and parameters.

136 Results

In the following sections, we determine the equilibria of the model in Eq. 12, namely, solutions
138 of $\hat{p}' = \hat{p}$, and analyze their local stability. We then analyze the evolution of a modifier of social
association. Finally, we compare derived conditions to outcomes of stochastic simulations with a
140 structured population.

Evolution of cooperation

142 To determine the equilibria in our model, we analyze the fixed points of Eq. 12. We define $f(\hat{p}) =$
 $\bar{w}(\hat{p}' - \hat{p})$. Then, using *SymPy* (Meurer et al., 2017), a Python library for symbolic mathematics, this
144 simplifies to

$$f(\hat{p}) = \bar{w}(\hat{p}' - \hat{p}) = \beta_1 \hat{p}^3 + \beta_2 \hat{p}^2 + \beta_3 \hat{p},\tag{13}$$

146 where

$$\begin{aligned}\beta_1 &= [c(1-v) - b(1-\alpha v)](T_A - T_B), \\ \beta_2 &= -\beta_1 - \beta_3, \\ \beta_3 &= \alpha b v T_A - c v(1 - T_B) + (T_A - T_B).\end{aligned}\tag{14}$$

148 If $T = T_A = T_B$ then $\beta_1 = 0$ and $\beta_3 = -\beta_2 = \alpha b v T - c v(1 - T)$. Thus, $f(\hat{p})$ becomes a quadratic
polynomial,

$$150 \quad f(\hat{p}) = \hat{p}(1-\hat{p})[\alpha b v T - c v(1 - T)].\tag{15}$$

Clearly the only two equilibria are the fixations $\hat{p} = 0$ and $\hat{p} = 1$. These equilibria are locally stable
152 if $f'(\hat{p}) < 0$ near the equilibrium (see Appendix A), where $f'(\hat{p}) = (1-2\hat{p})[\alpha b v T - c v(1 - T)]$, so
that

$$\begin{aligned}154 \quad f'(0) &= \alpha b v T - c v(1 - T), \\ f'(1) &= -\alpha b v T + c v(1 - T).\end{aligned}\tag{16}$$

In the general case where $T_A \neq T_B$, the coefficient β_1 is not necessarily zero, and $f(\hat{p})$ is a cubic
156 polynomial. Therefore, three equilibria may exist, two of which are $\hat{p} = 0$ and $\hat{p} = 1$, and the third
is

$$158 \quad \hat{p}^* = \frac{\beta_3}{\beta_1} = \frac{\alpha b v T_A - c v(1 - T_B) + (T_A - T_B)}{[c(1-v) - b(1-\alpha v)](T_A - T_B)}.\tag{17}$$

Note that the sign of the cubic (Eq. 13) at positive (negative) infinity is equal (opposite) to the sign of β_1 . If $T_A > T_B$, then

$$\beta_1 < [c(1 - \alpha v) - b(1 - \alpha v)](T_A - T_B) = (1 - \alpha v)(c - b)(T_A - T_B) < 0, \quad (18)$$

since $c < b$ and $\alpha v < 1$. Hence the signs of the cubic at positive and negative infinity are negative and positive, respectively. First, if $\beta_3 < \beta_1$ then $1 < \hat{p}^*$ and therefore $f'(0) < 0$ and $f'(1) > 0$; that is, fixation of the defector phenotype B is the only locally stable legitimate (i.e. between 0 and 1) equilibrium. Second, if $\beta_1 < \beta_3 < 0$ then $0 < \hat{p}^* < 1$ and therefore $f'(0) < 0$ and $f'(1) < 0$ so that both fixations are locally stable and \hat{p}^* separates the domains of attraction. Third, if $0 < \beta_3$ then $\hat{p}^* < 0$ and therefore $f'(0) > 0$ and $f'(1) < 0$; that is, fixation of the cooperator phenotype A is the only locally stable legitimate equilibrium.

Similarly, if $T_A < T_B$, then

$$\beta_1 > [c(1 - \alpha v) - b(1 - \alpha v)](T_A - T_B) = (1 - \alpha v)(c - b)(T_A - T_B) > 0, \quad (19)$$

since $c < b$ and $\alpha v < 1$, and the signs of the cubic at positive and negative infinity are positive and negative, respectively. First, if $\beta_3 < 0$ then $\hat{p}^* < 0$ and therefore $f'(0) < 0$ and $f'(1) > 0$; that is, fixation of the defector phenotype A is the only locally stable legitimate equilibrium. Second, if $0 < \beta_3 < \beta_1$ then $0 < \hat{p}^* < 1$ and therefore $f'(0) > 0$ and $f'(1) > 0$; that is, both fixations are locally unstable and \hat{p}^* is a stable polymorphic equilibrium. Third, if $\beta_1 < \beta_3$ then $\hat{p}^* > 1$ and therefore $f'(0) > 0$ and $f'(1) < 0$, and fixation of the cooperator phenotype A is the only locally stable legitimate equilibrium.

To present this analysis in terms of conditions on the cost of cooperation c , we define the cost boundaries, γ_1 and γ_2 , and the vertical transmission threshold, \hat{v} ,

$$\gamma_1 = \frac{bv\alpha T_A + (T_A - T_B)}{v(1 - T_B)}, \quad \gamma_2 = \frac{bv\alpha T_B + (1 + b)(T_A - T_B)}{v(1 - T_B) + (1 - v)(T_A - T_B)}, \quad \hat{v} = \frac{T_B - T_A}{1 - T_A}. \quad (20)$$

First, assume $T_A < T_B$. $\beta_3 < 0$ requires $\gamma_1 < c$. For $\beta_3 < \beta_1$ we need $c[v(1 - T_B) + (1 - v)(T_A - T_B)] > bv\alpha T_B + (1 + b)(T_A - T_B)$. Note that the expression in the square brackets is positive if and only if $v > \hat{v}$. Thus, for $\beta_3 < \beta_1$ we need $v > \hat{v}$ and $\gamma_2 < c$ or $v < \hat{v}$ and $c < \gamma_2$, and for $0 < \beta_3 < \beta_1$ we need $v > \hat{v}$ and $\gamma_2 < c < \gamma_1$, or $v < \hat{v}$ and $c < \min(\gamma_1, \gamma_2)$. For $\beta_1 < \beta_3$ we need $v > \hat{v}$ and $c < \gamma_2$ or $v < \hat{v}$ and $\gamma_2 < c$. However, some of these conditions cannot be met, since $v < \hat{v}$ implies $c < 1 < \gamma_2$.

Second, assume $T_A > T_B$. $\beta_3 > 0$ requires $\gamma_1 > c$. For $\beta_1 < \beta_3$ we need $c[v(1 - T_B) + (1 - v)(T_A - T_B)] < bv\alpha T_B + (1 + b)(T_A - T_B)$. Thus for $\beta_1 < \beta_3$ we need $v > \hat{v}$ and $c < \gamma_2$ or $v < \hat{v}$ and $c > \gamma_2$. But $\hat{v} < 0$ when $T_A > T_B$, and therefore we have $\beta_1 < \beta_3$ if $c < \gamma_2$. Similarly, we have $\beta_3 < \beta_1$ if $c > \hat{\gamma}_2$.

The following result summarizes the possible outcomes of the model (Eq. 12).

Result 1 (Equilibria and stability). *With vertical, horizontal, and oblique transmission, the cultural evolution of a cooperator phenotype will follow one of the following scenarios depending on the cost boundaries γ_1 and γ_2 and the vertical transmission threshold \hat{v} (Eq. 20):*

1. Fixation of cooperation: if (i) $T_A \geq T_B$ and $c < \gamma_1$; or if (ii) $T_A < T_B$ and $v > \hat{v}$ and $c < \gamma_2$.
2. Fixation of defection: if (iii) $T_A \geq T_B$ and $\gamma_2 < c$; or if (iv) $T_A < T_B$ and $\gamma_1 < c$.
3. Stable coexistence: if (v) $T_A < T_B$ and $v < \hat{v}$ and $c < \gamma_1$; or if (vi) $T_A < T_B$ and $v > \hat{v}$ and $\gamma_2 < c < \gamma_1$.
4. Unstable coexistence: if (vii) $T_A > T_B$ and $\gamma_1 < c < \gamma_2$.

These conditions are illustrated in Figure 3ab.

Much of the literature on evolution of cooperation focuses on conditions for an initially rare cooperative phenotype to invade a population of defectors. The following remarks on such a condition.

202

Remark 1 (Condition for cooperation to increase from rarity: cost of cooperation). *If the initial frequency of cooperation is very close to zero, then its frequency will increase if the cost of cooperation is low enough,*

206

$$c < \gamma_1 = \frac{bv\alpha T_A + (T_A - T_B)}{v(1 - T_B)} . \quad (21)$$

This unites the conditions for fixation of cooperation and for stable coexistence, which both lead to instability of the fixation state associated with defection, $\hat{p} = 0$.

Importantly, increasing social association α increases the boundary cost ($\partial\gamma_1/\partial\alpha > 0$), making it easier for cooperation to increase from rarity. Similarly, increasing the horizontal transmission of cooperation, T_A , increases the boundary ($\partial\gamma_1/\partial T_A > 0$), facilitating the evolution of cooperation. However, increasing the horizontal transmission of defection, T_B , can increase or decrease the cost boundary, but it increases the cost boundary when it is already above one ($c < 1 < \gamma_1$): $\partial\gamma_1/\partial T_B$ is positive when $T_A > \frac{1}{1+\alpha bv}$, which gives $\gamma_1 > 1/v$. Therefore, increasing T_B decreases the cost boundary and limits the evolution of cooperation, but only if $T_A < \frac{1}{1+\alpha bv}$.

Increasing the vertical transmission rate, v , can either increase or decrease the boundary, depending on the horizontal transmission bias, $T_A - T_B$, because $\text{sign}(\partial\gamma_1/\partial v) = -\text{sign}(T_A - T_B)$. When $T_A < T_B$ we get $\partial\gamma_1/\partial v > 0$, and therefore as the vertical transmission rate increases, the cost boundary increases, making it easier for cooperation to evolve from rarity. In contrast, when $T_A > T_B$ we get $\partial\gamma_1/\partial v < 0$, and therefore as the vertical transmission rate increases, the cost boundary decreases, making it harder for cooperation to evolve from rarity.

In general, this condition cannot be formulated in the form of Hamilton's rule due to the horizontal transmission bias term, $T_A - T_B$. When there is no horizontal transmission bias, $T_A = T_B$, the next remark applies.

Remark 2 (Unbiased horizontal transmission). *If horizontal transmission is unbiased, $T = T_A = T_B$, then cooperation will take over the population from any initial frequency if the cost is low enough,*

228

$$c < b \cdot \frac{v\alpha T}{1 - T} . \quad (22)$$

This condition is obtained by setting $T = T_A = T_B$ in inequality 21 and can be interpreted as a version of Hamilton's rule ($c < b \cdot r$, inequality 1) or as a version of inequality 3, where $v\alpha T/(1 - T)$ can be regarded as the *effective relatedness* or *effective assortment*, respectively. Figure 5a illustrates this condition for $v = 1$. Note that the right-hand side equals γ_1 when $T = T_A = T_B$.

In the next remarks we examine the effect of social association on the evolution of cooperation.

234

Remark 3 (No social association). *Without social association ($\alpha = 0$), cooperation will increase from rarity if there is horizontal transmission bias for cooperation, $T_A > T_B$, and*

236

$$c < \frac{T_A - T_B}{v(1 - T_B)} . \quad (23)$$

Figure 2a illustrates this condition, which is obtained by setting $\alpha = 0$ in inequality 21. Importantly, the benefit of cooperation, b , does not affect the evolution of cooperation in the absence of social

240 association, and the outcome is determined only by cultural transmission.

242 **Remark 4** (Perfect social association). *With perfect social association ($\alpha = 1$), cooperation will increase from rarity if*

$$244 \quad c < \frac{bvT_A + (T_A - T_B)}{v(1 - T_B)}. \quad (24)$$

This condition is obtained by setting $\alpha = 1$ in inequality 21.

246 Moreover, without oblique transmission, $v = 1$, the only equilibria are the fixation states, $\tilde{p} = 0$ and $\tilde{p} = 1$, and cooperation will evolve from any initial frequency (i.e., $\tilde{p}' > \tilde{p}$) if inequality 24 applies.

248 This is similar to case of microbe-associated cooperation studied by Lewin-Epstein et al. (2017), and therefore when $v = 1$, this remark is equivalent to their eq. 1.

250 Given the previous two remarks for no association and perfect association ($\alpha = 0$ and $\alpha = 1$), it is interesting to examine the general effect of social association on the evolution of cooperation. Consider
252 the social association boundaries, a_1 and a_2 ,

$$a_1 = \frac{c \cdot v(1 - T_A) - (T_A - T_B)(1 + b - c)}{b \cdot v \cdot T_B}, \quad a_2 = \frac{c \cdot v(1 - T_B) - (T_A - T_B)}{b \cdot v \cdot T_A}. \quad (25)$$

254 Then the following applies.

256 **Remark 5** (Condition for cooperation to increase from rarity: social association). *Cooperation will increase from rarity if social association is high enough, specifically if $a_2 < \alpha$.*

258 Figures 2c and 3cd illustrate this condition. With horizontal transmission bias for cooperation, $T_A > T_B$, cooperation can fix from any initial frequency if $a_2 < \alpha$ (green area in positive x-axis). With horizontal
260 bias favoring defection, $T_A < T_B$, cooperation can fix from any frequency if social association is high, $a_1 < \alpha$ (green area with $T_A < T_B$), and can also increase when rare and reach stable coexistence with
262 defection if social association is intermediate, $a_2 < \alpha$ (yellow area). Without horizontal bias, $T_A = T_B$, fixation of cooperation occurs if social association is high enough, $\frac{c}{b \cdot v} \cdot \frac{1-T}{T} < \alpha$ (inequality 22; in this
264 case $a_1 = a_2$).

Interestingly, because $\text{sign}(\partial a_2 / \partial v) = \text{sign}(T_A - T_B)$, the effect of the vertical transmission rate v
266 on this condition depends on the horizontal transmission bias. That is, with horizontal bias for cooperation, $T_A > T_B$, evolution of cooperation is facilitated by oblique transmission, whereas with
268 horizontal bias for defection, $T_A < T_B$, evolution of cooperation is facilitated by vertical transmission. This is demonstrated in Figure 3cd.

270 The next remarks further inspect the roles of vertical and oblique transmission in the evolution of cooperation.

272 **Remark 6** (Necessary condition for fixation of cooperation: vertical transmission rate). *Fixation of
274 cooperation is possible only if the vertical transmission rate is high enough,*

$$v > \hat{v} = \frac{T_B - T_A}{1 - T_A}. \quad (26)$$

276 This condition does not guarantee fixation of cooperation. Rather, if this condition does not apply then cooperation cannot fix. If horizontal transmission is biased for cooperation, $T_A > T_B$, cooperation
278 can fix with any vertical transmission rate (because $\hat{v} < 0$). In contrast, if horizontal transmission is biased for defection, $T_A < T_B$, cooperation can fix only if the vertical transmission rate is high enough:
280 in this case oblique transmission can prevent fixation of cooperation (see Figure 3bd).

Without oblique transmission, the following applies.

Remark 7 (Condition for cooperation to increase from rarity: vertical and horizontal transmission).

Without oblique transmission ($v = 1$) cooperation will increase from rarity if

$$c < \frac{b\alpha T_A + (T_A - T_B)}{1 - T_B}, \quad (27)$$

which can also be written as

$$\frac{c(1 - T_B) - (T_A - T_B)}{bT_A} < \alpha. \quad (28)$$

These conditions is obtained by setting $v = 1$ in inequalities 21 and 25

Without vertical transmission we have the following result.

Remark 8 (Condition for global fixation of cooperation: oblique and horizontal transmission). *Without*

vertical transmission ($v = 0$), the frequency of the cooperator phenotype among adults increases every generation, $p' > p$ if there is a horizontal transmission bias in favor of cooperation, namely

$$T_A > T_B. \quad (29)$$

This condition is found by setting $v = 0$ in Eq. 6, which becomes $\hat{p} = p$, and then Eq. 8 becomes $p' = p^2(T_B - T_A) + p(1 + T_A - T_B)$. That is, in the absence of vertical transmission, selection plays no role in the evolution of cooperation (i.e. b and c do not affect p'). The dynamics are determined solely by differential horizontal transmission of the two phenotypes, namely, the relative tendency of each phenotype to be horizontally transmitted to peers. Note that without bias in horizontal transmission, $T_A = T_B$, phenotype frequencies are static, $p' = p$.

Finally, the last remark focuses on the coexistence of cooperation and defection.

Remark 9 (Coexistence of cooperation and defection). *Cooperation and defection can coexist at frequencies \tilde{p}^* and $1 - \tilde{p}^*$ (Eq. 17). When it occurs, the coexistence is stable if there is horizontal transmission bias for defection, $T_A < T_B$, and unstable if there is horizontal bias for cooperation, $T_A > T_B$.*

See yellow and blue area in Figure 2 and Figure 3 for cases of stable and unstable coexistence, respectively. When unstable coexistence occurs, cooperation will fix if its initial frequency is $p > \tilde{p}^*$, and defection will fix if its initial frequency is $1 - p > 1 - \tilde{p}^*$; this occurs when there is horizontal transmission bias for cooperation, $T_A > T_B$, and the cost is intermediate, $\gamma_1 < c < \gamma_2$. Figure 4a shows the mapping $\tilde{p} \rightarrow \tilde{p}'$.

Evolution of social association

We now analyze the evolution of social association under perfect vertical transmission, $v = 1$. We assume a population at a stable coexistence of the two phenotypes, cooperation A , and defection B , such that the frequency of A among parents is \tilde{p}^* (Eq. 17). Note that for stable coexistence to occur, we assume horizontal bias for defection, $T_A < T_B$, and an intermediate cost of cooperation, $\gamma_2 < c < \gamma_1$ (Eq. 20)

The stable population mean fitness is $\bar{w}^* = 1 + \tilde{p}^*(b - c)$, which is increasing in \tilde{p}^* . Therefore, to determine the effect of increasing α on the stable population mean fitness, we must analyze its effect

320 on \tilde{p}^* ,

$$\frac{\partial \tilde{p}^*}{\partial \alpha} = \frac{bT_A - c(1 - T_B) + (T_A - T_B)}{b(1 - \alpha)^2(T_B - T_A)}. \quad (30)$$

322 Note that stable coexistence implies $c < \gamma_1$, and because $\alpha < 1$, we have

$$c < \gamma_1 = \frac{b\alpha T_A + (T_A - T_B)}{1 - T_B} < \frac{bT_A + (T_A - T_B)}{1 - T_B}. \quad (31)$$

324 Therefore, the numerator in Eq. 30 is positive. Since $T_A < T_B$, the denominator in Eq. 30 is also
 326 positive, and hence the derivative $\partial \tilde{p}^* / \partial \alpha$ is positive. Thus, the population mean fitness increases as
 social association α increases.

We found that if social association increases, the population mean fitness also increases. But can
 328 an allele that increases social association evolve? To answer this question, we extend our model to
 include a “modifier locus” (Liberman and Feldman, 1986; Liberman, 1988) that determines social
 330 association, but has no direct effect on fitness. We assume that the modifier locus has two alleles, M
 and m , which induce social associations α_1 and α_2 , respectively. Suppose that the population evolves
 332 to a stable equilibrium when only allele M is present; we check the local stability of this equilibrium
 to the introduction of allele m at the modifier locus; This is called “external stability”, see Altenberg
 334 et al. (2017).

We define the frequencies of the pheno-genotypes AM , BM , Am , and Bm as $\tilde{\mathbf{p}} = (\tilde{p}_1, \tilde{p}_2, \tilde{p}_3, \tilde{p}_4)$. The
 336 frequencies of the pheno-genotypes in the next generation are defined by the recursion system

$$\begin{aligned} \bar{w}\tilde{p}'_1 &= \tilde{p}_1(\tilde{p}_1 + \tilde{p}_3)(1 + b - c)(1 - (1 - \alpha_1)(\tilde{p}_2 + \tilde{p}_4)T_B) \\ &\quad + \tilde{p}_1(\tilde{p}_2 + \tilde{p}_4)(1 - c)(1 - \alpha_1 T_B(\tilde{p}_1 + \tilde{p}_3) - T_B(\tilde{p}_2 + \tilde{p}_4)) \\ &\quad + \tilde{p}_2(\tilde{p}_1 + \tilde{p}_3)(1 + b)T_A(\tilde{p}_1 + \tilde{p}_3 + \alpha_1(\tilde{p}_2 + \tilde{p}_4)) \\ &\quad + \tilde{p}_2(\tilde{p}_2 + \tilde{p}_4)(\tilde{p}_1 + \tilde{p}_3)(1 - \alpha_1)T_A, \\ \bar{w}\tilde{p}'_2 &= \tilde{p}_2(\tilde{p}_2 + \tilde{p}_4)(1 - (1 - \alpha_1)(\tilde{p}_1 + \tilde{p}_3)T_A) \\ &\quad + \tilde{p}_2(\tilde{p}_1 + \tilde{p}_3)(1 + b)(1 - \alpha_1 T_A(\tilde{p}_2 + \tilde{p}_4) - T_A(\tilde{p}_1 + \tilde{p}_3)) \\ &\quad + \tilde{p}_1(\tilde{p}_2 + \tilde{p}_4)(1 - c)(\alpha_1 T_B + (1 - \alpha_1)(\tilde{p}_2 + \tilde{p}_4)T_B) \\ &\quad + \tilde{p}_1(\tilde{p}_1 + \tilde{p}_3)(1 + b - c)(1 - \alpha_1)(\tilde{p}_2 + \tilde{p}_4)T_B, \\ \bar{w}\tilde{p}'_3 &= \tilde{p}_3(\tilde{p}_1 + \tilde{p}_3)(1 + b - c)(1 - (1 - \alpha_2)(\tilde{p}_2 + \tilde{p}_4)T_B) \\ &\quad + \tilde{p}_3(\tilde{p}_2 + \tilde{p}_4)(1 - c)(1 - \alpha_2 T_B(\tilde{p}_1 + \tilde{p}_3) - T_B(\tilde{p}_2 + \tilde{p}_4)) \\ &\quad + \tilde{p}_4(\tilde{p}_1 + \tilde{p}_3)(1 + b)T_A(\tilde{p}_1 + \tilde{p}_3 + \alpha_2(\tilde{p}_2 + \tilde{p}_4)) \\ &\quad + \tilde{p}_4(\tilde{p}_2 + \tilde{p}_4)(\tilde{p}_1 + \tilde{p}_3)(1 - \alpha_2)T_A, \\ \bar{w}\tilde{p}'_4 &= \tilde{p}_4(\tilde{p}_2 + \tilde{p}_4)(1 - (1 - \alpha_2)(\tilde{p}_1 + \tilde{p}_3)T_A) \\ &\quad + \tilde{p}_4(\tilde{p}_1 + \tilde{p}_3)(1 + b)(1 - \alpha_2 T_A(\tilde{p}_2 + \tilde{p}_4) - T_A(\tilde{p}_1 + \tilde{p}_3)) \\ &\quad + \tilde{p}_3(\tilde{p}_2 + \tilde{p}_4)(1 - c)(\alpha_2 T_B + (1 - \alpha_2)(\tilde{p}_2 + \tilde{p}_4)T_B) \\ &\quad + \tilde{p}_3(\tilde{p}_1 + \tilde{p}_3)(1 + b - c)(1 - \alpha_2)(\tilde{p}_2 + \tilde{p}_4)T_B, \end{aligned} \quad (32)$$

338 where $\bar{w} = 1 + (b - c)(\tilde{p}_1 + \tilde{p}_3)$ is the population mean fitness.

The equilibrium when only allele M is present is $\tilde{\mathbf{p}}^* = (\tilde{p}^*, 1 - \tilde{p}^*, 0, 0)$, where

$$\tilde{p}^* = \frac{c(1 - T_B) - b\alpha_1 T_A - (T_A - T_B)}{b(1 - \alpha_1)(T_A - T_B)}, \quad (33)$$

342 which we get by setting $\alpha = \alpha_1$ and $v = 1$ in Eq. 17. The local stability of $\tilde{\mathbf{p}}^*$ to the introduction of
 allele m is determined by the linear approximation \mathbf{L}^* of the transformation in Eq. 32 near $\tilde{\mathbf{p}}^*$ (i.e.,
 the Jacobian of the transformation at the equilibrium). \mathbf{L}^* is known to have a block structure, with

344 the diagonal blocks occupied by the matrices \mathbf{L}_{in}^* and \mathbf{L}_{ex}^* . The latter is the external stability matrix:
 346 the linear approximation to the transformation near $\tilde{\mathbf{p}}^*$ involving only the pheno-genotypes Am and
 Bm . Because we assume that $\tilde{\mathbf{p}}^*$ is internally stable (i.e. locally stable to small perturbations in
 the frequencies of AM and BM), the stability of $\tilde{\mathbf{p}}^*$ is determined by the eigenvalues of the external
 348 stability matrix \mathbf{L}_{ex}^* ,

$$\mathbf{L}_{ex}^* = \frac{1}{\bar{w}^*} \begin{pmatrix} X & Y \\ Z & Q \end{pmatrix}. \quad (34)$$

350 The elements of \mathbf{L}_{ex}^* are derived from Eq. 32,

$$\begin{aligned} X &= \frac{\partial \bar{w} \tilde{p}'_3}{\partial \tilde{p}_3}(\tilde{\mathbf{p}}^*) = b(1 - (1 - \alpha_2)T_B(1 - \tilde{p}^*))\tilde{p}^* + (1 - c)(1 - T_B(1 - \tilde{p}^*)), \\ Y &= \frac{\partial \bar{w} \tilde{p}'_3}{\partial \tilde{p}_4}(\tilde{\mathbf{p}}^*) = bT_A(\tilde{p}^* + \alpha_2(1 - \tilde{p}^*))\tilde{p}^* + T_A\tilde{p}^*, \\ Z &= \frac{\partial \bar{w} \tilde{p}'_4}{\partial \tilde{p}_3}(\tilde{\mathbf{p}}^*) = (1 - c)T_B(1 - \tilde{p}^*) + bT_B(1 - \alpha_2)\tilde{p}^*(1 - \tilde{p}^*), \\ Q &= \frac{\partial \bar{w} \tilde{p}'_4}{\partial \tilde{p}_4}(\tilde{\mathbf{p}}^*) = 1 - T_A\tilde{p}^* + b(1 - T_A\tilde{p}^* - \alpha_2T_A(1 - \tilde{p}^*))\tilde{p}^*. \end{aligned} \quad (35)$$

352 If both eigenvalues of \mathbf{L}_{ex}^* are less than 1, then the equilibrium $\tilde{\mathbf{p}}^*$ is externally stable and allele
 m cannot invade the population of allele M . These eigenvalues are the root of the characteristic
 354 polynomial $\lambda^2 - \lambda(X + Q) + XQ - YZ$; the leading eigenvalue λ_1 is the largest of them,

$$\lambda_1 = \frac{X + Q + \sqrt{(X + Q)^2 - 4(XQ - YZ)}}{2}. \quad (36)$$

356 Using *SymPy* (Meurer et al., 2017), a Python library for symbolic mathematics, we find that $\lambda_1 > 1$
 if

$$358 \quad \alpha_2 > \frac{-b(T_A - T_B)\tilde{p}^* - (T_A - T_B) + c(1 - T_B)}{bT_A - b(T_A - T_B)\tilde{p}^*} = \alpha_1, \quad (37)$$

which gives the following result.

360 **Result 2** (Evolution of social association). *In a population at a stable coexistence between cooperation
 362 and defection, a modifier allele can successfully invade the population if it increases the social
 association. Such an invasion will, in turn, increase the frequency of cooperator as well as the
 364 population mean fitness.*

Let us assume that modifier alleles with increased and decreased social association can appear in
 366 the population from time to time. Then, this results suggests that social association will continue to
 (slowly) increase as long as cooperation and defection coexist. This coexistence requires $a_2 < \alpha < a_1$
 368 (Figure 2c), and so this process will proceed until $\alpha = a_1$ and cooperation takes over the population.

370 Population structure

Social association may also emerge from population structure. Consider a population colonizing a 2D
 372 grid of size 100-by-100, where each site is inhabited by one individual, similarly to Lewin-Epstein and
 Hadany (2020). Each individual is characterized by its phenotype: either cooperator, A , or defector, B .
 374 Initially, each site in the grid is randomly colonized with either a cooperator or a defector, with equal
 probability. In each generation, half of the individuals are randomly chosen to "initiate" interactions.

376 Initiators then interact with a random neighbor (i.e. individual in an adjacent site) in a prisoners' dilemma game (Table 1) and a random neighbor (with replacement) in horizontal cultural transmission
 378 (Figure 1). The expected number of each of these interactions per individual per generation is one. The effective social association α in this model is the probability that the same neighbor is picked for
 380 both interactions, or $\alpha = 1/m$, where m is the number of neighbors. On an infinite grid, $m = 8$, but on a finite grid m can be lower in edge neighborhoods close to the grid border. As before, T_A and T_B are
 382 the probability for successful horizontal transmission of phenotypes A and B , respectively.

The order the interactions across the grid at each generation is random. After all interactions take
 384 place, individual fitness is determined by $w = 1 + b \cdot n_b - c \cdot n_c$, where n_b is the number interactions of that individual with cooperative neighbors, and n_c is the total number of interactions that that
 386 individual had ($n_b \leq n_c$). Then, a new generation is generated, and sites can be settled by offspring of any parent, not just neighbor parents. Thus, selection is global, rather than local, in accordance with
 388 our deterministic model. The parent is randomly drawn with probability proportional to its fitness, divided by the average fitness of all potential parents. Offspring then have the same phenotype as their
 390 parents (i.e. $v = 1$).

The outcomes of stochastic simulations with a structured population are shown in Figure 6. The
 392 figure demonstrates that the highest cost of cooperation c that permits the evolution of cooperation agrees with the conditions derived above for a model without population structure or stochasticity. An
 394 example for stable coexistence is shown in Figure 8. Simulating a structured population with local selection (i.e. sites can only be settled by offspring of neighbor parents) had only a minor effect on
 396 the agreement with the derived conditions (Figure 7).

These comparisons between the deterministic unstructured model and the stochastic structured model
 398 show that the derived conditions can be useful for predicting the dynamics of complex scenarios. Moreover, our structured populations demonstrate how social association (α) can emerge from local
 400 interactions between individuals.

Discussion

402 We studied the evolution of cooperation under non-vertical transmission using deterministic discrete-time evolutionary models with fitnesses in the form of payoffs from a prisoner's dilemma game. Under
 404 a combination of vertical, oblique, and horizontal transmission, cooperation or defection can either fix or coexist, depending on the relationship between the cost and benefit of cooperation, the horizontal
 406 transmission bias, and the association between social interaction and horizontal transmission (Result 1). Importantly, cooperation can increase from rarity (i.e. invade a population of defectors) if and only if
 408 (inequality 21),

$$c \cdot v(1 - T_B) < b \cdot v\alpha T_A + (T_A - T_B) . \quad (38)$$

410 That is, if the effective cost of cooperation (left-hand side) is smaller then the effective benefit plus the horizontal transmission bias (right-hand side). This condition cannot be formulated in the form of
 412 Hamilton's rule, $c < b \cdot r$, due to the effect of biased horizontal transmission.

Remarkably, stable coexistence between cooperation and defection can be stable if horizontal trans-
 414 mission is biased for defection ($T_A < T_B$) and both the cost of cooperation and social association are intermediate (yellow areas in Figures 2-3).

416 We find that higher social association α leads to evolution of higher frequencies of cooperation (Remark 5, Figures 2c and 3cd), and that when both cooperation and defection coexist, higher social
 418 association is expected to evolve (Result 2). Without social association, the benefit of cooperation

cannot facilitate its evolution; cooperation can only succeed under horizontal transmission bias ($T_A > T_B$, Remark 3).

Indeed, horizontal transmission plays a major role in the evolution of cooperation: increasing the transmission of cooperation, T_A , or decreasing the transmission of defection, T_B , facilitates the evolution of cooperation (Remarks 1 and 5, Figures 2-3). The effect of oblique transmission is more complex (Corollary 5). When there is horizontal transmission bias for cooperation, $T_A > T_B$, increasing the rate of oblique transmission, $1 - v$, will facilitate the evolution of cooperation (Figure 3ac). In contrast, when the bias is for defection, $T_A < T_B$, high rates of vertical transmission, v , are advantageous for cooperation, and there must be sufficient rate of vertical transmission for cooperation to fix in the population ($v > \hat{v}$, see Remark 6, Figure 3bd).

The conditions derived from our deterministic model provide a good approximation to outcomes of simulations of a complex stochastic model with structured populations, in which individuals can only interact with and transmit to their neighbors (Figure 6). In these structured populations social association arises due to both social interactions and horizontal cultural transmission being local.

Feldman et al. (1985) studied the dynamics of an altruistic phenotype with vertical cultural transmission and a gene that modifies the transmission of the phenotype. Their results are very sensitive to this genetic modification: without it, the conditions for invasion of the altruistic phenotype reduce to Hamilton's rule. Further work is needed to incorporate such genetic modification of cultural transmission into our model.

Woodcock (2006) stressed the significance of non-vertical transmission for the evolution of cooperation. He executed simulations with prisoner's dilemma payoffs but without horizontal transmission or social association ($\alpha = 0$). Nevertheless, his results demonstrated that it is possible to sustain altruistic behavior via cultural transmission for a substantial length of time. Our results provide strong evidence for his hypothesis that horizontal transmission can have an important role in the evolution of cooperation.

To understand the role of horizontal transmission, we first review the role of *assortment*. Eshel and Cavalli-Sforza (1982) showed that altruism can evolve when there is enough *assortative meeting*, namely, a tendency for individuals to interact within their phenotypic group. Fletcher and Doebeli (2009) further argued that a general explanation for the evolution of altruism is given by *assortment*: the correlation between individuals that carry an altruistic trait and the amount of altruistic behavior in their interaction group (see also Bijma and Aanen (2010)). They therefore suggested that to explain the evolution of altruism, we should seek mechanisms that generate assortment, such as population structure, repeated interactions, and individual recognition. Our results highlight another mechanism for generating assortment: an association between social interactions and horizontal transmission that creates a correlation between one's partner for interaction and partner for transmission. This mechanism does not require population structure, repeated interactions, or individual recognition. We show that high levels of such social association greatly increase the potential for evolution of cooperation (Figures 2-3). With enough social association, cooperation can increase in frequency when initially rare even when there is horizontal transmission bias against it ($T_A < T_B$, see Remark 5).

How does non-vertical transmission generate assortment? Lewin-Epstein et al. (2017) and Lewin-Epstein and Hadany (2020) suggested that microbes that manipulate their hosts to act altruistically can be favored by selection, which may help to explain the evolution of cooperation. From the kin selection point-of-view, if microbes can be transmitted *horizontally* from one host to another during host interactions, then following horizontal transmission the recipient host will carry microbes that are closely related to those of the donor host, even when the two hosts are (genetically) unrelated. From the assortment point-of-view, infection by behavior-determining microbes during interactions effectively generates assortment because a recipient of help may be infected by a behavior-determining

466 microbe and consequently become a helper. Cultural horizontal transmission can similarly generate
 assortment between the cooperative phenotype and the benefit of cooperation if cultural transmission
 468 and helping interactions occur between the same individuals, i.e. when there is social association,
 because the recipient of help may be also the recipient of the cultural trait for cooperation. Thus, with
 470 horizontal transmission, “assortment between focal cooperative players and cooperative acts in their
 interaction environment” (Fletcher and Doebeli, 2009) is generated not because *the helper is likely to*
 472 *be helped*, but rather because *the helped is likely to become a helper*.

These conclusions highlight the importance of non-vertical cultural transmission in explaining complex
 474 evolutionary phenomena, and furthers our understating of the cultural evolution of cooperation.

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Appendices

480 Appendix A

Let $f(p) = \lambda \cdot (p' - p)$, where $\lambda > 0$, and 0 and 1 are equilibria, that is, $f(0) = 0$ and $f(1) = 0$.

482 Set $p > p^* = 0$. Using a linear approximation for $f(p)$ near 0, we have

$$p' < p \Leftrightarrow f(p)/p < 0 \Leftrightarrow \frac{f'(0) \cdot p + O(p^2)}{p} < 0 \Leftrightarrow f'(0) + O(p) < 0 . \quad (\text{A1})$$

484 Therefore, by definition of big-O notation, if $f'(0) < 0$ then there exists $\epsilon > 0$ such that for any
 $0 < p < \epsilon$, it is guaranteed that $0 < p' < p$; that is, p' is closer to zero than p .

486 Set $p < p^* = 1$ Using a linear approximation for $f(p)$ near 1, we have

$$1 - p' < 1 - p \Leftrightarrow -\frac{f(p)}{1 - p} < 0 \Leftrightarrow \frac{f'(1)(p - 1) + O((p - 1)^2)}{p - 1} < 0 \Leftrightarrow f'(1) - O(1 - p) < 0 . \quad (\text{A2})$$

488 Therefore, if $f'(1) < 0$ then there exists $\epsilon > 0$ such that for any $1 - \epsilon < 1 - p < 1$ we have $1 - p' < 1 - p$;
 that is, p' is closer to one than p .

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Tables

Table 1: Payoff matrix for prisoner's dilemma.

	$\phi_2 = A$	$\phi_2 = B$
$\phi_1 = A$	$1 + b - c$	$1 - c$
$\phi_1 = B$	$1 + b$	1

The fitness of phenotype ϕ_1 when interacting with phenotype ϕ_2 . A is a cooperative phenotype, B is a defector phenotype, b is the benefit gained by an individual interacting with a cooperator, and c is the cost of cooperation. $0 < b < c$.

Table 2: Interaction frequency, fitness, and transmission probabilities.

Phenotype ϕ_1	Phenotype ϕ_2	Frequency	Fitness of ϕ_1	$P(\phi_1 = A)$ via horizontal transmission:	
				from partner, α	from population, $(1 - \alpha)$
A	A	\hat{p}^2	$1 + b - c$	1	$\hat{p} + (1 - \hat{p})(1 - T_B)$
A	B	$\hat{p}(1 - \hat{p})$	$1 - c$	$1 - T_B$	$\hat{p} + (1 - \hat{p})(1 - T_B)$
B	A	$\hat{p}(1 - \hat{p})$	$1 + b$	T_A	$\hat{p}T_A$
B	B	$(1 - \hat{p})^2$	1	0	$\hat{p}T_A$

Table 3: Model variables and parameters.

Symbol	Description	Values
A	Cooperator phenotype	
B	Defector phenotype	
p	Frequency of phenotype A among adults	$[0, 1]$
\tilde{p}	Frequency of phenotype A among parents	$[0, 1]$
\hat{p}	Frequency of phenotype A among juveniles	$[0, 1]$
v	Vertical transmission rate	$[0, 1]$
c	Cost of cooperation	$(0, 1)$
b	Benefit of cooperation	$c < b$
α	Probability of social association	$[0, 1]$
T_A, T_B	Horizontal transmission rates of phenotype A and B	$[0, 1]$

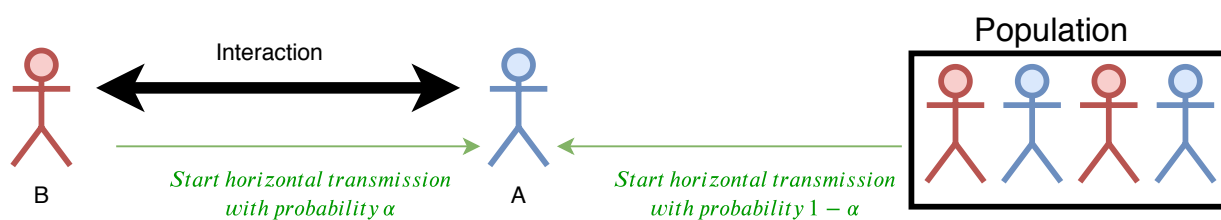


Figure 1: Cultural horizontal transmission with assortment. Transmission occurs between interacting partners with probability α (left) or between two random peers with probability $1 - \alpha$, where α is the *social association* parameter.

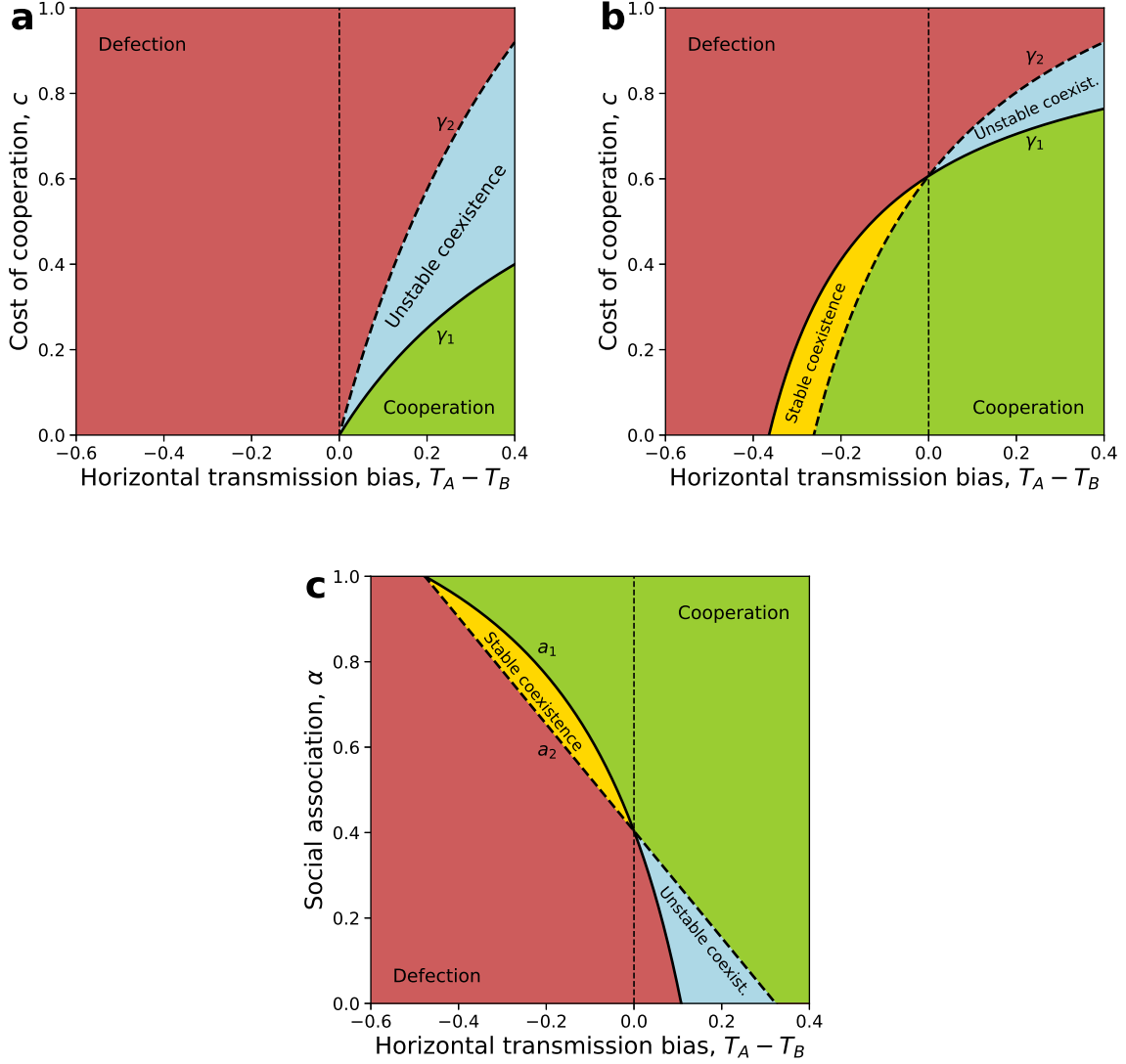


Figure 2: Evolution of cooperation under vertical and horizontal cultural transmission. The figure shows the global fixation of cooperation (green), global fixation of defection (red), fixation of either cooperation or defection depending on the initial conditions, i.e. unstable coexistence (blue), and stable coexistence of cooperation and defection (yellow). In all cases the horizontal bias ($T_A - T_B$) is on the x-axis. **(a-b)** The cost of cooperation c is on the y-axis; the cost boundaries γ_1 and γ_2 (Eq. 20) are the solid and dashed lines, respectively. **(c)** social association α is on the y-axis; the social association boundaries a_1 and a_2 (??) are the solid and dashed lines, respectively. Here, $b = 1.3$, $T_A = 0.4$. **(a)** $\alpha = 0$. **(b)** $\alpha = 0.7$. **(c)** $c = 0.35$.

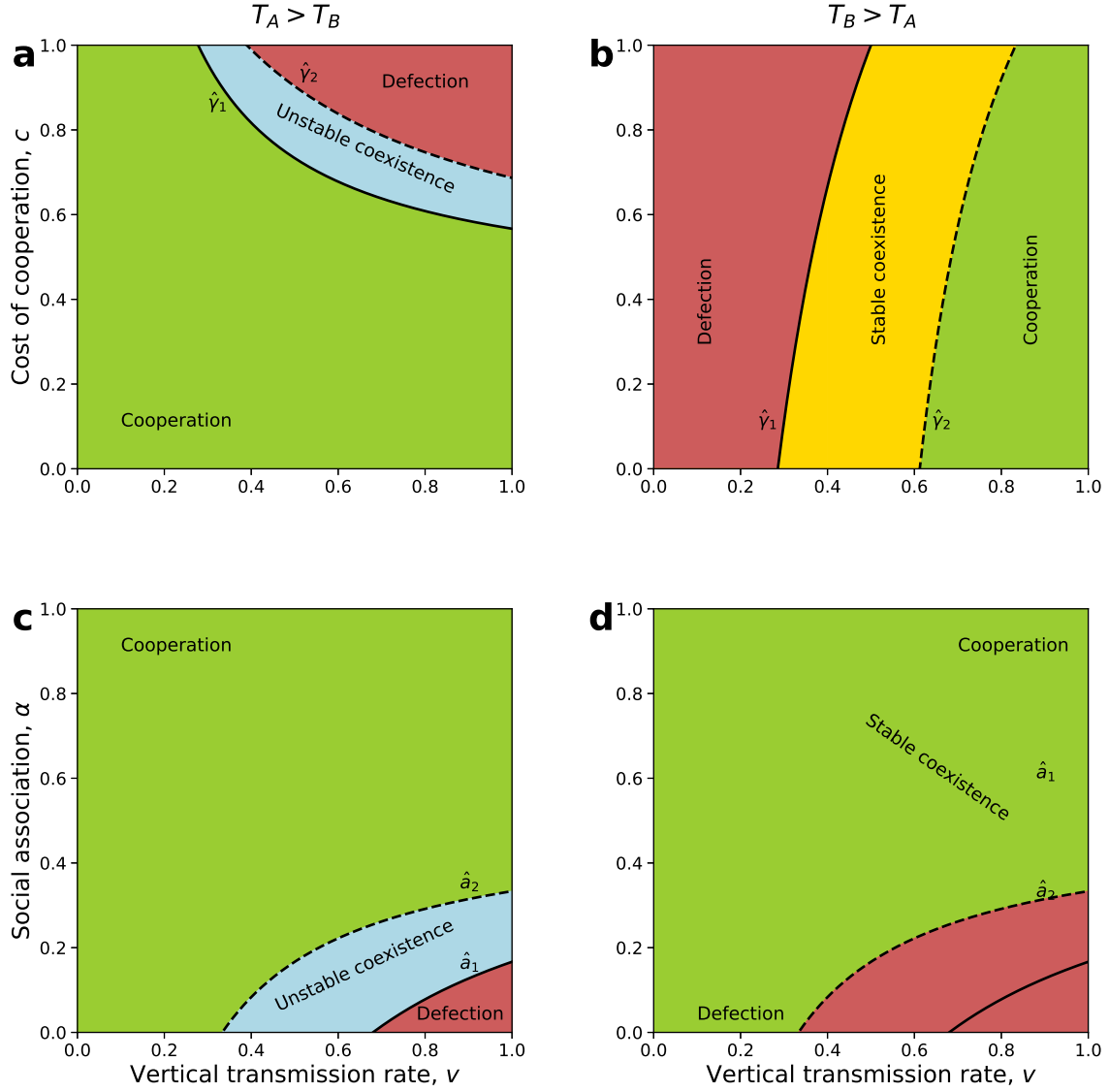


Figure 3: Evolution of cooperation under vertical, oblique, and horizontal cultural transmission.

The figure shows the global fixation of cooperation (green), global fixation of defection (red), fixation of either cooperation or defection depending on the initial conditions, i.e. unstable coexistence (blue), and stable coexistence of cooperation and defection (yellow). In all cases the vertical transmission rate ν is on the x-axis. **(a-b)** The cost of cooperation c is on the y-axis and the cost boundaries γ_1 and γ_2 (Eq. 20) are represented by the solid and dashed lines, respectively. **(c-d)** The social association α is on the y-axis and the social association boundaries a_1 and a_2 (Eq. 25) are represented by the solid and dashed lines, respectively. Horizontal transmission is biased in **(a,c)** for cooperation, $T_A > T_B$, and in **(b,d)** for defection, $T_A < T_B$. Here, $T_A = 0.5$, and **(a)** $b = 1.2$, $T_B = 0.4$, $\alpha = 0.4$; **(b)** $b = 2$, $T_B = 0.7$, $\alpha = 0.7$; **(c)** $b = 1.2$, $T_B = 0.4$, $c = 0.5$; **(d)** $b = 2$, $T_B = 0.7$, $c = 0.5$.

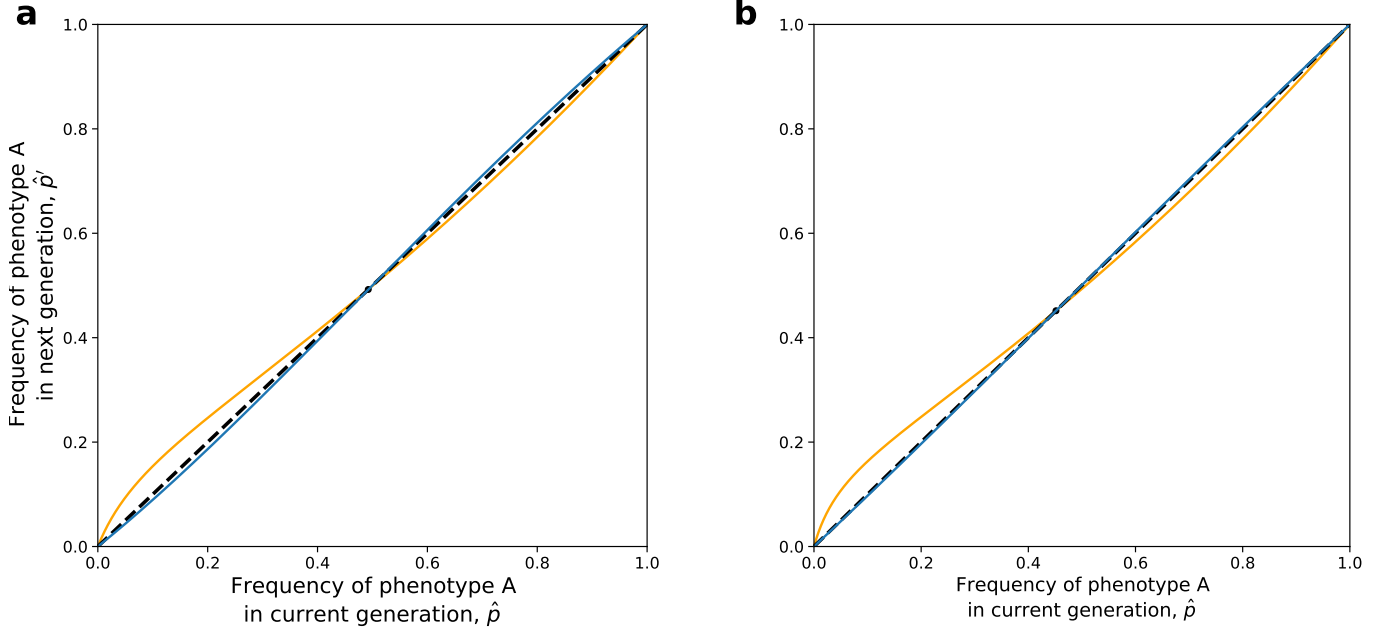


Figure 4: Stable and unstable coexistence between cooperation and defection. The curves show the frequency \hat{p}' of the cooperative phenotype A among juveniles in the next generation vs. that in current generation \hat{p} (Eq. 6). The dashed black line is $\hat{p}' = \hat{p}$. The curves and the dashed line intersect at the stable equilibrium \hat{p}^* (black circle). When $\hat{p} < \hat{p}^*$ the curve is above the dashed line, $\hat{p}' > \hat{p}$, and \hat{p} increases towards \hat{p}^* . When $\hat{p} > \hat{p}^*$ the curve is below the dashed line, $\hat{p}' < \hat{p}$, and \hat{p} decreases towards \hat{p}^* .

(a) There is no oblique transmission, $\nu = 1$. The orange curve, for which the polymorphic equilibrium is stable, is given by $T_A = 0.4$, $T_B = 0.9$, $b = 12$, $c = 0.35$, and $\alpha = 0.45$, which give $\gamma_2 < c < \gamma_1$ (Eq. 20). The blue curve, for which the equilibrium is unstable, is given by $T_A = 0.5$, $T_B = 0.1$, $b = 1.3$, $c = 0.904$, and $\alpha = 0.4$, which give $\gamma_1 < c < \gamma_2$.

(b) Oblique transmission exists. The orange curve is parameterized by $T_A = 0.4$, $T_B = 0.9$, $b = 20$, $c = 0.1$, $\alpha = 1$, and $\nu = 0.4$, which give $0 < \beta_3 < \beta_1$ (Eq. 14). The blue curve is parameterized by $T_A = 0.5$, $T_B = 0.4$, $b = 1.2$, $c = 0.487$, $\alpha = 0.09$ and $\nu = 0.6$, which give $\beta_1 < \beta_3 < 0$.

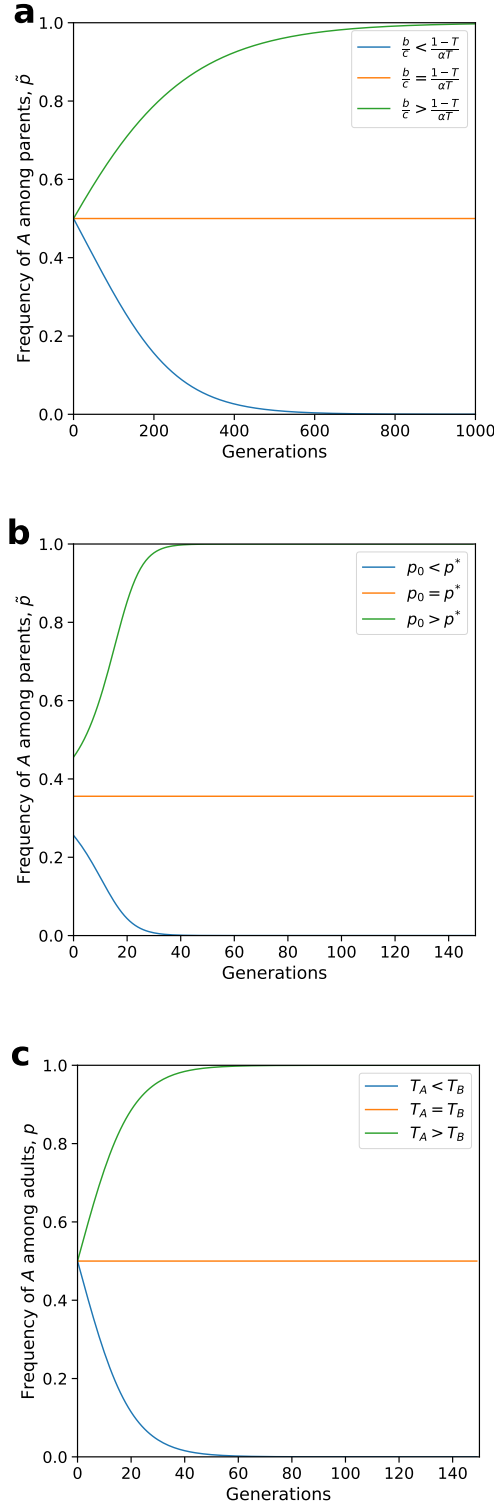


Figure 5: Dynamics of the frequency of cooperation. The frequency \tilde{p} of parents with cooperative phenotype A in (a-b) and the frequency p of adults with cooperative phenotype A in (c). The different lines correspond to parameter values that lead to fixation of cooperation (green), extinction of cooperation (red), or stable coexistence of cooperators and defectors (yellow). (a) $v = 1$, $T_A = T_B = T = 0.2$, $\alpha = 0.5 \neq 0$, $\tilde{p}_0 = 0.5$ and $c = 0.1$; (b) $v = 1$, $\alpha = 0$, $\tilde{p}^* \approx 0.35$, $T_A = 0.65$, $T_B = 0.1$, $b = 1.3$ and $c = 0.65$; (c) $v = 0$, $\alpha = 0.5$, $p_0 = 0.5$, $T_A = 0.5$, $b = 1.3$ and $c = 0.5$.

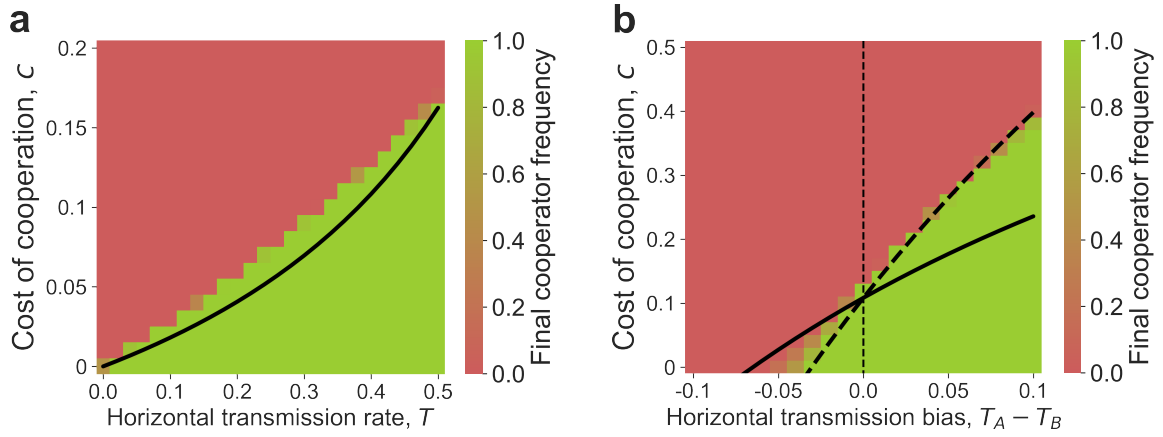


Figure 6: Evolution of cooperation in a structured population. The expected frequency of cooperators in a structured population after 10,000 generations is shown (red for 0%, green for 100%) as function of both the cost of cooperation (c) on the y-axis, and the symmetric horizontal transmission rate ($T = T_A = T_B$) on the x-axis of the left panel, or the transmission bias $T_A - T_B$ on the x-axis of the right panel. The population evolves on a 100-by-100 grid. Cooperation and horizontal transmission are both local between adjacent sites, and each site had 8 neighbors. Selection operates globally (see Figure 7 for results from a model with local selection). The black curves represent the cost boundaries for the evolution of cooperation in a well-mixed population with social association where $\alpha = 1/8$ in (a) Eq. 22 and (b) Eq. 20. Simulations were stopped at generation 10,000 or if one of the phenotypes fixed and 50 simulations were executed for each parameter set. Here, population size is 10,000 (100-by-100 grid), benefit of cooperation, $b = 1.3$, perfect vertical transmission $v = 1$. (a) Symmetric horizontal transmission, $T = T_A = T_B$. (b) Horizontal transmission rates $T_A = 0.4$, $0.3 < T_B < 0.5$.

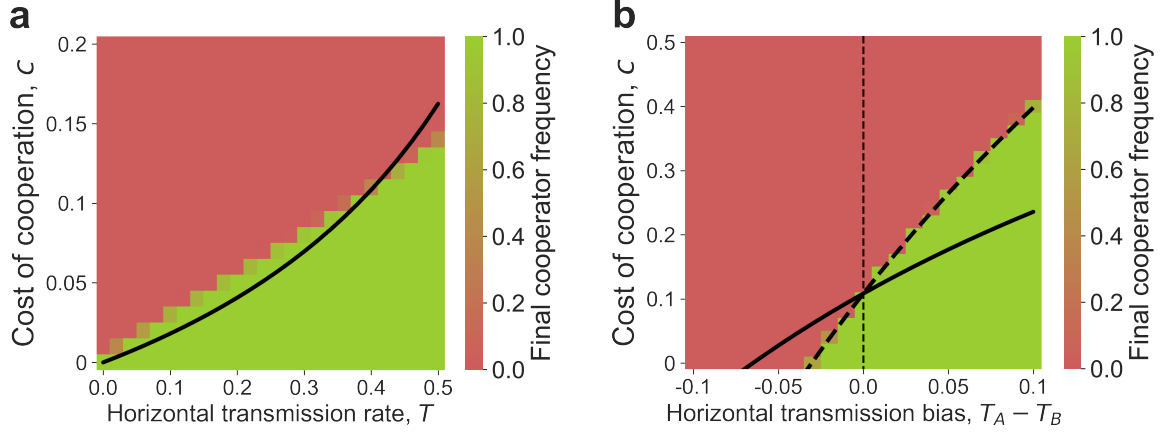


Figure 7: Evolution of cooperation in a structured population with local selection. The expected frequency of cooperators in a structured population after 10,000 generations is shown (red for 0%, green for 100%) as function of both the cost of cooperation (c) on the y-axis, and the symmetric horizontal transmission rate ($T = T_A = T_B$) on the x-axis of the left panel, or the transmission bias $T_A - T_B$ on the x-axis of the right panel. The population evolves on a 100-by-100 grid. Cooperation and horizontal transmission are both local between adjacent sites, and each site had 8 neighbors. Selection operates locally (see Figure 6 for results from a model with global selection). The black curves represent the cost boundaries for the evolution of cooperation in a well-mixed population with social association where $\alpha = 1/8$ in (a) Eq. 22 and (b) Eq. 20. Simulations were stopped at generation 10,000 or if one of the phenotypes fixed and 50 simulations were executed for each parameter set. Here, population size is 10,000 (100-by-100 grid), benefit of cooperation, $b = 1.3$, perfect vertical transmission $v = 1$. (a) Symmetric horizontal transmission, $T = T_A = T_B$. (b) Horizontal transmission rates $T_A = 0.4$, $0.3 < T_B < 0.5$.

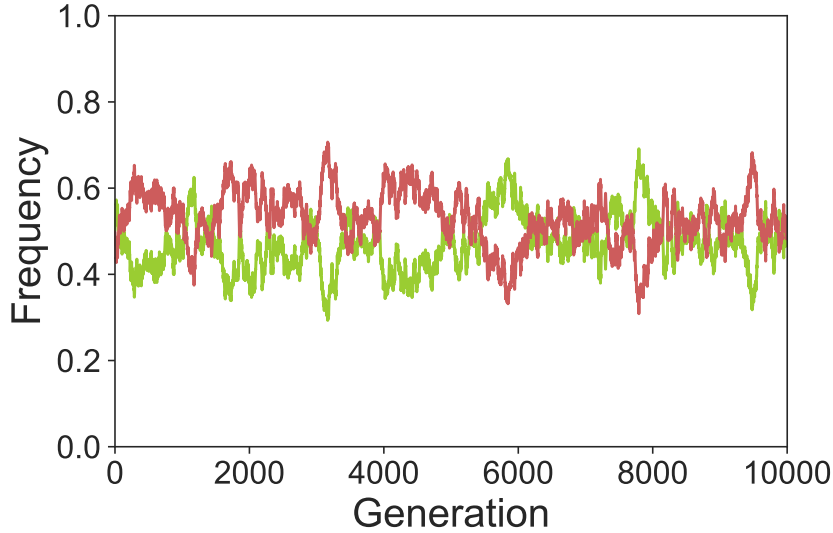


Figure 8: Stable coexistence of both phenotypes in a structured population. The frequency of cooperators (green) and defectors (red) in a structured finite population. Both phenotypes start at 50% frequency. The figure shows that neither phenotype is fixed throughout the simulation, maintaining stochastic coexistence. Here, population size is 10,000 (100-by-100 grid), selection operates globally, benefit and cost of cooperation $b = 1.3$ and $c = 0.02$, perfect vertical transmission $v = 1$, horizontal transmission rates $T_A = 0.4 < T_B = 0.435$.