

# Non-Vertical Cultural Transmission, Assortment, and the Evolution of Cooperation

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## Abstract

We construct models for the cultural evolution of cooperation in a well-mixed population with vertical, horizontal, and oblique transmission. Conditions are found for fixation and coexistence of cooperation and defection. Comparisons are drawn with Hamilton's rule and the concepts of relatedness and assortment. We show that assortment between cooperation and horizontal transmission facilitates the evolution of cooperation.

## 16 Introduction

Cooperative behavior can reduce an individual's fitness and increase the fitness of its conspecifics or competitors (Axelrod and Hamilton, 1981). Nevertheless, cooperative behavior appears to occur in many non-human animals (Dugatkin, 1997), including primates (Jaeggi and Gurven, 2013), rats (Rice and Gainer, 1962), birds (Stacey and Koenig, 1990; Krams et al., 2008), and lizards (Sinervo et al., 2006). Evolution of cooperative behavior remains an important conundrum in evolutionary biology. Since the work of Hamilton (1964) and Axelrod and Hamilton (1981), theories for the evolution of cooperative and altruistic behaviors have been intertwined often under the rubric of *kin selection*.

Kin selection theory posits that natural selection is more likely to favor cooperation between more closely related individuals. The importance of relatedness to the evolution of cooperation and altruism was demonstrated by Hamilton (1964), who showed that an allele that determines cooperative behavior will increase in frequency if the reproductive cost to the actor that cooperates,  $c$ , is less than the benefit to the recipient,  $b$ , times the relatedness,  $r$ , between the recipient and the actor. This relatedness coefficient  $r$  measures a probability that an allele sampled from the cooperator is identical by descent to one at the same locus in the recipient. This condition is known as *Hamilton's rule*:

$$c < b \cdot r. \quad (1)$$

Eshel and Cavalli-Sforza (1982) studied a related model for the evolution of cooperative behavior. Their model included *assortative meeting*, or non-random encounters, where a fraction  $m$  of individuals in the population each interact with an individual of the same phenotype, and a fraction  $1 - m$  interacts with a randomly chosen individual. Such assortative meeting may be due, for example, to population structure or active partner choice. In their model, cooperative behavior can evolve if<sup>1</sup> (Eshel and Cavalli-Sforza, 1982, eq. 3.2)

$$c < b \cdot m, \quad (2)$$

where  $b$  and  $c$  are the benefit and cost of cooperation. Here  $m$  in inequality 2 takes the role of the relatedness  $r$  in inequality 1.

The role of assortment in the evolution of altruism was studied by Fletcher and Doebeli (2009) in a *public goods* game. They found that for altruism to evolve, cooperative individuals must experience more cooperation, on average, than defecting individuals, and “thus, the evolution of altruism requires (positive) assortment between focal *cooperative* players and cooperative acts in their interaction environment.” This condition can be summarized by (Fletcher and Doebeli, 2009, eq. 2.3)

$$c < b \cdot (e_C - e_D), \quad (3)$$

where  $e_C$  is the probability that a cooperator receives help, and  $e_D$  is the probability that a defector receives help.<sup>2</sup>

Here we study the evolution of a cooperative behavior that is subject to *cultural transmission*, which allows an individual to acquire attitudes or behavioral traits from other individuals in its social group through imitation, learning, or other modes of communication (Cavalli-Sforza and Feldman, 1981; Richerson and Boyd, 2008). Feldman et al. (1985) introduced the first model for the evolution of altruism by cultural transmission. They demonstrated that if the fidelity of cultural transmission of

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<sup>1</sup>In an extended model, which allows an individual to encounter  $N$  individuals before choosing a partner, the righthand side is multiplied by  $E[N]$ , the expected number of encounters (Eshel and Cavalli-Sforza, 1982, eq. 4.6).

<sup>2</sup>inequality 3 generalizes inequality 1 and inequality 2 by substituting  $e_C = r + p$ ,  $e_D = p$  and  $e_C = m + (1 - m)p$ ,  $e_D = (1 - m)p$ , respectively, where  $p$  is the frequency of cooperators.

54 altruism is  $\varphi$ , then the condition for evolution of altruism in the case of sib-to-sib altruism is (Feldman  
et al., 1985, Eq. 16)

$$56 \quad c < b \cdot \varphi - \frac{1 - \varphi}{\varphi}. \quad (4)$$

Here  $\varphi$  takes the role of ‘relatedness’ ( $r$  in Eq. 1) or ‘assortment’ ( $m$  in Eq. 2), but the effective benefit  
58  $b \cdot \varphi$  is further reduced by  $(1 - \varphi)/\varphi$ . This shows that under a combination of genetic and vertical  
(parent-to-offspring) cultural transmission, the condition for the evolutionary success of altruism  
60 entails a modification of Hamilton’s rule (Eq. 1).

Cultural transmission may be either viewed as vertical, horizontal or oblique: vertical transmission  
62 occurs between parents and offspring, horizontal transmission occurs between individuals from the  
same generation, and oblique transmission occurs to offspring from the generation to which their  
64 parents belong (i.e. from non-parental adults). Evolution under either of these transmission models  
can be more rapid than under pure vertical transmission (Cavalli-Sforza and Feldman, 1981; Lycett  
66 and Gowlett, 2008; Ram et al., 2018). Both Woodcock (2006) and Lewin-Epstein et al. (2017)  
demonstrated that non-vertical transmission can help explain the evolution of cooperative behavior  
68 (the former using simulations with cultural transmission, the latter using a model where cooperation  
is mediated by microbes that manipulate their host’s behavior.) Specifically, some of the analyses  
70 by Lewin-Epstein et al. (2017) can be applied to cultural transmission, because models of cultural  
transmission are mathematically similar to those for transmission of infectious diseases (Cavalli-Sforza  
72 and Feldman, 1981).

Here we hypothesize that non-vertical cultural transmission can enhance the evolution of cooperation  
74 because it generates assortment between cooperative individuals. To test this hypothesis we construct  
a model in which behavioral changes can be mediated by cultural transmission that can occur during  
76 social interactions. For example, an individual that interacts with a cooperative individual might learn  
that cooperation is a valuable behavior, and will therefore teach its offspring to be cooperative. Our  
78 cultural evolution models include both vertical and non-vertical transmission of cooperation, and we  
investigate these models using mathematical analysis and simulations. Our results demonstrate that  
80 cultural transmission can facilitate the evolution of cooperation even when genetic transmission can  
not because it facilitates the generation of assortment, and that treatment of cooperation as a cultural  
82 trait, rather than a genetic one, can lead to a better understanding of its evolutionary dynamics.

## Models

84 Consider a very large population whose members are characterized by their phenotype  $\phi$ , which can  
be of two types,  $\phi = A$  for cooperators or  $\phi = B$  for defectors. An offspring inherits its phenotype  
86 from its parent via vertical transmission with probability  $v$  or from a random individual in the parental  
population via oblique transmission with probability  $(1 - v)$ . Following Ram et al. (2018), given that  
88 the parent phenotype is  $\phi$  and assuming uni-parental inheritance, the conditional probability that the  
phenotype  $\phi'$  of the offspring is  $A$  is

$$90 \quad P(\phi' = A \mid \phi) = \begin{cases} v + (1 - v)p, & \text{if } \phi = A \\ (1 - v)p, & \text{if } \phi = B \end{cases}, \quad (5)$$

where  $p = P(\phi = A)$  is the frequency of  $A$  among all adults in the parental generation.

92 Not all adults become parents due to natural selection, and we denote the frequency of phenotype  $A$   
among parents by  $\tilde{p}$ . Therefore, the frequency  $\hat{p}$  of phenotype  $A$  among juveniles (after selection and  
94 vertical and oblique transmission) is

$$\begin{aligned}\hat{p} &= \tilde{p}[v + (1 - v)p] + (1 - \tilde{p})[(1 - v)p] \\ &= v\tilde{p} + (1 - v)p.\end{aligned}\tag{6}$$

96 Individuals are assumed to interact according to a *prisoner's dilemma*. Specifically, individuals  
 98 interact in pairs; a cooperator suffers a fitness cost  $0 < c < 1$ , and its partner gains a fitness benefit  
 $b$ , where we assume  $c < b$ . Table 1 shows the payoff matrix, i.e. the fitness of an individual with  
 phenotype  $\phi_1$  when interacting with a partner of phenotype  $\phi_2$ .

	$\phi_2 = A$	$\phi_2 = B$
$\phi_1 = A$	$1 + b - c$	$1 - c$
$\phi_1 = B$	$1 + b$	$1$

**Table 1: Payoff matrix for prisoner's dilemma.** The fitness of phenotype  $\phi_1$  when interacting with phenotype  $\phi_2$ .  $A$  is a cooperative phenotype,  $B$  is a defector phenotype,  $b$  is the benefit gained by an individual interacting with a cooperator, and  $c$  is the cost of cooperation.  $b > c > 0$ .

100 Social interactions occur randomly: two individuals with phenotype  $A$  interact with probability  $\hat{p}^2$ ,  
 two individuals with phenotype  $B$  interact with probability  $(1 - \hat{p})^2$ , and two individuals with different  
 102 phenotypes interact with probability  $2\hat{p}(1 - \hat{p})$ .

Horizontal cultural transmission occurs between pairs of individuals from the same generation. It  
 104 occurs between social partners with probability  $\alpha$ , or between a random pair with probability  $1 - \alpha$   
 (see Figure 1). The assortment rate  $\alpha$  is therefore the fraction of population that receives (horizontal  
 106 transmission) from the social interaction partner, and  $1 - \alpha$  receives randomly. Horizontal transmission  
 is not always successful, as one partner may reject the other's phenotype. The probability for successful  
 108 horizontal transmission of phenotypes  $A$  and  $B$  are  $T_A$  and  $T_B$ , respectively (Table 2).

Therefore, the frequency  $p'$  of phenotype  $A$  among adults in the next generation, after horizontal  
 110 transmission, is

$$\begin{aligned}p' &= \hat{p}^2[\alpha + (1 - \alpha)(\hat{p} + (1 - \hat{p})(1 - T_B))] \\ &\quad + \hat{p}(1 - \hat{p})[\alpha(1 - T_B) + (1 - \alpha)(\hat{p} + (1 - \hat{p})(1 - T_B))] \\ &\quad + (1 - \hat{p})\hat{p}[\alpha T_A + (1 - \alpha)\hat{p}T_A] \\ &\quad + (1 - \hat{p})^2[(1 - \alpha)\hat{p}T_A],\end{aligned}\tag{7}$$

112 which simplifies to

$$p' = \hat{p}^2(T_B - T_A) + \hat{p}(1 + T_A - T_B).\tag{8}$$

Phenotype $\phi_1$	Phenotype $\phi_2$	Frequency	Fitness of $\phi_1$	$P(\phi_1 = A)$ via horizontal transmission:	
				from partner, $\alpha$	from population, $(1 - \alpha)$
$A$	$A$	$\hat{p}^2$	$1 + b - c$	$1$	$\hat{p} + (1 - \hat{p})(1 - T_B)$
$A$	$B$	$\hat{p}(1 - \hat{p})$	$1 - c$	$1 - T_B$	$\hat{p} + (1 - \hat{p})(1 - T_B)$
$B$	$A$	$\hat{p}(1 - \hat{p})$	$1 + b$	$T_A$	$\hat{p}T_A$
$B$	$B$	$(1 - \hat{p})^2$	$1$	$0$	$\hat{p}T_A$

**Table 2: Interaction frequency, fitness, and transmission probabilities.**

114 The frequency of  $A$  among parents (i.e. after selection) follows a similar dynamic, but also includes the effect of natural selection, and is therefore

$$\begin{aligned}
 \bar{w}\tilde{p}' &= \hat{p}^2(1+b-c)[\alpha + (1-\alpha)(\hat{p} + (1-\hat{p})(1-T_B))] \\
 &+ \hat{p}(1-\hat{p})(1-c)[\alpha(1-T_B) + (1-\alpha)(\hat{p} + (1-\hat{p})(1-T_B))] \\
 &+ (1-\hat{p})\hat{p}(1+b)[\alpha T_A + (1-\alpha)\hat{p}T_A] \\
 &+ (1-\hat{p})^2[(1-\alpha)\hat{p}T_A],
 \end{aligned}
 \tag{9}$$

where fitness values are taken from Table 1 and Table 2, and the population mean fitness is

$$\bar{w} = 1 + \hat{p}(b-c). \tag{10}$$

Eq. 9 can be simplified to

$$\begin{aligned}
 \bar{w}\tilde{p}' &= \hat{p}^2(1+b-c)(1 - (1-\hat{p})(1-\alpha)T_B) \\
 &+ \hat{p}(1-\hat{p})(1-c)(\hat{p}(1-\alpha)T_B + 1 - T_B) \\
 &+ (1-\hat{p})\hat{p}(1+b)(\hat{p}(1-\alpha) + \alpha)T_A \\
 &+ (1-\hat{p})^2\hat{p}(1-\alpha)T_A.
 \end{aligned}
 \tag{11}$$

where  $\hat{p} = v\tilde{p} + (1-v)p$ .

## 122 Results

### Oblique and Horizontal Transmission

124 With only oblique and horizontal transmission, i.e.  $v = 0$ , Eq. 6 becomes  $\hat{p} = p$  and Eq. 8 becomes

$$p' = p^2(T_B - T_A) + p(1 + T_A - T_B), \tag{12}$$

which gives the following result.

128 **Result 1** (Oblique and horizontal transmission of cooperation). *Without vertical transmission ( $v = 0$ ),*  
 130 *if there is a horizontal transmission bias in favor of cooperation, namely*

$$T_A > T_B, \tag{13}$$

132 *then  $p' > p$ , and the frequency of the cooperator phenotype among adults increases every generation.*

If  $T_A = T_B$ , then  $p' = p$  and the population is static. Therefore, in the absence of vertical transmission,  
 134 selection plays no role in the evolution of cooperation. Hence, cooperation will evolve if the cooperator phenotype has a horizontal transmission bias (see Figure 5c).

### 136 Vertical and Horizontal Transmission

With only vertical and horizontal transmission, i.e.  $v = 1$ , Eq. 6 becomes  $\hat{p} = \tilde{p}$ , and Eq. 11 for  
 138 the frequency of the cooperative phenotype among parents in the next generation  $\tilde{p}'$  can be written as

$$\begin{aligned}
 \bar{w}\tilde{p}' &= \tilde{p}^2(1+b-c)[1 - (1-\tilde{p})(1-\alpha)T_B] \\
 &+ \tilde{p}(1-\tilde{p})(1-c)[\tilde{p}(1-\alpha)T_B + 1 - T_B] \\
 &+ \tilde{p}(1-\tilde{p})(1+b)[\tilde{p}(1-\alpha) + \alpha]T_A \\
 &+ (1-\tilde{p})^2\tilde{p}(1-\alpha)T_A.
 \end{aligned}
 \tag{14}$$

Fixation of either cooperation,  $\tilde{p} = 1$ , or defection,  $\tilde{p} = 0$ , are equilibria of Eq. 14, that is, they solve  $\tilde{p}' = \tilde{p}$ . We therefore assume for the remainder of the analysis that  $0 < \tilde{p} < 1$ .

If  $\alpha = 1$ , then  $\tilde{p}' = \tilde{p}$  is reduced to

$$\tilde{p}(1 - \tilde{p})[(1 + b)T_A + (1 - c)(1 - T_B) - 1] = 0, \quad (15)$$

and there are no additional equilibria. For cooperation to take over the population (for  $\tilde{p} = 1$  to be globally stable) we require  $\tilde{p}' > \tilde{p}$ ; that is,

$$\tilde{p}^2(1 + b - c) + \tilde{p}(1 - \tilde{p})[(1 - c)(1 - T_B) + (1 + b)T_A] > \tilde{w}\tilde{p}. \quad (16)$$

Factoring out  $\tilde{p}(1 - \tilde{p})$  and setting  $\tilde{w} = 1 + \tilde{p}(b - c)$ , we find that  $\tilde{p}' > \tilde{p}$  if

$$(1 - c)(1 - T_B) + (1 + b)T_A > 1 \quad (17)$$

If  $\alpha < 1$ , divide both sides of Eq. 14 by  $\tilde{p}$  and set  $\tilde{w} = 1 + \tilde{p}(b - c)$ . Then  $\tilde{p}' > \tilde{p}$  if

$$\begin{aligned} 1 + \tilde{p}(b - c) &< \tilde{p}(1 + b - c)(1 - (1 - \tilde{p})(1 - \alpha)T_B) \\ &+ (1 - \tilde{p})(1 - c)(\tilde{p}(1 - \alpha)T_B + 1 - T_B) \\ &+ (1 - \tilde{p})(1 + b)(\tilde{p}(1 - \alpha) + \alpha)T_A \\ &+ (1 - \tilde{p})^2(1 - \alpha)T_A. \end{aligned} \quad (18)$$

Simplifying, we find that  $\tilde{p}' > \tilde{p}$  if and only if

$$c(1 - T_B) - b\alpha T_A - (T_A - T_B) < \tilde{p} \cdot b(1 - \alpha)(T_A - T_B). \quad (19)$$

Besides the fixation states  $\tilde{p} = 0$  and  $\tilde{p} = 1$ , there may be an actual polymorphic equilibrium of  $\tilde{p}' = \tilde{p}$  in Eq. 14, namely

$$\tilde{p}^* = \frac{c(1 - T_B) - b\alpha T_A - (T_A - T_B)}{b(1 - \alpha)(T_A - T_B)}, \quad (20)$$

which is legitimate if  $0 < \tilde{p}^* < 1$ .

Since all parameters are positive, we can apply inequality 19 and see that for  $\tilde{p}' > \tilde{p}$  we require that either

$$T_A > T_B \quad \text{and} \quad \tilde{p} > \tilde{p}^*, \quad \text{or} \quad (21)$$

$$T_A < T_B \quad \text{and} \quad \tilde{p} < \tilde{p}^*. \quad (22)$$

We define the initial frequency of the cooperator phenotype among parents,  $\tilde{p}_0$ , and the *cost boundaries*,

$$\gamma_1 = \frac{b\alpha T_A + (T_A - T_B)}{1 - T_B}, \quad \gamma_2 = \frac{b\alpha T_B + (1 + b)(T_A - T_B)}{1 - T_B}. \quad (23)$$

Then, applying Eqs. 20, 21, and 22, we summarize the possible outcomes in following result and corollaries.

**Result 2** (Vertical and horizontal transmission of cooperation). *With vertical and horizontal but without oblique transmission ( $v = 1$ ), the cultural evolution of a cooperator phenotype will follow one of the following scenarios, depending on the cost boundaries  $\gamma_1$  and  $\gamma_2$  (Eq. 23):*

1. Fixation of cooperation: if (i)  $T = T_A = T_B$  and  $c < b \cdot \alpha \frac{T}{1-T}$ ; or if (ii)  $T_A > T_B$  and  $c < \gamma_1$ ; or  
 170 if (iii)  $T_A < T_B$  and  $c < \gamma_2$ .

2. Fixation of defection: if (iv)  $T = T_A = T_B$  and  $c > b \cdot \alpha \frac{T}{1-T}$ ; or if (vi)  $T_A > T_B$  and  $\gamma_2 < c$ ; or  
 172 if (vi)  $T_A < T_B$  and  $\gamma_1 < c$ .

3. Fixation of either phenotype depending on initial frequency: if (vii)  $T_A > T_B$  and  $\gamma_1 < c < \gamma_2$ .

174 4. Coexistence of both phenotypes at  $\tilde{p}^*$ : if (viii)  $T_A < T_B$  and  $\gamma_2 < c < \gamma_1$ .

These conditions are illustrated in Figure 3b-c. Note that cooperation and defection can coexist stably  
 176 if there is horizontal bias for defection and the cost of cooperation is large but not too large. The  
 recursion dynamic for this case is illustrated in Figure 2.

178 Much of the literature on evolution of cooperation focuses on conditions for an initially rare cooper-  
 ative phenotype to invade a population of defectors. The next corollary deals with such a condition,  
 180 followed by a corollary that deals with symmetric horizontal transmission, i.e.  $T_A = T_B$ .

182 **Corollary 1** (Condition for cooperation to increase when initially rare). *If the initial frequency of the  
 cooperative phenotype is very close to zero,  $\tilde{p}_0 \approx 0$ , then this frequency will increase if*

$$184 \quad T_A > T_B \text{ and } c < \gamma_1, \quad \text{or} \quad T_A < T_B \text{ and } \gamma_2 < c < \gamma_1. \quad (24)$$

In general, these conditions cannot be formulated in the form of Hamilton's rule ( $c < b \cdot r$ ) due to the  
 186 horizontal transmission bias  $T_A - T_B$ . Without horizontal transmission bias, i.e., with  $T = T_A = T_B$ ,  
 these conditions reduce to a form of Hamilton's rule.

188 **Corollary 2** (Symmetric horizontal transmission). *If  $T = T_A = T_B$ , then cooperation will take over  
 190 the population if*

$$c < b \cdot \alpha \frac{T}{1-T}. \quad (25)$$

192 Inequality 25 is obtained by setting  $T = T_A = T_B$  in inequality 19 and can be interpreted as a version  
 of Hamilton's rule (inequality 1) or as a version of inequality 3, where  $\alpha T/(1-T)$  can be regarded as  
 194 the 'effective relatedness' or 'effective assortment', respectively. Figure 5a demonstrates this condition.

196 **Corollary 3** (No assortment of transmission and cooperation). *Without assortment ( $\alpha = 0$ ) and  
 with horizontal bias for cooperation ( $T_A > T_B$ ), then (1) if the cost is low compared to the bias  
 198 ( $c < (T_A - T_B)/(1 - T_B)$ ), then cooperation will fix from any positive frequency; (2) if the cost is low  
 compared to the benefit ( $c < (1 + b)(T_A - T_B)(1 - T_B)$ ), then cooperation will fix if the initial frequency  
 200 is high enough ( $\tilde{p}_0 > \tilde{p}^*$ ).*

Figure 3b illustrates these conditions, where the third equilibrium given by Eq. 20 becomes

$$202 \quad \tilde{p}^*(\alpha = 0) = \frac{c(1 - T_B) - (T_A - T_B)}{b(T_A - T_B)}, \quad (26)$$

and the cost boundaries are

$$204 \quad \gamma_1(\alpha = 0) = \frac{T_A - T_B}{1 - T_B}, \quad \gamma_2(\alpha = 0) = (1 + b) \frac{T_A - T_B}{1 - T_B}. \quad (27)$$

If  $T_A > T_B$  then  $0 < \gamma_1(\alpha = 0) < \gamma_2(\alpha = 0)$ . So either  $c < \gamma_1(\alpha = 0)$  or  $\gamma_1(\alpha = 0) < c < \gamma_2(\alpha = 0)$   
 206 will allow fixation of cooperation, the latter only if the initial frequency is high enough. If  $T_A < T_B$   
 then  $\gamma_2(\alpha = 0) < \gamma_1(\alpha = 0) < 0 < c$ , and defection will fix.

**Corollary 4** (Perfect assortment of transmission and cooperation). *With perfect assortment ( $\alpha = 1$ ), there are only two equilibria,  $\tilde{p} = 0$  and  $\tilde{p} = 1$ . The condition for evolution of cooperation (i.e. global stability of  $\tilde{p} = 1$ ) is found from inequality 17, namely*

$$c < \frac{b \cdot T_A + (T_A - T_B)}{1 - T_B}. \quad (28)$$

With perfect assortment horizontal transmission always co-occurs with the cooperative interaction. The same occurs in Lewin-Epstein et al. (2017), and therefore this corollary is equivalent to their result (see their eq. 1).

In terms of the cost boundaries, inequality 28 is equivalent to  $c < \gamma_1$ , and if  $T_A > T_B$  then that suffices for fixation of cooperation. If  $T_B > T_A$  then  $\gamma_2(\alpha = 1) < 0$  and again, inequality 28 is sufficient for increase in the frequency of A. Inequality 28 can be written as

$$1 - (1 - c)(1 - T_B) < (1 + b)T_A, \quad (29)$$

which provides an interesting interpretation for the success of cooperation. In the interaction between a cooperator and a defector,  $(1 - c)(1 - T_B)$  is the probability that the cooperator remains cooperative and also reproduces. Therefore,  $1 - (1 - c)(1 - T_B)$  is the probability that either the cooperator becomes a defector, or that it fails to reproduce. This is the *effective cost of cooperation* from this interaction, while  $(1 + b)T_A$  is the probability that the defector becomes cooperative and reproduces, which is the *effective benefit of cooperation* from this interaction. Thus, inequality 28 entails that cooperation can evolve if the effective cost of cooperation is less than the effective benefit.

Given the previous two corollaries for no assortment and perfect assortment ( $\alpha = 0$  and  $\alpha = 1$ ), it is interesting to examine the general effect of assortment on the evolution of cooperation. We denote the assortment boundaries by

$$a_1 = \frac{c(1 - T_B) - (1 + b)(T_A - T_B)}{b \cdot T_B}, \quad a_2 = \frac{c(1 - T_B) - (T_A - T_B)}{b \cdot T_A}. \quad (30)$$

Applying Eqs. 20, 21, and 22, we have the following corollary.

**Corollary 5** (Intermediate assortment of transmission and cooperation). *The cooperative phenotype will increase from rarity if the assortment rate is high enough, or specifically if*

$$\begin{aligned} T_A > T_B \text{ and } a_2 < \alpha, \quad \text{or} \\ T_A < T_B \text{ and } a_1 < \alpha. \end{aligned} \quad (31)$$

Figure 3c demonstrates these conditions. With horizontal bias for cooperation ( $T_A > T_B$ ; positive x-axis) cooperation fixes in the population from any initial positive frequency (green) if  $\alpha$  (y-axis) is above the orange line ( $a_1$ ), or if initially present at a high enough frequency (blue) if  $\alpha$  is above the blue line ( $a_2$ ). With horizontal bias for defection ( $T_B > T_A$ ; negative x-axis) cooperation fixes if  $\alpha$  is above the blue line ( $a_2$ ), but can be maintained in coexistence with defection (yellow) if  $\alpha$  is above the orange line ( $a_1$ ). Without horizontal bias ( $T_A = T_B$ ) fixation occurs if the assortment rate is high enough,  $\alpha > \frac{c}{b} \cdot \frac{1-T}{T}$  (inequality 25).

## With Vertical and Oblique Transmission

With both vertical and oblique transmission,  $0 < v < 1$ , the recursion system is more complex, and we focus on local rather than on global stability. To proceed, we note that Eq. 6 can give  $\hat{p}'$  as a function



246 of both  $p'$  and  $\tilde{p}'$ . Eq. 8 gives  $p'$  as a function of  $\tilde{p}$ , since  $\hat{p}$  is given in Eq. 6 as a function of  $\tilde{p}$  and Eq.  
 11 gives  $\tilde{p}'$  as a function of  $\hat{p}$ . Combining these equations, we find an equation for  $\hat{p}'$  as a function of  
 248  $\hat{p}$  (shown in Appendix A). We then determine the equilibria, namely, solutions of  $\hat{p}' = \hat{p}$ , and analyse  
 their local stability.

250 We apply Eqs. 6, 8, and 11 to obtain the function  $f(\hat{p})$  (see Appendix A):

$$f(\hat{p}) = \bar{w}(\hat{p}' - \hat{p}) = \beta_1 \hat{p}^3 + \beta_2 \hat{p}^2 + \beta_3 \hat{p}, \quad (32)$$

252 where

$$\begin{aligned} \beta_1 &= [c(1 - v) - b(1 - \alpha v)](T_A - T_B), \\ \beta_2 &= -\beta_1 - \beta_3, \\ \beta_3 &= \alpha b v T_A - c v(1 - T_B) + (T_A - T_B). \end{aligned} \quad (33)$$

254 If  $T = T_A = T_B$  then  $\beta_1 = 0$  and  $\beta_3 = -\beta_2 = \alpha b v T - c v(1 - T)$ , and  $f(\hat{p})$  becomes a quadratic  
 polynomial:

$$256 \quad f(\hat{p}) = \hat{p}(1 - \hat{p})[\alpha b v T - c v(1 - T)]. \quad (34)$$

Clearly the only two equilibria are the fixations  $\hat{p} = 0$  and  $\hat{p} = 1$ . These equilibria are locally stable if  
 258  $f'(\hat{p}) < 0$  near the equilibrium (Appendix B), and

$$f'(\hat{p}) = (1 - 2\hat{p})[\alpha b v T - c v(1 - T)], \quad (35)$$

260 with

$$\begin{aligned} f'(0) &= \alpha b v T - c v(1 - T), \\ f'(1) &= -\alpha b v T + c v(1 - T). \end{aligned} \quad (36)$$

262 Therefore with symmetric horizontal transmission, fixation of the cooperative phenotype ( $\hat{p} = 1$ )  
 occurs under the same condition as Corollary 1.1, namely Eq. 25.

264 In the general case where  $T_A \neq T_B$ , the coefficient  $\beta_1$  is not necessarily zero, and  $f(\hat{p})$  is a cubic  
 polynomial. Therefore, three equilibria may exist, two of which are  $\hat{p} = 0$  and  $\hat{p} = 1$ , and the third  
 266 is

$$\hat{p}^* = \frac{\beta_3}{\beta_1}. \quad (37)$$

268 Note that the sign of the cubic (Eq. 32) at positive (negative) infinity is equal (opposite) to the sign of  
 $\beta_1$ . If  $T_A > T_B$ , then

$$270 \quad \beta_1 < [c(1 - \alpha v) - b(1 - \alpha v)](T_A - T_B) = (1 - \alpha v)(c - b)(T_A - T_B) < 0, \quad (38)$$

since  $c < b$  and  $1 > \alpha v$ . Hence the signs of the cubic at positive and negative infinity are negative  
 272 and positive, respectively. First, if  $\beta_3 < \beta_1$  then  $1 < \hat{p}^*$  and therefore  $f'(0) < 0$  and  $f'(1) > 0$ ; that  
 is, fixation of the defector phenotype  $B$  is the only locally stable legitimate (i.e. between 0 and 1)  
 274 equilibrium. Second, if  $\beta_1 < \beta_3 < 0$  then  $0 < \hat{p}^* < 1$  and therefore  $f'(0) < 0$  and  $f'(1) < 0$ , that  
 is, both fixations are locally stable and  $\hat{p}^*$  separates the domains of attraction. Third, if  $0 < \beta_3$  then  
 276  $\hat{p}^* < 0$  and therefore  $f'(0) > 0$  and  $f'(1) < 0$ ; that is, fixation of the cooperators phenotype  $A$  is the  
 only locally stable legitimate equilibrium.

278 Similarly, if  $T_B > T_A$ , then

$$\beta_1 > [c(1 - \alpha v) - b(1 - \alpha v)](T_A - T_B) = (1 - \alpha v)(c - b)(T_A - T_B) > 0, \quad (39)$$

280 since  $c < b$  and  $1 > \alpha v$ , and the signs of the cubic at positive and negative infinity are positive and  
 negative, respectively. First, if  $\beta_3 < 0$  then  $\hat{p}^* < 0$  and therefore  $f'(0) < 0$  and  $f'(1) > 0$ ; that is,

fixation of the defector phenotype  $A = B$  is the only locally stable legitimate equilibrium. Second, if  $0 < \beta_3 < \beta_1$  then  $0 < \hat{p}^* < 1$  and therefore  $f'(0) > 0$  and  $f'(1) > 0$ ; that is, both fixations are locally unstable and  $\hat{p}^*$  is a stable polymorphic equilibrium. Third, if  $\beta_1 < \beta_3$  then  $\hat{p}^* > 1$  and therefore  $f'(0) > 0$  and  $f'(1) < 0$ , and fixation of the cooperator phenotype  $A$  is the only locally stable legitimate equilibrium.

The following result summarizes the possible outcomes.

**Result 3** (Vertical, oblique, and horizontal transmission of cooperation). *With vertical, horizontal, and oblique transmission, the cultural evolution of a cooperator phenotype will follow one of the following scenarios depending on the polynomial coefficients  $\beta_1$  and  $\beta_3$  (Eq. 33):*

1. Fixation of cooperation: if (i)  $T = T_A = T_B$  and  $c < b \cdot \frac{\alpha T}{1-T}$ ; or if (ii)  $T_A > T_B$  and  $0 < \beta_3$ ; or if (iii)  $T_A < T_B$  and  $\beta_1 < \beta_3$ .
2. Fixation of the defection: if (iv)  $T = T_A = T_B$  and  $c > b \cdot \frac{\alpha T}{1-T}$ ; or if (v)  $T_A > T_B$  and  $\beta_1 < \beta_3 < 0$ ; or if (vi)  $T_A < T_B$  and  $\beta_3 < 0$ .
3. Coexistence of both phenotypes at  $\hat{p}^*$ : if (vii)  $T_A < T_B$  and  $0 < \beta_3 < \beta_1$ .
4. Fixation of either phenotype depending on initial frequency: if (viii)  $T_A > T_B$  and  $\beta_3 < \beta_1$ .

## Discussion

We hypothesized that non-vertical transmission can explain the evolution of cooperation and evaluated this hypothesis using a deterministic discrete-time evolutionary model with fitnesses in the form of payoffs from a prisoner's dilemma game. Under oblique and horizontal cultural transmission, a horizontal transmission bias for the cooperative phenotype was found to be sufficient and necessary for evolution of cooperation (Result 1). Under horizontal and vertical cultural transmission, cooperation or defection can either fix or coexist at a stable polymorphism, depending on the relationship between the cost and benefit of cooperation, the horizontal bias, and the assortment between cooperation and transmission (Result 2). Under a combination of vertical, oblique, and horizontal transmission the dynamics are more complicated, but all outcomes can still occur, depending on parameter values (Result 3). Remarkably, stable coexistence between cooperation and defection can be maintained if horizontal transmission is biased for defection ( $T_B > T_A$ ) and the cost of cooperation and assortment ratio are intermediate (yellow areas Figure 3).

Eshel and Cavalli-Sforza (1982) showed that with assortative meeting, namely, a probability  $m$  that individuals interact within their phenotypic group, cooperation can evolve if  $c < b \cdot m$ . Fletcher and Doebeli (2009) have further argued that a simple and general explanation for the evolution of altruism is given by assortment between individuals that carry an altruistic trait, and the altruistic behavior of others. They therefore suggested that to explain the evolution of altruism, we should seek mechanisms that generate such assortment, and that one such major mechanism is population structure. Our results highlight another possibility for assortment, namely, individuals socially interacting at rate  $\alpha$  with their horizontal transmission partners. This mechanism does not require population structure. Rather, it depends on the co-occurrence of two social processes: cultural transmission and cooperation. We show that high levels of assortment significantly increase the potential for evolution of cooperation (Figure 3). With enough assortment ( $\alpha > (c(1 - T_B) + (T_B - T_A))/bT_A$ ), cooperation can increase in frequency when initially rare even when there is horizontal bias against it ( $T_B > T_A$ , see Result 2).

This study was partially inspired by the work of Lewin-Epstein et al. (2017) and Lewin-Epstein and Hadany (2020), who hypothesised that microbes that manipulate their hosts to act altruistically can be favored by selection, and may play a role in the evolution of cooperation. Indeed, it has been

326 shown that microbes can mediate behavioral changes in their hosts (Dobson, 1988; Poulin, 2010).  
 Therefore, natural selection on microbes may favor manipulation of the host so that it cooperates with  
 328 others. Microbes can be transmitted *horizontally* from one host to another during host interactions,  
 and following horizontal transfer, the recipient host may carry microbes that are closely related to  
 330 the microbes of the donor host, even when the two hosts are (genetically) unrelated (Lewin-Epstein  
 et al., 2017). Although this result can be described from a kin selection point-of-view, we are  
 332 more interested in the assortment point-of-view: infection by behavior-determining microbes during  
 interactions effectively generates assortment, as a cooperating donor can transmit to the recipient  
 334 both the benefit of the cooperative behavior and behavior-determining microbe. Horizontal cultural  
 transmission can similarly generate assortment between the cooperative phenotype and the benefit  
 336 of cooperation if cultural transmission and helping interactions occur between the same individuals,  
 which in our model occurs with rate  $\alpha$ . So, because non-vertical transmission can effectively generate  
 338 assortment, we suggest a revision of the conclusion made by Fletcher and Doebeli (2009)<sup>3</sup>: what is  
 necessary for the evolution of altruism is assortment between focal *pheno-genotype* and phenotypic  
 340 help.

Feldman et al. (1985) studied the dynamics of an altruistic phenotype with vertical cultural transmission  
 342 and a gene that modifies the transmission of the phenotype. Their results are very sensitive to this  
 genetic modification: without it, the conditions for invasion of the altruistic phenotype reduce to  
 344 Hamilton's rule. Further work is needed to include such genetic modification of cultural transmission  
 to our model.

346 An important implication of our results is that cooperation can evolve even without population struc-  
 ture, repeated interactions, or individual recognition. This highlights the potential importance of  
 348 non-vertical cultural transmission for explaining complex evolutionary phenomena, and furthers our  
 understating of the cultural evolution of cooperation.

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<sup>3</sup>“what is necessary for the evolution of altruism is assortment between focal genotype and phenotypic help, rather than the assortment among genetic types often emphasized in kin selection theory.”

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# Appendices

## 354 Appendix A

356 We want to find the frequency of juveniles with phenotype  $A$  in next generation  $\hat{p}'$  as a function of frequency of juveniles with phenotype  $A$  in the current generation  $\hat{p}$ . Starting from Eq. 6,

$$\hat{p}' = v\tilde{p}' + (1 - v)p', \quad (\text{A1})$$

358 we substitute  $p'$  using Eq. 8 and  $\tilde{p}'$  using Eq. 11, we have

$$\begin{aligned} \hat{p}' = & \frac{v}{\bar{w}} \left\{ \hat{p}^2(1 + b - c) \left[ 1 - (1 - \hat{p})(1 - \alpha)T_B \right] \right\} \\ & + \frac{v}{\bar{w}} \left\{ \hat{p}(1 - \hat{p})(1 - c) \left[ \hat{p}(1 - \alpha)T_B + 1 - T_B \right] \right\} \\ & + \frac{v}{\bar{w}} \left\{ \hat{p}(1 - \hat{p})(1 + b) \left[ \hat{p}(1 - \alpha) + \alpha \right] T_A \right\} \\ & + \frac{v}{\bar{w}} (1 - \hat{p})^2 \hat{p}(1 - \alpha)T_A \\ & + (1 - v)\hat{p}^2(T_B - T_A) + (1 - v)\hat{p}(1 + T_A - T_B), \end{aligned} \quad (\text{A2})$$

360 where  $\bar{w} = 1 + \hat{p}(b - c)$ . We define  $f(\hat{p})$  as

$$f(\hat{p}) = \bar{w}(\hat{p}' - \hat{p}) \quad (\text{A3})$$

362 Using *SymPy* (Meurer et al., 2017), a Python library for symbolic mathematics, we simplify Eq. A3 to eqs. 32-33.

## 364 Appendix B

366 Denote  $f(p) = \lambda(p' - p)$ , where  $\lambda > 0$ , and assume  $f(p^*) = 0$ ; i.e.,  $p^*$  is an equilibrium. We want a condition for  $|p' - p^*| < |p - p^*|$ .

368 If  $p > p^* = 0$ , we want a condition for  $p' < p$ , or  $\frac{p'}{p} < 1$ , or  $\lambda \frac{p' - p}{p} < 0$ , or  $\frac{f(p)}{p} < 0$ . Using a linear approximation for  $f(p)$  near 0, we have

$$\begin{aligned} p' < p & \Leftrightarrow \\ \frac{f'(0) \cdot p + O(p^2)}{p} < 0 & \Leftrightarrow \\ f'(0) + O(p) & < 0. \end{aligned} \quad (\text{B1})$$

370 Therefore, by definition of big-O notation, if  $f'(0) < 0$  then there exists  $\epsilon > 0$  such that for any  $0 < p < \epsilon$ , it is guaranteed that  $0 < p' < p$ , that is,  $p'$  is closer than  $p$  to zero.

372 If  $p < p^* = 1$ , we want a condition for  $1 - p' < 1 - p$ , or  $\frac{1-p'}{1-p} < 1$ , or  $\lambda \frac{-(p'-p)}{1-p} < 0$ , or  $-\frac{f(p)}{1-p} < 0$ .  
 Using a linear approximation for  $f(p)$  near 1, we have

$$\begin{aligned}
 &1 - p' < 1 - p \Leftrightarrow \\
 374 \quad &\frac{f'(1)(p-1) + O((p-1)^2)}{p-1} < 0 \Leftrightarrow \quad (B2) \\
 &f'(1) - O(1-p) < 0.
 \end{aligned}$$

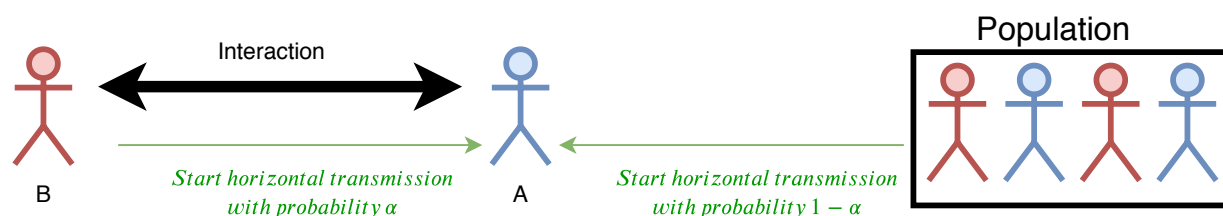
Therefore, if  $f'(1) < 0$  then there exists  $\epsilon > 0$  such that for any  $1 - \epsilon < 1 - p < 1$  it is guaranteed that  
 376  $1 - p' < 1 - p$ , that is,  $p'$  is closer than  $p$  to one.

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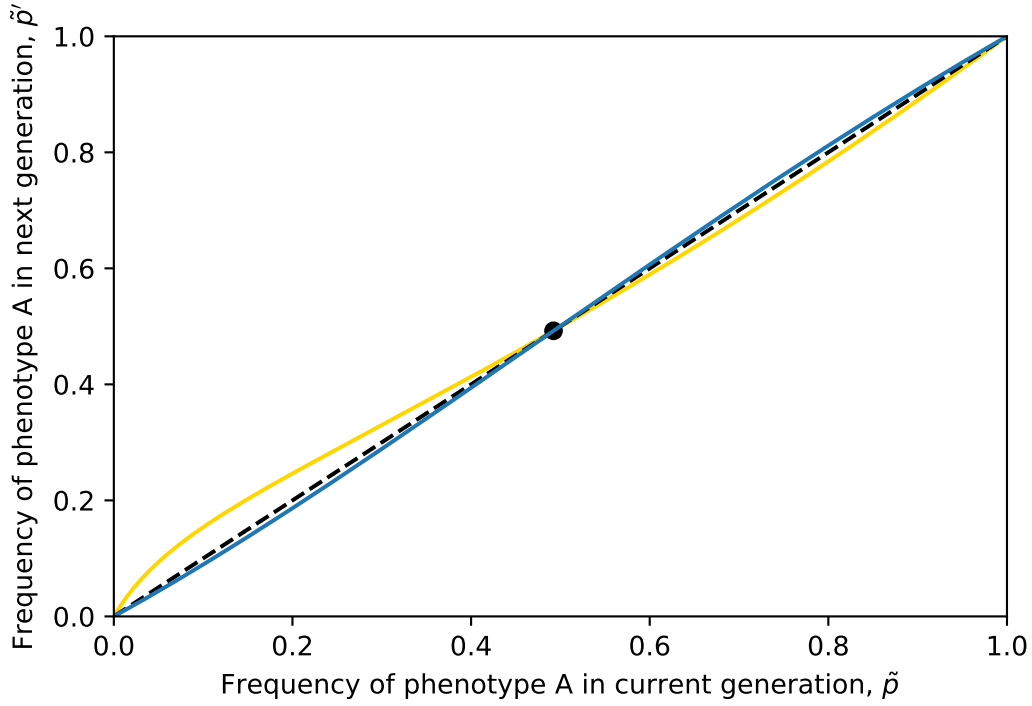
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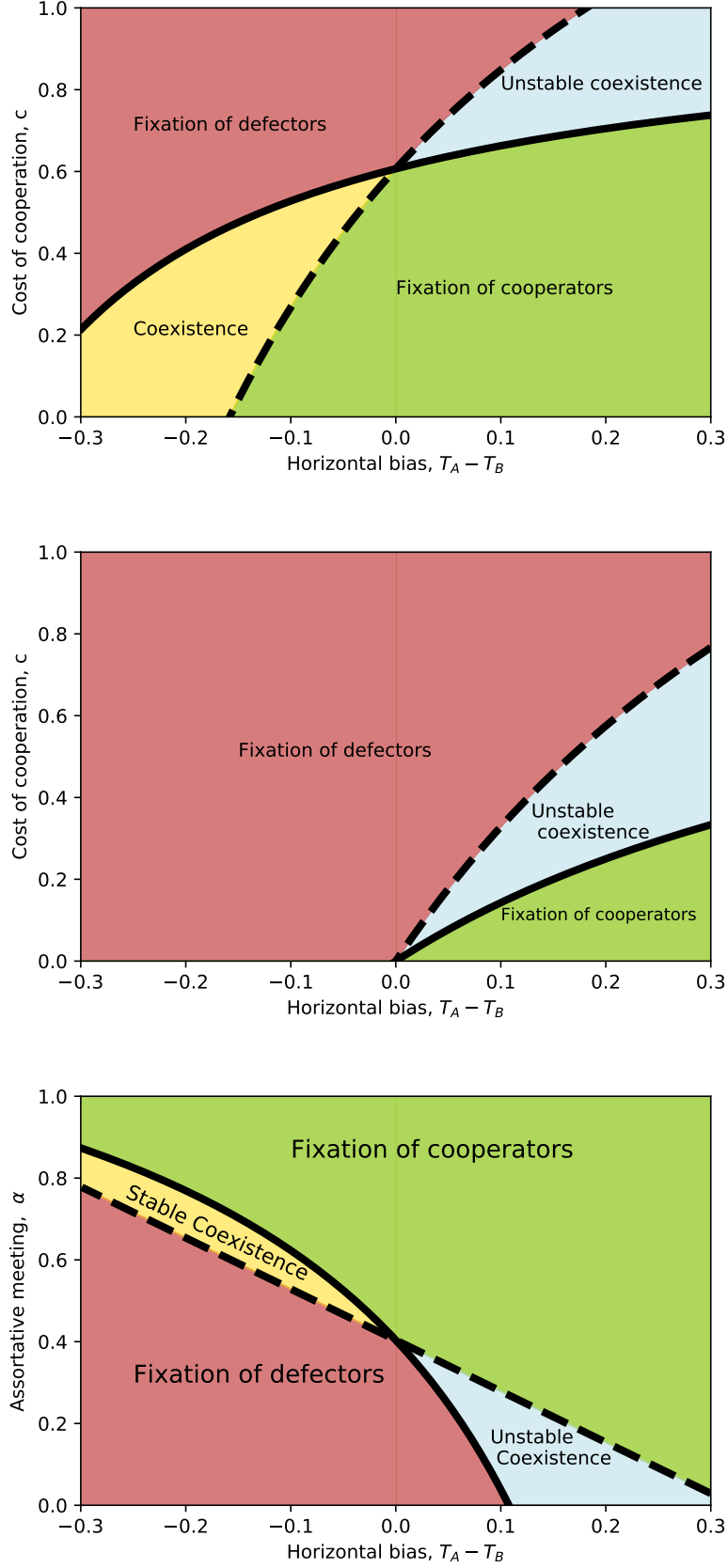
## Figures



**Figure 1: Cultural horizontal transmission with assortment.** Transmission occurs between interacting partners with probability  $\alpha$  (left) or between two random peers with probability  $1 - \alpha$ .

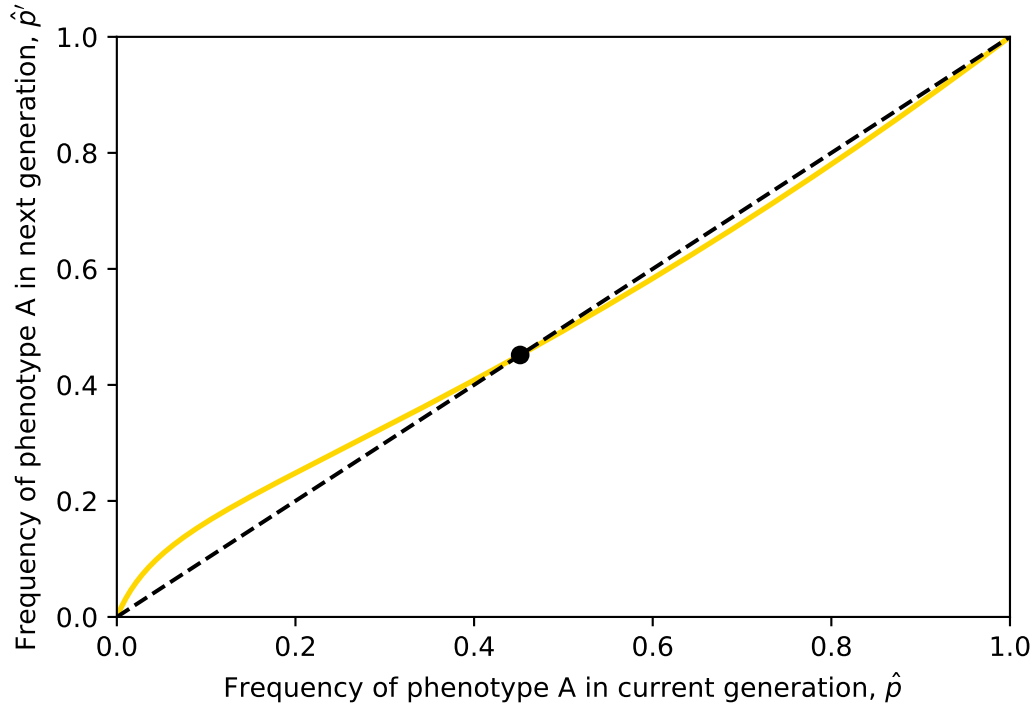


**Figure 2: Stable and unstable coexistence between cooperation and defection without oblique transmission.** The curves show the frequency of the cooperative phenotype  $A$  among parents in the next generation  $\tilde{p}'$  vs. the current generation  $\tilde{p}$  (Eq. 14). The dashed black line is  $\tilde{p}' = \tilde{p}$ . The curves and the dashed line intersect at the polymorphic equilibrium  $\tilde{p}^*$  (black circle). When the curves are above the dashed line,  $\tilde{p}' > \tilde{p}$ , then  $\tilde{p}$  increase. When the curves are below the dashed line,  $\tilde{p}' < \tilde{p}$ , then  $\tilde{p}$  decreases. The yellow curve, for which the polymorphic equilibrium is stable, is given by  $T_A = 0.4$ ,  $T_B = 0.9$ ,  $b = 12$ ,  $c = 0.35$ , and  $\alpha = 0.45$ , which give  $\gamma_2 < c < \gamma_1$  (Eq. 23) The blue curve, for which the equilibrium is unstable, is given by  $T_A = 0.5$ ,  $T_B = 0.1$ ,  $b = 1.3$ ,  $c = 0.904$ , and  $\alpha = 0.4$ , which give  $\gamma_1 < c < \gamma_2$ . In both cases there is no oblique transmission,  $\nu = 1$ ; see Figure 4 for  $\nu < 1$ .

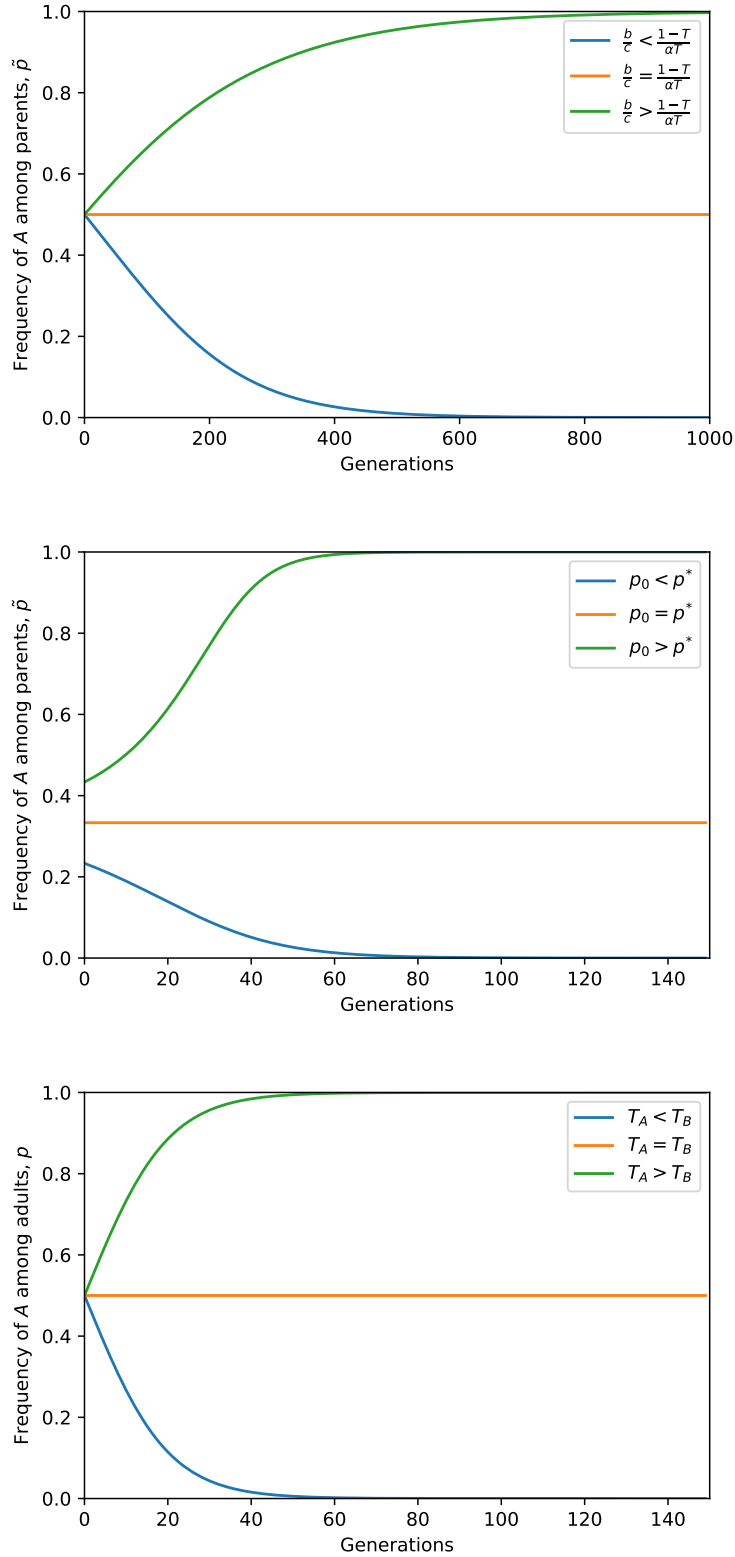


**Figure 3: Evolution of cooperation under vertical and horizontal cultural transmission.** The figure shows the global fixation of cooperation (green), global fixation of defection (red), fixation of either cooperation or defection depending on the initial conditions, i.e. unstable coexistence (blue), and stable coexistence of cooperation and defection (yellow). In all cases the horizontal bias ( $T_A - T_B$ ) is on the x-axis. **(a-b)** the cost of cooperation  $c$  is on the y-axis; the cost boundaries  $\gamma_1$  and  $\gamma_2$  (Eq. 23) are the solid and dashed lines. **(c)** the assortment  $\alpha$  is on the y-axis; the assortment boundaries  $a_1$  and  $a_2$  (Eq. 30) are the solid and dashed lines. Here,  $b = 1.3$ ,  $T_A = 0.4$ . **(a)**  $\alpha = 0.7$ . **(b)**  $\alpha = 0$ . **(c)**  $c = 0.35$ .





**Figure 4: Stable coexistence between cooperation and defection with oblique transmission.** The curve shows the frequency of the cooperative phenotype  $A$  among juveniles in the next generation  $\hat{p}'$  vs. the current generation  $\hat{p}$  (Eq. 6). The dashed black line is  $\hat{p}' = \hat{p}$ . The curve and the dashed line intersect at the stable equilibrium  $\hat{p}^*$  (black circle). When  $\hat{p} < \hat{p}^*$  then the curve is above the dashed line,  $\hat{p}' > \hat{p}$ , and  $\hat{p}$  increases towards  $\hat{p}^*$ . When  $\hat{p} > \hat{p}^*$  then the curve is below the dashed line,  $\hat{p}' < \hat{p}$ , and  $\hat{p}$  decreases towards  $\hat{p}^*$ . Here,  $T_A = 0.4$ ,  $T_B = 0.9$ ,  $b = 20$ ,  $c = 0.1$ ,  $\alpha = 1$ , and  $v = 0.4$ , which give  $0 < \beta_3 < \beta_1$  (Eq. 33).



**Figure 5: Dynamics of the frequency of cooperation.** The frequency  $\tilde{p}$  of parents with cooperative phenotype A in (a-b) and the frequency  $p$  of adults with cooperative phenotype A in (c). The different lines correspond to parameter values that lead to fixation of cooperation (green), extinction of cooperation (red), or stable coexistence of cooperators and defectors (yellow). (a)  $v = 1$ ,  $T_A = T_B = T$ ,  $\alpha \neq 0$ ; (b)  $v = 1$ ,  $\alpha = 0$ ; (c)  $v = 0$ .