TITLE

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Abstract

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Introduction

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Models and Methods

Model. We model SARS-CoV-2 infection dynamics by following the number of susceptible S, exposed E, reported infected I_r, and unreported infected I_u individuals in a population of size N. This model distinguishes between reported and unreported infected individuals: the reported infected are those that have enough symptoms to eventually be tested and thus appear in daily case reports, to which we fit the model.

Susceptible (S) individuals become exposed due to contact with reported or unreported infected individuals (I_r or I_u) at a rate β_t or $\mu\beta_t$. The parameter $0 < \mu < 1$ represents the decreased transmission rate from unreported infected individuals, who are often subclinical or even asymptomatic. The transmission rate β_t may change over time t due to behavioral changes of both susceptible and infected individuals. Exposed individuals, after an average incubation period of Z days, become reported infected with probability α_t or unreported infected with probability $(1-\alpha_t)$. The reporting rate α_t may also change over time due to changes in human behavior. Infected individuals remain infectious for an average period of D days, after which they either recover, or becomes ill enough to be quarantined. They therefore no longer infect other individuals, and therefore the model does not track their frequency. The model is described by the following equations:

$$
\begin{aligned}
\frac{dS}{dt} &= -\beta_t S \frac{I_p}{N} - \mu\beta_t S \frac{I_s}{N} \\
\frac{dE}{dt} &= \beta_t S \frac{I_p}{N} + \mu\beta_t S \frac{I_s}{N} - \frac{E}{Z} \\
\frac{dI_r}{dt} &= \alpha_t \frac{E}{Z} - \frac{I_r}{D} \\
\frac{dI_u}{dt} &= (1-\alpha_t)\frac{E}{Z} - \frac{I_r}{D}.
\end{aligned}
\tag{1}
$$

This model is inspired by Li et al.[1] and Pei and Shaman[2], who used a similar model with multiple regions and constant transmission β and reporting rate α to infer COVID-19 dynamics in China and the continental US, respectively.

Likelihood function. In this model, $Y_t = \alpha_t E(t)/Z$ is the *expected* number of new reported infected individuals on day t. We also define $\tilde{Y}_t$ to be the expected cumulative number of reported infected individuals up to day t,

$$
\tilde{Y}_t = \sum_{i=1}^{t} Y_i
$$

We denote by X_t the number of reported cases in day t and by

$$
\tilde{X}_t = \sum_{i=1}^{t} X_i,
$$

the cumulative number of reported cases until day t (with $X_0 = 0$). We assume that reported infected individuals yet to be reported, i.e. individuals in $\tilde{Y}_t$, are reported in the daily case report of day t with probability p_t, which may change over time (note that t is a specific date, and not the elapsed time since infection). Therefore, the number of reported cases in day t is

$$
X_t \sim Bin\big(n_t, p_t\big),
$$

where n_t the *realized* number of reported infected individuals yet to appear in daily reports by day t. Given $\tilde{X}_{t-1}$, we assume n_t is Poisson distributed

$$
\big(n_t \mid \tilde{X}_{t-1}\big) \sim Poi\Big(\tilde{Y}_t - \tilde{X}_{t-1}\Big), \quad n_1 \sim Poi(Y_1).
$$

Therefore, $\left(X_t \mid \tilde{X}_{t-1}\right)$ is a binomial conditioned on a Poisson, which reduces to a Poisson with

$$\left(X_t \mid \tilde{X}_{t-1}\right) \sim Poi\Big(\big(\tilde{Y}_t - \tilde{X}_{t-1}\big)p_t\Big), \quad X_1 \sim Poi(Y_1 p_1).$$

Therefore, for given model parameters $\theta = (Z, D, \mu, \{\beta_t\}, \{\alpha_t\}, \{p_t\}, S(0), E(0), I_r(0), I_u(0))$, which also include the initial conditions, it is possible to compute $\{E(t)\}_{t=1}^{T}$ and $\{Y_t\}_{t=1}^{T}$. Then, since $\tilde{X}_{t-1}$ is a function of $X_1, \ldots, X_{t-1}$, we can write the probability of a observed daily case reports $\mathbf{X} = (X_1, \ldots, X_T)$ as

$$\mathbb{L}(\theta \mid \mathbf{X}) = P(\mathbf{X} \mid \theta) = P(X_1 \mid \theta)P(X_2 \mid X_1, \theta) \ldots P(X_T \mid X_1, \ldots X_{T-1}, \theta), \tag{2}$$

which is a *likelihood function*.

Results

Discussion

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Acknowledgements

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