

The role of aneuploidy in the evolution of cancer drug resistance

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Abstract

12 Introduction

Aneuploidy in cancer. Chromosomal instability (CIN) is the mitotic process in which cells suffer from chromosome mis-segregation that leads to aneuploidy, where cells are characterized by structural changes of the chromosomes and copy number alterations (Schukken and Foijer, 2018). Interestingly, aberrations in chromosome copy number have been shown to allow cancer cells to survive under stressful conditions such as drug therapy. Indeed, cancer cells are often likely to be aneuploid, and aneuploidy is associated with poor patient outcomes (Ben-David and Amon, 2020).

The role of chromosomal instability (CIN) in the emergence of cancer has been studied extensively in the past decades (Christine et al., 2018, Komarova et al., 2003, Michor et al., 2005, Nowak et al., 2002, Pavelka et al., 2010, Zhu et al., 2018). One hypothesis is that CIN facilitates tumor genesis by accelerating the removal of tumor suppression genes (TSG) and subsequent appearance of cancer. The deletion of tumor suppression genes can happen in two ways: two point mutations deleting both alleles of the TSG (assuming a diploid genotype), or one point mutation and one chromosomal loss event. Initial theoretical studies have shown that aneuploidy can have a significant role in the deletion of the the tumor suppressing genes when compared to two consecutive point mutations (Komarova et al., 2008, 2003, Michor et al., 2005, Nowak et al., 2002). However, when taking into account that the appearance of aneuploidy requires a mutation to trigger CIN, the probability that CIN precedes tumor genesis is highly unlikely.

Evolutionary rescue. Populations adapted to a certain environment are vulnerable to environmental changes, which might cause extinction of the population. Examples of such environmental changes include climate change, invasive species or the onset of drug therapies. Adaptation is a race against time as the population size decreases in the new environment (Tanaka and Wahl, 2022). *Evolutionary rescue* is the process where the population acquires a trait that increases fitness in the new environment such that extinction is averted. It is mathematically equivalent to the problem of crossing of fitness valley (Weissman et al., 2009, 2010). There are three potential ways for a population to survive environmental change: migration to a new habitat similar to the one before the onset of environmental change (Cobbold and Stana, 2020); adaptation by phenotypic plasticity without genetic modification (Carja and Plotkin, 2017, 2019, Levien et al., 2021); and adaptation through genetic modifications, e.g., mutation (Uecker and Hermisson, 2011, 2016, Uecker et al., 2014).

Models of evolutionary rescue usually assume that the fitness of the wildtype and mutant are homogeneous in time. An exception was given by Marrec and Bitbol (2020), who modeled the fitness of the wildtype and mutant as time dependent. Additionally, Uecker and Hermisson (2011) investigated the probability of fixation of a beneficial mutation in a variable environment with arbitrary time-dependent selection coefficient and population size. Most models focus on the probability that at least one mutation rescues the population. How multiple mutations contribute to the survival of the population is less explored, but Wilson et al. (2017) have shown that evolutionary rescue is significantly enhanced by soft selective sweeps when multiple mutations contribute. Evolutionary rescue that requires two successive mutations has been investigated using diffusion approximation by Martin et al. (2013).

Methods

52 Evolutionary model

We follow the number of cancer cells that have one of three different genotypes at time t : wildtype, w_t ; aneuploid, a_t ; and mutant, m_t . These cells divide and die with rates λ_k and μ_k (for $k = w, a, m$). The difference between the division and death rate is $\Delta_k = \lambda_k - \mu_k$. We assume the population of cells is under a strong stress, such as drug therapy, to which the wildtype genotype is susceptible and

therefore $\Delta_w < 0$, whereas the mutant is resistant to the stress, $\Delta_m > 0$. We analyze three scenarios:
 58 in the first, aneuploid cells are partially resistant, $\Delta_m > \Delta_a > 0$; in the second, aneuploid cells are
 tolerant, $0 > \Delta_a > \Delta_w$ (see Brauner et al., 2016, for the distinction between susceptible, resistant,
 60 and tolerant); in the third, aneuploid cells are non-growing or "barely growing", that is, either slightly
 tolerant or slightly resistant, such that $\Delta_a \approx 0$. Wildtype cells may missegregate to become aneuploids
 62 at rate u . Both aneuploid and wildtype cells may mutate to become mutants at rate v , which we assume
 is lower than the division rates, $v < \min(\lambda_w, \lambda_a, \lambda_m)$. See Figure 1 for an illustration of the model.

64 Stochastic simulations

Simulations are performed using a *Gillespie algorithm* (Gillespie, 1976, 1977) implemented in Python
 66 (Van Rossum and Others, 2007). The simulation monitors the number of cells of each type: wildtype,
 aneuploid, and mutant. The wildtype population initially consists of w_0 cells, whereas the other cell
 68 types are initially absent.

The state of the stochastic system at time t is represented by the triplet (w_t, a_t, m_t) . The following
 70 describes the events that may occur (right column), the rates at which they occur (middle column),
 and the effect these events have on the state (Figure 1):

72	$(+1, 0, 0) :$	$\lambda_w w_t$	(birth of wildtype cell) ,
	$(-1, 0, 0) :$	$\mu_w w_t$	(death of wildtype cell) ,
74	$(-1, +1, 0) :$	$u w_t$	(wildtype cell becomes aneuploid) ,
	$(-1, 0, +1) :$	$v w_t$	(wildtype cell becomes mutant) ,
76	$(0, +1, 0) :$	$\lambda_a a_t$	(birth of aneuploid cell) ,
	$(0, -1, 0) :$	$\mu_a a_t$	(death of aneuploid cell) ,
78	$(0, -1, +1) :$	$v a_t$	(aneuploid cell becomes mutant) ,
	$(0, 0, +1) :$	$\lambda_m m_t$	(birth of mutant cell) ,
80	$(0, 0, -1) :$	$\mu_m m_t$	(death of mutant cell) .

Each iteration of the simulation loop starts with computing the rates v_j of each event j . We then
 82 draw the time until the next event, Δt , from an exponential distribution whose rate parameter is the
 sum of the rates of all events, such that $\Delta t \sim \text{Exp}(\sum_j v_j)$. Then, we randomly determine which event
 84 occurred, where the probability for event j is $p_j = v_j / \sum_i v_i$. Finally, we update the number of cells of
 each type according to the event that occurred and update the time from t to $t + \Delta t$. We repeat these
 86 iterations until either the population becomes extinct (the number of cells of all types is zero) or the
 number of mutant cells is high enough so that its extinction probability is $< 0.1\%$, that is until

$$88 \quad m_t > \left\lceil -\frac{3 \log 10}{\log \left(\frac{\mu_m}{\lambda_m} \right)} \right\rceil + 1,$$

τ -leaping. When simulations are slow (e.g. due to large population size), we utilize τ -leaping
 90 (Gillespie, 2001), where change in number of cells of genotype i in a fixed time interval Δt is
 Poisson distributed with mean $v_i \Delta t$. If the change in number of cells is negative and larger than the
 92 subpopulation size then the subpopulation size is updated to be zero.

Density-dependent growth. In our analysis we assume that lineages produced by cells from the
 94 initial population divide and die independently of each other, which may be unrealistic, as cells
 usually compete for resources. A more realistic model includes competition for limited resources and
 96 spatial structure, which may play an important role in the development of cancer (e.g., Martens et al.,

2011). To simulate birth and death rates that depend on the number of cells in the population, we
 98 transform the rates of division and death to the following:

$$\begin{aligned}
 &\lambda'_w = \lambda_w, \\
 &\mu'_w = \mu_w, \\
 &\lambda'_a = C_1 + (\lambda_a - \mu_a) \left(1 - \frac{w + a + m}{K}\right), \\
 &\mu'_a = C_1, \\
 &\lambda'_m = C_2 + (\lambda_m - \mu_m) \left(1 - \frac{w + a + m}{K}\right), \\
 &\mu'_m = C_2,
 \end{aligned}$$

where $C_1, C_2 > 0$ are constants and K is the maximum carrying capacity.

106 Code and data availability.

All source code is available online at <https://github.com/yoavram-lab/EvolutionaryRescue>.

108 Results

Evolutionary rescue probability

110 In our model, *evolutionary rescue* occurs when resistant cells appear and fixate ($m_t \gg 1$) in the
 population before the population becomes extinct ($w_t = a_t = m_t = 0$). Aneuploidy may contribute
 112 to evolutionary rescue by either preventing (when $\Delta_a > 0$) or delaying (when $0 > \Delta_a > \Delta_w$) the
 extinction of the population before mutant cells appear and fixate.

114 To estimate the rescue probability p_{rescue} , we assume independence between clonal lineages starting
 from an initial population of N wildtype cells (we check the effect of density-dependent growth on
 116 our results below). Thus, the rescue probability is given by

$$p_{\text{rescue}} = 1 - (1 - p_w)^N \quad (2a)$$

$$\approx 1 - e^{-Np_w}, \quad (2b)$$

where p_w is the probability that the lineage of a single wild-type cell avoids extinction, and the
 120 approximation $(1 - p_w) \approx e^{-p_w}$ assumes that p_w (but not necessarily Np_w) is small.

The probability for evolutionary rescue p_{rescue} (eq. (2b)) is $p_{\text{rescue}} \approx Np_w$ when $N < 1/p_w$ and
 122 $p_{\text{rescue}} \approx 1$ when $N > 1/p_w$. Therefore, $N^* = 1/p_w$ is the threshold tumor size for rescue above which
 rescue is very likely (Figure 4). Therefore, we can write

$$124 \quad p_{\text{rescue}} = 1 - e^{-N/N^*}. \quad (3)$$

In the Appendix below, we use the theory of multi-type branching processes to find approximate
 126 expressions eqs. (21), (28) and (31) for p_w in different regimes. Substituting these into $N^* = 1/p_w$,
 we find approximations for the threshold tumor size, N^* .

128 In the following, an important quantity is $T^* = (4v\lambda_a\Delta_m/\lambda_m)^{-1/2}$, which is the critical time an
 130 aneuploid lineage needs to survive to produce a resistant mutant that avoids random extinction. First,
 if aneuploidy is very rare ($uT^* < 1$), or if aneuploidy is rare ($u < -\Delta_a$) and very sensitive to the drug
 132 ($\Delta_a T^* < -1$), then rescue will likely occur by a direct resistance mutation in a sensitive cell, such that

$$N_m^* \approx \frac{|\Delta_w|}{v} \cdot \frac{\lambda_m}{\Delta_m}. \quad (4)$$

134 Otherwise, aneuploidy is frequent enough ($u > \max(-\Delta_a, 1/T^*)$) to affect the evolution of the tumor.
 135 The threshold tumor size, N^* , can then be approximated by one of the following cases, depending on
 136 $\Delta_a T^*$, the change in the log of the aneuploid population size during the critical time,

$$N_a^* \approx \frac{|\Delta_w|}{u} \cdot \begin{cases} \frac{|\Delta_a|}{v} \cdot \frac{\lambda_m}{\Delta_m}, & \Delta_a T^* \ll -1, \\ 2\lambda_a T^*, & -1 \ll \Delta_a T^* \ll 1, \\ \frac{\lambda_a}{\Delta_a}, & \Delta_a T^* \gg 1. \end{cases} \quad (5)$$

138 Comparing these approximations to results of stochastic evolutionary simulations, we find that the
 approximations perform very well (Figures 2 to 4).

140 **Density-dependent growth.** In our analysis we used branching processes, which assume that growth
 (division and death) are density-independent. However, growth may be limited by resources (oxygen,
 142 nutrients, etc.) and therefore depend on cell density. We therefore performed stochastic simulations
 of a logistic growth model with carrying capacity K (see Methods). We find that our approximations
 144 agree with results of simulations with density-dependent growth for biologically relevant parameter
 values (Figure 4).

146 **Standing vs. de-novo genetic variation** In the above we assumed that upon beginning of drug
 therapy, the initial tumor consisted entirely of wildtype cells. However, aneuploid cells are likely
 148 generated even before onset of treatment at some rate $\tilde{u} \leq u$ (because the treatment itself may promote
 generation of aneuploid cells REF), which are likely to have a deleterious effect (REF). But if the
 150 number of cells in the tumor N is large, as expected if drug treatment is applied, there may already be
 a fraction $f = \tilde{u}/s$ of aneuploid cells in the population, where s is the cost of aneuploidy (REF).

152 In this scenario, the probability of evolutionary rescue by cells with aneuploidy from the initial
 population is

$$154 \quad p_{sgv} = 1 - (1 - p_a)^{fN} \approx 1 - e^{-fNp_a}.$$

The total probability of evolutionary rescue is given by

$$156 \quad \begin{aligned} p_{rescue} &= p_{sgv} + (1 - p_{sgv}) p_{de-novo} \\ &= 1 - \exp\left(-[(1 - f)p_w + fp_a]N\right). \end{aligned} \quad (6)$$

158 The fraction of cases in which the population is rescued by pre-existing aneuploid cells (i.e.,
 standing genetic variation) is given by $F(f) = \frac{p_{sgv}}{p_{total}}$ (Figure 5).

160 Effect of aneuploidy on the probability of evolutionary rescue

To determine the extent to which aneuploidy may affect evolutionary rescue, we define H to be the
 162 ratio of the rescue probability with and without aneuploidy ($u > 0$ and $u = 0$, respectively),

$$H = \frac{p_{rescue}(u > 0)}{p_{rescue}(u = 0)}. \quad (7)$$

164 Plugging in our approximations from eq. (2a), we have

$$H = \begin{cases} \frac{1 - \exp\left[\frac{N}{\Delta_w - u - v} \left(v \frac{\Delta_m}{\lambda_m} + \frac{u(\Delta_a - v)}{2\lambda_a} + u \sqrt{\frac{v\Delta_m}{\lambda_a \lambda_m}} \right)\right]}{1 - \exp\left[\frac{vN\Delta_m}{(\Delta_w - v)\lambda_m}\right]}, & 4\lambda_a v p_m > (\Delta_a - v)^2, \\ \frac{1 - \exp\left[\frac{v\Delta_m N}{\lambda_m \Delta_w} \left(1 - \frac{u}{\Delta_a}\right)\right]}{1 - \exp\left(\frac{v\Delta_m N}{\lambda_m \Delta_w}\right)}, & \Delta_a < 0 \quad \text{and} \quad 4\lambda_a v p_m < (\Delta_a - v)^2, \\ \frac{1 - \exp\left[\frac{N}{\Delta_w} \left(\frac{u\Delta_a}{\lambda_a} + \frac{uv\Delta_m}{\lambda_m \Delta_a} + \frac{v\Delta_m}{\lambda_m}\right)\right]}{1 - \exp\left[\frac{v\Delta_m N}{\lambda_m \Delta_w}\right]}, & \Delta_a > 0 \quad \text{and} \quad 4\lambda_a v p_m < (\Delta_a - v)^2. \end{cases} \quad (8)$$

166 We find that the rescue ratio increase with the aneuploidy growth rate Δ_a , because the better
 167 aneuploid cells are in growth, the better they are at rescuing the population (when they provide partial
 168 resistance) or delaying the extinction of the population (when they provide tolerance). However, the
 169 rescue decreases with the wildtype growth rate Δ_w , because the better the wildtype is at growth, the
 170 less is depends on aneuploidy for rescue or delay, and the more likely it is to directly produce mutant
 171 cells, rather than relying on aneuploid cells for producing mutant cells (Figure 6). The effect of the
 172 initial tumor size N is the similar to that of the wildtype growth rate. Importantly, in large tumors, the
 ratio converges to unity, that is, aneuploidy does not affect the probability for evolutionary rescue.

174 Evolutionary rescue time

Even when evolutionary rescue occurs, it may take a long time. We therefore wish to estimate the
 176 mean waiting time for rescue and the effect aneuploidy may have on it. We calculate the mean time for
 the appearance of the first mutant that rescues the cell population. This can occur either through the
 178 evolutionary trajectory *wildtype* \rightarrow *mutant* or through the trajectory *wildtype* \rightarrow *aneuploid* \rightarrow
mutant. We start with the former.

180 Assuming no aneuploidy ($u = 0$), we define T_1 to be the time at which the first mutant cell appears
 that will avoid extinction and will therefore rescue the population. Note that if extinction occurs, that
 182 is the frequency of mutants after a very long time is zero, $m_\infty = 0$, then it is implied that $T_1 = \infty$, and
 vice versa if $T_1 < \infty$ then $m_\infty > 0$.

184 The number of successful mutants generated until time t can be approximated by an inhomogeneous
 Poisson process with rate $R(t) = u p_a w_t$, where $w_t = N e^{\Delta_w t}$ is the number of wildtype cells at time t .
 186 Note that

$$\int_0^t R(z) dz = u p_a N \frac{\exp[\Delta_w t] - 1}{\Delta_w} \approx u p_a N t, \quad (9)$$

188 by integrating the exponential and because $\frac{\exp[\Delta_w t] - 1}{\Delta_w} = \frac{1 + \Delta_w t + O(t^2) - 1}{\Delta_w} = t + O(t^2)$. The probability
 density function of T_1 is thus $R(t) \exp\left(-\int_0^t R(z) dz\right)$. Therefore, the probability density function of
 190 the conditional random variable $(T_1 \mid T_1 < \infty)$ is $f_1(t) = \frac{R(t) \exp\left(-\int_0^t R(z) dz\right)}{P_{\text{rescue}}}$.

192 We are interested in the mean conditional time, $\tau_1 = \mathbb{E}[T_1 \mid T_1 < \infty]$, which is given by

$$\tau_1 = \int_0^\infty t f_1(t) dt = \frac{\int_0^\infty t R(t) \exp\left(-\int_0^t R(z) dz\right) dt}{P_{\text{rescue}}} = \frac{\int_0^\infty \exp\left(-\int_0^t R(z) dz\right) dt}{P_{\text{rescue}}} \quad (10)$$

194 after applying integration by parts. Therefore, plugging eqs. (2b) and (9) in eq. (10),

$$\tau_1 = \frac{\int_0^\infty e^{-u N p_a \frac{e^{\Delta_w t} - 1}{\Delta_w}} dt}{1 - (1 - p_w)^N} \approx \frac{\int_0^\infty \exp(-u p_a N t) dt}{1 - e^{-N p_w}} \approx \quad (11)$$

$$(1 + e^{-Np_w}) \int_0^\infty e^{-up_a Nt} dt = \frac{1 + e^{-Np_w}}{up_a N}, \quad (12)$$

where we use the approximations $\frac{e^{\Delta_w \tau} - 1}{\Delta_w} = \frac{1 + \Delta_w \tau + O(\tau^2) - 1}{\Delta_w} = \tau + O(\tau^2)$ and $(1 - e^{-Np_w})^{-1} \approx 1 + e^{-Np_w}$ and integrate the exponent. Figure 7B show the agreement between this approximating and simulation results for intermediate and large tumor sizes.

When $Nu \gg 1$ the aneuploid frequency dynamics is roughly deterministic and therefore can be approximated by

$$a_t \approx \frac{Nue^{\Delta_w t}}{\Delta_w - \Delta_a} \left[1 - e^{(\Delta_w - \Delta_a)t} \right]. \quad (13)$$

As a result, when $N \gg 1$ the number of successful mutants created by direct mutation and via aneuploidy can be approximated by inhomogeneous Poisson processes with the rates

$$r_1(t) = vp_m \int_0^t a_z dz = \frac{uvNp_m}{\Delta_w - \Delta_a} \left(\frac{e^{\Delta_w t} - 1}{\Delta_w} - \frac{e^{\Delta_a t} - 1}{\Delta_a} \right), \quad (14)$$

$$r_2(t) = vp_m \int_0^t w_z dz = vNp_m \frac{e^{\Delta_w t} - 1}{\Delta_w}. \quad (15)$$

For large initial population sizes we assume that the two processes are independent and as a result, they can be merged into a single Poisson process with rate $(r_1 + r_2)(t)$. Consequently, the mean time to the appearance of the first rescue mutant is

$$\tau_2 = \frac{\int_0^\infty e^{-(r_1(t) + r_2(t))} dt}{1 - (1 - p_w)^N} = \frac{\int_0^\infty \exp \left[-\frac{uvNp_m}{\Delta_w - \Delta_a} \left(\frac{e^{\Delta_w t} - 1}{\Delta_w} - \frac{e^{\Delta_a t} - 1}{\Delta_a} \right) - vNp_m \frac{e^{\Delta_w t} - 1}{\Delta_w} \right] dt}{1 - (1 - p_w)^N}, \quad (16)$$

which we plot in Figure 7A as a function of the initial population size, N .

We wish to obtain a simpler formula for τ_2 , similar to eq. (11). We thus have the following expansions,

$$\begin{aligned} \frac{e^{\Delta_w t} - 1}{\Delta_w} &= \frac{1 + \Delta_w t + O(t^2) - 1}{\Delta_w} = t + O(t^2), \\ \frac{e^{\Delta_a t} - 1}{\Delta_a} &= \frac{1 + \Delta_a t + O(t^2) - 1}{\Delta_a} = t + O(t^2), \end{aligned}$$

which we use to derive a first-order approximation for τ_2 ,

$$\tau_2 \approx (1 + e^{-Np_w}) \int_0^\infty e^{-uNp_m t} dt = \frac{(1 + e^{-Np_w})}{uNp_m}, \quad (17)$$

Figure 7A shows that the approximation eq. (17) has a good fit with simulation results for large initial wildtype population size ($N > 2 \cdot 10^7$). An approximation that uses the second-order terms, $\frac{\Delta_w^2 t^2}{2}$ and $\frac{\Delta_a^2 t^2}{2}$, does not perform better (Figure 7A).

Importantly, in Figure 7 we observe that in initially large tumors, direct mutation drives evolutionary rescue while aneuploidy plays a role for intermediate sized tumors. This is consistent with Figure 6B,D, where aneuploidy only increases the probability of evolutionary rescue in small and intermediate tumors ($N < 10^7$).

Effect of aneuploidy on mean evolutionary rescue time

Next, we wish to find the ratio of the mean rescue time with and without aneuploidy ($u > 0$ and $u = 0$, respectively),

$$\frac{\tau_2}{\tau_1} = H^{-1} \cdot \frac{\int_0^\infty \exp \left[-\frac{uvNp_m}{\Delta_w - \Delta_a} \left(\frac{e^{\Delta_w t} - 1}{\Delta_w} - \frac{e^{\Delta_a t} - 1}{\Delta_a} \right) - vNp_m \frac{e^{\Delta_w t} - 1}{\Delta_w} \right] dt}{\int_0^\infty e^{-vNp_m \frac{e^{\Delta_w t} - 1}{\Delta_w}} dt}, \quad (18)$$

where H is the ratio of the rescue probability with and without aneuploidy, see eq. (7). Figure 8 shows how this ratio, τ_2/τ_1 , changes as a function of the initial tumor size, N , for different aneuploid growth rates, Δ_a .

Discussion

We have modeled a tumor—a population of cancer cells—exposed to drug therapy that causes the population to decline in size towards potential extinction. The cancer cell population can be "evolutionary rescued", or escape extinction, by two paths. In the direct path, a sensitive cell acquires a mutation that confers resistance that allows it to rapidly grow. In the indirect path, a sensitive cell first becomes aneuploid, which diminishes the effect of the drug, and then an aneuploid cell acquires a mutation that confers resistance (Figure 1).

Using multitype branching processes, we derived the probability of evolutionary rescue of the population of cancer cells under different scenarios for the effect of aneuploidy, ranging from tolerance to partial resistance. We obtained exact and approximate expressions for the probability of evolutionary rescue (eq. (2a)). Our results show that the probability of evolutionary rescue increases with the initial tumor size N , the sensitive growth rate Δ_w , the mutation rate v , and the aneuploidy rate u .

When aneuploid cells are partially resistant to the drug ($\Delta_w \ll 0 \ll \Delta_a \ll \Delta_m$), evolutionary rescue can be approximated by a one-step process in which aneuploidy itself rescues the population (Figure 2). When aneuploidy only provides tolerance to the drug ($\Delta_w \ll \Delta_a \ll 0 \ll \Delta_m$), it cannot rescue the population. Instead, it acts as a *stepping stone* through which the resistant mutant can appear more rapidly, given that the aneuploid cell population declines slower than the sensitive cell population. In this case, aneuploidy provides two benefits. First, it delays the extinction of the population—providing more time for appearance of the resistance mutation. Second, it increases the population size relative to a sensitive population—providing more cells in which mutations can occur, i.e., it increases the mutation supply, Nv .

We find that aneuploidy can have a significant effect on evolutionary rescue (Figures 6 and 8). For example, when aneuploidy cells are "barely-resistant" (they grow at a very low rate, $\Delta_a = 10^{-3}$) the probability of evolutionary rescue is 1,000-fold higher with aneuploidy than without it (for parameters previously described in cancer, see Table 1). Interestingly, aneuploidy is unlikely to contribute to evolutionary rescue in primary tumors in which the number of cells is large enough ($N > 10^7$) for the appearance of resistant mutation directly in sensitive cells before these cells become extinct (Figure 6). However, aneuploidy can have a crucial role in evolutionary rescue of secondary tumors, in which the number of sensitive cells may be below the detection threshold of $\sim 10^7$ (Bozic et al., 2013). Given the fact that the mean time for such secondary tumors to overcome chemotherapy can be of the order of 100 days (Figure 7), this can explain the reappearance of cancer even after initial remission. Indeed, we find that the tumor size can decrease by orders of magnitude before it is rescued (Figure 9).

We hypothesized that presence of *standing variation*—the existence of a subpopulation of aneuploid cancer cells before therapy begins—can facilitate evolutionary rescue by reducing the waiting time for the appearance of aneuploid cells. Indeed, we observe that even when a small fraction of the initial

tumor is aneuploid, evolutionary rescue is more likely to occur through this existing standing variation, rather than through *de novo* aneuploid cells (Figure 5).

We have assumed that cancer cell lineages are independent of each other. However, this may not be the case, as cancer cells compete for resources (e.g., blood supply). Nevertheless, we find that when the carrying capacity is large our approximation for the probability of evolutionary rescue agrees with results of stochastic simulations with density-dependent growth (Figure 4). Future work may focus on scenarios with small carrying capacity by analyzing density-dependent branching processes.

Our model predictions may be tested by experiments (Martin et al., 2013). For example, to study the effects of initial tumor size on the probability of evolutionary rescue, a large culture mass can be propagated from a single cancer cell in permissive conditions and then diluted to a range of starting tumor sizes. Afterwards, these tumors may be exposed to anti-cancer drugs that induces aneuploidy or to saline solution for control. Cell density can then be measured and compared to the predictions of our model.

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	Name	Value	Units	References
N	Initial tumor size	$10^7 - 10^9$	cells	Del Monte (2009)
λ_w	Wildtype division rate	0.14	1/days	(Bozic et al., 2013)
μ_w	Wildtype death rate	0.17	1/days	Bozic et al. (2013)
λ_a	Aneuploid division rate*	0.14	1/days	-
μ_a	Aneuploid death rate*	0.13 – 0.17	1/days	-
λ_m	Mutant division rate	0.14	1/days	Bozic et al. (2013)
μ_m	Mutant death rate	0.13	1/days	Bozic et al. (2013)
u	Missegregation rate	$10^{-3} - 10^{-2}$	1/cell division	Bakker et al. (2023), Nowak et al. (2004)
v	Mutation rate	$10^{-7} - 10^{-9}$	1/gene/cell division	Nowak et al. (2004)

Table 1: Model parameters. Aneuploid birth rate λ_a is set to the same value as the wildtype and mutant birth rates, λ_w and λ_m . Aneuploid death rate μ_a is set to an intermediate value between the wildtype and mutant death rates, μ_w and μ_m .

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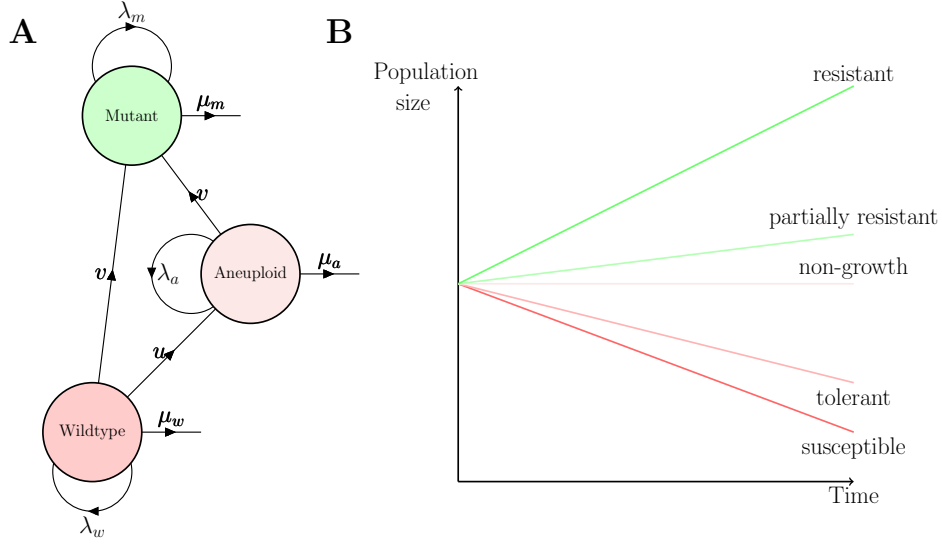


Figure 1: Model illustration. (A) A population of cancer cells is composed of wildtype, aneuploid, and mutant cells, which divide with rates λ_w , λ_a , and λ_m and die at rates μ_w , μ_a , and μ_m , respectively. Wildtype cells can become aneuploid at rate u . Both aneuploid and wildtype cells can acquire a beneficial mutation with rate v . Color denotes the relative growth rates of the three genotypes such that $\lambda_w - \mu_w < \lambda_a - \mu_a < \lambda_m - \mu_m$. (B) The wildtype and the mutant are susceptible and resistant, respectively, to the drug. The aneuploid may be tolerant, non-growing, or or partially resistant.

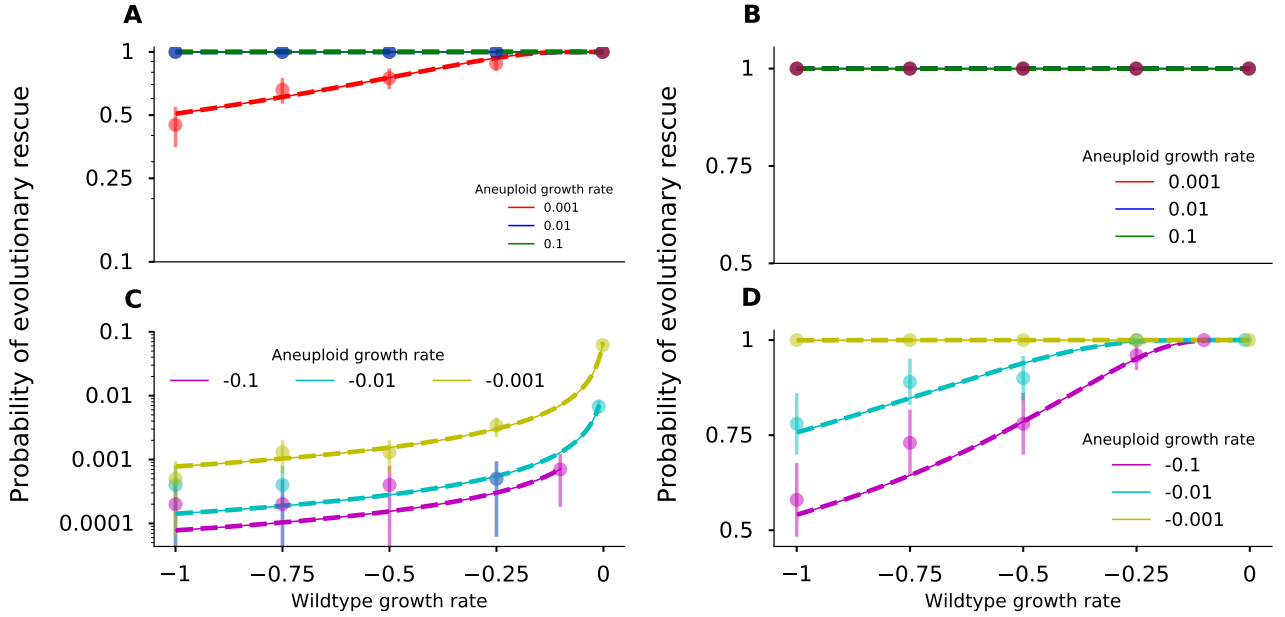


Figure 2: Evolutionary rescue probability with partially resistant or tolerant aneuploid cells. Rescue probability is very high when aneuploidy provides partial resistance ($\lambda_a = 0.01$), in an initially small tumor (A, $N = 10^4$) and even more so in an initially large tumor (B, $N = 10^8$). When aneuploidy provides tolerance (C, $N = 10^4$; D, $N = 10^8$), the rescue probability is much lower. In both scenarios, rescue probability increase with both the wildtype growth rate (x-axis) and the aneuploidy growth rate (colors). Markers represent simulation results with 95% CI; solid and dashed lines for the exact formula (eq. (20) in eq. (2a)); dashed lines for the approximate formula (eq. (5)), demonstrating that they all agree. Parameters: division rate $\lambda_w = \lambda_a = \lambda_m = 0.14$ (so that growth rate changes due to variable death rate); mutant death rate $\mu_m = 0.13$ (so that mutant growth rate $\Delta_m = 0.01$); aneuploidy rate $u = 10^{-2}$; mutation rate $v = 10^{-7}$.

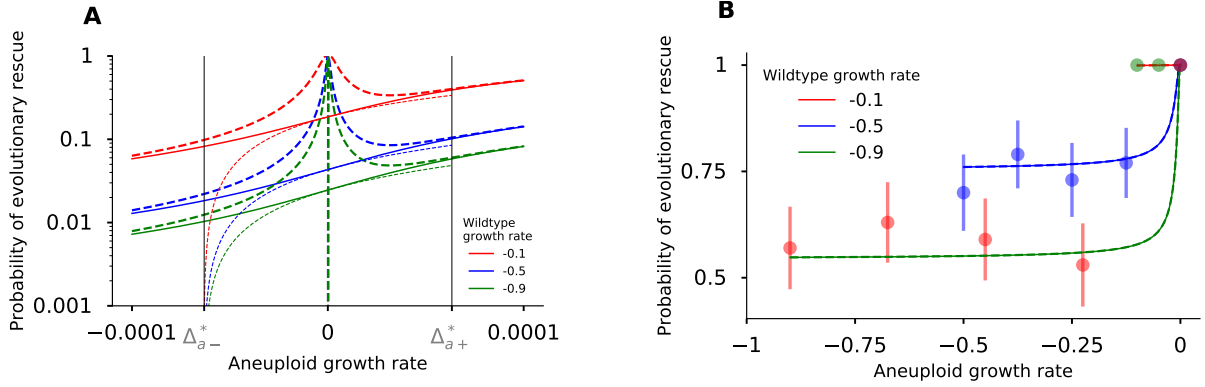


Figure 3: Evolutionary rescue probability with tolerant or non-growing aneuploid cells. Rescue probability grows with the aneuploid growth rate Δ_a (x-axis), and is much higher in an initially large tumor than in a small one (**A**: $N = 10^4$; **B**: $N = 10^8$). Markers for simulation results with 95% CI; solid lines for the exact formula (eq. (20) in eq. (2a)); dashed lines for the approximate formula (eq. (5)). The approximation agrees with the simulation and exact solution when the initial tumor size is large (panel B). When the tumor size is small (panel A), we switch between the approximation for tolerant and for non-growing aneuploid cells; the switch occurs at $1/T^*$. Parameters: $\lambda_w = \lambda_a = \lambda_m = 0.14$; $\mu_m = 0.13$; $u = 10^{-2}$; $v = 10^{-7}$.

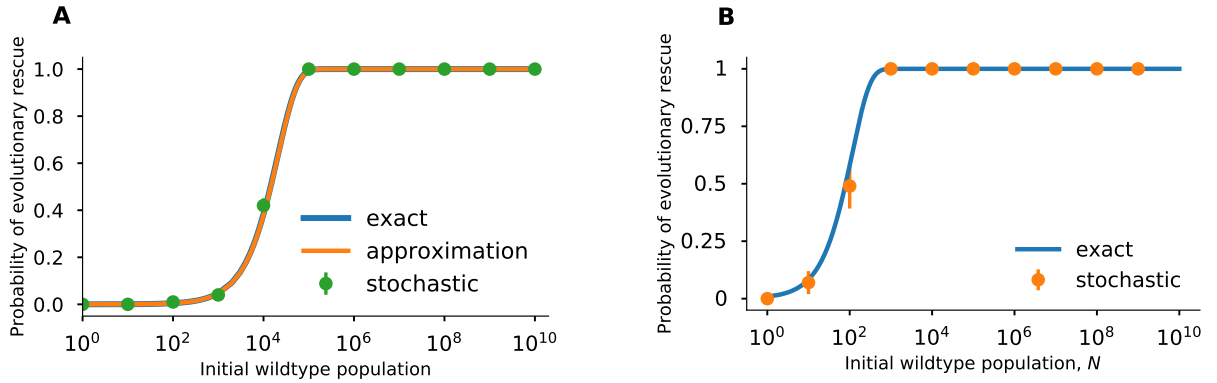


Figure 4: Evolutionary rescue probability for variable initial tumor size. (**A**) Comparison of simulation results (markers with 95% CI, too small to appear with 10^5 simulations per marker), the exact formula (blue line, eq. (20) in eq. (2a)) and the approximate formula (orange line, eq. (5)). (**B**) Comparison of results of simulations with density-dependent growth (markers with 95% CI) and the exact formula (blue line, eq. (20) in eq. (2a)) with maximum carrying capacity $K = 10^9$. Parameters: $\lambda_w = \lambda_a = \lambda_m = 0.14$; $\mu_w = 0.17$; (A) $\mu_a = 0.15$, (B) $\mu_a = 0.135$; $\mu_m = 0.13$; $u = 10^{-2}$; $v = 10^{-7}$.

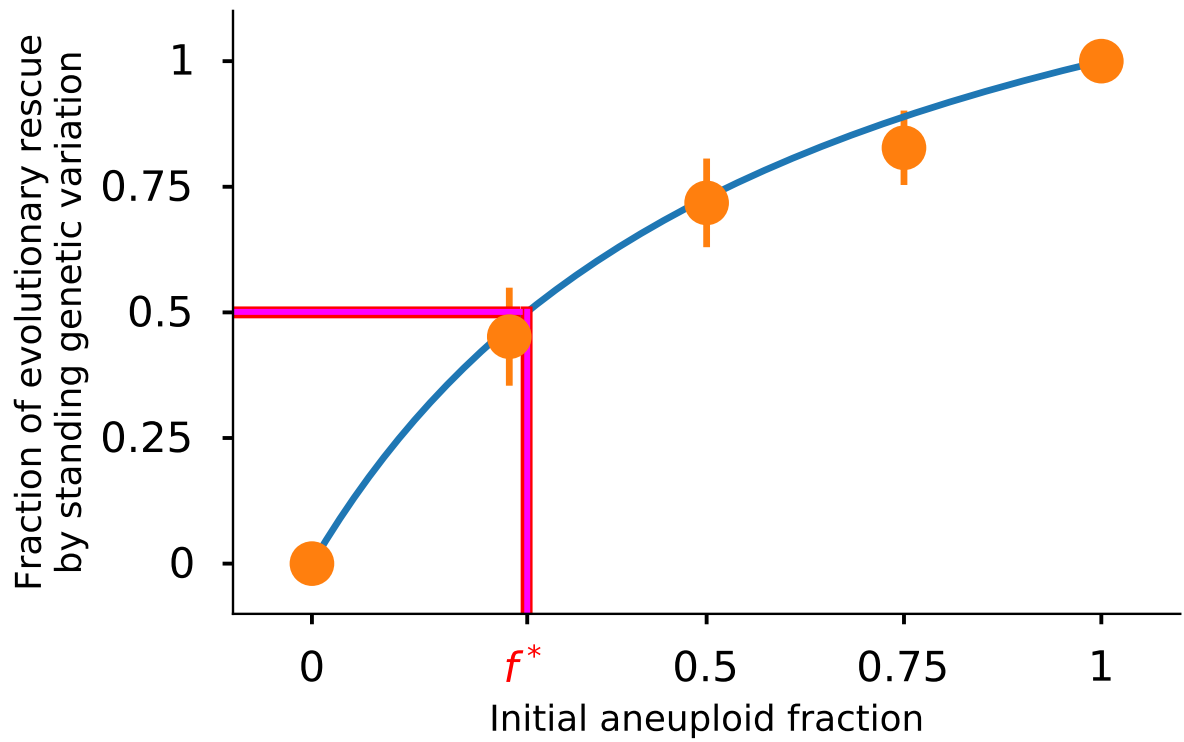


Figure 5: Effect of standing variation on evolutionary rescue. In aneuploid cells already exist in the population at the onset of drug therapy as standing genetic variation, then evolutionary rescue is more likely... Parameters: $\lambda_w = \lambda_a = \lambda_m = 0.14$; $\mu_w = 0.17$; $\mu_a = 0.145$; $\mu_m = 0.13$; $u = 10^{-2}$; $v = 10^{-7}$.

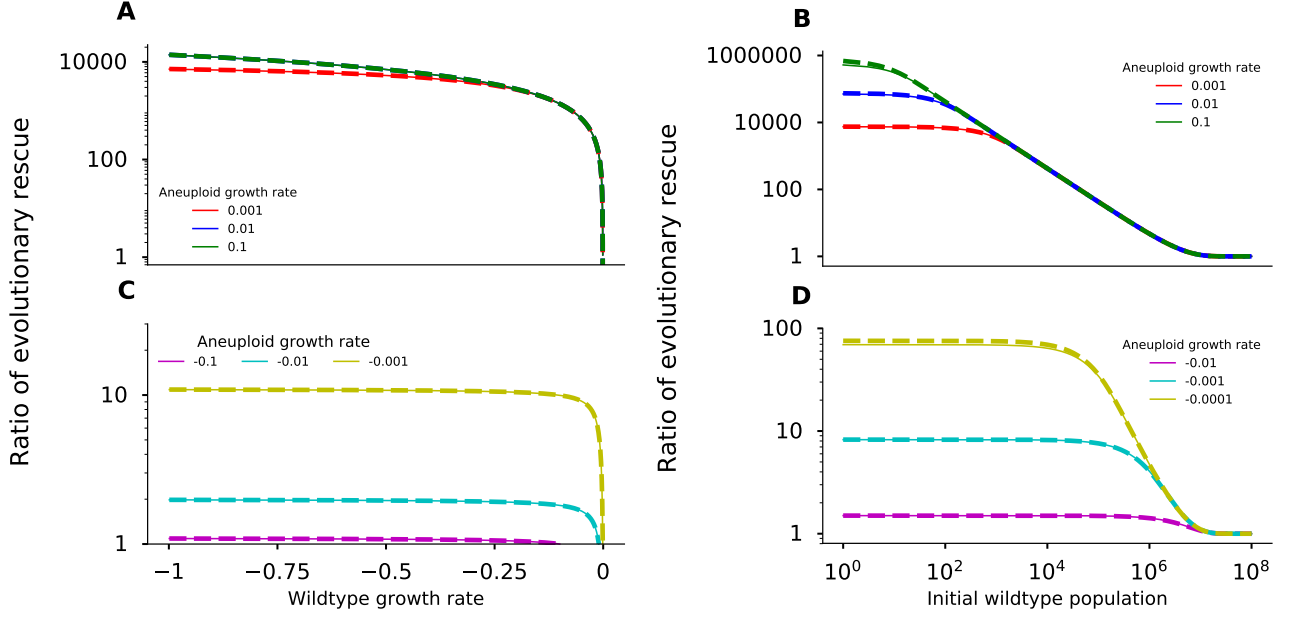


Figure 6: Effect of aneuploidy on evolutionary rescue. The ratio of rescue probability with and without aneuploid (H , eq. (8)) increases with the aneuploid growth rate (colors) and decreases with the wildtype growth rates and initial tumor size (x-axes), except for large tumors where the ratio converges to unity. (A, B) Aneuploidy provides partial resistance. (C, D) Aneuploidy provides tolerance. Solid and dashed lines apply p_{rescue} from the exact formula of (eq. (20) in eq. (2a)); dashed lines apply p_{rescue} from the approximate formula (eq. (5)), with good agreement. Parameters: $N = 10^4$; $\lambda_w = \lambda_a = \lambda_m = 0.14$; (B) $\mu_w = 0.17$; $\mu_m = 0.13$; $u = 10^{-2}$; $v = 10^{-7}$.

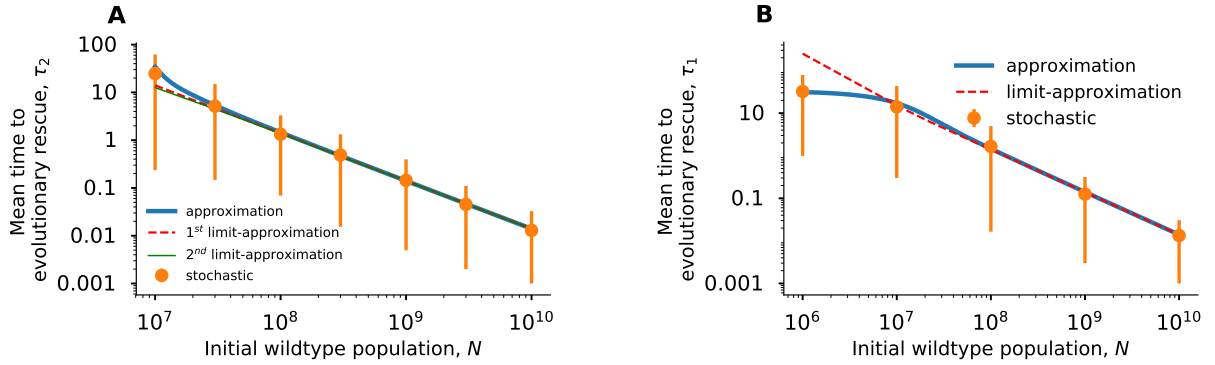


Figure 7: Evolutionary rescue time. Shown is the mean time for appearance of a resistance mutation the leads to evolutionary rescue (**left**) with ($u > 0$) and (**right**) without ($u = 0$ aneuploidy). Our inhomogeneous Poisson-process approximations (solid blue lines, right: eq. (10), left: eq. (16)) is in agreement with simulation results (orange markers with 95% CI). Our 1st-order (dashed red lines, right: eq. (11), left: ??) and 2nd-order (green line, left: eq. (17)) approximations work well when the initial tumor size is large (here $> 10^8$ cells). Parameters: $\lambda_w = \lambda_a = \lambda_m = 0.14$; $\mu_w = 0.17$; (A) $\mu_a = 0.145$; $\mu_m = 0.13$; $u = 10^{-2}$; $v = 10^{-7}$.

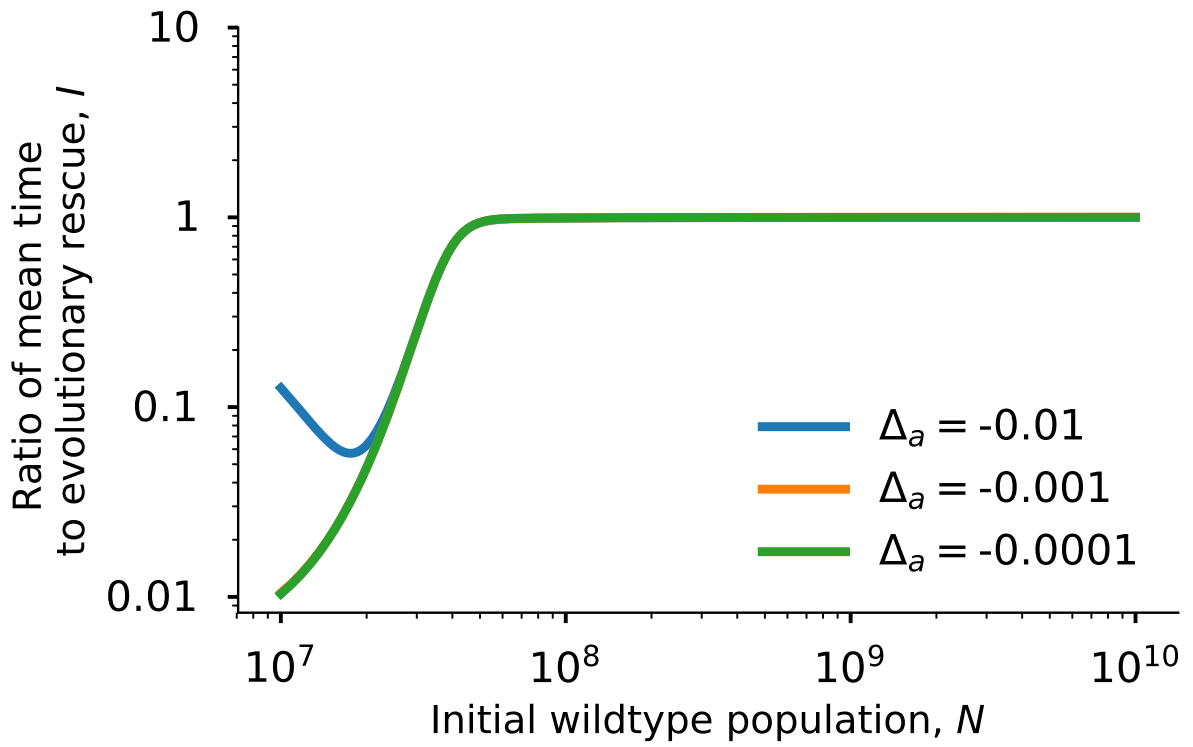


Figure 8: Ratio of evolutionary rescue time with and without aneuploidy. The ratio of the mean time to appearance of a resistance mutation that leads to evolutionary rescue with ($u > 0$) and without ($u = 0$) aneuploidy for variable initial tumor sizes (eq. (18)) when aneuploidy provides tolerance to the drug ($\Delta_a < 0$). When the initial tumor size is not large ($< 10^8$), aneuploidy can decrease the rescue time by 10-100-fold.

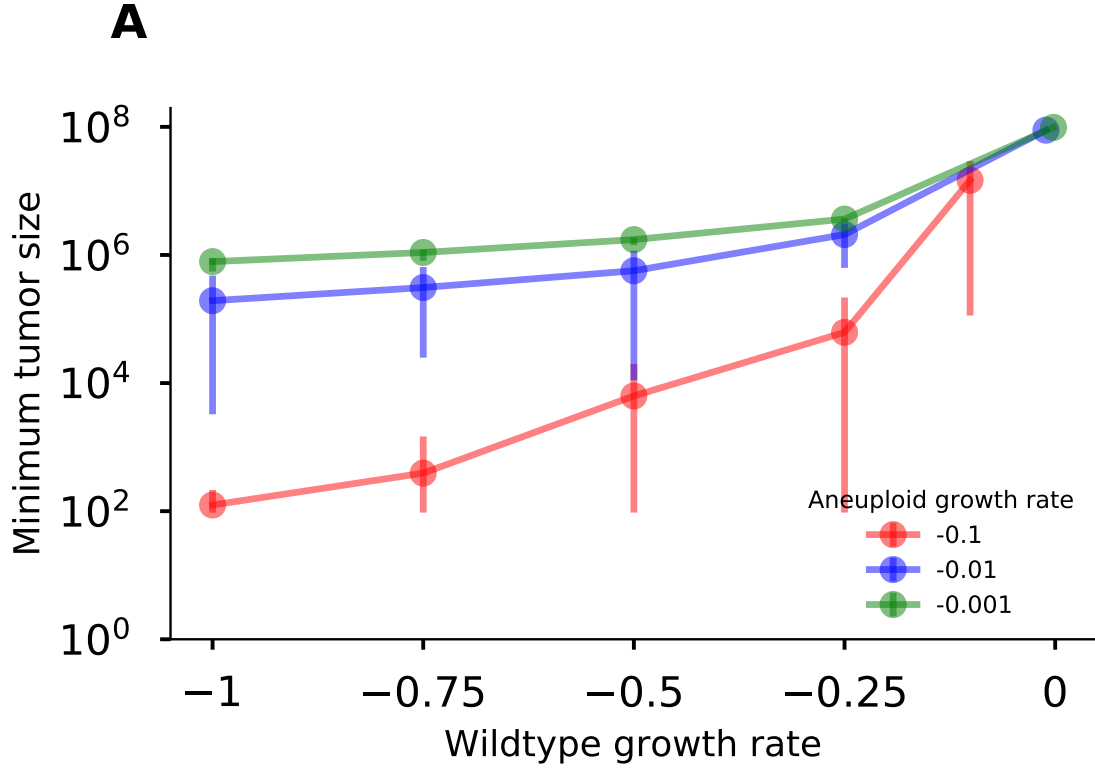


Figure 9: TODO. TODO

Appendices

Survival probability of a single lineage

To analyze evolutionary rescue in this model, we use the framework of *multitype branching processes* (Harris et al., 1963, Rybnikov et al., 2021). This allows us to find explicit expressions for the *survival probability*: the probability that a lineage descended from a single cell does not become extinct.

Let p_w , p_a , and p_m be the survival probabilities of a population consisting initially of single wildtype cell, aneuploid cell, or mutant cell, respectively. The complements $1 - p_w$, $1 - p_a$, and $1 - p_m$ are the extinction probabilities, which satisfy each its respective equation,

$$\begin{aligned}
 1 - p_w &= \frac{\mu_w}{\lambda_w + \mu_w + u + v} + \frac{u}{\lambda_w + \mu_w + u + v} (1 - p_a) + \\
 &\quad \frac{\lambda_w}{\lambda_w + \mu_w + u + v} (1 - p_w)^2 + \frac{v}{\lambda_w + \mu_w + u + v} (1 - p_m), \\
 1 - p_a &= \frac{\mu_a}{\lambda_a + \mu_a + v} + \frac{v}{\lambda_a + \mu_a + v} (1 - p_m) + \frac{\lambda_a}{\lambda_a + \mu_a + v} (1 - p_a)^2, \\
 1 - p_m &= \frac{\mu_m}{\lambda_m + \mu_m} + \frac{\lambda_m}{\lambda_m + \mu_m} (1 - p_m)^2.
 \end{aligned} \tag{19}$$

The survival probabilities are given by the smallest solution for each quadratic equation (Uecker

et al., 2015). Therefore we have

$$\begin{aligned} p_w &= \frac{\lambda_w - \mu_w - u - v + \sqrt{(\lambda_w - \mu_w - u - v)^2 + 4\lambda_w (up_a + vp_m)}}{2\lambda_w}, \\ p_a &= \frac{\lambda_a - \mu_a - v + \sqrt{(\lambda_a - \mu_a - v)^2 + 4\lambda_a vp_m}}{2\lambda_a}, \\ p_m &= \frac{\lambda_m - \mu_m}{\lambda_m}. \end{aligned} \quad (20)$$

Note that the equation for p_w depends on both p_a and p_m , and the equation for p_a depends on p_m . To proceed, we can plug the solution for p_m and p_a into the solution for p_w . We perform this for three different scenarios.

Scenario 1: Aneuploid cells are partially resistant

We first assume that aneuploidy provides partial resistance to drug therapy, $\lambda_a > \mu_a$, and that this resistance is significant, $(\lambda_a - \mu_a - v)^2 > 4\lambda_a vp_m$. We thus rewrite eq. (20) as

$$\begin{aligned} p_w &= \frac{\lambda_w - \mu_w - u - v}{2\lambda_w} \left(1 - \sqrt{1 + \frac{4\lambda_w (vp_m + up_a)}{(\lambda_w - \mu_w - u - v)^2}} \right), \text{ and} \\ p_a &= \frac{\lambda_a - \mu_a - v}{2\lambda_a} \left(1 + \sqrt{1 + \frac{4\lambda_a vp_m}{(\lambda_a - \mu_a - v)^2}} \right). \end{aligned}$$

Using the quadratic Taylor expansion $\sqrt{1+x} = 1 + x/2 + O(x^2)$ and assuming $u, v \ll 1$, we obtain the following approximation for the survival probability of a population initially consisting of a single wildtype cell,

$$p_w \approx -\frac{vp_m + up_a}{\lambda_w - \mu_w - u - v} \quad (21)$$

$$\approx -\frac{1}{\lambda_w - \mu_w} \left[\frac{u(\lambda_a - \mu_a)}{\lambda_a} + \frac{uv(\lambda_m - \mu_m)}{\lambda_m(\lambda_a - \mu_a)} + \frac{v(\lambda_m - \mu_m)}{\lambda_m} \right] \quad (22)$$

Now uv is very small, and if we assume $v \ll u$, we have

$$p_w \approx \frac{u}{|\Delta_w|} \cdot \frac{\Delta_a}{\lambda_a}. \quad (23)$$

Second-order approximation. To improve our approximation, we can consider the second term of the Taylor series expansion,

$$\left(1 + \frac{4\lambda_a vp_m}{(\lambda_a - \mu_a - v)^2} \right)^{\frac{1}{2}} = 1 + \frac{2\lambda_a vp_m}{(\lambda_a - \mu_a - v)^2} - \frac{(\lambda_a vp_m)^2}{4(\lambda_a - \mu_a - v)^4} + \dots,$$

which gives us the following approximation,

$$p_a \approx \frac{\lambda_a - \mu_a - v}{\lambda_a} + \frac{vp_m}{\lambda_a - \mu_a - v} - \frac{\lambda_a (vp_m)^2}{8(\lambda_a - \mu_a - v)^3}. \quad (24)$$

We therefore have

$$p_w \approx -\frac{1}{\lambda_w - \mu_w - u - v} \left[\frac{u(\lambda_a - \mu_a - v)}{\lambda_a} + \frac{uv(\lambda_m - \mu_m)}{\lambda_m(\lambda_a - \mu_a - v)} + \frac{v(\lambda_m - \mu_m)}{\lambda_m} - \frac{uv^2\lambda_a(\lambda_m - \mu_m)^2}{8\lambda_m^2(\lambda_a - \mu_a - v)^3} \right]$$

$$\approx -\frac{1}{\lambda_w - \mu_w} \left[\frac{u(\lambda_a - \mu_a)}{\lambda_a} + \frac{uv(\lambda_m - \mu_m)}{\lambda_m(\lambda_a - \mu_a)} + \frac{v(\lambda_m - \mu_m)}{\lambda_m} - \frac{uv^2\lambda_a(\lambda_m - \mu_m)^2}{8\lambda_m^2(\lambda_a - \mu_a)^3} \right], \quad (25)$$

and using $\Delta_k = \lambda_k - \mu_k$, we can write the above equation as

$$p_w \approx -\frac{1}{\Delta_w} \left(\frac{u\Delta_a}{\lambda_a} + \frac{uv\Delta_m}{\lambda_m\Delta_a} + \frac{v\Delta_m}{\lambda_m} - \frac{uv^2\lambda_a\Delta_m^2}{8\lambda_m^2\Delta_a^3} \right). \quad (26)$$

Scenario 2: Aneuploid cells are tolerant.

We now assume that aneuploidy provides tolerance to drug therapy, that is, the number of aneuploid cells significantly declines over time, but at a lower rate than the number of wildtype cells, $\lambda_w - \mu_w < \lambda_a - \mu_a < 0$. We also assume that the decline are significant, $(\lambda_a - \mu_a - v)^2 > 4\lambda_a v p_m$. We rewrite eq. (20) as

$$p_w = \frac{\lambda_w - \mu_w - u - v}{2\lambda_w} \left(1 - \sqrt{1 + \frac{4\lambda_w(v p_m + u p_a)}{(\lambda_w - \mu_w - u - v)^2}} \right), \text{ and} \quad (27)$$

$$p_a = \frac{\lambda_a - \mu_a - v}{2\lambda_a} \left(1 - \sqrt{1 + \frac{4\lambda_a v p_m}{(\lambda_a - \mu_a - v)^2}} \right).$$

Since $u, v \ll 1$, the term in the root can be approximated using a 1st-order Taylor expansion. So, substituting the expressions for p_a and p_m , we have

$$p_w \approx -\frac{v p_m + u p_a}{\lambda_w - \mu_w - u - v}$$

$$\approx \frac{1}{\lambda_w - \mu_w - u - v} \left[\frac{uv(\lambda_m - \mu_m)}{\lambda_m(\lambda_a - \mu_a - v)} - \frac{v(\lambda_m - \mu_m)}{\lambda_m} \right]$$

$$\approx \frac{v(\lambda_m - \mu_m)}{\lambda_m(\lambda_w - \mu_w)} \left[\frac{u}{(\lambda_a - \mu_a)} - 1 \right] \quad (28)$$

$$= \frac{v\Delta_m}{\lambda_m|\Delta_w|} \left(\frac{u}{|\Delta_a|} + 1 \right).$$

Scenario 3: Aneuploid cells are non-growing

We now assume that the growth rate of aneuploid cells is close to zero (either positive or negative), such that $(\lambda_a - \mu_a - v)^2 < 4\lambda_a v p_m$. We rewrite eq. (20) as

$$p_a = \frac{\lambda_a - \mu_a - v + 2\sqrt{\lambda_a v p_m} \left(1 + \frac{(\lambda_a - \mu_a - v)^2}{4\lambda_a v p_m} \right)^{\frac{1}{2}}}{2\lambda_a}. \quad (29)$$

Using a following Taylor series expansion for small $(\lambda_a - \mu_a - v)^2 / 4\lambda_a v p_m$,

$$\left(1 + \frac{(\lambda_a - \mu_a - v)^2}{4\lambda_a v p_m} \right)^{\frac{1}{2}} = 1 + \frac{(\lambda_a - \mu_a - v)^2}{8\lambda_a v p_m} + \dots,$$

we obtain the approximation

$$\begin{aligned}
p_a &\approx \frac{\lambda_a - \mu_a - v + 2\sqrt{\lambda_a v p_m} \left[1 + \frac{(\lambda_a - \mu_a - v)^2}{8\lambda_a v p_m} \right]}{2\lambda_a} \\
&= \frac{\lambda_a - \mu_a - v + 2\sqrt{\lambda_a v p_m} + \frac{(\lambda_a - \mu_a - v)^2}{4\sqrt{\lambda_a v p_m}}}{2\lambda_a} \\
&= \frac{\left(\lambda_a - \mu_a - v + 2\sqrt{\lambda_a v p_m} \right)^2 + 4\lambda_a v p_m}{8\lambda_a \sqrt{\lambda_a v p_m}} \\
&= \frac{4\lambda_a v p_m + 4\lambda_a v p_m \left(1 + \frac{\lambda_a - \mu_a - v}{2\sqrt{\lambda_a v p_m}} \right)^2}{8\lambda_a \sqrt{\lambda_a v p_m}} \\
&= \frac{1}{2\lambda_a} \left(\lambda_a - \mu_a - v + 2\sqrt{\lambda_a v p_m} \right).
\end{aligned} \tag{30}$$

Plugging this in eq. (28), the survival probability of a population starting from one wildtype individual is

$$\begin{aligned}
p_w &\approx -\frac{1}{\lambda_w - \mu_w - u - v} \left[v \frac{\lambda_m - \mu_m}{\lambda_m} + \frac{u}{2\lambda_a} \left(\lambda_a - \mu_a - v + 2\sqrt{\lambda_a v p_m} \right) \right] \\
&= -\frac{1}{\lambda_w - \mu_w - u - v} \left[v \frac{\lambda_m - \mu_m}{\lambda_m} + \frac{u}{2\lambda_a} (\lambda_a - \mu_a - v) + u \sqrt{\frac{v (\lambda_m - \mu_m)}{\lambda_a \lambda_m}} \right].
\end{aligned} \tag{31}$$