Supporting material for

Predicting microbial relative growth in a mixed culture from growth curve data

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# Supporting Text 1: Monoculture model

We derive our growth models from a resource consumption perspective (Otto and Day, 2007, 365; Gopalsamy, 1986). We denote by the density of a limiting resource, and by the density of the cell population, both in total mass per unit of volume.

We assume that the culture is well-mixed and homogeneous and that the resource is depleted by the growing cell population without being replenished. Therefore, the intake of resources occurs when cells meet resource via a mass action law with resource intake rate . Once inside the cell, resources are converted to cell mass at a conversion rate of . Cell growth is assumed to be proportional to , whereas resource intake is proportional to a power of cell density, . We denote .

We can describe this process with differential equations for and :

|  |  |  |
| --- | --- | --- |
|  |  | [A1a]  [A1b] |

These equations can be converted to equations in and :

,

which yields

|  |  |  |
| --- | --- | --- |
|  |  | [A2a]  [A2b] |

with .

To solve this system, we use a conservation law approach by setting (Dilao and Domingos, 1999). We find that *M* is constant

,

and we can substitute in eq. A2b to get

|  |  |  |
| --- | --- | --- |
|  | . | [A3] |

Substituting again , and defining , we get

|  |  |  |
| --- | --- | --- |
|  | , | [A4] |

which is the Richards differential equation (Richards, 1959), with the maximum population density *K* and the specific growth rate in low density. To the best of our knowledge this the first derivation of the Richards differential equation from a resource consumption perspective.

We solve eq. A4 via eq. A3, which is a logistic equation and therefore has a known solution. Setting the initial cell density we have

.

Eq. A4 is an autonomous differential equation ( doesn't explicitly depend on ). To include a lag phase, Baranyi and Roberts (1994) suggested to add an adjustment function , which makes the equation non-autonomous (explicitly dependent on ):

|  |  |  |
| --- | --- | --- |
|  | . | [A5] |

Baranyi and Roberts suggested a Michaelis-Menten type of function (Baranyi, 1997)

,

which has two parameters: *q0* is the initial physiological state of the population, and *m* is rate at which the physiological state adjusts to growth conditions. Integrating gives

.

Therefore, integrating eq. A5 produces eq. 2.

The term is used to describe the deceleration in the growth of the population as it approaches the maximum density . When , the deceleration is the same as in the standard logistic model and the density at the time of the maximum population growth is half the maximum density, . When or , the deceleration is slower or faster, respectively, and the density at the time of the maximum growth rate is (Richards 1959, substituting *)*.

We use six forms of the Baranyi-Roberts model (Figure S2, Table S1). The full model is described by eq. 2 and has six parameters. A five-parameter form of the model assumes , as in the standard logistic model, but still incorporates the adjustment function and therefore includes a lag phase. Another five-parameter form has both rate parameters set to the same value (), which was suggested to make the fitting procedure more stable (Baranyi, 1997; Clark *et al.*, 2010). A four-parameter form has both of the previous constraints, setting and (Baranyi, 1997). Another four-parameter form of the model has no lag phase, with 1, which yields the Richards model (Richards, 1959), also called the -logistic model (Gilpin and Ayala, 1973), or the generalized logistic model. This form of the model is useful in cases where there is no observed lag phase: either because the population adjusts very rapidly or because it is already adjusted prior to the growth experiment, possibly by pre-growing it in fresh media before the beginning of the experiment. The last form is the standard logistic model, in which and .

# Supporting Text 2: Mixed culture model

We consider the case in which two species or strains grow in the same culture, competing for a single limiting resource, similarly to eq. A1:

|  |  |  |
| --- | --- | --- |
|  |  | [B1a]  [B1b]  [B1c] |

We define , and (where *j* is 1 when *i* is 2 and vice versa) to find that and is constant. We then substitute into the differential equations for . Denoting and , we get

|  |  |  |
| --- | --- | --- |
|  |  | [B2a]  [B2b] |

where .

We get a similar result if each strain is limited by a different resource that both strains consume:

|  |  |  |
| --- | --- | --- |
|  |  | [B3a]  [B3b]  [B3c]  [B3d] |

Here, we notice first that and therefore is a constant. We then substitute in eqs. B3 and continue as above. This changes the definition of .

If the intake rates depend only on the resource then

|  |  |  |
| --- | --- | --- |
|  |  | [B4a]  [B4b] |

Then we define and and again continue as above.

# Supporting figures



Figure S1. Fluorescence microscopy of *E. coli* strains carrying GFP or RFP. Image of a mixture of DH5α-GFP and TG1-RFP cells.



Figure S2. Growth models hierarchy. The Baranyi-Roberts model and five nested models defined by fixing one or two parameters. See Supporting Text 1 and Table S1 for more details.

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Figure S3. Mixed culture growth predictions with confidence intervals. The green and red lines and markers correspond to the dashed green and red lines and the markers in Figure 3D-F, respectively. The gray area shows the 95% confidence interval, calculated using bootstrap (1000 samples).

# Supporting tables

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Model name | **# Parameters** | **Free Parameters** | **Fixed Parameters** | **References** |
| Baranyi Roberts 1994 | 6 |  | - | (Baranyi and Roberts, 1994) |
| Baranyi 1997 | 5 |  |  | - |
| Baranyi Roberts 1994 | 5 |  |  | - |
| Richards 1959 | 4 |  |  | (Richards, 1959) |
| Baranyi 1997 | 4 |  |  | (Baranyi, 1997) |
| Logistic | 3 |  |  | (Verhulst, 1838) |

Table S1. Growth models. The table lists the growth models used for fitting growth curve data. All models are defined by eqs. 1 and 2, by fixing specific parameters. is the initial population density; is the maximum population density; is the specific growth rate in low density; is the surface to mass ratio; is the initial physiological state; is the physiological adjustment rate. Note that when , the value of is irrelevant. See also the hierarchy diagram in Figure S2 and a detailed discussion in Supporting text 1.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | Experiment A | | | | Experiment B | | Experiment C | |
| *Strain*  *Parameter* | | **GFP** | | **RFP** | **GFP** | **RFP** | **GFP** | **RFP** |
|  | 0.125 | | 0.124 | | 0.286 | 0.23 | 0.188 | 0.204 |
|  | 0.528 | | 0.65 | | 0.619 | 0.628 | 0.633 | 0.741 |
|  | 0.376 | | 0.587 | | 0.304 | 0.484 | 8 | 8 |
|  | 2.636 | | 1\* | | 2.484 | 1.491 | 1\* | 0.164 |
|  | 0.032 | | 0.008 | | **-\*** | **-\*** | 0.039 | 0.393 |
|  | 0.937 | | 3.735 | | **-\*** | **-\*** | 0.188 | 0.104 |

Table S2. Estimated parameters from growth model fitting. \* denotes fixed parameters; - denotes invalid parameter values.