The code used to complete this problem set is attached in the appendix below.

The expected value function can be written as:

$$\mathbb{E}_\epsilon\left[V(i,c,p,\epsilon)\right] = \mathbb{E}_\epsilon\left[\max_{a\in\{0,1\}} U(a|i,c,p,\epsilon) + \beta\sum_{c',p'}\mathbb{E}_{\epsilon'}\left[V(i',c',p',\epsilon')\right]\Pr(c',p'|c,p,a)\right]$$

$$\mathbb{E}_\epsilon\left[V(s,\epsilon)\right] = \mathbb{E}_\epsilon\left[\max_{a\in\{0,1\}} U(a|s,\epsilon) + \beta\sum_{s'}\mathbb{E}_{\epsilon'}\left[V(s',\epsilon')\right]\Pr(s'|s,a)\right]$$

$$\overline{V}(s) = \mathbb{E}_\epsilon\left[\max_{a\in\{0,1\}} U(a|s,\epsilon) + \beta\sum_{s'}\overline{V}(s')\Pr(s'|s,a)\right]$$

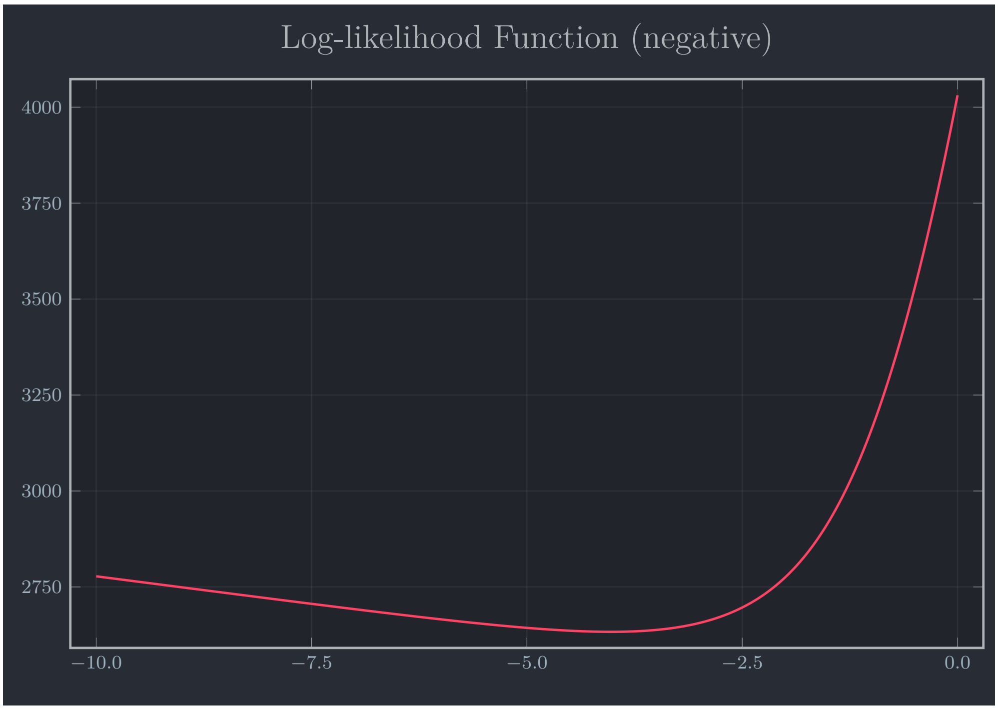
The numerical value for each state variable, s, is in the first column below, with the implied value function from $\hat{P}(s)$ in the second column. As you can see, the expected values are much higher using $\hat{P}(s)$, for each state.

$$\begin{bmatrix} 61.13 & 60.91 \\ 65.01 & 64.8 \\ 68.48 & 68.27 \\ 71.67 & 71.44 \\ 74.63 & 74.38 \\ 77.39 & 77.1 \\ 79.96 & 79.59 \\ 82.26 & 81.74 \\ 84.07 & 83.38 \\ 58.49 & 58.27 \\ 63.13 & 62.91 \\ 67.01 & 66.8 \\ 70.48 & 70.27 \\ 73.67 & 73.43 \\ 76.63 & 76.38 \\ 79.39 & 79.1 \\ 81.96 & 81.59 \\ 84.26 & 83.64 \\ 63.24 & 63.0 \\ 66.89 & 66.68 \\ 70.2 & 69.98 \\ 73.26 & 73.01 \\ 76.11 & 75.84 \\ 78.77 & 78.44 \\ 81.2 & 80.74 \\ 83.28 & 82.68 \\ 84.28 & 83.31 \\ 61.03 & 60.81 \\ 65.24 & 65.03 \\ 68.89 & 68.68 \\ 72.2 & 71.98 \\ 75.26 & 75.02 \\ 78.11 & 77.84 \\ 80.77 & 80.4 \\ 83.2 & 82.77 \\ 85.28 & 84.68 \end{bmatrix} \tag{1}$$

The log-likelihood function is

$$\mathcal{L}(s_i|\lambda) = \sum_i a_i \log\left(P(s_i)\right) + (1 - a_i)\log\left(1 - P(s_i)\right)$$

Solving this log-likelihood function using a nested fixed point algorithm yields a result of $\hat{\lambda} = -4.024$. The negative log-likelihood function is plotted below, displaying a unique minimum between -10 and 0.



Appendix

The first codefile named “runfile.jl" runs the code.

The second codefile named “functions.jl" contains the relevant functions.