

Econ899 PS2b

November 22, 2021

1. See the julia code
2. For $T_i = 1$, the probability can be written as follows

$$Pr\left(\eta_{i0} < \frac{-\alpha_0 - X_i\beta - Z_{it}\gamma}{\sigma_0}\right) \quad (1)$$

For $T_i = 2$, the probability is

$$Pr(\epsilon_{i0} < \alpha_0 + X_i\beta + Z_{it}\gamma, \epsilon_{i1} < -\alpha_1 - X_i\beta - Z_{it}\gamma) \quad (2)$$

The conditional probability can be written as follows

$$\begin{aligned} & Pr(\epsilon_{i1} < -\alpha_1 - X_i\beta - Z_{it}\gamma | \epsilon_{i0} < \alpha_0 + X_i\beta + Z_{it}\gamma) \\ &= Pr\left(\eta_{i0} + \rho\sigma_0\eta_{i1} < -\alpha_1 - X_i\beta - Z_{it}\gamma | \eta_{i0} < \frac{\alpha_0 + X_i\beta + Z_{it}\gamma}{\sigma_0}\right) \\ &= Pr\left(\eta_{i1} < \frac{-\alpha_1 - X_i\beta - Z_{it}\gamma - \eta_{i0}^*}{\rho\sigma_0} \middle| \eta_{i0} < \frac{\alpha_0 + X_i\beta + Z_{it}\gamma}{\sigma_0}\right) \end{aligned}$$

where η^* is a random variable from $\eta_{i0} < \frac{\alpha_0 + X_i\beta + Z_{it}\gamma}{\sigma_0}$. Then, the probability of $T_i = 2$ is

$$Pr\left(\eta_{i1} < \frac{-\alpha_1 - X_i\beta - Z_{it}\gamma - \eta_{i0}^*}{\rho\sigma_0} \middle| \eta_{i0} < \frac{\alpha_0 + X_i\beta + Z_{it}\gamma}{\sigma_0}\right) Pr\left(\eta_{i0} < \frac{\alpha_0 + X_i\beta + Z_{it}\gamma}{\sigma_0}\right) \quad (3)$$

Similarly, we can define probability in $T_i = 3, 4$. Based on these probabilities, we can define the log likelihood. The code is GHKLL2.

3. See the code AcceptRejectLL()
4. We got the following likelihoods:
 - Quarature method:-40992
 - GHK method:-60572
 - Accept/Reject method:-75167

The value varies, but the order (-1e5) is the same.

5. We got the following result.

$$\begin{aligned} \alpha_0 &= 3.12, \alpha_1 = 0.88, \alpha_2 = 2.29 \\ \text{score}_0 &= 0.00, \text{ratespread} = -0.25, \text{largeloan} = -0.79, \text{mediumloan} = -0.39 \\ \text{irefinance} &= -0.04, \text{ager} = -0.42, \text{cltv} = 0.34, \text{dti} = 0.67, \text{cu} = -0.43 \\ \text{firstmort} &= 0.59, \text{iFHA} = -0.05, \text{openyear2} = -0.70, \text{openyear3} = 0.07 \\ \text{openyear4} &= 0.12, \text{openyear5} = 0.02 \\ \text{score0} &= 0.30, \text{score1} = -0.16, \text{score2} = 0.35, \rho = 0.57 \end{aligned}$$