Econ899 PS2b

November 22, 2021

1. See the julia code
2. For $T_i = 1$, the probability can be written as follows

$$Pr\left(\eta_{i0} < \frac{-\alpha_0 - X_i\beta - Z_{it}\gamma}{\sigma_0}\right) \tag{1}$$

For $T_i = 2$, the probability is

$$Pr\left(\epsilon_{i0} < \alpha_0 + X_i\beta + Z_{it}\gamma, \epsilon_{i1} < -\alpha_1 - X_i\beta - Z_{it}\gamma\right) \tag{2}$$

The conditional probability can be written as follows

$$\begin{aligned}
&Pr\left(\epsilon_{i1} < -\alpha_1 - X_i\beta - Z_{it}\gamma | \epsilon_{i0} < \alpha_0 + X_i\beta + Z_{it}\gamma\right) \\
&= Pr\left(\eta_{i0} + \rho\sigma_0\eta_{i1} < -\alpha_1 - X_i\beta - Z_{it}\gamma | \eta_{i0} < \frac{\alpha_0 + X_i\beta + Z_{it}\gamma}{\sigma_0}\right) \\
&= Pr\left(\eta_{i1} < \frac{-\alpha_1 - X_i\beta - Z_{it}\gamma - \eta^*_{i0}}{\rho\sigma_0} \middle| \eta_{i0} < \frac{\alpha_0 + X_i\beta + Z_{it}\gamma}{\sigma_0}\right)
\end{aligned}$$

where η^* is a random variable from $\eta_{i0} < \frac{\alpha_0 + X_i\beta + Z_{it}\gamma}{\sigma_0}$. Then, the probability of $T_i = 2$ is

$$Pr\left(\eta_{i1} < \frac{-\alpha_1 - X_i\beta - Z_{it}\gamma - \eta^*_{i0}}{\rho\sigma_0} \middle| \eta_{i0} < \frac{\alpha_0 + X_i\beta + Z_{it}\gamma}{\sigma_0}\right) Pr\left(\eta_{i0} < \frac{\alpha_0 + X_i\beta + Z_{it}\gamma}{\sigma_0}\right) \tag{3}$$

Similarly, we can define probability in $T_i = 3, 4$. Based on these proabilities, we can define the log likelihood. The code is GHKLL2.

3. See the code AcceptRejectLL()
4. We got the following likelihoods:
   - Quarature method:
   - GHK method:
   - Accept/Reject method:

   The value varies, but the order (-1e5) is the same.
5. We got the following result. Unfortunately, these values are different from results of STATA code.

$$\begin{aligned}
\alpha_0 = 3.12, \alpha_1 = 0.88, \alpha_2 = 2.29 \\
score_0 = 0.00, ratespread = -0.25, largeloan = -0.79, mediumloan = -0.39 \\
irefinance = -0.04, ager = -0.42, cltv = 0.34, dti = 0.67, cu = -0.43 \\
firstmort = 0.59, iFHA = -0.05, openyear2 = -0.70, openyear3 = 0.07 \\
openyear4 = 0.12, openyear5 = 0.02 \\
score0 = 0.30, score1 = -0.16, score2 = 0.35, \rho = 0.57
\end{aligned}$$