Econ899b PS4

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December 15, 2021

1. The expected value function can be written as follows:

$$\begin{aligned}
\bar{V} &= E_\epsilon[V(i,c,p,\epsilon)] \\
&= E_\epsilon[\max_a U(a|i,c,p,\epsilon) + \beta \sum E[V(i',c',p',\epsilon'] Pr(c',p'|c,p,a)]] \\
&= E_\epsilon[\max_a \mathbb{1}(a=1)(\alpha c - p) + \mathbb{1}(a=1)\mathbb{1}(i>1)\alpha c + \mathbb{1}(a=1)\mathbb{1}(i=1)\lambda \mathbb{1}(c>0) + \epsilon(a) \\
&\quad + \beta \sum E[V(i',c',p',\epsilon'] Pr(c',p'|c,p,a)]]
\end{aligned}$$

Let $v(a,i,c,p) = \mathbb{1}(a=1)(\alpha c - p) + \mathbb{1}(a=1)\mathbb{1}(i>1)\alpha c + \mathbb{1}(a=1)\mathbb{1}(i=1)\lambda\mathbb{1}(c>0) + \beta \sum E[V(\cdot)]Pr(c',p'|c,p,a)]$. Then,

$$E_\epsilon[\max_a v(a,i,c,p) + \epsilon(a)] = \ln(\sum_a \exp(v(\cdot))) + \gamma$$

The Table is (1).

I	C	P	EV
0	0	4	61.12785224254987
1	0	4	65.01024882086212
2	0	4	68.48210863284643
3	0	4	71.66875693503552
4	0	4	74.63022715292597
5	0	4	77.39429444746555
6	0	4	79.95878691331467
7	0	4	82.2632871179156
8	0	4	84.07324859344968
0	1	4	58.49101865421765
1	1	4	63.12785224254987
2	1	4	67.01024882086212
3	1	4	70.48210863284643
4	1	4	73.66875693503552
5	1	4	76.63022715292594
6	1	4	79.39429444746558
7	1	4	81.95878691331467
8	1	4	84.2632871179156
0	0	1	63.24416119042413
1	0	1	66.89462073690096
2	0	1	70.20312375430869
3	0	1	73.26050214629285
4	0	1	76.11022495925096
5	0	1	78.76597733882473
6	0	1	81.20090619366847
7	0	1	83.28155981827642
8	0	1	84.27758186000644
0	1	1	61.025485862098925
1	1	1	65.24416119042414
2	1	1	68.89462073690099
3	1	1	72.20312375430869
4	1	1	75.26050214629282
5	1	1	78.11022495925093
6	1	1	80.76597733882475
7	1	1	83.20090619366847
8	1	1	85.28155981827642

(1)

2. The result is (2). The difference between the true an implied is too small.

I	C	P	EV	$\hat{EV}$
0	0	4	61.12785224254987	60.71685415008766
1	0	4	65.01024882086212	64.5936112359602 1
2	0	4	68.48210863284643	68.05323953115087
3	0	4	71.66875693503552	71.21078777903563
4	0	4	74.63022715292597	74.128187694844
5	0	4	77.39429444746555	76.80253283552119
6	0	4	79.95878691331467	79.20529891994728
7	0	4	82.2632871179156	81.24124984308239
8	0	4	84.07324859344968	82.64595938571622
0	1	4	58.49101865421765	58.0797578508909
1	1	4	63.12785224254987	62.71421645611459
2	1	4	67.01024882086212	66.592322238919
3	1	4	70.48210863284643	70.05278380353798
4	1	4	73.66875693503552	73.2073673946673
5	1	4	76.63022715292594	76.12787686979966
6	1	4	79.39429444746558	78.77931511201233
7	1	4	81.95878691331467	81.06049632164435
8	1	4	84.2632871179156	83.17556592207703
0	0	1	63.24416119042413	62.829230140790884
1	0	1	66.89462073690096	66.46700616878599
2	0	1	70.20312375430869	69.75722184911152
3	0	1	73.26050214629285	72.78251015847485
4	0	1	76.11022495925096	75.56841086673374
5	0	1	78.76597733882473	78.10017998173997
6	0	1	81.20090619366847	80.29875755014329
7	0	1	83.28155981827642	82.06923576093345
8	0	1	84.27758186000644	82.65905072376842
0	1	1	61.025485862098925	60.61447742610977
1	1	1	65.24416119042414	64.82752674093717
2	1	1	68.89462073690099	68.47022110976287
3	1	1	72.20312375430869	71.76423459119239
4	1	1	75.26050214629282	74.78554697997075
5	1	1	78.11022495925093	77.49062843672597
6	1	1	80.76597733882475	80.07072261396762
7	1	1	83.20090619366847	82.28185545507503
8	1	1	85.28155981827642	83.02656430722752

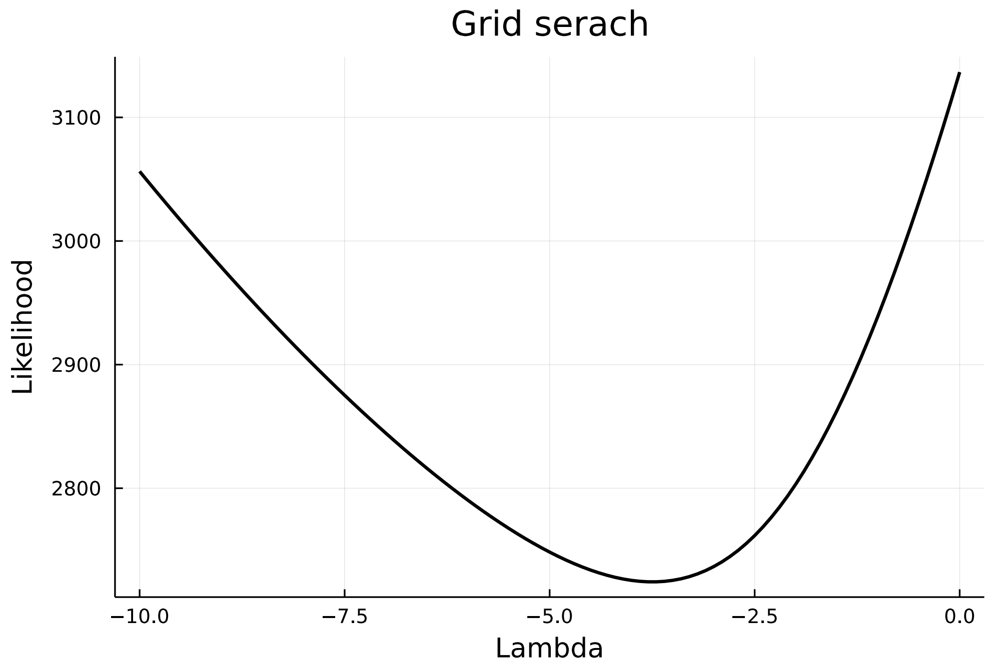
(2)

3. The log-likelihood function is

$$\sum_i a_i \log Pr_i(s) + (1 - a_i) \log(1 - Pr_i(s))$$

where $Pr_i(s)$ is the conditional choice probability.

4. The loglikihood has the following shape. And, the results are the following: MLE: -4.300628278478276 NFP: -4.300628278478276. Although these values are not matched with results, the value is relatively similar with the true value. Since MLE and NFP yield the same result, there would be a bug in my code.


Grid serach
Likelihood
3100
3000
2900
2800
−10.0
−7.5
−5.0
−2.5
0.0
Lambda