ECON 899 – Problem Set 3

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The optimal labor supply

Houshold's problem is

$$\max_{c,l} \frac{\left[c^\gamma (1-l)^{1-\gamma}\right]^{1-\sigma}}{1-\sigma} \quad \text{s.t. } c = (1-\theta)we(z,\eta_j)l + (1+r)a - a' \tag{1}$$

The first order condition yields

$$c = \frac{\gamma}{1-\gamma}(1-\theta)we(z,\eta_j)(1-l) \tag{2}$$

Using this and the budget constraint, we get the optimal l in the PS.
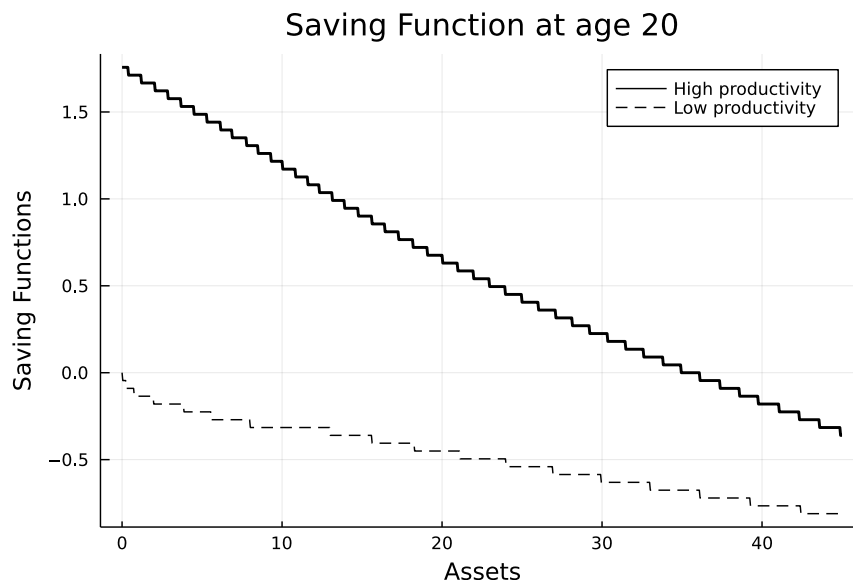
Exercise 1

Solve the dynamic programming problem of retirees and workers. Plot the value function over a for a retired agent at the model-age 50. Is it increasing and concave? Plot the savings function for a worker at the model-age 20, $a'_{20}(z,a)$. Is saving increasing in a? Is it increasing in z?

<u>Answer:</u> The value function over a for a retired agent at the model-age 50 is given below:
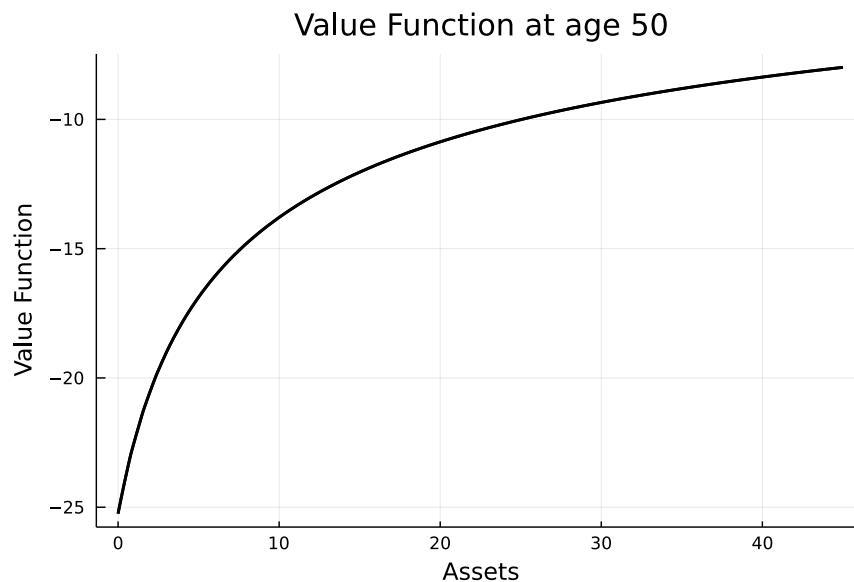
It is increasing and concave.

The savings function for a worker at the model-age 20, $a'_{20}(z,a) = a' - a$ is the following.

Saving Function at age 20

The saving is decreasing in a and increasing in z. ∎

Exercise 2

After solving for agent's dynamic programming problem, solve for the steady-state distribution of agents over age, productivity and asset holdings, $F_j(z, a)$. Find first the relative sizes of each cohort of age j (denoted by μ_j) using the expression below:

$$\mu_{i+1} = \frac{\mu_i}{1 + N}, \text{ for } i = 1, \dots, N - 1$$

with any $\mu_1 = \tilde{\mu}_1 > 0$. Then normalize μ, so that it sums up to 1 across all age groups. Finally, start with the newborn generation with zero wealth: given its distribution, $F_1(z^H, 0) = \mu_1 0.2037$ and $F_1(z^L, 0) = \mu_1 0.7963$, compute the distribution of agents over asset holdings at subsequent ages by applying the optimal decision rules.

Answer: See the code and report the results in Exercise 3 based on Exercise 2. ∎

Exercise 3

1. First, solve for the benchmark model with social security. Is this economy dynamically efficient (compare the interest rate with the implicit return from social security, which is equal to the population growth rate)? Now eliminate social security by setting $\theta = 0$. Observe how aggregate capital accumulation and labor supply change as a result of the tax reform. Provide intuition in terms of insurance and output efficiency. How does aggregate welfare change? Who benefits and who loses due to this reform? How does the reform affect cross-sectional wealth inequality? You can use table 1 to support your answers.

   Answer: Table 1 summarises the results.This economy is not efficient because it is not equal to the population growth. The total welfare decrease ∎

2. In the second experiment, there is no idiosyncratic risk. Assume that at each age j, $z^L = z^H = 0.5$. First, compute the aggregate variables for the case with social security. How does the aggregate capital stock change relative to the benchmark model? Provide intuition in terms of capital as a buffer stock. Then, eliminate social security. How does the aggregate welfare change? What can you conclude about social security as an insurance device against idiosyncratic risk? Comment on the extent, to which these welfare comparisons across steady states are meaningful or misleading.

   Answer: The welfare decreases. The social security decreases comparing with benchmark model. ∎

3. Consider the case, when labor supply is exogenous ($\gamma = 1$). Compare the distortionary effect of social security on the aggregate labor supply. How does the support for social security change with exogenous labor supply?

Table 1: Results of policy experiments

	Benchmark Model		No risk, $z^L = z^H = 0.5$		Exogenous labor, $\gamma = 1$	
capital, K	3.365	4.537	1.051	1.287	7.094	9.46
labor, L	0.343	0.364	0.161	0.17	0.754	0.754
wage, w	1.455	1.587	1.256	1.327	1.434	1.591
interest, r	0.024	0.012	0.049	0.038	0.026	0.011
pension benefit, b	0.225	0.0	0.091	0.0	0.488	0.0
total welfare, W	-35.781	-37.311	-44.976	-44.957	-23.27	-25.471
cv(wealth)	—	0.353	—	0.119	—	0.316

Answer:

This economy is not efficient.

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