Assume that households have log preferences, the production technology satisfies $Y_t = Z_t K_t^\alpha$ where $\alpha = 0.36$; and capital depreciates at rate $\delta = 0.025$. We will assume technology shocks follow a 2 state Markov Process. The transition matrix is calibrated to NBER business cycle data where we take an expansion to be an instance of a positive technology shock and recession to be an instance of a negative technology shock. The transition matrix is given by

$$\begin{bmatrix} 0.977 & 0.023 \\ 0.074 & 0.926 \end{bmatrix}$$

Comments on computation time: *The stochastic code on Julia converged after 434 iterations and took 4.95 seconds to run.*

1. State the dynamic programming problem.

   Answer: The dynamic programming problem can be stated as:

$$V(K, Z) = \max_{C,K'} \{\log(C) + \beta \mathbb{E}[V(K', Z')|Z]\} \text{ s.t. } C + K' = ZK^\alpha + (1 - \delta)K$$

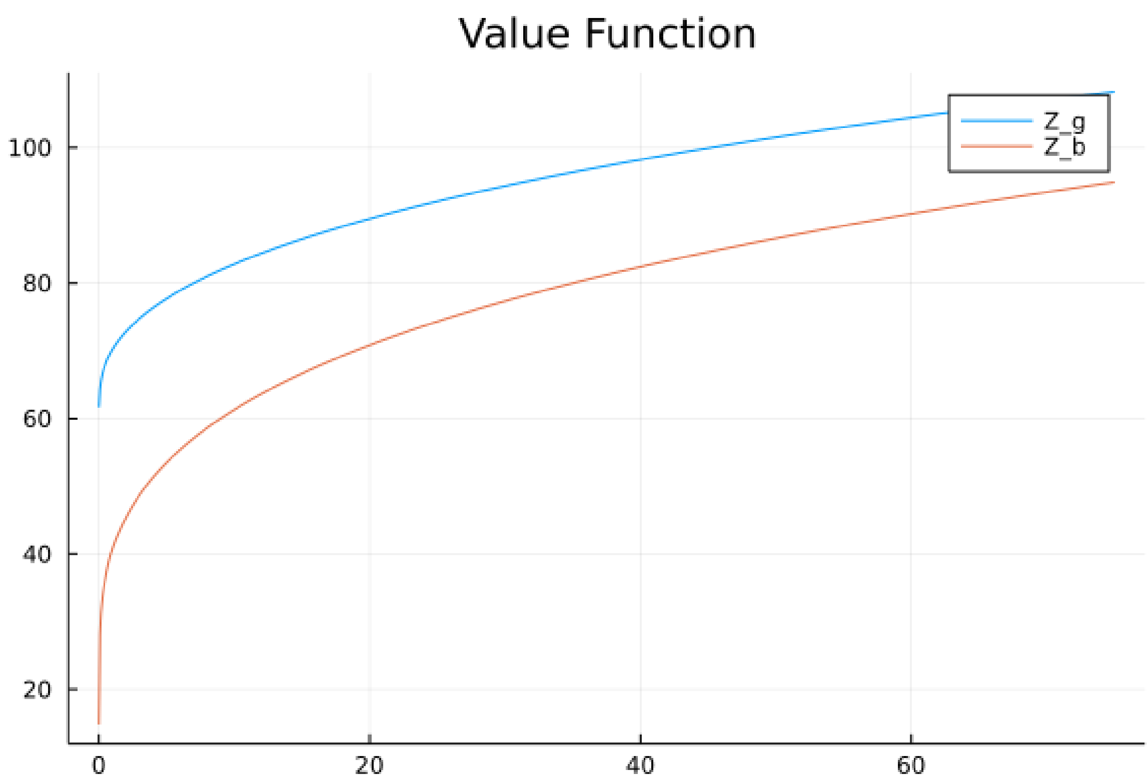
   which can be rewritten as an dynamic optimization problem of one variable as

$$V(K, Z) = \max_{K'} \{\log(ZK^\alpha + (1 - \delta)K - K') + \beta \mathbb{E}[V(K', Z')|Z]\}$$

   ∎

2. Plot the value function K over each state Z. Is it increasing (i.e. is $V(K_{i+1}, Z) \geq V(K_i, Z)$) for $K_{i+1} > K_i$? Is it concave?

   Answer:



   The value function appears to be strictly increasing and concave. ∎

3. Is the decision rule increasing in K and Z (i.e. is $K'(K_{i+1}, Z) \geq K'(K_i, Z)$ for $K_{i+1} > K_i$ and is $K'(K, Z^g) \geq K'(K, Z^b)$)? Is savings increasing in K and Z (to see this, plot the change in the decision rule $K'(K, Z) - K$ across K for each possible exogenous state Z)?