Objectives

In this lab, we focus on **building regression models** to analyze wage differences. Specifically, we explore:

- **Transforming variables** (e.g., log transformation of wages)
- **Incorporating categorical variables** (e.g., gender effects)
- **Adding interaction terms** to capture varying relationships (e.g., how wage growth differs by gender)
- **Modeling non-linearity** (e.g., quadratic terms for age)

Regression models help us **quantify relationships** between variables, make **predictions**, and test **hypotheses**. By systematically incorporating transformations, categorical variables, and interactions, we improve our ability to **interpret patterns** in data and make **data-driven decisions**.

Instructions

- Complete all **subsection tasks** in the Jupyter Notebook.
- Save your **final notebook as a PDF** with all code executed and outputs visible.
- Upload the PDF to **Gradescope** (link on Brightspace).

Prelims

Let us first mount folder with files and change the working directory to where the files are. Make sure to replace the folder name below with the path in your Google Drive.

```
from google.colab import drive
drive.mount('/content/drive')

import os

# Replace the path the actual folder name
os.chdir('/content/drive/MyDrive/DSO530Public/data')

# Confirm that the files are accessible
os.listdir()
```

```
Drive already mounted at /content/drive; to attempt to forcibly remount, call drive.mount("/content/drive", force_remount=True).
['wages.csv',
 'census2000.csv',
 'anscombe.csv',
 'confood.csv',
 'pickup.csv',
 'sales.csv',
 'grades.csv',
 'telemarketing.csv',
 'imports.csv',
 'diamonds.csv',
 'supervisor.csv',
 'lab1',
 '<path_to_output_folder>']
```

Load packages

```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import statsmodels.api as sm
import statsmodels.formula.api as smf
```

This code snippet below is setting the default size for plots (figures) that are created using the `matplotlib.pyplot` library.

```
# set the figure width and height
fig_height = 6
fig_width = fig_height * 1.618
plt.rcParams['figure.figsize'] = (fig_width, fig_height)
```

Convert Notebook to PDF

You can use the following to convert your Notebook to a PDF

```
!apt-get install -y pandoc
!apt-get install -y texlive-xetex texlive-fonts-recommended texlive-plain-generic
```

```
Reading package lists... Done
Building dependency tree... Done
Reading state information... Done
pandoc is already the newest version (2.9.2.1-3ubuntu2).
```

```
0 upgraded, 0 newly installed, 0 to remove and 18 not upgraded.
Reading package lists... Done
Building dependency tree... Done
Reading state information... Done
texlive-fonts-recommended is already the newest version (2021.20220204-1).
texlive-plain-generic is already the newest version (2021.20220204-1).
texlive-xetex is already the newest version (2021.20220204-1).
0 upgraded, 0 newly installed, 0 to remove and 18 not upgraded.
```

Do not forget to change the path below.

```
!jupyter nbconvert --to pdf "<path_to_my_notebook>/Case\ Study\ In\ Interacations\ \(lab\).ipynb" --output-dir="<path_to_output_folder>"
```

```
                copy of reveal.js: e.g., "reveal.js".
                If a relative path is given, it must be a subdirectory of the
                current directory (from which the server is run).
                See the usage documentation
                (https://nbconvert.readthedocs.io/en/latest/usage.html#reveal-js-html-slideshow)
                for more details.
        Default: ''
        Equivalent to: [--SlidesExporter.reveal_url_prefix]
    --nbformat=<Enum>
        The nbformat version to write.
                Use this to downgrade notebooks.
        Choices: any of [1, 2, 3, 4]
        Default: 4
        Equivalent to: [--NotebookExporter.nbformat_version]

    Examples
    --------

        The simplest way to use nbconvert is

                > jupyter nbconvert mynotebook.ipynb --to html

                Options include ['asciidoc', 'custom', 'html', 'latex', 'markdown', 'notebook', 'pdf', 'python', 'qtpdf', 'qtpng', 'rst',

                > jupyter nbconvert --to latex mynotebook.ipynb

                Both HTML and LaTeX support multiple output templates. LaTeX includes
                'base', 'article' and 'report'.  HTML includes 'basic', 'lab' and
                'classic'. You can specify the flavor of the format used.

                > jupyter nbconvert --to html --template lab mynotebook.ipynb

                You can also pipe the output to stdout, rather than a file

                > jupyter nbconvert mynotebook.ipynb --stdout

                PDF is generated via latex

                > jupyter nbconvert mynotebook.ipynb --to pdf

                You can get (and serve) a Reveal.js-powered slideshow

                > jupyter nbconvert myslides.ipynb --to slides --post serve

                Multiple notebooks can be given at the command line in a couple of
                different ways:

                > jupyter nbconvert notebook*.ipynb
                > jupyter nbconvert notebook1.ipynb notebook2.ipynb

                or you can specify the notebooks list in a config file, containing::

                    c.NbConvertApp.notebooks = ["my_notebook.ipynb"]

                > jupyter nbconvert --config mycfg.py

    To see all available configurables, use `--help-all`.
```

Case study in interaction

Use census data to explore the relationship between **log wage rate** (`log(income/hours)`) and **age**—a proxy for experience.

Let us first load the data.

```
# Load the data
census = pd.read_csv("census2000.csv")
census.head()
```

	age	sex	marital	race	education	income	hours
0	48	M	Married	White	3.hsgrad	52000	2600
1	24	M	Divorced	White	2.high	35000	2080
2	19	F	Single	Black	3.hsgrad	2400	240
3	18	M	Single	Black	2.high	6100	1500
4	28	M	Married	Other	4.assoc	22000	2080

다음 단계: census 변수로 코드 생성 추천 차트 보기 New interactive sheet

```
#  give names of all columns
census.columns
```

```
Index(['age', 'sex', 'marital', 'race', 'education', ' income', 'hours'], dtype='object')
```

```
# rename columns " income" to "income"
census.rename(columns={" income": "income"}, inplace=True)
```

```
# summarize numerical variables
census.describe()
```

	age	income	hours
count	31402.000000	31402.000000	31402.000000
mean	40.335361	32099.931629	1973.679638
std	12.843879	37412.099182	703.049875
min	18.000000	1.000000	24.000000
25%	30.000000	12000.000000	1680.000000
50%	40.000000	25000.000000	2080.000000
75%	49.000000	40000.000000	2304.000000
max	93.000000	372000.000000	5096.000000

```
# what levels do categorical variables have
for col in ["sex", "marital", "race", "education"]:
    print(col, census[col].unique())
```

```
sex ['M' 'F']
marital ['Married' 'Divorced' 'Single' 'Separated' 'Widow']
race ['White' 'Black' 'Other' 'Asian' 'NativeAmerican']
education ['3.hsgrad' '2.high' '4.assoc' '5.bachs' '7.profdeg' '8.phd' '6.mstr'
 '1.grade' '0.none']
```

Census Data Variables

- **age** : The age of the individual in years.
- **sex** : The gender of the individual (typically `"M"` for male and `"F"` for female).
- **`marital`** : Marital status (e.g., `"Single"`, `"Married"`, `"Divorced"`, etc.).
- **`race`** : Racial or ethnic background (e.g., `"White"`, `"Black"`, `"Other"`, etc.).
- **`education`** : The highest level of education attained (e.g., `"0.none"`, `"1.grade"`, etc.).
- **`income`** : Total annual income earned by the individual (in dollars).
- **`hours`** : Total number of hours worked in a year.

These variables are used to analyze wage disparities based on age, gender, education, and other demographic factors.

We focus on **active, full-time** workers:

- `hours > 500` : Excludes part-time or sporadic workers.
- `income > 5000` : Removes extremely low earners to avoid distortions.
- `age < 60` : Excludes retirees and older individuals who may work part-time.

```
# Filter workers based on conditions
workers = (census["hours"] > 500) & (census["income"] > 5000) & (census["age"] < 60)
```

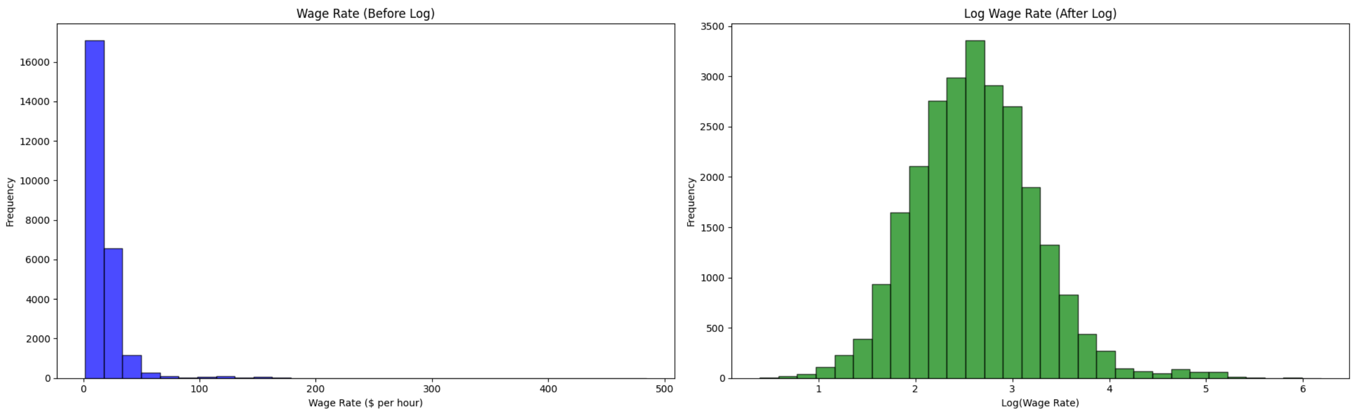
We take the log of the wage rate (income per hour) for several reasons:

- Normalizes the distribution: Income data is typically right-skewed. Taking the log makes it more symmetric.
- Reduces heteroskedasticity: Variability in wages often increases with higher incomes. Logging the wage rate stabilizes variance and makes regression assumptions more valid.

```
# Compute log wage rate
log_WR = np.log(census["income"] / census["hours"])[workers]
```

Compare the histograms

```
# Compute wage rate
WR = (census["income"] / census["hours"])[workers]

# Plot histograms before and after log transformation
fig, axes = plt.subplots(1, 2, figsize=(fig_width*2, fig_height))

# Histogram of raw wage rate
axes[0].hist(WR[workers], bins=30, color="blue", alpha=0.7, edgecolor="black")
axes[0].set_title("Wage Rate (Before Log)")
axes[0].set_xlabel("Wage Rate ($ per hour)")
axes[0].set_ylabel("Frequency")

# Histogram of log wage rate
axes[1].hist(log_WR[workers], bins=30, color="green", alpha=0.7, edgecolor="black")
axes[1].set_title("Log Wage Rate (After Log)")
axes[1].set_xlabel("Log(Wage Rate)")
axes[1].set_ylabel("Frequency")

# Adjust layout and show
plt.tight_layout()
plt.show()
```



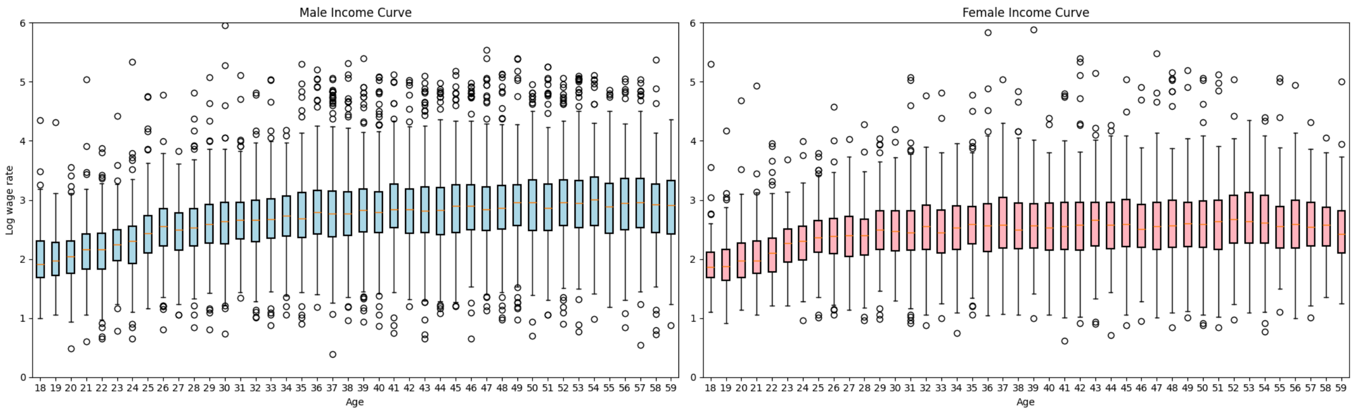
The right histogram looks more "normal".

We will investigate such transformations in more details in the next class.

Next, let us visualize the distribution of log wage rates (`log_WR`) across different age groups for each gender.

```
age = census["age"][workers]
sex = census["sex"][workers]

# Boxplots for male and female income
fig, axes = plt.subplots(1, 2, figsize=(fig_width*2, fig_height))

# Male income boxplot
male_log_WR = log_WR[sex == "M"]
male_age = age[sex == "M"]
box = axes[0].boxplot([male_log_WR[male_age == a] for a in sorted(male_age.unique())],
                positions=sorted(male_age.unique()), patch_artist=True )
# Change box colors
for patch in box['boxes']:
    patch.set(facecolor="lightblue", edgecolor="black", linewidth=1.5)  # Set fill and edge color
axes[0].set_title("Male Income Curve")
axes[0].set_xlabel("Age")
axes[0].set_ylabel("Log wage rate")
axes[0].set_ylim(0, 6)

# Female income boxplot
female_log_WR = log_WR[sex == "F"]
female_age = age[sex == "F"]
box = axes[1].boxplot([female_log_WR[female_age == a] for a in sorted(female_age.unique())],
                positions=sorted(female_age.unique()), patch_artist=True)
# Change box colors
for patch in box['boxes']:
    patch.set(facecolor="lightpink", edgecolor="black", linewidth=1.5)  # Set fill and edge color
axes[1].set_title("Female Income Curve")
```

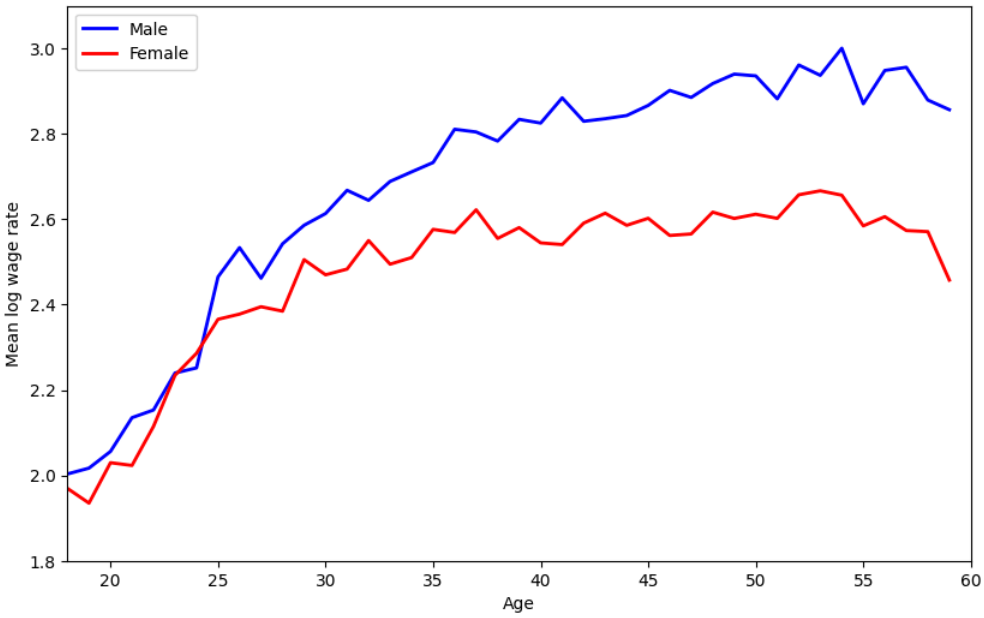
```
axes[1].set_xlabel("Age")
axes[1].set_ylim(0, 6)

plt.tight_layout()
plt.show()
```



Let us obtain a nonparametric fit.

This fit shows the average log wage rate for men and women across different age groups.

```
# Compute mean wage at each age level
men = sex == "M"
malemean = log_WR[men].groupby(age[men]).mean()
femalemean = log_WR[~men].groupby(age[~men]).mean()

# Plot mean log wage rates
plt.figure()
plt.plot(malemean.index, malemean.values, color="blue", linewidth=2, label="Male")
plt.plot(femalemean.index, femalemean.values, color="red", linewidth=2, label="Female")
plt.xlabel("Age")
plt.ylabel("Mean log wage rate")
plt.xlim(18, 60)
plt.ylim(1.8, 3.1)
plt.legend()

plt.show()
```



A discrepancy between mean $\log(WR)$ for men and women.

- Female wages flatten at about 30, while men's keep rising.

Can we build a simple, interpretable model that summarizes the observation above?

What is the **goal**?

When constructing regression models, the objective depends on the type of question we want to answer.

1. Relationship-type questions and inference?
   - Are women paid differently than men on average?
   - Does age/experience differently affect men and women?
2. Data summarization?
   - Matched the dynamics/trends
   - Describe a past phenomenon
3. Prediction?
   - Need a fair, objective criterion that matches the idea of predicting the future. Avoid overfitting.

```
# let us create a new DataFrame that has log_WR, age, men for workers
workers_df = pd.DataFrame({"log_WR": log_WR, "age": age, "men": men})
workers_df.head()
```

	log_WR	age	men
0	2.995732	48	True
1	2.822980	24	True
3	1.402824	18	True
4	2.358675	28	True
6	2.407465	40	False

다음 단계: workers_df 변수로 코드 생성 추천 차트 보기 New interactive sheet

```
def add_means_to_plot(axis, malemean, femalemean):
  """Adds the mean log wage rates for males and females to an existing plot.

  This function takes two pandas Series, `malemean` and `femalemean`,
  representing the mean log wage rates for males and females at different ages,
  and adds them to the current matplotlib plot.  It assumes that the indices
  of these Series correspond to age values and that a plot is already active.

  Args:
      axis: matplotlib axis to plot on. If None, uses current axis.
      malemean: A pandas Series containing the mean log wage rates for males at different ages.
      femalemean: A pandas Series containing the mean log wage rates for females at different ages.
  """
  axis.plot(malemean.index, malemean.values, color="blue", linewidth=2, linestyle="--", label="Male Mean")
  axis.plot(femalemean.index, femalemean.values, color="red", linewidth=2, linestyle="--", label="Female Mean")
  axis.set_xlabel("Age")
  axis.set_ylabel("Mean log wage rate")
  axis.set_xlim(18, 60)
  axis.set_ylim(1.8, 3.1)
  axis.legend()
```

This code defines a function called `add_means_to_plot`. Its purpose is to add lines representing the mean (average) log wage rates for males and females to an existing plot. We will see it in action below.

Model Building

We will build few models together. We start from the simplest possible model.

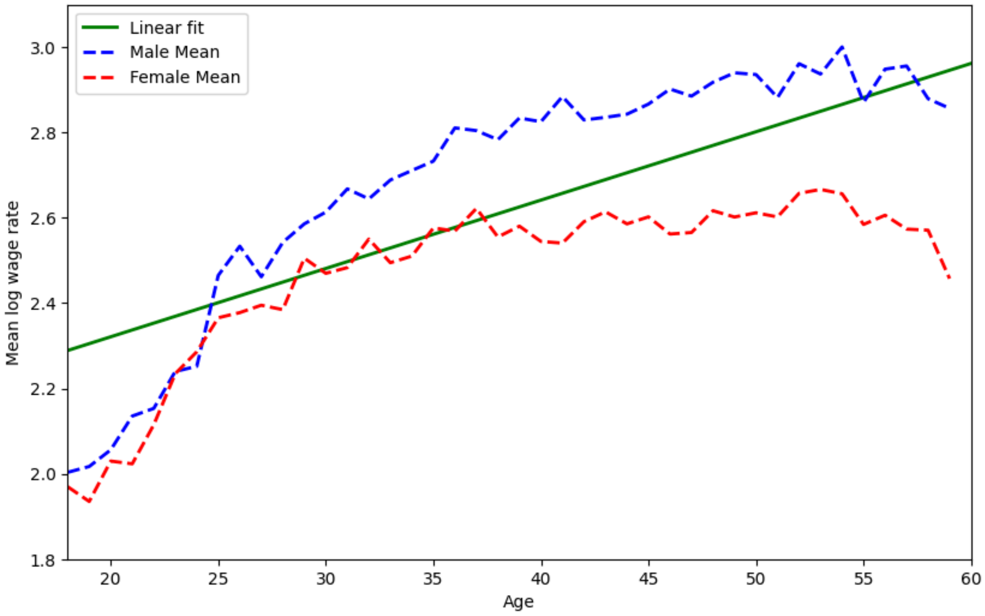
For reference on how to use formulas used in model fitting in Python, you can check https://www.statsmodels.org/dev/example_formulas.html

Simplest Model

First we can start by a simple linear model.

```
grid = np.linspace(18, 60, 100)

# Fit a linear model of log_WR onto age
wagereg1 = smf.ols("log_WR ~ age", data=workers_df).fit()

# visualize the fit
fig, ax = plt.subplots()
```

```
ax.plot(grid, wagereg1.predict(pd.DataFrame({"age": grid})), color="green", linewidth=2, label="Linear fit")
add_means_to_plot(ax, malemean, femalemean)
plt.show()
```



You get one line for both men and women. This model does not provide a good explanation.

```
# print the summary for the fit
print( wagereg1.summary() )
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                 log_WR   R-squared:                       0.074
Model:                            OLS   Adj. R-squared:                  0.074
Method:                 Least Squares   F-statistic:                     2016.
Date:                Thu, 06 Feb 2025   Prob (F-statistic):               0.00
Time:                        04:08:06   Log-Likelihood:                -23321.
No. Observations:               25403   AIC:                         4.665e+04
Df Residuals:                   25401   BIC:                         4.666e+04
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Intercept      2.0006      0.014    139.480      0.000       1.973       2.029
age            0.0160      0.000     44.902      0.000       0.015       0.017
==============================================================================
Omnibus:                     1319.359   Durbin-Watson:                   1.725
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             2445.807
Skew:                           0.398   Prob(JB):                         0.00
Kurtosis:                       4.296   Cond. No.                         152.
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

We have learned the following model.

```
# print E[log(WR)] = intercept + slope x age
intercept = wagereg1.params["Intercept"]
slope = wagereg1.params["age"]
print(f"E[log(WR)] = {intercept:.3f} + {slope:.3f} x age")
```

```
E[log(WR)] = 2.001 + 0.016 x age
```

Adding Sex Effect

Task:

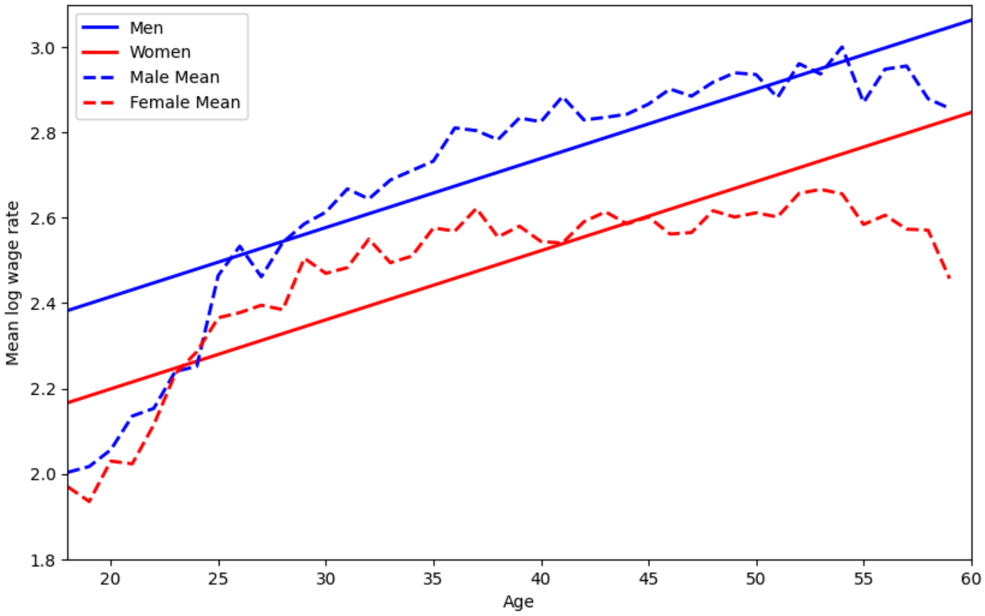
Fit a model that includes the effect of `sex`.

That is, build a model E[log(WR)] = b0 + b1 x age + b2 x ind(men = T)

```
#TODO: build a linear model for log_WR using age and sex
wagereg2 = smf.ols("log_WR ~ age + men", data=workers_df).fit()
```

```
# TODO: Visualize the fit
fig, ax = plt.subplots()
```

```
# Predict for men
ax.plot(grid, wagereg2.predict(pd.DataFrame({"age": grid, "men": True})), color="blue", linewidth=2, label="Men")

# Predict for women
ax.plot(grid, wagereg2.predict(pd.DataFrame({"age": grid, "men": False})), color="red", linewidth=2, label="Women")

# Add the mean lines
add_means_to_plot(ax, malemean, femalemean)
plt.show()
```



In the plot above, you should see that:

- The male wage line is shifted up from the female line.

```
# print the summary for the fit
print(wagereg2.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                 log_WR   R-squared:                       0.103
Model:                            OLS   Adj. R-squared:                  0.103
Method:                 Least Squares   F-statistic:                     1454.
Date:                Thu, 06 Feb 2025   Prob (F-statistic):               0.00
Time:                        04:08:06   Log-Likelihood:                -22914.
No. Observations:               25403   AIC:                         4.583e+04
Df Residuals:                   25400   BIC:                         4.586e+04
Df Model:                           2
Covariance Type:            nonrobust
================================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
--------------------------------------------------------------------------------
Intercept      1.8750      0.015    126.880      0.000       1.846       1.904
men[T.True]    0.2162      0.008     28.748      0.000       0.201       0.231
age            0.0162      0.000     46.124      0.000       0.016       0.017
==============================================================================
Omnibus:                     1292.485   Durbin-Watson:                   1.696
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             2514.597
Skew:                           0.376   Prob(JB):                         0.00
Kurtosis:                       4.345   Cond. No.                         161.
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

Task:

What model did you estimate above?

```
# TODO: print the coefficients of the model E[log(WR)] = b0 + b1 x age + b2 x ind(men = T)
intercept = wagereg2.params["Intercept"]
slope_age = wagereg2.params["age"]
slope_men = wagereg2.params["men[T.True]"]

print(f"E[log(WR)] = {intercept:.3f} + {slope_age:.3f} x age + {slope_men:.3f} x men[T.True]")
```

```
E[log(WR)] = 1.875 + 0.016 x age + 0.216 x men[T.True]
```

Add Interactions between sex and age

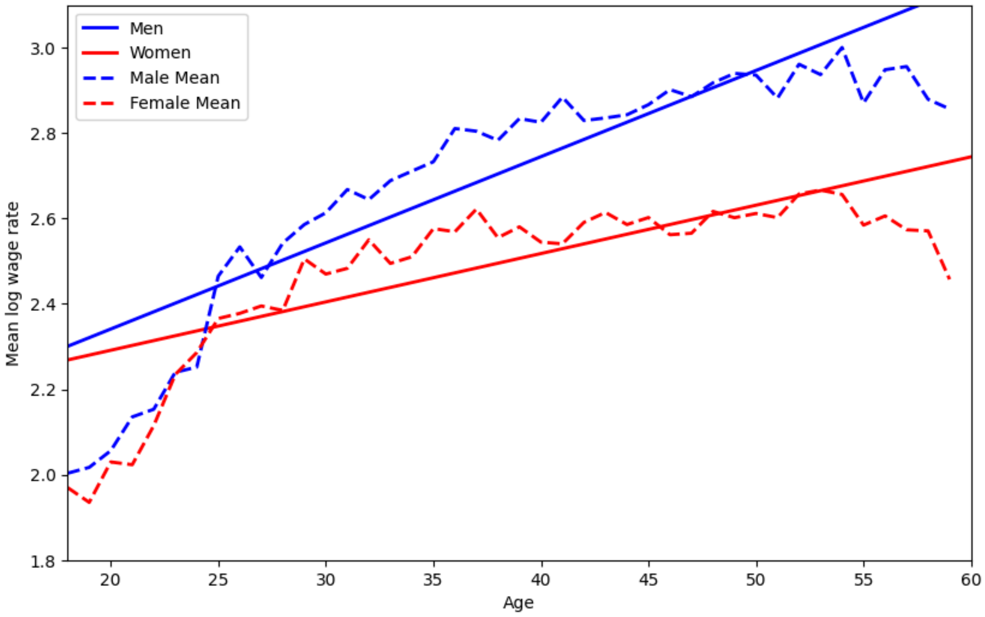
Task:

Fit a model that includes interactions between `sex` and `age`.

That is, build a model E[log(WR)] = b0 + b1 x age + (b2 + b3 x age) x men[T.True]

```
# TODO: Fit a model that includes interactions between `sex` and `age`.
wagereg3 = smf.ols("log_WR ~ age + C(men)*age", data=workers_df).fit()
```

```
# TODO: Visualize the fit
fig, ax = plt.subplots()

# Predict for men
ax.plot(grid, wagereg3.predict(pd.DataFrame({"age": grid, "men": True})), color="blue", linewidth=2, label="Men")

# Predict for women
ax.plot(grid, wagereg3.predict(pd.DataFrame({"age": grid, "men": False})), color="red", linewidth=2, label="Women")

# Add the mean lines
add_means_to_plot(ax, malemean, femalemean)
plt.show()
```



In the plot above, you should see that:

- The interaction term gives us different slopes for each sex.

```
# print the summary for the fit
print(wagereg3.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                 log_WR   R-squared:                       0.108
Model:                            OLS   Adj. R-squared:                  0.108
Method:                 Least Squares   F-statistic:                     1028.
Date:                Thu, 06 Feb 2025   Prob (F-statistic):               0.00
Time:                        04:08:07   Log-Likelihood:                -22835.
No. Observations:               25403   AIC:                         4.568e+04
Df Residuals:                   25399   BIC:                         4.571e+04
Df Model:                           3
Covariance Type:            nonrobust
=======================================================================================
                          coef    std err          t      P>|t|      [0.025      0.975]
---------------------------------------------------------------------------------------
Intercept               2.0649      0.021     97.931      0.000       2.024       2.106
C(men)[T.True]         -0.1275      0.028     -4.503      0.000      -0.183      -0.072
age                     0.0113      0.001     21.700      0.000       0.010       0.012
C(men)[T.True]:age      0.0089      0.001     12.588      0.000       0.007       0.010
==============================================================================
Omnibus:                     1301.800   Durbin-Watson:                   1.696
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             2568.966
Skew:                           0.374   Prob(JB):                         0.00
Kurtosis:                       4.366   Cond. No.                         418.
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

Task:

What model did you estimate above?

```
# TODO: print the coefficients of the model E[log(WR)] = b0 + b1 x age + (b2 + b3 x age) x men[T.True]
intercept = wagereg3.params["Intercept"]
slope_age = wagereg3.params["age"]
slope_men = wagereg3.params["C(men)[T.True]"]
slope_interaction = wagereg3.params["C(men)[T.True]:age"]

print(f"E[log(WR)] = {intercept:.3f} + {slope_age:.3f} x age + ({slope_men:.3f} + {slope_interaction:.3f} x age) x men[T.True]")
```

```
E[log(WR)] = 2.065 + 0.011 x age + (-0.128 + 0.009 x age) x men[T.True]
```

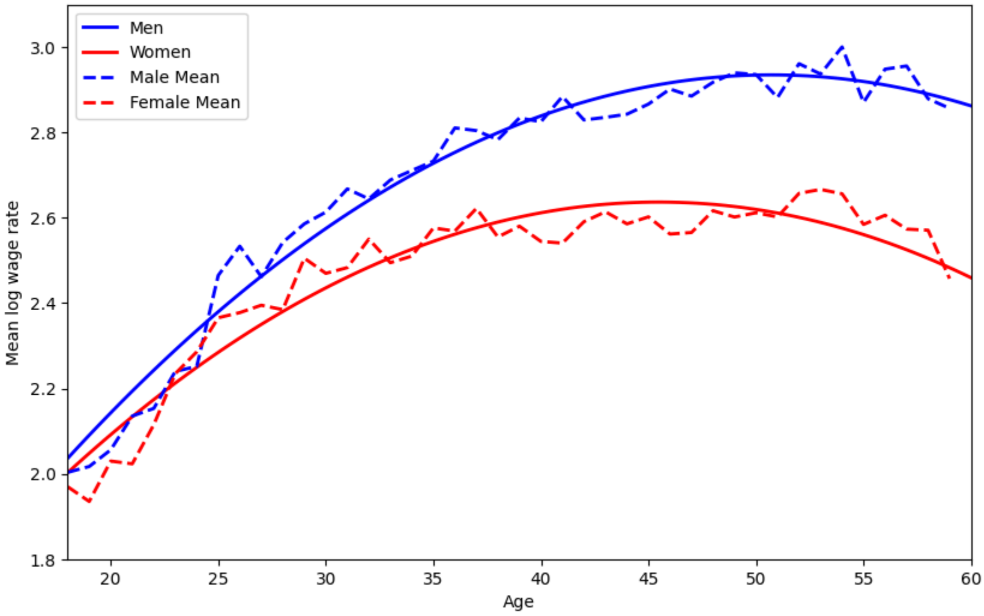
Add quadratics to the model

Task:

Fit a model that also includes a quadratic term age^2.

```
# TODO: Fit a model that includes interactions between `sex` and `age` and quadratic term age^2
wagereg4 = smf.ols("log_WR ~ age + I(age**2) + C(men)*age", data=workers_df).fit()
```

```
# TODO: Visualize the fit
fig, ax = plt.subplots()

# Predict for men
ax.plot(grid, wagereg4.predict(pd.DataFrame({"age": grid, "men": True})), color="blue", linewidth=2, label="Men")

# Predict for women
ax.plot(grid, wagereg4.predict(pd.DataFrame({"age": grid, "men": False})), color="red", linewidth=2, label="Women")

# Add the mean lines
add_means_to_plot(ax, malemean, femalemean)
plt.show()
```



In the plot above, you should see that:

- age^2 allows us to capture a nonlinear wage curve.

```
# print the summary for the fit
print(wagereg4.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                 log_WR   R-squared:                       0.131
Model:                            OLS   Adj. R-squared:                  0.131
Method:                 Least Squares   F-statistic:                     959.8
Date:                Thu, 06 Feb 2025   Prob (F-statistic):               0.00
Time:                        04:08:07   Log-Likelihood:                -22503.
No. Observations:               25403   AIC:                         4.502e+04
Df Residuals:                   25398   BIC:                         4.506e+04
Df Model:                           4
Covariance Type:            nonrobust
======================================================================================
                         coef    std err          t      P>|t|      [0.025      0.975]
--------------------------------------------------------------------------------------
Intercept              0.8989      0.050     18.148      0.000       0.802       0.996
C(men)[T.True]        -0.1246      0.028     -4.459      0.000      -0.179      -0.070
age                    0.0765      0.003     29.831      0.000       0.071       0.081
C(men)[T.True]:age     0.0088      0.001     12.655      0.000       0.007       0.010
I(age ** 2)           -0.0008   3.24e-05    -25.940      0.000      -0.001      -0.001
```

```
==============================================================================
Omnibus:                     1338.503   Durbin-Watson:                   1.709
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             2775.627
Skew:                           0.369   Prob(JB):                         0.00
Kurtosis:                       4.442   Cond. No.                     2.50e+04
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 2.5e+04. This might indicate that there are
strong multicollinearity or other numerical problems.
```

Task:

What model did you estimate above?

```
# TODO: print the coefficients of the model  E[log(WR)] = b0 + b1 x age + b4 x age^2 + (b2 + b3 x age) x men[T.True]
intercept = wagereg4.params["Intercept"]
slope_age = wagereg4.params["age"]
slope_men = wagereg4.params["C(men)[T.True]"]
slope_interaction = wagereg4.params["C(men)[T.True]:age"]
slope_age_squared = wagereg4.params["I(age ** 2)"]

print(f"E[log(WR)] = {intercept:.3f} + {slope_age:.3f} x age + {slope_age_squared:.3f} x age^2 + ({slope_men:.3f} + {slope_interaction:.3f
```

```
E[log(WR)] = 0.899 + 0.076 x age + -0.001 x age^2 + (-0.125 + 0.009 x age) x men[T.True]
```

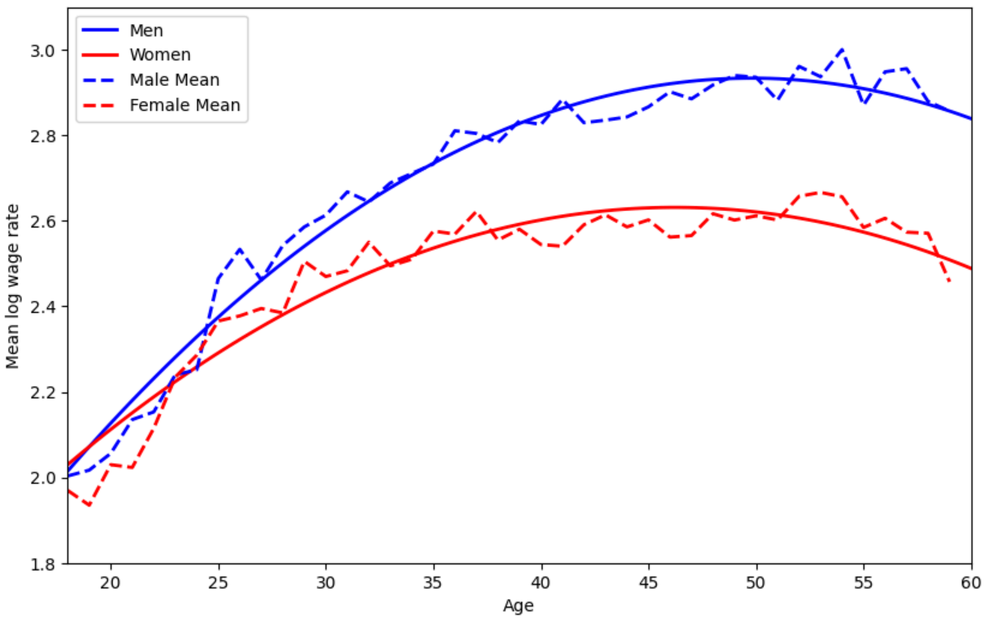
Add an interaction term on the curvature

Task:

Fit a model that also includes an interaction between age^2 and `sex`.

```
# TODO: Fit a model that also includes an interaction between $age^2$ and `sex`.
wagereg5 = smf.ols("log_WR ~ age + I(age**2) + C(men)*(age + I(age**2))", data=workers_df).fit()
```

```
# TODO: Visualize the fit
fig, ax = plt.subplots()

# Predict for men
ax.plot(grid, wagereg5.predict(pd.DataFrame({"age": grid, "men": True})), color="blue", linewidth=2, label="Men")

# Predict for women
ax.plot(grid, wagereg5.predict(pd.DataFrame({"age": grid, "men": False})), color="red", linewidth=2, label="Women")

# Add the mean lines
add_means_to_plot(ax, malemean, femalemean)
plt.show()
```



In the plot above, you should see that:

- This full model provides a generally decent looking fit.

```
# print the summary for the fit
print(wagereg5.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                 log_WR   R-squared:                       0.132
Model:                            OLS   Adj. R-squared:                  0.131
Method:                 Least Squares   F-statistic:                     769.1
Date:                Thu, 06 Feb 2025   Prob (F-statistic):               0.00
Time:                        04:08:08   Log-Likelihood:                -22500.
No. Observations:               25403   AIC:                         4.501e+04
Df Residuals:                   25397   BIC:                         4.506e+04
Df Model:                           5
Covariance Type:            nonrobust
==================================================================================================
                                 coef    std err          t      P>|t|      [0.025      0.975]
--------------------------------------------------------------------------------------------------
Intercept                      1.0189      0.071     14.395      0.000       0.880       1.158
C(men)[T.True]                -0.3391      0.095     -3.584      0.000      -0.525      -0.154
age                            0.0698      0.004     18.290      0.000       0.062       0.077
C(men)[T.True]:age             0.0208      0.005      4.074      0.000       0.011       0.031
I(age ** 2)                   -0.0008   4.88e-05    -15.461      0.000      -0.001      -0.001
C(men)[T.True]:I(age ** 2)    -0.0002   6.53e-05     -2.373      0.018      -0.000   -2.69e-05
==============================================================================
Omnibus:                     1339.467   Durbin-Watson:                   1.709
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             2772.257
Skew:                           0.369   Prob(JB):                         0.00
Kurtosis:                       4.440   Cond. No.                     6.41e+04
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 6.41e+04. This might indicate that there are
strong multicollinearity or other numerical problems.
```

Task:

What model did you estimate above?

```
# TODO: print the coefficients of the estimated model

intercept = wagereg5.params["Intercept"]
slope_age = wagereg5.params["age"]
slope_men = wagereg5.params["C(men)[T.True]"]
slope_interaction = wagereg5.params["C(men)[T.True]:age"]
slope_age_squared = wagereg5.params["I(age ** 2)"]
slope_interaction_squared = wagereg5.params["C(men)[T.True]:I(age ** 2)"]

print(f"E[log(WR)] = {intercept:.2f} + {slope_age:.2f} x age + {slope_age_squared:.4f} x age^2 + ({slope_men:.2f} + {slope_interaction:.2f
```

```
E[log(WR)] = 1.02 + 0.07 x age + -0.0008 x age^2 + (-0.34 + 0.02 x age + -0.00015 x age^2) x men[T.True]
```

Optional

Task:

Consider building a model that has an interaction between age and edu.

```
# add edu into workers_df
workers_df["edu"] = census["education"][workers]
```

```
workers_df["edu"].unique()
```

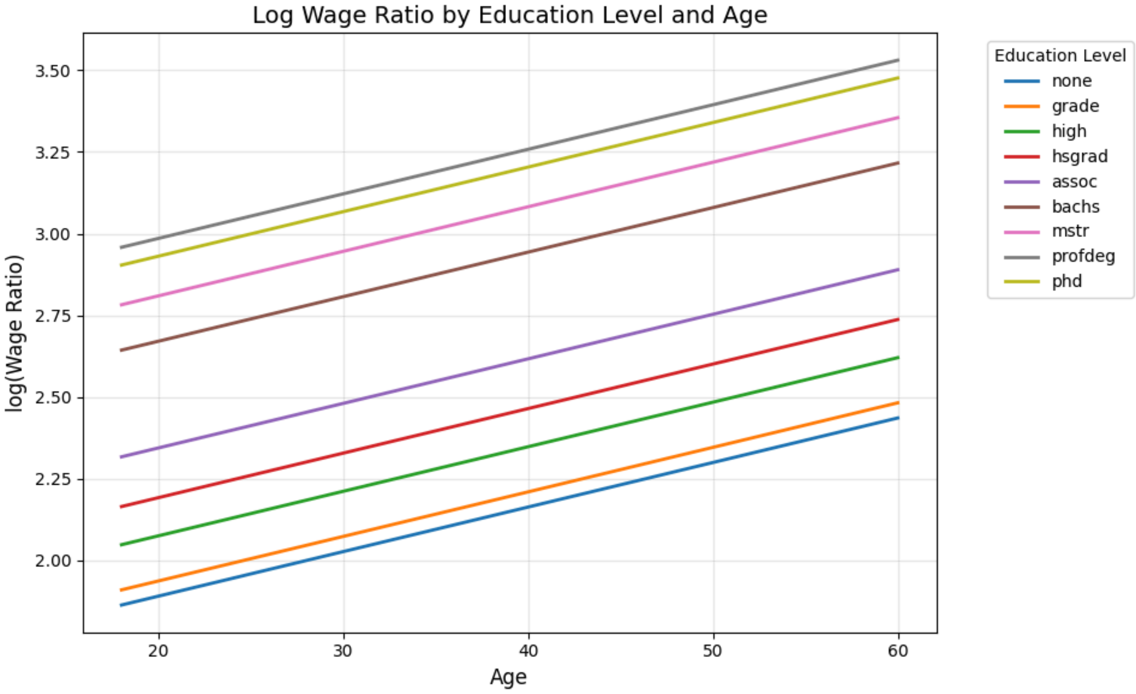
```
array(['3.hsgrad', '2.high', '4.assoc', '5.bachs', '7.profdeg', '8.phd',
       '6.mstr', '1.grade', '0.none'], dtype=object)
```

You can first condsider visualizing average `log_WR` as a function of `edu` and `age`.

```
#TODO: build a linear model for log_WR using edu and age.
wagereg6 = smf.ols("log_WR ~ edu + age", data=workers_df).fit()
```

```
# TODO: Visualize the fit
fig, ax = plt.subplots()

# Define colors for different education levels
colors = ['#1f77b4', '#ff7f0e', '#2ca02c', '#d62728', '#9467bd',
          '#8c564b', '#e377c2', '#7f7f7f', '#bcbd22']

# Education levels in order
edu_levels = ['0.none', '1.grade', '2.high', '3.hsgrad', '4.assoc',
              '5.bachs', '6.mstr', '7.profdeg', '8.phd']

# Plot prediction for each education level
for i, edu in enumerate(edu_levels):
    predictions = wagereg6.predict(pd.DataFrame({
        "age": grid,
        "edu": [edu] * len(grid)
    }))
```

```
    ax.plot(grid, predictions, color=colors[i],
            linewidth=2, label=edu.split('.')[1])

# Customize the plot
ax.set_xlabel("Age", fontsize=12)
ax.set_ylabel("log(Wage Ratio)", fontsize=12)
ax.set_title("Log Wage Ratio by Education Level and Age", fontsize=14)
ax.legend(title="Education Level", bbox_to_anchor=(1.05, 1),
         loc='upper left')
ax.grid(True, alpha=0.3)

plt.tight_layout()
plt.show()
```



Subsequently, try building a simple, interpretable model for the average.

```
# print the summary for the fit
print(wagereg6.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                 log_WR   R-squared:                       0.228
Model:                            OLS   Adj. R-squared:                  0.228
Method:                 Least Squares   F-statistic:                     833.4
Date:                Thu, 06 Feb 2025   Prob (F-statistic):               0.00
Time:                        04:08:09   Log-Likelihood:                -21004.
No. Observations:               25403   AIC:                         4.203e+04
Df Residuals:                   25393   BIC:                         4.211e+04
Df Model:                           9
Covariance Type:            nonrobust
==================================================================================
                     coef    std err          t      P>|t|      [0.025      0.975]
----------------------------------------------------------------------------------
Intercept          1.6174      0.045     36.345      0.000       1.530       1.705
edu[T.1.grade]     0.0464      0.047      0.981      0.326      -0.046       0.139
edu[T.2.high]      0.1847      0.044      4.173      0.000       0.098       0.271
edu[T.3.hsgrad]    0.3015      0.043      6.983      0.000       0.217       0.386
edu[T.4.assoc]     0.4538      0.043     10.523      0.000       0.369       0.538
edu[T.5.bachs]     0.7805      0.043     17.951      0.000       0.695       0.866
edu[T.6.mstr]      0.9193      0.045     20.497      0.000       0.831       1.007
edu[T.7.profdeg]   1.0953      0.050     21.808      0.000       0.997       1.194
edu[T.8.phd]       1.0408      0.056     18.700      0.000       0.932       1.150
age                0.0136      0.000     41.422      0.000       0.013       0.014
==============================================================================
Omnibus:                     1446.835   Durbin-Watson:                   1.845
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             3965.288
Skew:                           0.306   Prob(JB):                         0.00
Kurtosis:                       4.836   Cond. No.                     1.50e+03
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 1.5e+03. This might indicate that there are
strong multicollinearity or other numerical problems.
```

```
# TODO: print the coefficients of the model.
intercept = wagereg6.params["Intercept"]
slope_age = wagereg6.params["age"]

# Extract education coefficients
edu_coeffs = {level: wagereg6.params[f"edu[T.{level}]"] for level in edu_levels[1:]}
```

```
# Print the model equation
print(f"E[log(WR)] = {intercept:.3f} + {slope_age:.3f} × age +")
for edu, coef in edu_coeffs.items():
    print(f"            {coef:.3f} × edu[T.{edu}] +")
```

```
E[log(WR)] = 1.617 + 0.014 × age +
            0.046 × edu[T.1.grade] +
            0.185 × edu[T.2.high] +
            0.301 × edu[T.3.hsgrad] +
            0.454 × edu[T.4.assoc] +
            0.780 × edu[T.5.bachs] +
            0.919 × edu[T.6.mstr] +
            1.095 × edu[T.7.profdeg] +
            1.041 × edu[T.8.phd] +
```