Evaluation of imputation techniques with varying percentage of missing data

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Abstract

Missing data is a common problem which has consistently plagued statisticians and applied analytical researchers. While replacement methods like mean-based or hot deck imputation have been well researched, emerging imputation techniques enabled through improved computational resources have had limited formal assessment. This study formally considers five more recently developed imputation methods – *Amelia, Mice, mi, Hmisc* and *missForest* – compares their performances using RMSE against actual values and against the well established mean-based replacement approach. The RMSE measure was consolidated by method using a ranking approach. Our results indicate that the *missForest* algorithm performed best and the *mi* algorithm performed worst.

Keywords: Missing Data; Missing at Random; Missing Completely at Random; Imputation; Multiple Imputation; MICE; mi; Amelia; missForest; Harrell Miscellaneous; RMSE

1 Introduction

Missing data is a common problem encountered when analyzing data. Efron (1994) defined missing data as a difficult problem due to the absence of some data elements in the familiar data structure. Graham (2009) stated that the missing values significantly impact results drawn from the dataset. There are three major concerns related to missing data reduction in statistical power, biased parameter estimation, and sample not being representative of the population.

Musil, Warner, Yobas, and Jones (2002) imputed missing at random data to compare five different approaches (list-wise deletion, mean substitution, simple regression, regression with an error term, and the expectation maximization [EM] algorithm) and compared the effects on descriptive statistics and correlation coefficients for imputed data and the complete data. They found mean substitution having the least impact whereas the regression with an error term and EM algorithm produced results closest to actual values.

Baker, White, and Mengersen (2014) sought to find an appropriate imputation technique for health survey data and found that mean imputation was more accurate than multivariate normal and conditional autoregressive prior distribution-based imputation. Bono, Ried, Kimberlin, and Vogel (2007) investigated patient data and imputed with item-mean, pearson mean, regression, and hot-deck imputation. Unlike Musil et al. (2002), they found that all imputed values are comparable to the complete case mean values except regression imputation.

On the other hand, Huisman (2000) replaced values with different imputation techniques on 4 different datasets and evaluated performance using Cronbach's alpha (Tatsuoka, Lord, Novick, and Birnbaum (1971)) and Loevinger's H-coefficient (Mokken (1997)). They found

that corrected variable mean (based on ability of the variable, i.e. score based on the observed values in the given variable when compared against the mean score of these values) substitution outperformed other imputation techniques.

Barnes, Lindborg, and Seaman (2006) tested multiple imputation methods (regression-based MI methods, Bayesian least square (BLS), predictive mean matching (PMM), local random residual (LRR), modified propensity scores (MPS), completion score (CS)) and a last observation carried forward on small samples in clinical trials using simulation study and found BLS performed best and CS second best.

Pantanowitz and Marwala (2009) assessed the impact of imputation on missing data with statistical analysis on classification problem. They imputed using random forests, auto-associative neural networks with genetic algorithms, auto-associative neuro-fuzzy configurations, and two random Forests and neural network. However, they did not find the impact of imputations significant.

Myrtveit, Stensrud, and Olsson (2001) evaluated four imputation techniques (listwise deletion(LD), mean imputation (MeI), similar response pattern imputation (SRPI), and full information maximum likelihood (FIML)) from a software cost modeling perspective. They found that FIML performs better when data is not MCAR but the other techniques are biased with MCAR data.

Van Hulse and Khoshgoftaar (2008) applied five imputation techniques (MeI, Regression Imputation (RI), Instance-based learning imputation (RI), REPTree imputation (RTI) and Bayesian multiple imputation (BMI)) on noisy software measurement data. They evaluated imputed values based on the impact of noise on imputation effectiveness using ANOVA and found that BMI and RI had the best results whereas MeI performed worst.

Junninen, Niska, Tuppurainen, Ruuskanen, and Kolehmainen (2004) applied univariate (linear, spline, and nearest neighbor interpolation), multivariate (regression-based imputation (REGEM), nearest neighbor (NN), self-organizing map (SOM) and multi-layer perceptron (MLP)), and hybrid methods using historical simulated missing data patterns on air quality data and evaluated imputed values based on statistical measures - Index of agreement, R^2 , RMSE, and Absolute MSE with bootstrapped standard errors. They found that the performance of multivariate methods can be improved slightly by using hybridization and more substantial multiple imputation where final imputed values are derived from various multivariate imputation results.

Ambler, Omar, and Royston (2007) compared the imputation techniques and evaluated the imputed values using measure of agreement, rank correlation, RMSE, regression-based calibration measure and regression coefficients, and confidence interval coverage. Their experiment started with imputed values for the predictor variable. They found MICE outperformed with respect to the model estimation.

Most formal research to date has been done within a particular context e.g. model, software measurement, classification problem, etc. or application domain e.g. health, survey,

clinical trials, quality measurement, etc. Table1 provides the literature summary. The need to evaluate emerging imputation techniques in generalized contexts motivated this study. In this research, the objective is to evaluate the performance of six different imputation methods based on their imputed values against the original values independent of context. The experiment is designed in such a way that it started with the complete data set then different percentages, ranging from 5% to 25% in increments of 5%, of the data were deleted randomly, and five different imputation techniques were used to impute the missing values. These imputed values were compared against the actual values using RMSE. Section 2 discusses categories of missing data and different imputation techniques used in this experiment. Section 3 describes the data used for the study. Section 4 discusses the design of the experiment. Section 5 discusses the findings of the experiment. Section 6 provides conclusions based on the experiments.

2 Theoretical Concepts

In this study, Missing Completely at Random missing data is considered. Six different imputation methods were used to impute values including single-value imputation using the mean. The six techniques evaluated here are *mean imputation, multiple imputation by chained equation, multiple imputation with diagnostics, amelia, harrell miscellaneous,* and *miss-Forest*. The three assumptions of missingness are *missing at random, missing completely at random,* and *missing not at random*.

2.1 Missing Data

Efron (1994) defined missing data as a problem caused by the absence of a familiar data structure. Rubin (1976) classified missing data into three classes based on the likelihood of missing.

- Missing At Random (MAR) Missing probability may depend on observed data but not on unobserved data (Little and Rubin (2002)), i.e. systematically related to observed data and not to unobserved data. Van Buuren (2012) defined MAR data as when the probability of missing is the same within observed data variables.
- Missing Completely at Random (MCAR) Missing probability is independent of observed and unobserved data (Little and Rubin (2002)). It is considered special case of MAR when there is no systematic difference between the variables with missing data and variables with complete data. Van Buuren (2012) defined MCAR data when the probability of missing is the same for all variables.
- Missing Not At Random (MNAR)- Missing probability does not depend on unobserved data. Mack et al defined MNAR as when missing data is not related to any measurable events or factors (Mack, Su, and Westreich (2018)). Van Buuren (2012) defined MNAR data as when it is neither MAR nor MCAR data.

Table 1: Literature Summary

Techniques Evaluated	Key Findings	Citations
List-wise deletion, Mean imputation, simple regression, regression with an error term and expectation maximization algorithm	- Better performance: Regression with an error term and EM algo- rithm - Worst Performance: mean Impu- tation	Musil et al. (2002)
Mean Imputation, Multivariate Normal and conditional auto-regressive prior distribution	Better Performance: mean ImputationLesson: Choose imputation based on the application	Baker et al. (2014)
Mean Imputation (Item-mean,Pearson mean), regression, and hot-deck imputation	- Better Performance: mean and Hot-deck imputation - Worse Performance: Regression based	Bono et al. (2007)
Random draw substitution, Mean Imputation (item mean imputation, Pearson mean and corrected item mean imputation), item correlation imputation, hotdeck imputation (hot-deck next case and hot-deck nearest neighbor)	- Best Performance: Corrected variable mean	Huisman (2000)
Regression-based Mean Imputation, Bayesian least square(BLS), predictive mean matching(PMM), local random residual(LRR), modified propensity scores(MPS), completion score(CS)) and a last observation carried forward on small samples	- Best Performance: BLS and CS	Barnes et al. (2006)
Random forests, auto-associative neural networks with genetic algorithms, auto-associative neuro-fuzzy configurations, and two random forests and neural network	- Imputation didn't give significant results	Pantanowitz and Mar- wala (2009)
Listwise deletion(LD), mean imputation , similar response pattern imputation (SRPI), and full information maximum likelihood (FIML)	FIML performs better with non MCAR dataOther techniques: Biased with MCAR data	Myrtveit et al. (2001)
Mean Imputation, Regression Imputation (RI), Instance-based learning imputation (RI), REPTree imputation (RTI) and Bayesian multiple imputation(BMI)	- Worse Performance: Mean Im-	Van Hulse and Khosh- goftaar (2008)
Univariate (linear, spline, and nearest neighbor interpolation), multivariate (regression-based imputation (REGEM), nearest neighbor (NN), selforganizing map (SOM) and multi-layer perceptron (MLP)), and hybrid methods using historical simulated missing data	- Lesson: Hybridization improves performance of multivariate methods	Junninen et al. (2004)
Mean imputation, Conditional mean imputation, Multiple Imputations - Hotdecking, Hotdecking by covariate pattern, Hotdecking by observation, Hot- decking including outcome, Multiple imputation by Chained Equation (MICE)	- Best Performance: MICE with least biased estimate - Better Performance: Conditional mean imputation but inappropri- ate for variable selection methods	Ambler et al. (2007)

2.2 Imputation Techniques

Imputation is the process of replacing missing values with appropriate meaningful estimates. Researchers are frequently tempted to delete the observations or variables with missing values, however, this has potential to lead to information loss impacting results. Another consideration is also pairwise deletion in which the analysis is done with complete cases of relevant variables; as a result, the sample size differs for different dependent variables. Six different imputations are investigated, discussed as below:

2.2.1 Mean Imputation

Missing values are replaced with the column-based mean i.e. individual variable mean value. This method is the easiest but not very accurate. It does not take into account the correlation between the explanatory variables. One may also consider median or mode substitution if the data is skewed.

2.2.2 Multiple Imputation by Chained Equation (MICE)

MICE works in iterations in which imputations are done for each variable one by one. Azur, Stuart, Frangakis, and Leaf (2011) defined the approach - First, initiate by replacing all missing values with individual predictor variables, referring to mean imputation as "place holders". Second, to impute a variable, v, mean values are replaced back missing values, variable v is now a dependent variable and is regressed over the other variables working as independent variables, missing values are then replaced by the predictions based on the regression model. Third, To impute other missing variables, imputed values of variable v where v acts as independent variable whereas rest of the variables will be used with mean substitution. Fourth, second & third steps are repeated until each variable is imputed with predictions. These steps are one iteration or cycle. The number of iterations/cycles are repeated, and imputations are updated.

2.2.3 Multiple Imputation with Diagnostics (mi)

Su, Gelman, Hill, and Yajima (2011) stated that *mi* imputation technique is derived from MICE but with one of the key differences that it imputes from conditional distribution of a variable whereas other variables are either imputed or observed. The benefit of mi over MICE is that it has the capability to deal with irregularities in the data, e.g. multicollinearity¹ within a dataset. To impute a variable, the procedure is split into four steps (Su et al. (2011))- First, setup analyzes missing data patterns to recognize problems in data structure, performs preprocessing and identifies conditional models. Second, iterates over MICE based imputations but with a conditional model, and tests imputed values for conditionality, acceptability, and convergence. Third, analysis obtains multiple imputed complete datasets and pools them for the complete case analysis. Fourth, validation analyzes sensitivity, performs cross-validation, and tests for compatibility.

¹A problem where two or more predictors are highly linearly associated

2.2.4 Amelia

Honaker, King, and Blackwell (2011) mentioned that the algorithm assumes complete data that follows a multivariate normal distribution. The algorithm works well if the data is sampled from other distributions (Schafer (1997, 1999)). It imputes based on a Bootstrapping and Expectation-Maximization (EMB) algorithm. First, EMB algorithm draws from the posterior by integrating the EM algorithm with bootstrapping where each bootstrapped sample introduces uncertainty and the EM algorithm finds the mode of the posterior (Dempster, Laird, and Rubin (1977)), which allows for fundamental uncertainty (Honaker and King (2010)). Second, EMB is applied to generate parameters for the complete data. Third, imputes with the values drawn from conditional distribution on observed and complete data parameters, $\theta = (\mu, \Sigma)$ - linear regression with parameters (Honaker et al. (2011)). Fourth, pools the multiple imputations using an average of the estimates.

2.2.5 Harrell Miscellaneous (Hmisc)

Frank E Harrell Jr (2010), and Harrell and Dupont (2016) stated that the algorithm works at simple imputation based on mean/mode/median as well as multiple imputation based on additive regression, bootstrapping, and predictive mean matching approaches. To impute missing values in each variable, First,i nitiate the missing values from randomized sample of non-missing values of size 'm'. Second, fit the flexible additive model to find the optimum transformation. Identity transformation can be forced as well. Third, apply the flexible fitted model to make predictions for the non-missing observed values. Fourth, impute the missing value with the observed value, where the predicted transformed value is nearest to the predicted transformed value of the missing value. Fifth, to impute other variables, randomly draw the imputed values of the variable. For n iterations, the first set of 'x' iterations are the burn-in set.

2.2.6 missForest

Stekhoven (2012) defined this as a non-parametric approach in which variables are pairwise independent. The algorithm is based on the random forest approach (Breiman (2001)). For each variable, *missForest* generates random forest with the observed values and predicts to impute missing values. The algorithm repeats itself until the number of iterations is maximized or stopping criterion is met. Oshiro, Perez, and Baranauskas (2012) recommended forest with trees between 64 and 128.

2.3 Computation

In terms of computation, single-value imputation i.e. *mean* imputation is the fastest algorithm. The algorithms *Amelia*, *mi* and Hmisc imputed quickly. *MICE* algorithm relatively more time to impute. None of these techniques were computationally demanding. However, *missForest* algorithm is computationally intensive and time consuming. For the

purpose of this experiment, minimum recommended number of trees, 64 trees, were generated in forest (Oshiro et al. (2012)). The *missForest* algorithm run on more than 20 cores on GPU in parallel. Despite using the GPU and 20 cores, the single run of one iteration took more than 45 minutes to impute whereas rest of the algorithms run five iterations in couple of minutes on single core to impute.

3 Data

The current study uses data provided by a major U.S. Credit Bureau and includes consumer credit utilization data from 2008, with all personally identifiable fields omitted prior to receipt. The multiple data sets were appropriately merged resulting in 1.2 million records with 152 input variables and one binary dependent variable, deciding factor whether credit should be given or not. Missing values are part of the data: columns with more than 40% of missing data were removed from the analysis. For the purpose of this study, only complete records were considered. The data set was a reduced to almost a quarter of the data - 329,917 records with 153 variables. Finally, the data was reduced to 329,917 records with the 21 most prominent variables using LASSO with hyper-parameter λ chosen to be 0.01 using 10-fold cross-validation. For the analysis, only 21 selected variables were used. All variables are continuous.

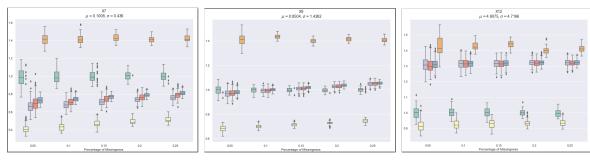
4 Experiment Design

The experiment is designed to discover the impact of the percent of missing data on the choice of imputation algorithm. The experiment is conducted on a data set with 329,917 records with 21 predictor variables. The experiment is designed such that 50 random samples of 50,000 records were generated from the data. Percentage of the data is deleted from each variable using the MCAR^{2,3} approach - 5%, 10%, 15%, 20%, and 25%. Each sample is standardized using the mean and standard deviation from the data with missing values (ignoring missing value). Standardization is done on the random sample after the MCAR process is applied. For calculation of RMSE, the actual values are also standardized with parameters based on the data with missing values. Missing values are imputed based on six different approaches - mean, MICE, mi, Amelia, Hmisc, and missForest. For each pair of missing percentage and imputation method, 5 iterations were computed apart from mean as it will remain constant for each iteration. RMSE measure of difference between the imputed values and actual values is used to compare the various imputation approaches.

²Amelia Algorithm assumes the missingness to MAR, but there is no way to check if the missing data is MAR. Therefore, in order to ensure MAR -special case of MAR applied, MCAR

³MCAR behavior in the data can be tested by turning the dataset into a binary dummy dataset where missing value takes 1 otherwise 0 and applying Welsh's t-test on each pair of variables in the dummy dataset.





(a) Mean substitution under-(b) Mean substitution performs at (c) Mean substitution out-performs par

Figure 1: BoxPlots based on RMSE Results

5 Results

The study showed that the percentage of missing data does not impact the choice of algorithm. For each imputation technique, RMSE was computed to analyze the performance of each iteration with respect to algorithm and missing percentage.

To visualize the results, box-plots were generated for each variable to show the performance of each imputation algorithm corresponding to different missing percentages. Figure 1 shows three of them to illustrate results The best performing algorithm was found to be *missForest*. However, it is computationally intensive and time consuming. The algorithm consistently gave the lowest RMSE in each case - implying the imputations were closest with *missForest* algorithm. The worst performing algorithm was found to be *mi*. The algorithm consistently gave the highest RMSE in each case - implying the imputations were farthest with mi algorithm. Three algorithms, MICE, Amelia and Hmisc, performed with minimal differences, indicating the imputations based on either of these approaches would not be far from each other. Always performed better than mi algorithm but worse than missForest. One of the three algorithms can be chosen for analysis as their results are not far from each other. Mean substitution either under-performed(Fig-1(a) or outperformed(Fig-1(c)) or was at par(Fig-1(b) with MICE, Amelia, and Hmisc algorithms. Mean substitution outperforms for 14 variables. Mean substitution underperforms for six variables. Mean substitution performs almost at par for one variable. The boxplots of other variables are shown in Appendix B.

In order to further explore the imputed values, an ANOVA test was applied to test if the imputations using MICE, Amelia, and Hmisc are statistically similar for each of 21 variables. *MICE*, *Amelia*, and *Hmisc* imputations are found to be statistically similar for four out of 21 variables. *MICE* and *Amelia* imputations are found to be statistically similar for five out of 21 variables. *MICE* and *Hmisc* imputations are found to be statistically similar for 11 out of 21 variables. *Amelia* and *Hmisc* imputations are found to be statistically similar

Table 2: Results consolidated based on Ranks 25% Missing Data 5% Missing Data 15% Missing Data Mean Mean Standard Deviation Standard Deviation Mean Standard Deviation Method/Rank Average Median Average Median Average Median Average Median Average Median Average Median Mean **MICE** 2 3 2 3 3 3 2 3 3 3 Mi 6 6 6 5 6 3 4 4 Amelia 4 4 Hmisc 3 3 3 3 -3 3 3 3 3 3 missForest 5 5 6 6

for five out of 21 variables.

For each variable, the multiple measures – mean, median, Q1, Q2, Q3, Q4 and standard deviation – were computed. These RMSE measures for six imputation techniques – computed for 5%, 15%, and 25% – given in Appendix A. Ranks for mean and standard deviation are assigned to each imputation technique ranging from one to six for each variable where Rank one indicates best and six indicates worst. These results are further consolidated based on these ranks. For each imputation technique, ranks were consolidated in Table 2 using average and median. The table shows that the *missForest* algorithm performs best and the *Mi* algorithm performs worst w.r.t. average and median of ranks of algorithms against RMSE values despite the wide range of its standard deviation indicated by higher rank of average and median of standard deviation.

6 Conclusion

The study was conducted to determine if there is an impact on choice of algorithm with change in missing percentage in the dataset. In this study, 50 random samples with 50,000 records were collected from a complete dataset with more than 300,000 records. The missing data was generated using the MCAR approach with varying percentages of missingness - 5%, 10%, 15%, 20%, and 25%. The missing data was imputed with five different algorithms in addition to mean-based imputation; five iterations were computed for each algorithm and missing percentage of missingness. The results showed that there is no impact on choice of algorithm with change in missing percentage. Based on RMSE, the study showed that *missForest* algorithm performed best whereas *mi* algorithm performed wost irrespective of the missing percentage. The performance of *MICE*, *Amelia*, and *Hmisc* was close to each other - RMSE of *MICE* & *Hmisc* are statistically similar for a little more than 50% of variables, and RMSE of *MICE* & *Amelia* and *Amelia* & *Hmisc* are statistically similar for 25% of variables based on ANOVA test results. These three algorithms and *mean* imputations always performed better than the *mi* algorithm but worse than the *missForest* algorithm.

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Appendices

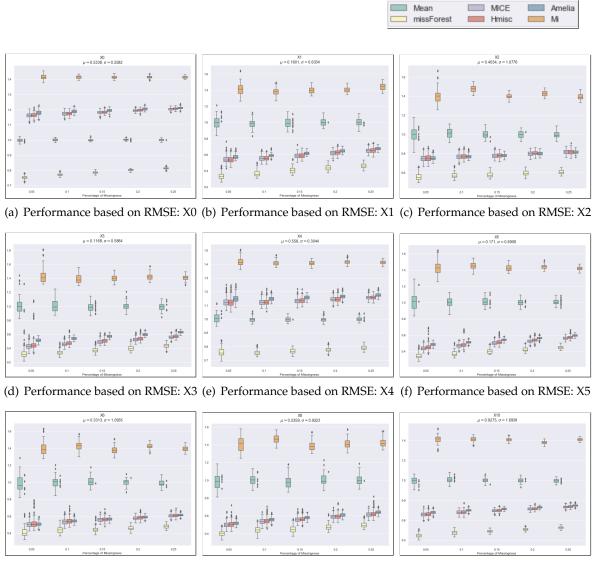
A Results based on RMSE

	ρ	0.0065	0.0076	0.0076	0.0080	0.0077	0.0135	0.0496	0.0277	0.0398	0.0185	0.0337	0.0791	0.0400	0.0236	0.0324	0.0178	0.0256	0.0220	0.0583	0.0272	0.0403	0.0185	0.0309	0.0888	0.0158	0.0158	0.0118	0.0149	0.0160	0.0401	0.0334	0.0196	0.0242	0.0147	0.0209	0.1379	0.0391	0.0210	0.0282	0.0172	0.0216
	Q4	1.0136	1.2245	1.4295	1.2378	1.2264	0.9999	1.1061	0.7330	1.5292	0.7263	0.7468	0.9984	1.0876	0.8743	1.4673	8098.0	0.9054	0.6520	1.1043	0.6257	1.4910	0.6718	0.6641	1.0945	1.0425	1.2146	1.4416	1.2269	1.2002	1.0414	1.0856	0.6115	1.4712	0.6342	0.6650	1.0856	1.0861	0.6716	1.4645	0.6612	0.6891
ntage	Q3	1.0038	1.2084	1.4197	1.2158	1.2110	0.8162	1.0366	0.6675	1.4718	0.6881	0.6753	0.4834	1.0185	0.8315	1.4138	0.8260	0.8324	0.6217	1.0243	0.5749	1.4308	0.6422	0.5914	0.4553	1.0084	1.1655	1.4215	1.1837	1.1686	0.8018	1.0277	0.5760	1.4350	0.6047	0.5845	0.4640	1.0148	0.6171	1.4128	0.6225	0.6241
25% Missing Percentage	Q2	0.9950	1.1976	1.4072	1.2060	1.2000	0.8090	9696.0	0.6297	1.4059	0.6607	0.6293	0.4408	0.9722	0.7984	1.3711	0.8000	0.7949	0.5885	0.9556	0.5423	1.3838	0.6173	0.5518	0.4133	0.9884	1.1455	1.4032	1.1653	1.1466	0.7800	0.9865	0.5479	1.4029	0.5849	0.5605	0.4324	0.9662	0.5910	1.3735	0.6033	0.5949
25% Mis	Q1	0.9831	1.1802	1.3988	1.1886	1.1865	0.7967	0.8838	0.5845	1.3606	0.6281	0.5700	0.3996	0.9250	0.7553	1.3329	0.7609	0.7500	0.5621	0.8446	0.4918	1.3047	0.5806	0.5170	0.3502	0.9715	1.1208	1.3806	1.1397	1.1164	0.7590	0.9350	0.5160	1.3627	0.5603	0.5305	0.4107	0.9021	0.5517	1.3311	0.5723	0.5577
	Median	1.0009	1.2043	1.4140	1.2110	1.2050	0.8131	0.9977	0.6471	1.4469	0.6740	0.6524	0.4596	0.9910	0.8161	1.3873	0.8121	0.8158	0.6000	0.9912	0.5553	1.4096	0.6271	0.5717	0.4347	0.9992	1.1567	1.4119	1.1747	1.1582	0.7894	1.0028	0.5631	1.4143	0.5962	0.5720	0.4479	0.9768	0.6035	1.3917	0.6117	0.6058
	ή	1.0000	1.2034	1.4141	1.2110	1.2053	0.8129	1.0046	0.6486	1.4400	0.6745	0.6535	0.4722	0.9958	0.8150	1.3924	0.8133	0.8151	0.6047	0.9894	0.5578	1.4073	0.6287	0.5748	0.4433	1.0006	1.1566	1.4120	1.1757	1.1584	6962'0	1.0062	0.5628	1.4182	0.5952	0.5730	0.4825	0.9854	0.6050	1.3927	0.6144	0.6088
	ο	0.0073	0.0100	6600.0	0.0103	0.0101	0.0077	0.0573	0.0322	0.0394	0.0193	0.0313	0.1592	0.0409	0.0270	0.0271	0.0184	0.0261	0.0430	9290.0	0.0316	0.0496	0.0218	0.0324	0.1470	0.0183	0.0206	0.0153	0.0162	0.0209	0.0233	0.0543	0.0238	0.0391	0.0165	0.0312	0.1311	0.0558	0.0259	0.0431	0.0187	0.0279
	Q4	1.0241	1.2071	1.4390	1.2188	1.2151	0.8036	1.1306	0.7217	1.4943	0.6740	0.6815	1.1060	1.1026	0.8577	1.4599	0.8229	0.8416	0.9715	1.1436	0.6166	1.5146	0.6284	0.6175	1.0732	1.0380	1.2198	1.4429	1.1939	1.2172	0.9975	1.1225	0.5765	1.5155	0.5839	0.6399	1.1225	1.1764	0.6453	1.5098	0.6100	0.6839
ıtage	Q 3	1.0015	1.1879	1.4200	1.2009	1.1902	0.7901	1.0407	0.6089	1.4259	0.6308	0.6123	0.4330	1.0263	0.7932	1.4139	0.7888	0.7937	0.5933	1.0243	0.5111	1.4315	0.5867	0.5262	0.3937	1.0090	1.1438	1.4197	1.1681	1.1481	0.7805	1.0489	0.5206	1.4536	0.5525	0.5312	0.4191	1.0452	0.5755	1.4032	0.5777	0.5787
15% Missing Percentage	Q2	0.9938	1.1749	1.4060	1.1872	1.1757	0.7808	0.9490	0.5641	1.3648	0.6019	0.5644	0.3741	0.9685	0.7577	1.3816	0.7653	0.7601	0.5540	0.9429	0.4715	1.3674	0.5554	0.4849	0.3503	0.9838	1.1184	1.3972	1.1456	1.1204	0.7526	0.9721	0.4865	1.3937	0.5323	0.4970	0.3770	0.9697	0.5417	1.3456	0.5521	0.5436
15% Mis	Q1	0.9863	1.1447	1.3916	1.1563	1.1611	0.7684	0.8822	0.5060	1.3258	0.5762	0.5198	0.3426	0.9255	0.7208	1.3359	0.7318	0.7118	0.5307	0.8627	0.4362	1.3057	0.5211	0.4367	0.3008	0.9661	1.0807	1.3686	1.1194	1.0877	0.7277	0.9119	0.4557	1.3585	0.5041	0.4527	0.3579	0.9085	0.5013	1.2868	0.5152	0.4821
	Median	0966.0	1.1814	1.4123	1.1952	1.1828	0.7854	0.9894	0.5850	1.3904	0.6149	0.5852	0.4027	0.9916	0.7726	1.3952	0.7762	0.7766	0.5695	0.9856	0.4889	1.3998	0.5705	0.5047	0.3720	0.9954	1.1328	1.4059	1.1570	1.1346	0.7623	1.0060	0.5010	1.4230	0.5425	0.5116	0.3946	1.0012	0.5573	1.3725	0.5650	0.5582
	π	0.9982	1.1812	1.4123	1.1941	1.1835	0.7858	0.9915	0.5870	1.3932	0.6165	0.5886	0.4477	6266:0	0.7769	1.3990	0.7780	0.7779	0.5775	0.9864	0.4940	1.4003	0.5714	0.5070	0.4099	9966.0	1.1323	1.4083	1.1566	1.1353	0.7659	1.0078	0.5038	1.4247	0.5424	0.5176	0.4268	1.0085	0.5584	1.3756	0.5655	0.5618
	ο	0.0115	0.0156	0.0176	0.0161	0.0156	0.0411	0.0788	0.0455	90200	0.0286	0.0420	0.1364	0.0793	0.0360	0.0745	0.0232	0.0369	0.0915	0.1333	0.0428	0.1011	0.0237	0.1042	0.1306	0.0365	0.0341	0.0274	0.0285	0.0350	0.0499	0.1016	0.0282	0.0804	0.0190	0.0495	0.0950	0.0975	0.0361	0.0745	0.0265	0.0692
	Q4	1.0234	1.2033	1.4544	1.2285	1.2151	1.0013	1.2137	0.7632	1.6400	82.29	0.6717	1.1355	1.1756	0.8519	1.6650	0.8310	0.8685	1.0808	1.4417	0.5921	1.8115	0.5803	1.0809	1.3226	1.1145	1.2487	1.5046	1.2546	1.2479	1.0583	1.2897	0.5284	1.6410	0.5380	0.6915	1.2193	1.2866	0.6347	1.6296	0.5932	0.7897
tage	8	1.0047	1.1725	1.4265	1.1895	1.1732	0.7596	1.0455	0.5665	1.4550	0.5895	0.5637	0.3569	1.0477	0.7703	1.4314	0.7666	0.7745	0.5776	1.0606	0.4539	1.4730	0.5305	0.4674	0.3473	1.0302	1.1392	1.4335	1.1650	1.1385	0.7784	1.0762	0.4572	1.4735	0.4986	0.4690	0.3668	1.0532	0.5259	1.4403	0.5218	0.5352
5% Missing Percentage	Q2	9066.0	1.1517	1.4028	1.1675	1.1541	0.7464	0.9430	0.5130	1.3593	0.5539	0.5086	0.3069	0.9509	0.7232	1.3487	0.7352	0.7245	0.5249	0.9295	0.4010	1.3537	0.4967	0.4131	0.2847	0.9791	1.1021	1.3970	1.1297	1.1000	0.7392	0.9286	0.4206	1.3703	0.4713	0.4320	0.3229	0.9187	0.4806	1.3375	0.4919	0.4805
5% Mis	Q1	0.9735	1.1130	1.3757	1.1324	1.1220	0.7213	0.8353	0.4323	1.2638	0.5239	0.4422	0.2594	0.8082	0.6594	1.2577	0.6933	0.6643	0.4938	0.8194	0.3251	1.3174	0.4474	0.3299	0.2151	0.9467	1.0614	1.3615	1.0849	1.0516	0.6931	0.8371	0.3798	1.2555	0.4348	0.3725	0.2788	0.8195	0.4261	1.2748	0.4380	0.4312
	Median	0.9961	1.1626	1.4123	1.1780	1.1633	0.7542	9266.0	0.5342	1.4129	0.5676	0.5340	0.3309	0.9954	0.7435	1.3986	0.7503	0.7500	0.5455	0.9971	0.4224	1.4060	0.5123	0.4349	0.3142	1.0023	1.1193	1.4106	1.1464	1.1184	0.7525	1.0158	0.4382	1.4256	0.4829	0.4475	0.3395	0.9590	0.5012	1.3821	0.5066	0.5031
	π	0.9972	1.1617	1.4140	1.1784	1.1639	0.7597	0.9985	0.5413	1.4107	0.5739	0.5389	0.3596	0.9964	0.7471	1.4014	0.7528	0.7505	0.5675	1.0174	0.4275	1.4310	0.5136	0.4579	0.3387	1.0067	1.1240	1.4159	1.1495	1.1232	0.7635	1.0153	0.4408	1.4287	0.4855	0.4569	0.3544	0.9892	0.5050	1.3959	0.5083	0.5197
	Method	Mean	MICE	Mi	Amelia	Hmisc	missForest	Mean	MICE	Mi	Amelia	Hmisc	missForest	Mean	MICE	Mi	Amelia	Hmisc	missForest	Mean	MICE	Mi	Amelia	Hmisc	missForest	Mean	MICE	Mi	Amelia	Hmisc	missForest	Mean	MICE	Mi	Amelia	Hmisc	missForest	Mean	MICE	Mi	Amelia	Hmisc
			į		A	-	mis		1		A	ш,	mis		I		A		mi		ľ		A	-	miš		į		A				J		A	-	mis		1		A	H
	Variable	X0						X						X2						X3						X4						X5						9X				

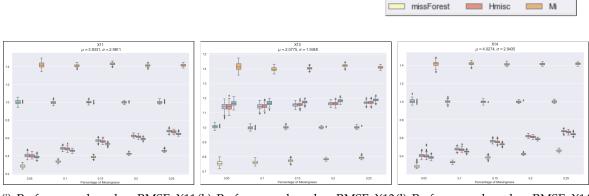
				5% Mis	5% Missing Percentage	ıntage					15% Miss	15% Missing Percentage	tage					25% Miss	25% Missing Percentage	tage		
Variable	Method	π	Median	Q1	Q2	8	Q4	σ	ή	Median	Q1	Ğ	Q3	Q4	ο	μ]	Median	Ŏ1	Q2	Q3	Q4	ο
	missForest	0.4284	0.3932	0.3224	0.3746	0.4408	1.0874	0.1187	0.4480	0.4394	0.3815	0.4266	0.4584	8286.0	0.0499	0.4847	0.4769	0.4327	0.4595	0.4942	1.0156	0.0597
X7	Mean	0.9928	0.9839	0.7673	0.9167	1.0594	1.1870	0.0937	8666.0	0.9889	0.8818	0.9524	1.0398	1.1414	0.0634	1.0002	0.9934	0.8868	9096.0	1.0342	1.1326	0.0544
	MICE	0.6641	0.099	0.5133	0.6152	0.7024	0.9439	0.0657	0.7131	0.7101	0.6127	0.6818	0.7427	0.8248	0.0433	0.7596	0.7567	0.6674	0.7366	0.7850	0.8622	0.0357
	Mi	1.4151	1.4132	1.2803	1.3655	1.4545	1.5587	9290.0	1.4304	1.4193	1.3437	1.4000	1.4626	1.5212	0.0445	1.4188	1.4121	1.3292	1.3951	1.4495	1.5271	0.0444
	Amelia	0.7309	0.7322	0.6488	0.6992	0.7602	0.8242	0.0379	0.7708	0.7707	0.7130	0.7519	0.7889	0.8300	0.0266	0.8105	9608.0	0.7468	0.7937	90880	9888.0	0.0259
	Hmisc	0.6959	0.6923	0.5355	0.6402	0.7462	1.0265	0.0779	0.7469	0.7450	0.6084	0.7184	0.7753	0.8831	0.0474	0.7873	0.7901	0.6574	0.7636	0.8093	0.9128	0.0389
	missForest	0.4284	0.4023	0.3286	0.3821	0.4320	1.1307	0.1191	0.5043	0.4589	0.3742	0.4456	0.4960	1.1414	0.1439	0.5184	0.5071	0.4500	0.4928	0.5381	0.9972	0.0588
X8	Mean	0.9913	0.9868	0.8116	0.9169	1.0447	1.1847	0.0930	0.9809	0.9725	0.8647	0.9232	1.0234	1.1813	0.0746	1.0013	0.9949	0.8995	0.9600	1.0383	1.1874	0.0594
	MICE	0.4993	0.4954	0.4206	0.4757	0.5166	0.6440	0.0334	0.5580	0.5534	0.4898	0.5381	0.5757	0.6752	0.0314	0.6141	0.6110	0.5435	0.5943	0.6326	0.7024	0.0291
	Mi	1.4089	1.4161	1.2495	1.3355	1.4684	1.5783	0.0882	1.3875	1.3776		1.3446		1.5442	0.0544	1.4204	1.4150	1.3423	1.3864	1.4521	1.5542	0.0441
	Amelia	0.5177	0.5153	0.4657	0.5026	0.5301	0.5954	0.0230	0.5793	0.5785				9/£9.0	0.0218	0.6392	0.6385	0.5870	0.6261	0.6535	0.7039	0.0225
	Hmisc	0.5057	0.5025	0.4244	0.4797	0.5251	0.7181	0.0374	0.5652	0.5603				0.6746	0.0343	0.6229	0.6175	0.5461	0.5989	0.6374	0.8167	0.0375
	missForest	0.4190	0.4016	0.3264	0.3803	0.4195	1.0568	0.1129	0.4498	0.4474		0.4199	0.4725	0.5642	0.0396	0.4990	0.4915	0.4433	0.4749	0.5164	0.9937	0.0453
6X	Mean	0.9971	1.0018	0.9087	0.9717	1.0239	1.0865	0.0388	0966.0	0.9962		0.9833	1.0110	1.0342	0.0196	9666:0	1.0003	0.9628	0.9879	1.0095	1.0471	0.0179
	MICE	0.9699	0.9693	0.8860	0.9468	0.9886	1.0546	0.0302	1.0100	1.0096	0.9492	9866.0	1.0220	1.0557	0.0181	1.0474	1.0480	1.0039	1.0372	1.0582	1.0926	0.0152
	Mi	1.4108	1.4121	1.2964	1.3766	1.4403	1.5301	0.0486	1.3989	1.4001	1.3568	1.3864	1.4110	1.4402	0.0186	1.4059	1.4055	1.3662	1.3938	1.4194	1.4547	0.0182
	Amelia	0.9805	0.9800	0.9271	0.9679	0.9949	1.0534	0.0204	1.0183	1.0182	0.9815	1.0103	1.0266	1.0573	0.0136	1.0552	1.0552	1.0154	1.0465	1.0632	1.0856	0.0125
	Hmisc	0.9721	0.9701	0.8866	0.9530	0.9919	1.0675	0.0286	1.0110	1.0104	0.9500	0.9997	1.0237	1.0556	0.0174	1.0493	1.0497	1.0017	1.0375	1.0610	1.0937	0.0165
	missForest	0.6838	0.6815	0.6195	0.6638	0.6995	0.9794	0.0392	0.7159	0.7145	0.6832	0.7051	0.7213	1.0147	0.0274	0.7504	0.7455	0.7087	0.7336	0.7563	1.0229	0.0416
X10	Mean	0.9935	0.9992	0.9036	0.9699	1.0213	1.0628	0.0395	1.0016	1.0016	0.9492	0.9865	1.0148	1.0598	0.0235	0.9938	0.9942	0.9539	0.9810	1.0068	1.0366	0.0188
	MICE	0.6581	0.6560	0.5934	0.6434	0.6737	0.7361	0.0224	0.6951	0.6949	0.6541	0.6871			0.0145	0.7338	0.7331	0.6999	0.7254	0.7421	0.7691	0.0128
	Mi	1.4102	1.4139	1.2939	1.3864	1.4345	1.5169	0.0403	1.4080	1.4087	1.3678	1.3960	1.4176	1.4512	0.0168	1.4079	1.4068	1.3786	1.3988	1.4171	1.4373	0.0131
	Amelia	0.6804	0.6818	0.6321	8299.0	0.6904	0.7294	0.0165	0.7144	0.7146			0.7225		0.0109	0.7483	0.7484	0.7221	0.7419	0.7540	0.7808	0.0103
	Hmisc	0.6612	0.6621	0.6017	0.6465	0.6745	0.7432	0.0225	0.6980	0.6977				0.7434	0.0147	0.7374	0.7358	0.7056	0.7279	0.7466	0.7792	0.0135
	missForest	0.4572	0.4450	0.4024	0.4333	0.4612	1.0017	0.0748	0.4934	0.4911				0.9592		0.5401	0.5267	0.4967	0.5168	0.5350	1.0202	0.0832
X11	Mean	1.0012	0.9995	0.9414	0.9841	1.0208	1.0571	0.0252	1.0018	1.0004			1.0106	1.0420		1.0011	0.9999	0.9784	9886.0	1.0099	1.0381	0.0145
	MICE	0.4052	0.4017	0.3559	0.3901	0.4194	0.4698	0.0216	0.5699	0.5689				0.6146		92290	0.6770	0.6413	0.6694	0.6859	0.7206	0.0122
	Mi	1.4172	1.4168	1.3442	1.3944	1.4413	1.4939	0.0299	1.4298	1.4282		1.4234		1.4630		1.4145	1.4132	1.3838	1.4054	1.4249	1.4458	0.0142
	Amelia	0.3872	0.3864	0.3474	0.3779	0.3972	0.4380	0.0151	0.5340	0.5342		0.5284		0.5594	9600.0	0.6441	0.6441	0.6209	0.6365	0.6499	0.6880	0.0104
	Hmisc	0.3997	0.3981	0.3649	0.3868	0.4118	0.4493	0.0173	0.5602	0.5594	0.5250	0.5503		0.6033	0.0151	0.6671	0.6672	0.6360	0.6564	0.6791	0.6993	0.0142
	missForest	0.2979	0.2847	0.2516	0.2728	0.2981	1.0180	0.0924	0.4012	0.3860		0.3805	0.3938	1.0169	0.0940	0.4690	0.4569	0.4357	0.4506	0.4631	1.0261	0.0777
X12	Mean	1.0022	1.0001	0.9252	0.9691	1.0249	1.1141	0.0403	1.0006	1.0033		96260		1.0643	0.0309	0.9957	0.9950	0.9601	0.9762	1.0136	1.0544	0.0220
	MICE	1.3083	1.3056	1.1982	1.2775	1.3395	1.4271	0.0448	1.3161	1.3168	1.2297	1.2967		1.3823	0.0284	1.3201	1.3208	1.2600	1.3054	1.3350	1.3807	0.0226
	Mi	1.4218	1.4102	1.2939	1.3844	1.4789	1.5636	0.0601	1.4389	1.4416	1.3828	1.4238		1.4860	0.0224	1.4104	1.4084	1.3634	1.3959	1.4252	1.4686	0.0229
	Amelia	1.3109	1.3077	1.2270	1.2891	1.3301	1.4122	0.0335	1.3179	1.3191	1.2477	1.3019	1.3336	1.3843	0.0229	1.3227	1.3232	1.2870	1.3092	1.3335	1.3719	0.0179
	Hmisc	1.3046	1.2969	1.1825	1.2722	1.3308	1.4349	0.0447	1.3139	1.3134	1.2275	1.2951	1.3324	1.3841	0.0275	1.3189	1.3182	1.2637	1.3057	1.3317	1.3705	0.0205
	missForest	0.9173	0.9112	0.8504	0.8895	0.9394	1.0446	0.0377	0.9276	0.9307	0.8622	0.9062	0.9488	6886.0	0.0287	0.9346	0.9331	0.8948	0.9187	0.9482	0.9959	0.0203
X13	Mean	1.0047	1.0039	0.9784	0.9968	1.0126	1.0334	0.0131	0.9995	0.9991		0.9937	1.0051	1.0239	0.0089	1.0006	1.0007	0.9858	0.9947	1.0063	1.0190	0.0084
	MICE	1.1408	1.1412	1.0622	1.1264	1.1580	1.1948	0.0235	1.1530	1.1531	1.1222	1.1442	1.1619	1.1835	0.0119	1.1682	1.1680	1.1385	1.1621	1.1747	1.2005	0.0097
	Mi	1.4127	1.4124	1.3569	1.3959	1.4355	1.4727	0.0245	1.4034	1.4027	1.3813	1.3979	1.4086	1.4266	8600.0	1.4085	1.4081	1.3881	1.4029	1.4153	1.4285	0.0085
	Amelia	1.1631	1.1628	1.1176	1.1509	1.1759	1.2096	0.0180	1.1726	1.1719	1.1428	1.1673	1.1787	1.1976	0.0100	1.1879	1.1874	1.1598	1.1815	1.1942	1.2187	0.0094
	Hmisc	1.1412	1.1418	1.0803	1.1247	1.1561	1.2170	0.0224	1.1541	1.1547	1.1182	1.1472	1.1621	1.1937	0.0120	1.1712	1.1711	1.1378	1.1634	1.1777	1.2018	0.0107
	missForest	0.7578	0.7548	0.7170	0.7423	0.7654	1.0181	0.0357	0.7725	0.7729	0.7474	0.7667	0.7780	0.7989	0.0093	0.7933	0.7928	0.7785	0.7864	0.7973	1.0118	0.0159
X14	Mean	1.0058	1.0037	0.9618	0.9883	1.0208	1.0886	0.0257	9266.0	9666.0	0.9607	0.9875	1.0071	1.0334	0.0159	0.9993	0.9974	0.9789	0.9903	1.0093	1.0262	0.0123
	MICE	0.4067	0.4045	0.3611	0.3920	0.4195	0.4722	0.0207	0.5662	0.5672	0.5331	0.5564	0.5739	0.6082	0.0134	0.6758	0.6754	0.6365	0.6674	0.6836	0.7087	0.0125
	Mi	1.4144	1.4177	1.3280	1.3981	1.4317	1.4838	0.0287	1.4112	1.4109	1.3826	1.4011	1.4195	1.4455	0.0140	1.4160	1.4163	1.3892	1.4074	1.4246	1.4376	0.0113
	Amelia	0.3863	0.3850	0.3492	0.3758	0.3965	0.4286	0.0151	0.5298	0.5293	0.4975	0.5242	0.5356	0.5622	8600.0	0.6422	0.6421	0.6110	0.6351	0.6485	0.6819	0.0106
																				i,	-	

				5% Mis	5% Missing Percentage	ntage					15% Mis.	15% Missing Percentage	ıtage				Ì	25% Mis	25% Missing Percentage	ıtage		
Variable	Method	π	Median	Q1	Q2	S)	Q4	ο	ή	Median	Q1	Q2	Q3	Q4	σ	ή	Median	Ŏ1	Q2	Q3	Q4	σ
	Hmisc	0.4052	0.4007	0.3591	0.3898	0.4138	0.5268	0.0260	0.5575	0.5563	0.5185	0.5479	0.5666	0.6064	0.0147	0.6650	0.6664	0.6330	0.6554	0.6752	0.6947	0.0139
	missForest	0.3118	0.2903	0.2615	0.2783	0.3004	1.0208	0.1191	0.3886	0.3843	0.3489	0.3786	0.3933	9866.0	0.0411	0.4603	0.4591	0.4359	0.4539	0.4675	0.4879	3.0095
X15	Mean	0.9979	0.9978	0.9238	0.9772	1.0218	1.1070	0.0343	0.9999	1.0017	0.9588	0.9857	1.0129	1.0436	0.0190	0.9946	0.9942	0.9648	0.9803	1.0016	1.0308	0.0166
	MICE	1.1375	1.1349	1.0275	1.1174	1.1560	1.2488	0.0330	1.1641	1.1624	1.0974	1.1492	1.1767	1.2689	0.0232	1.1815	1.1803	1.1386	1.1707	1.1916	1.2401	0.0158
	Mi	1.4055	1.4043	1.3098	1.3745	1.4382	1.4835	0.0403	1.4115	1.4126	1.3750	1.3995	1.4236	1.4537	0.0166	1.4190	1.4192	1.3807	1.4039	1.4305	1.4670	0.0183
	Amelia	1.1481	1.1457	1.0823	1.1300	1.1658	1.2421	0.0264	1.1668	1.1677	1.1224	1.1564	1.1765	1.2169	0.0158	1.1835	1.1831	1.1460	1.1740	1.1926	1.2290	0.0143
	Hmisc	1.1374	1.1331	1.0413	1.1113	1.1582	1.2983	0.0370	1.1634	1.1624	1.1177	1.1505	1.1771	1.2178	0.0200	1.1811	1.1797	1.1364	1.1695	1.1916	1.2316	0.0167
	missForest	0.7759	0.7646	0.7078	0.7422	0.7934	1.0241	0.0559	0.7970	0.7942	0.7590	0.7864	0.8052	9066.0	0.0221	0.8145	0.8118	0.7839	0.8042	0.8238	0.8562	0.0149
X16	Mean	1.0144	0.9966	0.9162	0.9817	1.0236	1.6872	0.1020	1.0121	1.0009	0.9482	0.9875	1.0119	1.3032	0.0618	0.9946	0.9953	0.9227	0.9855	1.0094	1.0483	0.0246
	MICE	1.1489	1.1312	1.0378	1.1093	1.1534	1.8268	0.1045	1.1591	1.1490	1.0765	1.1335	1.1604	1.4828	0.0568	1.1669	1.1680	1.0833	1.1565	1.1789	1.2753	0.0267
	Mi	1.4224	1.4127	1.3360	1.3818	1.4314	2.0013	0.0912	1.4183	1.4086	1.3634	1.4006	1.4226	1.6385	0.0466	1.4031	1.4016	1.3613	1.3921	1.4178	1.4415	9810.0
	Amelia	1.1542	1.1408	1.0561	1.1251	1.1548	1.7686	9680.0	1.1674	1.1549	1.1128	1.1461	1.1685	1.4107	0.0515	1.1725	1.1727	1.0877	1.1643	1.1831	1.2220	0.0192
	Hmisc	1.1450	1.1306	1.0227	1.1131	1.1512	1.7348	0.0929	1.1586	1.1490	1.0737	1.1343	1.1664	1.4089	0.0523	1.1649	1.1672	1.0652	1.1539	1.1772	1.2523	0.0251
	missForest	0.7819	0.7532	0.7051	0.7397	0.7783	1.5344	0.1201	0.7993	0.7822	0.7351	0.7731	0.7991	1.1224	0.0694	0.8012	0.8028	0.7448	0.7923	0.8125	0.8544	0.0202
X17	Mean	1.0000	1.0037	0.9592	09860	1.0179	1.0418	0.0218	0.9972	0.9950	0.9671	9886.0	1.0050	1.0294	0.0124	0.9977	0.9961	9696.0	9886.0	1.0058	1.0237	0.0121
	MICE	0.8675	0.8683	0.8232	0.8572	0.8789	0.9197	0.0169	0.9038	0.9028	0.8771	0.8955	0.9118	0.9309	0.0113	0.9445	0.9449	0.9140	0.9375	0.9507	0.9711	20102
	Mi	1.4110	1.4147	1.3578	1.3943	1.4294	1.4729	0.0242	1.4252	1.4247	1.3926	1.4180	1.4334	1.4516	0.0114	1.4091	1.4078	1.3754	1.3995	1.4178	1.4423	0.0132
	Amelia	0.8679	0.8683	0.8253	0.8550	0.8815	0.9094	0.0163	0.9032	0.9025	0.8827	0.8967	0.9091	0.9320	0.0094	0.9428	0.9422	0.9165	0.9350	0.9481	0.9850	0.0111
	Hmisc	0.8683	0.8688	0.8181	0.8570	0.8801	0.9266	0.0176	0.9049	0.9039	0.8780	0.8954	0.9130	0.9403	0.0122	0.9465	0.9461	0.9160	0.9408	0.9524	0.9782	0.0105
	missForest	0.6102	0.6055	0.5682	0.5888	0.6142	1.0133	0.0554	0.6342	0.6342	0.6187	0.6269	0.6410	0.6548	0.0091	0.6668	8999.0	0.6406	0.6596	0.6725	0.6923	0.0099
X18	Mean	1.0026	0.9965	0.9580	0.9829	1.0192	1.0682	0.0254	9266.0	0.9987	0.9713	0.9872	1.0074	1.0218	0.0133	1.0016	1.0011	0.9873	0.9940	1.0074	1.0292	0.0094
	MICE	0.8377	0.8371	0.7907	0.8240	0.8506	0.8917	0.0211	0.8752	0.8750	0.8464	0.8674	0.8833	0.9044	0.0119	0.9214	0.9209	0.8945	0.9153	0.9277	0.9456	9600°C
	Mi	1.4165	1.4182	1.3732	1.3967	1.4323	1.4740	0.0235	1.4130	1.4127	1.3817	1.4049	1.4225	1.4425	0.0124	1.4134	1.4123	1.3902	1.4086	1.4196	1.4392	0.0103
	Amelia	0.8387	0.8395	0.7924	0.8263	0.8503	0.8971	0.0175	0.8765	0.8761	0.8499	0.8704	0.8834	0.9050	6600.0	0.9217	0.9210	0.9005	0.9154	0.9275	0.9489	3.0088
	Hmisc	0.8403	0.8400	0.7863	0.8258	0.8554	0.9028	0.0217	0.8770	0.8768	0.8443	0.8686	0.8849	0.9068	0.0119	0.9226	0.9224	0.8953	0.9156	0.9291	0.9514	0.0094
	missForest	0.6033	0.5915	0.5497	0.5766	0.6073	1.0047	0.0683	0.6442	0.6235	0.5987	0.6153	0.6310	1.0138	0.0861	0.6657	0.6585	0.6404	0.6539	0.6643	1.0084	0.0488
X19	Mean	1.0015	1.0012	0.9610	0.9881	1.0086	1.0460	0.0185	0.9997	0.9992	0.9743	0.9922	1.0078	1.0263	0.0113	1.0017	1.0013	0.9878	0.9970	1.0055	1.0209	0.0082
	MICE	1.3533	1.3530	1.2819	1.3368	1.3696	1.4101	0.0237	1.3558	1.3560	1.3193	1.3461	1.3661	1.3929	0.0137	1.3612	1.3614	1.3370	1.3546	1.3678	1.3912	0.0109
	Mi	1.4142	1.4132	1.3448	1.3988	1.4304	1.4795	0.0253	1.4142	1.4129	1.3839	1.4069	1.4236	1.4432	0.0129	1.4121	1.4102	1.3911	1.4056	1.4186	1.4461	0.0099
	Amelia	1.3564	1.3555	1.2798	1.3421	1.3717	1.4118	0.0224	1.3558	1.3558	1.3074	1.3474	1.3640	1.4019	0.0130	1.3605	1.3606	1.3331	1.3538	1.3670	1.3887	0.0108
	Hmisc	1.3535	1.3529	1.2801	1.3373	1.3708	1.4173	0.0244	1.3550	1.3548	1.3168	1.3453	1.3643	1.3947	0.0143	1.3611	1.3608	1.3324	1.3543	1.3682	1.3951	0.0103
	missForest	0.9583	0.9563	0.9172	0.9435	0.9702	1.0460	0.0209	0.9601	0.9589	0.9373	0.9514	0.9680	0.9880	0.0111	0.9675	0.9672	0.9507	0.9618	0.9710	1.0085	3.0088
X20	Mean	0.9987	0.9992	0.9643	0.9904	1.0061	1.0303	0.0138	0.9994	0.9995	0.9822	0.9916	1.0055	1.0232	0.0095	1.0002	1.0006	0.9864	0.9955	1.0037	1.0224	0.0071
	MICE	0.8164	0.8171	0.7766	0.8071	0.8251	0.8532	0.0132	0.8398	0.8400	0.8204	0.8344	0.8445	0.8608	0800.0	0.8714	0.8713	0.8563	0.8670	0.8759	0.8893	0.0062
	Mi	1.4108	1.4096	1.3697	1.4006	1.4190	1.4762	0.0176	1.4113	1.4108	1.3890	1.4061	1.4174	1.4376	0.0095	1.4153	1.4151	1.3997	1.4094	1.4211	1.4344	0.0072
	Amelia	0.8507	0.8511	0.8153	0.8410	0.8605	0.8812	0.0135	8698.0	0.8705	0.8459	0.8656	0.8747	0.8949	0800.0	8968.0	0.8959	0.8789	0.8918	0.9016	0.9157	2900.0
	Hmisc	0.8190	0.8193	0.7823	0.8105	0.8279	0.8566	0.0126	0.8427	0.8428	0.8183	0.8371	0.8477	0.8637	0.0079	0.8740	0.8739	0.8548	0.8697	0.8783	0.8954	9900.0
	missForest	0.5262	0.5109	0.4911	0.5041	0.5172	1.0257	0.0867	0.5399	0.5396	0.5285	0.5357	0.5435	0.5533	0.0059	0.5734	0.5721	0.5595	0.5691	0.5743	0.9962	0.0271

B Box Plots based on RMSE



(g) Performance based on RMSE: X6 (h) Performance based on RMSE: X8 (i) Performance based on RMSE: X10

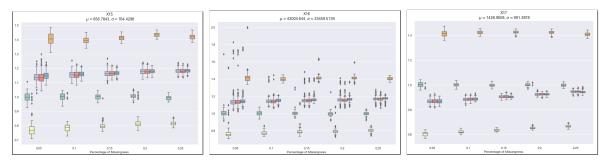


Mean

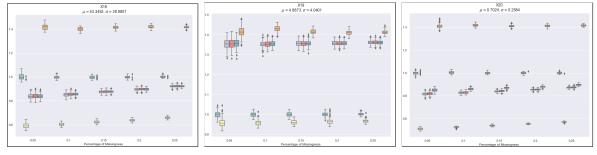
Amelia

MICE

(j) Performance based on RMSE: X11(k) Performance based on RMSE: X13(l) Performance based on RMSE: X14



(m) Performance based on RMSE:(n) Performance based on RMSE: X16(o) Performance based on RMSE: X17 X15



(p) Performance based on RMSE: X18(q) Performance based on RMSE: X19(r) Performance based on RMSE: X20