Design of an Active Suspension System Using Discrete Time Pole-Placement and Prediction State Observer

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***Abstract*—The design of controllers in discrete time state space has been well known for a very long time. The following project report summarizes the use of two common design techniques, called the Pole Placement Technique and State Observer Feedback Control for an active suspension system.**

***Index Terms*—Pole Placement, Reference Tracking, Integral Error Control, Full Order State Observer**

I. Introduction

An Active Suspension system is a type of suspension used to improve ride quality in automobiles. This is generally done by the utilization of actuators in conjunction with traditional spring damper suspension systems. The main idea is to control the vertical movement of the chassis with respect to the wheels. This is accomplished by using a series of sensors to measure the displacements and an onboard electronic controller to individually monitor the actuator movement at each wheel.

The design techniques discussed in this report are the Pole Placement Technique and the Full order State Observer for a deadbeat response. All these techniques are part of a standard Discrete control systems course and are widely known.

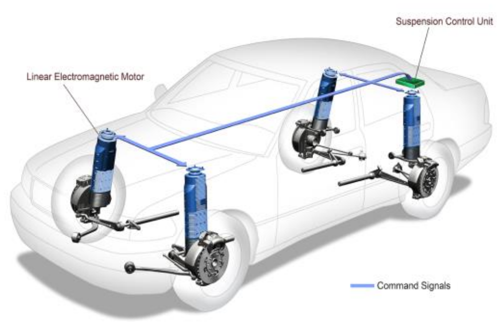

Fig-1 Electromagnetic Active Suspension System (Bose)

II. Mathematical Model

When an active suspension system is designed, we reduce an automobile to a 1/4th model to simplify the problem to a 1-D spring damper system. The model also contains an actuator to be able to control the vertical displacement of a vehicle.

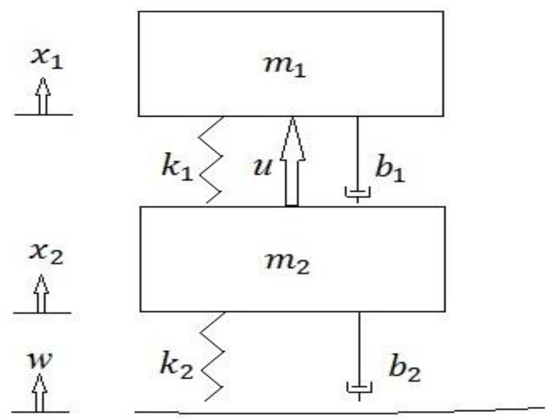

Fig-2 The 1/4th Model Approximation

A. Equations

TABLE I
VARIABLES & CONSTANTS

Symbol	Quantity	Values/Dimensions
x_1	Vehicle displacement	m
x_2	Suspension displacement	m
m_1	Vehicle mass	2500 kg
m_2	Suspension mass	320 kg
w	Road profile	m
k_1	Spring Constant Suspension	80,000 N/m
k_2	Spring Constant Tire	500,000 N/m
u	Actuator Force	N
b_1	Damping Ratio Suspension	350 N.s/m
b_2	Damping Ratio Tire	15,020 N.s/m

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1) General Dynamic Equations

$$m_1\ddot{x}_1 = -b_1(\dot{x}_1 - \dot{x}_2) - k_1(x_1 - x_2) + u \quad (1)$$

$$m_2\ddot{x}_2 = b_1(\dot{x}_1 - \dot{x}_2) + k_1(x_1 - x_2) + b_2(\dot{w} - \dot{x}_2) + k_2(w - x_2) - u \quad (2)$$

The same system can be represented in continuous time state space as,

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

Where

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \ddot{x}_1 \\ \dot{y}_1 \\ \ddot{y}_1 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-b_1 b_2}{m_1 m_2} & 0 & \left[\frac{b_1}{m_1}\left(\frac{b_1}{m_1} + \frac{b_1}{m_2} + \frac{b_2}{m_2}\right) - \frac{k_1}{m_1}\right] & \frac{-b_1}{m_1} \\ \frac{b_2}{m_2} & 0 & -\left(\frac{b_1}{m_1} + \frac{b_1}{m_2} + \frac{b_2}{m_2}\right) & 1 \\ \frac{k_2}{m_2} & 0 & -\left(\frac{k_1}{m_1} + \frac{k_1}{m_2} + \frac{k_2}{m_2}\right) & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ \frac{1}{m_1} & \frac{b_1 b_2}{m_1 m_2} \\ \frac{1}{m_1} + \frac{1}{m_2} & \frac{-k_2}{m_2} \end{bmatrix}$$

$$u = \begin{bmatrix} u \\ w \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 \end{bmatrix}$$

The above system can be discretized using MATLAB to a zero order hold equivalent discrete time state space equation.

$$x(k+1) = Gx(k) + Hu(k)$$
$$y(k) = Cx(k) + Du(k)$$

Since we can control only the control input "u", we modify the vector "H" so that we use only the first row in it.i.e we assume the system has no disturbance when designing the following controllers.

The block diagram representation of the open loop system is as shown in Fig 3.

B. Assumptions

In our analysis, we assume that the controlled input is a scalar quantity which is the actuation force. The output we measure is the relative displacement between the vehicle and the suspension. Although the system we use is a linear one, most real life suspension systems are non-linear.

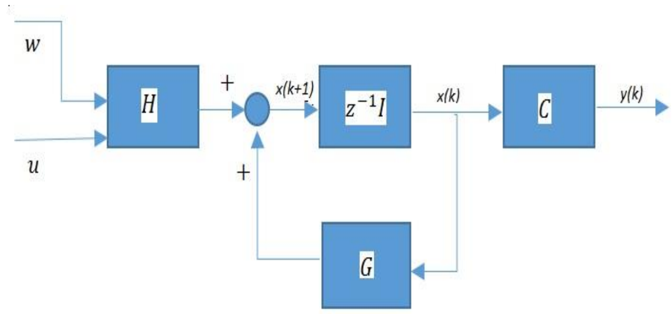

Fig-3 State Space Representation of the Suspension system

C. Open Loop System Characteristics

To simulate what would happen to our vehicle if it were to run into a 10cm high step, we could plot the open loop step response (Fig 4).

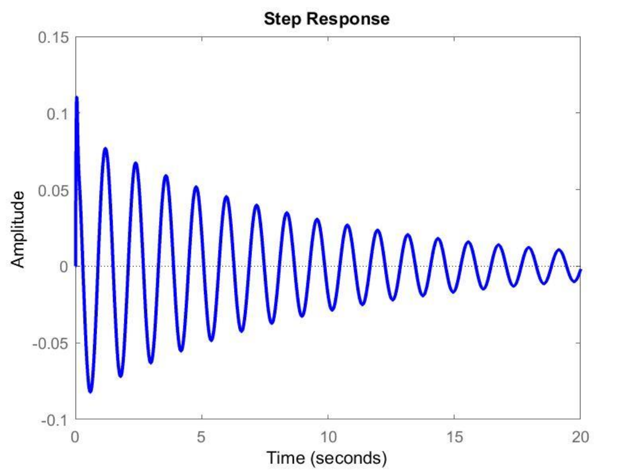

Fig-4 Open loop response of the uncontrolled system

We see that the vehicle enters into a damped sinusoidal oscillation which is as predicted by our model but would be a rather unpleasant driving experience.

III. Control Problem

We want to design a feedback controller so that when the road disturbance (W) is simulated by a unit step input, the output $(x_1 - x_2)$ has a settling time less than 5 seconds and an overshoot less than 5%. For example, when the vehicle runs onto a 10 cm high step, the body will oscillate within a range of +/- 5 mm and will stop oscillating within 5 seconds.

We will use a step input signal of 10cm to observe the controller effectiveness and study its transient characteristics.

IV. CONTROLLER DESIGN

A. *Design via Pole Placement (State feedback Control)*

Since all our states are available for measurement we can use the pole placement technique to determine the state feedback gain matrix needed to shift the poles of the system to the desired pole locations. The desired pole locations are obtained from the transient response characteristics of the given control problem. In addition to the transient response characteristics we add an additional requirement that the output follows the reference input at steady state.

As our discrete time system is complete state controllable, we can employ the pole placement technique.

The roots of the given characteristic equation given by $Det(zI - G) = 0$,is

$$\begin{bmatrix} 0.99 + 0.05i \\ 0.99 - 0.05i \\ 0.73 + 0.27i \\ 0.73 - 0.27i \end{bmatrix}$$

The desired closed loop poles are determined from the transient response characteristics.
We have,

$$\zeta = -\frac{\ln(\%OS)}{\sqrt{\pi^2 + \ln(\%OS)^2}} = 0.691$$

$$\omega_n = \frac{4}{\zeta T_s}$$

$$\omega_d = \omega_n\sqrt{1 - \zeta^2}$$

$$s_{1,2} = -\omega_n\zeta \pm \mathrm{i}\omega_d$$

$$z_{1,2} = e^{sT}$$

$$z_{1,2} = \begin{bmatrix} 0.8 + 0.17i \\ 0.8 - 0.17i \end{bmatrix}$$

In order to carry out pole placement technique, we need two additional poles as we are dealing with a 4th order system. We can choose two poles from the given characteristic equation, i.e

$$\begin{bmatrix} 0.73 + 0.27i \\ 0.73 - 0.27i \end{bmatrix}$$

As the additional poles.
The desired characteristic equation is obtained from the poles,

$$\begin{bmatrix} 0.80 + 0.17i \\ 0.80 - 0.17i \\ 0.73 + 0.27i \\ 0.73 - 0.27i \end{bmatrix}$$

The state feedback gain matrix is determined by,

$$K = (\underline{\alpha} - \underline{a})(\tilde{A}^{-T})(C^{-1})$$

$$K = 10^6[1.99 \quad 0.1 \quad -0.02 \quad -0.008]$$

The state feedback gain changes the steady state gain of the entire system. Hence we need to have an adjustable gain *'h'*, so that the unit step response at infinite time is unity. This can be determined both in z-plane as well as in state space.

$$h = \frac{1}{C(I - G + HK)^{-1}H}$$

which gives ,

$$h = 2.05(10^6)$$

The system with the controller is now shown as below.

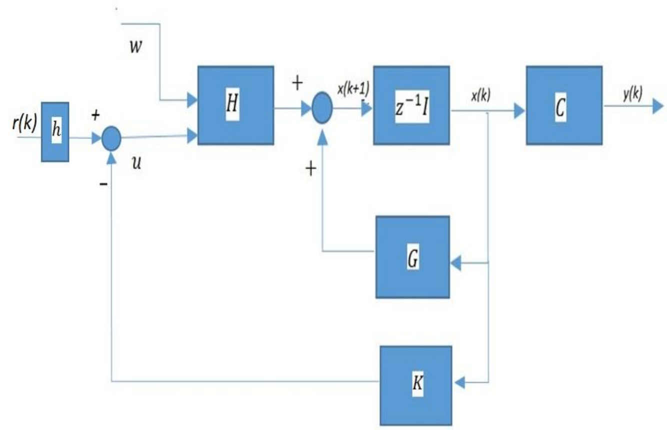

Fig-5 State feedback Control for Active Suspension System

The response of the closed loop system to a 10cm high step was simulated and the result is as shown in Fig 6.

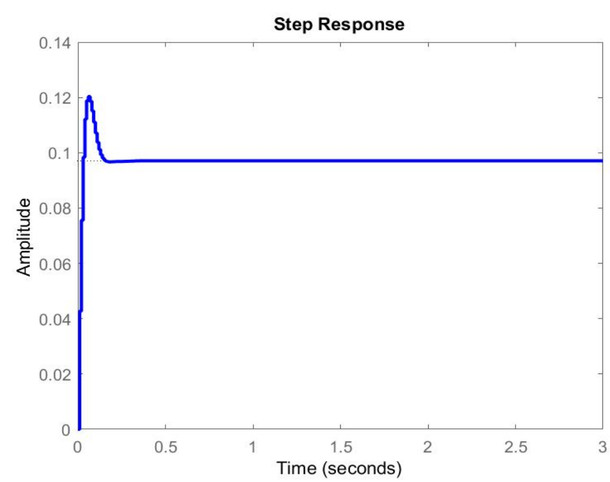

Fig-6 Step response of State feedback controlled system

The response of the system shows that both settling time and the maximum overshoot criteria has been met and thus there is no more damped oscillation of the vehicle body.
Although there is a finite distance between the vehicle mass and the suspension in steady state (0.1 m), that can be reset to zero by moving the actuator by the same amount.

B. Design via Pole Placement (Integral Error feedback)

The Integral Error feedback controller is generally used when there is a constant unknown disturbance and it is required that the output follow the reference input at steady state. The disturbance we account for in this controller is different from the road profile that we use in our model.
This is also a very useful controller that takes care of mechanical wear in the suspension system that results from repeated use.
The setup of this control loop is very similar to the state feedback controller.
The system used in Part A of controller design is extended by adding a state, called the integral of error.

$$q(k+1) = q(k) + T\{r(k) - y(k)\}$$

The new system of 5th order is given by the set of equations,

$$p(k+1) = \bar{G}p(k) + \bar{H}u(k) + \begin{bmatrix} 0 \\ T \end{bmatrix} r(k) + \begin{bmatrix} m(k) \\ 0 \end{bmatrix}$$

$$u(k) = -[k_x \quad k_q] \begin{bmatrix} x(k) \\ q(k) \end{bmatrix}$$

$$p(k+1) = \begin{bmatrix} x(k+1) \\ q(k+1) \end{bmatrix}$$

$$\bar{G} = \begin{bmatrix} G & 0 \\ -TC & 1 \end{bmatrix}$$

$$\bar{H} = \begin{bmatrix} H \\ 0 \end{bmatrix}$$

In the following analysis we use $\bar{G}, \bar{H}$ as our state matrix and the input matrix. The matrix H is a single column matrix with respect to control input "u".
The roots of $Det(zI - \bar{G})$ are,

$$\begin{bmatrix} 0.99 + 0.05i \\ 0.99 - 0.05i \\ 1 + 0i \\ 0.73 - 0.27i \\ 0.73 + 0.27i \end{bmatrix}$$

We need to choose 5 poles for the desired characteristic equation. We take two poles from the ransient response characteristics, two poles from $Det(zI - \bar{G})$, and one pole midway between the origin and the dominant closed loop poles, ie at z=0.5.

The desired closed loop pole locations are,

$$\begin{bmatrix} 0.73 - 0.27i \\ 0.73 + 0.27i \\ 0.8 + 0.17i \\ 0.8 - 0.17i \\ 0.5 + 0i \end{bmatrix}$$

The state feedback gain matrix becomes,

$$K = (\underline{\alpha} - \underline{a})(\tilde{A}^{-T})(C^{-1})$$

Which gives the following value for state feedback gain,

$$K = 10^8 [\, 0.065 \quad -0.004 \quad 0.004 \quad 0.0006 \quad -1.03]$$

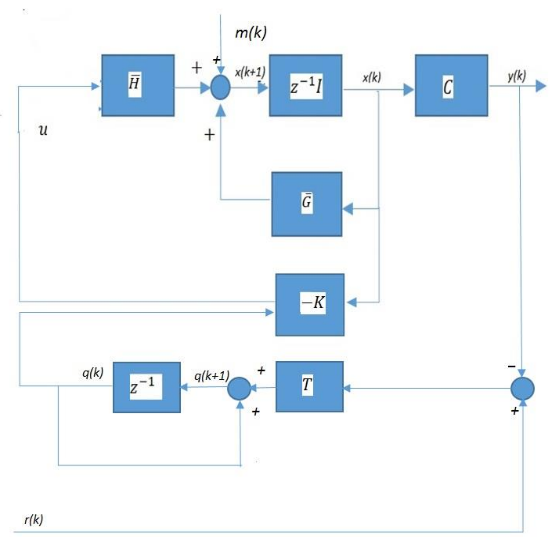

Fig-7 Integral Error Feedback Control

C. Design of a Full Order State Observer

In the following subsection we will design a full order prediction state observer. The necessary and sufficient condition for its design is that the system under analysis is complete state observable.

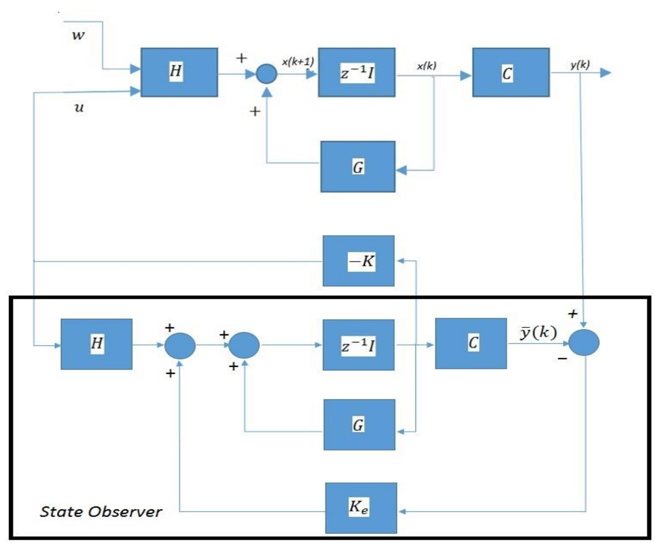

Fig-8 Observed State Feedback Control

The full order state observer is a controller that is useful in applications where it is very tough to measure some of the states. In that case an estimate of the states has to be made by analyzing the output and control input.

The pole placement design and the observer design are independent of each other. The pole placement design is done so that the system satisfies performance requirements. The poles of the observer are chosen so that the observer response happens faster than system response.
In this section, an observer for our system is designed so that the error vector exhibits a deadbeat response.
The gain for the error dynamic equation in the state observer is given by

$$K_e = {O_d}^{-1}.\tilde{A}^{-1}.(\underline{\alpha} - \underline{a})^T$$

$$\underline{\alpha} = [\ 0\ 0\ 0\ 0]$$

The value of the state feedback gain for the error dynamics is

$$K_e = 10^2[\ -0.04 \quad 8.17 \quad 0.035 \quad 5.11]$$

V. Conclusion

In the project, the design of an active suspension system was completed using the Pole Placement technique and the Full order Prediction Observer technique. The design parameters for each case was obtained to satisfy the required transient response characteristics. Future work could involve extending the system into multiple dimensions and also obtaining optimized values for the state feedback gain matrix in each case.

References

[1] Dr. Pan Agathoklis , Lecture Notes in *Control theory and Systems II, ELEC 460, Spring 2017, University of Victoria*.
[2] Katsuhiko Ogata, *Discrete Time -Control Systems, 2nd Ed,* Prentice Hall International.
[3] Katsuhiko Ogata, *Modern Control Engineering, 5th Ed,* Prentice Hall.
[4] Prof. Bill Messner & Prof. Dawn Tilbury, Mathematical models and examples, *Control Tutorials for MATLAB and Simulink,* University of Michigan/Carnegie Mellon University. *http://ctms.engin.umich.edu/CTMS/index.php?aux*