LQR Trajectory Tracking for Non Holonomic Mobile Robots

by

Chinnappa Alapanda
V00864775
B.Eng,PES Institute of Technology, 2014

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Advanced Control Theory

in the Department of Mechanical Engineering

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Course Instructor

Dr. Yang Shi, Professor
(Department of Mechanical Engineering)

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ABSTRACT

This project addresses the problem of solving a trajectory tracking problem for non holonomic wheeled mobile robots. The inherent nature of the kinematics of these robots are that they are non linear and non-integrable. The method used here is called the approximate linearization along the required trajectory. Then an optimal control law based on LQR is used to solve the trajectory tracking problem in the error state space.The robot is tasked to drive in common trajectories and the controller is checked for its ability to handle sensor noise and model uncertainties.

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Dr.Yang Shi, for giving us the freedom to choose our topics and guidance through all technical difficulties

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The divine gift does not come from a higher power but from our own minds
Dr. Robert Ford

DEDICATION

To my parents

Chapter 1

Introduction

Mobile robots are growing in ubiquity thanks to the growth of warehouse fullfillment centers across the world.The major control component in these robots are trajectory tracking. A motion planning framework, part of a larger navigation stack determines the trajectory and the objective of the controller is to make sure, the robot follows it with minimum deviation.

Non holonomic robots are a class of mechanical systems whose analysis is done from concepts from classical mechanics. Their dynamics are governed by a set of non-integrable kinematic equations.As a result it can reach any possible location in cartesian space but it cannot follow any given trajectory exactly.The controllers designed for these systems are called non holonomic controllers.Most solutions to the trajectory tracking problem are non linear, but some of them do represent a special class of linearized methods.

1.1 Motivation

This project deals with the use of linear control theory to address a fundamental problem faced in the trajectory tracking of non holonomic mobile robots. This class of robots are an intersting problem to look at because of their ubiquity. The main learning objective of this course project is to get a glimpse into the existing approaches to the mobile robot trajectory tracking problem and to identify problems that could be addressed as part of a bigger project.

1.2 Literature Review and Sources

Several approaches to the trajectory tracking problem for non holonomic mobile robots have become popular.

In [1],a cascaded systems approach is used to develop a full order and a reduced order observer for trajectory tracking.The limitation of this approach was that it could not follow linear trajectories.

In [2], the authors propose a controller based on dynamic feedback linearization. This is an exact linearization procedure and is a standard in many text books on control of mobile robots.

In[3] the authors use linearization about the desired trajectory to reduce the system of non linear equations to a LTV system and apply path tracking to control the robot.The controlled inputs in this approach are right and left wheel velocities.The controller worked well for low robot velocities and high sampling time, but the error would grow if either were to change too much.

The approach used in this report is very similar to the one developed by [3]. But instead of considering the input to be wheel velocities,here we use linear and angular velocities.

In fact the controller used in this project is derived from the one developed in [4].A very good introduction to how the controller is implemented is found in the thesis [5].

The controllability analysis presented in the report is derived from [6].

1.3 Overview of Report

Here is what the project report includes:

Chapter 1 A brief introduction to the problem and the main motivation in picking it up.

Chapter 2 Modelling and analysis of non holonomic robots is presented. Controllability of the error model and the method of linearization about the desired trajectory is discussed.

Chapter 3 An introduction to LQR optimal control Law is presented.

Chapter 4 All simulation results are presented for the controller design in SIMULINK.

Chapter 5 Conclusions and Further Work is explained.

Chapter 2

The Problem to be Solved

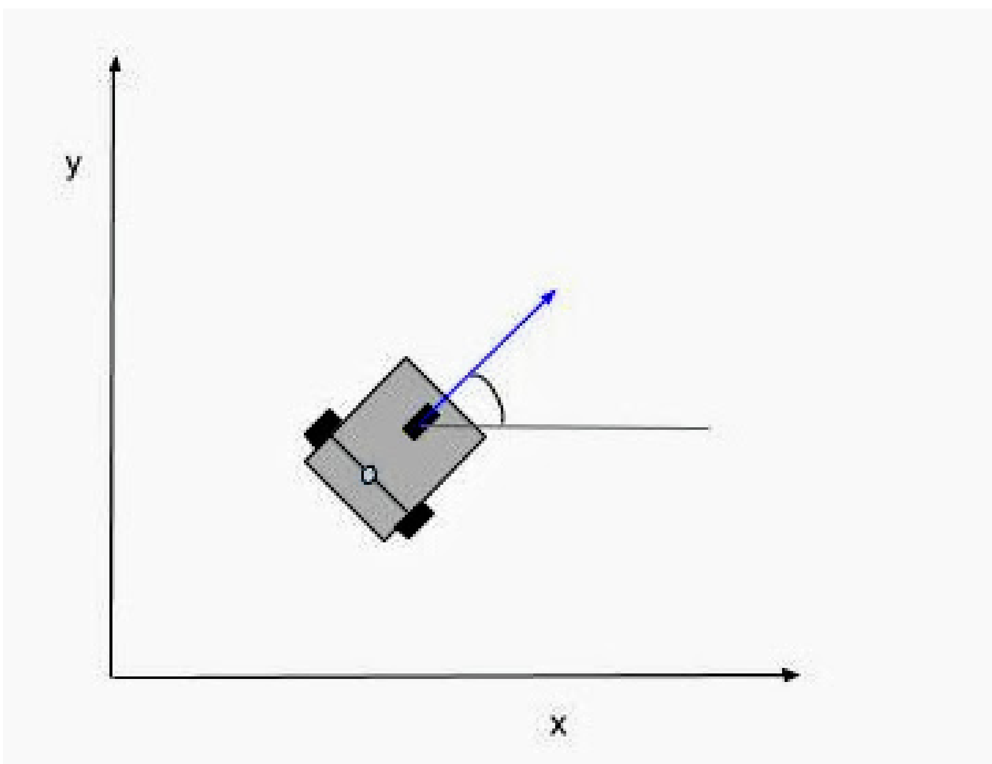

Figure 2.1: The Differential Drive Robot

2.1 Differential Drive Robot

A differential drive robot essentially consists of two drive wheels fixed to a common shaft. An additional castor wheel is provided to support the vehicle as shown in figure.

The higher level control inputs to the robot are the linear and angular velocities. The low level controller on the robot then converts these commands into rotational displacements of the two wheels. An assumption that must hold good in design on kinematic controllers is that there is no slip between the wheels and the contact surface.

In a standard robot navigation stack, a motion planner determines the trajectory that the robot has to follow based on obstacles and vision data. The controller designed is tasked with the objective of tracking the path with time constraints. In order to develop a controller for the tracking we need to look at the model of the robot.

The governing differential equation of the differential drive robot is given by,

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} v\cos\theta \\ v\sin\theta \\ \omega \end{bmatrix} \tag{2.1}$$

As we can see the equations are non linear. In order to use Linear systems theory, we need to get a linear approximate model.

2.2 Linearization about a trajectory

We want our robot to follow a given trajectory and the trajectory has to satisfy the non holonomic constraints. We can take the error between the desired trajectory and current trajectory to model a linear system.

Define

$$\boldsymbol{x}(t), \boldsymbol{x}_d(t)$$

as the system and reference states. Where

$$\boldsymbol{x}(t) = \begin{bmatrix} x(t) & y(t) & \theta(t) \end{bmatrix}^T, \boldsymbol{x}_d(t) = \begin{bmatrix} x_d(t) & y_d(t) & \theta_d(t) \end{bmatrix}^T$$

and

$$\boldsymbol{u}(t) = \begin{bmatrix} u(t) & \omega(t) \end{bmatrix}^T, \boldsymbol{u}_d(t) = \begin{bmatrix} u_d(t) & \omega_d(t) \end{bmatrix}^T$$

The kinematic model of error thus becomes,

$$\dot{\boldsymbol{e}}(t) = \boldsymbol{A}(t) \cdot \boldsymbol{e}(t) + \boldsymbol{B}(t) \cdot \bar{\boldsymbol{u}}(t) \tag{2.2}$$

Here,

$$\boldsymbol{e}(t) = \boldsymbol{x}(t) - \boldsymbol{x_d}(t), \bar{\boldsymbol{u}}(t) = \boldsymbol{u}(t) - \boldsymbol{u_d}(t)$$

$$\boldsymbol{A}(t), \boldsymbol{B}(t)$$

represent the Jacobians of 2.1 with respect to

$$\boldsymbol{x}(t), \boldsymbol{u}(t)$$

respectively.

$$\boldsymbol{A}(t) = \begin{bmatrix} 0 & 0 & -v\sin\theta \\ 0 & 0 & v\cos\theta \\ 0 & 0 & 0 \end{bmatrix}$$

$$\boldsymbol{B}(t) = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix}$$

As the linearized system is an LTV system, we know the controllability is determined by the gramian. But there is an easier way to check controllablity if we change the state space equation such that its reference frame is on the moving robot.With the transformation,

$$\boldsymbol{e_R}(t) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \boldsymbol{e}(t)$$

the new state and input matrices become,

$$\bar{\boldsymbol{A}}(t) = \begin{bmatrix} 0 & \omega & 0 \\ -\omega & 0 & v \\ 0 & 0 & 0 \end{bmatrix}$$

$$\bar{\boldsymbol{B}}(t) = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Assuming

$$v, \omega$$

are constant, the system becomes time invariant. The controllability matrix then becomes,

$$C = \begin{bmatrix} B & AB & A^2B \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & -\omega_d^2 & v_d\omega_d \\ 0 & 0 & -\omega_d & v_d & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{2.3}$$

As we can see the system is controllable for linear feedbacks, in the case of straight line and curved trajectories only if the velocities

$$v_d, \omega_d$$

are non-zero constants.It is possible to have a constant

$$v_d$$

and a zero

$$\omega_d$$

, in which case the controllability matrix still has full row rank. This is the biggest limitation to this approach. To control for time varying velocities,non linear feedback should be used. There is an additional requirement on this type of controller, the error between the initial state and initial desired state must be zero, ie

$$\boldsymbol{e}(0) = 0$$

.

Hence, we will use a linear state feedback controller with constant velocity to track our given trajectory.

Chapter 3

Optimal Control Law

3.1 LQR Tracking Controller

Linear Quadratic Regulator is a part of optimal control theory. It is also an important area of state space controller design. Consider our error equation,

$$\dot{\boldsymbol{e}}(t) = \boldsymbol{A}(t) \cdot \boldsymbol{e}(t) + \boldsymbol{B}(t) \cdot \bar{\boldsymbol{u}}(t)$$

which was discussed in the previous chapter. We choose to implement a control law as described in Divelbiss(1997).

The performance index to be optimized is

$$J = \int_0^\infty (\boldsymbol{e}(t)^T Q \boldsymbol{e}(t) + \bar{\boldsymbol{u}}(t)^T R \bar{\boldsymbol{u}}(t)) \tag{3.1}$$

where Q and R are Time invariant and diagonal. The choice of these matrices is done based on trial and error.

The control law that optimizes the above equation is,

$$\boldsymbol{u}(t) = -K\boldsymbol{e}(t) = -K(\boldsymbol{x}(t) - \boldsymbol{x}\boldsymbol{d}(t)) \tag{3.2}$$

and the control gain is obtained from the equation,

$$K = R^{-1} B^T P \tag{3.3}$$

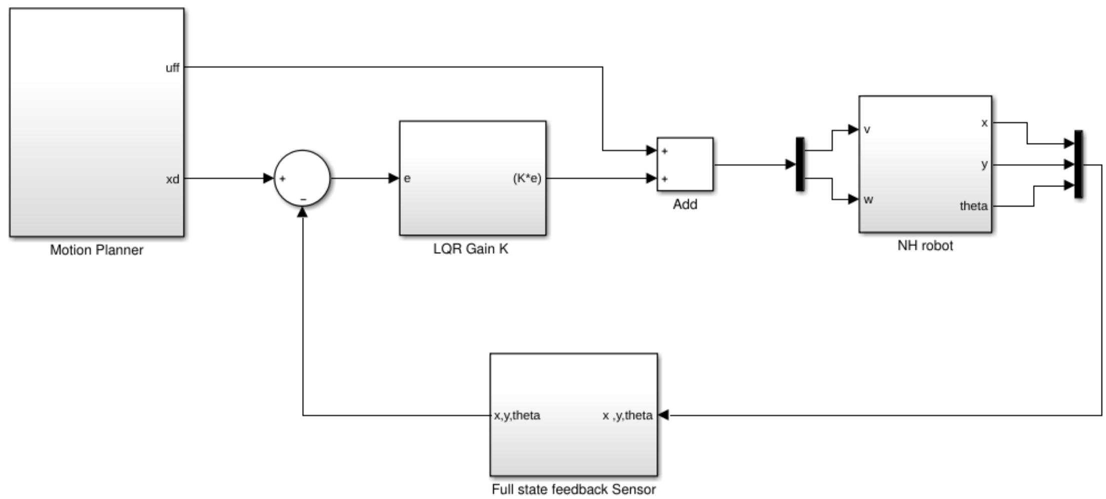

Figure 3.1: Linear Quadratic Regulator

with P being solved by using the equation,

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \tag{3.4}$$

The important thing to realize is that, although the gains and optimal control law is obtained for the linearized model, the same control law is used as input to the non linear model as shown in figure below.

The total control law thus becomes a combination of the feedforward term and the feedback term that linearizes the error to zero(Talati 2007).

$$\boldsymbol{u}(t) = \boldsymbol{u}_{ff} + K(\boldsymbol{x}(t) - \boldsymbol{x}_d(t)) \tag{3.5}$$

While performing simulations for the calculation of the LTV gains,matlab function

$$\boldsymbol{lqr}(A, B, Q, R)$$

can be used.

3.2 Trajectory Generation

Although the LQR tracking controller can be used for any arbitrary trajectory, generating a trajectory for simulation is a tough task. Its almost as complicated as writing your own motion planner. A simple way to go about it is to feed the desired angular and linear velocities to the model of the differential drive robot. The state of the robot thus generated in open loop is then used as the desired trajectory.

Chapter 4

Simulations

In this chapter, all the simulation results are presented and explained so that the results can be recreated. To simulate a real robot, we need to add some non linearities and errors to the model of the robot.Then we use the gains obtained from the linearized model. The feedback then ensures that the non linearity is taken care of. Since the model of the robot is kinematic, no model uncertainties that can be added.

For a given trajectory, we consider two cases,

1. Robot simulated with Random Noise

2. Robot simulated with Random Disturbance

The noise and disturbance added are random numbers in the range,

$$N(t) \in [0, 0.05\boldsymbol{x}_d(t)]$$

$$D(t) \in [0, 0.1\boldsymbol{u}_f(t)]$$

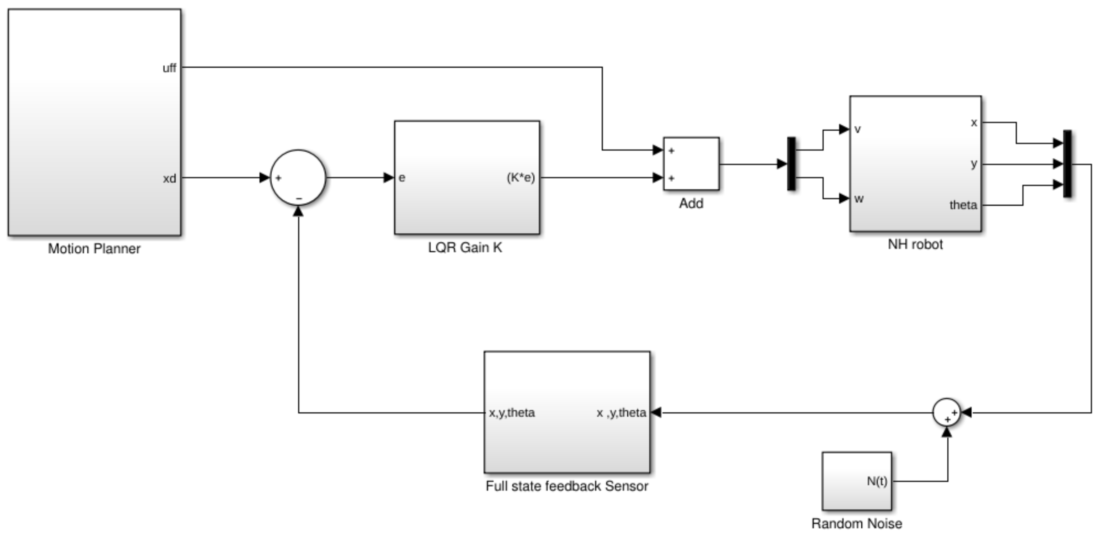

Figure 4.1: Modelling Sensor Noise

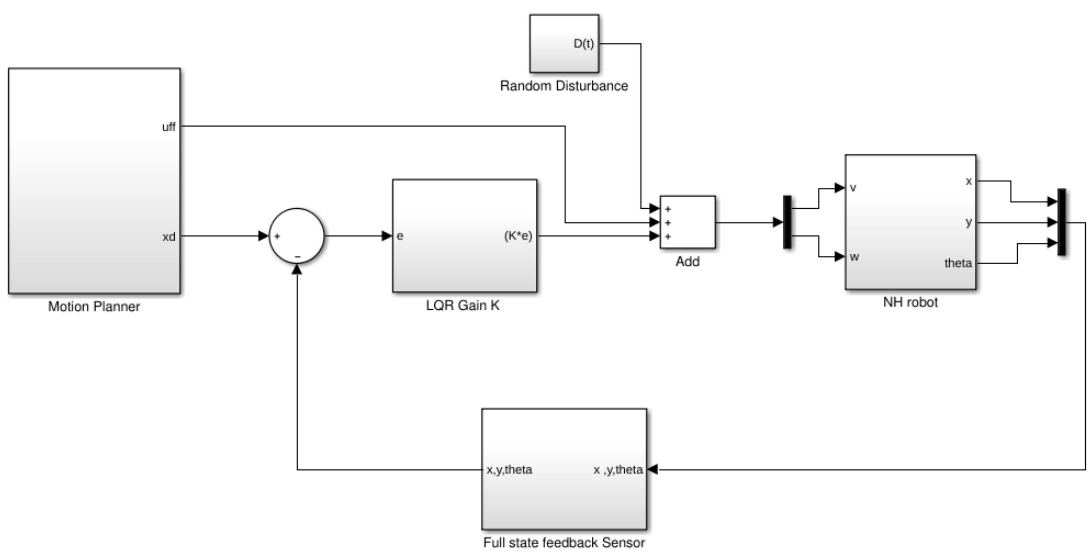

Figure 4.2: Modelling External Disturbance

4.1 Straight Line Tracking

For the straight line tracking problem, we consider the following conditions:

The Q and R weights used are

$$Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1e6 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

The Q is chosen so that the error in y- direction is minimized. R is chosen as the identity matrix as the effort for the inputs are not considered here.

h

Variable	Description	Value
$x(0)$	Pos,x initial	0
$y(0)$	Pos,y initial	0
$\theta(0)$	Orientation, initial	0
v_d	Vel, desired	1
ω_d	Ang Vel, desired	0

Table 4.1: Initial conditions for Line Tracking

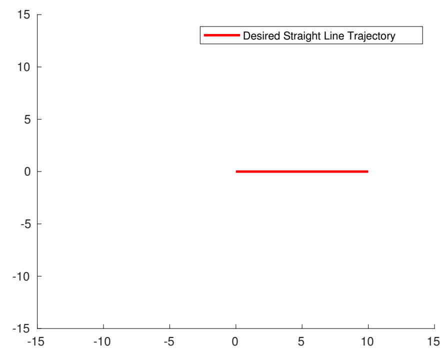

Figure 4.3: Desired Trajectory St Line

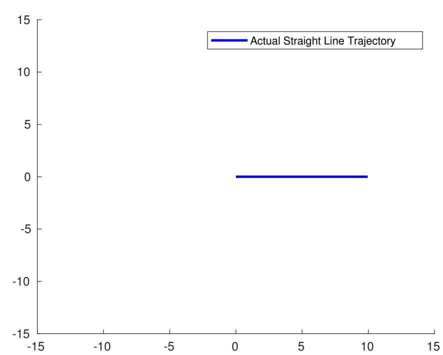

Figure 4.4: Actual Trajectory St Line with Noise

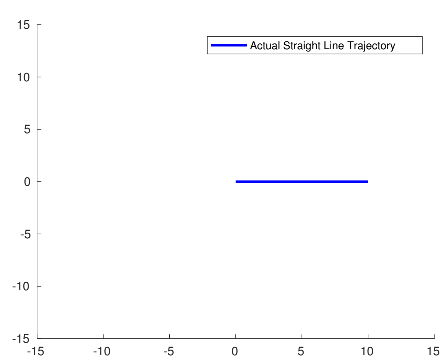

Figure 4.5: Actual Trajectory St Line with Disturbance

4.2 Circular Arc Tracking

Variable	Description	Value
$x(0)$	Pos,x initial	0
$y(0)$	Pos,y initial	0
$\theta(0)$	Orientation, initial	0
v_d	Vel, desired	1
ω_d	Ang Vel, desired	1

Table 4.2: Initial conditions for Arc Tracking

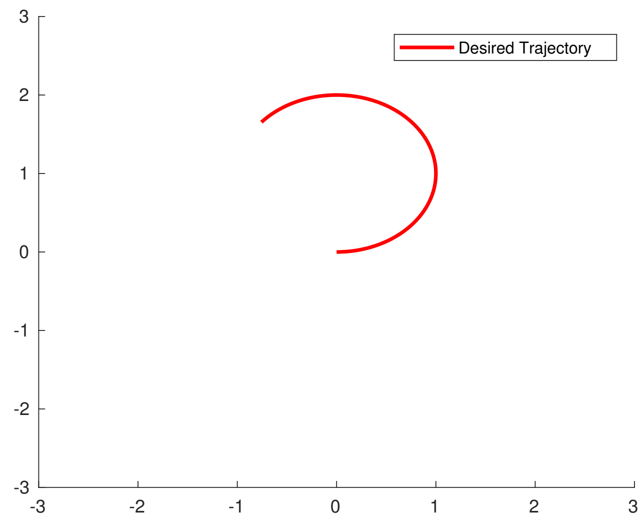

Figure 4.6: Desired Trajectory Arc

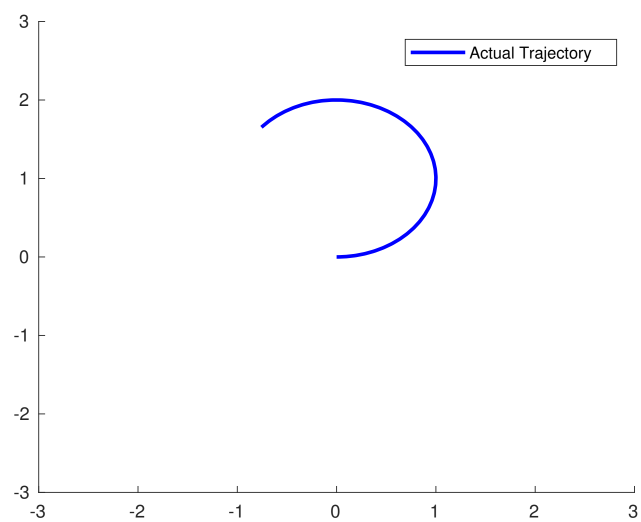

Figure 4.7: Actual Trajectory Arc with Noise

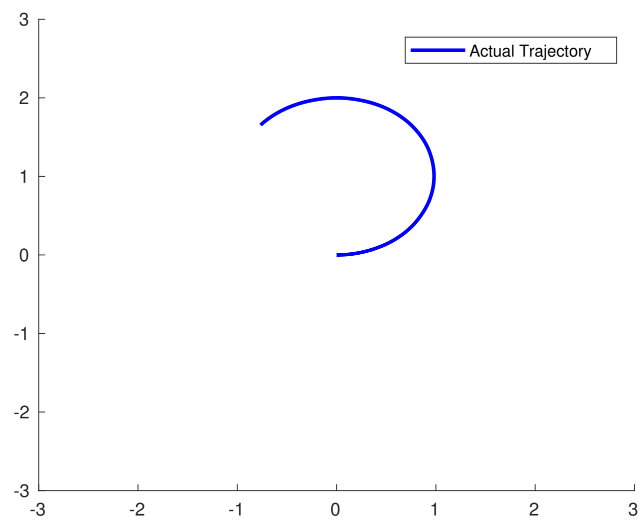

Figure 4.8: Actual Trajectory Arc with Disturbance

Chapter 5

Conclusion and Future Work

Although LQR is a reliable method to guarantee trajectory tracking,it has a few limitations. The most important one being the assumption that linear and angular velocities can not be zero, i.e the trajectory can never have stationary points. This is a major challenge to non holonomic robots that have limitations in the velocity space as a result of their dynamics.

The development of an LQR trajectory tracking algorithm for a non linear robot via the method of approximate linearization about the trajectory is used[Refer text book].Simulations have been performed to check the trajectory tracking effectiveness for two cases, a straight line and a circular arc. The controller is also checked for its effectiveness in handling noise and disturbances.

The report also contained a brief introduction and review of existing approches to trajectory tracking. Future work can include comparing this controller with other non-linear tracking controllers and using the algorithm on actual robots.

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