1 Lecture 11

Eigenvalue Problems

Definition 1.1. ALgebraic multiplicity of an eigenvalue counts the number of times the eigenvalue occurs as a root of the characteristic equation. Geometric multiplicity counts how many linearly independent eigenvectors correspond to an eigenvalue. It is a proveable fact that algebraic multiplicity is at least as much as the geometric multiplicity. If algebraic multiplicity is greater than geometric multiplicity, we say that the matrix is defective.

Theorem 1.2. Similar matrices share the same eigenvalues.

Theorem 1.3. If a matrix is defective, then it cannot have an eigenvector basis.

Proposition 1.4. A matrix is diagonalizable iff it has an eigenvector basis.

Proposition 1.5. The following matrix transformations change eigenvalues and eigenvectors as follows:

- $A \to (A - \sigma I)$ causes $\lambda \to \lambda - \sigma$.
- $A \to A^{-1}$ causes $\lambda \to \frac{1}{\lambda} >$
- $A \to A^k$ causes $\lambda \to \lambda^k$.
- If $A = PXP^{-1}$, then X has the same eigenvalues but every eigenvector v now becomes $P^{-1}v$.

Proposition 1.6. Suppose that we perturb a diagonal matrix A with some matrix E. Then the distance between any eigenvalue u of $A+E$ to an eigenvalue of A λ_k closest to u is bounded by $k(A)\|E\|$.