

Problem (1)

Define X_i as the value chosen in the i th iteration. It follows that

$$X = \frac{1}{k} \sum_{i=1}^k X_i$$

From this, we can compute $\mathbb{E}[X]$ and $\text{Var}[X]$.

$$\begin{aligned} \mathbb{E}[X] &= \frac{1}{k} \mathbb{E} \left[\sum_{i=1}^k X_i \right] \\ &= \frac{1}{k} \sum_{i=1}^k \mathbb{E}[X_i] \\ &= \frac{1}{k} \sum_{i=1}^k \mathbb{E}[X_i] \\ &= \frac{1}{k} \sum_{i=1}^k \sum_{j=1}^n z_j \frac{1}{n} \\ &= \frac{1}{k} \sum_{i=1}^k \sum_{j=1}^n z_j \frac{1}{n} \\ &= \frac{1}{k} \sum_{i=1}^k \alpha \\ &= \alpha \end{aligned}$$

$$\text{Var}[X] = \mathbb{E}[X^2] - \mathbb{E}[X]^2$$

$$\text{Var}[X] = \sum_{i=1}^n \frac{1}{n} z_i^2 - \alpha^2$$

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$$Hf = \frac{\partial}{\partial w} \nabla f = \frac{\partial}{\partial w} \frac{-1}{n} \sum_{i \in [n]} \frac{y^i(x^i)}{(1 + \exp(y^i w^T x^i))}$$