

Universitas Indonesia

Patience from Yusuf

Thanks to our handsome members:

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Contest (1)

Contents.txt	10 lines
1. Contest	
2. Mathematics	
3. Data structures	
4. Numerical	
5. Number Theory	
6. Combinatorial	
7. Graph	
8. Geometry	
9. Strings	
10. Various	

template.cpp	15 lines
#include <bits/stdc++.h>	
using namespace std;	
#define rep(i, a, b) for(int i = a; i < (b); ++i)	
#define trav(a, x) for(auto& a : x)	
#define all(x) x.begin(), x.end()	
#define sz(x) (int)(x).size()	
typedef long long ll;	
typedef pair<int, int> pii;	
typedef vector<int> vi;	
int main() {	
cin.sync_with_stdio(0); cin.tie(0);	
cin.exceptions(cin.failbit);	
}	

.bashrc	3 lines
alias c='g++ -Wall -Wconversion -Wfatal-errors -g -std=c++14 \	
-fsanitize=undefined,address'	
xmodmap -e 'clear lock' -e 'keycode 66=less greater' #caps =◇	

compile.bat	1 lines
g++ -std=c++11 -Wall -O2 -o %1 %1.cpp -Wl,--stack,268435456	

Mathematics (2)

2.1 Equations

$$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The extremum is given by $x = -b/2a$.

$$\begin{aligned} ax + by &= e & x &= \frac{ed - bf}{ad - bc} \\ cx + dy &= f & y &= \frac{af - ec}{ad - bc} \end{aligned} \Rightarrow$$

In general, given an equation $Ax = b$, the solution to a variable x_i is given by

$$x_i = \frac{\det A'_i}{\det A}$$

Contents template.bashrc compile

where A'_i is A with the i 'th column replaced by b .

2.2 Recurrences

If $a_n = c_1a_{n-1} + \dots + c_ka_{n-k}$, and r_1, \dots, r_k are distinct roots of $x^k + c_1x^{k-1} + \dots + c_k$, there are d_1, \dots, d_k s.t.

$$a_n = d_1r_1^n + \dots + d_kr_k^n.$$

Non-distinct roots r become polynomial factors, e.g.
 $a_n = (d_1n + d_2)r^n.$

2.3 Trigonometry

$$\begin{aligned} \sin(v + w) &= \sin v \cos w + \cos v \sin w \\ \cos(v + w) &= \cos v \cos w - \sin v \sin w \end{aligned}$$

$$\begin{aligned} \tan(v + w) &= \frac{\tan v + \tan w}{1 - \tan v \tan w} \\ \sin v + \sin w &= 2 \sin \frac{v + w}{2} \cos \frac{v - w}{2} \\ \cos v + \cos w &= 2 \cos \frac{v + w}{2} \cos \frac{v - w}{2} \end{aligned}$$

$$(V + W) \tan(v - w)/2 = (V - W) \tan(v + w)/2$$

where V, W are lengths of sides opposite angles v, w .

$$\begin{aligned} a \cos x + b \sin x &= r \cos(x - \phi) \\ a \sin x + b \cos x &= r \sin(x + \phi) \end{aligned}$$

where $r = \sqrt{a^2 + b^2}, \phi = \text{atan2}(b, a)$.

2.4 Geometry

2.4.1 Triangles

Side lengths: a, b, c

Semiperimeter: $p = \frac{a + b + c}{2}$

Area: $A = \sqrt{p(p - a)(p - b)(p - c)}$

Circumradius: $R = \frac{abc}{4A}$

Inradius: $r = \frac{A}{p}$

Length of median (divides triangle into two equal-area triangles): $m_a = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$

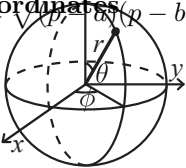
Length of bisector (divides angles in two):

$$s_a = \sqrt{bc \left[1 - \left(\frac{a}{b + c} \right)^2 \right]}$$

Law of sines: $\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c} = \frac{1}{2R}$
Law of cosines: $a^2 = b^2 + c^2 - 2bc \cos \alpha$
Law of tangents: $\frac{a + b}{a - b} = \frac{\tan \frac{\alpha + \beta}{2}}{\tan \frac{\alpha - \beta}{2}}$

$$4A = 2ef \cdot \sin \theta = F \tan \theta = \sqrt{4e^2f^2 - F^2}$$

For cyclic quadrilaterals the sum of opposite angles is 180° ,
 $ef = ac + bd$, and $A = \sqrt{(p - a)(p - b)(p - c)(p - d)}$.



$$\begin{aligned} x &= r \sin \theta \cos \phi & r &= \sqrt{x^2 + y^2 + z^2} \\ y &= r \sin \theta \sin \phi & \theta &= \text{acos}(z/\sqrt{x^2 + y^2 + z^2}) \\ z &= r \cos \theta & \phi &= \text{atan2}(y, x) \end{aligned}$$

2.5 Derivatives/Integrals

$$\begin{aligned} \frac{d}{dx} \arcsin x &= \frac{1}{\sqrt{1 - x^2}} & \frac{d}{dx} \arccos x &= -\frac{1}{\sqrt{1 - x^2}} \\ \frac{d}{dx} \tan x &= 1 + \tan^2 x & \frac{d}{dx} \arctan x &= \frac{1}{1 + x^2} \\ \int \tan ax &= -\frac{\ln |\cos ax|}{a} & \int x \sin ax &= \frac{\sin ax - ax \cos ax}{a^2} \\ \int e^{-x^2} &= \frac{\sqrt{\pi}}{2} \text{erf}(x) & \int x e^{ax} dx &= \frac{e^{ax}}{a^2} (ax - 1) \end{aligned}$$

Integration by parts:

$$\int_a^b f(x)g(x)dx = [F(x)g(x)]_a^b - \int_a^b F(x)g'(x)dx$$

2.6 Sums

$$(n+1)^{k+1} - 1 = \sum_{m=1}^n ((m+1)^{k+1} - m^{k+1})$$
$$\sum_{m=1}^n ((m+1)^{k+1} - m^{k+1}) = \sum_{p=0}^k \binom{k+1}{p} (1^p + 2^p + \dots + n^p)$$
$$\sum_{k=0}^n k^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$$
$$\sum_{k=0}^n k^5 = \frac{2n^6 + 6n^5 + 5n^4 - n^2}{12}$$
$$\sum_{k=0}^n kx^k = \frac{x - (n+1)x^{n+1} + nx^{n+2}}{(x-1)^2}$$

$$\sum_{k=0}^n k \binom{n}{k} = n2^{n-1}$$
$$\sum_{j=0}^k \binom{m}{j} \binom{n-m}{k-j} = \binom{n}{k}$$
$$\sum_{m=0}^n \binom{m}{k} = \binom{n+1}{k+1}$$
$$\sum_{j=0}^m \binom{m}{j}^2 = \binom{2m}{m}$$
$$\sum_{i=0}^n i^2 \binom{n}{i}^2 = n^2 \binom{2n-2}{n-1}$$
$$\sum_{k=-a}^a (-1)^k \binom{2a}{k+a}^3 = \frac{(3a)!}{a!^3}$$
$$\sum_{k=0}^n k^2 \binom{n}{k} = (n+n^2)2^{n-2}$$
$$\sum_{m=0}^n \binom{m}{j} \binom{n-m}{k-j} = \binom{n+1}{k+1}$$
$$\sum_{k=0}^{\lfloor \frac{n}{2} \rfloor} \binom{n-k}{k} = F(n+1)$$
$$\sum_{i=0}^n i \binom{n}{i}^2 = \frac{n}{2} \binom{2n}{n}$$
$$\sum_{k=q}^n \binom{n}{k} \binom{k}{q} = 2^{n-q} \binom{n}{q}$$

2.7 Series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots, \quad (-\infty < x < \infty)$$
$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots, \quad (-1 < x \leq 1)$$
$$\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{2x^3}{32} - \frac{5x^4}{128} + \dots, \quad (-1 \leq x \leq 1)$$
$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots, \quad (-\infty < x < \infty)$$
$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots, \quad (-\infty < x < \infty)$$

2.8 Probability theory

Let X be a discrete random variable with probability $p_X(x)$ of assuming the value x . It will then have an expected value (mean) $\mu = \mathbb{E}(X) = \sum_x x p_X(x)$ and variance $\sigma^2 = V(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2 = \sum_x (x - \mathbb{E}(X))^2 p_X(x)$ where σ is the standard deviation. If X is instead continuous it will have a probability density function $f_X(x)$ and the sums above will instead be integrals with $p_X(x)$ replaced by $f_X(x)$.

Expectation is linear:

$$\mathbb{E}(aX + bY) = a\mathbb{E}(X) + b\mathbb{E}(Y)$$

For independent X and Y ,

$$V(aX + bY) = a^2V(X) + b^2V(Y).$$

2.8.1 Discrete distributions

Binomial distribution

The number of successes in n independent yes/no experiments, each which yields success with probability p is $\text{Bin}(n, p)$, $n = 1, 2, \dots$, $0 \leq p \leq 1$.

$$p(k) = \binom{n}{k} p^k (1-p)^{n-k}$$
$$\mu = np, \sigma^2 = np(1-p)$$

$\text{Bin}(n, p)$ is approximately $\text{Po}(np)$ for small p .

First success distribution

The number of trials needed to get the first success in independent yes/no experiments, each wich yields success with probability p is $\text{Fs}(p)$, $0 \leq p \leq 1$.

$$p(k) = p(1-p)^{k-1}, \quad k = 1, 2, \dots$$

$$\mu = \frac{1}{p}, \sigma^2 = \frac{1-p}{p^2}$$

Poisson distribution

The number of events occurring in a fixed period of time t if these events occur with a known average rate κ and independently of the time since the last event is $\text{Po}(\lambda)$, $\lambda = t\kappa$.

$$p(k) = e^{-\lambda} \frac{\lambda^k}{k!}, \quad k = 0, 1, 2, \dots$$
$$\mu = \lambda, \sigma^2 = \lambda$$

2.8.2 Continuous distributions

Uniform distribution

If the probability density function is constant between a and b and 0 elsewhere it is $\text{U}(a, b)$, $a < b$.

$$f(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & \text{otherwise} \end{cases}$$

$$\mu = \frac{a+b}{2}, \sigma^2 = \frac{(b-a)^2}{12}$$

Exponential distribution

The time between events in a Poisson process is $\text{Exp}(\lambda)$, $\lambda > 0$.

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

$$\mu = \frac{1}{\lambda}, \sigma^2 = \frac{1}{\lambda^2}$$

Normal distribution

Most real random values with mean μ and variance σ^2 are well described by $\mathcal{N}(\mu, \sigma^2)$, $\sigma > 0$.

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

If $X_1 \sim \mathcal{N}(\mu_1, \sigma_1^2)$ and $X_2 \sim \mathcal{N}(\mu_2, \sigma_2^2)$ then

$$aX_1 + bX_2 + c \sim \mathcal{N}(\mu_1 + \mu_2 + c, a^2\sigma_1^2 + b^2\sigma_2^2)$$

Data structures (3)

OrderStatisticTree.h

Description: A set (not multiset!) with support for finding the n'th element, and finding the index of an element.
Time: $\mathcal{O}(\log N)$
<ext/pb.ds/assoc.container.hpp>, <ext/pb.ds/tree.policy.hpp>
7 lines

using namespace __gnu_pbds;

#define fbo find_by_order
#define ook order_of_key

typedef long long ll;
typedef tree<ll, null_type, less<ll>, rb_tree_tag,
tree_order_statistics_node_update> pbds;

LineContainer.h

Description: Container where you can add lines of the form $kx+m$, and query maximum values at points x . Useful for dynamic programming.
Usage: For minimum: change m, c to negative.
Then, the result of the query change to $-result$.
Time: $\mathcal{O}(\log N)$

35 lines

```
const ll is_query = -(1LL<<62);
struct Line {
    ll m, b;
    mutable function<const Line*> succ;
    bool operator<(const Line& rhs) const {
        if (rhs.b != is_query) return m < rhs.m;
        const Line* s = succ();
        if (!s) return 0;
        ll x = rhs.m;
        return b - s->b < (s->m - m) * x;
    }
};
struct HullDynamic : public multiset<Line> { // will maintain upper hull for maximum
    bool bad(iterator y) {
        auto z = next(y);
        if (y == begin()) {
            if (z == end()) return 0;
            return y->m == z->m && y->b <= z->b;
        }
        auto x = prev(y);
        if (z == end()) return y->m == x->m && y->b <= x->b;
        return (x->b - y->b)*(z->m - y->m) >= (y->b - z->b)*(y->m - x->m); // beware overflow!
    }
    void insert_line(ll m, ll b) {
        auto y = insert({ m, b });
        y->succ = [=] { return next(y) == end() ? 0 : &*next(y); };
        if (bad(y)) { erase(y); return; }
        while (next(y) != end() && bad(next(y))) erase(next(y));
        while (y != begin() && bad(prev(y))) erase(prev(y));
    }
    ll eval(ll x) { // becareful when there is no line returned.
        auto l = *lower_bound((Line) { x, is_query });
        return l.m * x + l.b;
    }
};
```

Treap.h

Description: A short self-balancing tree. It acts as a sequential container with log-time splits/joins, and is easy to augment with additional data.
Time: $\mathcal{O}(\log N)$

55 lines

```
struct Node {
    Node *l = 0, *r = 0;
    int val, y, c = 1;
    Node(int val) : val(val), y(rand()) {}
    void recalc();
};

int cnt(Node* n) { return n ? n->c : 0; }
void Node::recalc() { c = cnt(l) + cnt(r) + 1; }

template<class F> void each(Node* n, F f) {
    if (n) { each(n->l, f); f(n->val); each(n->r, f); }
}

pair<Node*, Node*> split(Node* n, int k) {
    if (!n) return {};
    if (cnt(n->l) >= k) { // "n->val" >= v for lower_bound(v)
        auto pa = split(n->l, k);
        n->l = pa.second;
```

```
        n->recalc();
        return {pa.first, n};
    } else {
        auto pa = split(n->r, k - cnt(n->l) - 1);
        n->r = pa.first;
        n->recalc();
        return {n, pa.second};
    }
}

Node* merge(Node* l, Node* r) {
    if (!l) return r;
    if (!r) return l;
    if (l->y > r->y) {
        l->r = merge(l->r, r);
        l->recalc();
        return l;
    } else {
        r->l = merge(l, r->l);
        r->recalc();
        return r;
    }
}

Node* ins(Node* t, Node* n, int pos) {
    auto pa = split(t, pos);
    return merge(merge(pa.first, n), pa.second);
}

// Example application: move the range [l, r) to index k
void move(Node& t, int l, int r, int k) {
    Node *a, *b, *c;
    tie(a,b) = split(t, l); tie(b,c) = split(b, r - l);
    if (k <= l) t = merge(ins(a, b, k), c);
    else t = merge(a, ins(c, b, k - r));
}
```

ImplicitAVL.h

Description: Implicit AVL. Delete array at index $id1$, and insert at indices $id2$. Preorder to restore the array.
Time: $\mathcal{O}(Q \log N)$

145 lines

```
int n, x, id1, id2;
struct node{
    int key, height, size;
    struct node* left;
    struct node* right;
};

inline int getH(struct node *cur){
    if (cur == NULL) return 0;
    return cur -> height;
}

inline int getB(struct node* cur) {
    if (cur == NULL) return 0;
    return getH(cur->left) - getH(cur->right);
}

inline int getS(struct node* cur) {
    if (cur == NULL) return 0;
    return cur -> size;
}

inline void updateH(struct node* cur) {
    int a = getH(cur->left);
    int b = getH(cur->right);
    cur->height = ((a > b) ? a : b) + 1;
}
```

```
inline void updateS(struct node* cur) {
    cur->size = 1 + getS(cur->left) + getS(cur->right);
}

inline struct node* newnode(int key) {
    struct node* cur = new node();
    cur -> key = key;
    cur -> left = NULL;
    cur -> right = NULL;
    cur -> height = 1;
    cur -> size = 1;
    return cur;
}

inline struct node* RotateRight(struct node* y) {
    struct node* x = y->left;
    struct node* xR = x->right;
    x->right = y;
    y->left = xR;
    updateH(y); updateH(x);
    updateS(y); updateS(x);
    return x;
}

inline struct node* RotateLeft(struct node* y) {
    struct node* x = y->right;
    struct node* xL = x->left;
    x->left = y;
    y->right = xL;
    updateH(y); updateH(x);
    updateS(y); updateS(x);
    return x;
}

struct node* minval(struct node* cur) {
    struct node* proc = cur;
    while(proc->left != NULL) proc=proc->left;
    return proc;
}
```

```
int value;

struct node* refresh(struct node* cur) {
    if (cur == NULL) return cur;
    updateH(cur); updateS(cur);
    int balance = getB(cur);
    if (balance > 1 && getB(cur->left) >= 0) return RotateRight(cur);
    if (balance > 1 && getB(cur->left) < 0) {cur->left = RotateLeft(cur->left); return RotateRight(cur);}
    if (balance <-1 && getB(cur->right) <= 0) return RotateLeft(cur);
    if (balance <-1 && getB(cur->right) > 0) {cur->right = RotateRight(cur->right); return RotateLeft(cur);}
    return cur;
}

struct node* DKTHS(struct node* cur, int K, bool left) {
    if (left) {
        if (cur -> left != NULL) cur -> left = DKTHS(cur -> left, K, left);
        else {
            struct node* temp = cur -> left? cur->left : cur -> right;
            if (temp == NULL) {
                temp = cur;
                cur = NULL;
            } else *cur = *temp;
            free(temp);
        }
    }
```

```
    }
} else if (cur == NULL) return cur;
else if (K <= getS(cur->left)) cur -> left = DKTHS(cur->left,
    K, 0);
else if (K == getS(cur->left) + 1) {
    value = cur->key;
    if ((cur->left==NULL)|| (cur->right==NULL)) {
        struct node* temp = cur -> left? cur->left : cur -> right
            ;
        if (temp == NULL) {
            temp = cur;
            cur = NULL;
        } else *cur = *temp;
        free(temp);
    } else {
        node* temp = minval(cur->right);
        cur->key = temp->key;
        cur->right = DKTHS(cur->right, cur->key, 1);
    }
} else cur -> right = DKTHS (cur->right, K - getS(cur->left) -
    1, 0);
return refresh(cur);
}
struct node* KTHS(struct node* cur, int K, bool left) {
    if (cur == NULL) return newnode(value);
    if (left) {
        if (cur == NULL) return newnode(value);
        else cur -> left = KTHS(cur->left, K, 1);
    }
    else if (K <= getS(cur->left)) cur -> left = KTHS(cur->left,
        K, 0);
    else if (K == getS(cur->left) + 1) {
        if (cur -> right == NULL) cur->right = KTHS(cur->right,K,0)
            ;
        else cur->right = KTHS(cur->>right, K, 1);
    } else cur -> right = KTHS(cur->right, K - getS(cur->left) -
        1, 0);
    return refresh(cur);
}
int cnt(0);
void preOrder(struct node* cur) {
    if (cur == NULL) return;
    preOrder(cur->left);
    A[++cnt] = cur->key;
    // printf("%d ",cur->key);
    preOrder(cur->right);
}

int main() {
    scanf("%d %d",&n, &x);
    struct node* root = NULL;
    for(int i = 1; i <= n; i++) {
        value = i;
        root = KTHS(root, i, 0);
    }
    for(int i = 1; i <= x; i++) {
        scanf("%d %d",&id1, &id2);
        root = DKTHS (root, id1, 0);
        root = KTHS (root, id2 - 1, 0);
    }
}
```

AVL.h

Description: Not Implicit. insert, delete, count, kth

42 lines

```
node* insert(node* cur, int key) {
    if (cur == NULL) return newnode(key);
    if (key == cur -> key) return cur;
    if (key < cur -> key) cur -> left = insert(cur->left, key);
```

AVL RMQ Polynomial PolyRoots PolyInterpolate

```
    else cur -> right = insert(cur->right, key);
    return balance(cur);
}

node* Delete(node* cur, int key) {
    if (cur == NULL) return cur;
    if (key < cur -> key) cur->left = Delete(cur->left, key);
    else if (key > cur -> key) cur->right = Delete(cur->right,
        key);
    else {
        if ((cur->left==NULL)|| (cur->right==NULL)) {
            node* temp = cur -> left? cur->left : cur -> right;
            if (temp == NULL) {
                temp = cur;
                cur = NULL;
            } else *cur = *temp;
            free(temp);
        } else {
            node* temp = minval(cur->right);
            cur -> key = temp -> key;
            cur -> right = Delete(cur->right, temp->key);
        }
    }
    if (cur == NULL) return cur;
    updateH(cur); updateS(cur);
    return balance(cur);
}

int KTHS(node* cur, int K) {
    if (K <= getS(cur->left)) return KTHS(cur->left, K);
    else if (K == getS(cur->left) + 1) return cur->key;
    else return KTHS (cur->right, K - getS(cur->left) - 1);
}

int COUNT(node* cur, int K) { // how many <= K
    if (cur == NULL) return 0;
    if (K < cur->key) return COUNT (cur->left, K);
    else return getS (cur->left) + 1 + COUNT (cur->right, K);
}
```

RMQ.h

Description: Range Minimum Queries on an array. Returns min(V[a, V[a + 1], ... V[b - 1]) in constant time. Usage: RMQ rmq(values); rmq.query(inclusive, exclusive); Time: $\mathcal{O}(|V| \log |V| + Q)$

17 lines

```
template<class T>
struct RMQ {
    vector<vector<T>>> jmp;
    RMQ(const vector<T>& V) {
        int N = sz(V), on = 1, depth = 1;
        while (on < sz(V)) on *= 2, depth++;
        jmp.assign(depth, V);
        rep(i,0,depth-1) rep(j,0,N)
            jmp[i+1][j] = min(jmp[i][j],
                jmp[i][min(N - 1, j + (1 << i))]);
    }
    T query(int a, int b) {
        assert(a < b); // or return inf if a == b
        int dep = 31 - __builtin_clz(b - a);
        return min(jmp[dep][a], jmp[dep][b - (1 << dep)]);
    }
};
```

Numerical (4)

Polynomial.h

17 lines

```
struct Poly {
    vector<double> a;
    double operator()(double x) const {
        double val = 0;
        for(int i = sz(a); i--;) (val *= x) += a[i];
        return val;
    }
    void diff() {
        rep(i,1,sz(a)) a[i-1] = i*a[i];
        a.pop_back();
    }
    void divroot(double x0) {
        double b = a.back(), c; a.back() = 0;
        for(int i=sz(a)-1; i--;) c = a[i], a[i] = a[i+1]*x0+b, b=c;
        a.pop_back();
    }
};
```

PolyRoots.h

Description: Finds the real roots to a polynomial. Usage: poly_roots({{2,-3,1}},-1e9,1e9) // solve $x^2-3x+2 = 0$ Time: $\mathcal{O}(n^2 \log(1/\epsilon))$

23 lines

```
vector<double> poly_roots(Poly p, double xmin, double xmax) {
    if (sz(p.a) == 2) { return {-p.a[0]/p.a[1]}; }
    vector<double> ret;
    Poly der = p;
    der.diff();
    auto dr = poly_roots(der, xmin, xmax);
    dr.push_back(xmin-1);
    dr.push_back(xmax+1);
    sort(all(dr));
    rep(i,0,sz(dr)-1) {
        double l = dr[i], h = dr[i+1];
        bool sign = p(l) > 0;
        if (sign ^ (p(h) > 0)) {
            rep(it,0,60) { // while (h - l > 1e-8)
                double m = (l + h) / 2, f = p(m);
                if ((f <= 0) ^ sign) l = m;
                else h = m;
            }
            ret.push_back((l + h) / 2);
        }
    }
    return ret;
}
```

PolyInterpolate.h

Description: Given n points (x[i], y[i]), computes an n-1-degree polynomial p that passes through them: $p(x) = a[0] * x^0 + \dots + a[n - 1] * x^{n-1}$. For numerical precision, pick $x[k] = c * \cos(k/(n - 1) * \pi), k = 0 \dots n - 1$. Time: $\mathcal{O}(n^2)$

13 lines

```
typedef vector<double> vd;
vd interpolate(vd x, vd y, int n) {
    vd res(n), temp(n);
    rep(k,0,n-1) rep(i,k+1,n)
        y[i] = (y[i] - y[k]) / (x[i] - x[k]);
    double last = 0; temp[0] = 1;
    rep(k,0,n) rep(i,0,n) {
        res[i] += y[k] * temp[i];
        swap(last, temp[i]);
        temp[i] -= last * x[k];
    }
    return res;
}
```

Integrate.h

Description: Simple integration of a function over an interval using Simpson's rule. The error should be proportional to h^4 , although in practice you will want to verify that the result is stable to desired precision when epsilon changes.

```
double quad(double (*f)(double), double a, double b) {
    const int n = 1000;
    double h = (b - a) / 2 / n;
    double v = f(a) + f(b);
    rep(i,1,n*2)
        v += f(a + i*h) * (i&1 ? 4 : 2);
    return v * h / 3;
}
```

Determinant.h

Description: Calculates determinant of a matrix. Destroys the matrix.
Time: $\mathcal{O}(N^3)$

```
double det(vector<vector<double>>& a) {
    int n = sz(a); double res = 1;
    rep(i,0,n) {
        int b = i;
        rep(j,i+1,n) if (fabs(a[j][i]) > fabs(a[b][i])) b = j;
        if (i != b) swap(a[i], a[b]), res *= -1;
        res *= a[i][i];
        if (res == 0) return 0;
        rep(j,i+1,n) {
            double v = a[j][i] / a[i][i];
            if (v != 0) rep(k,i+1,n) a[j][k] -= v * a[i][k];
        }
    }
    return res;
}
```

IntDeterminant.h

Description: Calculates determinant using modular arithmetics. Modulos can also be removed to get a pure-integer version.
Time: $\mathcal{O}(N^3)$

```
const ll mod = 12345;
ll det(vector<vector<ll>>& a) {
    int n = sz(a); ll ans = 1;
    rep(i,0,n) {
        rep(j,i+1,n) {
            while (a[j][i] != 0) { // gcd step
                ll t = a[i][i] / a[j][i];
                if (t) rep(k,i,n)
                    a[i][k] = (a[i][k] - a[j][k] * t) % mod;
                swap(a[i], a[j]);
                ans *= -1;
            }
        }
        ans = ans * a[i][i] % mod;
        if (!ans) return 0;
    }
    return (ans + mod) % mod;
}
```

Simplex.h

Description: Solves a general linear maximization problem: maximize $c^T x$ subject to $Ax \leq b, x \geq 0$. Returns -inf if there is no solution, inf if there are arbitrarily good solutions, or the maximum value of $c^T x$ otherwise. The input vector is set to an optimal x (or in the unbounded case, an arbitrary solution fulfilling the constraints). Numerical stability is not guaranteed. For better performance, define variables such that $x = 0$ is viable.

Usage: vvd A = {{1,-1}, {-1,1}, {-1,-2}};
vvd b = {1,1,-4}, c = {-1,-1}, x;
T val = LPSolver(A, b, c).solve(x);
Time: $\mathcal{O}(NM * \#pivots)$, where a pivot may be e.g. an edge relaxation. $\mathcal{O}(2^n)$ in the general case.

```
typedef double T; // long double, Rational, double + modP>...
typedef vector<T> vd;
typedef vector<vd> vvd;

const T eps = 1e-8, inf = 1/.0;
#define MP make_pair
#define ltj(X) if(s == -1 || MP(X[j],N[j]) < MP(X[s],N[s])) s=j

struct LPSolver {
    int m, n;
    vi N, B;
    vvd D;

    LPSolver(const vvd& A, const vd& b, const vd& c) :
        m(sz(b)), n(sz(c)), N(n+1), B(m), D(m+2, vd(n+2)) {
        rep(i,0,m) rep(j,0,n) D[i][j] = A[i][j];
        rep(i,0,m) { B[i] = n+i; D[i][n] = -1; D[i][n+1] = b[i]; }
        rep(j,0,n) { N[j] = j; D[m][j] = -c[j]; }
        N[n] = -1; D[m+1][n] = 1;
    }

    void pivot(int r, int s) {
        T *a = D[r].data(), inv = 1 / a[s];
        rep(i,0,m+2) if (i != r && abs(D[i][s]) > eps) {
            T *b = D[i].data(), inv2 = b[s] * inv;
            rep(j,0,n+2) b[j] -= a[j] * inv2;
            b[s] = a[s] * inv2;
        }
        rep(j,0,n+2) if (j != s) D[r][j] *= inv;
        rep(i,0,m+2) if (i != r) D[i][s] *= -inv;
        D[r][s] = inv;
        swap(B[r], N[s]);
    }

    bool simplex(int phase) {
        int x = m + phase - 1;
        for (;;) {
            int s = -1;
            rep(j,0,n+1) if (N[j] != -phase) ltj(D[x]);
            if (D[x][s] >= -eps) return true;
            int r = -1;
            rep(i,0,m) {
                if (D[i][s] <= eps) continue;
                if (r == -1 || MP(D[i][n+1] / D[i][s], B[i])
                    < MP(D[r][n+1] / D[r][s], B[r])) r = i;
            }
            if (r == -1) return false;
            pivot(r, s);
        }
    }

    T solve(vd &x) {
        int r = 0;
        rep(i,1,m) if (D[i][n+1] < D[r][n+1]) r = i;
        if (D[r][n+1] < -eps) {
            pivot(r, n);
            if (!simplex(2) || D[m+1][n+1] < -eps) return -inf;
            rep(i,0,m) if (B[i] == -1) {
                int s = 0;
                rep(j,1,n+1) ltj(D[i]);
                pivot(i, s);
            }
        }
    }
}
```

```
bool ok = simplex(1); x = vd(n);
rep(i,0,m) if (B[i] < n) x[B[i]] = D[i][n+1];
return ok ? D[m][n+1] : inf;
}
};
```

SolveLinear.h

Description: Solves $A * x = b$. If there are multiple solutions, an arbitrary one is returned. Returns rank, or -1 if no solutions. Data in A and b is lost.
Time: $\mathcal{O}(n^2m)$

```
typedef vector<double> vd;
const double eps = 1e-12;

int solveLinear(vector<vd>& A, vd& b, vd& x) {
    int n = sz(A), m = sz(x), rank = 0, br, bc;
    if (n) assert(sz(A[0]) == m);
    vi col(m); iota(all(col), 0);

    rep(i,0,n) {
        double v, bv = 0;
        rep(r,i,n) rep(c,i,m)
            if ((v = fabs(A[r][c])) > bv)
                br = r, bc = c, bv = v;
        if (bv <= eps) {
            rep(j,i,n) if (fabs(b[j]) > eps) return -1;
            break;
        }
        swap(A[i], A[br]);
        swap(b[i], b[br]);
        swap(col[i], col[bc]);
        rep(j,0,n) swap(A[j][i], A[j][bc]);
        bv = 1/A[i][i];
        rep(j,i+1,n) {
            double fac = A[j][i] * bv;
            b[j] -= fac * b[i];
            rep(k,i+1,m) A[j][k] -= fac*A[i][k];
        }
        rank++;
    }

    x.assign(m, 0);
    for (int i = rank; i--;) {
        b[i] /= A[i][i];
        x[col[i]] = b[i];
        rep(j,0,i) b[j] -= A[j][i] * b[i];
    }
    return rank; // (multiple solutions if rank < m)
}
```

SolveLinear2.h

Description: To get all uniquely determined values of x back from SolveLinear, make the following changes:

```
"SolveLinear.h"
7 lines

rep(j,0,n) if (j != i) // instead of rep(j,i+1,n)
// ... then at the end:
x.assign(m, undefined);
rep(i,0,rank) {
    rep(j,rank,m) if (fabs(A[i][j]) > eps) goto fail;
    x[col[i]] = b[i] / A[i][i];
fail:; }
```

SolveLinearBinary.h

Description: Solves $Ax = b$ over \mathbb{F}_2 . If there are multiple solutions, one is returned arbitrarily. Returns rank, or -1 if no solutions. Destroys A and b .
Time: $\mathcal{O}(n^2m)$

```
typedef bitset<1000> bs;
```

```
int solveLinear(vector<bs>& A, vi& b, bs& x, int m) {
    int n = sz(A), rank = 0, br;
    assert(m <= sz(x));
    vi col(m); iota(all(col), 0);
    rep(i,0,n) {
        for (br=i; br<n; ++br) if (A[br].any()) break;
        if (br == n) {
            rep(j,i,n) if(b[j]) return -1;
            break;
        }
        int bc = (int)A[br]._Find_next(i-1);
        swap(A[i], A[br]);
        swap(b[i], b[br]);
        swap(col[i], col[bc]);
        rep(j,0,n) if (A[j][i] != A[j][bc]) {
            A[j].flip(i); A[j].flip(bc);
        }
        rep(j,i+1,n) if (A[j][i]) {
            b[j] ^= b[i];
            A[j] ^= A[i];
        }
        rank++;
    }

    x = bs();
    for (int i = rank; i--;) {
        if (!b[i]) continue;
        x[col[i]] = 1;
        rep(j,0,i) b[j] ^= A[j][i];
    }
    return rank; // (multiple solutions if rank < m)
}
```

MatrixInverse.h

Description: Invert matrix A . Returns rank; result is stored in A unless singular (rank < n). Can easily be extended to prime moduli; for prime powers, repeatedly set $A^{-1} = A^{-1}(2I - AA^{-1}) \pmod{p^k}$ where A^{-1} starts as the inverse of $A \pmod p$, and k is doubled in each step.

Time: $\mathcal{O}(n^3)$

```
int matInv(vector<vector<double>>& A) {
    int n = sz(A); vi col(n);
    vector<vector<double>> tmp(n, vector<double>(n));
    rep(i,0,n) tmp[i][i] = 1, col[i] = i;

    rep(i,0,n) {
        int r = i, c = i;
        rep(j,i,n) rep(k,i,n)
            if (fabs(A[j][k]) > fabs(A[r][c]))
                r = j, c = k;
        if (fabs(A[r][c]) < 1e-12) return i;
        A[i].swap(A[r]); tmp[i].swap(tmp[r]);
        rep(j,0,n)
            swap(A[j][i], A[j][c]), swap(tmp[j][i], tmp[j][c]);
        swap(col[i], col[c]);
        double v = A[i][i];
        rep(j,i+1,n) {
            double f = A[j][i] / v;
            A[j][i] = 0;
            rep(k,i+1,n) A[j][k] -= f*A[i][k];
            rep(k,0,n) tmp[j][k] -= f*tmp[i][k];
        }
        rep(j,i+1,n) A[i][j] /= v;
        rep(j,0,n) tmp[i][j] /= v;
        A[i][i] = 1;
    }

    for (int i = n-1; i > 0; --i) rep(j,0,i) {
```

```
        double v = A[j][i];
        rep(k,0,n) tmp[j][k] -= v*tmp[i][k];
    }

    rep(i,0,n) rep(j,0,n) A[col[i]][col[j]] = tmp[i][j];
    return n;
}
```

4.1 Fourier transforms

FastFourierTransform.h

Description: Computes $f(k) = \sum_x f(x) \exp(-2\pi i k x / N)$ for all k . Useful for convolution: $\text{conv}(a, b) = c$, where $c[x] = \sum a[i]b[x-i]$. a and b should be of roughly equal size. For convolutions of integers, consider using a number-theoretic transform instead, to avoid rounding issues. This works safely when the coefficient result is under 10^{14} .

Time: $\mathcal{O}(N \log N)$

```
const double PI = acos(-1);
typedef complex<double> base;
void fft (vector<base> & a, bool invert) {
    int n = (int) a.size();
    for (int i=1, j=0; i<n; ++i) {
        int bit = n >> 1;
        for (; j>=bit; bit>>=1)
            j -= bit;
        j += bit;
        if (i < j)
            swap (a[i], a[j]);
    }
    for (int len=2; len<=n; len<=1) {
        double ang = 2*PI/len * (invert ? -1 : 1);
        base wlen (cos(ang), sin(ang));
        for (int i=0; i<n; i+=len) {
            base w (1);
            for (int j=0; j<len/2; ++j) {
                base u = a[i+j], v = a[i+j+len/2] * w;
                a[i+j] = u + v;
                a[i+j+len/2] = u - v;
                w *= wlen;
            }
        }
    }
    if (invert)
        for (int i=0; i<n; ++i)
            a[i] /= n;
}

void multiply (const vector<int> &a, const vector<int> &b,
    vector<int> &res){
    vector<base> fa (a.begin(), a.end()), fb (b.begin(), b.end()
    );
    size_t n = 1;
    while (n < max (a.size(), b.size())) n <= 1;
    fa.resize (n), fb.resize (n);
    fft (fa, false), fft (fb, false);
    for (size_t i=0; i<n; ++i)
        fa[i] *= fb[i];
    fft (fa, true);
    res.resize (n);
    for (size_t i=0; i<n; ++i)
        res[i] = int (fa[i].real() + 0.5); // <- beware if res[i]
        negative, floor function retard
    }
```

NumberTheoreticTransform.h

Description: Can be used for convolutions modulo specific nice primes of the form $2^a b + 1$, where the convolution result has size at most 2^a . For other primes/integers, use two different primes and combine with CRT. May return negative values.

Time: $\mathcal{O}(N \log N)$

```
268 lines
// just check if g^((p-1)/r) % p != 1 for every possible r (r =
// prime factor of p-1)
// instead of find g such that g^1, g^2, g^3, ... g^p-1 are the
// permutation of {1, 2, 3, ..., p-1}.
bool check_generator_version_langrage(ll g) {
    for(int i = 1; i <= top; i++) {
        ll value = power(g, (p-1) / prime_factors[i]);
        if (value == 1) return false;
    }
    return true;
}

const int mod = 7340033; // c * 2^k + 1
const ll root = 5; // root = g ^ c % mod
const ll root_1 = 4404020; // root_1 = (root)^-1 % mod
const ll root_pw = 1<<20; // root_pw = (1 << k)
int rev[7340033];
ll getmod(ll a, ll tmod) {return ((a%tmod)+tmod)%tmod;}
void fft (vector<ll> &a, bool invert) {
    int n = (int) a.size();
    for (int i=1, j=0; i<n; ++i) {
        int bit = n >> 1;
        for (; j>=bit; bit>>=1)
            j -= bit;
        j += bit;
        if (i < j)
            swap (a[i], a[j]);
    }
    for (int len=2; len<=n; len<=1) {
        ll wlen = invert ? root_1 : root;
        for (int i=len; i<root_pw; i<=1)
            wlen = ll (wlen * ll1 * wlen % mod);
        for (int i=0; i<n; i+=len) {
            ll w = 1;
            for (int j=0; j<len/2; ++j) {
                ll u = a[i+j], v = ll (a[i+j+len/2] * ll1 * w % mod);
                a[i+j] = getmod(u+v,mod);
                a[i+j+len/2] = getmod(u-v,mod);
                w = ll (w * ll1 * wlen % mod);
            }
        }
    }
    if (invert) {
        for (int i=0; i<n; ++i)
            a[i] = int (a[i] * ll1 * rev[n] % mod);
    }
}

void precalc(){ // calculate inverse of MOD in O(MOD)
    rev[1] = 1;
    for (int i=2; i<mod; i++)
        rev[i] = (mod - (mod/i) * rev[mod%i] % mod) % mod;
}

void multiply (const vector<ll> &a, const vector<ll> &b,
    vector<ll> &res) {
    vector<ll> fa (a.begin(), a.end()), fb (b.begin(), b.end()
    );
    size_t n = 1;
    while (n < max (a.size(), b.size())) n <= 1;
    fa.resize (n), fb.resize (n);
    fft (fa, false), fft (fb, false);
    for(i,n)
        fa[i] *= fb[i];
    fft (fa, true);
```

```

    res.resize (n);
    forn(i,n) // for(i=0;i<n;)
        res[i] = fa[i] % mod;
}

namespace fft{
    typedef complex<double> base;
    void fft(vector<base> &a, bool inv){
        int n = a.size(), j = 0;
        vector<base> roots(n/2);
        for(int i=1; i<n; i++){
            int bit = (n >> 1);
            while(j >= bit){
                j -= bit;
                bit >>= 1;
            }
            j += bit;
            if(i < j) swap(a[i], a[j]);
        }
        double ang = 2 * acos(-1) / n * (inv ? -1 : 1);
        for(int i=0; i<n/2; i++){
            roots[i] = base(cos(ang * i), sin(ang * i));
        }
        for(int i=2; i<n; i<=1){
            int step = n / i;
            for(int j=0; j<n; j+=i){
                for(int k=0; k<i/2; k++){
                    base u = a[j+k], v = a[j+k+i/2] * roots[step * k];
                    a[j+k] = u+v;
                    a[j+k+i/2] = u-v;
                }
            }
        }
        if(inv) for(int i=0; i<n; i++) a[i] /= n;
    }

    vector<lint> multiply(vector<lint> &v, vector<lint> &w){
        vector<base> fv(v.begin(), v.end()), fw(w.begin(), w.end())
        ;
        int n = 1;
        while(n < max(v.size(), w.size())) n <= 1;
        n <= 1;
        fv.resize(n);
        fw.resize(n);
        fft(fv, 0);
        fft(fw, 0);
        for(int i=0; i<n; i++) fv[i] *= fw[i];
        fft(fv, 1);
        vector<lint> ret(n);
        for(int i=0; i<n; i++) ret[i] = (lint)round(fv[i].real());
        return ret;
    }

    vector<lint> multiply(vector<lint> &v, vector<lint> &w, lint
        mod){
        int n = 1;
        while(n < max(v.size(), w.size())) n <= 1;
        n <= 1;
        vector<base> v1(n), v2(n);
        vector<base> r1(n), r2(n);
        for(int i=0; i<v.size(); i++){
            v1[i] = base(v[i] >> 15, v[i] & 32767);
        }
        for(int i=0; i<w.size(); i++){
            v2[i] = base(w[i] >> 15, w[i] & 32767);
        }
        fft(v1, 0);
        fft(v2, 0);
        for(int i=0; i<n; i++){
            int j = (i ? (n - i) : i);

```

```

            base ans1 = (v1[i] + conj(v1[j])) * base(0.5, 0);
            base ans2 = (v1[i] - conj(v1[j])) * base(0, -0.5);
            base ans3 = (v2[i] + conj(v2[j])) * base(0.5, 0);
            base ans4 = (v2[i] - conj(v2[j])) * base(0, -0.5);
            r1[i] = (ans1 * ans3) + (ans1 * ans4) * base(0, 1);
            r2[i] = (ans2 * ans3) + (ans2 * ans4) * base(0, 1);
        }
        fft(r1, 1);
        fft(r2, 1);
        vector<lint> ret(n);
        for(int i=0; i<n; i++){
            lint av = (lint)round(r1[i].real());
            lint bv = (lint)round(r1[i].imag()) + (lint)round(r2[i].
                real());
            lint cv = (lint)round(r2[i].imag());
            av %= mod, bv %= mod, cv %= mod;
            ret[i] = (av << 30) + (bv << 15) + cv;
            ret[i] %= mod;
            ret[i] += mod;
            ret[i] %= mod;
        }
        return ret;
    }
}

struct NTT {
public:
    NTT(const int _mod) : mod(_mod) {
        proot = find_primitive_root();
        int totient = mod-1;
        max_len = 1;
        while ((totient & max_len) == 0)
            max_len <= 1;
    }

    void fft(vector<int> &a, bool invert) {
        int n = (int) a.size();
        if (n & (n-1)) {
            int pw = 1;
            while (pw < n) pw <= 1;
            n = pw;
            a.resize(n, 0);
        }
        assert(n <= max_len);

        for (int i=1, j=0; i<n; ++i) {
            int bit = n >> 1;
            for (; j>=bit; bit>>=1)
                j -= bit;
            j += bit;
            if (i < j)
                swap (a[i], a[j]);
        }

        int root = powmod(proot, (mod-1) / n);
        if (invert)
            root = inverse(root);

        for (int len = 2; len <= n; len <= 1) {
            int wlen = root;
            for (int i = len; i < n; i <= 1)
                wlen = int (wlen * 111 * wlen % mod);
            for (int i = 0; i < n; i += len) {
                int w = 1;
                for (int j = 0; j < len/2; ++j) {
                    int u = a[i+j], v = int (a[i+j+len/2] * 111 * w % mod
                        );
                    a[i+j] = u+v < mod ? u+v : u+v-mod;
                    a[i+j+len/2] = u-v >= 0 ? u-v : u-v+mod;

```

```

                    w = int (w * 111 * wlen % mod);
                }
            }
        }
        if (invert) {
            int nrev = inverse(n);
            for (int i = 0; i < n; ++i)
                a[i] = int (a[i] * 111 * nrev % mod);
        }
    }

    vector<int> multiply(vector<int> a, vector<int> b) {
        int len = a.size() + b.size() - 1;
        int pw = 1;
        while (pw < len) pw <= 1;
        len = pw;
        a.resize(len, 0);
        b.resize(len, 0);
        fft(a, false);
        fft(b, false);
        for (int i = 0; i < len; ++i)
            a[i] = 111 * a[i] * b[i] % mod;
        fft(a, true);
        while (len > 0 && a[len-1] == 0)
            --len;
        a.resize(len);
        return a;
    }

    long long powmod(long long b, long long p) {
        long long r = 1;
        for (; p; p >>= 1, b = b * b % mod)
            if (p & 1)
                r = r * b % mod;
        return r;
    }

    long long inverse(int x) {
        return powmod(x, mod-2);
    }

private:
    const int mod;
    int proot;
    int max_len = 1;

    int find_primitive_root() {
        int totient = mod - 1;
        vector<int> checks;
        int tmp = totient;
        for (int p = 2; p * p <= tmp; ++p) {
            if (tmp % p) continue;
            checks.push_back(totient/p);
            while ((tmp % p) == 0) tmp /= p;
        }
        if (tmp > 1)
            checks.push_back(totient/tmp);
        for (int r = 2; ; ++r) {
            bool ok = 1;
            for (int f : checks) {
                if (powmod(r, f) == 1) {
                    ok = 0;
                    break;
                }
            }
            if (ok)
                return r;
        }
        assert(false);
    }

```



```
    }
};

const int mod = 998244353;

NTT ntt(mod);
```

FastSubsetTransform.h
Description: Transform to a basis with fast convolutions of the form $c[z] = \sum_{z=x\oplus y} a[x] \cdot b[y]$, where \oplus is one of AND, OR, XOR. The size of a must be a power of two.
Usage: N should be in the form $1<<X$
Time: $\mathcal{O}(N \log N)$

```
template <typename T>
struct FWT {
    void fwt(vector<T>&io, int n,int t) {
        for (int d = 1; d < n; d <= 1) {
            for (int i = 0, m = d<<1; i < n; i += m) {
                for (int j = 0; j < d; j++) {
                    T x = io[i+j], y = io[i+j+d];
                    if(t==0)io[i+j] = (x+y)%MOD, io[i+j+d] = (x+MOD-y)%MOD; // xor
                    else if(t==1)io[i+j] = (x+y)%MOD; // and
                    else io[i+j+d] = (x+y)%MOD; // or
                }
            }
        }
    }
    void ufwt(vector<T>&io, int n,int t) {
        for (int d = 1; d < n; d <= 1) {
            for (int i = 0, m = d<<1; i < n; i += m) {
                for (int j = 0; j < d; j++) {
                    T x = io[i+j], y = io[i+j+d];
                    if(t==0)io[i+j] = (x+y)*dua%MOD, io[i+j+d] = (x-y+MOD)*dua%MOD; // xor
                    else if(t==1)io[i+j] = (x-y+MOD)%MOD; // and
                    else io[i+j+d] = (y-x+MOD)%MOD; // or
                }
            }
        }
    }
    // a, b are two polynomials and n is size which is power of two
    void convolution(vector<T>&a, vector<T>&b, int n,int t) {
        fwt(a, n,t);
        fwt(b, n,t);
        for (int i = 0; i < n; i++)
            a[i] = a[i]*b[i]%MOD;
        ufwt(a, n,t);
    }
};
```

35 lines

```
ull res = k / m * sumsq(to) + c / m * to;
k %= m; c %= m;
if (k) {
    ull to2 = (to * k + c) / m;
    res += to * to2;
    res -= divsum(to2, m-1 - c, m, k) + to2;
}
return res;
}

ll modsum(ull to, ll c, ll k, ll m) {
    c = ((c % m) + m) % m;
    k = ((k % m) + m) % m;
    return to * c + k * sumsq(to) - m * divsum(to, c, k, m);
}
```

ModSqrt.h
Description: Tonelli-Shanks algorithm for modular square roots.
Time: $\mathcal{O}(\log^2 p)$ worst case, often $\mathcal{O}(\log p)$

```
"ModPow.h"
30 lines

ll sqrt(ll a, ll p) {
    a %= p; if (a < 0) a += p;
    if (a == 0) return 0;
    assert(modpow(a, (p-1)/2, p) == 1);
    if (p % 4 == 3) return modpow(a, (p+1)/4, p);
    // a^(n+3)/8 or 2^(n+3)/8 * 2^(n-1)/4 works if p % 8 == 5
    ll s = p - 1;
    int r = 0;
    while (s % 2 == 0)
        ++r, s /= 2;
    ll n = 2; // find a non-square mod p
    while (modpow(n, (p - 1) / 2, p) != p - 1) ++n;
    ll x = modpow(a, (s + 1) / 2, p);
    ll b = modpow(a, s, p);
    ll g = modpow(n, s, p);
    for (;;) {
        ll t = b;
        int m = 0;
        for (; m < r; ++m) {
            if (t == 1) break;
            t = t * t % p;
        }
        if (m == 0) return x;
        ll gs = modpow(g, 1 << (r - m - 1), p);
        g = gs * gs % p;
        x = x * gs % p;
        b = b * g % p;
        r = m;
    }
}
```

5.2 Primality

MillerRabinAndPollardRho.h
Description: Miller-Rabin primality probabilistic test. if $n < 3,825,123,056,546,413,051$, it is enough to test $a = 2, 3, 5, 7, 11, 13, 17, 19$, and 23 . This pollard rho and miller rabin both works for $n < 10^{18}$
Time: 15 times the complexity of $a^b \bmod c$.

62 lines

```
vector<long long> A({2, 3, 5, 7, 11, 13, 17, 19, 23});

long long largemul(long long a, long long b, long long n) {
    // assert(0 <= a && a < n && 0 <= b && b < n);
    long long r = 0;
    for (; b; b >= 1, a <= 1) {
        if (a >= n) a -= n;
        if (b & 1) {
            r += a;
            if (r >= n) r -= n;
        }
    }
}
```

```
    }
}
return r;
}

long long fastexp(long long a, long long b, long long n) {
    // assert(0 <= a && a < n && b >= 0);
    long long ret = 1;
    for (; b; b >= 1, a = largemul(a, a, n))
        if (b & 1) ret = largemul(ret, a, n);
    return ret;
}

bool mrtest(long long n) {
    if (n == 1) return false;
    long long d = n-1;
    int s = 0;
    while ((d & 1) == 0) {
        s++;
        d >>= 1;
    }
    s--;
    if (s < 0) s = 0;
    for (int j = 0; j < (int)A.size(); j++) {
        if (A[j] >= n) continue;
        long long ad = fastexp(A[j], d, n);
        if (ad == 1) continue;
        bool notcomp = false;
        long long a2rd = ad;
        for (int r = 0; r <= s; r++) {
            if(a2rd == n-1) {notcomp = true; break;}
            a2rd = largemul(a2rd, a2rd, n);
        }
        if (!notcomp) {
            return false;
        }
    }
    return true;
}

long long gcd(long long a, long long b) { return a ? gcd(b % a, a) : b; }

long long pollard_rho(long long n) {
    int i = 0, k = 2;
    long long x = 3, y = 3; // random seed = 3, other values possible
    while (1) {
        i++;
        x = largemul(x, x, n)-1; // generating function
        if (x < 0) x += n;
        long long d = gcd(llabs(y - x), n); // the key insight
        if (d != 1 && d != n) return d;
        if (i == k) y = x, k <= 1;
    }
}
```

5.3 Divisibility

euclid.h
Description: Finds the Greatest Common Divisor to the integers a and b . Euclid also finds two integers x and y , such that $ax + by = \gcd(a,b)$. If a and b are coprime, then x is the inverse of $a \pmod b$.

8 lines

```
ll gcd(ll a, ll b) { return __gcd(a, b); }

// beware overflow when get the actual result (x * c/gcd). Use
// bezout identity to make the result small. (by using mod b/ gcd(a,b))
ll euclid(ll a, ll b, ll &x, ll &y) {
    if (b) { ll d = euclid(b, a % b, y, x);
```

```
    return y -= a/b * x, d; }
    return x = 1, y = 0, a;
}
```

5.3.1 Bézout’s identity

For $a \neq, b \neq 0$, then $d = gcd(a,b)$ is the smallest positive integer for which there are integer solutions to

$ax + by = d$

If (x,y) is one solution, then all solutions are given by

$\left(x + \frac{kb}{gcd(a,b)}, y - \frac{ka}{gcd(a,b)}\right), \quad k \in \mathbb{Z}$

phiFunction.h
Description: Euler’s totient or Euler’s phi function is defined as $\phi(n) := \#$ of positive integers $\leq n$ that are coprime with n . The cototient is $n - \phi(n)$. $\phi(1) = 1, p$ prime $\Rightarrow \phi(p^k) = (p - 1)p^{k-1}, m, n$ coprime $\Rightarrow \phi(mn) = \phi(m)\phi(n)$. If $n = p_1^{k_1} p_2^{k_2} \dots p_r^{k_r}$ then $\phi(n) = (p_1 - 1)p_1^{k_1-1} \dots (p_r - 1)p_r^{k_r-1}$. $\phi(n) = n \cdot \prod_{p|n} (1 - 1/p)$.
 $\sum_{d|n} \phi(d) = n, \sum_{1 \leq k \leq n, gcd(k,n)=1} k = n\phi(n)/2, n > 1$
Euler’s thm: a, n coprime $\Rightarrow a^{\phi(n)} \equiv 1 \pmod n$.
Fermat’s little thm: p prime $\Rightarrow a^{p-1} \equiv 1 \pmod p \forall a$.

```
const int LIM = 5000000;
int phi[LIM];
```

```
void calculatePhi() {
    rep(i,0,LIM) phi[i] = i&1 ? i : i/2;
    for(int i = 3; i < LIM; i += 2)
        if(phi[i] == i)
            for(int j = i; j < LIM; j += i)
                (phi[j] /= i) *= i-1;
}
```

5.4 Fractions

5.5 Chinese remainder theorem

chinese.h
Description: Chinese Remainder Theorem.
Time: $\log(m + n)$
"euclid.h" 22 lines
// Chinese remainder theorem (special case): find z such that
// z % x = a, z % y = b. Here, z is unique modulo M = lcm(x,y)
.
// Return (z,M). On failure, M = -1.
PII chinese_remainder_theorem(int x, int a, int y, int b) {
 int s, t;
 int d = extended_euclid(x, y, s, t);
 if (a%d != b%d) return make_pair(0, -1);
 return make_pair(mod(s*b*x+t*a*y,x*y)/d, x*y/d);
}
// Chinese remainder theorem: find z such that
// z % x[i] = a[i] for all i. Note that the solution is
// unique modulo M = lcm_i (x[i]). Return (z,M). On
// failure, M = -1. Note that we do not require the a[i]’s
// to be relatively prime.
PII chinese_remainder_theorem(const VI &x, const VI &a) {
 PII ret = make_pair(a[0], x[0]);
 for (int i = 1; i < x.size(); i++) {
 ret = chinese_remainder_theorem(ret.second, ret.first, x[i]
], a[i]);
 }

phiFunction chinese

```
    if (ret.second == -1) break;
}
return ret;
}
```

5.6 Pythagorean Triples

The Pythagorean triples are uniquely generated by

$a = k \cdot (m^2 - n^2), \quad b = k \cdot (2mn), \quad c = k \cdot (m^2 + n^2),$

with $m > n > 0, k > 0, m \perp n$, and either m or n even.

5.7 Primes

$p = 962592769$ is such that $2^{21} \mid p - 1$, which may be useful. For hashing use 970592641 (31-bit number), 31443539979727 (45-bit), 3006703054056749 (52-bit). There are 78498 primes less than 1 000 000. 664 579 primes under 10^7 , 5 761 455 primes under 10^7 , 50.847.534 primes under 10^9 .

Primitive roots exist modulo any prime power p^a , except for $p = 2, a > 2$, and there are $\phi(\phi(p^a))$ many. For $p = 2, a > 2$, the group $\mathbb{Z}_{2^a}^\times$ is instead isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_{2^{a-2}}$.

5.8 Estimates

$\sum_{d|n} d = O(n \log \log n)$.

The number of divisors of n is at most around 100 for $n < 5e4$, 500 for $n < 1e7$, 2000 for $n < 1e10$, 200 000 for $n < 1e19$.

5.9 Mobius Inversion

5.9.1 Definition

$$\mu(n) = \begin{cases} 0 & n \text{ isn't square free} \\ 1 & \text{if } n \text{ is equal to } 1 \\ (-1)^k & n \text{ has } k \text{ distinct prime} \end{cases}$$

If f ang d are arithmetic function satisfying

$$g(n) = \sum_{d|n} f(d)$$

Then

$$f(n) = \sum_{d|n} \mu(d)g\left(\frac{n}{d}\right)$$

5.9.2 Example

Find:

$$\sum_{i=1}^n \sum_{j=i+1}^n gcd(i, j)$$

Can be transformed into:

$$G = \sum_{g=1}^n h(g) * cnt(g)$$

In this example, $h(g) = g$. Present $h(g)$ as mobius with:

$$rep(i, 1, n)rep(j, i, n, i)f[j] = h[i] * \mu\left(\frac{j}{i}\right)$$

Then:

$$G = \sum_{g=1}^n \left(\sum_{d|g} f(d)\right) * cnt(g)$$

$$G = \sum_{d=1}^n f(d) * cnt2(d)$$

here $cnt2(d)$ = how many d such that $gcd(i, j)$ is multiple of d. This is easy to count:

$$G = \sum_{d=1}^n f(d) \left(\frac{n}{d}\right)^2$$

$f(d)$ is happened to be totient here. Note that we can calculate f using 2 for loops above.

5.9.3 Example 2

Find triplet (i, j, k) such that $gcd(a[i], a[j], a[k]) = 1$. Same as step above, but $h(g) = 1$ only if $g = 1$ else $h(g) = 0$.

After some calculation, $cnt2(d) = gcd(a[i], a[j], a[k])$ is multiple of d.

If $dp[x]$ is the number of i such that $x|a[i]$, then $cnt2[d] = C(dp[x], 3)$

5.9.4 Example 3

Find out sum of $lcm(x, y)$ for each pair (x, y) in range $(1, n)$ Answer:

$$g(l) = \sum_{d|l} \mu(d)ld$$

$$f(n) = \sum_{l=1}^n \left(\frac{n}{l} + 1\right)^2 g(l)$$

5.9.5 Helper $O(2\sqrt{n})$

Loop through different values of n/d.
for(int i = 1; la; i <= n; i = la + 1) la = n / (n / i);
n/x yields the same value for $i <= x <= la$

Combinatorial (6)

6.1 Permutations

6.1.1 Factorial

6.1.2 Cycles

Let $g_S(n)$ be the number of n -permutations whose cycle lengths all belong to the set S . Then

$$\sum_{n=0}^\infty g_S(n) \frac{x^n}{n!} = \exp\left(\sum_{n\in S} \frac{x^n}{n}\right)$$

6.1.3 Derangements

Permutations of a set such that none of the elements appear in their original position.

$$D(n) = (n-1)(D(n-1)+D(n-2)) = nD(n-1)+(-1)^n = \left\lfloor \frac{n!}{e} \right\rfloor$$

6.1.4 Burnside’s lemma

Given a group G of symmetries and a set X , the number of elements of X up to symmetry equals

$$\frac{1}{|G|} \sum_{g\in G} |X^g|,$$

where X^g are the elements fixed by g ($g.x = x$).

If $f(n)$ counts ”configurations” (of some sort) of length n , we can ignore rotational symmetry using $G = \mathbb{Z}_n$ to get

$$g(n) = \frac{1}{n} \sum_{k=0}^{n-1} f(\gcd(n, k)) = \frac{1}{n} \sum_{k|n} f(k) \phi(n/k).$$

6.2 Partitions and subsets

6.2.1 Partition function

Number of ways of writing n as a sum of positive integers, disregarding the order of the summands.

$$p(0) = 1, \; p(n) = \sum_{k\in\mathbb{Z}\setminus\{0\}} (-1)^{k+1} p(n - k(3k - 1)/2)$$

$$p(n) \sim 0.145/n \cdot \exp(2.56\sqrt{n})$$

n	0	1	2	3	4	5	6	7	8	9	20	50	100
$p(n)$	1	1	2	3	5	7	11	15	22	30	627	~2e5	~2e8

6.2.2 Binomials

binomialModPrime.h
Description: Lucas’ thm: Let n, m be non-negative integers and p a prime. Write $n = n_kp^k + \dots + n_1p + n_0$ and $m = m_kp^k + \dots + m_1p + m_0$. Then $\binom{n}{m} \equiv \prod_{i=0}^k \binom{n_i}{m_i} \pmod{p}$. fact and invfact must hold pre-computed factorials / inverse factorials, e.g. from ModInverse.h.
Time: $\mathcal{O}(\log_p n)$

```
// in case MOD is small yet a and b can be larger than it
int comb (int a,int b) {
    int res = 1;
    while(a > 0 && b > 0) { // beware of a%mod < b%mod, should
        return 0
        res = (res * C[a % MOD][b % MOD]) % MOD;
        a /= MOD; b /= MOD;
    }
    return res;
}
```

6.3 General purpose numbers

6.3.1 Stirling numbers of the first kind

Number of permutations on n items with k cycles.

$$c(n, k) = c(n - 1, k - 1) + (n - 1)c(n - 1, k), \; c(0, 0) = 1$$
$$\sum_{k=0}^n c(n, k)x^k = x(x + 1) \dots (x + n - 1)$$

$$c(8, k) = 8, 0, 5040, 13068, 13132, 6769, 1960, 322, 28, 1$$
$$c(n, 2) = 0, 0, 1, 3, 11, 50, 274, 1764, 13068, 109584, \dots$$

6.3.2 Eulerian numbers

Number of permutations $\pi \in S_n$ in which exactly k elements are greater than the previous element. k j:s s.t. $\pi(j) > \pi(j + 1)$, $k + 1$ j:s s.t. $\pi(j) \geq j$, k j:s s.t. $\pi(j) > j$.

$$E(n, k) = (n - k)E(n - 1, k - 1) + (k + 1)E(n - 1, k)$$

$$E(n, 0) = E(n, n - 1) = 1$$

$$E(n, k) = \sum_{j=0}^k (-1)^j \binom{n+1}{j} (k+1-j)^n$$

6.3.3 Stirling numbers of the second kind

Partitions of n distinct elements into exactly k groups.

$$S(n, k) = S(n - 1, k - 1) + kS(n - 1, k)$$

$$S(n, 1) = S(n, n) = 1$$

$$S(n, k) = \frac{1}{k!} \sum_{j=0}^k (-1)^{k-j} \binom{k}{j} j^n$$

6.3.4 Bell numbers

Total number of partitions of n distinct elements. $B(n) = 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, \dots$. For p prime,

$$B(p^m + n) \equiv mB(n) + B(n + 1) \pmod{p}$$

6.3.5 Catalan numbers

$$C_n = \frac{1}{n+1} \binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n+1} = \frac{(2n)!}{(n+1)!n!}$$

$$C_0 = 1, \; C_{n+1} = \frac{2(2n+1)}{n+2} C_n, \; C_{n+1} = \sum C_i C_{n-i}$$

$$C_n = 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, \dots$$

- sub-diagonal monotone paths in an $n \times n$ grid.
- strings with n pairs of parenthesis, correctly nested.
- binary trees with with $n + 1$ leaves (0 or 2 children).
- ordered trees with $n + 1$ vertices.
- ways a convex polygon with $n + 2$ sides can be cut into triangles by connecting vertices with straight lines.
- permutations of $[n]$ with no 3-term increasing subseq.

Graph (7)

7.1 Fundamentals

bellmanFord.h
Description: Calculates shortest path in a graph that might have negative edge distances. Propagates negative infinity distances (sets dist = -inf), and returns true if there is some negative cycle. Unreachable nodes get dist = inf.
Time: $\mathcal{O}(EV)$

```
#define ll T; // or whatever
struct Edge { int src, dest; T weight; };
struct Node { T dist; int prev; };
struct Graph { vector<Node> nodes; vector<Edge> edges; };

const T inf = numeric_limits<T>::max();
bool bellmanFord2(Graph& g, int start_node) {
    trav(n, g.nodes) { n.dist = inf; n.prev = -1; }
    g.nodes[start_node].dist = 0;

    rep(i,0,sz(g.nodes)) trav(e, g.edges) {
        Node& cur = g.nodes[e.src];
        Node& dest = g.nodes[e.dest];
        if (cur.dist == inf) continue;
        T ndist = cur.dist + (cur.dist == -inf ? 0 : e.weight);
        if (ndist < dest.dist) {
            dest.prev = e.src;
            dest.dist = (i >= sz(g.nodes)-1 ? -inf : ndist);
        }
    }
    bool ret = 0;
    rep(i,0,sz(g.nodes)) trav(e, g.edges) {
        if (g.nodes[e.src].dist == -inf)
            g.nodes[e.dest].dist = -inf, ret = 1;
    }
}
```

```
    return ret;
}

7.2 Euler walk
EulerWalk.h
Description: Eulerian undirected/directed path/cycle algorithm.
Usage: For eulerian path, should pass cur with odd degree to eulerian().
Time:  $\mathcal{O}(E)$  where E is the number of edges.
21 lines
```

```
void eulerian(int cur){
    stack<int> st;
    vector<int> ans;
    st.push(cur);
    //V is multiset
    while(!st.empty()){
        int cur = st.top();
        if(V[cur].size()){
            auto it = V[cur].begin();
            st.push(*it);
            V[cur].erase(it);
            //use this for bidirectional graph
            //if(V[*it].count(cur)){
            //    V[*it].erase(V[*it].find(cur));
            //}
        }else{
            ans.pb(cur);
            st.pop();
        }
    }
}
```

7.3 Network flow

```
MinCostMaxFlow.h
Description: Min-cost max-flow. Not handling negative cycle. When there is negative cycle, there is no answer for MCMF.
Time: Approximately  $\mathcal{O}(E^2)$ 
70 lines
```

```
struct edge{
    int to, rev;
    int flow, cap;
    int cost;
};
vector<edge> G[500];
inline void add(int s, int t, int capa, int costs) {
    edge a = {t, G[t].size(), 0, capa, costs};
    edge b = {s, G[s].size(), 0, 0, -costs};
    G[s].push_back(a);
    G[t].push_back(b);
}
inline bool SPFA() {
    for(int i = 0; i <= sink; i++) dist[i] = INF, flag[i] = false, bt[i] = -1, idx[i] = -1;
    dist[source] = 0;
    queue<int> q;
    q.push(source);
    flag[source] = true;
    while(!q.empty()) {
        int now = q.front();
        q.pop();
        flag[now] = false;
        int size = G[now].size();
        for(int i = 0; i < size; i++) {
            int to = G[now][i].to;
            int cost = G[now][i].cost;
            int capa = G[now][i].cap;
            if (capa > 0 && dist[to] > dist[now] + cost) {
                dist[to] = dist[now] + cost;
```

```
                bt[to] = now;
                idx[to] = i;
                if (!flag[to]) {
                    flag[to] = true;
                    q.push(to);
                }
            }
        }
    }
    return bt[sink] != -1;
}

pair<int,int> MCMF() {
    pair<int,int> res; res.first = 0, res.second = 0;
    while(true) {
        if (!SPFA()) break;
        int mins = INF;
        int ptr = sink;
        int total = 0;
        while(ptr != source) {
            int from = bt[ptr];
            int id = idx[ptr];
            if (G[from][id].cap < mins)
                mins = G[from][id].cap;
            total += G[from][id].cost;
            ptr = from;
        }
        res.first += mins;
        res.second += total * mins;
        ptr = sink;
        while(ptr != source) {
            int from = bt[ptr];
            int id = idx[ptr];
            int rev = G[from][id].rev;
            G[from][id].cap -= mins;
            G[ptr][rev].cap += mins;
            ptr = from;
        }
    }
    return res;
}
```

```
Dinic.h
Description: Flow algorithm with guaranteed complexity  $\mathcal{O}(V^2E)$ .
59 lines
```

```
struct edge{
    int to, rev;
    int flow, cap;
};
vector<edge> G[MAXE];
inline void add(int s, int t, int cap) {
    edge a = {t, G[t].size(), 0, cap};
    edge b = {s, G[s].size(), 0, 0};
    G[s].push_back(a);
    G[t].push_back(b);
}
inline bool search() {
    for(int i = 0; i <= n + 1; i++) dist[i] = -1;
    dist[source] = 0;
    int tail = 0;
    q[tail] = source;
    for(int head = 0; head <= tail; head++) {
        int u = q[head];
        int sz = G[u].size();
        for(int i = 0; i < sz; i++) {
            int v = G[u][i].to;
            if (dist[v] < 0 && G[u][i].flow < G[u][i].cap) {
                dist[v] = dist[u] + 1;
                q[++tail] = v;
```

```
            }
        }
    }
    return dist[sink] >= 0;
}
int dinic(int now, int flo) {
    if (now == sink)
        return flo;
    int size = G[now].size();
    for(int &i = work[now]; i < size; i++) {
        int to = G[now][i].to, flow = G[now][i].flow, cap = G[now][i].cap, rev = G[now][i].rev;
        if (flow >= cap) continue;
        if (dist[to] == dist[now] + 1) {
            int fflow = dinic(to, min(flo, cap - flow));
            if (fflow) {
                G[now][i].flow += fflow;
                G[to][rev].flow -= fflow;
                return fflow;
            }
        }
    }
    return 0;
}
inline int maxflow() {
    int ans = 0;
    while(search()) {
        for(int i = 0; i <= n + 1; i++) work[i] = 0;
        while(true) {
            int res = dinic(source, INF);
            if (res == 0) break;
            ans += res;
        }
    }
    return ans;
}
```

```
MinCut.h
Description: After running max-flow, the left side of a min-cut from s to t is given by all vertices reachable from s, only traversing edges with positive residual capacity.
```

```
GlobalMinCut.h
Description: Stoer-Wagner. Find a global minimum cut in an undirected graph, as represented by an adjacency matrix.
Time:  $\mathcal{O}(V^3)$ 
31 lines
```

```
pair<int, vi> GetMinCut(vector<vi>& weights) {
    int N = sz(weights);
    vi used(N), cut, best_cut;
    int best_weight = -1;

    for (int phase = N-1; phase >= 0; phase--) {
        vi w = weights[0], added = used;
        int prev, k = 0;
        rep(i,0,phase){
            prev = k;
            k = -1;
            rep(j,1,N)
                if (!added[j] && (k == -1 || w[j] > w[k])) k = j;
            if (i == phase-1) {
                rep(j,0,N) weights[prev][j] += weights[k][j];
                rep(j,0,N) weights[j][prev] = weights[prev][j];
                used[k] = true;
                cut.push_back(k);
                if (best_weight == -1 || w[k] < best_weight) {
                    best_cut = cut;
```

```
        best_weight = w[k];
    }
} else {
    rep(j,0,N)
        w[j] += weights[k][j];
    added[k] = true;
}
}
}
return {best_weight, best_cut};
}
```

7.4 Matching

hopcroftKarp.h

Description: Find a maximum matching in a bipartite graph.
Usage: node from 1..n || 0 is NIL
left side 1..n || right side n+1..n+m
G = {0} U {1..n} U {n+1..n+m}
Time: $\mathcal{O}(\sqrt{VE})$

```
bool bfs() {
    queue<int> q;
    for(int i = 1 ; i <= n ; i++)
        if(match[i] == 0) {
            dist[i] = 0;
            q.push(i);
        }
    else
        dist[i] = INF;
    dist[0] = INF;

    while(!q.empty()) {
        int cur = q.front();
        q.pop();
        if(cur) {
            for(int nex : adj[cur]) {
                if(dist[match[nex]] == INF) {
                    dist[match[nex]] = dist[cur] + 1;
                    q.push(match[nex]);
                }
            }
        }
    }
    return dist[0] != INF;
}

int dfs(int now) {
    if(now == 0) return 1; // found 1 augmenting path
    for(int nex : adj[now]) {
        if(dist[match[nex]] == dist[now] + 1 && dfs(match[nex])) {
            match[nex] = now;
            match[now] = nex;
            return 1;
        }
    }
    dist[now] = INF;
    return 0;
}

int hopcroftKarp() {
    int ret = 0;
    memset(match, 0, sizeof match);
    while(bfs()) {
        for(int i = 1 ; i <= n ; i++)
            if(match[i] == 0)
                ret += dfs(i);
    }
    return ret;
}
```

DFSMatching.h

Description: This is a simple matching algorithm but should be just fine in most cases. Graph g should be a list of neighbours of the left partition. n is the size of the left partition and m is the size of the right partition. If you want to get the matched pairs, $match[i]$ contains match for vertex i on the right side or -1 if it's not matched.
Time: $\mathcal{O}(EV)$ where E is the number of edges and V is the number of vertices.

```
vi match;
vector<bool> seen;
bool find(int j, const vector<vi>& g) {
    if (match[j] == -1) return 1;
    seen[j] = 1; int di = match[j];
    trav(e, g[di])
        if (!seen[e] && find(e, g)) {
            match[e] = di;
            return 1;
        }
    return 0;
}

int dfs_matching(const vector<vi>& g, int n, int m) {
    match.assign(m, -1);
    rep(i,0,n) {
        seen.assign(m, 0);
        trav(j,g[i])
            if (find(j, g)) {
                match[j] = i;
                break;
            }
    }
    return m - (int)count(all(match), -1);
}
```

Hungarian.h

Description: Min cost bipartite matching. Negate costs for max cost. Be-careful of INF.
Usage: $k = \max(N,M)$ where N is left and M is right.
Time: $\mathcal{O}(N^3)$

```
const int INF = 1e9;
int a[N][N],u[N],v[N],ans[N],minv[N],p[N],way[N];
bool used[N];
int Assign(){
    for(int i = 1 ; i <= k ; i++){
        p[0] = i;
        int j0 = 0;
        for(int j = 1 ; j <= k ; j++)
            minv[j] = INF,used[j] = 0;
        do{
            used[j0] = 1;
            int i0 = p[j0],delta = INF, j1;
            for(int j = 1 ; j <= k ; j++)
                if(!used[j]){
                    int cur = val[i0][j] - u[i0] - v[j];
                    if(cur < minv[j])
                        minv[j] = cur,way[j] = j0;
                    if(minv[j] < delta)
                        delta = minv[j], j1 = j;
                }
            for(int j = 0 ; j <= k ; j++)
                if(used[j])
                    u[p[j]] += delta, v[j] -= delta;
                else
                    minv[j] -= delta;
            j0 = j1;
        }while(p[j0] != 0);
    }
    do{
```

```
        int j1 = way[j0];
        p[j0] = p[j1];
        j0 = j1;
    }while(j0);
}

for(int i = 1 ; i <= k ; i++)
    ans[p[i]] = i;
int ret = 0;
for(int i = 1 ; i <= k ; i++)
    ret += val[i][ans[i]]; // i is matched with job ans[i]
return ret;
}
```

GeneralMatching.h

Description: Matching for general graphs. Fails with probability N/mod .
Time: $\mathcal{O}(N^3)$

```
int lca(vector<int>&match, vector<int>&base, vector<int>&p,int a,int b){
    vector<bool> used(SZ(match));
    while(true){
        a = base[a];
        used[a]=true;
        if(match[a]==-1)break;
        a = p[match[a]];
    }
    while(true){
        b = base[b];
        if(used[b])return b;
        b = p[match[b]];
    }
    return-1;
}

void markPath(vector<int>&match, vector<int>&base, vector<bool>&blossom, vector<int>&p,int v,int b,int children){
    for(; base[v] != b; v = p[match[v]]){
        blossom[base[v]] = blossom[base[match[v]]]=true;
        p[v] = children;
        children = match[v];
    }
}

int findPath(vector<vector<int>>&graph, vector<int>&match, vector<int>&p,int root){
    int n = SZ(graph);
    vector<bool> used(n);
    FORIT(it, p)*it =-1;
    vector<int> base(n);
    for(int i =0; i < n;++i) base[i]= i;
    used[root]=true;
    int qh =0;
    int qt =0;
    vector<int> q(n);
    q[qt++]= root;
    while(qh < qt){
        int v = q[qh++];
        FORIT(it, graph[v]){
            int to =*it;
            if(base[v]== base[to]|| match[v]== to)continue;
            if(to == root || match[to]!=-1&& p[match[to]]!=-1){
                int curbase = lca(match, base, p, v, to);
                vector<bool> blossom(n);
                markPath(match, base, blossom, p, v, curbase, to);
                markPath(match, base, blossom, p, to, curbase, v);
                for(int i =0; i < n;++i){
                    if(blossom[base[i]]){
                        base[i]= curbase;
                        if(!used[i]){
                            used[i]=true;
                            q[qt++] = i;
                        }
                    }
                }
            }
        }
    }
}
```

```
int main() {
    dfs(1,0);
    idx=0;
    for(auto i:comps){
        idx++;
        for(int j:i){
            if(ap[j]){
                ve[j].pb(idx);
            }
            else{
                di[j]=idx;
            }
        }
    }
}
```

```
for(int i=1;i<=n;i++){
    if(ap[i]){
        di[i]=++idx;
        ya[idx]=true;
        for(int j:ve[i]){
            G[idx].pb(j);
            G[j].pb(idx);
        }
    }
}
```

2sat.h

Description: Calculates a valid assignment to boolean variables a, b, c,... to a 2-SAT problem, so that an expression of the type $(a \vee b) \wedge \neg (a \vee c) \wedge \neg (d \vee b) \wedge \dots$ becomes true, or reports that it is unsatisfiable. Negated variables are represented by bit-inversions ($\sim x$).

Usage: TwoSat ts(number of boolean variables);
ts.either(0, ~3); // Var 0 is true or var 3 is false
ts.set_value(2); // Var 2 is true
ts.at_most_one({0,~1,2}); // ≤ 1 of vars 0, ~1 and 2 are true
ts.solve(); // Returns true iff it is solvable
ts.values[0..N-1] holds the assigned values to the vars

Time: $\mathcal{O}(N + E)$, where N is the number of boolean variables, and E is the number of clauses.

```
struct TwoSat {
    int N;
    vector<vi> gr;
    vi values; // 0 = false, 1 = true

    TwoSat(int n = 0) : N(n), gr(2*n) {}

    int add_var() { // (optional)
        gr.emplace_back();
        gr.emplace_back();
        return N++;
    }

    void either(int f, int j) {
        f = max(2*f, -1-2*f);
        j = max(2*j, -1-2*j);
        gr[f^1].push_back(j);
        gr[j^1].push_back(f);
    }

    void set_value(int x) { either(x, x); }

    void at_most_one(const vi& li) { // (optional)
        if (sz(li) <= 1) return;
        int cur = ~li[0];
        rep(i,2,sz(li)) {
            int next = add_var();
            either(cur, ~li[i]);
            either(cur, next);
            either(~li[i], next);
            cur = ~next;
        }
        either(cur, ~li[1]);
    }

    vi val, comp, z; int time = 0;
    int dfs(int i) {
        int low = val[i] = ++time, x; z.push_back(i);
        trav(e, gr[i]) if (!comp[e])
            low = min(low, val[e] ? dfs(e));
        ++time;
        if (low == val[i]) do {
            x = z.back(); z.pop_back();
            comp[x] = time;
        }
```

```
        if (values[x>>1] == -1)
            values[x>>1] = !(x&1);
    } while (x != i);
    return val[i] = low;
}

bool solve() {
    values.assign(N, -1);
    val.assign(2*N, 0); comp = val;
    rep(i,0,2*N) if (!comp[i]) dfs(i);
    rep(i,0,N) if (comp[2*i] == comp[2*i+1]) return 0;
    return 1;
}
};
```

hierholzer.h

Description: Invert matrix A. Returns rank; result is stored in A unless singular (rank < n). Can easily be extended to prime moduli; for prime powers, repeatedly set $A^{-1} = A^{-1}(2I - AA^{-1}) \pmod{p^k}$ where A^{-1} starts as the inverse of A mod p, and k is doubled in each step.

Time: $\mathcal{O}(n^3)$

```
void printCircuit(vector<vector<int>> adj)
{
    unordered_map<int, int> edge_count;

    for (int i = 0; i < adj.size(); i++)
    {
        edge_count[i] = adj[i].size();
    }
    stack<int> curr_path;
    vector<int> circuit;

    curr_path.push(0);
    int curr_v = 0; // Start vertex

    while (!curr_path.empty())
    {
        if (edge_count[curr_v])
        {
            curr_path.push(curr_v);
            int next_v = adj[curr_v].back();
            edge_count[curr_v]--;
            adj[curr_v].pop_back();
            curr_v = next_v;
        }
        else
        {
            circuit.push_back(curr_v);
            curr_v = curr_path.top();
            curr_path.pop();
        }
    }
    for (int i = circuit.size() - 1; i >= 0; i--)
    {
        cout << circuit[i];
        if (i)
            cout << " -> ";
    }
}
```

7.6 Heuristics

MaximalCliques.h

Description: Runs a callback for all maximal cliques in a graph (given as a symmetric bitset matrix; self-edges not allowed). Possible optimization: on the top-most recursion level, ignore 'cands', and go through nodes in order of increasing degree, where degrees go down as nodes are removed.

```
Time:  $\mathcal{O}\left(3^{n/3}\right)$ , much faster for sparse graphs 12 lines

typedef bitset<128> B;
template<class F>
void cliques(vector<B>& eds, F f, B P = ~B(), B X={}, B R={}) {
    if (!P.any()) { if (!X.any()) f(R); return; }
    auto q = (P | X)._Find_first();
    auto cands = P & ~eds[q];
    rep(i,0,sz(eds)) if (cands[i]) {
        R[i] = 1;
        cliques(eds, f, P & eds[i], X & eds[i], R);
        R[i] = P[i] = 0; X[i] = 1;
    }
}
```

7.7 Trees

DSUOnTree.h

Description: DSU On Tree, find how many nodes colored c in a subtree

```
int cnt[maxn];
bool big[maxn];
void add(int v, int p, int x){
    cnt[ col[v] ] += x;
    for(auto u: g[v])
        if(u != p && !big[u])
            add(u, v, x)
}

void dfs(int v, int p, bool keep){
    int mx = -1, bigChild = -1;
    for(auto u : g[v])
        if(u != p && sz[u] > mx)
            mx = sz[u], bigChild = u;
    for(auto u : g[v])
        if(u != p && u != bigChild)
            dfs(u, v, 0); // run a dfs on small childs and clear them from cnt
    if(bigChild != -1)
        dfs(bigChild, v, 1), big[bigChild] = 1; // bigChild marked as big and not cleared from cnt
    add(v, p, 1);
    //now cnt[c] is the number of vertices in subtree of vertex v that has color c. You can answer the queries easily

    if(bigChild != -1)
        big[bigChild] = 0;
    if(keep == 0)
        add(v, p, -1);
}
```

LinkCutTree.h

Description: Represents a forest of unrooted trees. You can add and remove edges (as long as the result is still a forest), and check whether two nodes are in the same tree. Aggregate path is work on node, not on edge. To make it work on edge, for each edge (u,v), add (u, e) and (e, v).

Usage: LinkCut graph(n+2);

Time: All operations take amortized $\mathcal{O}(\log N)$.

```
107 lines
struct Node { // Splay tree. Root's pp contains tree's parent.
    Node *p = 0, *pp = 0, *c[2];
    bool flip = 0;
    // path length aggregate
    int dist = 1;
    Node() { dist = 1; c[0] = c[1] = 0; fix(); }
    void fix() {
        if (c[0]) c[0]->p = this;
        if (c[1]) c[1]->p = this;
        // (+ update sum of subtree elements etc. if wanted)
        // below is for path length aggregate
        dist = 1;
```

```
    if (c[0]) dist += c[0]->dist;
    if (c[1]) dist += c[1]->dist;
}
void push_flip() {
    if (!flip) return;
    flip = 0; swap(c[0], c[1]);
    if (c[0]) c[0]->flip ^= 1;
    if (c[1]) c[1]->flip ^= 1;
}
int up() { return p ? p->c[1] == this : -1; }
void rot(int i, int b) {
    int h = i ^ b;
    Node *x = c[i], *y = b == 2 ? x : x->c[h], *z = b ? y : x;
    if ((y->p == p)) p->c[up()] = y;
    c[i] = z->c[i ^ 1];
    if (b < 2) {
        x->c[h] = y->c[h ^ 1];
        z->c[h ^ 1] = b ? x : this;
    }
    y->c[i ^ 1] = b ? this : x;
    fix(); x->fix(); y->fix();
    if (p) p->fix();
    swap(pp, y->pp);
}
void splay() {
    for (push_flip(); p; ) {
        if (p->p) p->p->push_flip();
        p->push_flip(); push_flip();
        int c1 = up(), c2 = p->up();
        if (c2 == -1) p->rot(c1, 2);
        else p->p->rot(c2, c1 != c2);
    }
}
Node* first() {
    push_flip();
    return c[0] ? c[0]->first() : (splay(), this);
}
};

struct LinkCut {
    vector<Node> node;
    LinkCut(int N) : node(N) {}

    void link(int u, int v) { // add an edge (u, v)
        assert(!connected(u, v));
        make_root(&node[u]);
        node[u].pp = &node[v];
    }

    void cut(int u, int v) { // remove an edge (u, v)
        Node *x = &node[u], *top = &node[v];
        make_root(top); x->splay();
        assert(top == (x->pp ? x->c[0]));
        if (x->pp) x->pp = 0;
        else {
            x->c[0] = top->p = 0;
            x->fix();
        }
    }

    bool connected(int u, int v) { // are u, v in the same tree?
        Node* nu = access(&node[u])->first();
        return nu == access(&node[v])->first();
    }

    void make_root(Node* u) {
        access(u);
        u->splay();
        if (u->c[0]) {
            u->c[0]->p = 0;
            u->c[0]->flip ^= 1;
            u->c[0]->pp = u;
        }
    }
};
```

```
    u->c[0] = 0;
    u->fix();
}
}
Node* access(Node* u) {
    u->splay();
    // destroy right child
    if (u->c[1]) { u->c[1]->p = 0; u->c[1]->pp = u; }
    u->c[1] = 0;
    u->fix();
    //
    while (Node* pp = u->pp) {
        pp->splay(); u->pp = 0;
        if (pp->c[1]) {
            pp->c[1]->p = 0; pp->c[1]->pp = pp; }
        pp->c[1] = u; pp->fix(); u = pp;
    }
    return u;
}

// use this to aggregate:
int aggregate(int a, int b) {
    make_root(&node[a]);
    return access(&node[b])->aggr;
}
};
```

MatrixTree.h

Description: To count the number of spanning trees in an undirected graph G : create an $N \times N$ matrix mat , and for each edge $(a,b) \in G$, do $mat[a][a]++$, $mat[b][b]++$, $mat[a][b]--$, $mat[b][a]--$. Remove the last row and column, and take the determinant.

Centroid.h

Description: Build Centroid Structure 79 lines

```
void dfs0(int now,int pa,int lev){
    P[now][0]=pa;
    level[now]=lev;
    for(set<int>::iterator it=ad[now].begin();it!=ad[now].end();
        it++){
        if(pa!=*it){
            dfs0(*it,now,lev+1);
        }
    }
}

int dist(int aa,int bb){
    return level[aa]+level[bb]-2*level[findLCA(aa,bb)];
}

int nn=0;

void dfs1(int now,int pa){
    sz[now]=1;
    nn++;
    for(set<int>::iterator it=ad[now].begin();it!=ad[now].end();
        it++){
        if(pa!=*it){
            dfs1(*it,now);
            sz[now]+=sz[*it];
        }
    }
}

int dfs2(int now,int pa){
```

```
for(set<int>::iterator it=ad[now].begin();it!=ad[now].end();
    it++){
    if(pa!=*it && sz[*it]>nn/2){
        return dfs2(*it,now);
    }
}
return now;
}

void centroid(int now,int pa){
    nn=0;
    dfs1(now,now);
    int center=dfs2(now,now);
    if(pa==-1)pa=center;
    par[center]=pa;
    for(set<int>::iterator it=ad[center].begin();it!=ad[center].
        end();it++){
        ad[*it].erase(center);
        centroid(*it,center);
    }
    ad[center].clear();
}

void update(int now){
    int x=now;
    while(1){
        while(!pq[x].empty() && !white[pq[x].top().s])pq[x].pop();
        pq[x].push(mp(dist(now,x),now));
        ans[x]=pq[x].top().f;
        if(x==par[x])break;
        x=par[x];
    }
}

void update1(int now){
    int x=now;
    while(1){
        while(!pq[x].empty() && !white[pq[x].top().s])pq[x].pop();
        ans[x]=pq[x].top().f;
        if(x==par[x])break;
        x=par[x];
    }
}

int query(int now){
    int x=now,ret=INF;
    while(1){
        ret=min(ret,dist(now,x)+ans[x]);
        if(x==par[x])break;
        x=par[x];
    }
    return ret;
}
```

Geometry (8)

8.1 Geometric primitives

Point.h

Description: Class to handle points in the plane. T can be e.g. double or long long. (Avoid int.) 25 lines

```
template<class T>
struct Point {
    typedef Point P;
    T x, y;
    explicit Point(T x=0, T y=0) : x(x), y(y) {}
    bool operator<(P p) const { return tie(x,y) < tie(p.x,p.y); }
```



```
bool operator==(P p) const { return tie(x,y)==tie(p.x,p.y); }
P operator+(P p) const { return P(x+p.x, y+p.y); }
P operator-(P p) const { return P(x-p.x, y-p.y); }
P operator*(T d) const { return P(x*d, y*d); }
P operator/(T d) const { return P(x/d, y/d); }
T dot(P p) const { return x*p.x + y*p.y; }
T cross(P p) const { return x*p.y - y*p.x; }
T cross(P a, P b) const { return (a-*this).cross(b-*this); }
T dist2() const { return x*x + y*y; }
double dist() const { return sqrt((double)dist2()); }
// angle to x-axis in interval [-pi, pi]
double angle() const { return atan2(y, x); }
P unit() const { return *this/dist(); } // makes dist()==1
P perp() const { return P(-y, x); } // rotates +90 degrees
P normal() const { return perp().unit(); }
// returns point rotated 'a' radians ccw around the origin
P rotate(double a) const {
    return P(x*cos(a)-y*sin(a),x*sin(a)+y*cos(a)); }
};
```

lineDistance.h

Description: Returns the signed distance between point p and the line containing points a and b. Positive value on left side and negative on right as seen from a towards b. a==b gives nan. P is supposed to be Point<T> or Point3D<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long. Using Point3D will always give a non-negative distance.

```
"Point.h" 11 lines
template<class P>
double lineDist(const P& a, const P& b, const P& p) {
    return (double) (b-a).cross(p-a) / (b-a).dist();
}

// return the distance and point at c, cannot 3D
double lineDist(const P& a, const P& b, const P& p, P& c) {
    double u = (p-a).dot(b-a) / (b-a).dist2();
    c = a + ((b - a) * u);
    return (c - p).dist();
}
```

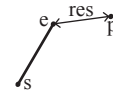
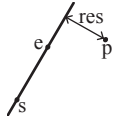
SegmentDistance.h

Description: Returns the shortest distance between point p and the line segment from point s to e.

Usage: Point<double> a, b(2,2), p(1,1);
bool onSegment = segDist(a,b,p) < 1e-10;

```
"Point.h" 20 lines
typedef Point<double> P;
double segDist(P& s, P& e, P& p) { // beware overflow! better use the other one
    if (s==e) return (p-s).dist();
    auto d = (e-s).dist2(), t = min(d,max(.0, (p-s).dot(e-s)));
    return ((p-s)*d-(e-s)*t).dist()/d;
}
```

```
// return the distance and point at c, cannot 3D:
double segDist(const P& a, const P& b, const P& p, P& c) {
    double u = (p-a).dot(b-a) / (b-a).dist2();
    if (u < 0.0) {
        c = a;
        return (p - a).dist();
    } else if (u > 1.0) {
        c = b;
        return (p - b).dist();
    }
    c = a + ((b - a) * u);
}
```



```
return (c - p).dist();
}
```

SegmentIntersection.h

Description: If a unique intersestion point between the line segments going from s1 to e1 and from s2 to e2 exists r1 is set to this point and 1 is returned. If no intersection point exists 0 is returned and if infinitely many exists 2 is returned and r1 and r2 are set to the two ends of the common line. The wrong position will be returned if P is Point<int> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or long long. Use segmentIntersectionQ to get just a true/false answer.

Usage: Point<double> intersection, dummy;
if (segmentIntersection(s1,e1,s2,e2,intersection,dummy)==1)
cout << "segments intersect at " << intersection << endl;
"Point.h" 27 lines

```
template<class P>
int segmentIntersection(const P& s1, const P& e1,
    const P& s2, const P& e2, P& r1, P& r2) {
    if (e1==s1) {
        if (e2==s2) {
            if (e1==e2) { r1 = e1; return 1; } //all equal
            else return 0; //different point segments
        } else return segmentIntersection(s2,e2,s1,e1,r1,r2); //swap
    }
    //segment directions and separation
    P v1 = e1-s1, v2 = e2-s2, d = s2-s1;
    auto a = v1.cross(v2), a1 = v1.cross(d), a2 = v2.cross(d);
    if (a == 0) { //if parallel
        auto b1=s1.dot(v1), c1=e1.dot(v1),
            b2=s2.dot(v1), c2=e2.dot(v1);
        if (a1 || a2 || max(b1,min(b2,c2))>min(c1,max(b2,c2)))
            return 0;
        r1 = min(b2,c2)<b1 ? s1 : (b2<c2 ? s2 : e2);
        r2 = max(b2,c2)>c1 ? e1 : (b2>c2 ? s2 : e2);
        return 2-(r1==r2);
    }
    if (a < 0) { a = -a; a1 = -a1; a2 = -a2; }
    if (0<a1 || a<-a1 || 0<a2 || a<-a2)
        return 0;
    r1 = s1-v1*(a2/a); // beware overflow 3 point product!
    return 1;
}
```



SegmentIntersectionQ.h

Description: Like segmentIntersection, but only returns true/false. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or long long.

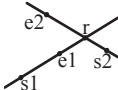
```
"Point.h" 16 lines
template<class P>
bool segmentIntersectionQ(P s1, P e1, P s2, P e2) {
    if (e1 == s1) {
        if (e2 == s2) return e1 == e2;
        swap(s1,s2); swap(e1,e2);
    }
    P v1 = e1-s1, v2 = e2-s2, d = s2-s1;
    auto a = v1.cross(v2), a1 = d.cross(v1), a2 = d.cross(v2);
    if (a == 0) { // parallel
        auto b1 = s1.dot(v1), c1 = e1.dot(v1),
            b2 = s2.dot(v1), c2 = e2.dot(v1);
        return !a1 && max(b1,min(b2,c2)) <= min(c1,max(b2,c2));
    }
    if (a < 0) { a = -a; a1 = -a1; a2 = -a2; }
    return (0 <= a1 && a1 <= a && 0 <= a2 && a2 <= a);
}
```

lineIntersection.h

Description: If a unique intersestion point of the lines going through s1,e1 and s2,e2 exists r is set to this point and 1 is returned. If no intersection point exists 0 is returned and if infinitely many exists -1 is returned. If s1==e1 or s2==e2 -1 is returned. The wrong position will be returned if P is Point<int> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or long long.

Usage: point<double> intersection;
if (1 == LineIntersection(s1,e1,s2,e2,intersection))
cout << "intersection point at " << intersection << endl;
"Point.h" 9 lines

```
template<class P>
int lineIntersection(const P& s1, const P& e1, const P& s2,
    const P& e2, P& r) {
    if ((e1-s1).cross(e2-s2)) { //if not parallell
        r = s2-(e2-s2)*(e1-s1).cross(s2-s1)/(e1-s1).cross(e2-s2);
        return 1;
    } else
        return -(e1-s1).cross(s2-s1)==0 || s2==e2;
}
```



sideOf.h

Description: Returns where p is as seen from s towards e. 1/0/-1 ⇔ left/on line/right. If the optional argument eps is given 0 is returned if p is within distance eps from the line. P is supposed to be Point<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long.

Usage: bool left = sideOf(p1,p2,q)==1;
"Point.h" 11 lines

```
template<class P>
int sideOf(const P& s, const P& e, const P& p) {
    auto a = (e-s).cross(p-s);
    return (a > 0) - (a < 0);
}
template<class P>
int sideOf(const P& s, const P& e, const P& p, double eps) {
    auto a = (e-s).cross(p-s);
    double l = (e-s).dist()*eps;
    return (a > l) - (a < -l);
}
```

onSegment.h

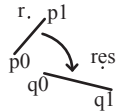
Description: Returns true iff p lies on the line segment from s to e. Intended for use with e.g. Point<long long> where overflow is an issue. Use (segDist(s,e,p)<=epsilon) instead when using Point<double>.

```
"Point.h" 5 lines
template<class P>
bool onSegment(const P& s, const P& e, const P& p) {
    P ds = p-s, de = p-e;
    return ds.cross(de) == 0 && ds.dot(de) <= 0;
}
```

linearTransformation.h

Description: Apply the linear transformation (translation, rotation and scaling) which takes line p0-p1 to line q0-q1 to point r.

```
"Point.h" 6 lines
typedef Point<double> P;
P linearTransformation(const P& p0, const P& p1,
    const P& q0, const P& q1, const P& r) {
    P dp = p1-p0, dq = q1-q0, num(dp.cross(dq), dp.dot(dq));
    return q0 + P((r-p0).cross(num), (r-p0).dot(num))/dp.dist2();
}
```



RotationalSweeping.h

Description: Rotational sweeping based on origin. Remove the origin. Useful for many calculations such as total area of possible triangle.
Time: $\mathcal{O}(N \log N)$

65 lines

```
11 cross2(point p, point r) {
    return p.x * r.y - p.y * r.x;
}

// 2 points seen on the same angle will be sorted based on
// closest to origin.
bool cmp(point a, point b) {
    if (a.y == 0 && b.y == 0) {
        if (a.x * b.x >= 0) return a.x*a.x+a.y*a.y < b.x*b.x+b.y*b.y;
    }
    return a.x > b.x; // x+ first
}

// return a vector rotated ccw, from positive x-axis, and id
// being removed from vector.
vector<point> rotational_sweep(vector<point> &all, int id) {
    vector<point> up, low;
    for(int i = 0; i < sz(all); i++) if (i != id) {
        point p = all[i];
        p.x -= all[id].x, p.y -= all[id].y;
        if (p.y >= 0) up.push_back(p); // point on y==0 will
            // be treated on top
        else low.push_back(p);
    }
    sort(up.begin(), up.end(), cmp);
    sort(low.begin(), low.end(), cmp);
    vector<point> res = up;
    for(auto cur : low) res.push_back(cur);
    return res;
}

// apply this : last podal lie on the same line.
// note : true if segment a..b contain origin, or a.dist2() < b
// .dist2()
inline bool collinear_sweep(point a, point b) {
    if (cross2(a,b) != 0) return false;
    if ((a.x < 0 && b.x > 0) || (a.y < 0 && b.y > 0)) return true
    ;
    if ((a.x == 0 && a.y == 0) || (b.x == 0 && b.y == 0)) return
    true;
    return a.x*a.x+a.y*a.y < b.x*b.x+b.y*b.y;
}

void process(vector<point> &cur, int id) { // total area of
    // possible triangle:
    vector<point> rot = rotational_sweep(cur, id);
    int size = sz(rot);

    11 sum_rx = rot[0].x, sum_ry = rot[0].y;

    // do rotate with i being the podal, r with the last podal
    // important: check whether there is collinear (:check the
    // quadrant!)
    for(int i = 0, r = 0; i < size; i++) {
        while(cross2(rot[i], rot[(r+1)%size]) > 0 ||
            collinear_sweep(rot[i], rot[(r+1)%size])) { // toggle
            // this

            r = (r + 1)%size;
            // do something
        }
    }
}
```

```
        sum_rx += rot[r].x, sum_ry += rot[r].y;
    }
    // process i..r:
    ans += rot[i].x * sum_ry - rot[i].y * sum_rx;
    // after process i..r, do cancel:
    sum_rx -= rot[i].x, sum_ry -= rot[i].y;
    if (i == r) {
        r = (i+1)%size;
        sum_rx = rot[r].x, sum_ry = rot[r].y;
    }
}
}
```

RotationSweepSwap.h

Description: Find whether exists triangle whose area is exactly S.

85 lines

```
struct point{
    11 x, y;
    point(){}
    point(11 _x, 11 _y) {
        x = _x, y = _y;
    }
};
11 cross(point a, point b) {
    return a.x * b.y - a.y * b.x;
}
11 area(point p, point q, point r) {
    return abs(cross(p,q) + cross(q,r) + cross(r,p));
}
struct seg{
    int i, j;
    point a, b, vec;
    seg() {}
    seg(int _i, int _j, point _a, point _b, point _vec) {
        i = _i, j = _j, a = _a, b = _b, vec = _vec;
    };
};
const int N = 2005;
11 n, s;
point arr[N];
bool cmp(point a, point b) {
    if (a.y == b.y) return a.x < b.x;
    return a.y < b.y;
}
bool cmp2(seg a, seg b) {
    return cross(a.vec, b.vec) > 0;
}
vector<seg> segment;
int ranked[N], id[N];
int main() {
    scanf("%lld %lld",&n, &s); s *= 2;
    for(int i = 1; i <= n; i++) {
        scanf("%lld %lld",&arr[i].x, &arr[i].y);
    }
    sort(arr + 1, arr + n + 1, cmp);
    for(int i = 1; i <= n; i++) for(int j = 1; j < i; j++) {
        segment.push_back(seg(i, j, arr[i], arr[j], point(arr[i].x-
            arr[j].x, arr[i].y-arr[j].y)));
    }
    sort(segment.begin(), segment.end(), cmp2);
    for(int i = 1; i <= n; i++) ranked[i] = id[i] = i;
    for(auto cur : segment) {
        int a = id[cur.j], b = id[cur.i];
        int l = 1, r = a-1;
        while(l <= r) {
            int mid = (l + r) >> 1;
            11 hasil = area(arr[ranked[mid]], cur.a, cur.b);
            if (hasil == s) {
                printf("Yes\n");
            }
        }
    }
}
```

```
        printf("%lld %lld\n",arr[ranked[mid]].x, arr[ranked[mid]
            ].y);
        printf("%lld %lld\n",cur.a.x, cur.a.y);
        printf("%lld %lld\n",cur.b.x, cur.b.y);
        return 0;
    } else if (hasil < s) {
        r = mid-1;
    } else {
        l = mid+1;
    }
}
l = b+1, r = n;
while(l <= r) {
    int mid = (l + r) >> 1;
    11 hasil = area(arr[ranked[mid]], cur.a, cur.b);
    if (hasil == s) {
        printf("Yes\n");
        printf("%lld %lld\n",arr[ranked[mid]].x, arr[ranked[mid]
            ].y);
        printf("%lld %lld\n",cur.a.x, cur.a.y);
        printf("%lld %lld\n",cur.b.x, cur.b.y);
        return 0;
    } else if (hasil > s) {
        r = mid-1;
    } else {
        l = mid+1;
    }
}
assert(a + 1 == b);
swap(ranked[a], ranked[b]);
swap(id[cur.i],id[cur.j]);
}
printf("No\n");
return 0;
}
```

8.2 Circles

CircleIntersection.h

Description: Computes a pair of points at which two circles intersect. Returns false in case of no intersection.

14 lines

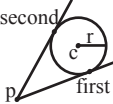
```
"Point.h"
typedef Point<double> P;
bool circleIntersection(P a, P b, double r1, double r2,
    pair<P, P>* out) {
    P delta = b - a;
    assert(delta.x || delta.y || r1 != r2);
    if (!delta.x && !delta.y) return false;
    double r = r1 + r2, d2 = delta.dist2();
    double p = (d2 + r1*r1 - r2*r2) / (2.0 * d2);
    double h2 = r1*r1 - p*p*d2;
    if (d2 > r*r || h2 < 0) return false;
    P mid = a + delta*p, per = delta.perp() * sqrt(h2 / d2);
    *out = {mid + per, mid - per};
    return true;
}
```

circleTangents.h

Description:

Returns a pair of the two points on the circle with radius r centered around c whos tangent lines intersect p. If p lies within the circle NaN-points are returned. P is intended to be Point<double>. The first point is the one to the right as seen from the p towards c.

```
Usage: typedef Point<double> P;
pair<P,P> p = circleTangents(P(100,2),P(0,0),2);
"Point.h"
template<class P>
```

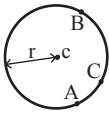


```
pair<P,P> circleTangents(const P &p, const P &c, double r) {
    P a = p-c;
    double x = r*r/a.dist2(), y = sqrt(x-x*x);
    return make_pair(c+a*x+a.perp()*y, c+a*x-a.perp()*y);
}
```

circumcircle.h

Description:

The circumcircle of a triangle is the circle intersecting all three vertices. ccRadius returns the radius of the circle going through points A, B and C and ccCenter returns the center of the same circle.



```
"Point.h" 9 lines
typedef Point<double> P;
double ccRadius(const P& A, const P& B, const P& C) {
    return (B-A).dist()*(C-B).dist()*(A-C).dist()/
        abs((B-A).cross(C-A))/2;
}
P ccCenter(const P& A, const P& B, const P& C) {
    P b = C-A, c = B-A;
    return A + (b*c.dist2()-c*b.dist2()).perp()/b.cross(c)/2;
}
```

MinimumEnclosingCircle.h

Description: Computes the minimum circle that encloses a set of points.

Time: expected $\mathcal{O}(n)$

40 lines

```
namespace cover_2d{
    double eps = 1e-9;
    using Point = complex<double>;
    struct Circle{ Point p; double r; };
    double dist(Point p, Point q){ return abs(p-q); }
    double area2(Point p, Point q){ return (conj(p)*q).imag(); }
    bool in(const Circle& c, Point p){ return dist(c.p, p) < c.r
        + eps; }
    Circle INVAL = Circle{Point(0, 0), -1};
    Circle mCC(Point a, Point b, Point c){
        b -= a; c -= a;
        double d = 2*(conj(b)*c).imag(); if(abs(d)<eps) return
            INVAL;
        Point ans = (c*norm(b) - b*norm(c)) * Point(0, -1) / d;
        return Circle{a + ans, abs(ans)};
    }
    Circle solve(vector<Point> p) {
        mt19937 gen(0x94949); shuffle(p.begin(), p.end(), gen);
        Circle c = INVAL;
        for(int i=0; i<p.size(); ++i) if(c.r<0 ||!in(c, p[i])){
            c = Circle{p[i], 0};
            for(int j=0; j<=i; ++j) if(!in(c, p[j])){
                Circle ans{(p[i]+p[j])*0.5, dist(p[i], p[j])*0.5};
                if(c.r == 0) { c = ans; continue; }
                Circle l, r; l = r = INVAL;
                Point pq = p[j]-p[i];
                for(int k=0; k<=j; ++k) if(!in(ans, p[k])) {
                    double a2 = area2(pq, p[k]-p[i]);
                    Circle c = mCC(p[i], p[j], p[k]);
                    if(c.r<0) continue;
                    else if(a2 > 0 && (l.r<0||area2(pq, c.p-p[i]) > area2
                        (pq, l.p-p[i]))) l = c;
                    else if(a2 < 0 && (r.r<0||area2(pq, c.p-p[i]) < area2
                        (pq, r.p-p[i]))) r = c;
                }
                if(l.r<0&&r.r<0) c = ans;
                else if(l.r<0) c = r;
                else if(r.r<0) c = l;
                else c = l.r<=r.r?l:r;
            }
        }
    }
```

```
        return c;
    }
};
```

8.3 Polygons

insidePolygon.h

Description: Returns true if p lies within the polygon described by the points between iterators begin and end. If strict false is returned when p is on the edge of the polygon. Answer is calculated by counting the number of intersections between the polygon and a line going from p to infinity in the positive x-direction. The algorithm uses products in intermediate steps so watch out for overflow. If points within epsilon from an edge should be considered as on the edge replace the line "if (onSegment..." with the comment below it (this will cause overflow for int and long long).

Usage: typedef Point<int> pi;
vector<pi> v; v.push_back(pi(4,4));
v.push_back(pi(1,2)); v.push_back(pi(2,1));
bool in = insidePolygon(v.begin(),v.end(), pi(3,4), false);
Time: $\mathcal{O}(n)$

```
"Point.h", "onSegment.h", "SegmentDistance.h" 14 lines
template<class It, class P>
bool insidePolygon(It begin, It end, const P& p,
    bool strict = true) {
    int n = 0; //number of isects with line from p to (inf,p.y)
    for (It i = begin, j = end-1; i != end; j = i++) {
        //if p is on edge of polygon
        if (onSegment(*i, *j, p)) return !strict;
        //or: if (segDist(*i, *j, p) <= epsilon) return !strict;
        //increment n if segment intersects line from p
        n += (max(i->y, j->y) > p.y && min(i->y, j->y) <= p.y &&
            ((*j-*i).cross(p-*i) > 0) == (i->y <= p.y));
    }
    return n&1; //inside if odd number of intersections
}
```

PolygonArea.h

Description: Returns twice the signed area of a polygon. Clockwise enumeration gives negative area. Watch out for overflow if using int as T!

```
"Point.h" 6 lines
template<class T>
T polygonArea2(vector<Point<T>>& v) {
    T a = v.back().cross(v[0]);
    rep(i,0,sz(v)-1) a += v[i].cross(v[i+1]);
    return a;
}
```

PolygonCenter.h

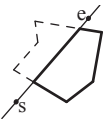
Description: Returns the center of mass for a polygon.

```
"Point.h" 10 lines
typedef Point<double> P;
Point<double> polygonCenter(vector<P>& v) {
    auto i = v.begin(), end = v.end(), j = end-1;
    Point<double> res{0,0}; double A = 0;
    for (; i != end; j=i++) {
        res = res + (*i + *j) * j->cross(*i);
        A += j->cross(*i);
    }
    return res / A / 3;
}
```

PolygonCut.h

Description:

Returns a vector with the vertices of a polygon with everything to the left of the segment going from s to e cut away.



Usage: vector<P> p = ...;
p = polygonCut(p, P(0,0), P(1,0));

"Point.h", "lineIntersection.h" 15 lines

```
typedef Point<double> P;
vector<P> polygonCut(const vector<P>& poly, P s, P e) {
    vector<P> res;
    rep(i,0,sz(poly)) {
        P cur = poly[i], prev = i ? poly[i-1] : poly.back();
        bool side = s.cross(e, cur) < 0;
        if (side != (s.cross(e, prev) < 0)) {
            res.emplace_back();
            lineIntersection(s, e, cur, prev, res.back());
        }
        if (side)
            res.push_back(cur);
    }
    return res;
}
```

ConvexHull.h

Description:

Returns a vector of indices of the convex hull in counter-clockwise order. Points on the edge of the hull between two other points are not considered part of the hull.

Usage: vector<P> ps, hull;
trav(i, convexHull(ps)) hull.push_back(ps[i]);

Time: $\mathcal{O}(n \log n)$

"Point.h" 20 lines

```
typedef Point<ll> P;
pair<vi, vi> ulHull(const vector<P>& S) {
    vi Q(sz(S)), U, L;
    iota(all(Q), 0);
    sort(all(Q), [&S](int a, int b){ return S[a] < S[b]; });
    trav(it, Q) {
        #define ADDP(C, cmp) while (sz(C) > 1 && S[C[sz(C)-2]].cross(\
            S[it], S[C.back()]) cmp 0) C.pop_back(); C.push_back(it);
        ADDP(U, <=); ADDP(L, >=);
    }
    return {U, L};
}
```



```
vi convexHull(const vector<P>& S) {
    vi u, l; tie(u, l) = ulHull(S);
    if (sz(S) <= 1) return u;
    if (S[u[0]] == S[u[1]]) return {0};
    l.insert(l.end(), u.rbegin()+1, u.rend()-1);
    return l;
}
```

PolygonDiameter.h

Description: Calculates the max squared distance of a set of points.

"ConvexHull.h" 19 lines

```
vector<pii> antipodal(const vector<P>& S, vi& U, vi& L) {
    vector<pii> ret;
    int i = 0, j = sz(L) - 1;
    while (i < sz(U) - 1 || j > 0) {
        ret.emplace_back(U[i], L[j]);
        if (j == 0 || (i != sz(U)-1 && (S[L[j]] - S[L[j-1]])
            .cross(S[U[i+1]] - S[U[i]]) > 0)) ++i;
        else --j;
    }
    return ret;
}
```

```
pii polygonDiameter(const vector<P>& S) {
    vi U, L; tie(U, L) = ulHull(S);
    pair<ll, pii> ans;
```

```
trav(x, antipodal(S, U, L))
    ans = max(ans, {(S[x.first] - S[x.second]).dist2(), x});
return ans.second;
}
```

PointInsideHull.h

Description: Determine whether a point t lies inside a given polygon (counter-clockwise order). The polygon must be such that every point on the circumference is visible from the first point in the vector. It returns 0 for points outside, 1 for points on the circumference, and 2 for points inside. **Time:** $\mathcal{O}(\log N)$

"Point.h", "sideOf.h", "onSegment.h"22 lines

```
typedef Point<ll> P;
int insideHull2(const vector<P>& H, int L, int R, const P& p) {
    int len = R - L;
    if (len == 2) {
        int sa = sideOf(H[0], H[L], p);
        int sb = sideOf(H[L], H[L+1], p);
        int sc = sideOf(H[L+1], H[0], p);
        if (sa < 0 || sb < 0 || sc < 0) return 0;
        if (sb==0 || (sa==0 && L == 1) || (sc == 0 && R == sz(H)))
            return 1;
        return 2;
    }
    int mid = L + len / 2;
    if (sideOf(H[0], H[mid], p) >= 0)
        return insideHull2(H, mid, R, p);
    return insideHull2(H, L, mid+1, p);
}
```

```
int insideHull(const vector<P>& hull, const P& p) {
    if (sz(hull) < 3) return onSegment(hull[0], hull.back(), p);
    else return insideHull2(hull, 1, sz(hull), p);
}
```

LineHullIntersection.h

Description: Line-convex polygon intersection. The polygon must be ccw and have no colinear points. isct(a, b) returns a pair describing the intersection of a line with the polygon: $\bullet (-1, -1)$ if no collision, $\bullet (i, -1)$ if touching the corner i , $\bullet (i, i)$ if along side $(i, i + 1)$, $\bullet (i, j)$ if crossing sides $(i, i + 1)$ and $(j, j + 1)$. In the last case, if a corner i is crossed, this is treated as happening on side $(i, i + 1)$. The points are returned in the same order as the line hits the polygon. **Time:** $\mathcal{O}(N + Q \log n)$

"Point.h"63 lines

```
ll sgn(ll a) { return (a > 0) - (a < 0); }
typedef Point<ll> P;
struct HullIntersection {
    int N;
    vector<P> p;
    vector<pair<P, int>> a;

    HullIntersection(const vector<P>& ps) : N(sz(ps)), p(ps) {
        p.insert(p.end(), all(ps));
        int b = 0;
        rep(i,1,N) if (P{p[i].y,p[i].x} < P{p[b].y, p[b].x}) b = i;
        rep(i,0,N) {
            int f = (i + b) % N;
            a.emplace_back(p[f+1] - p[f], f);
        }
    }

    int qd(P p) {
        return (p.y < 0) ? (p.x >= 0) + 2
            : (p.x <= 0) * (1 + (p.y <= 0));
    }
}
```

```
int bs(P dir) {
    int lo = -1, hi = N;
    while (hi - lo > 1) {
        int mid = (lo + hi) / 2;
        if (make_pair(qd(dir), dir.y * a[mid].first.x) <
            make_pair(qd(a[mid].first), dir.x * a[mid].first.y))
            hi = mid;
        else lo = mid;
    }
    return a[hi%N].second;
}
```

```
bool isign(P a, P b, int x, int y, int s) {
    return sgn(a.cross(p[x], b)) * sgn(a.cross(p[y], b)) == s;
}
```

```
int bs2(int lo, int hi, P a, P b) {
    int L = lo;
    if (hi < lo) hi += N;
    while (hi - lo > 1) {
        int mid = (lo + hi) / 2;
        if (isign(a, b, mid, L, -1)) hi = mid;
        else lo = mid;
    }
    return lo;
}
```

```
pii isct(P a, P b) {
    int f = bs(a - b), j = bs(b - a);
    if (isign(a, b, f, j, 1)) return {-1, -1};
    int x = bs2(f, j, a, b)%N,
        y = bs2(j, f, a, b)%N;
    if (a.cross(p[x], b) == 0 &&
        a.cross(p[x+1], b) == 0) return {x, x};
    if (a.cross(p[y], b) == 0 &&
        a.cross(p[y+1], b) == 0) return {y, y};
    if (a.cross(p[f], b) == 0) return {f, -1};
    if (a.cross(p[j], b) == 0) return {j, -1};
    return {x, y};
}
```

8.4 Misc. Point Set Problems

closestPair.h
Description: $i1, i2$ are the indices to the closest pair of points in the point vector p after the call. The distance is returned. **Time:** $\mathcal{O}(n \log n)$

"Point.h"58 lines

```
template<class It>
bool it_less(const It& i, const It& j) { return *i < *j; }
template<class It>
bool y_it_less(const It& i, const It& j) {return i->y < j->y;}

template<class It, class IIt> /* IIt = vector<It>::iterator */
double cp_sub(IIt ya, IIt yaend, IIt xa, It &i1, It &i2) {
    typedef typename iterator_traits<It>::value_type P;
    int n = yaend-ya, split = n/2;
    if (n <= 3) { // base case
        double a = (*xa[1]-*xa[0]).dist(), b = 1e50, c = 1e50;
        if (n==3) b=(*xa[2]-*xa[0]).dist(), c=(*xa[2]-*xa[1]).dist()
            ;
        if (a <= b) { i1 = xa[1];
            if (a <= c) return i2 = xa[0], a;
            else return i2 = xa[2], c;
        } else { i1 = xa[2];
            if (b <= c) return i2 = xa[0], b;
            else return i2 = xa[1], c;
        }
    }
}
```

```
vector<It> ly, ry, stripy;
P splitp = *xa[split];
double splitx = splitp.x;
for(IIt i = ya; i != yaend; ++i) { // Divide
    if(*i != xa[split] && (**i-splitp).dist2() < 1e-12)
        return i1 = *i, i2 = xa[split], 0;// nasty special case!
    if (**i < splitp) ly.push_back(*i);
    else ry.push_back(*i);
} // assert((signed)lefty.size() == split)
It j1, j2; // Conquer
double a = cp_sub(ly.begin(), ly.end(), xa, i1, i2);
double b = cp_sub(ry.begin(), ry.end(), xa+split, j1, j2);
if (b < a) a = b, i1 = j1, i2 = j2;
double a2 = a*a;
for(IIt i = ya; i != yaend; ++i) { // Create strip (y-sorted)
    double x = (*i)->x;
    if (x >= splitx-a && x <= splitx+a) stripy.push_back(*i);
}
for(IIt i = stripy.begin(); i != stripy.end(); ++i) {
    const P &p1 = **i;
    for(IIt j = i+1; j != stripy.end(); ++j) {
        const P &p2 = **j;
        if (p2.y-p1.y > a) break;
        double d2 = (p2-p1).dist2();
        if (d2 < a2) i1 = *i, i2 = *j, a2 = d2;
    }
}
return sqrt(a2);
}
```

```
template<class It> // It is random access iterators of point<T>
double closestpair(It begin, It end, It &i1, It &i2) {
    vector<It> xa, ya;
    assert(end-begin >= 2);
    for (It i = begin; i != end; ++i)
        xa.push_back(i), ya.push_back(i);
    sort(xa.begin(), xa.end(), it_less<It>);
    sort(ya.begin(), ya.end(), y_it_less<It>);
    return cp_sub(ya.begin(), ya.end(), xa.begin(), i1, i2);
}
```

VoronoiDiagram.h

Description: Voronoi Diagram using Half Plane Intersection. For each point, do process. **Time:** $\mathcal{O}(n^2 \log n)$

<Point.h>, <RotationalSweep.h>60 lines

```
struct line{ LD m, c; P where; bool straight; };
line make_line(P p) {
    line ret; ret.where = p;
    if (fabs(p.y)<EPS) ret.straight = true, ret.c = p.x/2.0, ret.m = 1e9;
    else {
        ret.straight = false, ret.m = ((LD)-p.x)/p.y;
        ret.c = (-ret.m/2.0) * p.x + p.y/2.0;
    }
    return ret;
}

bool fine(line p, line q, line r) { // pot_p-q has different region with (0,0) seen from r
    LD px, py;
    if (p.where.cross(q.where)<=EPS||q.where.cross(r.where)<=EPS)
        return true;
    if (p.straight) px = p.c, py=q.m*p.c+q.c;
    else if (q.straight) px = q.c, py=p.m*q.c+p.c;
    else {px=(p.c-q.c)/(q.m-p.m); py=p.m*px+p.c;}
    if (r.straight && fabs(r.c-px) < EPS) return false;
    if (!r.straight && fabs(py-(r.m*px+r.c)) < EPS) return false;
    return r.straight? !((r.c+EPS>0)^(r.c+EPS>=px)):!((0<=EPS+r.c)^(py-r.m*px<=EPS+r.c));
}
```

```

}
void process(vector<point> &po, int id) { // rotational up: x
    ==0+ to x>0-. dn: x==0- to x<0+
    vector<point> rot = rotational_sweep(po, id); // if collinear
        , put largest distance first
    deque<line> res;
    for(point curr: rot) {
        line cur = make_line(curr);
        P bef = sz(res)>=1?res[sz(res)-1].where:P(0,0), aft = cur.
            where;
        if (sz(res)>=1 && bef.cross(aft) < -EPS) { res.push_back(
            cur); continue; }
        if (sz(res)>=1 && fabs(bef.cross(aft)) <= EPS &&
            fabs(bef.dist()-aft.dist()-(aft-bef).dist()) > EPS)
            res.pop_back();
        while(sz(res) >= 2 && !fine(res[sz(res)-2], res[sz(res)-1],
            cur))
            res.pop_back();
        res.push_back(cur);
    }
    while(sz(res)>=3) {
        bool e = false;
        while(sz(res)>=3&&!fine(res[sz(res)-2],res[sz(res)-1],res
            [0])) res.pop_back(),e=true;
        while(sz(res)>=3&&!fine(res.back(), res[0], res[1])) res.
            pop_front(),e=true;
        if (!e) break;
    }
    vector<P> v; res.push_back(res[0]);
    for(int i = 0; i < sz(res)-1; i++) {
        line cur = res[i], nxt = res[i+1];
        if (cur.where.cross(nxt.where) <= EPS) {
            LD zz = max(fabs(cur.where.x), fabs(cur.where.y));
            LD zz2 = max(fabs(nxt.where.x), fabs(nxt.where.y));
            zz = (zz > MAX) ? 1.0 : ceil(MAX/zz);
            zz2 = (zz2 > MAX) ? 1.0 : ceil(MAX/zz2);
            P mid1 = (cur.where/2.0).perp() * zz + (cur.where/2.0);
            P mid2 = (nxt.where/2.0).perp().perp().perp() * zz2 + (
                nxt.where/2.0);
            v.push_back(mid1+po[id]), v.push_back(mid2+po[id]);
        } else {
            P pot; // assert(!cur.straight || !nxt.straight);
            if (cur.straight) pot.x = cur.c, pot.y = nxt.m*cur.c+nxt.
                c;
            else if (nxt.straight) pot.x = nxt.c, pot.y = cur.m*nxt.c
                +cur.c;
            else { pot.x = (nxt.c-cur.c)/(cur.m-nxt.m); pot.y = cur.m
                *pot.x+cur.c; }
            v.push_back(pot+po[id]);
        }
    }
}
```

FastDelaunay.h

Description: Fast Delaunay triangulation. There must be no duplicate points. If all points are on a line, no triangles will be returned. Should work for doubles as well, though there may be precision issues in 'circ'. Returns triangles in order {t[0][0], t[0][1], t[0][2], t[1][0], ... }, all counter-clockwise. Time: $\mathcal{O}(n \log n)$

"Point.h"90 lines

```
typedef Point<ll> P;
typedef struct Quad* Q;
typedef __int128_t ll1; // (can be ll if coords are < 2e4)
P arb(LLONG_MAX,LLONG_MAX); // not equal to any other point

struct Quad {
    bool mark; Q o, rot; P p;
    P F() { return r()->p; }
```

```

    Q r() { return rot->rot; }
    Q prev() { return rot->o->rot; }
    Q next() { return rot->r()->o->rot; }
};

bool circ(P p, P a, P b, P c) { // is p in the circumcircle?
    ll1 p2 = p.dist2(), A = a.dist2()-p2,
        B = b.dist2()-p2, C = c.dist2()-p2;
    return p.cross(a,b)*C + p.cross(b,c)*A + p.cross(c,a)*B > 0;
}

Q makeEdge(P orig, P dest) {
    Q q0 = new Quad(0,0,0,orig), q1 = new Quad(0,0,0,arb),
        q2 = new Quad(0,0,0,dest), q3 = new Quad(0,0,0,arb);
    q0->o = q0; q2->o = q2; // 0-0, 2-2
    q1->o = q3; q3->o = q1; // 1-3, 3-1
    q0->rot = q1; q1->rot = q2;
    q2->rot = q3; q3->rot = q0;
    return q0;
}

void splice(Q a, Q b) {
    swap(a->o->rot->o, b->o->rot->o); swap(a->o, b->o);
}

Q connect(Q a, Q b) {
    Q q = makeEdge(a->F(), b->p);
    splice(q, a->next());
    splice(q->r(), b);
    return q;
}

pair<Q,Q> rec(const vector<P>& s) {
    if (sz(s) <= 3) {
        Q a = makeEdge(s[0], s[1]), b = makeEdge(s[1], s.back());
        if (sz(s) == 2) return { a, a->r() };
        splice(a->r(), b);
        auto side = s[0].cross(s[1], s[2]);
        Q c = side ? connect(b, a) : 0;
        return {side < 0 ? c->r() : a, side < 0 ? c : b->r() };
    }

    #define H(e) e->F(), e->p
    #define valid(e) (e->F().cross(H(base)) > 0)
    Q A, B, ra, rb;
    int half = (sz(s) + 1) / 2;
    tie(ra, A) = rec({s.begin(), s.begin() + half});
    tie(B, rb) = rec({s.begin() + half, s.end()});
    while ((B->p.cross(H(A)) < 0 && (A = A->next()) ||
        (A->p.cross(H(B)) > 0 && (B = B->r()->o)));
        Q base = connect(B->r(), A);
        if (A->p == ra->p) ra = base->r();
        if (B->p == rb->p) rb = base;

    #define DEL(e, init, dir) Q e = init->dir; if (valid(e)) \
        while (circ(e->dir->F(), H(base), e->F())) { \
            Q t = e->dir; \
            splice(e, e->prev()); \
            splice(e->r(), e->r()->prev()); \
            e = t; \
        }
    for (;) {
        DEL(LC, base->r(), o); DEL(RC, base, prev());
        if (!valid(LC) && !valid(RC)) break;
        if (!valid(LC) || (valid(RC) && circ(H(RC), H(LC))))
            base = connect(RC, base->r());
        else
            base = connect(base->r(), LC->r());
    }
    return { ra, rb };
}
```

```

vector<P> triangulate(vector<P> pts) {
    sort(all(pts)); assert(unique(all(pts)) == pts.end());
    if (sz(pts) < 2) return {};
    Q e = rec(pts).first;
    vector<Q> q = {e};
    int qi = 0;
    while (e->o->F().cross(e->F(), e->p) < 0) e = e->o;
#define ADD { Q c = e; do { c->mark = 1; pts.push_back(c->p); \
    q.push_back(c->r()); c = c->next(); } while (c != e); }
    ADD; pts.clear();
    while (qi < sz(q)) if (!(e = q[qi++])->mark) ADD;
    return pts;
}
```

8.5 3D

PolyhedronVolume.h

Description: Magic formula for the volume of a polyhedron. Faces should point outwards.

6 lines

```
template<class V, class L>
double signed_poly_volume(const V& p, const L& trilst) {
    double v = 0;
    trav(i, trilst) v += p[i.a].cross(p[i.b]).dot(p[i.c]);
    return v / 6;
}
```

Point3D.h

Description: Class to handle points in 3D space. T can be e.g. double or long long.

32 lines

```
template<class T> struct Point3D {
    typedef Point3D P;
    typedef const P& R;
    T x, y, z;
    explicit Point3D(T x=0, T y=0, T z=0) : x(x), y(y), z(z) {}
    bool operator<(R p) const {
        return tie(x, y, z) < tie(p.x, p.y, p.z); }
    bool operator==(R p) const {
        return tie(x, y, z) == tie(p.x, p.y, p.z); }
    P operator+(R p) const { return P(x+p.x, y+p.y, z+p.z); }
    P operator-(R p) const { return P(x-p.x, y-p.y, z-p.z); }
    P operator*(T d) const { return P(x*d, y*d, z*d); }
    P operator/(T d) const { return P(x/d, y/d, z/d); }
    T dot(R p) const { return x*p.x + y*p.y + z*p.z; }
    P cross(R p) const {
        return P(y*p.z - z*p.y, z*p.x - x*p.z, x*p.y - y*p.x);
    }
    T dist2() const { return x*x + y*y + z*z; }
    double dist() const { return sqrt((double)dist2()); }
    //Azimuthal angle (longitude) to x-axis in interval [-pi, pi]
    double phi() const { return atan2(y, x); }
    //Zenith angle (latitude) to the z-axis in interval [0, pi]
    double theta() const { return atan2(sqrt(x*x+y*y),z); }
    P unit() const { return *this/(T)dist(); } //makes dist()==1
    //returns unit vector normal to *this and p
    P normal(P p) const { return cross(p).unit(); }
    //returns point rotated 'angle' radians ccw around axis
    P rotate(double angle, P axis) const {
        double s = sin(angle), c = cos(angle); P u = axis.unit();
        return u.dot(u)*(1-c) + (*this)*c - cross(u)*s;
    }
};
```

sphericalDistance.h

Description: Returns the shortest distance on the sphere with radius r adius between the points with azimuthal angles (longitude) ϕ_1 (ϕ_1) and ϕ_2 (ϕ_2) from x axis and zenith angles (latitude) θ_1 (θ_1) and θ_2 (θ_2) from z axis. All angles measured in radians. The algorithm starts by converting the spherical coordinates to cartesian coordinates so if that is what you have you can use only the two last rows. $dx \times radius$ is then the difference between the two points in the x direction and $d \times radius$ is the total distance between the points.

```
8 lines
double sphericalDistance(double f1, double t1,
    double f2, double t2, double radius) {
    double dx = sin(t2)*cos(f2) - sin(t1)*cos(f1);
    double dy = sin(t2)*sin(f2) - sin(t1)*sin(f1);
    double dz = cos(t2) - cos(t1);
    double d = sqrt(dx*dx + dy*dy + dz*dz);
    return radius*2*asin(d/2);
}
```

Strings (9)

KMP.h
Description: $pi[x]$ computes the length of the longest prefix of t that ends at x (abacaba -> 0010123). Can be used to find all occurrences of a string.
Time: $\mathcal{O}(n)$

```
15 lines
int kmp(const string &T, const string &P) {
    if (P.empty()) return 0;
    vector<int> pi(P.size(), 0);
    for (int i = 1, k = 0; i < P.size(); ++i) {
        while (k && P[k] != P[i]) k = pi[k - 1];
        if (P[k] == P[i]) ++k;
        pi[i] = k;
    }
    for (int i = 0, k = 0; i < T.size(); ++i) {
        while (k && P[k] != T[i]) k = pi[k - 1];
        if (P[k] == T[i]) ++k;
        if (k == P.size()) return i - k + 1;
    }
    return -1;
}
```

Manacher.h
Description: For each position in a string, computes $p[0][i]$ = half length of longest even palindrome around pos i , $p[1][i]$ = longest odd (half rounded down).
Time: $\mathcal{O}(N)$

```
11 lines
void manacher(const string& s) {
    int n = sz(s);
    vi p[2] = {vi(n+1), vi(n)};
    rep(z,0,2) for (int i=0,l=0,r=0; i < n; i++) {
        int t = r-i+!z;
        if (i<r) p[z][i] = min(t, p[z][l+t]);
        int L = i-p[z][i], R = i+p[z][i]-!z;
        while (L>=1 && R+1<n && s[L-1] == s[R+1])
            p[z][i]++, L--, R++;
        if (R>r) l=L, r=R;
    }
}
```

MinRotation.h
Description: Finds the lexicographically smallest rotation of a string.
Usage: rotate(v.begin(), v.begin()+min.rotation(v), v.end());
Time: $\mathcal{O}(N)$

```
8 lines
int min_rotation(string s) {
    int a=0, N=sz(s); s += s;
    rep(b,0,N) rep(i,0,N) {
        if (a+i == b || s[a+i] < s[b+i]) {b += max(0, i-1); break;}
    }
}
```

```

        if (s[a+i] > s[b+i]) { a = b; break; }
    }
    return a;
}
```

NextPermutation.h
Description: Finds the lexicographically smallest next permutation. Intu- itively, find largest index i such that $a[i] < a[i+1]$ then, find largest index j such that $j > i$ and $a[j] > a[i]$ Swap($a[j]$, $a[i-1]$). Then reverse suffix start at i .
Usage: return the array x and the permutation
Time: $\mathcal{O}(N)$

```
12 lines
bool nextPermutation(int x[], int n) {
    int k = -1;
    for (int i = n - 2; k == -1 && i >= 0; --i)
        if (x[i] < x[i + 1]) k = i;
    if (k == -1) return false;
    int l = -1;
    for (int i = n - 1; l == -1 && i > k; --i)
        if (x[k] < x[i]) l = i;
    swap(x[k], x[l]);
    reverse(x + k + 1, x + n);
    return true;
}
```

SuffixArray.h
Description: Compute Suffix Array of Strings.
Usage: compute.lcp(x,y) run in log N.
buildlcp() run in N and return $lcp[i] = lcp SA[i]$ and $SA[i-1]$.
Time: $\mathcal{O}(|N| \log |N|)$

```
76 lines
class Element_suffix{
public:
    int rank_now, rank_pref, pos;
};

class Suffix{
private:
    inline bool same_rank(Element_suffix a, Element_suffix b) {
        return a.rank_now == b.rank_now && a.rank_pref == b.
            rank_pref;
    }
    inline void reset_freq(bool is_sort_now) {
        for(int i = 0; i <= end; i++) freq[i] = 0;
        for(int i = 0; i < n; i++) freq[ is_sort_now ? suf[i].
            rank_now+1 : suf[i].rank_pref+1 ]++;
        start[0] = 0;
        for(int i = 1; i <= end; i++) {
            start[i] = freq[i-1];
            freq[i] += freq[i-1];
        }
    }
public:
    int sorted[20][MAX], freq[MAX], start[MAX], SA[MAX], end, n;
    Element_suffix suf[MAX], tmp[MAX];
    void build_suffix() {
        n = strlen(s);
        if (n == 1) {
            SA[0] = 0;
            return;
        }
        end = max(n, 1 << 8);
        for(int i = 0; i < n; i++) sorted[0][i] = (int)s[i];
        int step = 1;
        for(int cnt = 1; cnt < n; step++, cnt *= 2) {
            for(int i = 0; i < n; i++) {
                suf[i].rank_pref = sorted[step-1][i];
            }
        }
    }
}
```

```

        suf[i].rank_now = (i + cnt < n) ? sorted[step-1][i+cnt]
            : -1;
        suf[i].pos = i;
    }
    reset_freq(1);
    for(int i = 0; i < n; i++) tmp[start[suf[i].rank_now
        +1]++] = suf[i];
    reset_freq(0);
    for(int i = 0; i < n; i++) suf[start[tmp[i].rank_pref
        +1]++] = tmp[i];
    for(int i = 0; i < n; i++) {
        sorted[step][suf[i].pos] = (i && same_rank(suf[i], suf[
            i-1])) ? sorted[step][suf[i-1].pos] : i;
    }
    } step--;
    for(int i = 0; i < n; i++) SA[sorted[step][i]] = i;
}
};
int compute_lcp(int x, int y) {
    int ans = 0;
    for(int k = 20; k >= 0; k--) {
        int s = (1 << k);
        if (x + s - 1 < n && y + s - 1 < n && sorted[k][x] ==
            sorted[k][y]) {
            ans += s;
            x += s;
            y += s;
        }
    }
    return ans;
}
void buildLCP(){
    phi[SA[0]] = -1;
    for(int i = 1 ; i < len ; i++)
        phi[SA[i]] = SA[i - 1];
    for(int i = 0, l = 0 ; i < len ; i++){
        if(phi[i] == -1)
            PLCP[i] = 0;
        else{
            while(s[i + l] == s[phi[i] + l]) l++;
            PLCP[i] = l;
            l = max(0,l - 1);
        }
    }
    for(int i = 0 ; i < len ; i++)
        LCP[i] = PLCP[SA[i]];
}
```

AhoCorasick.h
Description: Aho-Corasick tree is used for dictionary matching. Initialize the tree like the example at main below.
Time: Function create is $\mathcal{O}(26N)$ where N is the sum of length of patterns. Becareful if the pattern allow duplicate. If not, the worst case is $N\sqrt{N}$.

```
63 lines
const int N = 1e5 + 5;

struct AhoCorasick {
    int trie[N][26];
    int fail[N];
    int saiz;

    AhoCorasick() {
        memset(trie[0],-1,sizeof trie[0]);
        saiz = 0;
    }

    void add(string str) {
        int cur = 0;
        for(int i = 0 ; i < str.length() ; i++) {
```

```
//checkChar(str[i]);
int nex = str[i] - 'a';
if(trie[cur][nex] == -1) {
    trie[cur][nex] = ++saiz;
    memset(trie[saiz],-1,sizeof trie[saiz]);
}
cur = trie[cur][nex];
}

void build() {
    queue<int> q;
    fail[0] = 0;

    for(int i = 0 ; i < 26 ; i++)
        if(trie[0][i] == -1)
            trie[0][i] = 0;
        else {
            int nex = trie[0][i];
            fail[nex] = 0;
            q.push(nex);
        }

    while(!q.empty()) {
        int now = q.front();
        q.pop();

        for(int i = 0 ; i < 26 ; i++)
            if(trie[now][i] == -1)
                trie[now][i] = trie[fail[now]][i];
            else {
                int nex = trie[now][i];
                fail[nex] = trie[fail[now]][i];
                q.push(nex);
            }
    }

    int getIndex(string str) {
        int cur = 0;
        for(int i = 0 ; i < str.length() ; i++) {
            //checkChar(str[i]);
            cur = trie[cur][str[i] - 'a'];
        }
        return cur;
    }
};
```

Various (10)

10.1 Known Problems

StableMarriage.h
Description: While there is a free man m: let w be the most preferred woman to whom he has not yet proposed, and propose m to w. If w is free, or is engaged to someone whom she prefers less than m, match m with w, else deny proposal.

FlowShopScheduling.h
Description: Schedule N jobs on 2 machines to minimize completion time. i-th job takes ai and bi time to execute on 1st and 2nd machine, respectively. Each job must be first executed on the first machine, then on second. Both machines execute all jobs in the same order. solution -> sort jobs by key ai < bi ? ai : (oo-bi), i.e. first execute all jobs with ai < bi in order of increasing ai, then all other jobs in order of decreasing bi.

KonigTheorem.h
Description: Consider a bipartite graph where the vertices are partitioned into left (L) and right (R) sets. Suppose there is a maximum matching which partitions the edges into those used in the matching (Em) and those not (E0). Let T consist of all unmatched vertices from L, as well as all vertices reachable from those (starting from vertices of T) by going left-to-right along edges from E0 and right-to-left along edges from Em . This essentially means that for each unmatched vertex in L, we add into T all vertices that occur in a path alternating between edges from E0 and Em. minimum vertex cover : vertices in T are added if they are in R and subtracted if they are in L to obtain the minimum vertex cover. There is a code above.

MoserCircle.h
Description: Determine the number of pieces into which a circle is divided if n points on its circumference are joined by chords with no three internally concurrent. Solution: g(n) = nC4 + nC2 + 1.

EulerFaceFormula.h
Description: V – E + F = 2 [V: vertices E: edges F: faces]

CayleyFormula.h
Description: There are n^{n-2} spanning trees of a complete graph with n la-beled vertices. Spanning Tree of Complete Bipartite Graph is $N^{M-1} * M^{N-1}$.

PickTheorem.h
Description: Pick’s Theorem: A = i + b/2 – 1. A is Area, I is internal points, and B is Border points .

JosephusProblem.h
Description: There are n person in a table waiting to be executed. Person 1 hold a knife. Each step whoever has the knife, kill the person next to him. Who’s alive at the end?

ErdosGallai.h
Description: Given degree of n nodes. Is it possible to build the graph?

```
tmp += min(d[i], k);
if (sum > tmp) {
    can = false;
    break;
}
}
if (can) printf("Possible\n");
else printf("Not possible\n");
```

10.2 Polynomial

order unit(x,p) is smallest z such that

$$x^z = 1 \bmod p$$

if order unit is p-1, then x is primitive root of p.
FFT mod besar: pecah jadi 2.

$$A = A1 + 2^{16} * A2$$

$$B = B1 + 2^{16} * B2$$

$$AB = A1B1 + 2^{16}(A1B2 + A2B1) + 2^{32}A2B2$$

$$x^{-n} \bmod Q(X)$$

merupakan banyaknya cara tepat n. dimana Q(X) adalah 1/Q(X) pada GF diatas.

$$x^{-1} \bmod Q(X)$$

bernilai $+cx^{i-1}$ untuk setiap $-cx^i$ di Q(X). untuk x^{-n} , tinggal DnC.

10.3 Minimum-Cut Problems

10.3.1 Open-pit Mining

Given DAG with value on each node (value can be negative). We can choose a node X to be removed from graph and then we get the value of that node to the score. Before removing node X, X SHOULD HAVE ZERO in-degree. After removing node X, all edges that outcoming from X become removed. We can do that until the graph has no node, or we can stop at any time. What is the maximum score? Solution: Sum of positive value on each node plus minimum cut. For every edge (a,b), Connect b to a with capacity INF. Connect every source to positive node with capacity abs(value). Connect every negative node to sink with capacity abs(value).

10.3.2 Landscaping

Given $N \times N$ grid, each cell is either low or high land. There are N trucks going straight from top to bottom and N truck going straight from left to right. Also a cost C is added if a truck hit a land from low to high or high to low. We can also toggle the high of the cell and a cost D will be added. What is the minimum of total cost? final position of $f2N$ trucks should be at bottom and right. Solution: Connect every cell to its neighbor with capacity C . Connect every low cell to source with capacity D . Connect every high cell to sink with capacity D .

10.4 Desperate Optimization

FastRead.h

Description: Fast Read for Int/Long long

Usage: fastRead_int(x)

8 lines

```
inline void fastRead_int(int &x) {
    register int c = getchar_unlocked();
    x = 0;
    for(;; ((c<48 || c>57) && c != '-'); c = getchar_unlocked())
        ;
    for(;; c>47 && c<58 ; c = getchar_unlocked()) {
        x = (x<<1) + (x<<3) + c - 48;
    }
}
```

FastMod.h

Description: Fast MOD

Usage: rem(a*b). Dont forget to use manual C++ MOD at the end USE MM as (2^61) / MOD for safety

3 lines

```
inline int rem(long long a) {
    return a-mod*((a>>29)*MM>>32);
}
```

ClockTime.h

Description: Elapsed time from the beginning of running program

Usage: cek_time()

5 lines

```
clock_t first_attempt = clock();
inline void cek_time(){
    clock_t cur = clock()- first_attempt;
    cerr<<"TIME : "<<(double) cur/CLOCKS_PER_SEC<<endl;
}
```

10.5 Misc. algorithms

Karatsuba.h

Description: Faster-than-naive convolution of two sequences: $c[x] = \sum a[i]b[x-i]$. Uses the identity $(aX + b)(cX + d) = acX^2 + bd + ((a + c)(b + d) - ac - bd)X$. Doesn't handle sequences of very different length well. See also FFT, under the Numerical chapter.

Time: $\mathcal{O}(N^{1.6})$

10.6 Dynamic programming

DivideAndConquerDP.h

Description: Given $a[i] = \min_{l \leq i \leq k} (f(i, k))$ where the (minimal) optimal k increases with i , computes $a[i]$ for $i = L..R - 1$.

Time: $\mathcal{O}((N + (hi - lo)) \log N)$

18 lines

```
struct DP { // Modify at will:
```

```
int lo(int ind) { return 0; }
int hi(int ind) { return ind; }
ll f(int ind, int k) { return dp[ind][k]; }
void store(int ind, int k, ll v) { res[ind] = pii(k, v); }

void rec(int L, int R, int LO, int HI) {
    if (L >= R) return;
    int mid = (L + R) >> 1;
    pair<ll, int> best (LLONG_MAX, LO);
    rep(k, max(LO, lo(mid)), min(HI, hi(mid)))
        best = min(best, make_pair(f(mid, k), k));
    store(mid, best.second, best.first);
    rec(L, mid, LO, best.second+1);
    rec(mid+1, R, best.second, HI);
}

void solve(int L, int R) { rec(L, R, INT_MIN, INT_MAX); }
```

KnuthDP.h

Description: When doing DP on intervals: $a[i][j] = \min_{i < k < j} (a[i][k] + a[k][j]) + f(i, j)$, where the (minimal) optimal k increases with both i and j , one can solve intervals in increasing order of length, and search $k = p[i][j]$ for $a[i][j]$ only between $p[i][j - 1]$ and $p[i + 1][j]$. This is known as Knuth DP. Sufficient criteria for this are if $f(b, c) \leq f(a, d)$ and $f(a, c) + f(b, d) \leq f(a, d) + f(b, c)$ for all $a \leq b \leq c \leq d$. Consider also: LineContainer (ch. Data structures), monotone queues, ternary search.

Time: $\mathcal{O}(N^2)$

10.6.1 Slope Trick Narrow Rectangle

Define $dp[i][x]$: the minimum cost to move the first i rectangles such that the last (the i -th) rectangle's leftmost coordinate is x . This will lead to a solution for partial score. Now, see $dp[i]$ as a function: it returns $dp[i][x]$ for given x . It turns out that this function is a polyline consisting of $2i + 3$ sections, and the slopes of the sections are $-i - 1, -i, \dots, i, i + 1$ from left to right. Thus, this polyline can be represented using $2i + 3$ integers $l_0, l_1, \dots, l_i, r_0, r_1, \dots, r_i, c$.

- In the interval $(-\infty, l_i]$, the slope of the polyline is $-i - 1$.
- In the interval $[l_i, l_i - 1]$, the slope of the polyline is $-i$.
- ...
- In the interval $[l_1, l_0]$, the slope of the polyline is -1 .
- In the interval $[l_0, r_0]$, the polyline is constant, and the value is c .
- In the interval $[r_0, r_1]$, the slope of the polyline is 1 .
- ...
- In the interval $[r_i - 1, r_i]$, the slope of the polyline is i .
- In the interval $[r_i, \infty)$, the slope of the polyline is $i + 1$.

Now we compute the polylines in the order $dp[0], dp[1], \dots, dp[N]$. We should keep two sets (or priority queues) representing l_0, \dots, l_i and r_0, \dots, r_i meanwhile, and the solution works in $\mathcal{O}(N \log N)$.

NarrowRectangle.h

Description: Slope Trick At Coder Narrow Rectangle Solution

33 lines

```
priority_queue<LL> kiri;
priority_queue<LL, vector<LL>, greater<LL> > kanan;
int main() {
    ios_base::sync_with_stdio(false); cin.tie(0); cout.tie(0);
    cin >> n;
    for (int i = 1; i <= n; i++) cin >> l[i] >> r[i];
    LL ans = 0LL, lz1 = 0LL, lzt = 0LL;
    kiri.push(l[1]);
    kanan.push(l[1]);
    for (int i = 2; i <= n; i++) {
        lzt += r[i-1] - l[i-1];
        lz1 -= r[i] - l[i];
        LL tmp1 = kiri.top() + lz1, tmp2 = kanan.top() + lzt;
        if (l[i] <= tmp1) {
            kiri.pop();
            kiri.push(l[i] - lz1);
            kiri.push(l[i] - lzt);
            kanan.push(tmp1 - lzt);
            ans += llabs(tmp1 - l[i]);
        } else if (l[i] >= tmp2) {
            kanan.pop();
            kanan.push(l[i] - lzt);
            kanan.push(l[i] - lzt);
            kiri.push(tmp2 - lzt);
            ans += llabs(l[i] - tmp2);
        } else {
            kiri.push(l[i] - lzt);
            kanan.push(l[i] - lzt);
        }
    }
    cout << ans << '\n';
    return 0;
}
```

10.6.2 Slope Trick Explanation

Define $f_i(x)$ as the minimum number of moves to change the first i elements into a non-decreasing sequence such that $a_i \leq x$. It is easy to see that by definition we have the recurrences: $f_i(X) = \min_{Y \leq X} (|a_i - Y|)$ when $i = 1$ and $f_i(X) = \min_{Y \leq X} (f_{i-1} + |a_i - Y|)$

Now, note that $f_i(X)$ is non-increasing, since it is at most the minimum among all the values of f for smaller X by definition. We store a set of integers that denotes where the function f_i change slopes. More formally, we consider the function $g_i(X) = f_i(X + 1) - f_i(X)$. The last element of the set will be the smallest j such that $g_i(j) = 0$, the second last element will be the smallest j such that $g_i(j) = -1$, and so on. (note that the set of slope changing points is bounded).

Let $Opt(i)$ denote a position where $f_i(X)$ achieves its minimum. (i.e. $g_i(Opt(i)) = 0$) The desired answer will be $f_n(Opt(n))$. We'll see how to update these values quickly.

There are two cases to consider:

Case 1: $Opt(i - 1) \leq a_i$

Here, the slope at every point before a_i decreases by 1. Thus, we push a_i into the slope array as this indicates that we decreases the slope at all the slope changing points by 1, and the slope changing point for $slope = 0$ is a_i , i.e. $Opt(i) = a_i$. Thus, this case is settled.

Case 2: $Opt(i - 1) > a_i$

Now, we insert a_i into the set, since it decreases the slope at all the slope changing points before a_i by 1. Furthermore, we insert a_i again because it increases the slope at the slope changing points between a_i and $Opt(i - 1)$ by 1. Now, we can just take $Opt(i) = Opt(i - 1)$ since the slope at $Opt(i - 1)$ is still 0. Finally, we remove $Opt(i - 1)$ from the set because it's no longer the first point where the slope changes to 0. (it's the previous point where the slope changes to -1 and the slope now becomes 0 because of the addition of a_i) Thus, the set of slope changing points is maintained. We have $f_i(Opt(i)) = f_{i-1}(Opt(i - 1)) + |Opt(i - 1) - a_i|$. Thus, we can just use a priority queue to store the slope changing points and it is easy to see that the priority queue can handle all these operations efficiently (in $O(\log N)$ time).

SimpleSlopeTrick.h

Description: Simple Slope Trick

```
int main()
{
    int n, t;
    long long ans = 0;
    std::priority_queue<int> Q;
    scanf("%d%d", &n, &t);
    Q.push(t);
    for(int i=1; i<n; i++)
    {
        scanf("%d", &t); t-=i;
        Q.push(t);
        if(Q.top() > t)
        {
            ans += Q.top() - t;
            Q.pop();
            Q.push(t);
        }
    }
    printf("%lld", ans);
    return 0;
}
```

10.7 Optimization tricks

10.7.1 Bit hacks

- $x \& -x$ is the least bit in x .
- `for (int x = m; x;) { --x &= m; ... }` loops over all subset masks of m (except m itself).
- `c = x&-x, r = x+c; ((r^x) >> 2)/c | r` is the next number after x with the same number of bits set.
- `rep(b,0,K) rep(i,0,(1 << K)) if (i & 1 << b) D[i] += D[i^(1 << b)];` computes all sums of subsets.

TortoiseHare.h

Description: Tortoise Hare.

Usage: Cycle Finding

Time: $O(N)$

```
class Main {
    static int Z, I, M, L, mu, lambda;
    static int f(int x) { return (Z * x + I) % M; }
    static void floydCycleFinding(int x0) { // function "int f(int x)" must be defined earlier
        // 1st part: finding k*mu, hare's speed is 2x tortoise's
        int tortoise = f(x0), hare = f(f(x0)); // f(x0) is the node next to x0
        while (tortoise != hare) { tortoise = f(tortoise); hare = f(f(hare)); }
        // 2nd part: finding mu, hare and tortoise move at the same speed
        mu = 0; hare = x0;
        while (tortoise != hare) { tortoise = f(tortoise); hare = f(hare); mu++; }
        // 3rd part: finding lambda, hare moves, tortoise stays
        lambda = 1; hare = f(tortoise);
        while (tortoise != hare) { hare = f(hare); lambda++; }
    }
    public static void main(String[] args) {
        Scanner sc = new Scanner(System.in);
        for (int caseNo = 1; ; caseNo++) {
            Z = sc.nextInt();
            I = sc.nextInt();
            M = sc.nextInt();
            L = sc.nextInt();
            if (Z == 0 && I == 0 && M == 0 && L == 0) break;
            floydCycleFinding(L);
            System.out.printf("Case %d: %d\n", caseNo, lambda);
        }
    }
}
```

BIT Range Sum Update

Update Query

Update(BITree1, l, val)

Update(BITree1, r+1, -val)

UpdateBIT2(BITree2, l, val*(l-1))

UpdateBIT2(BITree2, r+1, -val*r)

Range Sum

getSum(BITTree1, k) *k) - getSum(BITTree2, k)