Q → How many multiples of x are there from 1 to N, for given input x, N.

$N = 10$ 1 2 3 4 5 6 7 8 9 10

$x = 3$ Ans = 3

$x = 4$ Ans = 2

$$Ans = floor\left(\frac{N}{x}\right)$$

Q → Count the number of trailing 0s in N!

328700

Factorial → $N! = 1 * 2 * 3 * 4 \ldots * (N-1) * N$

$N=6$ $6! = 1*2*3*4*5*6 = 720$ Ans = 1

$N=11$ $11! = 1*2*3* \ldots *10*11 = 39916800$ Ans = 2

8 digits

$N = 53!$ → 427 . . . 000000000000 Ans = 12

70 digits

not possible to store in int/long.

$N=11$ $11! \rightarrow 1 * 2 * 3 * 4 * 5 * 6 * 7 * 8 * 9 * 10 * 11$

$4 \to 2*2$, $6 \to 2*3$, $8 \to 2*2*2$, $10 \to 2*5$

39916800

$x * 10$ → adds 1 trailing 0 in x.

$10 \to 2*5$

Ans = # times we multiply 5

Ans = # multiples of 5 from 1 to N

$N = 11$ → # multiples of 5 = $\frac{11}{5}$ = 2

$N = 53$ → # multiples of 5 = $\frac{53}{5}$ = 10 → 10 + 2

5 10 15 20 25 30 35 40 45 50

$25 \to 5*5$, $50 \to 5*5*2$

multiples of 5 → 1
multiples of 25 → 1 extra 5

5^1
5^2
5^3 125 → 5 * 5 * 5
250 → 5 * 5 * 5 * 2
375 → 5 * 5 * 5 * 3
5^4 625 → 5 * 5 * 5 * 5

$5^1 \quad 5^2 \quad 5^3 \ldots$

N = 53 → # multiples of 5^1 = 53/5 = 10 → {5, 10, 15, 20, 25, 30, 35, 40, 45, 50}
# multiples of 5^2 = 53/25 = 2 → {25, 50}
# multiples of 5^3 = 53/125 = 0
12

N = 720 # multiples of 5^1 = 720/5 = 144
# multiples of 5^2 = 720/25 = 28
# multiples of 5^3 = 720/125 = 5
# multiples of 5^4 = 720/625 = 1
# multiples of 5^5 = 720/3125 = 0
178 (Ans)

x * 25
x * 5 * 5

$5^K = N$
$\Rightarrow K = \log_5(N)$

$5^1 \longrightarrow 5^K <= N$

$\boxed{N <= 10^9}$

```
ans = 0
for (i = 5 ; i <= N ; i *= 5)
        ans += N/i
return ans
```

TC = O($\log_5$(N))   SC = O(1)

Modular Arithmatic (%)

A%B → Remainder when A/B.

$\boxed{0 <= A\%B <= (B-1)}$

Repeated subtraction of B from A.

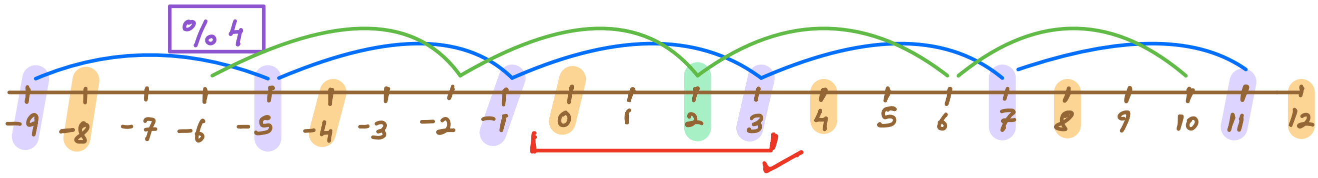
10 % 2 = 0
10 % 3 = 1
20 % 6 = 2

50 % 5 = 0
100 % 15 = 10

100 − 15 = 85 − 15 = 70 − 15 = 55
55 − 15 = 40 − 15 = 25 − 15 = 10

$\boxed{x \% 1 = 0}$ $\boxed{x \% x = 0}$

$\boxed{x \% y = 0}$ → y is a factor of x.



[-3 0]

$x \% 4 = 0$

↳ x is a multiple of 4.

% 4 → all numbers at jump of 4 are equal.

$7 \% 4 = 3$

$11 \% 4 = 3$

$3 \% 4 = 3$

$\boxed{\begin{array}{l} -9 \% 4 = 3 \\ -5 \% 4 = 3 \\ -1 \% 4 = 3 \end{array}}$

-1 (c++, java, js...) ✗

3 (python..) ✓

[0 3]

$\boxed{\begin{array}{l} (A < B) \rightarrow A \% B = A \\ A >= 0 \end{array}}$

$(A < 0)$

$(A \% B + B) \% B$

$-3 + 4 = 1$

$-2 + 4 = 2$

$-1 + 4 = 3$

$0 + 4 = 4 \longrightarrow \underline{0}$

Properties

int → $\approx 2*10^9$

$a + b$, $a * b$ > can overflow

1) $(a + b) \% m = ((a \% m) + (b \% m)) \% m$

2) $(a * b) \% m = ((a \% m) * (b \% m)) \% m$

> handles overflow if m is not too large.

3) $(a - b) \% m = ((a \% m) - (b \% m) + m) \% m$

$\underbrace{}_{0} - \underbrace{}_{(m-1)} + m$

[0 m-1]

$a = 17$ $(a-b) \% m = (17 - 8) \% 5$

$b = 8$ $= 9 \% 5 = \underline{4}$ ✓

$m = 5$

$a \% m = 17 \% 5 = 2$

$b \% m = 8 \% 5 = 3$

$(2 - 3) \% 5 = \underline{-1 \% 5} \rightarrow \underline{-1}$ (not python)

$m = 2*10^9$

$a \% m \rightarrow (2*10^9 - 1)$ max value

adding or multiply → can overflow

$m = 10^8$ $a \% m \rightarrow 10^8 - 1$

int → 4 Bytes → 32 bits

$2^{31}-1$ max int value

$\approx \underline{2*10^9}$

$$\boxed{\begin{array}{l} a + b \quad > 2*10^9 \\ \text{int} \quad \text{int} \end{array}}$$ overflow

$\overbrace{(a+b)}^{1}\overbrace{\%m}^{2}$

$m = \underline{10^6}$ (example)

$\overbrace{(a\%m)}^{1}\overbrace{+}^{3}\overbrace{(b\%m)}^{2}\ \overbrace{\%m}^{4}$

$(10^6-1) + (10^6-1)$ → not overflow

max

---

4) $(a^b)\%m = (a\%m)^b\%m$

$\boxed{a < m}$

ans = 1 $\quad$ a = $\underline{a\%m}$

for i → 1 to b

ans = (ans * a) % m

return ans

small a & b → ✓ $\quad 3^4 \to \underline{81}$

large values → overflow $\quad \underline{300}^{40}$ x

// take care of overflow

ans at every step $\underline{\underline{< m}}$

$(\underbrace{ans\%m * a\%m}_{\text{extra}})\%m$

ans & a → int

m → 1000 ✓

m → 10^7 x → long ✓

TC = O(b)

SC = $\underline{O(1)}$

$\boxed{\log_x y = 3 \to x^3 = y}$

$\boxed{\text{10:45 PM}}$

$3^{20}\%11$ → 20 steps

$2^6 = (2^2)^3 = 4^3$

$\underline{64} \qquad \underline{64}$

$2^5 = 2*2^4$

$3^{20}\%11 \to (3^2)^{10}\%11$

$9^{10}\%11 \to (9^2)^5\%11$

$(81\%11)^5\%11$

$= 4^5\%11 \to 4*(4^2)^2\%11$

$4*(16\%11)^2\%11$

$4*(5)^2\%11 \to 4*(25\%11)^1\%11$

$4*3\%11$

$= 12\%11 = \underline{1}$ ✓

$81 = 7*11 + 4$

q $\qquad$ r

$\boxed{a\%m}$

$a^b\%m$

- $(a^2\%m)^{b/2}\%m$, b is even
- $a*(a^2\%m)^{b/2}\%m$, $\underline{\text{b is odd}}$

int division

$5/2 = \underline{2}$

steps $\rightarrow$ $\log_2 b$

```
long fastPower (a, b, m) {   // M > 1
    if (b == 0)
        return 1;
    if (b % 2 == 0)
        return fastPower((a*a)%m, b/2, m)
    else
        return (a * fastPower((a*a)%m, b/2, m)) % m
}
```

TC = $O(\log_2 b)$

SC = $O(\log_2 b)$ → recursion

H.W → Try iterative code.

---

5) $(a/b)\%m = ((a\%m) * (b^{-1}\%m))\%m$

b^{-1} ↓ inverse mod of b wrt m.

$\frac{10}{4} = 2.5$

remainder ↓ integers

$\boxed{\frac{1}{b} = b^{-1}}$

$\boxed{\text{only possible if } \gcd(b,m) = 1}$

$\left(\frac{b}{b}\right)\%m = 1\%m = 1$

$(b * b^{-1})\%m = 1$

b^{-1} ↓ any integer that can be placed here.

b = 5   m = 7

$(5 * 3)\%7 = 1$

3 → inverse mod of 5 wrt 7.

$5^{-1} \text{mod}(7) = 3$

$(5*1)\%7 = 5$

$(5*2)\%7 = 10\%7 = 3$

$(5*3)\%7 = 15\%7 = 1$ ✓

b = 4   m = 5

$(4 * 4)\%5 = 1$

$(4*4)\%5 = 16\%5 = 1$

$4\%5 = 4$

$(4*x)\%5 \rightarrow (4*(x\%5))\%5$

[0 4]

range of inverse mod → [0 m-1]

TC = $O(M)$

$\boxed{M = 10^9 + 7} \rightarrow$ prime number ✓

$\boxed{10^9 + 9} \rightarrow$ H.W

Fermat Theorem

$$\boxed{a^{p-1} \equiv 1 \pmod{p}}$$

a → any int

p → prime

$x \equiv y \pmod{m}$

$x \% m = y \% m$

$a^{p-1} \% p = 1 \% p = 1$

$(a^{p-1} * a^{-1}) \% p = (1 * a^{-1}) \% p$

$\Rightarrow (a^{p-2}) \% p = (a^{-1}) \% p$

fastPower $TC = O(\log p)$

$300^{-1} \bmod (10^9 + 7) \rightarrow 300^{10^9+5} \% (10^9 + 7)$ ✓

$2 \% 5 = 7 \% 5$

$(2 * 3) \% 5 = (7 * 3) \% 5$

$\Rightarrow \underline{1} \qquad \underline{1}$

Q → Find $3^{1002} \% 11$?

11 → prime

$3^{11-1} \% 11 = 1 \Rightarrow 3^{10} \% 11 = \underline{1}$

$(\underbrace{3^{10} * 3^{10} * 3^{10} \ldots 3^{10}}_{100 \text{ times}} * 3^2) \% 11$

$\boxed{1002 \% 10}$

$((\cancel{3^{10} \% 11})^{1} * (\cancel{3^{10} \% 11})^{1} \ldots * (\cancel{3^{10} \% 11})^{1} * (3^2 \% 11)) \% 11 \rightarrow 9 \% 11 = \underline{9}$ ✓

$$\boxed{a^b \% p = a^{b \% (p-1)} \% p} \longrightarrow \text{prime}$$
