

GCD → Greatest Common Divisor (≥ 0) ✓
HCF → Highest Common Factor

$$\text{gcd}(15, 25) = 5$$

$$\text{gcd}(50, 115) = 5$$

$$\text{gcd}(20, 16) = \underline{4}$$

$$1 \leq \text{factor of } a \leq a$$

$$TC = O(\min(a, b))$$

```
if (a == 0 || b == 0)
    return a + b
```

$$a + b > 0$$

```
gcd = 1
for i → 2 to min(a, b)
    if (a % i == 0 && b % i == 0)
        gcd = i
return gcd
```

$$\text{gcd}(100, 50)$$

Properties

$$1) \text{gcd}(a, b) = \text{gcd}(b, a)$$

$$2) \text{gcd}(1, a) = 1$$

only divisor of 1 = 1
 $\& a \% 1 = 0$
 $\Rightarrow \text{gcd}(1, a) = \underline{1}$

$$3) \text{gcd}(0, a) = a$$

$$a > 0$$

$$0 \% x = 0 \quad \checkmark$$

\Rightarrow all numbers are divisor of 0
 $\&$ greatest divisor of $a = a$
 $\Rightarrow \text{gcd}(0, a) = \underline{a} \quad \checkmark$

$$4) \text{gcd}(a, b) = \text{gcd}(a, b - a)$$

$$b > a$$

$$\text{gcd}(10, 15) = \text{gcd}(10, 5)$$

To prove $\text{gcd}(a, b) = \text{gcd}(a, b - a)$

let $\text{gcd}(a, b) = d \rightarrow a \% d = 0 \quad b \% d = 0$
 $a = d * k_1 \quad b = d * k_2$

$$15 \% 5 = 0$$

$$15 = 5 * \underline{3}_K$$

$$(b - a) = (k_2 - k_1) d \rightarrow (b - a) \% d = 0$$

let $\gcd(a, b-a) = m \rightarrow a \% m = 0 \quad (b-a) \% m = 0$
 $a = m * T_1 \quad b-a = m * T_2$
 $a + (b-a) = b = (T_1 + T_2) m \rightarrow b \% m = 0$

If $a \% d = 0$ & $(b-a) \% d = 0 \Rightarrow d$ is a common factor of a & $b-a$

But, m is $\gcd(a, b-a)$

$$\Rightarrow m \geq d$$

If $a \% m = 0$ & $b \% m = 0 \Rightarrow m$ is a common factor of a & b

But, d is $\gcd(a, b)$

$$\Rightarrow d \geq m$$

$$d = m \Rightarrow \gcd(a, b) = \gcd(a, b-a)$$

Hence Proved!



5) $\gcd(a, b) = \gcd(a, b-a)$
 $= \gcd(a, b-a-a)$
 $= \gcd(a, b-a-a-a)$
 \vdots
 ≥ 0

$$\gcd(a, b) = \gcd(a, b \% a)$$

$$\gcd(a, b) = \gcd(b, a)$$

$$\gcd(0, a) = a$$

$$b < a \Rightarrow \text{swap } a \& b$$

$$\gcd(a, b) = \gcd(b \% a, a)$$

$$\gcd(100, 60) = \gcd(60 \% 100, 100) \\ = \gcd(60, 100)$$

Euclid Theorem

$$\begin{aligned} \gcd(100, 60) \\ &= \gcd(60, 100) \\ &= \gcd(60, 100 \% 60 = 40) \\ &= \gcd(40, 60) \\ &= \gcd(40, 60 \% 40 = 20) \\ &= \gcd(20, 40) \\ &= \gcd(20, 40 \% 20 = 0) \\ &= \gcd(0, 20) = \underline{20} \checkmark \end{aligned}$$

```

int gcd(a, b) {
    if (a == 0)
        return b;
    return gcd(b % a, a);
}

```

Time Complexity

H.W → Try iterative code.

$\text{gcd}(5000, 10000)$
 \downarrow
 $\text{gcd}(10000 \% 5000 = 0, 5000)$
 \downarrow
5000 2 steps

$\text{gcd}(64, 50)$
 $\downarrow 1$
 $\text{gcd}(50 \% 64 = 50, 64)$
 $\downarrow 2$
 $\text{gcd}(64 \% 50 = 14, 50)$
 $\downarrow 3$
 $\text{gcd}(50 \% 14 = 8, 14)$
 $\downarrow 4$
 $\text{gcd}(14 \% 8 = 6, 8)$
 $\downarrow 5$
 $\text{gcd}(8 \% 6 = 2, 6)$
 $\downarrow 6$
 $\text{gcd}(6 \% 2 = 0, 2) \rightarrow \underline{2} \checkmark$

increase in i/p \nRightarrow increase in # steps

$\text{gcd}(a, b) \rightarrow \text{gcd}(b \% a, a)$

$b \rightarrow b \% a < a$ $b = 10$

$5 > 3$
 $-5 < -3$

$a < b/2$
 $b \% a < a < b/2$

$\Rightarrow b \% a < b/2$

$a = 3$
 $b \% a < a < b/2$
 $10 \% 3 < 3 < 10/2$
 $10 \% 3 < 10/2$
 $1 < 5$

$a = b/2$
 $b \% a = 0$
 $a = 5$
 $b \% a = 10 \% 5 = \underline{0}$

$b \% a < b/2$
 $b \rightarrow < b/2$

$a > b/2$
 $-a < -b/2$
 $+b$ on both sides
 $b - a < b - b/2$
 $b - a < b/2$

$\frac{x-x}{2} = \frac{x}{2}$

$a = 8$
 $8 > 10/2$
 $-8 < -10/2$

$b \% a \leq b - a < b/2$
 $b \% a < b/2$

$10 - 8 < 10 - 10/2$
 $10 - 8 < 10/2$
 $10 \% 8 \leq 10 - 8 < 10/2$
 $10 \% 8 < 10/2$
 $2 < 5$

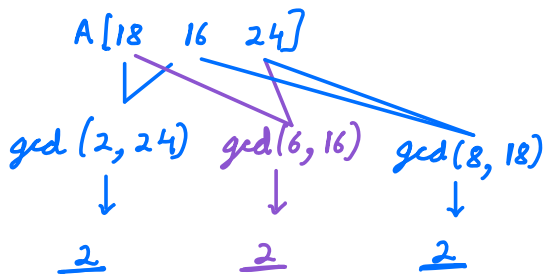
$$b \rightarrow b/2 \Rightarrow \# \text{ steps} = \log_2(b)$$

$$TC < O(\log_2(b))$$

linear \rightarrow log ✓

6) $\text{gcd}(a, b, c) \rightarrow$

- $\text{gcd}(a, \text{gcd}(b, c))$ ✓
- $\text{gcd}(b, \text{gcd}(a, c))$ ✓
- $\text{gcd}(c, \text{gcd}(a, b))$ ✓



Gcd of all array elements \rightarrow

```
ans = A[0]
for i  $\rightarrow$  1 to (N-1)
    ans = gcd(ans, A[i])
return ans
```

#elements of array

$$TC = O(N \log(A[i]))$$

value of elements

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Q \rightarrow Check if there is a subsequence in the array with $\text{gcd} = 1$.
 continuous or non-continuous

$A = [2, 30, 14, 72, 60] \rightarrow \text{Ans} = \text{false}$

$\rightarrow A = [6, 15, 30, 10, 150] \rightarrow \text{Ans} = \text{true}$

3

$$\text{gcd}(6, 15, 10) = 1$$

$$\text{gcd}(6, 15, 10, 30) = 1$$

$$\text{gcd}(6, 15, 10, 30, 150) = 1$$

$$\text{gcd}(\underbrace{-, -, -, -, -}_1, 150) = 1$$

if $\text{gcd}(\text{any subsequence}) = 1$

$$\Rightarrow \text{gcd}(\text{all elements}) = 1$$

full array \rightarrow subsequence ✓

if $\text{gcd}(\text{all elements}) = 1 \Rightarrow \text{Ans} = \text{true}$
 else $\Rightarrow \text{Ans} = \text{false}$

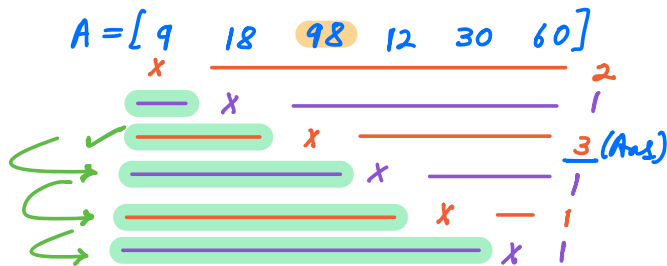
4 elements

✓ ✓ ✓ ✓
 x x x x

$2 * 2 * 2 * 2 \rightarrow 2^4 \Rightarrow \text{total subsequence} = \underline{2^N}$
 (including empty.)

non-empty $\Rightarrow \underline{2^N - 1}$

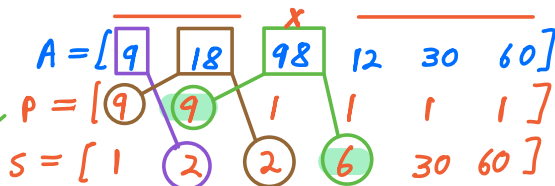
Q \rightarrow Given an integer array, find max gcd of all elements of the array after deleting exactly one elements.



Bruteforce \rightarrow calculate gcd of all elements except $A[i] \quad \forall i$.

$TC = O(N * N \log(A[i]))$

$A = [20 \quad 64 \quad 24 \quad 100 \quad 50]$ $\text{Ans} = \underline{4}$



$P[i] = \text{gcd}(P[i-1], A[i])$

$S[i] = \text{gcd}(S[i+1], A[i])$

$\text{gcd of all elements excluding } A[i] = \text{gcd}(P[i-1], S[i+1])$

$i = 0 \rightarrow S[1]$

$i = N-1 \rightarrow P[N-2]$

$TC = O(N \log(A[i]))$

$SC = O(N)$

Q \rightarrow N players are playing a game.

$A[i] \rightarrow$ health/power of i^{th} player

If player i attacks player $j \rightarrow (i \rightarrow j)$
 if $(A[j] \leq A[i]) \rightarrow$ player j is dead $A[j] = 0$ ✓
 if $(A[j] > A[i]) \rightarrow A[j] = A[j] - A[i]$ ✓

Find min possible health of last surviving player.

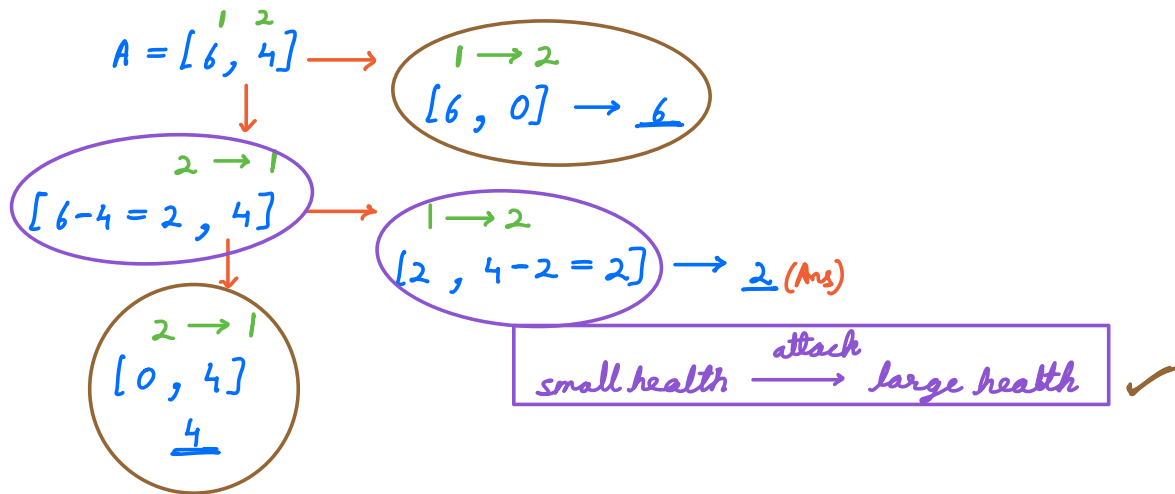


Diagram illustrating the attack process for $A = [9, 6, 15]$:

$$A = [9, 6, 15] \rightarrow [3, 6, 9] \rightarrow [3, 3, 6] \rightarrow [3, 0, 3] \rightarrow 3 \text{ ✓}$$

Diagram illustrating the attack process for a and b :

$$\begin{array}{l} a \rightarrow a \\ b \rightarrow b - a \\ a \leq b \end{array} \rightarrow \text{gcd}(a, b)$$

Ans = gcd of all elements ✓