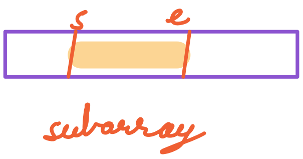
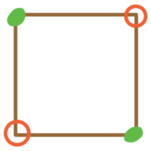
# 2D Arrays / Matrix

Q → Given a matrix of integers A[N][M]. Find the sum of given submatrix.





$$A = \begin{array}{c|cccc} & 0 & 1 & 2 & 3 \\ 0 & 1 & 3 & 5 & 2 \\ 1 & 4 & 8 & 5 & 0 \\ 2 & 10 & 20 & -1 & 3 \end{array}_{3*4}$$

Top Left + Bottom Right
OR
Top Right + Bottom Left

Top left → (0, 1)
Bottom right → (1, 2)
Ans = 3 + 5 + 8 + 5 = 21

I/P
$$A = \begin{array}{c|cccc} & 0 & 1 & 2 & 3 \\ 0 & 1 & 3 & 5 & 2 \\ 1 & 4 & 8 & 5 & 0 \\ 2 & 10 & 20 & -1 & 3 \end{array}_{3*4}$$

I/P { Top left → (sx, sy) → (1, 0)
Bottom Right → (ex, ey) → (2, 2)

Ans = 4 + 8 + 5 + 10 + 20 − 1 = 46

```
ans = 0
for i → sx to ex
    for j → sy to ey
        ans += A[i][j]
return ans
```

TC = O(N*M) ✓
SC = O(1)

Q → Find submatrix sum for multiple queries.

$$A = \begin{array}{c|cccc} & 0 & 1 & 2 & 3 \\ 0 & 1 & 3 & 5 & 2 \\ 1 & 4 & 8 & 5 & 0 \\ 2 & 10 & 20 & -1 & 3 \end{array}_{3*4}$$

I/P → Queries (Top left, Bottom Right)

sx, sy, ex, ey
0 , 0    0 , 2 → 1+3+5 = 9
1 , 1    2 , 3 → 35

Bruteforce → ∀ queries, travel & find sum.
TC = O(Q * N * M)   SC = O(1)

Optimization ideas → Prefix Sum
- row wise } ✓
- column wise
- complete matrix ←

$$A = \begin{array}{cccc} 0 & 1 & 2 & 3 \\ \hline 1 & 3 & 5 & 2 \end{array}$$

$p \rightarrow [\,1 \quad 4 \quad 9 \quad 11\,]$

previously calculated value

$\boxed{P[i] = P[i-1] + A[i]}$

$P[i] \rightarrow$ sum from $\underline{0 \rightarrow i}$

$s = 1$

$e = 3$

$\rightarrow P[3] - P[1-1] = 11 - 1 = \underline{10}$

$\boxed{Ans = P[e] - P[s-1]}$

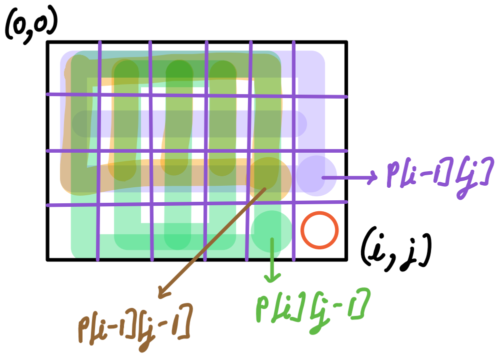
$$A = \begin{array}{c|cccc} & 0 & 1 & 2 & 3 \\ \hline 0 & 1 & 3 & 5 & 2 \\ 1 & 4 & 8 & 5 & 0 \\ 2 & 10 & 20 & -1 & 3 \end{array}$$

Top Left    Bottom Right

$P[i][j] \rightarrow$ sum from $\underline{(0,0)} \longrightarrow \underline{(i,j)}$

$P[1][1] = 1 + 3 + 4 + 8 = \underline{16}$

$P[2][1] = 1 + 3 + 4 + 8 + 10 + 20 = \underline{46}$



$$\boxed{\begin{aligned} P[i][j] = {} & P[i-1][j] + P[i][j-1] \\ & - P[i-1][j-1] + A[i][j] \end{aligned}}$$ ✓

$TC = \underline{O(N*M)}$   $SC = \underline{O(N*M)} \rightarrow \underline{O(1)}$ (modify A)

$$A = \begin{array}{c|cccc} & 0 & 1 & 2 & 3 \\ \hline 0 & 1 & 3 & 5 & 2 \\ 1 & 4 & 8 & 5 & 0 \\ 2 & 10 & 20 & -1 & 3 \end{array}$$

$\boxed{P[0][0] = A[0][0]}$

$$P = \begin{array}{c|cccc} & 0 & 1 & 2 & 3 \\ \hline 0 & 1 & 4 & 9 & 11 \\ 1 & 5 & 16 & 26 & 28 \\ 2 & 15 & 46 & 55 & 60 \end{array}$$

$\rightarrow \boxed{P[i][j-1] + A[i][j]}$

$\downarrow \boxed{P[i-1][j] + A[i][j]}$

1D $A[] \rightarrow \boxed{Ans = P[e] - P[s-1]}$

$$A = \begin{array}{c|cccc} & 0 & 1 & 2 & 3 \\ \hline 0 & 1 & 3 & 5 & 2 \\ 1 & 4 & 8 & 5 & 0 \\ 2 & 10 & 20 & -1 & 3 \end{array}$$

(2,1)

$$P = \begin{array}{c|cccc} & 0 & 1 & 2 & 3 \\ \hline 0 & 1 & 4 & 9 & 11 \\ 1 & 5 & 16 & 26 & 28 \\ 2 & 15 & 46 & 55 & 60 \end{array}$$

Top left $\rightarrow (0,1)$

Bottom right $\rightarrow (2,2)$

$Ans = 55 - 15 = \underline{40}$

Top left $\rightarrow \underline{(1,1)}$

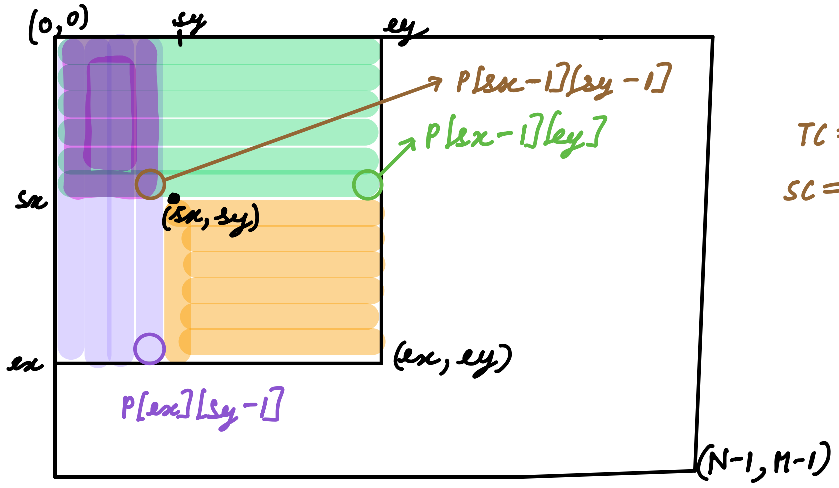
Bottom Right $\rightarrow \underline{(2,3)}$

$>$ $P[2][3] - P[2][0] - P[0][3] + P[0][0]$

$60 - 15 - 11 + 1 = \underline{35}$

Top left → (sx, sy)
Bottom Right → (ex, ey)

$$\boxed{\begin{aligned} Ans = {} & P[ex][ey] - P[ex][sy-1] \\ & - P[sx-1][ey] + P[sx-1][sy-1] \end{aligned}}$$ ✓



$TC = O(N*M + Q)$

$SC = O(N*M)$ / $O(1)$

Q → Sum of all submatrix sum.

$$A = \begin{array}{c|cc} & 0 & 1 \\ \hline 0 & 9 & 6 \\ 1 & 5 & 4 \end{array} \rightarrow$$

$[9]$ 9 $\quad$ $[6]$ 6 $\quad$ $[5]$ 5

$[9 \;\; 6]$ 15 $\quad$ $\begin{bmatrix}6\\4\end{bmatrix}$ 10 $\quad$ $[5 \;\; 4]$ 9

$\begin{bmatrix}9\\5\end{bmatrix}$ 14

$[4]$ 4

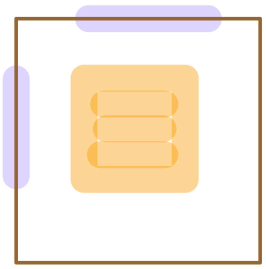
$\begin{bmatrix}9 & 6\\5 & 4\end{bmatrix}$ 24

contribution technique
↓
Ans = Σ contribution of all elements

Count of submatrix

Ans = 9 + 15 + 14 + 24 +
6 + 10 + 5 + 9 + 4 = 96 ✓

$\#$ subarrays $= \frac{N(N+1)}{2}$



$\#$ submatrix $= \frac{N(N+1)\,M(M+1)}{4}$

$$A = \begin{array}{c|cccc} & 0 & 1 & 2 & 3 \\ \hline 0 & 1 & 3 & 5 & 2 \\ 1 & 4 & 8 & 5 & 0 \\ 2 & 10 & 20 & -1 & 3 \\ 3 & 3 & 2 & 1 & 0 \\ 4 & 5 & -1 & -2 & 3 \end{array}$$

→ # subarray for column (continuous segment of col)

$= \frac{\underline{4*(4+1)}}{2} = \underline{10}$

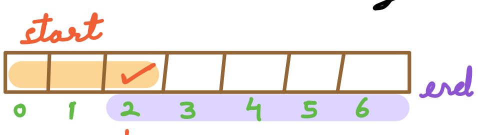
# submatrix = 15 * 10 = $\underline{150}$

↳ # subarray of row = $\frac{\underline{5(5+1)}}{2} = \underline{15}$ (continuous segment of row)

contribution of 1 element → A[i][j] * $\underline{\text{\# submatrix it is part of}}$

## # submatrix containing A[i][j]

10:50 PM



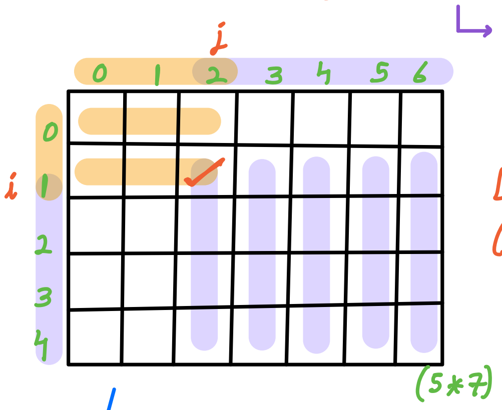
[L R] → R-L+1

$\underline{\phantom{R-L+1}}$

(i+1) * (N-i) ←

# subarray containing A[2] = 3 * 5 = $\underline{15}$ [0 i] [i N-1]

↳ (# start position * # end position)



Top-left

[0 i] [0 j]

(i+1) * (j+1)

Bottom-Right

[i N-1] [j M-1]

(N-i) * (M-j)

# submatrix = (i+1)*(j+1)*(N-i)*(M-j) ←

start → (0,0) (0,1) (0,2)

(1,0) (1,1) (1,2) } → (1+1)*(2+1) = $\underline{6}$

$\underline{120}$

1,2

end → (1,2) (1,3) (1,4) (1,5) (1,6) } (5-1)*(7-2) = $\underline{20}$

(2,2) ______

(3,2) ______

(4,2) ______

$$\boxed{Ans = \sum_i \sum_j A[i][j] * (i+1) * (j+1) * (N-i) * (M-j)}$$

TC = O(N*M)

SC = $\underline{O(1)}$

Q → Check if any element X is present in a row wise & column wise sorted matrix.

inc.

L → R

T ↓ B inc.

$$A = \begin{bmatrix} -5 & -2 & 1 & 3 \\ -4 & 0 & 3 & 4 \\ -3 & 2 & 6 & 8 \end{bmatrix}$$

| | |
|---|---|
| X = 0 | Ans = true |
| X = 2 | Ans = true |
| X = 7 | Ans = false |

Bruteforce → Travel & check the matrix → TC = O(N*M)

SC = O(1)

$\boxed{\text{Binary Search}}$ ← X

inc.

L → R

T ↓ B inc

$$A = \begin{bmatrix} -5 & -2 & 1 & 3 \\ -4 & 0 & 3 & 4 \\ -3 & 2 & 6 & 8 \end{bmatrix}$$

X = 2

dec

L ← R

T ↓ B inc

$$A = \begin{bmatrix} -5 & -2 & 1 & 3 \\ -4 & 0 & 3 & 4 \\ -3 & 2 & 6 & 8 \end{bmatrix}$$

A[i][j] = 3 > X

A[i][j] = 1 < X

3 > X

0 < X

A[i][j] = 2 = X ✓

```
if (A[i][j] > x)
        discard column → go left
if (A[i] [j] < x)
        discard row → go down
```

dec

L ← R

T ↓ B inc

$$A = \begin{array}{c|cccc} & 0 & 1 & 2 & 3 \\ \hline 0 & -5 & -2 & 1 & 3 \\ 1 & -4 & 0 & 3 & 4 \\ 2 & -3 & 2 & 6 & 8 \end{array}$$

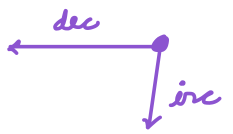
✓

X = -9

| A[i][j] | | X |
|---|---|---|
| 3 | > | X |
| 1 | > | X |
| -2 | > | -9 |
| -5 | > | -9 |

Ans = false

```
i = 0    j = M-1
while (i < N && j >= 0) {
    if (A[i][j] == x)
            return true
    else if (A[i][j] > x)
            j -= 1  ✓     M-1 → 0
    else
            i += 1  ✓     0 → N-1
}
return false
```



TC = $\underline{O(N+M)}$

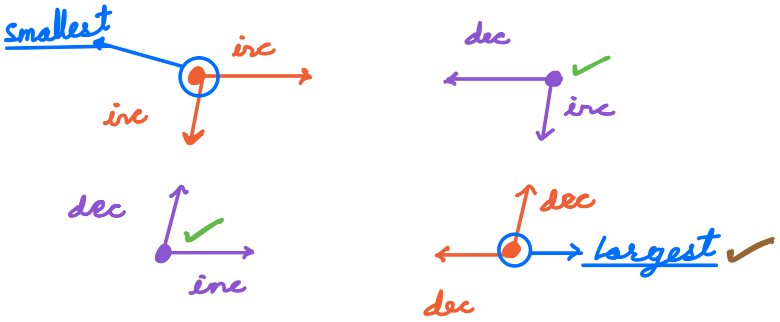
SC = $\underline{O(1)}$

$L \xrightarrow{\text{inc.}} R$

$$A = \begin{bmatrix} -5 & -2 & 1 & 3 \\ -4 & 0 & 3 & 4 \\ -3 & 2 & 6 & 8 \end{bmatrix}$$

$T \xrightarrow{\text{inc.}} B$

row wise & column wise sorted matrix.



$L \xrightarrow{\text{inc.}} R$

$$A = \begin{array}{c|cccc} 0 & -5 & -2 & 0 & 2 \\ 1 & -4 & 0 & 3 & 4 \\ 2 & 2 & 3 & 6 & 8 \\ \hline & 0 & 1 & 2 & 3 \end{array}$$

$T \xrightarrow{\text{inc.}} B$

$x = 1$