GCD → Greatest Common Divisor (≥ 0) ✓

HCF → Highest common Factor

$gcd(15, 25) = 5$

15: 1, 3, 5, 15
25: 1, 5, 25

$gcd(50, 115) = 5$

$gcd(20, 16) = 4$

```
if (a == 0 || b == 0)
    return a + b
```

$\boxed{a+b > 0}$

$\boxed{1 \leq \text{factor of } a \leq a}$

```
gcd = 1
for i → 2 to min(a, b)
    if (a % i == 0 && b % i == 0)
        gcd = i
return gcd
```

$TC = O(\min(a, b))$

$gcd(100, 50)$
50 50 → $gcd = 50$

Properties

1) $gcd(a, b) = gcd(b, a)$

2) $gcd(1, a) = 1$

only divisor of $1 = 1$
& $a \% 1 = 0$
$\Rightarrow gcd(1, a) = 1$

3) $gcd(0, a) = a$
$\boxed{a > 0}$

$0 \% x = 0$ ✓
⇒ all numbers are divisor of 0
& greatest divisor of $a = a$
$\Rightarrow gcd(0, a) = a$ ✓

4) $\boxed{gcd(a, b) = gcd(a, b-a) \quad \boxed{b > a}}$

$gcd(10, 15) = gcd(10, 5)$

To prove $gcd(a, b) = gcd(a, b-a)$

Let $gcd(a, b) = d$ → $a \% d = 0$ $\quad b \% d = 0$ $\quad 15 \% 5 = 0$

$a = d * K_1$ $\quad b = d * K_2$ $\quad 15 = 5 * \underline{3}_K$

$(b-a) = (K_2 - K_1)\, d \longrightarrow (b-a) \% d = 0$

Let $\underline{gcd(a, b-a) = m} \rightarrow$ $a \% m = 0$ $\quad (b-a) \% m = 0$

$a = m * T_1 \qquad b-a = m * T_2$

$\cancel{a} + (b - \cancel{a}) = b = (T_1 + T_2)\, m \longrightarrow b \% m = 0$

If $a \% d = 0$ & $(b-a) \% d = 0 \Rightarrow d$ is a commonfactor of a & $b-a$

But, m is $gcd(a, b-a)$

$\Rightarrow \boxed{m >= d}$

If $a \% m = 0$ & $b \% m = 0 \Rightarrow m$ is a commonfactor of a & b

But, d is $gcd(a, b)$

$\Rightarrow \boxed{d >= m}$

$\boxed{d = m} \Rightarrow \boxed{gcd(a,b) = gcd(a, b-a)}$

<u>Hence Proved!</u> ☺

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5) $gcd(a, b) = gcd(a, b-a)$

$= gcd(a, b-a-a)$

$= gcd(a, \underline{b-a-a-a})$ $\boxed{>= 0}$

$\boxed{gcd(a,b) = gcd(a, b \% a)}$

$\boxed{gcd(a,b) = gcd(b, a)}$

$\boxed{gcd(0, a) = a}$

$\boxed{gcd(a,b) = gcd(b \% a, a)}$

$\boxed{b < a \Rightarrow \text{swap } a \& b}$

$gcd(100, 60)$

$= gcd(60, 100)$

$= gcd(60, 100 \% 60 = 40)$

$= gcd(40, 60)$

$= gcd(40, 60 \% 40 = 20)$

$= gcd(20, 40)$

$= gcd(20, 40 \% 20 = 0)$

$= gcd(0, 20) = \underline{20}$ ✓

$gcd(100, 60) = gcd(60 \% 100, 100)$

$= gcd(60, 100)$

<u>Euclid Theorem</u>

```
int gcd(a, b) {
    if(a == 0)
        return b
    return gcd(b%a, a)
}
```

H.W → Try iterative code.

$\underset{a}{gcd(64}, \underset{b}{50)}$

$\downarrow 1$

$gcd(50 \% 64 = 50, 64)$

$\downarrow 2$

$gcd(64 \% 50 = 14, 50)$

$\downarrow 3$

$gcd(50 \% 14 = 8, 14)$

$\downarrow 4$

$gcd(14 \% 8 = 6, 8)$

$\downarrow 5$

$gcd(8 \% 6 = 2, 6)$

$\downarrow 6$

$gcd(6 \% 2 = 0, 2) \xrightarrow{7} \underline{2}$ ✓

Time Complexity

$gcd(5000, 10000)$

$\downarrow$

$gcd(10000 \% 5000 = 0, 5000)$

$\downarrow$

$\underline{5000}$ 2 steps

$\boxed{\text{increase in i/p} \not\Rightarrow \text{increase in \# steps}}$

$gcd(a, b) \longrightarrow gcd(b\%a, a)$

$b \longrightarrow \boxed{b\%a < a}$ $\boxed{b = 10}$

$\boxed{\begin{matrix} 5 > 3 \\ -5 < -3 \end{matrix}}$

$a < b/2$

$b\%a < a < b/2$

$\Rightarrow \boxed{b\%a < b/2}$

$a = 3$

$b\%a < a < b/2$

$10\%3 < 3 < 10/2$

$10\%3 < 10/2$

$\underline{1 < 5}$

$a = b/2$

$\boxed{b\%a = 0}$

$a = 5$

$b\%a = 10\%5 = \underline{0}$

$a > b/2$

$-a < -b/2$

+b on both sides

$b - a < b - b/2$

$b - a < b/2$

$\boxed{x - \frac{x}{2} = \frac{x}{2}}$

$a = 8$

$8 > 10/2$

$-8 < -10/2$

$b\%a <= b - a < b/2$

$\boxed{b\%a < b/2}$

$10 - 8 < 10 - 10/2$

$10 - 8 < 10/2$

$10\%8 <= 10 - 8 < 10/2$

$10\%8 < 10/2$

$\underline{2 < 5}$

$\boxed{b\%a < b/2}$

$\boxed{b \longrightarrow < b/2}$

$b \to b/2 \Rightarrow$ #steps $= \log_2(b)$

$\boxed{TC < O(\log_2(b))}$ $\boxed{\text{linear} \to \log}$ ✓

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6) $gcd(a, b, c) \to$ $gcd(a, gcd(b, c))$ ✓
$gcd(b, gcd(a, c))$ ✓
$gcd(c, gcd(a, b))$ ✓

A [18 16 24]

$gcd(2, 24)$ $gcd(6, 16)$ $gcd(8, 18)$

$\downarrow$ $\downarrow$ $\downarrow$

2 2 2

Gcd of all array elements →

```
ans = A[0]
for i → 1 to (N-1)
    ans = gcd(ans, A[i])
return ans
```

$TC = O(N \log_2(A[i]))$

#elements of array

value of elements

$\boxed{10:42 \text{ PM IST}}$

Q → check ✓ if there is a subsequence in the array with gcd = 1.

continuous or non-continuous

$A = [2, 30, 14, 72, 60] \to$ Ans = false

→ $A = [6, 15, 30, 10, 150] \to$ Ans = true

$gcd(6, 15) = 3$

$gcd(6, 15, 10) = 1$

$gcd(6, 15, 10, 30) = 1$

$gcd(6, 15, 10, 30, 150) = 1$

$gcd(\underbrace{-, -, -, -, -}_{1}, 18) = 1$

if $gcd(\text{any subsequence}) = 1$

$\Rightarrow gcd(\text{all elements}) = 1$

full array → subsequence ✓

$\boxed{\begin{array}{l} \text{if } \gcd(\text{all elements}) = 1 \Rightarrow Ans = true \\ \qquad\qquad\qquad \text{else} \Rightarrow Ans = false \end{array}}$

---

4 elements

_ _ _ _

✓ ✓ ✓ ✓
x x x x

$2 * 2 * 2 * 2 \rightarrow 2^4 \Rightarrow$ total subsequence $= \underline{2^N}$

(including empty)

non-empty $\Rightarrow \underline{2^N - 1}$

Q → Given an integer array, find max gcd of all elements of the array after deleting exactly one elements.

A = [9 18 98 12 30 60]

x ——— 2
— x ——— 1
— x ——— 3 (Ans)
— x ——— 1
— x — 1
— x 1

Bruteforce → calculate gcd of all elements except A[i] ∀i.

$TC = \underline{O(N * N \log(A[i]))}$

A = [20 64 24 100 50] Ans = $\underline{4}$

A = [9 18 98 12 30 60]

P = [9 9 1 1 1 1]

S = [1 2 2 6 30 60]

$P[i] = \gcd(P[i-1], A[i])$

$S[i] = \gcd(S[i+1], A[i])$

gcd of all elements excluding $A[i] = \gcd(P[i-1], S[i+1])$

$i = 0 \longrightarrow S[1]$

$i = N-1 \longrightarrow P[N-2]$

$TC = O(N \log(A[i]))$

$SC = \underline{O(N)}$

Q → N players are playing a game.

A[i] → health/power of i^{th} player

If player i attacks player j $\rightarrow$ $(i \rightarrow j)$

if $(A[j] <= A[i]) \longrightarrow$ player j is dead $\underline{A[j] = 0}$ ✓

if $(A[j] > A[i]) \longrightarrow$ $\boxed{A[j] = A[j] - A[i]}$ ✓

Find min possible health of last surviving player.

$A = [\overset{1}{6}, \overset{2}{4}] \longrightarrow$ $1 \rightarrow 2$ $[6, 0] \longrightarrow \underline{6}$

$2 \rightarrow 1$ $[6-4=2, 4] \longrightarrow$ $1 \rightarrow 2$ $[2, 4-2=2] \longrightarrow \underline{2}$ (Ans)

$2 \rightarrow 1$ $[0, 4]$ $\underline{4}$

$\boxed{\text{small health} \xrightarrow{\text{attack}} \text{large health}}$ ✓

$A = [\overset{1}{9}, \overset{2}{6}, \overset{3}{15}] \longrightarrow [\overset{1}{3}, \overset{2}{6}, \overset{3}{9}] \longrightarrow [\overset{1}{3}, \overset{2}{3}, \overset{3}{6}] \longrightarrow [\overset{1}{3}, \overset{2}{0}, \overset{3}{3}]$ $\underline{3}$ ✓

$a \rightarrow a$
$b \rightarrow b-a$
$a <= b$
$\rightarrow \underline{gcd(a, b)}$

Ans = $\underline{\text{gcd of all elements}}$ ✓