count of factors

N = 10 {1, 2, 5, 10} 10 → 5 (÷2)

N = 24 {1, 2, 3, 4, 6, 8, 12, 24}

10 3 8 2 4 7 6 5 1 3 6

Q → Given a number N, count no. of factors of N.

smallest factor of N = 1
largest factor of N = N
} ✓

[1 N]
Range

%

a += 1
a = a + 1

```
count (N) {
    factors = 0
    for i → 1 to N
    |       if (N % i == 0)
    |           factors += 1
    return factors
}
```

iterations = N

10^8 iterations → 1 second

$N = 10^9$ → 10^9 iterations

$10^9 = 10 * 10^8$

⇒ 10 seconds

$a^{n+m} = a^n * a^m$

$N = 10^{18}$ → 10^{18} iterations

$= 10^{10} * 10^8$ ⇒ 10^{10} seconds

10^{10} sec → $\frac{10^{10}}{60*60}$ hours → $\frac{10^{10}}{60*60*24}$ days → $\frac{10^{10}}{60*60*24*365}$ years

$\approx 2.7 * 10^6$ ≈ 115741 = 317 years

f → kids → grandkids → ... 6 or 7th generation ✓

optimizing code → reduce # iterations

Observations

N = 10 {1, 2, 5, 10}

a	b	
1 * 10		2
2 * 5		2

N = 100

a	b	
1 * 100		→ 2
2 * 50		→ 2
4 * 25		→ 2
5 * 20		→ 2
10 * 10		→ 1
		9

$N = a * b$, where $a <= b$

N/a

min value of a = 1

max value of a = $\sqrt{N}$

$a <= b$

$\Rightarrow a <= N/a$

$\Rightarrow a^2 <= N \Rightarrow \boxed{a <= \sqrt{N}}$

```
count(N) {
    factors = 0
    for a → 1 to √N
        if (N % a == 0)
            if (a == N/a)
                factors += 1
            else
                factors += 2
    return factors
}
```

iterations = $\sqrt{N}$

N = 30

a = 1 2 3 4 5 6 → stop

factors = 2 + 2 + 2 + 2 = 8

5 * 6 = 30

```
for (a = 1; a <= √N; a++)
    a * a <= N
```

$N = 10^{18} \rightarrow \sqrt{10^{18}}$ iterations

$= 10^9$ iterations → 10 sec

317 years → 10 sec ✓

₹10 → 2 per

₹1000 → ?

$\frac{1000}{10} = 100$

10:35 pm

$10^8 \rightarrow$ 1 sec

$10^{18} \rightarrow \frac{10^{18}}{10^8} = 10^{10}$

$10^{18} = 10^2 * 10^8 * (10^8) \rightarrow$ sec

$= 10^2 * 10^8$ sec

Sum of Natural Numbers

Gauss

$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = 55$

$S = 1 + 2 + 3 + \ldots + 98 + 99 + 100$

$S = 100 + 99 + 98 + \ldots + 3 + 2 + 1$

$2S = 101 + 101 + 101 + \ldots \rightarrow 100 \text{ times}$

$2S = 100 * 101 \Rightarrow S = \frac{100 * 101}{2} = 5050$ ✓

$S = 1 + 2 + 3 + \ldots + (N-2) + (N-1) + N$

$S = N + (N-1) + (N-2) + \ldots + 3 + 2 + 1$

$2S = (N+1) + (N+1) + (N+1) + \ldots (N+1) + (N+1) + (N+1)$

$\Rightarrow 2S = N * (N+1) \Rightarrow \boxed{S = \frac{N*(N+1)}{2}}$ ✓

Sum of N whole numbers

0, 1, 2, 3, 4

$0 + 1 + 2 + 3 + \ldots (N-1) = \frac{(N-1) * N}{2}$ ✓

Integer Division

```
int a = 7;
print(a/2);
```

python → a//2

$7/2 \rightarrow 3$ $\quad 5/2 = 2$ $\quad 6/2 = 3$

$5.0/2 = 2.5$

Q → Given a +ve number N, count the no. of times it is divided by 2 to reach 1.

$N = 8 \quad \frac{8}{2} = 4 \rightarrow \frac{4}{2} = 2 \rightarrow \frac{2}{2} = 1 \quad$ Ans = 3

$N = 20$ $\quad \frac{20}{2} = 10 \longrightarrow \frac{10}{2} = 5 \longrightarrow \frac{5}{2} = 2 \longrightarrow \frac{2}{2} = 1$ $\quad$ Ans = 4

$N = 12$ $\quad \frac{12}{2} = 6 \longrightarrow \frac{6}{2} = 3 \longrightarrow \frac{3}{2} = 1$ $\quad$ Ans = 3

N	#steps
✓ 1 2^0	0
✓ 2 2^1	1
3	1
✓ 4 2^2	2
5	2
6	2
7	2
✓ 8 2^3	3
9	3
10	3
⋮	
15	3
✓ 16 2^4	4
17	4

$N = 2^k \longrightarrow$ #steps $= k$

$k = \log_2(N)$

$\log_b x = y \Rightarrow b^y = x$

$2^k <= N < 2^{k+1} \longrightarrow$ #steps $= k$

$k <= \log_2(N) < (k+1)$

#steps $= \text{floor}(\log_2(N))$ ✓

$2^3 <= 10 < 2^4$

$\log_2(10) = 3.32..$

$\log_3 27 = ?$ $\quad 3^3 = 27$

$\log_2 16 = 4$ $\quad 2^4 = 16$

$\log_2(2^k) = k$

$\log_2(2^3) = x$

$2^x = 2^3$

Q → Given a perfect sq. N, find sqrt (N) ?

$N = 49$ $\quad \sqrt{49} = 7$ $\quad 7*7 = 49$ $\quad 10*10 = 100$

$N = 50 \longrightarrow$ invalid input ←

$1 <= \sqrt{N} <= N$

```
for i → 1 to N
    if (i*i == N)
        return i
```

#iterations $= \sqrt{N}$

$N = 25$ $\quad i \rightarrow 1\ 2\ 3\ 4\ 5$ ✓ $\quad i \rightarrow 1 \text{ —— } \sqrt{N}$

$N = 9$ $\quad i = 1\ 2\ 3$ ✓

H.W → 1) Basic log properties
2) AP & GP
3) Basic P&C

AP → Arithmatic Progression
GP → Geometric Progression
P → Permutation ${}^{n}P_{r}$
C → Combinations ${}^{n}C_{r}$

1) 2 to 3 Hours → on our own ←
2) Watch hints / videos → 1 hour ✓
3) Ask TA ✓

$\log_2 16 = 4$ $\log_2 64 = 6$
$\sqrt{16} = 4$ $\sqrt{64} = 8$

$2^3 = 8$ $\log_2 8 = 3$