

8 bit number

Power of left shift

| | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 0 |
|-------------------------|------------------|---|---|---|---|---|---|---|
| $N = 45 \rightarrow$ | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 |
| $(1 \ll 2) \rightarrow$ | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| OR | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 |
| | $\rightarrow 45$ | | | | | | | |

| | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 0 |
|-------------------------|------------------|---|---|---|---|---|---|---|
| $45 \rightarrow$ | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 |
| $(1 \ll 4) \rightarrow$ | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 1 |
| | $\rightarrow 61$ | | | | | | | |
| | $45 + 2^4$ | | | | | | | |

$N \mid (1 \ll i)$

$\left\{ \begin{array}{l} N \Rightarrow i^{\text{th}} \text{ bit is already set (1)} \\ N + 2^i \Rightarrow i^{\text{th}} \text{ bit is unset (0)} \end{array} \right\}$

}

$i^{\text{th}} \text{ bit becomes 1.}$

| | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 0 |
|-------------------------|------------------|---|---|---|---|---|---|---|
| $N = 45 \rightarrow$ | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 |
| $(1 \ll 2) \rightarrow$ | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| XOR | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 |
| | $\rightarrow 41$ | | | | | | | |
| | $45 - 2^2$ | | | | | | | |

| | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 0 |
|-------------------------|------------------|---|---|---|---|---|---|---|
| $45 \rightarrow$ | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 |
| $(1 \ll 4) \rightarrow$ | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 1 |
| | $\rightarrow 61$ | | | | | | | |
| | $45 + 2^4$ | | | | | | | |

$N \wedge (1 \ll i)$

$\left\{ \begin{array}{l} N - 2^i \Rightarrow i^{\text{th}} \text{ bit is set (1)} \\ N + 2^i \Rightarrow i^{\text{th}} \text{ bit is unset (0)} \end{array} \right\}$

}

$\text{Toggle } i^{\text{th}} \text{ bit.}$

| | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 0 |
|-------------------------|-----------------|---|---|---|---|---|---|---|
| $N = 45 \rightarrow$ | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 |
| $(1 \ll 2) \rightarrow$ | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| AND | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| | $\rightarrow 4$ | | | | | | | |
| | 2^2 | | | | | | | |

| | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 0 |
|-------------------------|-----------------|---|---|---|---|---|---|---|
| $45 \rightarrow$ | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 |
| $(1 \ll 4) \rightarrow$ | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | $\rightarrow 0$ | | | | | | | |

$N \& (1 \ll i)$

$\left\{ \begin{array}{l} 2^i \Rightarrow i^{\text{th}} \text{ bit is set} \\ 0 \Rightarrow i^{\text{th}} \text{ bit is unset} \end{array} \right\}$

Given an integer N.

| | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 0 |
|----------------------|--------------|---|---|---|---|---|---|---|
| $N = 12 \rightarrow$ | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 |
| $i = 5$ | \downarrow | | | | | | | |
| | 0 | 0 | 0 | 1 | 1 | 0 | 0 | |

1) Set i^{th} bit of $N \rightarrow N = \underline{N | (1 \ll i)}$
no change if it is already set.

2) Toggle i^{th} bit of $N \rightarrow N = N \wedge (1 \ll i)$

3) Unset i^{th} bit of $N \rightarrow \text{if } (\text{checkBit}(N, i))$
no change if it is already unset. $N = N \wedge (1 \ll i)$
else // no change

4) check if i^{th} bit is set \rightarrow
 $N | (1 \ll i) == N \Rightarrow i^{\text{th}}$ bit is set
 $N \wedge (1 \ll i) < N \Rightarrow i^{\text{th}}$ bit is set
 $N \& (1 \ll i) == 2^i \Rightarrow i^{\text{th}}$ bit is set

Q \rightarrow check if i^{th} bit is set in N without left shift operator.

$N=45 \rightarrow$

| | | | | | | | | |
|--|---|---|---|---|---|---|---|---|
| | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 0 |
| | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 |

 $i=3 \rightarrow \text{Ans} = \text{True}$
 $i=4 \rightarrow \text{Ans} = \text{false}$

$N \& 1 \rightarrow$
 $1 \rightarrow 0^{\text{th}}$ bit is 1
 $0 \rightarrow 0^{\text{th}}$ bit is 0

$N=45 \rightarrow$

| | | | | | | | | |
|--|---|---|---|---|---|---|---|---|
| | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 0 |
| | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 |

 $N \gg 3 \rightarrow$

| | | | | | | | | |
|--|---|---|---|---|---|---|---|---|
| | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 0 |
| | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 |

$(N \gg i) \& 1 \rightarrow$
 $1 \Rightarrow i^{\text{th}}$ bit is set
 $0 \Rightarrow i^{\text{th}}$ bit is unset

Q \rightarrow Given an integer N , count no. of set bits in N .

$N=45 \rightarrow 00101101 \quad \text{Ans} = \underline{4}$

$N = 10 \rightarrow 0000101\textcircled{0}$ Ans = 2

$N = 10$
 $N = N \gg 1$
 print(N)

$N = 10$
 $N = N + 2$
 print(N) $\rightarrow 12$

$N = 10 \rightarrow 0000101\textcircled{0}$ Ans = 2
 $N \gg 1 \quad 0000010\textcircled{1}$
 $N \gg 2 \quad 0000001\textcircled{0}$
 $N \gg 3 \quad 00000001$
 $N \gg 4 \quad 00000000 \rightarrow \text{stop}$

$N \& 1 == 1 \quad \checkmark$

```
ans = 0
while (N > 0) {
    if ((N & 1) == 1) {
        ans += 1
    }
    N = N >> 1 // N = N / 2
}
return ans
```

TC = $O(\log_2(N))$
 SC = $O(1)$

int $\rightarrow 32$
 long $\rightarrow 64$

$\log_2(2^{32}) = 32$
 \uparrow
 2^{32}

10:30 PM

8 bit number

7 6 5 4 3 2 1 0
1 1 1 1 1 1 1 1

[0 — 255]

$$2^7 + 2^6 + 2^5 + \dots + 2^0 = \frac{2^0 \times (2^8 - 1)}{2 - 1} = 2^8 - 1 = 255$$

Negative Numbers (2's Complement)

signed bit \leftarrow MSB \rightarrow most significant bit
 (-ve) \uparrow
 $2^7 > (2^6 + 2^5 + 2^4 + 2^3 + 2^2 + 2^1 + 2^0)$
 $\frac{2^0(2^7 - 1)}{2 - 1} = 2^7 - 1$

1 \rightarrow -ve
 0 \rightarrow +ve

$N = \textcircled{45} \rightarrow$

7 6 5 4 3 2 1 0
 0 0 1 0 1 1 0 1

flip all bits \rightarrow 1 1 0 1 0 0 1 0 \rightarrow 1's Complement

add 1 \rightarrow + 0 0 0 0 0 0 0 1

-45 \leftarrow 1 1 0 1 0 0 1 1 \rightarrow 2's Complement

$$(-2^7) + 2^6 + 2^4 + 2^1 + 2^0 = 211$$

$$-2^7 + (2^6 + 2^4 + 2^1 + 2^0) = -128 + (64 + 16 + 2 + 1)$$

$$= -128 + 83 = \underline{-45}$$

$(-12) \rightarrow (?)_2$

12 \rightarrow 7 6 5 4 3 2 1 0
0 0 0 0 1 1 0 0

flip \rightarrow 1 1 1 1 0 0 1 1
add 1 \rightarrow 0 0 0 0 0 0 0 1

1 1 1 1 0 1 0 0

$$-2^7 + 2^6 + 2^5 + 2^4 + 2^2 = -128 + (64 + 32 + 16 + 4)$$

$$= -128 + 116 = \underline{-12}$$

45 - 12

45 + (-12)

discard \leftarrow 1

7 6 5 4 3 2 1 0
1 1 1 1 1
45 \rightarrow 0 0 1 0 1 1 0 1
-12 \rightarrow 1 1 1 1 0 1 0 0
0 0 1 0 0 0 0 1

$$2^5 + 2^0 = 32 + 1 = \underline{33}$$

$$\boxed{45 - 12 = 33}$$



Ranges

8 bit system

-2^7 2^6 2^5 2^4 2^3 2^2 2^1 2^0
7 6 5 4 3 2 1 0

min \rightarrow 1 0 0 0 0 0 0 0 $\rightarrow -2^7 = \underline{-128}$

max \rightarrow 0 1 1 1 1 1 1 1 $\rightarrow 2^7 - 1 = \underline{127}$

Range -128 to 127

Integers

31 30 29 28 --- 0
 32 bits min $\rightarrow 1\ 0\ 0\ 0\ \dots\ 0 \rightarrow -2^{31} = -2147483648 = -2 \times 10^9$
 max $\rightarrow 0\ 1\ 1\ 1\ \dots\ 1 \rightarrow 2^{31} - 1 = 2147483647 = \underline{2 \times 10^9}$

long

63 62 61 --- 0
 64 bits min $\rightarrow 1\ 0\ 0\ \dots\ 0 \rightarrow -2^{63} \approx -9 \times 10^{18}$
 max $\rightarrow 0\ 1\ 1\ \dots\ 1 \rightarrow 2^{63} - 1 = \underline{9 \times 10^{18}}$

Q \rightarrow Given a integer array,
 find the sum of all array elements.

$\xrightarrow{\text{long}}$
~~int~~ ans = 0
 for i \rightarrow 0 to (N-1)
 ans += A[i]
 return ans

Constraints

$1 \leq N \leq 10^5$
 $1 \leq A[i] \leq 10^6$

max total sum = $10^6 + 10^6 + 10^6 \dots + 10^6$
 10^5 times
 $= 10^6 \times 10^5 = \underline{10^{11}}$ ✓

constraints \rightarrow datatypes
 \rightarrow TLE

Q \rightarrow Find $a * b$ for given integers a & b.

int ans = a * b X
 return ans

\downarrow
 long ans = a * b X
 return ans

\downarrow
 overflow at multiplication ✓
 long ans = long(a * b) X
 return ans

||

max a $\rightarrow 2 \times 10^9$
 max b $\rightarrow 2 \times 10^9$
 max a * b $\rightarrow \underline{4 \times 10^{18}} \rightarrow$ long

int * int \rightarrow int

↓
long ans = long(a) * b ✓
return ans

long * int → long ✓

long ans = a
ans *= b ✓
return ans

long ans = (1L * a) * b ✓
return ans

long ans = a * 1L * b ✓
return ans
