

Q → Given an integer array of size N,
find the sum of elements from index L to R.

A = [-6, 3, 2, 4, 5, -2, 1, 9] L=1 R=4

$$\text{Ans} = 3 + 2 + 4 + 5 = \underline{14}$$

ans = 0

for i → L to R

ans += A[i]

return ans;

TC = O(N)

SC = O(1)

Find sum of elements from index L to R for multiple queries.

A = [-6, 3, 2, 4, 5, -2, 1, 9]

I/P { L = [1, 3, 0], R = [4, 6, 5] } → Q

Bruteforce → TC = O(Q * N)
SC = O(1)

$$3 + 2 + 4 + 5 = \underline{14}$$

$$4 + 5 - 2 + 1 = \underline{8}$$

$$-6 + 3 + 2 + 4 + 5 - 2 = \underline{6}$$

Cricket

Sairam

Over → 1 2 3 4 5 6 7 8 9 10
Scoreboard → 0 16 22 30 45 51 70 75 90 104 120

Scoreboard[i]
runs scored
from 1st to ith over.

Score of 10th over = 120 - 104 = 16
Score of last 6 overs = 120 - 45 = 75
Score of 4th to 7th over = 75 - 30 = 45

TC = O(1)
per query

A = [-6, 3, 2, 4, 5, -2, 1, 9]
Prefix Sum = [-6, -3, -1, 3, 8, 6, 7, 16]

L = [1, 3, 0]
R = [4, 6, 5]

P[i] = sum of elements from start till ith index. ✓

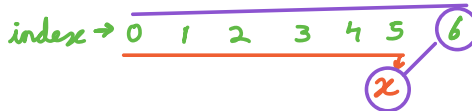
$$P[i] = P[i-1] + A[i]$$

TC per query = $O(1)$

TC = $O(N)$ SC = $O(N)$

$$A[i] = P[i] - P[i-1]$$

$$\begin{aligned} Q_1 &\rightarrow (1, 4) \rightarrow P[4] - P[1-1] = 8 - (-6) = \underline{14} \\ Q_2 &\rightarrow (3, 6) \rightarrow P[6] - P[3-1] = 7 - (-1) = \underline{8} \\ Q_3 &\rightarrow (0, 5) \rightarrow P[5] = \underline{6} \end{aligned}$$



$\rightarrow P[0] = A[0] \quad // P[] \text{ size } N$
 for $i \rightarrow 1$ to $(N-1)$
 $P[i] = P[i-1] + A[i]$ } TC = $O(N)$ SC = $O(N)$

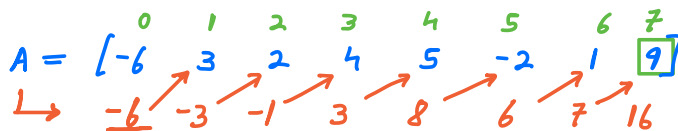
for $i \rightarrow 0$ to $(Q-1)$
 $l = L[i]$
 $r = R[i]$
 if $(l == 0)$
 print $(P[r])$
 else
 print $(P[r] - P[l-1])$

$L[i] \text{ --- } R[i]$

TC = $O(Q)$ SC = $O(1)$

Total TC = $O(N+Q)$
 SC = $O(N)$

TC = $O(Q \times N)$ \rightarrow TC = $O(N+Q)$ \rightarrow modify input ✓
 SC = $O(1)$ SC = $O(N)$ SC = $O(1)$



$A[i]$ = sum of elements from start till i^{th} index.

$$A[i] = A[i-1] + A[i]$$

for $i \rightarrow 1$ to $(N-1)$ } TC = $O(N)$ SC = $O(1)$
 $A[i] = A[i-1] + A[i]$

$$A = \begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ [-6 & 3 & 2 & 4 & 5 & -2 & 1 & 9] \end{matrix}$$

$$\text{Prefix Sum} = [-6 \quad -3 \quad -1 \quad 3 \quad 8 \quad 6 \quad 7 \quad 16]$$

$P[i] = \text{sum of elements from start till } i^{\text{th}} \text{ index.}$

$$A = \begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ [-6 & 3 & 2 & 4 & 5 & -2 & 1 & 9] \end{matrix}$$

$$\text{Suffix Sum} = [16 \quad 22 \quad 19 \quad 17 \quad 13 \quad 8 \quad 10 \quad 9]$$

$S[i] = \text{sum of elements from } i^{\text{th}} \text{ index till end.}$

$$S[i] = S[i+1] + A[i]$$

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$$S[N-1] = A[N-1]$$

for $i \rightarrow (N-2) \text{ to } 0$

$$S[i] = S[i+1] + A[i]$$

Q → Given an integer array of size N ,
find the first (smallest) equilibrium index.

↓ index k

sum of elements from index 0 to $(k-1)$ = sum of elements from index $(k+1)$ to $(N-1)$
 Left Sum Right Sum

$$A = [-7 \quad 1 \quad 5 \quad 2 \quad -4 \quad 3 \quad 0]$$

$$\begin{matrix} -7+1+5 & -4+3+0 \\ = -1 & = -1 \end{matrix} \quad \text{Ans} = \underline{3}$$

$$A = [-7 \quad 1 \quad 5 \quad 2 \quad -4 \quad 3 \quad 0]$$

$$-7+1+5+2-4+3 = 0 \quad \underline{0}$$

$$\rightarrow A = [5 \quad 1 \quad 3 \quad -6 \quad 5 \quad -2 \quad -1]$$

$$\underline{0} \quad 1+3-6+5-2-1 = \underline{0}$$

Bruteforce → First index k s.t $\text{sum}(0 \text{---}(k-1)) = \text{sum}(k+1 \text{---}(N-1))$.

$$P[k-1] = S[k+1]$$

$$\begin{array}{ccccccc} & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ A = & [-7 & 1 & 5 & 2 & -4 & 3 & 0] \\ P = & [-7 & -6 & -1 & 1 & -3 & 0 & 0] \\ S = & [0 & 7 & 6 & 1 & -1 & 3 & 0] \end{array}$$

Steps → 1) calculate $P[]$ & $S[]$.

$$\left. \begin{array}{l} TC = O(N+N) = O(N) \\ SC = O(N+N) = O(N) \end{array} \right\}$$

2) Find first k s.t $P[k-1] = S[k+1]$.

$$TC = O(N)$$

$$SC = O(1)$$

$$\begin{array}{l} \text{Total } TC = O(N) \checkmark \\ SC = O(N) \end{array}$$

corner cases → $k=0$ or $(N-1)$

$$P[k-1] = S[k+1]$$

$$+ A[k] \quad + A[k]$$

$$P[k] = S[k]$$

$k \text{---} (N-1)$

$$P[N-1] - P[k-1] \checkmark$$

$$\begin{array}{ccccccc} & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ A = & [-7 & 1 & 5 & 2 & -4 & 3 & 0] \\ P = & [-7 & -6 & -1 & 1 & -3 & 0 & 0] \\ S = & [0 & 7 & 6 & 1 & -1 & 3 & 0] \end{array}$$

$$P[6] - P[3-1]$$

$$\text{Vi check if } P[k] == P[N-1] - P[k-1]$$

$$\begin{array}{l} \text{modify i/p} \rightarrow TC = O(N) \\ SC = O(1) \end{array}$$

H.W → Solve with same TC & SC without modifying i/p.

Q → Given an integer array of size N , find sum of elements for queries →

Type $\frac{1}{2}$ L R start end

Type 1 → Sum of even index elements from L to R

Type 2 → Sum of odd index elements from L to R

$$A = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ 8 & -3 & 5 & -7 & 1 & 4 & 2 \end{bmatrix}$$

$$\text{evenP} = [8 \quad 8 \quad 13 \quad 13 \quad 14 \quad 14 \quad 16]$$

$$\text{oddP} = [0 \quad -3 \quad -3 \quad -10 \quad -10 \quad -6 \quad -6]$$

$$\begin{array}{cccc}
 & a_1 & a_2 & a_3 & a_4 \\
 \text{Type} \rightarrow & [1 & 2 & 2 & 1] \\
 L \rightarrow & [1 & 2 & 4 & 3] \\
 R \rightarrow & [4 & 6 & 5 & 3]
 \end{array}$$

$$\begin{array}{cccc}
 \text{S} + 1 = 6 & -7 + 4 = -3 & 4 & 0
 \end{array}$$

$$A = \begin{bmatrix} 2 & 3 & 1 & -1 & 0 & 8 & 5 & 4 \end{bmatrix}$$

$$\text{even } P = \begin{bmatrix} 2 & 2 & 3 & 3 & 3 & 3 & 8 & 8 \end{bmatrix}$$

$$\text{odd } P = \begin{bmatrix} 0 & 3 & 3 & 2 & 2 & 10 & 10 & 14 \end{bmatrix}$$

$A = [8, -3, 5, -7, 1, 4, 2]$
 $evenP = [8, 8, 13, 13, 14, 14, 16]$
 $oddP = [0, -3, -3, -10, -10, -6, -6]$

$TC = O(N)$
 $SC = O(N)$

$TC = O(1)$
 per query

$evenP[4] - evenP[1-1] = 14 - 8 = 6$

$oddP[6] - odd[2-1] = -6 - (-3) = -3$

$$\text{Total TC} = \underline{O(N + Q)} \quad \text{SC} = \underline{O(N)}$$

$$A = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ 8 & -3 & 5 & -7 & 1 & 4 & 2 \end{bmatrix}$$

$$\text{even } \rightarrow \underline{P} = [8 \quad -3 \quad 13 \quad -10 \quad 14 \quad -6 \quad 16]$$

$$P[i] = P[i-2] + A[i] \rightarrow i > 1$$

Type $\begin{cases} 1) \text{ even} \rightarrow R \text{ or } (L-1) \text{ is odd then do } (R-1) \text{ or } ((L-1)-1) \\ 2) \text{ odd} \rightarrow R \text{ or } (L-1) \text{ is even then do } (R-1) \text{ or } ((L-1)-1) \end{cases}$

$\approx 2 \times 10^9$ long.