Range → int → $-2*10^9$ to $2*10^9$

long → $-9*10^{18}$ to $9*10^{18}$

% → modulo operator (remainder)

Dividend = **Divisor** * Quotient + **Remainder**

10 % 4 $\quad 10 = 4 * \left(\frac{10}{4}\right) + r$

$\Rightarrow 10 = 4*2 + r \Rightarrow r = \underline{2}$ $\quad 10 \% 4 = \underline{2}$

13 % 5 $\quad 13 = 5 * \left(\frac{13}{5}\right) + r \Rightarrow 13 = 5*2 + r \Rightarrow r = \underline{3}$

Remainder = Repeated subtraction of divisor from dividend till number is $\underline{>= 0}$.

10 % 4 → $10 - 4 = 6 - 4 = \underline{2}$

18 % 5 → $18 - 5 = 13 - 5 = 8 - 5 = \underline{3}$

Range of A % B

Dividend

Divisor

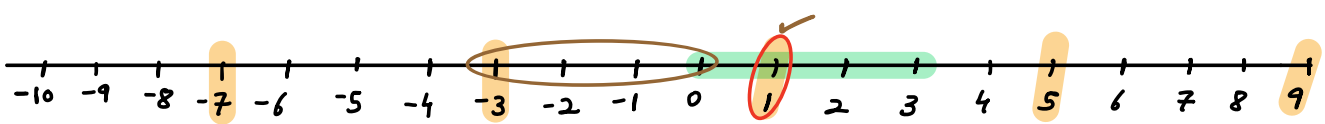
min = **0** $\quad$ max = **B-1**

Negative Numbers

-60 % 9 ⟶ $\underline{-6}$ $\quad$ Python → $\underline{3}$

$-60 = 9 * \left(\frac{-60}{9}\right) + r \Rightarrow -60 = 9*(-6) + r$

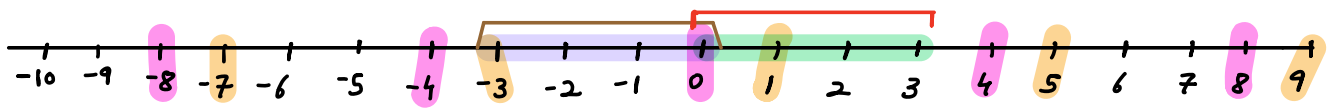
$\Rightarrow -60 = -54 + r \Rightarrow r = \underline{-6}$ ✓



$9 \% 4 = 5 \% 4 = 1 \% 4 = -3 \% 4 = -7 \% 4$

↳ Every number at a jump of 4 is equal under %4 ←

A % B → Every number at a jump of B is equal.



%4 → [0 3]

-10 % 4 → -2	-5 % 4 → -1	0 % 4 → 0	5 % 4 → 1
-9 % 4 → -1	-4 % 4 → 0	1 % 4 → 1	6 % 4 → 2
-8 % 4 → 0	-3 % 4 → -3	2 % 4 → 2	7 % 4 → 3
-7 % 4 → -3	-2 % 4 → -2	3 % 4 → 3	8 % 4 → 0
-6 % 4 → -2	-1 % 4 → -1	4 % 4 → 0	9 % 4 → 1

x % 4

$x < 0 \rightarrow [-3 \quad 0] \longrightarrow (x \% 4 < 0) \rightarrow x \% 4 + 4$

$x >= 0 \rightarrow [0 \quad 3]$ ✓

(python take care of this internally)

A % B → [0 B-1]

((A % B) + B) % B ←

$((x \% 4) + 4) \% 4 \rightarrow [0 \quad 3]$

$(7 \% 4) + 4 = 7 \% 4 = 3$

$3 \% 4 \neq -3 \% 4 \rightarrow 1 \% 4$

$2 \% 5 \neq -2 \% 5 \rightarrow 3 \% 5$

A % B → Range [0 B-1]

(A % B + B) % B

[-(B-1) 0] + B → [1 B] % B → [0 B-1]

[0 B-1] + B → [B 2B-1] % B → [0 B-1]

$a = 10^{12}$ $b = 10^{12}$ $a * b \rightarrow 10^{24}$ $\% 10^9 + 7$ → prime

Properties

1) $(a + b) \% m = (a \% m + b \% m) \% m$

$a = 17$ $b = 8$ $m = 5$

$(17+8) \% 5 = 25 \% 5 = 0$

$17 \% 5 + 8 \% 5 = 2 + 3 = 5$

$(2+3) \% 5 = 0$

2) $(a * b) \% m = (a \% m * b \% m) \% m$

3) $(a - b) \% m = (a \% m - b \% m + m) \% m$

$0 - (m-1) + m$

$m-1 \quad 0 + m \rightarrow 2m-1$

$a = 17$ $b = 8$ $m = 5$

$(17 - 8) \% 5 = 9 \% 5 = 4$ ✓

$17 \% 5 = 2$ $8 \% 5 = 3$

$(2 - 3) \% 5 = -1 + 5 = 4$

4) $(a / b) \% m \longrightarrow$ Advance content

$\frac{20}{3} \rightarrow 3 * \left(\frac{20}{3}\right) + 2$ (6 above the bracket)

$3 * 6.67 + 0$

$q \rightarrow$ decimal number x

$\frac{20}{3} = 6.67$

$6 \rightarrow q$

$2 \rightarrow r$

Q Calculate $\rightarrow a^b \% m$

$a * a * a * a \ldots a$ (b times)

long ans = 1

for i → 1 to b

ans = ans * a → overflow

return ans % m

$a = 2$ $b = 10$ $2^{10} \rightarrow 1024$

$a = 2$ $b = 100$ 2^{100}

```
long ans = 1          a = a%m  → 10^9        m → 10^9
for i → 1 to b
    ans = (ans * a) % m
return ans
```

range of ans → [0 ___ m-1] ✓ ≈ 10^9

$10^9 * 10^9$ → long

$(ans \%. m * a \%. m) \%. m$ ✓

$a^b \%. m = (a \%. m)^b \%. m$ ✓

TC = O(b)

SC = O(1)

a = 3 b = 4 m = 7

ans = 1 (ans * a) %. m

i → 1 → (1 * 3) %. 7 = 3

2 → (3 * 3) %. 7 = 9 %. 7 = 2

3 → (2 * 3) %. 7 = 6

4 → (6 * 3) %. 7 = 18 %. 7 = 4

10:50 PM

$9 \%. 7 = 2$

$9^2 \%. 7 = 81 \%. 7 = 4$

$9^2 \%. 7 \rightarrow 4$ ✓

Q → What is the remainder when N is divided by M. (N > 0)

Ans = N % M

N → = 10^5 digits

325 → 3 digits

N is stored in a array where A[i] → digit of N

N = 3 2 8 7 0 6 5 1 3 8 4 2 3 3 5 4 4 2 2 2 6 0 0 0 1 3 4 5

28 digits

Special cases

M = 1 → N % M = 0

M = 2 → N % 2 → 1 (last digit = {1, 3, 5, 7, 9})

0 (last digit = {2, 4, 6, 8, 0})

M = 3 → N % 3 = (sum of digits) % 3

2481 = 2 * 1000 + 4 * 100 + 8 * 10 + 1

= 2 * (999 + 1) + 4 * (99 + 1) + 8(9 + 1) + 1

$= (2*999 + 4*99 + 8*9) + (2+4+8+1)$

$= (9*K + \text{sum of digits}) \% 3$

$(\text{sum of digits}) \% 3$

$M = 9$

$N \% 9 = (\text{sum of digits}) \% 9$

10^5 digits → 99999 . . . 9 (10^5 times)

$(9+9+9+\_\_+9)$ 10^5 times

$= 9 * 10^5$ → int

$M = 4$ → $N \% 4 =$ last two digits $\% 4$

$N = 86124 = (861*100 + 24) \% 4$

H.W → $M = 8, 5, 6$

Any value of $M = 10^9$

$0 <= A[i] <= 9$

0	1	2	3	4
7	8	9	3	4

$M = 6$

$(7*10^4 + 8*10^3 + 9*10^2 + 3*10^1 + 4*10^0) \% 6$

$((7*10^4) \% 6 + (8*10^3) \% 6 + (9*10^2) \% 6 + (3*10^1) \% 6 + 4*10^0 \% 6) \% 6$

$7*10^4$, $8*10^3$, $9*10^2$, $3*10$, 4

$((7*(10^4 \% 6)) \% 6 + (8*(10^3 \% 6)) \% 6 + (9*(10^2 \% 6)) \% 6 + (3*(10^1 \% 6)) \% 6 + (4*(10^0 \% 6)) \% 6) \% 6$

$A[500]$ → $A[0] * 10^{499} \% M + \ldots$

$10^0 \% 6$ → $10^1 \% 6$ → $10^2 \% 6$ → $10^3 \% 6$ → $10^4 \% 6$

$1 \rightarrow (1*10) \% 6 = 4 \rightarrow (4*10) \% 6 = 4 \rightarrow (4*10) \% 6 = 4$

$M = 7$

$1 \rightarrow (1*10) \% 7 = 3 \rightarrow (3*10) \% 7 = 2 \rightarrow (2*10) \% 7 = 6 \ldots$

10^0 $10^1 \% 7$ $10^2 \% 7$ $10^3 \% 7 \ldots$ $10^{499} \% 7$

```
ans = 0      p = 1
for i → N-1 to 0                 // 10^499          TC = O(N)   SC = O(1)
  → ans = (ans + A[i] * p) % M
     p = (p * 10) % M
return ans
```

```
long p = 1
for i → 1 to 499                              10^499 % M ✓
    ans = (p * a) % m
                ↓
                10
return ans
```