Holsenbeck_S_7

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2017-10-26

Homework 7

Create Table

```
ageIQ <- tibble::tribble(~Age, ~IQ, 23L, 100L, 18L, 105L, 10L, 95L, 45L, 120L)
ageIQ.s <- cbind(ageIQ, Sums = rowSums(ageIQ))
ageIQ.s <- rbind(ageIQ.s, Sums = colSums(ageIQ.s))
summary(ageIQ)
```

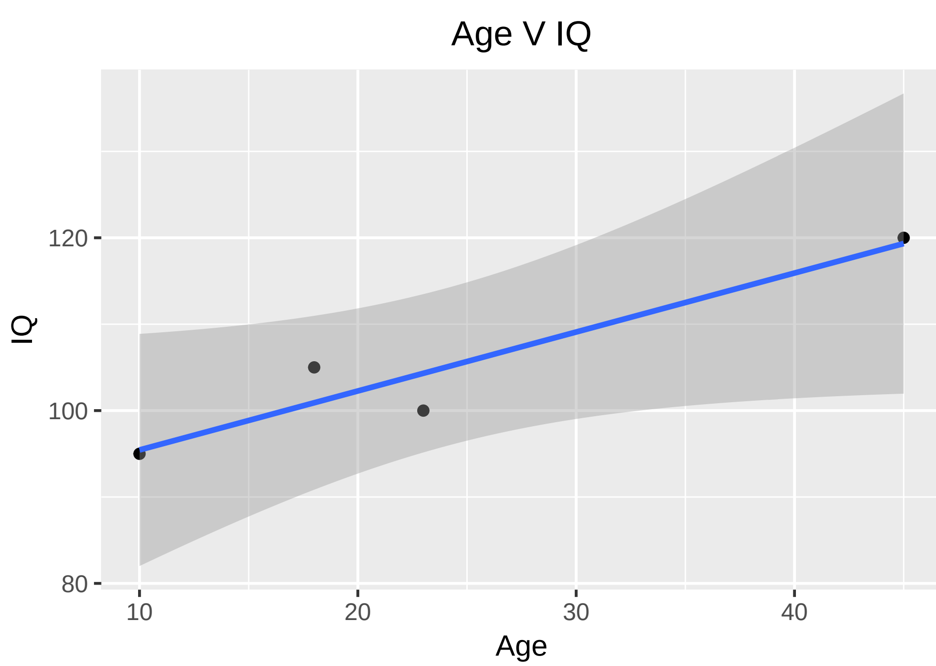
```
##       Age              IQ
##  Min.   :10.0   Min.   : 95.00
##  1st Qu.:16.0   1st Qu.: 98.75
##  Median :20.5   Median :102.50
##  Mean   :24.0   Mean   :105.00
##  3rd Qu.:28.5   3rd Qu.:108.75
##  Max.   :45.0   Max.   :120.00
```

1

Plot these four points using R.

1

```
ggplot(data = ageIQ, mapping = aes(x = Age, y = IQ)) + geom_point() + geom_smooth(method = lm) +
    ggtitle("Age V IQ") + theme(plot.title = element_text(hjust = 0.5))
```



2

Calculate the covariance between age and IQ.

A)

$$\text{Cov}(x,y) = \frac{1}{(n-1)}\sum_i (x_i - \bar{x})(y_i - \bar{y})$$

$$\text{Cov}(x,y) = \frac{1}{(4-1)}\sum_i [(23-24)(100-105) + (18-24)(105-105) + ... + (45-24)(120-105)]$$

$$\text{Cov}(x,y) = 153.\bar{3}$$

2

```r
ageIQ.cov <- ageIQ %>% mutate(S = (Age - mean(ageIQ$Age)) * (IQ - mean(ageIQ$IQ))) %>%
    colSums()
c(ageIQ.cov[3]/3, cov(ageIQ$Age, ageIQ$IQ))
```

```
##        S
## 153.3333 153.3333
```

3

Calculate their correlation. What does the number you get indicate?

A)

$$r = \frac{\text{Cov}(x,y)}{s_x s_y}$$

$$r = \frac{153.\bar{3}}{14.9\bar{8})(10.80123)}$$

$$r = .947$$

The closeness of the r value to positive 1 indicates a strong positive correlation between age & IQ.

3

```r
(r <- 153.33333/(sd(ageIQ$Age) * sd(ageIQ$IQ)))
```

```
## [1] 0.9470957
```

```r
c(r, cor(ageIQ$Age, ageIQ$IQ))
```

```
## [1] 0.9470957 0.9470957
```

4

Calculate the regression coefficients β_0 and β_1 and write out the equation of the best-fit line relating age and IQ.

A)

$$
\begin{aligned}
r = \frac{\text{Cov}(x,y)}{s_x s_y} &= \beta_1 \frac{s_x}{s_y} \\
\frac{r}{\frac{s_x}{s_y}} &= \beta_1 \\
\beta_1 &= \frac{0.9470957}{\frac{14.9\bar{8}}{10.80123}} \\
\beta_1 &= 0.6824926 \\
\beta_0 &= \bar{y} - \beta_1 \bar{x} \\
\beta_0 &= 105 - 0.6824926 * 24 \\
\beta_0 &= 88.62018
\end{aligned}
$$

Line of best fit:

$$
\begin{aligned}
\hat{y}_i &= \beta_0 + \beta_1 x_i \\
\hat{y}_i &= 88.62018 + 0.6824926 x_i
\end{aligned}
$$

```
(b1 <- r/(sd(ageIQ$Age)/sd(ageIQ$IQ)))
```

```
## [1] 0.6824926
```

```
(b0 <- mean(ageIQ$IQ) - b1 * mean(ageIQ$Age))
```

```
## [1] 88.62018
```

5

Calculate the predicted $\hat{y}_i$ for each x_i.

A)

$$
\begin{aligned}
104.3175 &= (88.62018) + (0.6824926)23 \\
100.9050 &= (88.62018) + (0.6824926)18 \\
95.4451 &= (88.62018) + (0.6824926)10 \\
119.3323 &= (88.62018) + (0.6824926)45
\end{aligned}
$$

5

```
(y.i <- ageIQ %>% mutate(`y^i` = b0 + b1 * Age))
```

```
## # A tibble: 4 x 3
##     Age    IQ    `y^i`
##   <int> <int>    <dbl>
## 1    23   100 104.3175
## 2    18   105 100.9050
## 3    10    95  95.4451
## 4    45   120 119.3323
```

6

Calculate R^2 from the TSS/SSE equation. How does it relate to the correlation? What does the number you get indicate?

A)

$$R^2 = \frac{TSS - SSE}{TSS}$$

$$\text{Where } TSS = \sum_i (y_i - \bar{y})^2 \text{ and } SSE = \sum_i (y_i - \hat{y}_i)^2$$

$$TSS = \sum_i (y_i - \bar{y})^2$$

$$TSS = \sum_i (100 - 105)^2 + (105 - 105)^2 + (95 - 105)^2 + (120 - 105)^2$$

$$TSS = 350$$

$$SSE = \sum_i (y_i - \hat{y}_i)^2$$

$$SSE = \sum_i (100 - 104.3175)^2 + (105 - 100.9050)^2 + (95 - 95.4451)^2 + (120 - 119.332)^2$$

$$SSE = 36.05341$$

$$R^2 = \frac{TSS - SSE}{TSS}$$

$$R^2 = \frac{350 - 36.05341}{350}$$

$$R^2 = 0.8969903$$

The R^2 value describes the proportion of the variation in the Y data that is explained by the values for X, in other words, how well the line of best fits predicts the data. The R^2 value is also sometimes called the proportional reduction in error and is described as the proportional reduction of error in the variation in Y that would be explained by the line of best fit. A value of 0.897 suggests that ~89.7% of the variation in Y can be explained by X.

6

```r
(TSS <- ageIQ %>% mutate(tss = ((IQ - mean(ageIQ$IQ))^2)))
```

```
## # A tibble: 4 x 3
##     Age    IQ   tss
##   <int> <int> <dbl>
## 1    23   100    25
## 2    18   105     0
## 3    10    95   100
## 4    45   120   225
```

```r
sum(TSS$tss)
```

```
## [1] 350
```

```r
(SSE <- y.i %>% mutate(sse = (IQ - `y^i`)^2))
```

```
## # A tibble: 4 x 4
##     Age    IQ    `y^i`        sse
##   <int> <int>    <dbl>      <dbl>
## 1    23   100 104.3175 18.6408704
## 2    18   105 100.9050 16.7686597
## 3    10    95  95.4451  0.1981176
## 4    45   120 119.3323  0.4457647
```

```r
sum(SSE$sse)
```

```
## [1] 36.05341
```

```r
(`r^2` <- (sum(TSS$tss) - sum(SSE$sse))/sum(TSS$tss))
```

```
## [1] 0.8969903
```

7

Calculate the standard error of β_1, and use that to test (using the t test) whether β_1 is significant.

A)

$$se_{\hat{y}} = \sqrt{\frac{\sum(y_i - \hat{y}_i)^2}{n-2}}$$
$$se_{\hat{y}} = \sqrt{\frac{SSE}{n-2}}$$
$$se_{\hat{y}} = \sqrt{\frac{36.05341}{4-2}}$$
$$se_{\hat{y}} = 4.245787$$
$$se_{\beta_0} = se_{\hat{y}}\sqrt{\frac{\sum x_i^2}{n\sum(x_i - \bar{x})^2}}$$
$$se_{\beta_0} = 4.245787\sqrt{\frac{2978}{4(674)}}$$
$$se_{\beta_1} = 4.245787\frac{1}{\sqrt{674}}$$
$$se_{\beta_1} = 0.1635416$$

T-Test: $H_0 : \beta_1 = 0$
$H_a : \beta_1 \neq 0$

$$t_{stat} = \frac{\beta_1 - \mu_0}{se_{\beta_1}}$$
$$t_{stat} = \frac{0.6824926 - 0}{0.1635416}$$
$$t_{stat} = 4.173205$$
$$df = n - k - 1$$
$$df = 4 - 1 - 1 = 2$$
$$t_{crit} = 4.302653$$

We fail to reject the null hypothesis at the 95% confidence level with a 2-tailed test. The positive correlation between age and IQ is not statistically significant. The failure to reject the null in this t-test of significance indicates that despite our Pearson coeffecient r and proportional reduction in error R^2 suggesting that the linear regression model matches the data with reasonable accuracy, the positive correlation between age and IQ is not significant, and the positive correlation in this instance may be due to chance.

7

```r
(sey <- sqrt(sum(SSE$sse)/2))
```

```
## [1] 4.245787
```

```r
(se <- ageIQ %>% mutate(`xi^2` = Age^2, `xi-x^2` = (Age - mean(ageIQ$Age))^2))
```

```
## # A tibble: 4 x 4
##     Age    IQ `xi^2` `xi-x^2`
##   <int> <int>  <dbl>    <dbl>
## 1    23   100    529        1
## 2    18   105    324       36
## 3    10    95    100      196
## 4    45   120   2025      441
```

```r
(seb0 <- sey * sqrt(sum(se$`xi^2`)/(4 * sum(se$`xi-x^2`))))
```

```
## [1] 4.462319
```

```r
(seb1 <- sey * (1/sqrt(sum(se$`xi-x^2`))))
```

```
## [1] 0.1635416
```

```r
(ts <- (b1 - 0)/seb1)
```

```
## [1] 4.173205
```

```r
qt(0.975, 2)
```

```
## [1] 4.302653
```

8

Calculate the p-value for β_1 and interpret it.

A) The $p_{value} = 0.05290431$ suggests that if sample data was gathered on age and IQ with $n > 100$, about ~5.2% of those trials would have a mean less than or equal to the one in this trial. Since $\alpha = .05$, we conclude that at the 95% confidence level $\alpha/p = .05$ with a 2-tailed test, the result for this trial is not statistically significant.

8

```r
(pv <- 2 * pt(ts, 2, lower.tail = F))
```

```
## [1] 0.05290431
```

9

Calculate the 95% CI for β_1 and interpret it.

A)

$$\text{CI} = 0.6824926 \pm 4.173205 * 0.1635416 = [1.110223e^{-16}, 1.364985]$$

The 95% confidence interval for β_1 indicates that for each year of age we would expect that IQ would increase between $1.110223e^{-16}$ and 1.364985. One end is very close to 0, suggesting that as each year passes, IQ may not increase hardly at all, and the other end suggesting a gain of up to 1.36 IQ points per year.

9

```r
(ci <- c(b1 + ts * seb1, b1 - ts * seb1))
```

```
## [1] 1.364985e+00 1.110223e-16
```

10

Confirm your results by regressing IQ on Age using R.

A) The results of the lm function match the previous calculations. **10**

```r
summary(lm(IQ ~ Age, data = ageIQ))
```

```
##
## Call:
## lm(formula = IQ ~ Age, data = ageIQ)
##
## Residuals:
##       1       2       3       4
## -4.3175  4.0950 -0.4451  0.6677
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  88.6202     4.4623  19.860  0.00253 **
## Age           0.6825     0.1635   4.173  0.05290 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 4.246 on 2 degrees of freedom
## Multiple R-squared:  0.897,  Adjusted R-squared:  0.8455
## F-statistic: 17.42 on 1 and 2 DF,  p-value: 0.0529
```

11

Plot your points again using R, including the linear fit line with its standard error.

A) The line of best fit and standard error was included in the initial plot.

12

What are your final conclusions about the relationship between age and IQ?

A) $r = .947$ indicating there is a strong positive correlation between age and IQ. $R^2 = 0.897$ indicates that approximately 89.7% of the error has been reduced and is explained by the line of best fit. The t-test at the 95% confidence level yielding a p-value of .053 indicates that despite the strength of the correlation indicated by the r and R^2 values, this correlation is not signficant, may not actually be as positive as the line of best fit in this trial suggests.