optimazation Assignment

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October 2022

Problem Statement **- Suppose the cubic** $P(x) = x^3 - px + q$ **has three distinct real roots where p>0 and q>0.Then which of the following holds?**

Solution

$$P(x) = x^3 - px + q \tag{0.0.1}$$

$P'(x) = 3x^2 - p$
f(x) consists of minima and maxima,
Using gradient ascent method we can find its minima ,

$$x_{n+1} = x_n - \alpha \nabla f(x_n) \tag{0.0.2}$$
$$\implies x_{n+1} = x_n - \alpha \left(3x_n^2 - 1\right) \tag{0.0.3}$$

Using gradient descent method we can find its maxima ,

$$x_{n+1} = x_n + \alpha \nabla f(x_n) \tag{0.0.4}$$
$$\implies x_{n+1} = x_n + \alpha \left(3x_n^2 - 1\right) \tag{0.0.5}$$

Taking $p = 1, q = 1, x_0 = 0.1, \alpha = 0.001$ and precision = 0.00000001, values obtained using python are:

$$\boxed{\text{Minima} = 0.6154} \tag{0.0.6}$$
$$\boxed{\text{Minima Point} = 0.5777} \tag{0.0.7}$$

$$\boxed{\text{Maxima} = 1.3846} \tag{0.0.8}$$
$$\boxed{\text{Maxima Point} = -0.5777} \tag{0.0.9}$$