Optical Surface States : Computational Modelling and Experimental Study of Tamm State

(8th Semester open lab experiment report)

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Abstract

The following experiment concerns itself with the study of Surface States in periodic stratified medium (Distributed Bragg Reflectors). The symmetry between the periodic potential in Condensed matter Physics and its optical analogue in terms of refractive indices is exploited to develop a theoretical framework.

A python program is developed to study the properties of DBR, Tamm plasmon excitation and behaviour of resonant wavelength on varying parameters such as incident angles, for a range of wavelength of incident light(mostly visible range).

Experimentation includes observation of characteristics of DBR, qualitative observation of Tamm state excitation and dependence of Resonant wavelength on incident angle of light for both s and p polarisation. Then the theoretical and experimental observations were compared and the underlying factors of errors were discussed.

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1. Introduction

Study of Surface States offer great experimental opportunity. The dielectric/metal interface is nowadays extensively studied and new experiments are being designed to dwell in the mysteries of Surface states. The symmetry between the periodic potential in Condensed Matter Physics and its optical analogue in terms of alternate layers of different refractive indices can be exploited to develop a theoretical framework in which computational modelling and subsequent experimentation can be done. For this purpose, DBR is used as the Photonic Crystal. DBR or Distributed Bragg Reflector is a periodic arrangement of layers of different refractive indices. This system is analogous to periodic potential in a crystal. Hence, one can use this analogy and obtain similar dispersion relations. The forbidden band of a crystal corresponds to the range of frequencies at which the reflectivity of DBR is very high. Thus, for application purposes, DBRs in which high reflectivity are required. Using Transfer Matrix method, one can program and simulate the experimental results. Thus, the theoretical framework for the same is presented in the following chapter.

1.1 Photonic Crystals

A photonic crystal is a periodic optical nanostructure that affects the motion of photons in much the same way that ionic lattices affect electrons in solids.
Photonic crystals are composed of periodic dielectric, metallo-dielectric—or even superconductor microstructures or nanostructures that affect electromagnetic wave propagation in the same way that the periodic potential in a semiconductor crystal affects electrons by defining allowed and forbidden electronic energy bands. Photonic crystals contain regularly repeating regions of high and low dielectric constant. Photons (behaving as waves) either propagate through this structure or not, depending on their wavelength. Wavelengths that propagate are called modes, and groups of allowed modes form bands.

Photonic crystals can be fabricated for one, two, or three dimensions. One-dimensional photonic crystals can be made of layers deposited or stuck together.Two-dimensional ones can be made by photolithography, or by drilling holes in a suitable substrate. Fabrication methods for three-dimensional ones include drilling under different angles, stacking multiple 2-D layers on top of each other, direct laser writing, or, for example, instigating self-assembly of spheres in a matrix and dissolving the spheres.

The periodic arrangement of atoms in a crystal provides a potential (V(x)) characterized by spatial periodicity

$$V(x+a) = V(x)$$

where a is a constant vector for a given crystal. Corresponding to this translational symmetry, Noether's theorem assures existence of a conserved quantity, known as crystal momentum(k). The eigenstates of Hamiltonian for this potential has a special structure. According to Bloch-Floquet theorem, the solutions are of the form

$$\psi_k(r) = u_k(r)exp(ik.r)$$

where $u_k(r+a) = u_k(r)$. Let's restrict our consideration to one-dimensional situation only. The periodicity in real space, renders the momentum space to be periodic. A period in the momentum space is known as Brillouin Zone. The spectrum of this potential is obtained by solving the Schrödinger equation for each wave vector k , and hence we obtain the band structure.

The dispersion relation ($E(=\hbar\omega)$ and k) the band structure reveals that some energy states

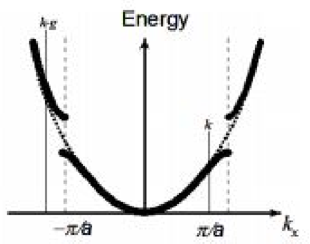


Fig. 1.1

are not allowed,and hence there is a gap between two energy bands known as band gap. When white light is incident on a stratified periodic structure, certain ranges of frequencies are reflected almost completely with reflectivity approaching unity. Due to this high reflectivity, these are also known as distributed Bragg Reflectors(DBR). Like the Schrödinger equation,

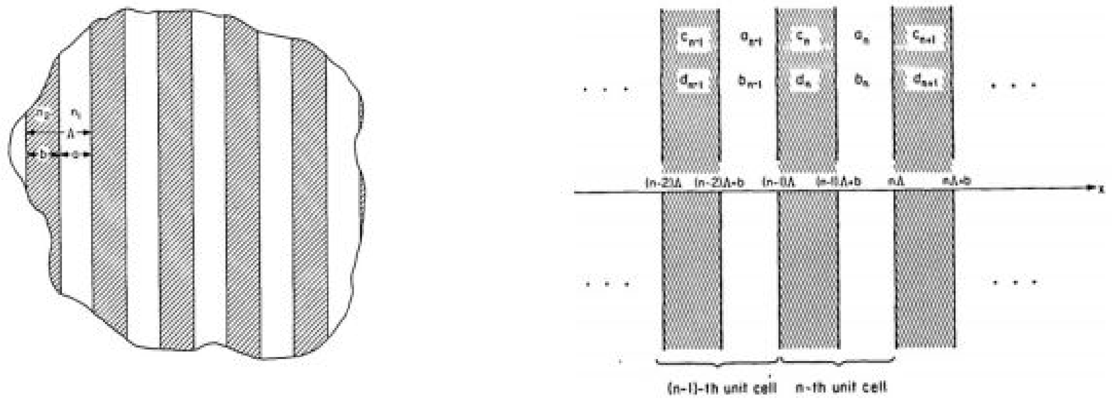


Fig. 1.2

Maxwell's equations are also linear. Systems with no charge or current are known as sourceless. For the sourceless Ampere's and Faraday's law, an eigenvalue equation similar to that of Bloch Schrödinger equation is obtained. Akin to quantum mechanical Bloch States, the Bloch electromagnetic fields can be determined. Bloch -Floquet theorem applies, if there is a periodicity in the refractive index

$$n(x+a) = n(x)$$

The frequency ranges over which the reflectivity approaches unity corresponds to the bandgap of the dispersion relation obtained in this case. The translational symmetry of the refractive index in this situation is the reason of high reflectivity for certain frequency ranges. Abrupt change on this periodicity results in interesting phenomena.

1.2 Surface States

Once the symmetry is broken, the Bloch solution for periodic media gets modified. In particular, if the symmetry breaking occurs due to a layer of metal coated onto the surface of PhC, then a localized electric field of a particular wavelength in the photonic band gap is formed at the metal-PhC interface. These solutions decay exponentially along the normal to the crystal surface, hence they are localized at the surface and are called surface modes. Surface waves are those modes which propagate along the surface, while the ones which cannot, are known as surface states. Abrupt termination of periodic potential results in two kinds of solutions, depending on the kind of material used for periodic layers.

If the material is metallic or semiconducting, then we obtain the first kind of solutions, which are characterized by exponentially decaying continuation of the Bloch waves. On the other hand, if we use periodic dielectric layers act as a medium, we obtain second kind of solution, which has a localized maximum at the interface which decays exponentially on either side of it. In case of metal coated periodic stratified dielectric media, it is the localization of electromagnetic field at the metal-PhC interface which decays exponentially on either its side. This state is characterized by a low reflectivity as well as low transmissivity at the Tamm wavelength. hence this state is non-propagating and is localized at the interface.

1.2.1 Tamm States

Particular surface states called optical Tamm states (OTSs) have attracted a lot of attention due to their huge potential for the fabrication of polariton lasers, optical data storage, and applications in analytical chemistry and medicine. Originally, Tamm states are electronic surface energies that are localized on crystal surfaces that were predicted by Tamm. Analogously, OTSs are surface modes that could be generated by perturbing properly periodic optical impedances at the surface of PhC. Unlike surface plasmon modes that are confined along the interface between a dielectric and a conductor, OTSs have a smaller wave vector in magnitude (parallel to the interface) compared with that of the light wave in vacuum. This enables a direct optical excitation of such modes for both transverse magnetic (TM) and transverse electric (TE) incident polarizations.

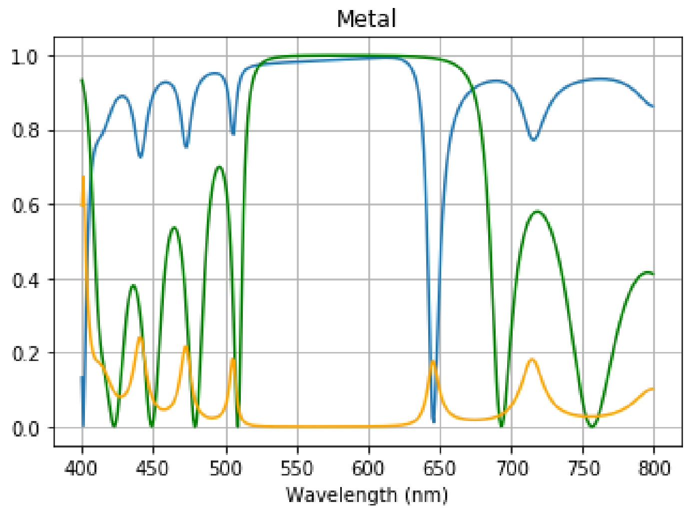


Fig. 1.3

Studying the reflectivity and transmittivity spectra, Tamm states in a metal-PhC can be identified. In the spectra given above, the green plot is that of reflectivity when no metal coating is there. The reflectivity (shown in blue color) increases as expected when metal is coated, but at 636 nm, in the photonic band gap, dip in reflectivity is observed. At this wavelength the transmissivity(in orange colour) is almost vanishing. To add to the confirmation of low values of reflectivity and transmissivity, we note that the absorption plot peaks at this particular wavelength. This allows us to infer that particular state has a localized field distribution inside the metal-PhC system. Thus the state at 636 nm is identified as a Tamm state. Field intensity distribution at this wavelength is localized at the interface of the metal and PhC.

1.3 Conventions and Setup

To determine the exact electromagnetic field we need the boundary conditions along with Maxwell's equations. For electromagnetic waves traveling from one medium to another separated by an infinite plane Maxwell's equationsalong with the boundary conditions reduce to Fresnel equations. Polarisation refers to the orientation of the electromagnetic field with respect to the infinite planar interface.
Consider an infinite planar interface passing through the origin and normal to $\widehat{x}$ We assume uniformity in the yz plane. The material on negative x-axis is of refractive index n_1, while that in the positive x-axis is n_2. The wavevector (k) of light is in the xy-plane, with the x-component of **k** being positive.
So the angle of incidence is θ_i, where

$$cos\theta_i = -\widehat{k}.\widehat{x}$$

The wavevector depends on the refractive index of the medium in which it is propagating. The magnitude of wavevector of light with angular frequency ω when it propagates through a medium of refractive index n is $k = n\omega/c$, where c is the speed of light in vacuum.
Let light be incident on the interface with angle of incidence θ_i. The component of wavevector perpendicular to the interface is $kcos\theta_i$, and that parallel to it is $ksin\theta_i = (n1\omega/c)sin\theta_i$. Snell's law ensures the parallel component being constant as light passes from the first medium to the second one

$$n_1 sin\theta_i = n_2 sin\theta_t$$

The electric field(**E**) and magnetic field(**H**) are perpendicular to the wavevector **k**. If the electromagnetic field is so oriented that the electric field is parallel to the planar interface then it is known as **s** polarisation. In this case, the magnetic field lies in the plane of incidence. On the other hand, if the magnetic field is parallel to the planar interface then it is known as **p** polarisation. In this case, the electric field is in the plane of incidence. The **E**-field component being parallel to the interface, in case of s-polarisation, is continuous across the sourceless media. For the case of p-polarisation, the same is true for **H**-field.
The coefficient of reflection is the ratio of reflected electric field amplitude to the incident field amplitude. Similarly, coefficient of transmission is defined as the ratio between the transmitted field amplitude to that of incident one. For light going from medium 1 to medium 2 we have

$$r_s = \frac{n_1 cos\theta_1 - n_2 cos\theta_2}{n_1 cos\theta_1 + n_2 cos\theta_2}$$
$$t_s = \frac{2n_1 cos\theta_1}{n_1 cos\theta_1 + n_2 cos\theta_2}$$
$$r_p = \frac{n_2 cos\theta_1 - n_1 cos\theta_2}{n_2 cos\theta_1 + n_1 cos\theta_2}$$
$$t_p = \frac{2n_1 cos\theta_1}{n_2 cos\theta_1 + n_1 cos\theta_2}$$

For a general periodic stratified medium, the wave amplitudes of neighbouring layers are related via the coefficients of reflection and transmission. We thus obtain a two-dimensional square matrix relating the amplitudes of electric field in the two medium. For multiple interfaces the wave amplitude on the either side of the layered material is are related by the product of matrices for each interface. Thus we obtain the reflectivity and transmissivity of the stratified medium. This method to compute the reflection and transmission coefficients is known as transfer matrix method. We have used this method in our computation.

Computational Modelling

The theoretical framework for Surface States can be realised using the computational power to get the reflectivity and transmittance of a Distributed Bragg's Reflector (DBR). Hence, a Python Program was developed to simulate the quantities of interest like reflectivity spectra, transmittance spectra and the corresponding phase difference. These quantities can be experimentally measured and thus are of great interest. Using such program, one can tweak different parameters and study their occurrence in actual experiments.

2.1 Working Formulae

Although the general treatment for the phenomena is presented in the theoretical framework section, here, the computational technique used to develop the program is briefly discussed.

2.1.1 Computing the Reflectance and Transmittance Spectrum

For light going from medium 1 to medium 2, the reflection and transmission coefficient are obtained by theory. One can extend this argument for an interface where light can come from both the sides.
Let the amplitude at the interface are given as E_{f1}, E_{f2}, E_{b1} and E_{b2} where following notation convention is used:

- Latin subscript f : Amplitute of forward travelling wave
- Latin subscript, b : Amplitude of backward travelling wave
- Numerical subscript : The medium of propagation

These fields are related by the coefficients of reflection and transmission as :

$$E_{b1} = E_{f1} r_{12} + E_{b2} f_{21}$$

$$E_{f2} = E_{f1} t_{12} + E_{b2} r_{21}$$

where t_{12}, r_{12} are the respective coefficients for going from medium 1 to medium 2.
Now, consider a scenario where N interfaces are present and they are labelled from 0 to N-1. The forward and backward going wave amplitude between two adjacent layer are related by a two dimensional matrix. Let v_n and w_n are the forward and backward going wave amplitudes for the n^{th} interface. They are related to that of $(n-1)^{th}$ layer as

$$\begin{bmatrix} v_n \\ w_n \end{bmatrix} = M_n \begin{bmatrix} v_{n+1} \\ w_{n+1} \end{bmatrix}$$

for n = 1, 2, . . . N-2, where

$$M_n = \begin{bmatrix} e^{-i\delta_n} & 0 \\ 0 & e^{i\delta_n} \end{bmatrix} \begin{bmatrix} 1 & r_{n,n+1} \\ r_{n,n+1} & 1 \end{bmatrix} \frac{1}{t_{n,n+1}}$$

In order to determine the relative amplitudes of reflected and transmitted waves from a DBR, the incident amplitude is taken to be unity, the reflected amplitude would be r and the transmitted amplitude would be t. Thus

$$\begin{bmatrix} 1 \\ r \end{bmatrix} = M \begin{bmatrix} t \\ 0 \end{bmatrix}$$

where $M = \frac{1}{t_{0,1}} \begin{bmatrix} 1 & r_{0,1} \\ r_{0,1} & 1 \end{bmatrix} M_1 M_2 .... M_{N-1}$
Finally, the coefficient of reflectivity and transmittivity, (r and t) are obtained in terms of the matrix elements:

$$t = \frac{1}{M_{00}}$$

$$r = \frac{M_{10}}{M_{00}}$$

Taking the square of magnitude of r and t gives reflectivity (R) and transmittivity (T). This formalism is governed by the fact that for a given wavelength and incident angle, coefficient of reflection and transmission are unique.

2.1.2 Debugging unphysical cases for complex Refractive Indices

Whenever the Refractive index takes complex values, the incident angle for the corresponding medium also becomes complex. Due to this, the reflectivity comes to be greater than unity. Snell's law requires :

$$n_0 sin\theta_0 = n_i sin\theta_i$$

The value of $nsin\theta$ is real for every layer. Hence,

$$\theta = sin^{-}\left(\frac{n_i}{n_0} sin\theta_i\right)$$

Thus, θ as well as $\pi - \theta$ solves the equation. Hence, choice of theta is crucial and is efficiently handled by the program.

2.1.3 Metal Coating

The refractive index of metal is given by the Lorentz-Drude Model. For Gold, the relevant parameters are collision wavelength (8935.20 nm) and Plasma wavelength (168.26 nm). Using these, one can calculate the refractive index as

$$n^2 = 1 - \frac{\omega_p^2}{\omega^2 - i\omega_c\omega}$$

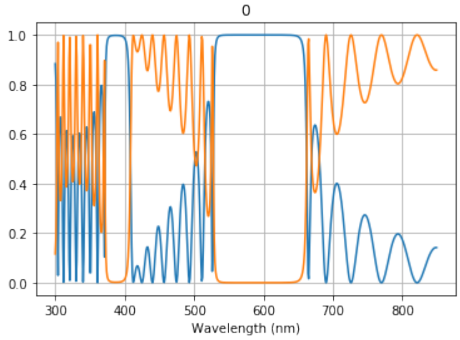
where ω,ω_p and ω_c are the angular frequency of incident light, plasma and collision angular frequency respectively.

2.2 Computational results

Different computational results were obtained using the python program by changing angle of incidence for normal DBR Spectra and metal coated DBR for resonance wavelength.

2.2.1 DBR Spectrum

Normal DBR (without metal), 8 bilayers, n_1 =2.869 , n_2 =1.465 , d_1 =150nm and d_2 =108nm . Plots of Reflectance(**R**) and Transmittance(**T**) vs Wavelength are shown for different angles.



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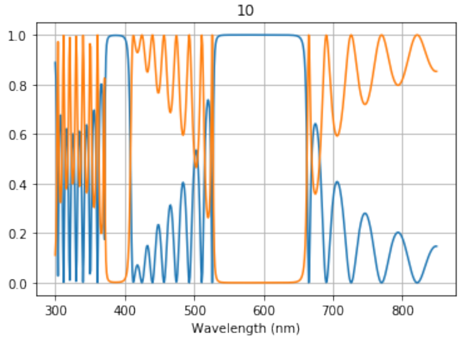
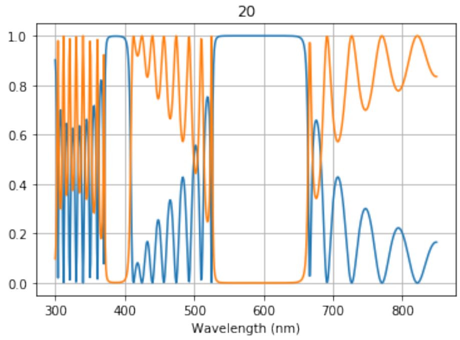
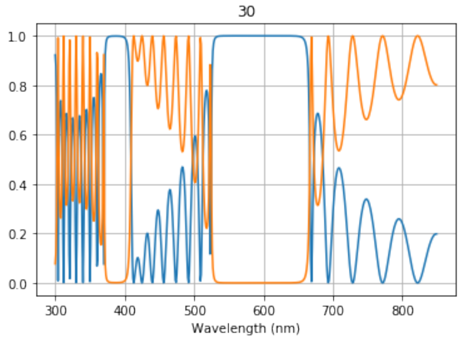


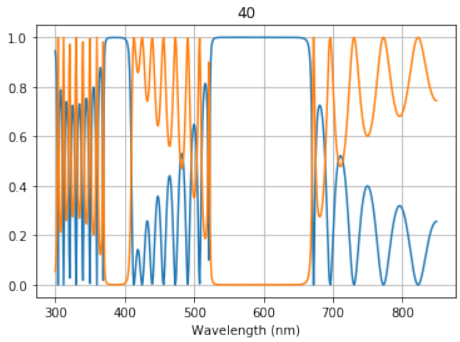
(b) 10 degree



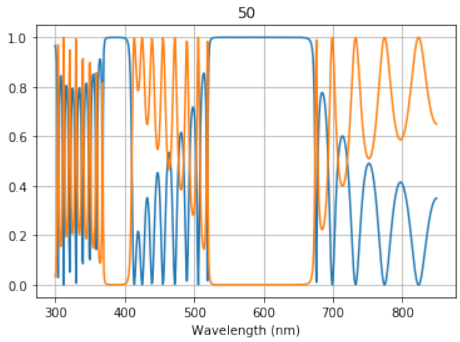
(c) 20 degree



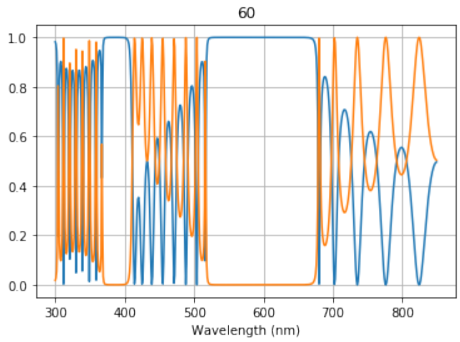
(d) 30 degree



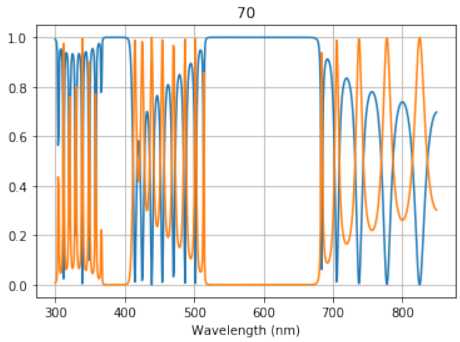
(e) 40 degree



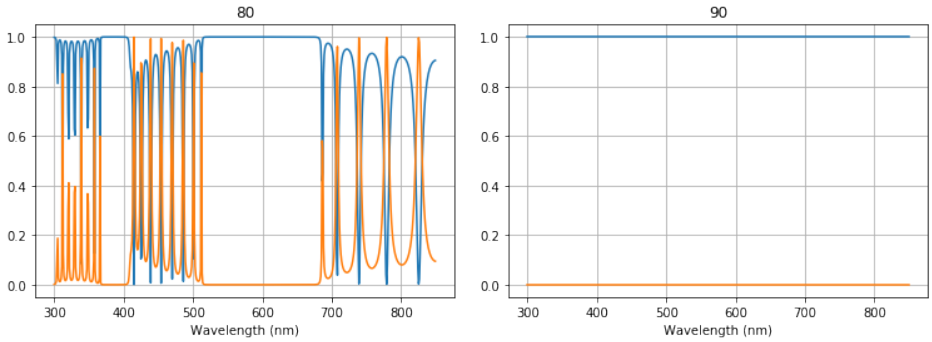
(f) 50 degree



(g) 60 degree



(h) 70 degree



(a) 80 degree (b) 90 degree

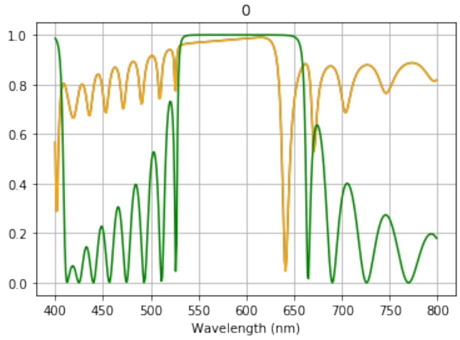
Fig. 2.2 R and T vs Wavelength for DBR

2.2.2 Metal coated DBR spectrum for s and p polarised light

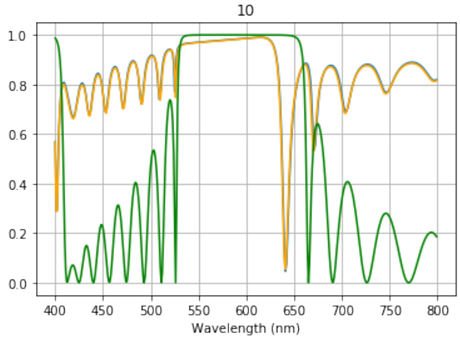
Now, when DBR is coated with a metal film, we get a dip in the reflectivity because of formation of Optical Tamm State at the interface of metal and dielectric media. The following simulation result is for the orientation in which light falls on the metal surface.

A resonant wavelength is observed where there is maximum absorption i.e. reflectivity spectrum exhibits a significant dip and corresponding transmission is not seen. Such optical state of the surface are tamm states which occur due to a phase matching criteria. Here, using computation, the criteria for observing Tamm states is studied.

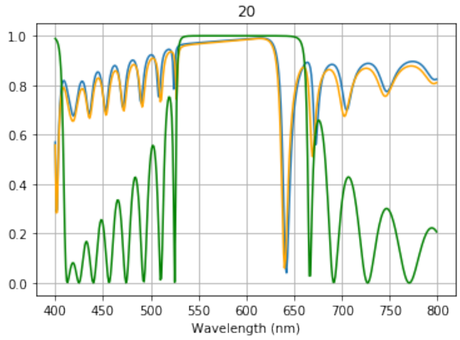
Metal thickness is d_m = 35nm, 8 bilayers, n_1 =2.869 , n_2 =1.465 , d_1 =150nm and d_2 =108nm . Plots of Reflectance(**R**) and Transmittance(**T**) vs Wavelength are shown for different angles.



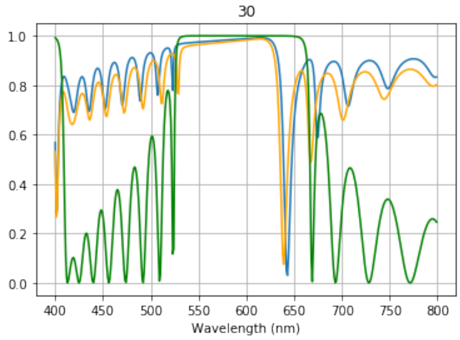
(a) 0 degree



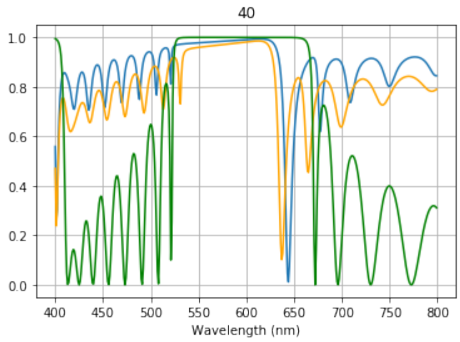
(b) 10 degree



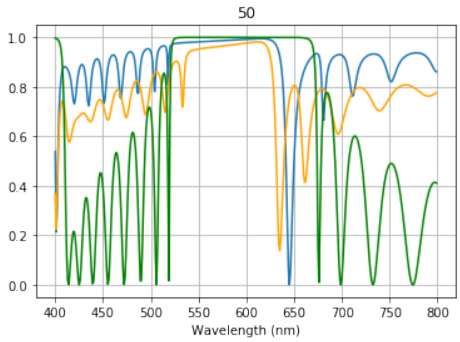
(c) 20 degree



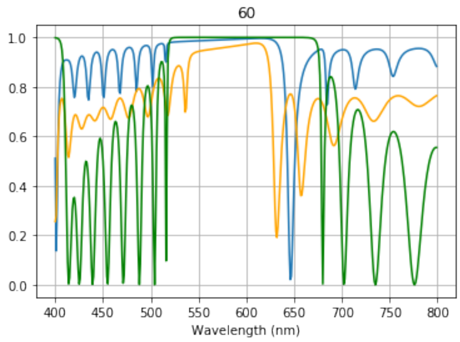
(d) 30 degree



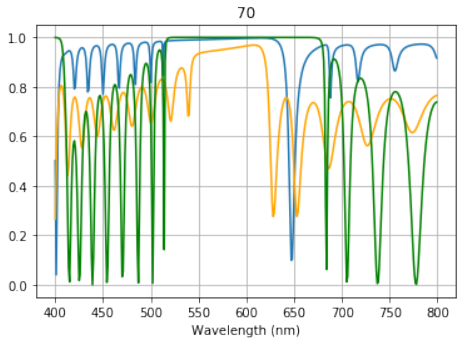
(e) 40 degree



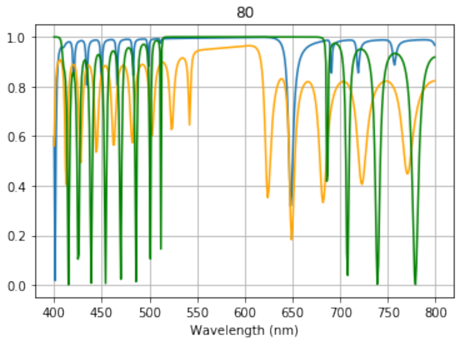
(f) 50 degree



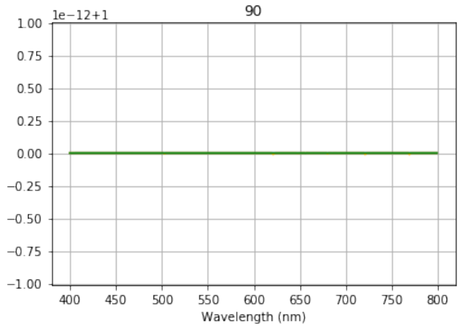
(g) 60 degree



(h) 70 degree



(a) 80 degree



(b) 90 degree

Fig. 2.4 R and T vs Wavelength for metal coated DBR

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The p polarised light is shown in orange and s polarised is in blue, green is the normal DBR spectrum.

Experimentation

The following set of experiments are to study and verify the concepts of developed theory in practice. First, a given DBR is characterised using the DBR Spectra Program by tweaking the parameters and determining the best fit. Then, qualitative observaton of Tamm States is done using a metal coated DBR. Finally, the dependence of incident angle on the resonant wavelength for s and p polarisation is studied.

3.1 Experimental Apparatus component

The experiment consisted of the following apparatus:

- White Light Source
- Collimating Convex Lens
- Aperture
- Polarising Beam Splitter
- Uncoated and Coated DBR (with stand of rotatable base)
- Spectrometer
- Stands and Breadboard
- Optical Fiber Cables

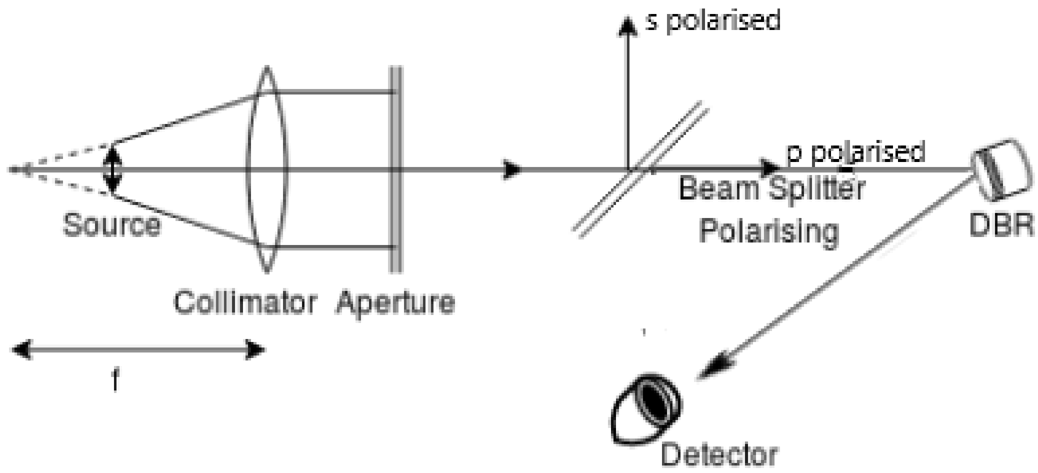


Fig. 3.1 Schematic Representation of Setup

3.1.1 General Procedure

The source of the white light is NOT a point source but gives a diverging beam. Hence, the virtual point source of this beam is taken to be at a distance of focal length of a collimating convex lens to get a parallel beam of light whose radius is decreased (homogenous intensity) using an aperture. Now this thin beam of white light is passed through a beam splitter.
Now, this light is passed through a Polarising beam splitter, which splits the original beam in two beams which are s polarised and p polarised. Then DBR is mounted on the rotation stage and placed in path of either s or p polarised light.
Then the reflected light from DBR is made to fall on the detector in which light travels through the optical cable to spectrometer which is connected to the laptop, and a plot is obtained in BWSpec software.
Then readings are taken for different angles and the data is saved and plotted.

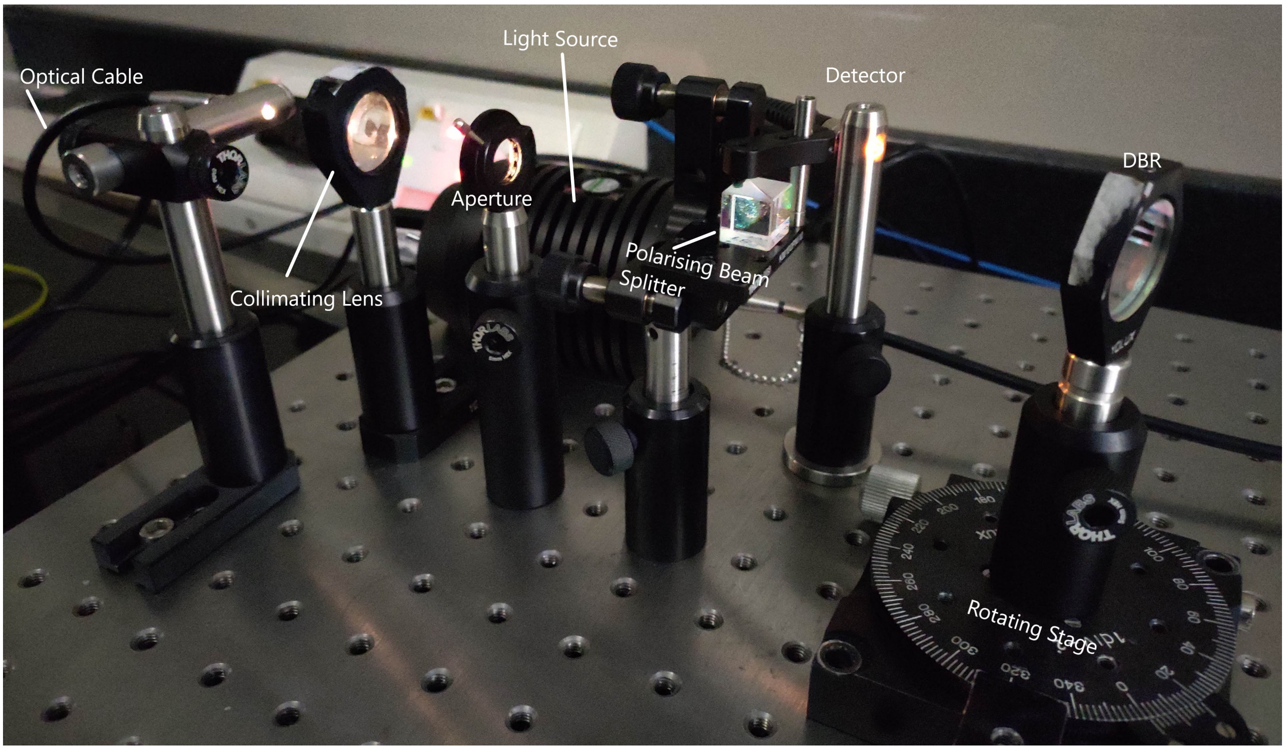


Fig. 3.2 Experimental Setup

3.2 Normal DBR Spectra

AIM : To obtain uncoated DBR spectra and compare the result with the simulated program.

3.2.1 Observations and Results

Here the beam splitter wasn't used, and the general procedure was followed for normal incidence to obtain the following data points for the reflectivity. The normalisation is done when comparison with simulated data is done so as to get the best fit such that band gap's reflectivity is unity.

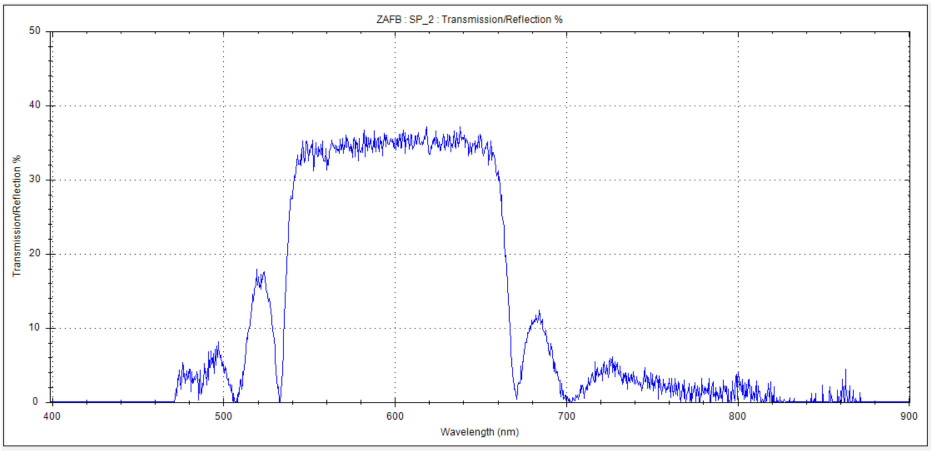


Fig. 3.3 Experimental graph

3.2.2 Analysis

By characterising a DBR, one means to determine the No. of Bilayers, the thickness and refractive index of both the constituent layers. First, the no of bilayers is fixed then tweaking the other parameters, the best fit parameters were determined. For N=7 the program gave the following result:

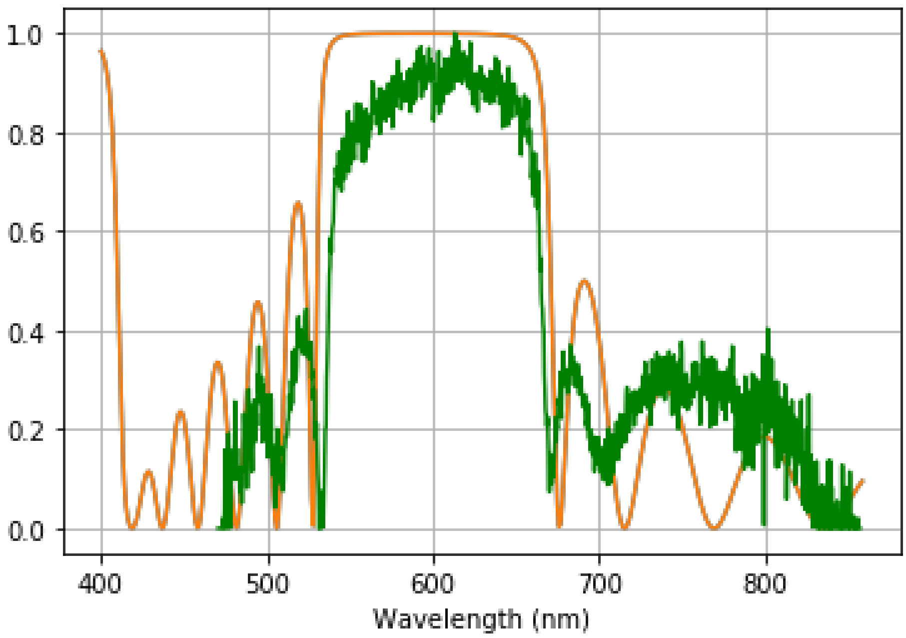


Fig. 3.4 Best fit simulated graph

3.3 Dependence of Resonant wavelength on incident angles and polarisation

AIM : To study the dependence of Resonant Wavelength on Incident Angles for both the polarisation.

3.3.1 Observation an Results

The general procedure is followed but here, the metal-coated DBR (30 nm Au) is placed on a rotating base and the Polarising Beam Splitter is used. The transmitted light from the Beam-splitter is p-polarised while the reflected light is s-polarised. Thus the spectrometer and the DBR are placed accordingly.

p polarisation

Data was taken for p polarised light at different wavelength and as expected, the resonance wavelength is moving towrards left, i.e. blue shift is happening.

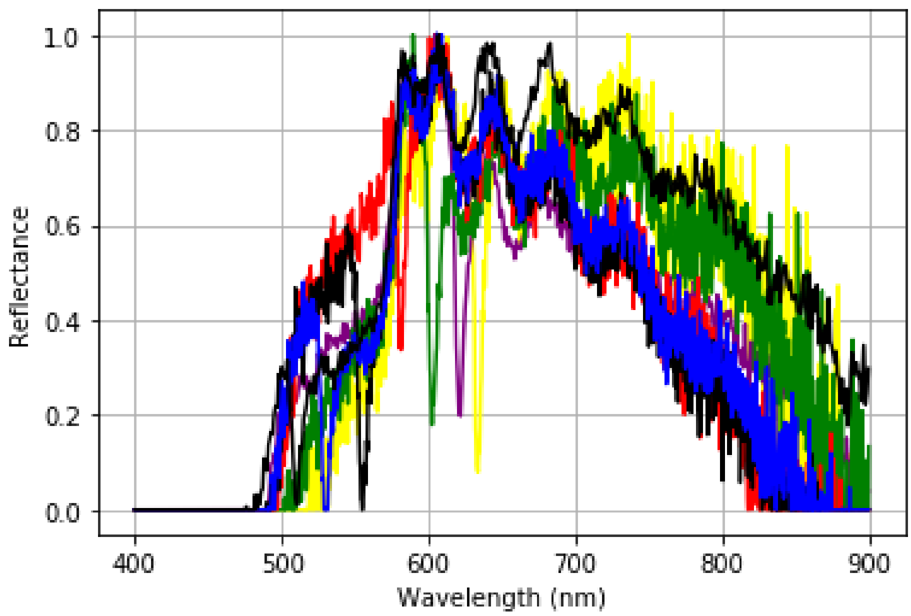
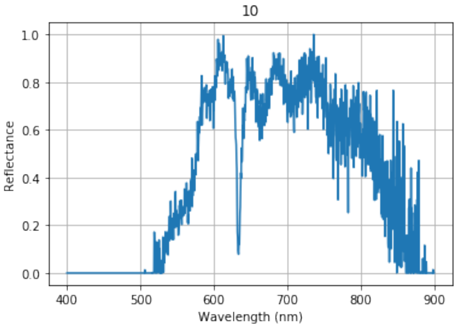
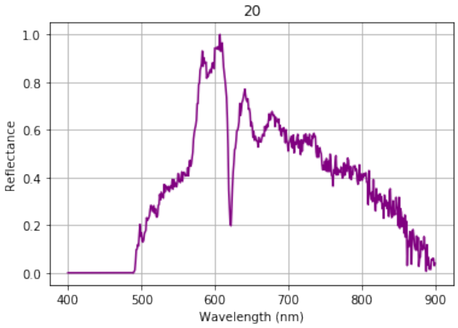


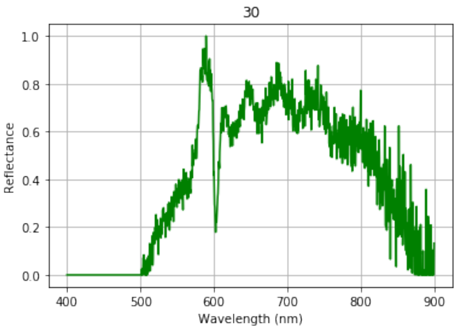
Fig. 3.5 Experimental Graphs for resonant wavelength at different angles



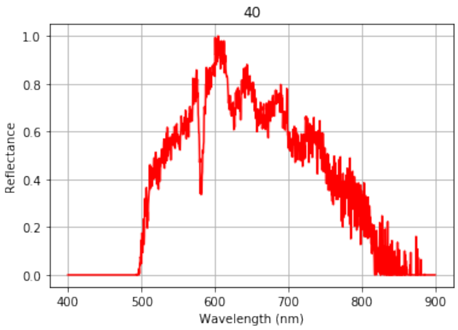
(a) 10 degree



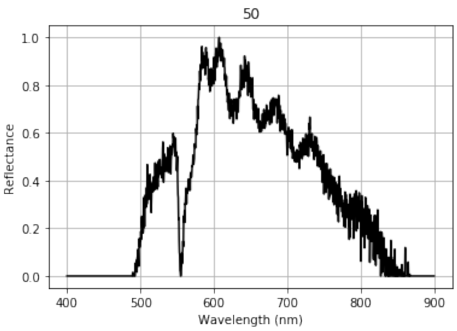
(b) 20 degree



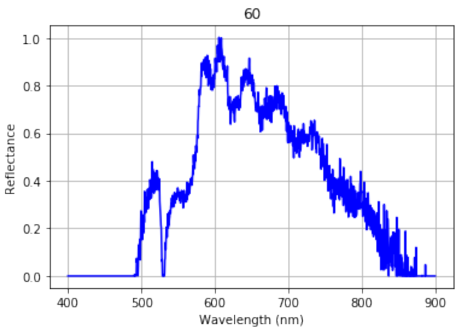
(c) 30 degree



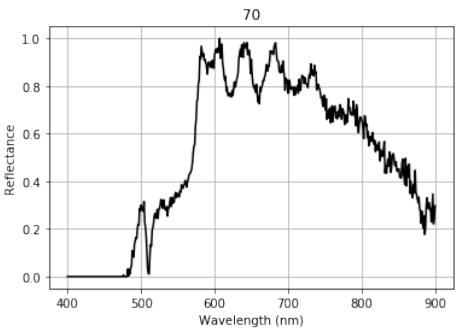
(d) 40 degree



(e) 50 degree



(f) 60 degree

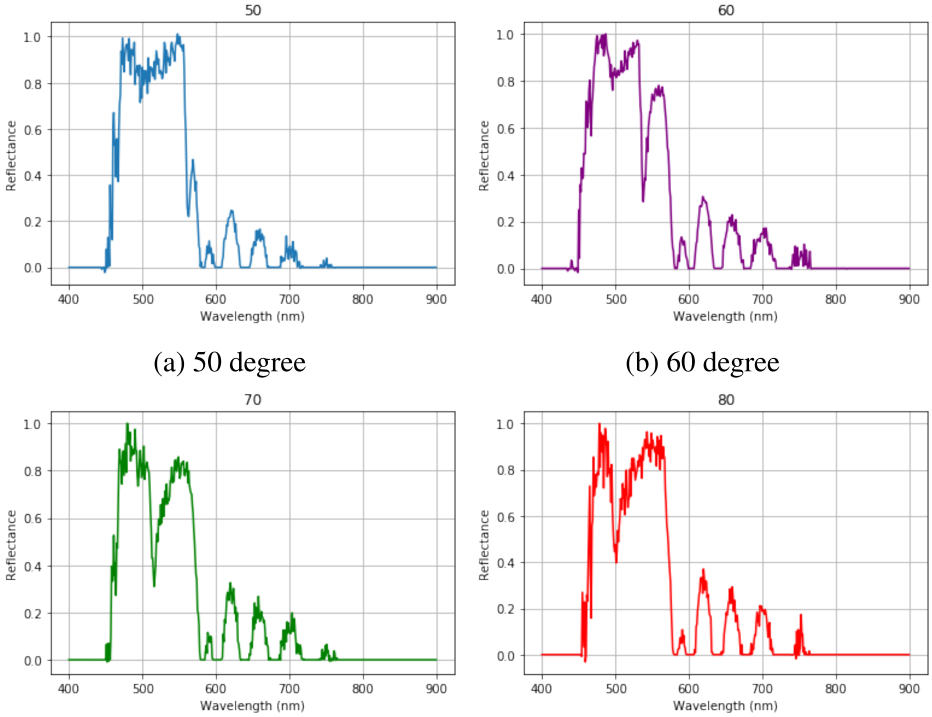


(g) 70 degree

Fig. 3.6 R and T vs Wavelength (Experimental) for metal coated DBR for different angles for p polarisation

s polarisation

Data was taken for s polarised light at different wavelength and as expected, the resonance wavelength is moving towrards left, i.e. blue shift is happening.



(a) 50 degree
(b) 60 degree
(c) 70 degree
(d) 80 degree

Fig. 3.7 R and T vs Wavelength (Experimental) for metal coated DBR for different angles for s polarisation

.

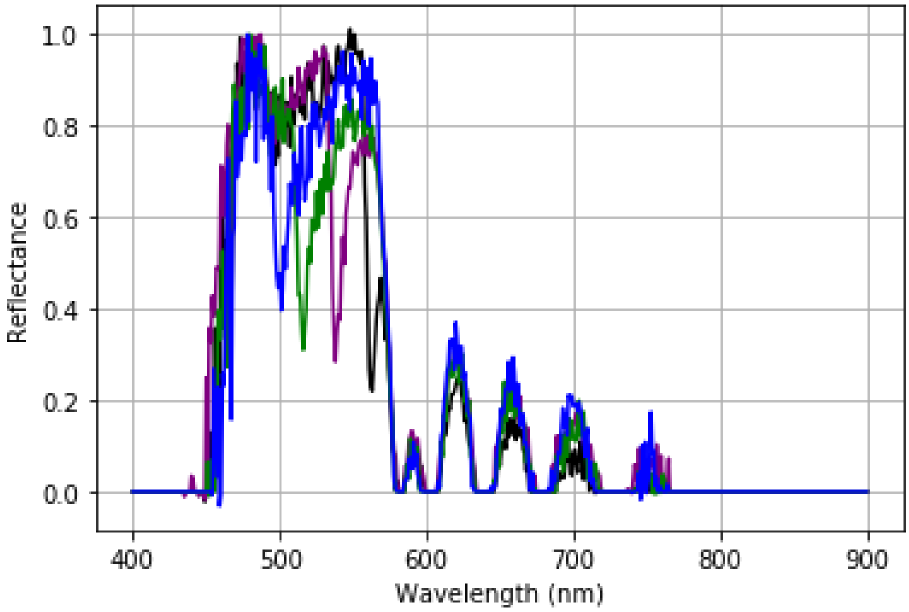


Fig. 3.8 Experimental Graphs for resonant wavelength at different angles for s polarisation

3.3.2 Tamm wavelength variation for s and p polarisation

The obtained data for the two are presented as below.

Incident Angle (Degrees)	p-polarisation	s-polarisation
10	633	
20	622	
30	603	
40	582	
50	556	563
60	530	538
70	510	516
80		501

Conclusion

Analogy between a system of periodic potential in Condensed Matter Physics and photonic band gap in Optics is studied extensively to build a theoretical framework for Surface states observed in Periodic arrangement of dielectrics of different refractive indices. This analogy opens a new window to study the Tamm Plasmons and design corresponding experiment and develop computational modelling which further can help us understand the physics of the interface states better.

Specifically, the dependence of Angle of Incidence on the Tamm resonant wavelength is studied for both s (TE Mode) and p-Polarisation (TM Mode). The comparisons were also studied. Furthermore, experimental analysis were done and characterisation of a given DBR was carried out using the python program. The major work included is the qualitative observation and confirmation of existence of Tamm state in a given 30-nm Au-coated DBR. The incident angle dependence on resonant wavelength were observed and it gave outstanding coherence with the computational results.

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Normal DBR Angle Variation

```
import numpy as np

import matplotlib.pyplot as plt

import math as math

import time

import cmath


a = 150*(10**(-9))

b = 108*(10**(-9))

i = (-1)**(0.5)

n1 = 2.869

n2 = 1.465

n = 8

c = 3*(10**(8))

w1 = 400*(10**(-9))

x = 0

h = np.arange(0, 10, 1)

for o in h:

    theta = np.pi - ((np.pi)/18)*o

    theta1 = np.arcsin(np.sin(theta)/n1)

    theta2 = np.arcsin(np.sin(theta)/n2)

    g = o*10

    #from n1 to n2

    r12 = (n1*np.cos(theta1) - n2*np.cos(theta2))/(n1*np.cos(theta1) + n2*np.cos(theta2))

    t12 = (2*n1*np.cos(theta1))/(n1*np.cos(theta1) + n2*np.cos(theta2))


    #from n2 to n1

    r21 = (n2*np.cos(theta2) - n1*np.cos(theta1))/(n1*np.cos(theta1) + n2*np.cos(theta2))

    t21 = (2*n2*np.cos(theta2))/(n1*np.cos(theta1) + n2*np.cos(theta2))


    #from air to n1

    r01 = (1*np.cos(theta) - n1*np.cos(theta1))/(1*np.cos(theta) + n1*np.cos(theta1))

    t01 = (2*np.cos(theta))/(1*np.cos(theta) + n1*np.cos(theta1))


    #from n2 to air
```

```
r20 = (n2*np.cos(theta2)-1*np.cos(theta))/(n2*np.cos(theta2)+1*np.cos(theta))

t20 = (2*n2*np.cos(theta2))/(1*np.cos(theta)+n2*np.cos(theta2))


#print (r12, t12, r21, t21, r01, t01)

d1 = a

d2 = b

Q12 = [[(1/t12)*1, (1/t12)*r12],

    [(1/t12)*r12, (1/t12)*1]]

Q21 = [[(1/t21)*1, (1/t21)*r21],

    [(1/t21)*r21, (1/t21)*1]]

Q01 = [[(1/t01)*1, (1/t01)*r01],

    [(1/t01)*r01, (1/t01)*1]]

Q20 = [[(1/t20)*1, (1/t20)*r20],

    [(1/t20)*r20, (1/t20)*1]]


#For p polarised

#from n1 to n2

r12p = (n2*np.cos(theta1) - n1*np.cos(theta2))/(n2*np.cos(theta1) + n1*np.cos(theta2))

t12p = (2*n1*np.cos(theta1))/(n2*np.cos(theta1) + n1*np.cos(theta2))


#from n2 to n1

r21p = (n1*np.cos(theta2) - n2*np.cos(theta1))/(n2*np.cos(theta1) + n1*np.cos(theta2))

t21p = (2*n2*np.cos(theta2))/(n2*np.cos(theta1) + n1*np.cos(theta2))


#from air to n1

r01p = (n1*np.cos(theta) - 1*np.cos(theta1))/(n1*np.cos(theta) + 1*np.cos(theta1))

t01p = (2*np.cos(theta))/(n1*np.cos(theta) + 1*np.cos(theta1))


#from n2 to air

r20p = ((1*np.cos(theta2))-n2*np.cos(theta))/((1*np.cos(theta2))+(n2*np.cos(theta)))

t20p = (2*n2*np.cos(theta2))/((n2*np.cos(theta))+(1*np.cos(theta2)))


Q12p = [[(1/t12p)*1, (1/t12p)*r12p],

      [(1/t12p)*r12p, (1/t12p)*1]]


Q21p = [[(1/t21p)*1, (1/t21p)*r21p],
```

```
        [(1/t21p)*r21p, (1/t21p)*1]]


Q01p = [[(1/t01p)*1, (1/t01p)*r01p],
        [(1/t01p)*r01p, (1/t01p)*1]]


Q20p = [[(1/t20p)*1, (1/t20p)*r20p],
        [(1/t20p)*r20p, (1/t20p)*1]]


w = np.arange(300*(10**(-9)), 850*(10**(-9)), 1*(10**(-9)))
lamlist, R1list, Tlist, Rplist = [], [], [], []


#DBR
for lam in w:
  delta1 = (2*np.pi*n1*d1)/lam
  delta2 = (2*np.pi*n2*d2)/lam


  E1 = [[np.exp(-i*delta1), 0],
       [0, np.exp(i*delta1)]]
  E2 = [[np.exp(-i*delta2), 0],
       [0, np.exp(i*delta2)]]
  Ma = np.matmul(E1, Q12)
  Mb = np.matmul(E2, Q21)
  M1 = np.matmul(Ma, Mb)
  M2 = np.linalg.matrix_power(M1, n)
  F1 = np.matmul(Ma, E2)
  F2 = np.matmul(F1, Q20)
  F3 = np.matmul(M2, F2)
  F4 = np.matmul(Q01, F3) #Final matrix M(as in report)



  Map = np.matmul(E1, Q12p)
  Mbp = np.matmul(E2, Q21p)
  M1p = np.matmul(Map, Mbp)
  M2p = np.linalg.matrix_power(M1p, n)
  F1p = np.matmul(Map, E2)
  F2p = np.matmul(F1p, Q20p)
```

```
    F3p = np.matmul(M2p, F2p)
    F4p = np.matmul(Q01p, F3p) #Final matrix M(as in report)

    r = (F4[1][0])/(F4[0][0])
    rp = (F4p[1][0])/(F4p[0][0])
    t = (1/(F4[0][0]))
    R = abs((r)**2)
    Rp = abs((rp)**2)
    T = abs((t)**2)
    R1list.append(R)
    Rplist.append(Rp)
    lamlist.append(lam*(10**(9)))
    Tlist.append(T)

plt.figure(o)
p1 = plt.plot(lamlist, R1list)
#p2 = plt.plot(lamlist, Rplist)
p3 = plt.plot(lamlist, Tlist)
title = plt.title(g)
xlegend = plt.xlabel('Wavelength (nm)')
ylegend = plt.ylabel('')
plt.grid()
```

Metal Surface DBR, Tamm Wavelength Angle Variation

```
import numpy as np
import matplotlib.pyplot as plt
import math as math
import time
import cmath
d2 = 108*(10**(-9))
d1 = 150*(10**(-9))
dm = 35*(10**(-9))
n2 = 1.465
n1= 2.869
n = 8
c = 3*(10**8)
i = (-1)**(0.5)
lamb_c = 8934.2*(10**(-9))
lamb_p = 168.26*(10**(-9))
w = np.arange(400*(10**(-9)), 800*(10**(-9)), 1*(10**(-9)))
h = np.arange(0, 10, 1)
for o in h:
    theta =((np.pi/18)*o)
    g = o*10
    lamlist, Rlist, Tlist, r1list, Rplist, Tplist = [], [], [], [], [], []
    for lam in w:
        nm = ((1-((1/lamb_p**2)/((1/lam**2)-((1j)/(lam*lamb_c)))))**(1/2))
        thetam = np.arcsin(np.sin(theta)/((nm)))
        theta1 = np.arcsin(np.sin(theta)/n1)
        theta2 = np.arcsin(np.sin(theta)/n2)



        #For s polarised
        #from n1 to n2
        r12 = (n1*np.cos(theta1) - n2*np.cos(theta2))/(n1*np.cos(theta1) + n2*np.cos(theta2))
        t12 = (2*n1*np.cos(theta1))/(n1*np.cos(theta1) + n2*np.cos(theta2))

        #from n2 to n1
```

```
r21 = (n2*np.cos(theta2) - n1*np.cos(theta1))/(n1*np.cos(theta1) + n2*np.cos(theta2))

t21 = (2*n2*np.cos(theta2))/(n1*np.cos(theta1) + n2*np.cos(theta2))


#from air to n1

r01 = (1*np.cos(theta) - n1*np.cos(theta1))/(1*np.cos(theta) + n1*np.cos(theta1))

t01 = (2*np.cos(theta))/(1*np.cos(theta) + n1*np.cos(theta1))


#from air to nm

r0m = (1*np.cos(theta) - nm*np.cos(thetam))/(1*np.cos(theta) + nm*np.cos(thetam))

t0m = (2*np.cos(theta))/(1*np.cos(theta) + nm*np.cos(thetam))


#from nm to n1

rm1 = (nm*np.cos(thetam) - n1*np.cos(theta1))/(nm*np.cos(thetam) + n1*np.cos(theta1))

tm1 = (2*nm*np.cos(thetam))/(nm*np.cos(thetam) + n1*np.cos(theta1))


#from n2 to air

r20 = ((n2*np.cos(theta2))-1*np.cos(theta))/((n2*np.cos(theta2))+(1*np.cos(theta)))

t20 = (2*n2*np.cos(theta2))/((1*np.cos(theta))+(n2*np.cos(theta2)))


Q12 = [[(1/t12)*1, (1/t12)*r12],

    [(1/t12)*r12, (1/t12)*1]]


Q21 = [[(1/t21)*1, (1/t21)*r21],

    [(1/t21)*r21, (1/t21)*1]]


Q01 = [[(1/t01)*1, (1/t01)*r01],

    [(1/t01)*r01, (1/t01)*1]]


Qm1 = [[(1/tm1)*1, (1/tm1)*rm1],

    [(1/tm1)*rm1, (1/tm1)*1]]


Q0m = [[(1/t0m)*1, (1/t0m)*r0m],

    [(1/t0m)*r0m, (1/t0m)*1]]


Q20 = [[(1/t20)*1, (1/t20)*r20],

    [(1/t20)*r20, (1/t20)*1]]
```

```
#For p polarised
#from n1 to n2
r12p = (-n2*np.cos(theta1) + n1*np.cos(theta2))/(n2*np.cos(theta1) + n1*np.cos(theta2))
t12p = (2*n1*np.cos(theta1))/(n2*np.cos(theta1) + n1*np.cos(theta2))

#from n2 to n1
r21p = (-n1*np.cos(theta2) + n2*np.cos(theta1))/(n2*np.cos(theta1) + n1*np.cos(theta2))
t21p = (2*n2*np.cos(theta2))/(n2*np.cos(theta1) + n1*np.cos(theta2))

#from air to n1
r01p = (-n1*np.cos(theta) + 1*np.cos(theta1))/(n1*np.cos(theta) + 1*np.cos(theta1))
t01p = (2*np.cos(theta))/(n1*np.cos(theta) + 1*np.cos(theta1))

#from air to nm
r0mp = (-nm*np.cos(theta) + 1*np.cos(thetam))/(nm*np.cos(theta) + 1*np.cos(thetam))
t0mp = (2*np.cos(theta))/(nm*np.cos(theta) + 1*np.cos(thetam))

#from nm to n1
rm1p = (-n1*np.cos(thetam) + nm*np.cos(theta1))/(n1*np.cos(thetam) + nm*np.cos(theta1))
tm1p = (2*nm*np.cos(thetam))/(n1*np.cos(thetam) + nm*np.cos(theta1))

#from n2 to air
r20p = ((-1*np.cos(theta2))+n2*np.cos(theta))/((1*np.cos(theta2))+(n2*np.cos(theta)))
t20p = (2*n2*np.cos(theta2))/((n2*np.cos(theta))+(1*np.cos(theta2)))

Q12p = [[(1/t12p)*1, (1/t12p)*r12p],
    [(1/t12p)*r12p, (1/t12p)*1]]

Q21p = [[(1/t21p)*1, (1/t21p)*r21p],
    [(1/t21p)*r21p, (1/t21p)*1]]

Q01p = [[(1/t01p)*1, (1/t01p)*r01p],
    [(1/t01p)*r01p, (1/t01p)*1]]
```

```
Qm1p = [[(1/tm1p)*1, (1/tm1p)*rm1p],
    [(1/tm1p)*rm1p, (1/tm1p)*1]]

Q0mp = [[(1/t0mp)*1, (1/t0mp)*r0mp],
    [(1/t0mp)*r0mp, (1/t0mp)*1]]

Q20p = [[(1/t20p)*1, (1/t20p)*r20p],
    [(1/t20p)*r20p, (1/t20p)*1]]


delta1 = (2*np.pi*n1*d1)/lam
delta2 = (2*np.pi*n2*d2)/lam
deltam = (2*np.pi*(nm)*dm)/lam

E1 = [[np.exp((1j)*delta1), 0],
    [0, np.exp(-(1j)*delta1)]]
E2 = [[np.exp((1j)*delta2), 0],
    [0, np.exp(-(1j)*delta2)]]
Em = [[np.exp((1j)*deltam), 0],
    [0, np.exp(-(1j)*deltam)]]


#For s polarised
Ma = np.matmul(E1, Q12)
Mb = np.matmul(E2, Q21)
M1 = np.matmul(Ma, Mb)
M2 = np.linalg.matrix_power(M1, n)
M3 = np.matmul(M2, E1)
M4 = np.matmul(M3, Q12)
M5 = np.matmul(M4, E2)
M6 = np.matmul(M5, Q20)
F1 = np.matmul(Qm1, M6)
F2 = np.matmul(Em, F1)
F3 = np.matmul(Q0m, F2)
F4 = np.matmul(Q01, M6)
```

```
#For p polarised
Map = np.matmul(E1, Q12p)
Mbp = np.matmul(E2, Q21p)
M1p = np.matmul(Map, Mbp)
M2p = np.linalg.matrix_power(M1p, n)
M3p = np.matmul(M2p, E1)
M4p = np.matmul(M3p, Q12p)
M5p = np.matmul(M4p, E2)
M6p = np.matmul(M5p, Q20p)
F1p = np.matmul(Qm1p, M6p)
F2p = np.matmul(Em, F1p)
F3p = np.matmul(Q0mp, F2p)
F4p = np.matmul(Q01p, M6p)



r = (F3[1][0])/(F3[0][0])
t = (1/(F3[0][0]))

r_p = (F3p[1][0])/(F3p[0][0])
t_p = (1/(F3p[0][0]))

r1 = (F4[1][0])/(F4[0][0])
Rm1 = abs((r1)**2)

R = abs((r)**2)
T = abs((t)**2)

R_p = abs((r_p)**2)
T_p = abs((t_p)**2)

Rlist.append(R)
lamlist.append(lam*(10**9))
Tlist.append(T)

Rplist.append(R_p)
```

```
    Tplist.append(T_p)


    r1list.append(Rm1)
plt.figure(o)
p1 = plt.plot(lamlist, Rlist)
p2 = plt.plot(lamlist, Rplist, color = 'Orange')
p3 = plt.plot(lamlist, r1list, color = 'Green')
#p4 = plt.plot(lamlist, Tlist, color = 'Orange')
title = plt.title(g)
xlegend = plt.xlabel('Wavelength (nm)')
ylegend = plt.ylabel('')
plt.grid()
```