Numerical problems of AM

1. A modulating signal $m(t)=10\cos(2\pi\times10^3 t)$ is amplitude modulated with a carrier signal $c(t)=50\cos(2\pi\times10^5 t)$. Find the modulation index, the carrier power, and the power required for transmitting AM wave.

$\mu=10/50=\mathbf{0.2}$

$Pc=Ac^2/2R = (50)^2/2(1) = \mathbf{1250\ Watt}$

$\Rightarrow Pt=Pc\,(1+\mu^2)/2 = 1250\,(1+(0.2)^2)/2 = \mathbf{1275\ Watt}$

2. The equation of amplitude wave is given by $s(t)=20[1+0.8\cos(2\pi\times10^3 t)]\cos(4\pi\times10^5 t)$. Find the carrier power, the total sideband power, and the band width of AM wave.

Given, the equation of Amplitude modulated wave is

$$s(t) = 20\left[1 + 0.8\cos\left(2\pi \times 10^3 t\right)\right]\cos\left(4\pi \times 10^5 t\right)$$

Re-write the above equation as

$$s(t) = 20\left[1 + 0.8\cos\left(2\pi \times 10^3 t\right)\right]\cos\left(2\pi \times 2 \times 10^5 t\right)$$

We know the equation of Amplitude modulated wave is

$$s(t) = A_c\left[1 + \mu\cos(2\pi f_m t)\right]\cos(2\pi f_c t)$$

By comparing the above two equations, we will get

Amplitude of carrier signal as $A_c = 20 volts$

Modulation index as $\mu = 0.8$

Frequency of modulating signal as $f_m = 10^3 Hz = 1KHz$

Frequency of carrier signal as $f_c = 2 \times 10^5 Hz = 200KHz$

The formula for Carrier power, P_c is

$$P_c = \frac{{A_c}^2}{2R}$$

Assume $R = 1\Omega$ and substitute A_c value in the above formula.

$$P_c = \frac{(20)^2}{2(1)} = 200W$$

Therefore, the **Carrier power**, P_c is **200watts**.

We know the formula for total side band power is

$$P_{SB} = \frac{P_c \mu^2}{2}$$

Substitute P_c and μ values in the above formula.

$$P_{SB} = \frac{200 \times (0.8)^2}{2} = 64W$$

Therefore, the **total side band power** is **64 watts.**

We know the formula for bandwidth of AM wave is

$$BW = 2f_m$$

Substitute f_m value in the above formula.

$$BW = 2(1K) = 2KHz$$

Therefore, the **bandwidth** of AM wave is **2 KHz.**

3. **When the modulation percentage is 75%, an AM transmitter radiates 10KW Power. How much of this is carrier Power?**

Solution: $P_t = 10\ KW$ and $m=0.75$

We know that

$$P_{t(total)} = \left(1+\frac{m^2}{2}\right)P_c$$

$$\therefore P_c = \frac{P_{t(total)}}{\left(1+\frac{m^2}{2}\right)}$$

$$\therefore P_c = \frac{10\times10^3}{\left(1+\frac{0.75^2}{2}\right)}$$

$$\therefore P_c = 7.8\times10^3 W$$

$$\therefore P_c = 7.8KW$$

4. **An AM transmitter radiates 20KW. If the modulation Index is 0.7. Find the carrier Power.**

Solution: $P_t = 20\ KW$ and $m=0.7$

We know that

$$P_{t\,(total)} = \left(1+\frac{m^2}{2}\right)P_c$$

$$\therefore P_c = \frac{P_{t\,(total)}}{\left(1+\frac{m^2}{2}\right)}$$

$$\therefore P_c = \frac{20\times10^3}{\left(1+\frac{0.7^2}{2}\right)}$$

$$\therefore P_c = 16.064\times10^3 W$$

$$\therefore P_c = 16.064 KW$$

5. **The total Power content of an AM signal is 1000W. Determine the power being transmitted at carrier frequency and at each side bands when modulation percentage is 100%.**

Solution: $P_t = 1000\ W$ **and** $m=1$

We know that

$$P_{t(total)} = \left(1+\frac{m^2}{2}\right)P_c \quad \Rightarrow \quad \therefore P_c = \frac{P_{t(total)}}{\left(1+\frac{m^2}{2}\right)}$$

$$\therefore P_c = \frac{1\times 10^3}{\left(1+\frac{1^2}{2}\right)} \quad \Rightarrow \quad \therefore P_c = 666.67W$$

$$P_{USB} = P_{LSB} = \left(\frac{m^2}{4}\right)P_c \quad \Rightarrow \quad P_{USB} = P_{LSB} = \left(\frac{1}{4}\right)\times 666.67$$

$$\therefore P_c = 166.67W$$

6. **A 500W, 100KHz carrier is modulated to a depth of 60% by modulating frequency of 1KHz. Calculate the total power transmitted. What are the sideband components of AM Wave?**

Solution: $P_c = 500\ W$, $f_c = 100\ KHz$, $m = 60\% = .6$ and $f_m = 1\ KHz$

We know that

$$P_{t\,(total)} = \left(1 + \frac{m^2}{2}\right) P_c \qquad \therefore = P_{t\,(total)} \left(1 + \frac{0.6^2}{2}\right) 500$$

$$\therefore P_{t\,(total)} = 590W$$

We know that

$$\therefore f_{USB} = f_c + f_m \qquad \therefore f_{LSB} = f_c - f_m$$

$$\therefore f_{USB} = 101\ KHz \qquad \therefore f_{LSB} = 99\ KHz$$

7. **A400W, 1MHz carrier is amplitude-modulated with a sinusoidal signal 0f 2500Hz. The depth of modulation is 75%. Calculate the sideband frequencies, bandwidth, and power in sidebands and the total power in modulated wave.**

Solution: $P_c = 400\ W$, $f_c = 1\ MHz$, $m = 75\% = .75$ and $f_m = 2.5\ KHz$

We know that

$$\therefore f_{USB} = f_c + f_m \qquad \therefore f_{LSB} = f_c - f_m \qquad \therefore BW = 2 f_m$$

$$\therefore f_{USB} = 1002.5\ KHz \qquad \therefore f_{LSB} = 997.5\ KHz \qquad \therefore BW = 2 \times 2.5 KHz = 5 KHz$$

We know that

$$P_{t\,(total)} = \left(1 + \frac{m^2}{2}\right) P_c \qquad \therefore P_{t\,(total)} = \left(1 + \frac{0.75^2}{2}\right) 400$$

$$\therefore P_{t\,(total)} = 512.5\ W$$

$$P_{USB} = P_{LSB} = \left(\frac{m^2}{4}\right) P_c \qquad P_{USB} = P_{LSB} = \left(\frac{0.75^2}{4}\right) 400 = 56.25 W$$

8. **A Carrier of 750 W,1MHz is amplitude modulated by sinusoidal signal of 2 KHz to a depth of 50%. Calculate Bandwidth, Power in side band and total power transmitted.**

Solution: $P_c = 750\ W,\ f_c = 1\ MHz,\ m = 50\% = .5$ and $f_m = 2\ KHz$

We know that

$$\therefore f_{USB} = f_c + f_m \qquad \therefore f_{LSB} = f_c - f_m$$

$$\therefore f_{USB} = 1002\ KHz \qquad \therefore f_{LSB} = 998\ KHz \qquad \therefore BW = 2 \times f_m = 4\ KHz$$

$$\therefore BW = f_{USB} - f_{LSB} = 4\ KHz$$

We know that

$$P_{t\,(total)} = \left(1 + \frac{m^2}{2}\right) P_c \qquad \therefore P_{t\,(total)} = \left(1 + \frac{0.5^2}{2}\right) 750$$

$$\therefore P_{t\,(total)} = 843.75W$$

$$P_{USB} = P_{LSB} = \left(\frac{m^2}{4}\right) P_c \qquad P_{USB} = P_{LSB} = \left(\frac{0.5^2}{4}\right) 750 = 46.875W$$

9. Calculate the percentage power saving when one side band and carrier is suppressed in an AM signal with modulation index equal to 1.

Solution: ***m = 1***

We know that

$$P_{t\,(total)} = \left(1 + \frac{m^2}{2}\right) P_c = \frac{3}{2} P_c$$

$$P_{suppressed} = P_c + P_{LSB}$$

$$\therefore P_{suppressed} = P_c + \left(\frac{m^2}{4}\right) P_c$$

$$\therefore P_{suppressed} = P_c + \left(\frac{1}{4}\right) P_c = \frac{5}{4} P_c$$

$$\text{Amount of power saved} = \frac{P_{suppressed}}{P_{total}}$$

$$\text{Amount of power saved} = \frac{\frac{5}{4} P_c}{\frac{3}{2} P_c} = \frac{10}{12} = 0.833 = 83.3\%$$

10. Calculate the percentage power saving when one side band and carrier is suppressed in an AM signal if percentage of modulation is 50%.

Solution: ***m = 0.5***

We know that

$$P_{t\,(total)} = \left(1 + \frac{m^2}{2}\right) P_c = \frac{9}{8} P_c$$

$$P_{suppressed} = P_c + P_{LSB}$$

$$\therefore\ P_{suppressed} = P_c + \left(\frac{m^2}{4}\right) P_c$$

$$\therefore\ P_{suppressed} = P_c + \left(\frac{1}{16}\right) P_c = \frac{17}{16} P_c$$

$$\text{Amount of power saved} = \frac{P_{suppressed}}{P_{total}}$$

$$\text{Amount of power saved} = \frac{\frac{9}{8} P_c}{\frac{17}{16} P_c} = \frac{9 \times 16}{8 \times 17} = 0.944 = 94.4\%$$

11. A Sinusoidal carrier frequency of 1.2MHz is amplitude modulated by a sinusoidal voltage of frequency 20KHz resulting in maximum and minimum modulated carrier amplitude of 110V & 90V respectively. Calculate

I. frequency of lower and upper side bands
II. unmodulated carrier amplitude
III. Modulation index
IV. Amplitude of each side band.

Solution: f_c =1.2 MHz, E_{max} = 110V, E_{min} = 90V and f_m =20 KHz

We know that

$$\therefore f_{USB} = f_c + f_m \qquad \therefore f_{LSB} = f_c - f_m$$

$$\therefore f_{USB} = 1220\ KHz \qquad \therefore f_{LSB} = 1180\ KHz$$

$$E_c = \frac{E_{max} + E_{min}}{2} = (110 + 90)/2 \qquad E_c = 100V$$

We also know that

$$E_m = \frac{E_{max} - E_{min}}{2}$$

$$\therefore E_m = (110 - 90)/2 = 10$$

$$m = \frac{E_{max} - E_{min}}{E_{max} + E_{min}} \qquad \therefore m = \frac{110 - 90}{110 + 90}$$

$m = 0.1$

12. An audio frequency signal 10 sin(2π×500t) is used to amplitude modulate a carrier of 50 sin(2π×10^{5}t).Calculate

I. frequency of side bands
II. Bandwidth
III. Modulation index
IV. Transmission efficiency
V. Total power delivered to a load of 600Ω.

Solution: f_m =500 Hz , E_m = 10V, f_c =100 KHz and E_c = 50V.

We know that $\therefore f_{USB} = f_c + f_m$ $\quad \therefore f_{LSB} = f_c - f_m$ $\quad \therefore BW = 2 f_m$

$\therefore f_{USB} = 100.5\ KHz$ $\quad \therefore f_{LSB} = 99.5\ KHz$ $\quad \therefore BW = 2 \times 500 Hz = 1 KHz$

We also know that $m = \frac{E_m}{E_c}$ $\quad m = \frac{10}{50}$ $\quad m = 0.2 = 50\%$

also carrier power $P_c = \frac{E_c^2}{2 \times R}$ $\quad P_c = \frac{2500}{2 \times 600} = 2.08$

and total power $P_t = \left(1 + \frac{m^2}{2}\right) P_c$ $\quad \therefore P_t = 2.125$

Transmission Efficiency

$\text{eff.} = \frac{m^2}{2 + m^2}$

$\text{eff.} = \frac{0.2^2}{2 + 0.2^2}$

$\text{eff.} = 0.196 = 1.96\%$

Self Study

Exercise Problems (Amplitude Modulation)

1. An AM voltage signal consists of a carrier wave $100\cos(2\pi \times 10^6 t)$ and a DSB-SC signal $(20\cos 6.28t + 50\cos 12.56t)\cos(2\pi \times 10^6 t)$,

(a) Draw the spectrum of the modulated message;

(b) Determine the carrier power, sideband power and the total power of the modulated signal.

2. An AM transmitter has a carrier power of 30W. The message signal is a sinusoidal signal and the percentage of modulation is 85%, i.e. $m = 0.85$. Calculate:

(a) the total power; and

(b) the power in one sideband.

Exercise Problems (Amplitude Modulation)

3. For DSB-SC modulation, $m(t) = 4 + 2\cos(2\pi \times 10^3 t)$, carrier wave $x_c(t) = 8\cos(2\pi \times 10^6 t)$

(a) Draw the frequency domain representation of $m(t)$, $x_c(t)$ and the modulation output $x(t)$;

(b) Draw the block diagram of the demodulation system diagram, if low pass filter is used in the demodulator, what is the minimum bandwidth required to fully recover the signal?

(c) What is the frequency and time domain representation of the demodulation output?

Exercise Problems (Amplitude Modulation)

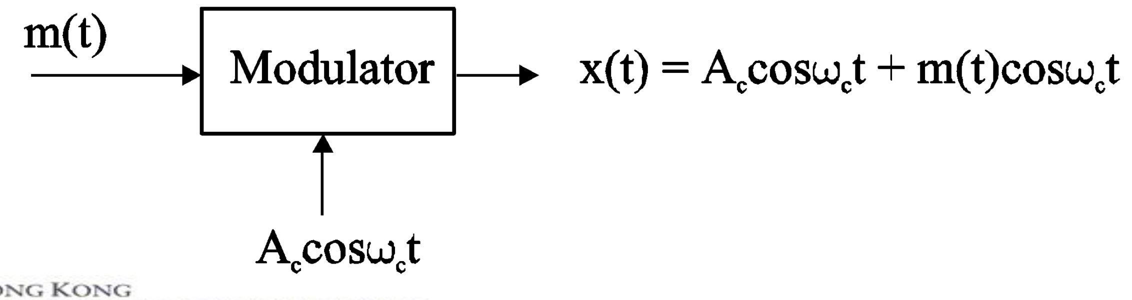
4. The figure below shows an amplitude modulator. Assuming sinusoidal carrier $x_c(t)$ and sinusoidal message signal $m(t)$, i.e., $x_c(t) = A_c\cos\omega_c t$, $\quad m(t) = A_m\cos\omega_m t$

The modulated signal $x(t)$ can be written as

$$x(t) = A_c[1 + (A_m/A_c)\cos\omega_m t]\cos\omega_c t$$

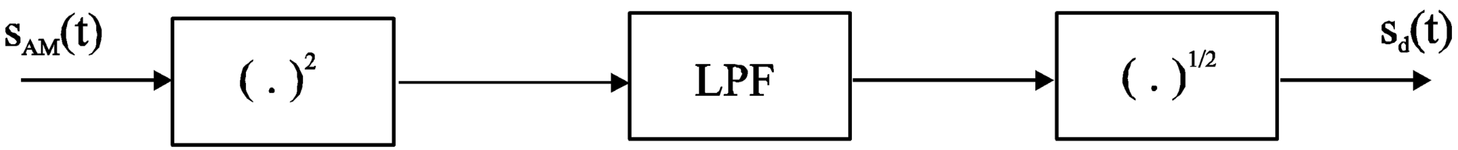
(a) For $A_m = A_c$, calculate the modulation index;

(b) Determine the fraction of total transmitted power concentrated in the modulation sideband for (1) $A_m = A_c$; (2) $A_m = A_c/2$; (3) $A_m = aA_c$, where $|a| < 1$;



Exercise Problems (Amplitude Modulation)

5. To prove that the following system can be used for AM signal demodulation and the bandwidth of the low-pass filter must be $2W_m$, where W_m is the highest frequency of the message signal.



Exercise Problems (Amplitude Modulation)

6. The following figure shows the output of a conventional amplitude modulator.

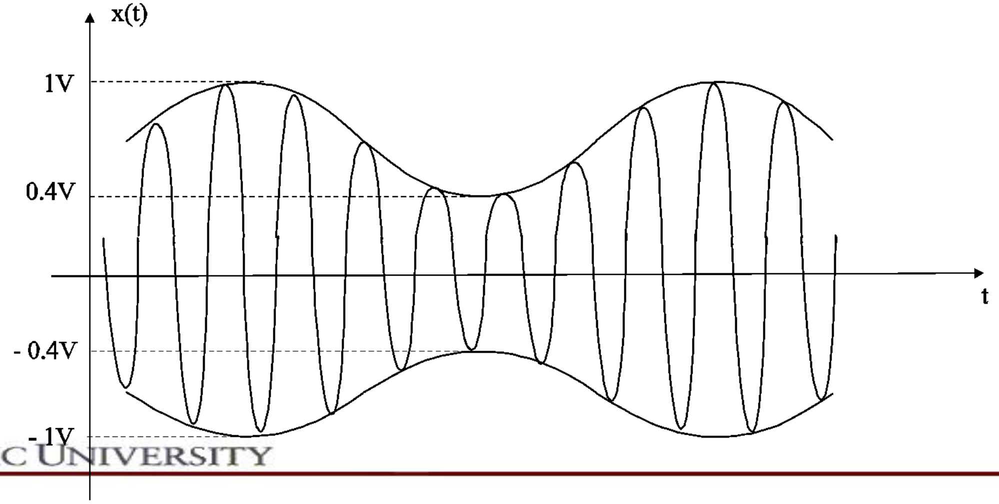
(1) What is the mathematical expression of x(t)?

(2) Calculate the modulation index;

(3) What is the amplitude of the sideband? What is the amplitude of the unmodulated carrier?

(4) Draw a two sided frequency domain representation of x(t);

(5) What is the ratio of power in the sidebands to the total power?



Solutions (Amplitude Modulation)

1. (a) $s_{AM}(t) = 100\cos(2\pi \times 10^6 t) + (20\cos 6.28t + 50\cos 12.56t)\cos(2\pi \times 10^6 t)$

$S_{AM}(\omega) = 100\pi[\delta(\omega + 2\pi \times 10^6) + \delta(\omega - 2\pi \times 10^6)] + 25\pi[\delta(\omega - 2\pi \times 10^6 + 12.56) + \delta(\omega - 2\pi \times 10^6 - 12.56) + \delta(\omega + 2\pi \times 10^6 + 12.56) + \delta(\omega + 2\pi \times 10^6 - 12.56)] + 10\pi[\delta(\omega - 2\pi \times 10^6 + 6.28) + \delta(\omega - 2\pi \times 10^6 - 6.28) + \delta(\omega + 2\pi \times 10^6 + 6.28) + \delta(\omega + 2\pi \times 10^6 - 6.28)]$

(b) $P_c = 5 \times 10^3$, $P_s = (20^2 + 50^2) / 4 = 2900 / 4 = 725$, $P_t = 5725$

(when a resistor of 1Ω is assumed the unit of power is W)

2. (a) $P_t = P_c(1 + m^2/2) = 30\ (1 + 0.852/2) = 40.8\text{W}$

P_s (double sideband) $= P_t - P_c = 40.8 - 30 = 10.8\text{W}$

P_s (single sideband) $= P_s / 2 = 10.8 / 2 = 5.4\text{W}$

Solutions (Amplitude Modulation)

3. (a) $m(t) = 4 + 2\cos(2\pi \times 10^3 t)$,

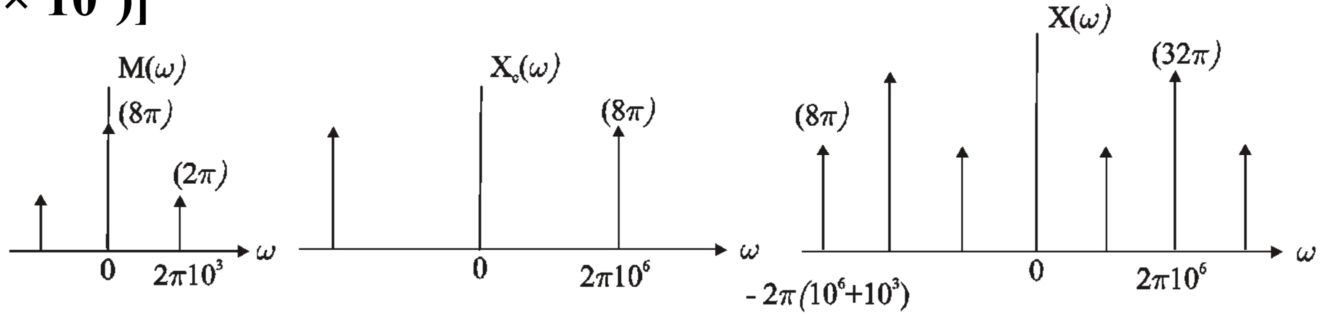
$M(\omega) = 8\pi\delta(\omega) + 2\pi[\delta(\omega + 2\pi \times 10^3) + \delta(\omega - 2\pi \times 10^3)]$

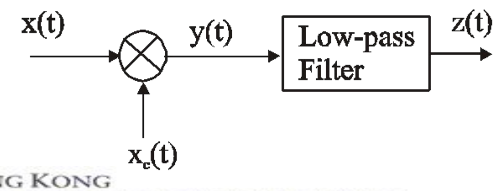
$x_c(t) = 8\cos(2\pi \times 10^6 t)$

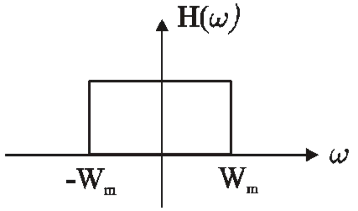
$X_c(\omega) = 8\pi[\delta(\omega + 2\pi \times 10^6) + \delta(\omega - 2\pi \times 10^6)]$

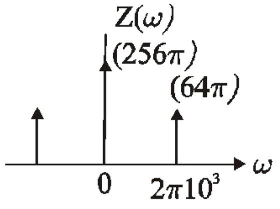
$x(t) = m(t)xc(t) = 32\cos(2\pi \times 10^6 t) + 16\cos(2\pi \times 10^3 t)\cos(2\pi \times 10^6 t)$

$X(\omega) = 32\pi[\delta(\omega + 2\pi \times 10^6) + \delta(\omega - 2\pi \times 10^6)] + 8\pi[\delta(\omega + 2\pi \times 10^6 - 2\pi \times 10^3) + \delta(\omega + 2\pi \times 10^6 + 2\pi \times 10^3) + \delta(\omega - 2\pi \times 10^6 - 2\pi \times 10^3) + \delta(\omega - 2\pi \times 10^6 + 2\pi \times 10^3)]$









Solutions (Amplitude Modulation)

(b) $W_m = 2\pi \times 10^3$ rad or $f_m = 1$kHz

(c) $y(t) = x(t)x_c(t) = [4 + 2\cos(2\pi \times 10^3 t)]\ [8\cos(2\pi \times 10^6 t)]2$

$= [4 + 2\cos(2\pi \times 10^3 t)] \times 32[1 + \cos(4\pi \times 10^6 t)]$

$= 128 + 64\cos(2\pi \times 10^3 t) + 128\cos(4\pi \times 10^6 t) + 64\cos(2\pi \times 10^3 t)\cos(4\pi \times 10^6 t)$

$= 128 + 64\cos(2\pi \times 10^3 t) + 128\cos(4\pi \times 10^6 t)$

$+ 32[\cos(4\pi \times 10^6 t + 2\pi \times 10^3 t) + \cos(4\pi \times 10^6 t - 2\pi \times 10^3 t)]$

After LPF, $z(t) = 128 + 64\cos(2\pi \times 10^3 t)$

$Z(\omega) = 256\pi\delta(\omega) + 64\pi[\delta(\omega + 2\pi \times 10^3)$

$+ \delta(\omega - 2\pi \times 10^3)]$

Solutions (Amplitude Modulation)

4. (a) $m = A_m / A_c = 1$

(b) Carrier power $P_c = A_c^{\,2} / 2$,

Sideband power $P_s = m^2 A_c^{\,2} / 4$

Total power $P_t = A_c^{\,2} / 2 + m^2 A_c^{\,2} / 4$, $\eta = P_s / P_t = m^2 / (m^2 + 2)$

(1) $m = 1$, $\eta = 1/3$

(2) $m = ½$, $\eta = 1/9$

(3) $m = a$, $\eta = a^2 / (a^2 + 2)$

5. $s_{AM}(t) = [A + m(t)]\cos\omega_c t$, $s_{AM}^{\;2}(t) = [A + m(t)]^2 \cos 2\omega_c t$

The output of LPF $= [A + m(t)]^2 / 2$,

$$s_d(t) = \frac{A + m(t)}{\sqrt{2}}$$

If $m(t) = A_m \cos\omega_m t$, the highest frequency is ω_m;

then $m^2(t) = A_m^{\;2}(1 + \cos 2\omega_m t)/2$, the highest frequency is $2\omega_m$.

thus the bandwidth of the low pass filter must be $2W_m$

Solutions (Amplitude Modulation)

6. (1) $x(t) = A(1 + m \cos\omega_m t) \cos\omega_c t$,

$|x(t)|_{max} = 1$, $|x(t)|_{min} = 0.4$,

i.e. $A(1 + m) = 1$, $A(1 - m) = 0.4$

$\Rightarrow m = (1 - 0.4) / (1 + 0.4) = 0.429$, $A = 1 / (1 + m) = 0.7$

$\Rightarrow x(t) = 0.7(1 + 0.429 \cos\omega_m t) \cos\omega_c t$

(2) $m = 0.429$

(3) Sideband amplitude:

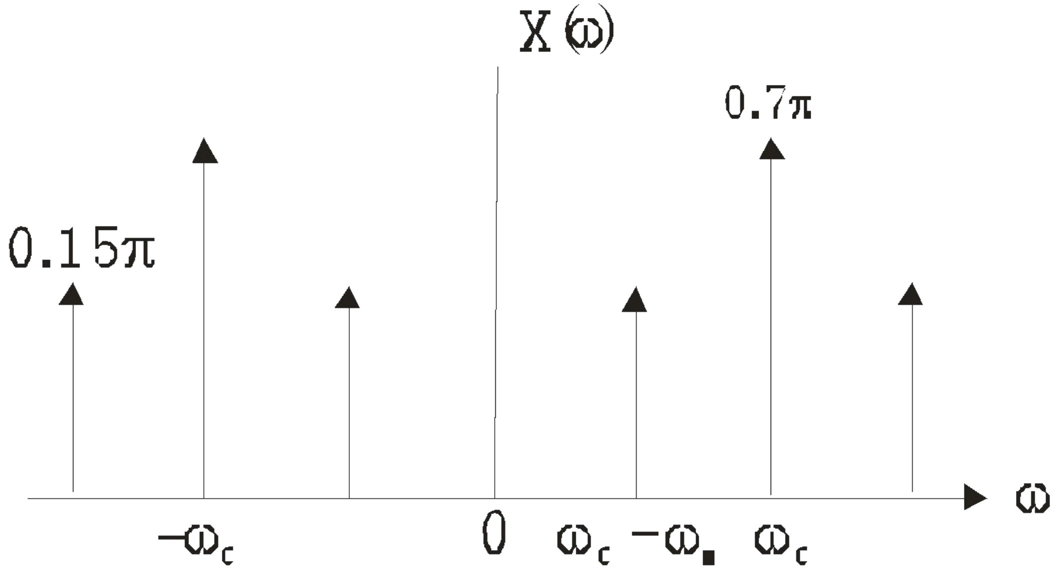
$A_m / 2 = mA / 2 = (0.7)(0.429) / 2 = 0.15$

Unmodulated carrier amplitude:

$A = 0.7$

Solutions (Amplitude Modulation)

(4)



(5) $A_m = (1 - 0.4) / 2 = 0.3$

Sideband power: $P_s = A_m^2 / 4 = 0.0225$

Total power: $P_t = P_s + A^2 / 2 = 0.0225 + (0.7)^2 / 2 = 0.0225 + 0.245 = 0.2675$

Ratio = 0.0225 / 0.2675 = 8.4%

Assignment:

1. The modulating signal m(t) is recovered from the DSBSC signal $s(t)=m(t)Cos(\omega_c t)$ by multiplying s(t) by a locally generated carrier $c'(t)=Cos(\omega_c t+\boldsymbol{\phi})$, where $\omega_c=2\pi f_c$, is the angular carrier frequency. The product of s(t) c'(t) is passed through a LPF which rejects the double frequency signal. Determine the maximum allowable value for the phase angle if the recovered signal is to be 95% of the maximum possible output. If the modulating signal is band limited to 10kHz, determine the minimum value of carrier frequency for which m(t) can be recovered by filtering.

2. A received single-tone sinusoidal modulated SSBSC signal $Cos\{(\omega_c+\omega_m)t\}$ has a normalized power of 0.5 watt. The signal is to be detected by carrier re-insertion technique. Find the amplitude of the carrier to be reinserted so that the power in the recovered signal at the demodulator output is 90% of the normalized power. The DC component can be neglected and $\omega_c=2\pi f_c$ and $\omega_m= 2\pi f_m$.

Note: workout example (3.44 and 3.48) of Analog Communication System (Seventh edition 2016) by Dr. Sanjay Sharma

Thank You.