Chapter-7 → Noise performance of band pass (Modulated) Communication System

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* Effect of Noise in Envelope and Synchronous demodulation of DSB-FC AM.

The received signal at the receiver or demodulator is the sum of signal and noise.

$$s(t) = [A_c + m(t)] \cos w_c t + n(t) \quad -(1)$$

The signal power at the input of the receiver is

$$P_i(t) = \frac{A_c^2}{2} + \frac{\overline{m^2(t)}}{2}$$

Noise power given as

$$P_{iN}(t) = \overline{n^2(t)}$$

Therefore, $(SNR)_i = \frac{\text{signal power } P_i(t)}{\text{Noise power } P_N(t)} = \frac{A_c^2/2 + \overline{m^2(t)}/2}{\overline{n^2(t)}}$

$$\boxed{\text{Signal to Noise Ratio } (SNR)_i = \frac{A_c^2 + \overline{m^2(t)}}{2\,\overline{n^2(t)}}} \quad -(2)$$

Detection for DSB-FC AM, two method is used

1. Envelope Detection.
2. Synchronous Detection.

1. Envelope Detection :→

For envelope detection, the received signal at the receiver or demodulator is

$$s(t) = [A_c + m(t)] \cos w_c t + n(t)$$

$$s(t) = [A_c + m(t)] \cos w_c t + n_c(t) \cos w_c t - n_s(t) \sin w_c t$$

$$s(t) = [A_c + m(t) + n_c(t)] \cos w_c t - n_s(t) \sin w_c t \quad -(1)$$

The envelope of the received signal is given as

$$V(t) = \sqrt{[A_c + m(t) + n_c(t)]^2 + n_s^2(t)} \quad -(2)$$

phase Angle of the envolope of received signal is

$$\varphi(t) = \tan^{-1}\left(\frac{n_s(t)}{A_c + m(t) + n_c(t)}\right) \quad -(3)$$

* For small Noise:→ $[A_c + m(t)] >> n(t)$

phase angle $\varphi(t) = 0$

and the output of ideal envelope detector

$$V(t) = A_c + m(t) + n_c(t) \quad -(4)$$

∴ Signal power $P_s = \overline{m^2(t)}$

Noise power $P_N = \overline{n_c^2(t)}$

$$\text{Signal to Noise power } (SNR) = \frac{P_s}{P_N} = \frac{\overline{m^2(t)}}{\overline{n_c^2(t)}} \quad -(5)$$

$\eta = \frac{\mu^2}{2+\mu^2} \times 100\%$

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Detection Gain is defined as ratio of the ~~out~~put SNR to the ~~in~~put SNR of detector.

$$\gamma = \frac{\overline{m^2(t)} / \overline{n_c^2(t)}}{\overline{A_c^2 + m^2(t)} / 2\overline{n^2(t)}}$$

As $\overline{n(t)} = \overline{n_c(t)}$

$$\boxed{\text{Detection Gain}(\gamma) = \frac{2\overline{m^2(t)}}{A_c^2 + \overline{m^2(t)}}}$$

which clearly illustrates that the detection Gain increase with decrease of the amplitude of carrier signal (A_c).

For $A_c \geq |m(t)|$

$$\text{efficiency}(\eta) = \frac{\overline{m^2(t)}}{A_c^2 + \overline{m^2(t)}}$$

Therefore Detection Gain $(\gamma) = 2\eta$

For 100% modulation index

$(\mu = 1)$, efficiency $\eta = \frac{1}{3}$

and $\boxed{\text{Detection Gain}(\gamma) = \frac{2}{3}}$ which again shows that even for 100% modulation, the detection gain is less than 1.

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2. Synchronous Detection

In coherent detection, the received modulated signal is multiplied with $\cos w_c t$ and then passed through LPF.

The output of synchronous detection is

$$v(t) = \frac{1}{2} m(t) + \frac{1}{2} n_c(t) \quad \text{— (1)}$$

output signal power $(P_{SO}) = \frac{\overline{m^2(t)}}{4}$ output Noise power $(P_{NO}) = \frac{\overline{n_c^2(t)}}{4}$

$\therefore$ Signal to Noise Ratio $(SNR)_o = \frac{P_{SO}}{P_{NO}}$

$$(SNR)_o = \frac{\overline{m^2(t)}}{\cancel{4}} \times \frac{\cancel{4}}{\overline{n_c^2(t)}}$$

$$(SNR)_o = \frac{\overline{m^2(t)}}{\overline{n_c^2(t)}} \quad \text{— (2)}$$

Again Detection Gain $(\gamma) = \frac{(SNR)_o}{(SNR)_i}$

$$\gamma = \frac{\overline{m^2(t)}}{\cancel{\overline{n_c^2(t)}}} \times \frac{2\,\cancel{\overline{n^2(t)}}}{A_c^2 + \overline{m^2(t)}}$$

$\because \overline{n(t)} = \overline{n_c(t)}$

$$\boxed{\text{Detection Gain } \gamma = \frac{2\overline{m^2(t)}}{A_c^2 + \overline{m^2(t)}}}$$

* Gain parameter for demodulation of Doubleside Band-Suppressed Carrier (DSB-SC)

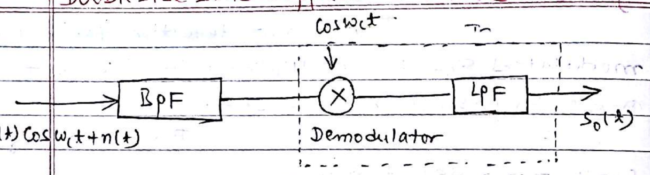


fig :⇒ DSB-SC De-Modulator

Above figure shows the block diagram of DSB-SC De-Modulator whose input signal to the receiver or demodulator is the sum of signal and noise.

$$s(t) = m(t)\cos w_c t + n(t) \quad — (1)$$

The signal power at the input of the receiver is

$$P_i(t) = \frac{\overline{m^2(t)}}{2}$$ and Noise power is $P_N(t) = \overline{n^2(t)}$

Input Signal to Noise Ratio of the demodulator

$$(SNR)_i = \frac{P_i(t)}{P_N(t)} = \frac{\overline{m^2(t)}}{2\,\overline{n^2(t)}} \quad — (2)$$

* Coherent or Synchronous demodulator

For Coherent detection the received signal is multiplied by a carrier ($\cos w_c t$) & passed through L.P.F

$$s(t) = m(t)\cos^2 w_c t + n(t)\cos w_c t$$

$$s(t) = m(t)\left(\frac{1+\cos 2w_c t}{2}\right) + \left(n_c(t)\cos w_c t - n_s(t)\sin w_c t\right)\cos w_c t$$

$$s(t) = \frac{1}{2}m(t) + \frac{1}{2}\cos 2w_c t + n_c(t)\cos^2 w_c t - n_s(t)\sin w_c t \cos w_c t$$

$$s(t) = \frac{1}{2}m(t) + \frac{1}{2}\cos 2w_c t + n_c(t)\left(\frac{1+\cos 2w_c t}{2}\right) - \frac{n_s(t)}{2}\sin 2w_c t$$

After passing through LPF

$$s_0(t) = \frac{1}{2}m(t) + \frac{1}{2}n_c(t) \quad — (3)$$

∴ output Noise power $P_{N_0} = \frac{\overline{n_c^2(t)}}{4}$

output signal power $P_{s_0} = \frac{\overline{m^2(t)}}{4}$

output signal to Noise Ratio of demodulator is

$$(SNR)_0 = \frac{P_{s_0}}{P_{N_0}} = \frac{\overline{m^2(t)}}{\overline{n_c^2(t)}}$$

$$\therefore \text{Detection Gain}(\gamma) = \frac{(SNR)_0}{(SNR)_i} = \frac{\overline{m^2(t)}}{\overline{n_c^2(t)}} \times \frac{2\,\overline{n^2(t)}}{\overline{m^2(t)}}$$

$$\because \overline{n(t)} = \overline{n_c(t)}$$

$$\therefore \boxed{\text{Detection Gain}(\gamma) = 2}$$

* Gain parameter for demodulation of (SSB) Single Side Band using synchronous demodulators.

Single side Band also known as Single side band suppressed carrier (SSB-SC) modulated signal is given as

$$s(t) = m(t)\cos\omega_c t \pm \hat{m}(t)\sin\omega_c t \quad -(1)$$

$\hat{m}(t)$ is Hilbert transform of message signal $m(t)$. '+' for LSB and '−' for USB.

The input signal to the receiver or demodulator is the sum of signal and noise given as

$$s(t) = [m(t)\cos\omega_c t \pm \hat{m}(t)\sin\omega_c t] + n(t) \quad -(2)$$

The signal power at the input of the receiver is

$$P_i(t) = \frac{\overline{m^2(t)}}{2} + \frac{\overline{\hat{m}^2(t)}}{2}$$

[Power of signal $m(t)$ is same as that of hilbert transform $\hat{m}(t)$]

$$P_i(t) = \overline{m^2(t)}$$

Noise power $P_{N_0}(t) = \overline{n^2(t)}$

Input signal to Noise Ratio $SNR = \frac{P_i(t)}{P_{N_0}(t)} = \frac{\overline{m^2(t)}}{\overline{n^2(t)}} \quad -(3)$

The received wave is multiplied by $(\cos\omega_c t)$ carrier and passed through LPF

$$s(t) = [m(t)\cos\omega_c t + \hat{m}(t)\sin\omega_c t]\cos\omega_c t + n(t)\cos\omega_c t$$

$$s(t) = [m(t)\cos\omega_c t \pm \hat{m}(t)\sin\omega_c t]\cos\omega_c t + [n_c(t)\cos\omega_c t - n_s(t)\sin\omega_c t]\cos\omega_c t$$

$$s(t) = [m(t)\cos^2\omega_c t \pm \hat{m}(t)\sin\omega_c t\cdot\cos\omega_c t] + n_c(t)\cos^2\omega_c t - n_s(t)\sin\omega_c t\cdot\cos\omega_c t$$

$$s(t) = \left[m(t)\left(\frac{1+\cos 2\omega_c t}{2}\right) \pm \hat{m}(t)\frac{\sin 2\omega_c t}{2}\right] + n_c(t)\left(\frac{1+\cos 2\omega_c t}{2}\right) - \frac{n_s(t)\sin 2\omega_c t}{2}$$

After passing through L.P.F we get,

$$s_0(t) = \frac{m(t)}{2} + \frac{n_c(t)}{2}$$

output signal power $P_{s_0} = \frac{\overline{m^2(t)}}{4}$

$$P_{s_0} = \frac{\overline{m^2(t)}}{4}$$

output Noise power $P_{N_0} = \frac{\overline{n_c^2(t)}}{4}$

output signal to Noise ratio of demodulator is

$$(SNR)_0 = \frac{P_{s_0}}{P_{N_0}} = \frac{\overline{m^2(t)}}{\cancel{4}} \times \frac{\cancel{4}}{\overline{n_c^2(t)}}$$

$$(SNR)_0 = \frac{\overline{m^2(t)}}{\overline{n_c^2(t)}} \quad -(4)$$

$\therefore$ Detection Gain $(\gamma) = \dfrac{(SNR)_o}{(SNR)_i}$

$$= \frac{\overline{m^2(t)}}{\cancel{\overline{n_c^2(t)}}} \times \frac{\cancel{\overline{n^2(t)}}}{\cancel{\overline{m^2(t)}}} \quad \left[\because \overline{n(t)} = \overline{n_c(t)}\right]$$

$$\boxed{\text{Detection Gain } (\gamma) = 1}$$

* Continue from envelope Detection of DSB-FC A.M

* Threshold effect in non linear demodulation of A.M.

For Large Noise case $n(t) >> [A_c + m(t)]$

The envelope detection of the received signal is

$$V(t) = \sqrt{[A_c + m(t) + n_c(t)]^2 + n_s^2(t)}$$

$$V(t) = \sqrt{(A_c + m(t))^2 + 2(A_c + m(t))\, n_c(t) + n_c^2(t) + n_s^2(t)}$$

since, $n(t) >> A_c + m(t)$ neglect the square terms

$$V(t) = \sqrt{n_c^2(t) + n_s^2(t) + 2(A_c + m(t))\, n_c(t)}$$

$$V(t) = \sqrt{n_c^2(t) + n_s^2(t)\left(1 + \frac{2(A_c + m(t))\, n_c(t)}{n_c^2(t) + n_s^2(t)}\right)}$$

$$V(t) = \sqrt{n_c^2(t) + n_s^2(t)}\,\sqrt{\left(1 + \frac{2[A_c + m(t)]\, n_c(t)}{n_c^2(t) + n_s^2(t)}\right)}$$

since $R(t)$ and $\theta(t)$ are the envelope and phase of $n(t)$ can be given as

$$R(t) = \sqrt{n_c^2(t) + n_s^2(t)}$$

$$\theta = \tan^{-1}\left(\frac{n_s(t)}{n_c(t)}\right)$$

$$\tan\theta = \frac{n_s(t)}{n_c(t)} = \frac{p}{b}$$

$$h = \sqrt{p^2 + b^2} = \sqrt{n_s^2(t) + n_c^2(t)}$$

$$\cos\theta = \frac{b}{h} = \frac{n_c(t)}{\sqrt{n_s^2(t) + n_c^2(t)}}$$

$$\therefore V(t) = R(t)\sqrt{1 + \frac{2[A_c + m(t)]}{\sqrt{n_c^2(t) + n_s^2(t)}} \times \frac{n_c(t)}{\sqrt{n_s^2(t) + n_c^2(t)}}}$$

$$V(t) = R(t)\sqrt{1 + \frac{2[A_c + m(t)]}{R(t)} \cos\theta(t)}$$

$$V(t) \approx R(t)\left[1 + \frac{A_c + m(t)}{R(t)}\right]\cos\theta(t)$$

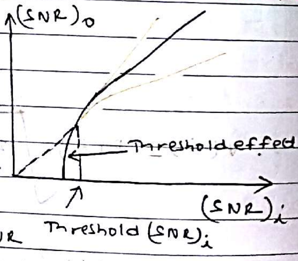
$$V(t) \approx R(t) + (A_c + m(t))\cos\theta(t)$$

$$\boxed{V(t) = R(t) + A_c\cos\theta(t) + m(t)\cos\theta(t)}$$

Above equation clearly illustrates that the output of the detector has no component strictly proportional to the message signal m(t).

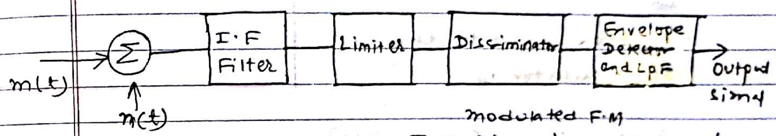
The only message signal term m(t) is multiplied by $\cos\theta(t)$. As $\theta(t)$ is the phase of bandpass noise n(t).

output or demodulated signal of envelope detector is completely different to that of message signal. ∴ This loss of message in Envelope detector operated at a large noise level is refered as Threshold Effect.



From the plot, we can see that the threshold effect deterioate the output SNR more rapidly than the Input SNR, when input SNR is below certain level (Threshold level).

* Effect of noise (Gain parameter) for non Coherent (limiter - discriminator - envelope detector) demodulation of F.M.



The modulated F.M signal present at the input of demodulator/receiver is

$$s(t) = A_c \cos\left[w_c t + 2\pi K_f \int_0^t m(t)\,dt\right] + n(t)$$

$$s(t) = A_c \cos\left[w_c t + 2\pi K_f \int_0^t m(t)\,dt\right] + n_c(t)\cos w_c t + n_s(t)\sin w_c t$$

$$s(t) = A_c \cos\left[w_c t + 2\pi K_f \int_0^t m(t)\,dt\right] + n_c(t)\cos w_c t + n_s(t)\sin w_c t$$

$$s(t) = A_c \cos[w_c t + \phi(t)] + n_c(t)\cos w_c t + n_s(t)\sin w_c t$$

$$\boxed{\text{where } \phi(t) = 2\pi K_f \int_0^t m(t)\,dt}$$

Input signal power

$$P_s = \frac{A_c^2}{2}$$

& Noise power (P_N) $= \frac{\overline{n_c^2(t)}}{2} + \frac{\overline{n_s^2(t)}}{2} = \frac{\overline{2n_c^2(t)}}{2} = n_c^2(t)$

Noise power $(P_N) = 2BN_0$

$$(SNR)_i = \frac{P_s}{P_N} = \frac{A_c^2}{4BN_0} \quad \text{—} (1)$$

Consider Ideal L.P.F with Bandwidth B at Tx & Rx. For baseband signal m(t) assumed to zero mean, wide sense stationary random process band limited to 'B' Hz.

For calculation of output signal power, we have to consider their is no input noise.

$$N_0 = 2\int_0^B \frac{N}{2}\, df = NB$$

Therefore input to the demodulator is

$$s_i(t) = A_c \cos[w_c t + \phi(t)]$$

The output of discriminator is

$$s_{dis}(t) = \frac{d s_i(t)}{dt} = \frac{d A_c \cos[w_c t + \phi(t)]}{dt}$$

$$s_{dis}(t) = A_c \sin[w_c t + \phi(t)] \left[w_c + \frac{d\phi(t)}{dt}\right]$$

As the envelope detector are assumed to be ideal and the low pass Filter (LPF) removes the component centred at 'w_c'

we get $s_o(t)\big|_{LPF} = A_c \frac{d\phi(t)}{dt} = A_c \frac{d\, 2\pi k_f \int_0^t m(t)\,dt}{dt}$

$$s_o(t)\big|_{LPF} = A_c 2\pi k_f m(t) = 2\pi A_c k_f m(t)$$

$\therefore$ output signal power $(P_s)_o = 4\pi^2 A_c^2 k_f^2 \overline{m^2(t)}$ —— (2)

Now, let us assume that the input is unmodulated F.M and noise

$$x_i(t) = A_c \cos w_c t + n_c(t)\cos w_c t + n_s(t)\sin w_c t$$

$$x_i(t) = [A_c + n_c(t)]\cos w_c t + n_s(t)\sin w_c t$$

$$x_i(t) = R_n(t)\cos[w_c t + \theta(t)] \quad \text{—— (1)}$$

where

$$R_n(t) = \sqrt{[A_c + n_c(t)]^2 + n_s^2(t)}$$

$$\theta(t) = \tan^{-1}\left(\frac{n_s(t)}{A_c + n_c(t)}\right)$$

As '$R_n(t)$' envelope is removed at discriminator, we assume high SNR

$$A_c >> n_c(t)$$

we get, $\theta(t) = \tan^{-1}\left(\frac{n_s(t)}{A_c}\right) \approx \frac{n_s(t)}{A_c}$

Input to the discriminator will be

$$x_{dis}(t) = \cos\left[w_c t + \frac{n_s(t)}{A_c}\right]$$

output of the discriminator is

$$y_{dis}(t) = \frac{d x_{dis}(t)}{dt} = \frac{d \cos[w_c t + n_s(t)/A_c]}{dt}$$

$$y_{dis}(t) = \sin\left(w_c t + \frac{n_s(t)}{A_c}\right)\left[w_c + \frac{d\, n_s(t)/A_c}{dt}\right]$$

For passing through ideal envelope detector and LPF, the output of demodulator is

$$y_o(t)\big|_{LPF} = \frac{1}{A_c}\frac{d n_s(t)}{dt}$$

since $n_s(t)$ has no mathematical representation, we now use spectral method or psdf approach.

$s_{ns}(t)$

$n_s(t) \longrightarrow$ Diffrentiator $\longrightarrow$ $s_{no}(f)$, $n_o(t)$

Therefore psdf of the signal at the output is

$$S_{no}(f) = S_{ns}(f)\,|H(f)|^2$$

As transfer function of a differentiator is

$$H(f) = j2\pi f$$

$$\therefore S_{no}(f) = N_0\,|j2\pi f|^2 = 4\pi^2 N_0 f^2 \quad (3)$$

which clearly shows that for higher frequency component, the signal will be covered with noise or will be deteriorated. To overcome this effect pre-emphasis and de-emphasis networks are used.

The Noise power is calculated

$$(P_N)_0 = \frac{1}{A_c^2}\int_{-B}^{B} S_{no}(f)\,df = \frac{1}{A_c^2}\,4\pi^2 N_0 \int_{-B}^{B} f^2\,df$$

$$(P_N)_0 = \frac{1}{A_c^2}\,4\pi^2 N_0 \left.\frac{f^3}{3}\right|_{-B}^{B}$$

$$(P_N)_0 = \frac{8\pi^2 N_0 B^3}{3A_c^2}$$

~~Now output signal to Noise ratio is~~

$$(SNR)_0 = \frac{(P_S)_0}{(P_N)_0} = \frac{\text{output signal power}}{\text{output Noise power}} = \frac{\cancel{4\pi^2} A_c^2 k_f^2 \overline{m^2(t)}}{2\,\cancel{8\pi^2} N_0 B^3} \times 3A_c^2$$

$$(SNR)_0 = \frac{3A_c^4 k_f^2 \overline{m^2(t)}}{2N_0 B^2}$$

$$\text{Detection Gain}(\gamma) = \frac{(SNR)_0}{(SNR)_i} = \frac{3A_c^4 k_f^2 \overline{m^2(t)}}{\cancel{2}N_0 B^2} \times \frac{\cancel{4}^{2}\cancel{N_0}}{\cancel{A_c^2}}$$

$$\boxed{\gamma = \frac{6A_c^2 k_f^2 \overline{m^2(t)}}{B^2}}$$

As Modulation index $\beta = \frac{\Delta f}{B} = \frac{k_f\,|m(t)|_{max}}{B}$

$$k_f = \beta \cdot B / |m(t)|_{max}$$

$$\therefore \text{Detection Gain}(\gamma) = \frac{6A_c^2 \beta^2 \cancel{B^2}\,\overline{m^2(t)}}{|m(t)|^2_{max}\,\cancel{B^2}}$$

$$\boxed{\text{Detection Gain}(\gamma) = \frac{6A_c^2 \beta^2 \overline{m^2(t)}}{|m(t)|^2_{max}}}$$

Detection Gain$(\gamma) \approx \beta^2$ which illustrates that for wide band F.M for $\beta >> 1$, the systems provides high gain on noise performance. Therefore F.M System is prefered over A.M in broadcasting.

* Threshold Effect in F·M

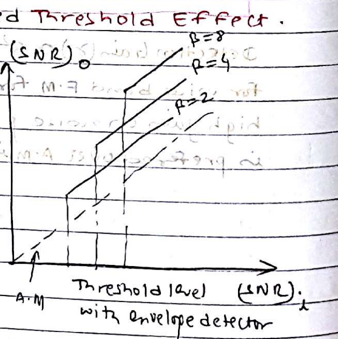
From above relation, we see the detection gain (r) is directly proportional to 'β^2'. with increase of 'β'. signal to Noise Ratio (SNR) increases.

But as mentioned by Carson's Rule B/w of F·M is

$\boxed{B_{FM} = 2(\beta+1) f_m}$ which shows increase of bandwidth with increase of modulation index 'β'. As the bandwidth increases the input Noise power also increases which will decrease input $(SNR)_i$.

For continuous increase of 'β' which relatively decreases input $(SNR)_i$, at certain point or level the output signal to Noise Ratio $(SNR)_0$ will reduce drastically which causes no reception of Frequency Modulated (F·M) signal. This point or level is called Threshold level and effect is called Threshold Effect.

Below this level for F·M 10dB i.e $(SNR)_i \le 10 dB$, the system will not receive the signal.



This threshold effect in F·M can be reduced by

a. using PLL in demodulation. PLL reduces the threshold level by 5-7 dB.

b. use of pre-emphasis and de-emphasis network. Improvement in system performance is about 6dB.

* Capture Effect :→

Capture effect is also known as locking property. If the input SNR of signal is below threshold it locks to the noise.

For example :→ If the interference is produced by another F·M wave with a frequency content close to the carrier frequency of the desired F·M wave.

When the interfered signal is stronger to the desired F·M wave. It suppresses the desired F·M wave and gets locked to the interfered signal.

Again the condition becomes exactly reversed when desired F·M signal is stronger to the interference signal. The interference signal are suppressed and gets locked to the desired F·M signal.

When both of desired and Interference signal are of nearly equal strength, the receiver fluctuates back and forth between them. This phenomena of locking is called capture Effect.

* SNR Improvement through pre-emphasis and De-emphasis

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Effect of noise increases with increase of frequency which is been discussed above. The signal to Noise Ratio (SNR) of the F.M reception and quality degrades due to increased frequency. This effect of degradation so called Threshold Effect are overcomed using pre-emphasis and de-emphasis circuit.

* pre-emphasis

Noise has greater effect on the higher modulating frequency. Such effect can be reduced, by emphasize the high frequency component prior to the modulation using pre-emphasis circuit.

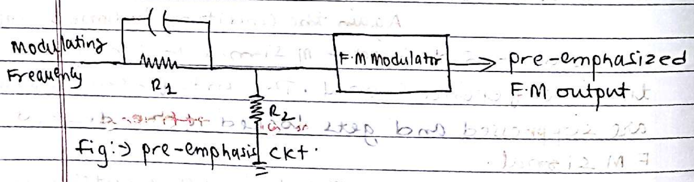

fig:-> pre-emphasis ckt.

The artificial boosting up of higher modulating frequency is achieved by increasing the amplitude of modulating frequency. This increases frequency deviation (Δf) and further increasing the modulation index (β) for higher modulating frequency.

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* De-emphasis

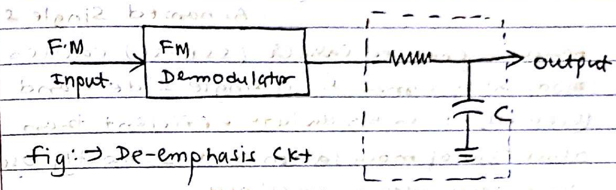


fig:-> De-emphasis ckt

The artificial boosting given to the higher modulating frequency in the pre-emphasis is nullified or compensated at the receiver called de-emphasis.

In pre-emphasis and de-emphasis ckt the low frequency and high frequency portion of the power spectral density of the message are equalized so that the message fully occupies the frequency band allocated to it. This process is then reversed at the de-emphasis ckt.

* Comparision of AM (DSB-FC, DSB-SC, SSB) and F.M Modulation

Modulation system are compared with respect to

a. Bandwidth Efficiency
b. Power Efficiency
c. System Complexity

a. **Bandwidth efficiency :→**

As named Single side band-suppressed carrier (SSB-SC) carries the modulated signal in a single side band (USB or LSB). It is bandwidth efficient than other type of modulation and is mostly used where bandwidth is concerned.

eg :→ Microwave link, Satellite Communication
TV (Vestigial side Band - suppressed carrier)

b. **Power Efficiency :→**

In terms of power Efficiency Frequency Modulation (F.M) has the highest efficiency and high level of noise immunity than other modulation scheme. DSB-AM are the least efficient system with regard to power efficiency.

eg. → F.M Stations or Broadcasting
→ space vechicle Communication.

c. **System Complexity :→**

DSB-AM are the simplest system than the system of other modulation technique. DSB-AM reception is least Complicated and used for commercial radio broadcasting. SSB-SC has the complex system as it requires precise filtering of side bands.

* **Noise performance of modulated digital system :**

Error probabilities

* **Error probabilities for coherent detection of ASK**

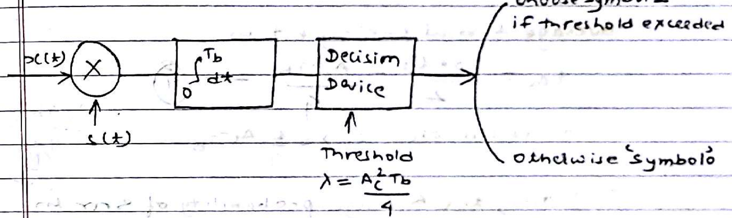


fig :→ coherent detection of binary ASK signal.

Let $s_0(t)$ and $s_1(t)$ denote the signal used to represent binary symbol '0' and '1' respectively.

For binary ASK signal

$$s_1(t) = A_c \cos(2\pi f_c t) \quad \text{symbol 1} \quad —(1)$$
$$s_0(t) = 0 \quad \text{symbol 0}$$

For ASK signal the signal energy is

$$E_0 = \int_0^{T_b} s_0^2(t)\, dt = 0 \quad —(2)$$ For transmission of symbol '0'

$$E_1 = \int_0^{T_b} s_1^2(t)\, dt$$

$$E_1 = \frac{A_c^2}{2} T_b \quad —(3)$$

For transmission of symbol 1

$$E_1 = \int_0^{T_b} A_c^2 \cos^2 2\pi f_c t\, dt$$
$$= A_c^2 \int_0^{T_b} \frac{1 + \cos 4\pi f_c t}{2} dt$$
$$= A_c^2 / T_b$$

$\cos 4\pi f_c t$ is filtered out

$N_0 \Rightarrow$ Input Noise at the receiving side

Therefore, for ASK signal, the transmitted signal energy alternates between 0 (when symbol 0 is sent) and $\frac{A_c^2 T_b}{2}$ (when symbol '1' is sent)

$\therefore$ Average signal energy bit is

$$E_{av} = \frac{E_0 + E_1}{2} = \frac{A_c^2 T_b}{4} \quad \text{--- (4)}$$

The threshold value is $\lambda = \frac{1}{4} A_c^2 T_b$ $\qquad \frac{A_c^2 T_b / 4}{N_0/2} = \frac{A_c^2 T_b}{2 N_0}$

Therefore, the Average probability of error in coherent binary ASK is

$$\boxed{P_e = \frac{1}{2} erfc\left(\frac{1}{2}\sqrt{\frac{A_c^2 T_b}{2N_0}}\right)}$$

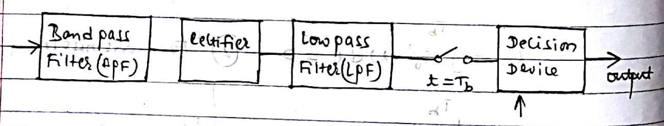
For the symmetric nature of Gaussian density function

$$\boxed{P_e = \frac{1}{2} erfc\left(\sqrt{\frac{A_c^2 T_b}{4N_0}}\right)}$$

Again for average signal energy per bit

$$\boxed{P_e = \frac{1}{2} erfc\left(\sqrt{\frac{E_{av}}{2N_0}}\right)}$$

* For Non-Coherent Detection



The error probability for Non-coherent detection is given as

$$\boxed{P_e = \frac{1}{2} \exp\left(-\frac{E_b}{2N_0}\right)}$$

$E_b \Rightarrow$ transmitted ASK signal energy per bit. $E_b = E_{av}$

$\frac{N_0}{2} \to$ p.s.d of channel noise assumed to be white Gaussian with zero mean

Error probability (P_e) for non coherent ASK is higher than that of Coherent detection but has simple arrangement of system.

* Error probability of Coherent detection of FSK

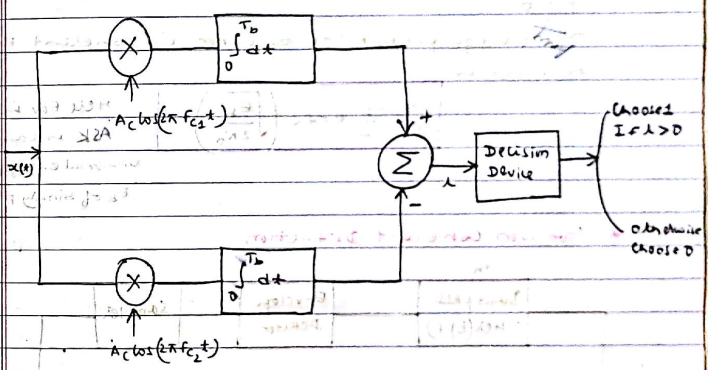


fig: $\Rightarrow$ Coherent Receiver of FSK

Let $s_0(t)$ and $s_1(t)$ represents the binary symbol 0 and 1 respectively given as

$$s_1(t) = A_c \cos(2\pi f_{c_1} t) \quad \text{Symbol 1}$$
$$s_0(t) = A_c \cos(2\pi f_{c_2} t) \quad \text{Symbol 0}$$

If f_{c_1} and f_{c_2} frequencies of the FSK signal are spaced far apart so that the signal $s_0(t)$ and $s_1(t)$ are treated as orthogonal to each other.

The average probability of error in the receiver is given as

$$P_e = \frac{1}{2}\, erfc\left(\sqrt{\frac{E_b(1-\rho)}{2N_0}}\right)$$

$\rho \Rightarrow$ absolute value of correlation coefficient

$N_0 \Rightarrow$ Noise spectral density.

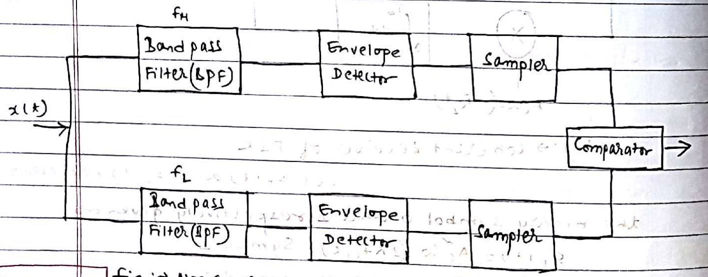
For orthogonal condition, the correlation parameter $\rho = 0$.

$\therefore$ The average probability of error in coherent FSK receiver is

$$\boxed{P_e = \frac{1}{2}\, erfc\left(\sqrt{\frac{E_b}{2N_0}}\right)}$$

Here E_{av} in binary ASK is same as the signal energy per bit E_b of binary FSK.

* For Non Coherent Detection



Theory has been explained already

fig :→ Non Coherent Detector

The average probability of error for non coherent detection binary FSK is

$$\boxed{P_e = \frac{1}{2}\exp\left(-\frac{E_b}{2N_0}\right)}$$

$E_b \Rightarrow$ transmitted FSK signal energy per bit

$\frac{N_0}{2} \Rightarrow$ p.s.d of the channel noise assumed to be white Gaussian with zero mean

* Error probability of Coherent detection of PSK

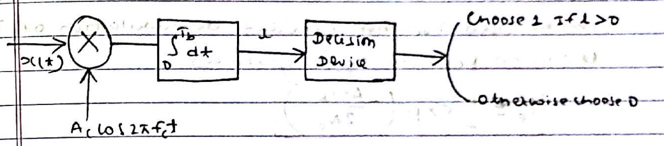


fig :→ Coherent receiver for PSK signal.

Let $S_0(t)$ and $S_1(t)$ represents the binary symbol 0 and 1 respectively.

For binary PSK signal

$S_1(t) = A_c \cos(2\pi f_c t)$ Symbol 1

$S_0(t) = A_c \cos(2\pi f_c t + \pi)$ Symbol 0

For PSK the coherent receiver are reduced to single path. This is because $S_0(t)$ is always negative to that of $S_1(t)$.

As we know the average probability of error in the receiver is given as

$$P_e = \frac{1}{2}\, erfc\left(\sqrt{\frac{E_b(1-\rho)}{2N_0}}\right)$$

$\rho \rightarrow$ absolute value of correlation coefficient

$N_0 \rightarrow$ Noise spectral density.

For PSK $\rho = -1$. Therefore average probability of error in coherent binary PSK received is

$$\boxed{P_e = \frac{1}{2}\, erfc\left(\sqrt{\frac{E_b}{N_0}}\right)}$$

The demodulation method used in PSK is always coherent. DPSK is the special case of PSK in which non-coherent detection method can be used. For non coherent demodulation of DPSK the error probability is given as

$$\boxed{P_e = \frac{1}{2}\exp\left(-\frac{A^2 T_b}{2N_0}\right)}$$

DPSK requires more power than PSK though DPSK are prefered because of its simplicity in demodulation.

* Comparision of modulated digital system in terms of bandwidth Efficiency, Power Efficiency and Complexity

System	Bandwidth	Required SNR(dB) for $P_e = 10^{-4}$	Equipment Complexity/uses
Coherent ASK	$\simeq 2r_b$	14.45dB	Moderate/ Generally not used
Non Coherent ASK	$\simeq 2r_b$	18.33dB	Minor/ Generally not used
Coherent FSK	$> 2r_b$	10.6dB	Major/ Generally not used.
Non Coherent FSK	$> 2r_b$	15.33dB	Minor/ For requirement in lowspeed data
Coherent PSK	$\simeq 2r_b$	8.45dB	Major/ For High speed Data.
DPSK	$\simeq 2r_b$	9.30dB	Moderate/ For Medium Speed Data.