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* Error Detection and Correction Algorithm

There are two important types of codes named as Block Codes and Convolution Codes used to detect and Correct the Error in Digital Communication System.

1) Linear Block Codes :→

In Linear Block codes each block of K message bits is encoded into block of n bits by adding [$q = n - k$] check bits.

Block of K message bit is represented by message data block

$M = m_1, m_2, m_3 \ldots m_K$ ~~consisting of~~ and check bits $C = c_1, c_2, c_3 \ldots c_q$

This means the q are the number of redundant bits added by the encoder. Code vector is represented by X.

$$X = (m_1, m_2 \ldots m_K \; c_1, c_2 \ldots c_q)$$

$$X = (M|C)$$

Generator matrix is represented by G which is

$$G = \left[\begin{array}{ccccc:cccc} 1 & 0 & 0 & \cdots & 0 & h_{11} & h_{21} & \cdots & h_{q1} \\ 0 & 1 & 0 & \cdots & 0 & h_{12} & h_{22} & \cdots & h_{q2} \\ \vdots & & & & & & & & \\ 0 & 0 & 0 & \cdots & 1 & h_{1k} & h_{2k} & \cdots & h_{qk} \end{array}\right]_{k \times n}$$

$I_k \ (k \times k)$ $\qquad$ $P_{k \times q}$

$$G = [I_k : P_{k \times q}]_{k \times n}$$

The Codeword or code vector is represented as

$\boxed{X = M \cdot G}$ where M is message which is generally $M = 2^k$ and G is Generator matrix.

Note:→ check bits play the role of error detection and Correction.

* Modulo Airthmetic Rules

$$\left[\begin{array}{ll} 0 \oplus 0 = 0 & 0 \oplus 1 = 1 \\ 1 \oplus 0 = 1 & 1 \oplus 1 = 0 \end{array}\right]$$

* The Generator matrix for a (6, 3) block Code is given below. Find the code word or Code vectors

$$G = \left[\begin{array}{ccc:ccc} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{array}\right]$$

I_k $\qquad$ $P_{k \times q}$

Here $k=3$, message is of 3 bits which has $2^k = 2^3 = 8$ possible message bit and 8 possible combination of code word

[000] [001] [010] [011] [100] [101] [110] [111]

* For $M = [000]$

Codevector $X = M \cdot G = [0\ 0\ 0] \begin{bmatrix} 1 & 0 & 0 & : & 0 & 1 & 1 \\ 0 & 1 & 0 & : & 1 & 0 & 1 \\ 0 & 0 & 1 & : & 1 & 1 & 0 \end{bmatrix}$

$X = [000000]$

* For $m = [001]$

$X = M \cdot G = [001] \begin{bmatrix} 1 & 0 & 0 & : & 0 & 1 & 1 \\ 0 & 1 & 0 & : & 1 & 0 & 1 \\ 0 & 0 & 1 & : & 1 & 1 & 0 \end{bmatrix}$

$X = [0\ 0\ 1\ 1\ 1\ 0]$

* For $m = [010]$

$X = M \cdot G = [010] \begin{bmatrix} 1 & 0 & 0 & : & 0 & 1 & 1 \\ 0 & 1 & 0 & : & 1 & 0 & 1 \\ 0 & 0 & 1 & : & 1 & 1 & 0 \end{bmatrix}$

$X = [010101]$

* For $m = [011]$

$X = M \cdot G = [011] \begin{bmatrix} 1 & 0 & 0 & : & 0 & 1 & 1 \\ 0 & 1 & 0 & : & 1 & 0 & 1 \\ 0 & 0 & 1 & : & 1 & 1 & 0 \end{bmatrix}$

$X = [011011]$

* For $M = [1\,0\,0]$

$$X = M \cdot G = [1\,0\,0] \begin{bmatrix} 1 & 0 & 0 & : & 0 & 1 & 1 \\ 0 & 1 & 0 & : & 1 & 0 & 1 \\ 0 & 0 & 1 & : & 1 & 1 & 0 \end{bmatrix}$$

$$X = [1\,0\,0\,0\,1\,1]$$

* For $M = [1\,0\,1]$

$$X = M \cdot G = [1\,0\,1] \begin{bmatrix} 1 & 0 & 0 & : & 0 & 1 & 1 \\ 0 & 1 & 0 & : & 1 & 0 & 1 \\ 0 & 0 & 1 & : & 1 & 1 & 0 \end{bmatrix}$$

$$X = [1\,0\,1\,1\,0\,1]$$

* For $M = [1\,1\,0]$

$$X = M \cdot G = [1\,1\,0] \begin{bmatrix} 1 & 0 & 0 & : & 0 & 1 & 1 \\ 0 & 1 & 0 & : & 1 & 0 & 1 \\ 0 & 0 & 1 & : & 1 & 1 & 0 \end{bmatrix}$$

$$X = [1\,1\,0\,1\,1\,0]$$

* For $M = [1\,1\,1]$

$$X = M \cdot G = [1\,1\,1] \begin{bmatrix} 1 & 0 & 0 & : & 0 & 1 & 1 \\ 0 & 1 & 0 & : & 1 & 0 & 1 \\ 0 & 0 & 1 & : & 1 & 1 & 0 \end{bmatrix}$$

$$X = [1\,1\,1\,0\,0\,0]$$

Message Block	Code vectors
000	[000000]
001	[001110]
010	[010101]
011	[011011]
100	[100011]
101	[101101]
110	[110110]
111	[111000]

* The Generator matrix for (7,3) block code is given below. Find the code word or code vector.

$$G = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{I_K} \vdots \underbrace{\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}}_{P_{K \times n}}$$

Here $K = 3$, Message is of 3 bits which has $2^K = 2^3 = 8$ possible message bit and 8 possible combination of code word.

[000] [001] [010] [011] [100] [101] [110] [111]

The code vector

$$X = M \cdot G$$

For M = [000]

$$X = [000] \cdot \begin{bmatrix} 1 & 0 & 0 & \vdots & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & \vdots & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & \vdots & 1 & 1 & 0 & 1 \end{bmatrix}$$

$$X = [0000000]$$

For M = 001

$$X = M \cdot G = [001] \begin{bmatrix} 1 & 0 & 0 & \vdots & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & \vdots & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & \vdots & 1 & 1 & 0 & 1 \end{bmatrix}$$

$$= [0011101]$$

For M = 010

$$X = M \cdot G = [010] \begin{bmatrix} 1 & 0 & 0 & \vdots & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & \vdots & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & \vdots & 1 & 1 & 0 & 1 \end{bmatrix}$$

$$X = \boxed{0101011}$$

For $M = 011$

$$X = M \cdot G = \begin{bmatrix} 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & : & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & : & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & : & 1 & 1 & 0 & 1 \end{bmatrix}$$

$$X = [0110110]$$

Message Block	Code vector
[000]	[0000000]
[001]	[0011101]
[010]	[0101011]
[011]	[0110110]
[100]	[1000111]
[101]	[1011010]
[110]	[1101100]
[111]	[1110001]

* The generator matrix for (7, 4) block code is given below. Find the codeword or code vector for [1101] [0110]

Here, $n = 7$, $k = 4$

$q = n - k = 7 - 4 = 3$

$$G = \begin{array}{c} \qquad I_k \qquad\qquad P_{k \times q} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & : & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & : & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & : & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & : & 1 & 1 & 1 \end{bmatrix} \end{array}$$

Here $k = 4$, Encoded message bit is 4 bit which has $2^k = 2^4 = 16$ possible combination of codeword or message bit.

M = [0000] [0001] [0010] [0011] [0100]
[0101] [0110] [0111] [1000] [1001]
[1010] [1011] [1100] [1101] [1110] [1111]

For M = 1101

$$X = M \cdot G = [1101] \begin{bmatrix} 1 & 0 & 0 & 0 & : & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & : & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & : & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & : & 1 & 1 & 1 \end{bmatrix}$$

$X = [1101001]$

For M = [0110]

$$X = M \cdot G = [0110] \begin{bmatrix} 1 & 0 & 0 & 0 & : & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & : & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & : & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & : & 1 & 1 & 1 \end{bmatrix}$$

$X = [0110011]$

* ## Parity check Matrix

The parity check matrix is used in the decoding operation. If c be the code vector which is (message + check) bits transmitted over a noisy channel.

Let R be the noise corrupted vector that was received. The vector R is the sum of original code vector C & the error vector E.

i.e $R = C + E$

$S = RH^T$ S is called error syndrome

$S = (C + E)H^T$

$S = CH^T + EH^T$

$S = EH^T \quad [\because CH^T = 0]$

H^T is the transpose of parity check matrix H. H is matrix associated with each (n, K) block codes. $(q = n - K)$

$$H = [P^T \; I_{n-K}]_{(n-K) \times n}$$

which is called Hamming codes define for (n, K) Linear Block codes

RH^T is 0 if no error

$RH^T \neq 0$ if their is error.

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* Parity Check Matrix (H) for Linear Block code.

For each block code there is $q \times n$ parity check matrix (H).

$$H = \left[P^T : I_q \right]_{q \times n}$$

where P^T is transpose of P submatrix.

* Hamming Codes

Hamming codes are defined as (n, k) Linear block codes. These codes satisfy the following conditions

1) Number of check bits $q \geq 3$
2) Block Length $n = 2^q - 1$
3) Number of message bits $k = n - q$
4) Minimum distance, $d_{min} = 3$

Coderate, $r = \frac{k}{n} = \frac{n-q}{n}$

$$r = 1 - \frac{q}{n}$$

$$r = 1 - \frac{q}{2^q - 1} \quad \left[r \simeq 1 \text{ if } q >> 1 \right]$$

* Error Detection and Correction capabilities of Hamming Codes

minimum distance, $d_{min} = 3$ of Hamming code.

It can be used to detect double errors and correct single errors.

* Basic Terms

a) Hamming weight (H.W) :→

No of 1's in code vector or word is called Hamming weight. It is represented by H.W

For eg :→ [1010001] [1011001]

H.W = 3 H.W = 4

b) Hamming Distance (d) :→

It is a number which gives the information about difference between two code vectors or code words i.e no of places where the bits differ

For eg :→ $x_1 = [1001011]$

$x_2 = [0100100]$

H.D = 6

c) Minimum Hamming Distance (dmin)

Minimum distance is denoted by dmin. It is the minimum or smallest distance between two code vectors or code words. $d_{min} \geq s+1$ ← No of error

$d_{min} \geq 2t+1$ ← correct No of error

* Syndrome Decoding : Method to Correct Errors

The parity check matrix is used in the decoding operation. If x be the code vector which is (message+check) bits transmitted over a noisy channel.

If y be the code vector received. Then

$x = y$ If there are no transmission error
$x \neq y$ Error

For every (n,k) linear block code, the parity check matrix (H) is

$$H = \left[P^T ; I_q \right]_{q \times n}$$

Transpose of H defined by H^T

$$H^T = \begin{bmatrix} P^T \\ \vdots \\ I_q \end{bmatrix}_{n \times q}$$

~~Therefore~~ $xH^T = 0$ (For all codevectors)

$yH^T = 0$ If $x = y$ No error
$yH^T \neq 0$ If $x \neq y$, Error. The non zero output of the product yH^T is called syndrome represented by S.

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$$S = y H^T$$

let us consider E is the error vector

$$y = x \oplus E$$

$$S = (x \oplus E) H^T$$

$$S = x H^T \oplus E H^T$$

$$\boxed{S = E H^T}$$

Thus indicates syndrome depends on error pattern only.

Error Correction using syndrome vector

To correct vector or to achieve the same transmitted code vector

$$\boxed{x = y \oplus E}$$

single bit error can be corrected using syndrome decoding.

X

The parity check matrix of a particular (7,4) linear block code is expressed as

$$H = \begin{array}{c} \quad\quad P^T \quad\quad\quad\quad I_q \\ \begin{bmatrix} 1 & 1 & 1 & 0 & : & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & : & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & : & 0 & 0 & 1 \end{bmatrix} \end{array}$$

i) Obtain the generator matrix (G)
ii) List all the code vectors
iii) Minimum distance between code vector
iv) How many errors can be detected?
How many errors can be corrected.
v) Show Encoder of (7,4) Hamming code.

$n = 7$, $k = 4$ $\quad q = n - k = 7 - 4 = 3$

There fore there are $M = 2^4 = 16$ combination of message bit.

$$P^T = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

$$P = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}_{k \times q}$$

$$G = \left[I_k : P_{k \times q} \right]_{k \times n}$$

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & : & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & : & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & : & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & : & 0 & 1 & 1 \end{bmatrix}_{k \times n}$$

Now, check vector

$$c = M \cdot P$$

$$c = (m_1\ m_2\ m_3\ m_4) \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$c_1 = 1 \times m_1 \oplus 1 \times m_2 \oplus 1 \times m_3 \oplus 0 \times m_4$$

$$c_1 = m_1 \oplus m_2 \oplus m_3$$

$$c_2 = 1 \times m_1 \oplus 1 \times m_2 \oplus 0 \times m_3 \oplus 1 \times m_4$$

$$c_2 = m_1 \oplus m_2 \oplus m_4$$

$$c_3 = 1 \times m_1 \oplus 0 \times m_2 \oplus 1 \times m_3 \oplus 1 \times m_4$$

$$c_3 = m_1 \oplus 0 \oplus m_3 \oplus m_4$$

$$c_3 = m_1 \oplus m_3 \oplus m_4$$

Therefore, $c_1 = m_1 \oplus m_2 \oplus m_3$

$$c_2 = m_1 \oplus m_2 \oplus m_4$$

$$c_3 = m_1 \oplus m_3 \oplus m_4$$

Codevector $x = [M \mid C] = [M : C]$

for $M = 0000$

$c_1 = 0$

$c_2 = 0$

$c_3 = 0$

Codevector

$$x = [0000 : 000]$$

For $M = 0001$

$$c_1 = m_1 \oplus m_2 \oplus m_3 = 0$$

$$c_2 = m_1 \oplus m_2 \oplus m_4 = 1$$

$$c_3 = m_1 \oplus m_3 \oplus m_4 = 1$$

Code vector $X = M/C = [M : C]$

$$X = [0001 : 011]$$

Message Block	Checkbits	Codevector	Weight of Codevector (Hamming weight)
$m_1\ m_2\ m_3\ m_4$	$c_1\ c_2\ c_3$	$m_1\ m_2\ m_3\ m_4\ c_1\ c_2\ c_3$	
0000	000	0000000	0
0001	011	0001011	3
0010	101	0010101	3
0011	110	0011110	4
0100	110	0100110	3
0101	101	0101101	4
0110	011	0110011	4
0111	000	0111000	3
1000	111	1000111	4
1001	100	1001100	3
1010	010	1010010	3
1011	001	1011001	4
1100	001	1100001	3
1101	010	1101010	4
1110	100	1110100	4
1111	111	1111111	7

As we know,

$d_{min} = 3$ ← No of error

$d_{min} \geq s + 1$

$3 \geq s+1$

$s \leq 2$

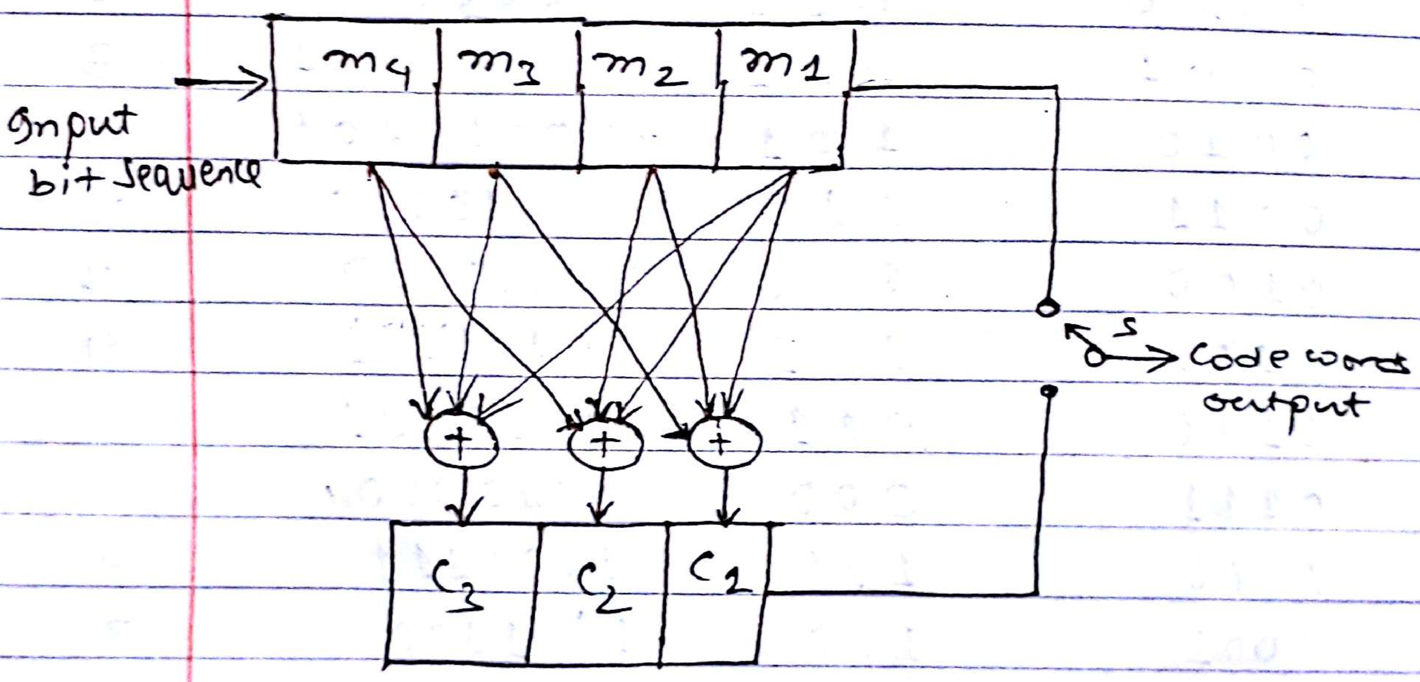
Hence two errors will be detected

Correct No of error

$d_{min} \geq 2t+1$

$3 \geq 2t+1$

$t \leq 1$ Therefore one error can be corrected



Check bit Register

X

* The parity check matrix of a (7, 4) Hamming code is expressed as under

$$H = \begin{bmatrix} 1 & 1 & 1 & 0 & \vdots & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & \vdots & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & \vdots & 0 & 0 & 1 \end{bmatrix}_{3 \times 7}$$

Evaluate the syndrome vector for single bit errors. If the transmitted code vector

~~This is a (7,4) linear block code~~

~~$n = 7,\ k = 4 \quad q = n - k = 7 - 4 = 3$~~

is $x = [1001110]$ and received vector $y = [1011110]$ prove $x = y \oplus E$

We know syndrome is q bit vector. Hence there will be $2^3 - 1 = 7$ Non-zero syndromes which shows single error pattern E is

Error vector : single bit error pattern	Syndrome vector
1 0 0 0 0 0 0	1 0 1
0 1 0 0 0 0 0	1 1 1
0 0 1 0 0 0 0	1 1 0
0 0 0 1 0 0 0	0 1 1
0 0 0 0 1 0 0	1 0 0
0 0 0 0 0 1 0	0 1 0
0 0 0 0 0 0 1	0 0 1

Transpose of H,

$$H^T = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$S = EH^T = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$S = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix}$$

Syndrome vector for first bit in error [1 0 1]

Syndrome vector for second bit in error

$$S = EH^T = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$S = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$$

$x = [1\,0\,0\,1\,1\,0] \quad y = [1\,0\,1\,1\,1\,1\,0]$

$$S = yH^T = [1\,0\,1\,1\,1\,1\,0] \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$S = [1\,1\,0]$$

$$S = yH^T = EH^T = [1\,1\,0]$$

$$E = [0\,0\,1\,0\,0\,0\,0]$$

$$x = y \oplus E = [1\,0\,1\,1\,1\,1\,0] + [0\,0\,1\,0\,0\,0\,0]$$

$$x = [1\,0\,0\,1\,1\,1\,0]$$

which is same as transmitted code vector

* For a (6,3) code, the parity check matrix is given by

$$H = \left[\begin{array}{ccc:ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

Determine whether a received code vector has no error. Received code vector is 100101

solution :→

$$H^T = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Received code vector $y = 100101$

$$S = yH^T = [100101] \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$S = [000]$$

• Syndrome vector is zero there are no errors in the received code vector.

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$\times$

* A $\overset{n,k}{(6,3)}$ Linear Block code is generated according to the following generator

$$G = \begin{array}{c} \begin{array}{cc} I_k & P_{k \times n} \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & : & 1 & 0 & 1 \\ 0 & 1 & 0 & : & 0 & 1 & 1 \\ 0 & 0 & 1 & : & 1 & 1 & 0 \end{bmatrix} \end{array}$$

For a particular codeword transmitted the received code vector is 100011. Find the corresponding data word transmitted

$$H = \begin{array}{c} \begin{array}{cc} P^T & I_q \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & : & 1 & 0 & 0 \\ 0 & 1 & 1 & : & 0 & 1 & 0 \\ 1 & 1 & 0 & : & 0 & 0 & 1 \end{bmatrix} \end{array}$$

$$H^T = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$y = [100011]$$

$$S = y \cdot H^T = [100011] \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$S = [110]$ which ~~[illegible]~~ has third bit of code vector corrupted.

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where

$E = [e_1\ e_2\ e_3\ e_4\ e_5\ e_6]$

$S = EH^T$

$$S = [e_1, e_2, e_3, e_4, e_5, e_6] \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = [1\,1\,0]$$

on solving we get

$e_1 + e_3 + e_4 = 1$

$e_2 + e_3 + e_5 = 1$

$e_1 + e_2 + e_6 = 0$

various error vector with single bit error are shown as

e_1	e_2	e_3	e_4	e_5	e_6
1	0	0	0	0	0
0	1	0	0	0	0
0	0	1	0	0	1
0	0	0	1	0	0
0	0	0	0	1	0
	0			0	1

As the error is in third bit, the correspond-ing error vector is

$$E = 001000$$

$$Y = 100011$$

$$X = Y + E$$

$$X = 101011$$

* For a (7, 4) block code is generated ~~[illegible]~~ according to the following parity check matrix H

$$H = \begin{array}{c} \quad P^T \qquad\qquad I_q \\ \begin{bmatrix} 1 & 1 & 1 & 0 & : & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & : & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & : & 0 & 0 & 1 \end{bmatrix} \end{array}$$

The received code word Y is 1000011 for a transmitted code word X. Find the corresponding data transmitted.

$$H^T = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$Y = \begin{bmatrix} 1000011 \end{bmatrix}$$

$$S = YH^T = [1000011] \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$S = [100]$$

$$S = EH^T = e_1 e_2 e_3 e_4 e_5 e_6 e_7 \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 100$$

$e_1 + e_2 + e_3 + e_5 = 1$

$e_1 + e_2 + e_4 + e_6 = 0$

$e_1 + e_3 + e_4 + e_7 = 0$

Various Error vector with single bit error are shown as

e_1	e_2	e_3	e_4	e_5	e_6	e_7
1	0	0	0	0	0	0
0	1	0	0	0	0	0
0	0	1	0	0	0	0
0	0	0	1	0	0	0
0	0	0	0	1	0	0
0	0	0	0	0	1	0
0	0	0	0	0	0	1

As the error is in fifth bit, the Corresponding error vector is

$$E = 0000100$$

$$X = Y + E = [1000011] + 0000100$$

$$X = [1000111]$$

Data word $(M) = 1000$

* Given a (6,3) Linear Block code with the following parity-check matrix H:

$$H = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

i) Find the Generator matrix G.

ii) Find the code word for the data bit 101.

soln :→

parity check matrix

$$H = \begin{array}{c} \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \\ \quad P_T \qquad\quad I_q \end{array}$$

$$n = 6, \; k = 3 \quad q = n - k = 6 - 3 = 3$$

$$G = [I_K \; P]$$

$$P^T = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \qquad P = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$G = [I_K \; P_{K \times q}]$$

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Code word for data bit 101

$$X = M \cdot G$$

$$X = 101 \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$X = [101010]$$

$$X = [101010]$$

* For the (7, 4) Hamming code, the parity check matrix H is given by

$$H = \begin{array}{c} \quad P^T \qquad\qquad I_q \\ \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

i) Construct the Generator matrix
ii) The code word that begins with 1010.
iii) If the received Codeword Y is 0111100, then decode the received codeword.

$$H = \begin{array}{c} \begin{array}{cc} P^T & I_q \end{array} \\ \begin{bmatrix} 1\,0\,1\,1 & 1\,0\,0 \\ 1\,1\,0\,1 & 0\,1\,0 \\ 0\,1\,1\,1 & 0\,0\,1 \end{bmatrix} \end{array}$$

$$P^T = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix} \qquad G = [I_K : P]$$

$$P = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} \qquad n = 7,\ k = 4$$

$$q = n - k = 7 - 4 = 3$$

$$G = \begin{bmatrix} 1\,0\,0\,0 & 1\,1\,0 \\ 0\,1\,0\,0 & 0\,1\,1 \\ 0\,0\,1\,0 & 1\,0\,1 \\ 0\,0\,0\,1 & 1\,1\,1 \end{bmatrix}$$

Code word for 1010

$$X = M \cdot G = 1010 \begin{bmatrix} 1000\ 110 \\ 0100\ 011 \\ 0010\ 101 \\ 0001\ 111 \end{bmatrix}$$

$$X = [1010011]$$

$y = [0\,1\,1\,1\,1\,1\,0\,0]$

$$S = yH^T = [0111100] \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$S = [1\,0\,1]$$

* For a (7, 4) block code generated by G below. Explain how the error syndrome helps in correcting a single error.

$$G = \begin{array}{cc} I_K & P_{K\times q} \\ \end{array}$$

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

we have $H = [P^T : I_q]$

$$H = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Let message be 1101

$X = M \cdot G =$

$$x$$

$$X = \begin{bmatrix} 1 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

Let $y = 1\,1\,1\,1\,0\,1\,0$ (3^{rd} bit in error)

$$s = yH^T \qquad H^T = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$s = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$s = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix}$$

$s = EH^T$ which is equal to third ~~bit~~ ~~column~~ row of H^T matrix which indicates their is error in third bit.

* Design a (7,3) block code generated by G. From this generated matrix. Calculate the total value of code vector X and also calculate the value of error in which 4th column of code vector is transmitted with error.

$$G = \begin{array}{c} \quad I_K \qquad\quad P_{K\times q} \\ \left[\begin{array}{ccc:cccc} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 \end{array}\right] \end{array}$$

$n = 7$, $K = 3$ check bits $q = n - K = 4$

Message of 3 bit which has $2^K = 8$ possible combination of code word.

Message bits M are

(000) (001) (010) (011) (100) (101) (110) (111)

Codevector $X = M \cdot G = [0\,0\,0] \begin{bmatrix} 1 & 0 & 0 & : & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & : & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & : & 1 & 1 & 0 & 1 \end{bmatrix}$

$X = [0000000]$

Code vector for $M = 001$

$X = M \cdot G = [001] \begin{bmatrix} 1 & 0 & 0 & : & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & : & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & : & 1 & 1 & 0 & 1 \end{bmatrix}$

$X = [0011101]$

For $M = 010$

code vector $X = M \cdot G = [010] \begin{bmatrix} 1 & 0 & 0 & : & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & : & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & : & 1 & 1 & 0 & 1 \end{bmatrix}$

$X = [0101011]$

Message Block	Code vector
[000]	[0000000]
[001]	[0011101]
[010]	[0101011]
[011]	[0110110]
[100]	[1000111]
[101]	[1011010]
[110]	[1101100]
[111]	[1110001]

$$H = [P^T : I_q]$$

$$P = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix} \quad P^T = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\begin{array}{cc} P^T & I_q \end{array}$$

$$H = \left[\begin{array}{ccc:cccc} 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \quad H^T = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For any code vector

$$x = [0011101]$$

$$E = [0010101]$$

$$S = EH^T = [0010101] \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$S = [1000]$$

It is equal to 4th row of H^T matrix. So signal is corrupted on the 4th column of code vector.

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Design a (7, 4) generator matrix and mathematically prove that 2nd bit of code vector which is corrupted by noise is equal to the 2nd row of H^T matrix.

Here, $n = 7$, $k = 4$

$q = n - k = 7 - 4 = 3$ check bits

Message of 4 bits which has $2^4 = 16$ possible combination of code word.

$M = [0000]\ [0001]\ [0010]\ [0011]$
$[0100]\ [0101]\ [0110]\ [0111]$
$[1000]\ [1001]\ [1010]\ [1011]$
$[1100]\ [1101]\ [1110]\ [1111]$

For $M = [1001]$

Code vector $X = M \cdot G$

$$X = [1001] \begin{bmatrix} 1000 : 011 \\ 0100 : 101 \\ 0010 : 110 \\ 0001 : 111 \end{bmatrix}$$

$$X = [1001100]$$

$$G = \begin{array}{c} \qquad I_k \qquad\qquad P_{k \times q} \\ \begin{bmatrix} 1000 : 011 \\ 0100 : 101 \\ 0010 : 110 \\ 0001 : 111 \end{bmatrix} \end{array}$$

$$H = [P^T : I_q]$$

$$P = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \qquad P^T = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

$$H = \begin{bmatrix} 0 & 1 & 1 & 1 & : & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & : & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & : & 0 & 0 & 1 \end{bmatrix}$$

$$H^T = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

If 2^{nd} bit is corrupted then

code vector $X = [1\,0\,0\,1\,1\,0\,0]$
$\uparrow$

$$E = [1\,1\,0\,1\,1\,0\,0]$$

$$S = EH^T = [1\,1\,0\,1\,1\,0\,0] \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$S = [1\,0\,1]$$

is same as the 2^{nd} row of H^T.
Therefore, in code vector X, 2^{nd} bit is corrupted by noise.

* Cyclic Codes :→

Cyclic codes is the subclass of Linear Block codes. They have the property that a cyclic shift of one code word produces another codeword.

For n-bit code vector

$$X = (x_{n-1}, x_{n-2} + x_{n-3} + \dots x_1, x_0)$$

Here, x_{n-1}, x_{n-2}, x_{n-3}, x_1, x_0 represent individual bits of the code vector X. If we shift the code vector cyclically then we get another code vector

$$X' = (x_{n-2}, x_{n-3}, \dots x_1, x_0, x_{n-1})$$

Similarly, cyclic code polynomial $c(x)$ can be generated by the data polynomial $d(x)$ of degree $(k-1)$ and a generator polynomial of $g(x)$ of degree $(n-k)$

$$c(x) = d(x)\, g(x)$$

where,

$$g(x) = x^{q} + g_{q-1} x^{q-1} + \dots g_1 x + 1$$

For finding rows of generating polynomial x^i

$$x^i g(x) = x^{i+q} + \overset{g_{q-1}}{x^{i+q-1}} \dots g_1 x^{i+1} + x^i$$

The generator polynomial of a (7, 4) cyclic code is $G(p) = p^3 + p + 1$. obtain all the code vectors for the code. (0, 1, 0, 1)

solution :→

Here, $n = 7$, $K = 4$ $q = n - K = 7 - 4 = 3$

There will be total $2^K = 2^4 = 16$ message vectors ~~for~~ of 4 each.

$x(M : C)$

$M = m_1 \, m_2 \, m_3 \, m_4$

$5K$

For $M = (0\,1\,0\,1)$
$\quad\quad\quad\;\; 3\,2\,1\,0$

$$m(p) = m_3 p^3 + m_2 p^2 + m_1 p + m_0$$

$$m(p) = p^2 + 1$$

Given, generator polynomial

$$G(p) = p^3 + p + 1$$

$p^3(1 \oplus 1)$

$$x(p) = m(p) \times G(p)$$

$p^3($

$$x(p) = (p^2 + 1)(p^3 + p + 1)$$

$$x(p) = p^5 + p^3 + p^2 + p^3 + p + 1$$

$$x(p) = p^5 + p^2 + p + 1$$

Degree of above polynomial is $n - 1 = 6$.

The code vector is

$$x(p) = 0 \times p^6 + 1 \times p^5 + 0 \times p^4 + 0 \times p^3 + 1 \times p^2 + 1 \times p + 1$$

$$X = (0\,1\,0\,0\,1\,1\,1)$$

7

* Find a Generator matrix corresponding to $g(x) = x^3 + x^2 + 1$ for a (7,4) Cyclic code.

solution :→

Here, $n = 7$, $k = 4$ $q = n - k = 3$

since, $k - 1 = 4 - 1 = 3$

value of $i = 0, 1, 2, 3$

$x^i g(x) = x^{i+q} + g_{q-1} x^{i+q-1} \ldots + x^i$

For $i = 3$,

$x^3 g(x) = x^6 + x^5 + x^3$

For $i = 2$

$x^2 g(x) = x^5 + x^4 + x^2$

For row 3, $i = 1$

$x g(x) = x^4 + x^3 + x$

For row 4, $i = 0$

$g(x) = x^3 + x^2 + 1$

In matrix form

$$G_{(4\times 7)} = \begin{array}{c} \begin{array}{ccccccc} x^6 & x^5 & x^4 & x^3 & x^2 & x & x^0 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}_{4\times 7} \end{array}$$

For a generator matrix $g(x) = x^3 + x^2 + 1$ for cyclic code (7, 4). Find code vectors for the following data vectors: 1010, 1111, 0001 and 1000.

Generator matrix $g(x) = x^2 + x^2 + 1$

For data vector $d = [1010]$

$$d(x) = x^3 + x$$

Code polynomial is

$$c(x) = d(x) \cdot g(x)$$

$$c(x) = (x^3 + x)(x^3 + x^2 + 1)$$

$$c(x) = x^6 + x^5 + x^3 + x^4 + x^2 + x$$

$$c(x) = x^6 + x^5 + x^3(1 \oplus 1) + x^4 + x$$

$$c(x) = x^6 + x^5 + x^4 + x$$

Hence Codevector $(1\,1\,1\,0\,0\,1\,0)$

For $d = [1111]$

$$d(x) = x^3 + x^2 + x + 1$$

$$c(x) = d(x) \cdot g(x)$$

$$= (x^3 + x^2 + x + 1)(x^3 + x)$$

$$= x^6 + x^5 + x^3 + x^5 + x^4 + x^2 + x^4 + x^3 + x + x^3 + x^2 + 1$$

$$= x^6 + x^5(1 \oplus 1) + x^3(1 \oplus 1 \oplus 1) + x^4(1 \oplus 1) + x^2(1 \oplus 1)$$

$$+ x + 1$$

$$c(x) = x^6 + x^3 + x + 1$$

Hence code vector $[1001011]$

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For $d = [0001]$

$d(x) = 1$

$c(x) = d(x) \cdot g(x)$

$= 1 \cdot (x^3 + x^2 + 1)$

$c(x) = x^3 + x^2 + 1$

Hence, codevector (0001101)

For $d = [1000]$

$d(x) = x^3$

$c(x) = d(x) \cdot g(x) = x^3(x^3 + x^2 + 1)$

$c(x) = x^6 + x^5 + x^3$

codevector $[1,101000]$ # Ans

X

* Find a generator matrix corresponding to $g(x) = x^3 + x^2 + 1$ for a (7, 4) cyclic code.

solution :→

Here $n = 7$, $k = 4$, $\cancel{q} = n - k = 3$

since $k - 1 = 4 - 1 = 3$

value of $i = 0, 1, 2, 3$

$$x^i g(x) = x^{i+\cancel{q}} + x^{i+\cancel{q}-1} \dots + x^i$$

For row 1, $i = 3$

$$x^3 g(x) = x^6 + x^5 + x^3$$

For row 2, $i = 2$

$$x^2 g(x) = x^5 + x^4 + x^2$$

For row 3, $i = 1$

$$x\, g(x) = x^4 + x^3 + x$$

For row 4, $i = 0$

$$g(x) = x^3 + x^2 + 1$$

In matrix form

$$G_{(4\times7)} = \begin{array}{c} \begin{matrix} x^6 & x^5 & x^4 & x^3 & x^2 & x & x^0 \end{matrix} \\ \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}_{4\times7} \end{array}$$

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* For a generator matrix $g(x) = x^3 + x^2 + 1$ for cyclic code (7, 4), Find code vectors for the following data vectors : 1010, 1111, 0001, and 1000

Here, $n = 7$, $k = 4$ $q = n - k = 3$

Generator matrix is

$$g(x) = x^3 + x^2 + 1$$

For data vector $d = [1010]$

$$d(x) = x^3 + x$$

Code polynomial is

$$c(x) = d(x)\, g(x)$$
$$= (x^3 + x)(x^3 + x^2 + 1)$$
$$= x^6 + x^5 + x^3 + x^4 + x^3 + x$$
$$= x^6 + x^5 + x^3(1 \oplus 1) + x^4 + x$$
$$\therefore c(x) = x^6 + x^5 + x^4 + x$$

Hence Code vector (1 1 1 0 0 1 0)

For $d = [1111]$

$$d(x) = x^3 + x^2 + x + 1$$

$$c(x) = d(x) \cdot g(x)$$
$$= (x^3 + x^2 + x + 1)(x^3 + x^2 + 1)$$
$$= x^6 + x^5 + x^3 + x^5 + x^4 + x^2 + x^4 + x^3 + x + x^3 + x^2 + 1$$
$$= x^6 + x^5(1 \oplus 1) + x^3(1 \oplus 1 \oplus 1) + x^4(1 \oplus 1) + x^2(1 \oplus 1) + x + 1$$

$= x^6 + x^3 + x + 1$

Hence, Code vector [1 0 0 1 0 1 1]

For d = [0 0 0 1]

$d(x) = 1$

$c(x) = d(x) \cdot g(x)$

$= 1 \cdot (x^3 + x^2 + 1)$

$c(x) = x^3 + x^2 + 1$

Hence, codevector [0 0 0 1 1 0 1]

For d = [1 0 0 0]

$d(x) = x^3$

$c(x) = d(x) \cdot g(x)$

$= x^3 [x^3 + x^2 + 1] = x^6 + x^5 + x^3$

$c(x) = x^6 + x^5 + x^3$

Hence, Code vector [1 1 0 1 0 0 0]

* Systematic cyclic codes.

Systematic Code

for code word polynomial $c(x)$ corresponding to data polynomial $d(x)$ is given by

$$c(x) = x^{n-k} d(x) + R(x)$$

Where $R(x)$ is remainder from dividing $x^{n-k} d(x)$ by $g(x)$

$$\boxed{R(x) = \frac{x^{n-k} d(x)}{g(x)}}$$

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* Construct a systematic (7, 4) cyclic Code using a generator polynomial

$$g(x) = x^3 + x^2 + 1$$

for data vector $d = [1010]$

$$d(x) = x^3 + x$$

Here, $n = 7$, $k = 4$

$$x^{n-k} d(x) = x^3(x^3 + x) = x^6 + x^4$$

$$x^3 - x^2 - 1$$

$$x^3 + x^2 + 1 \;\big)\; \cancel{x^6} + x^4$$

$$\cancel{x^6} + x^5 + x^3$$

$$- \quad - \quad -$$

$$-\cancel{x^5} + \cancel{x^4} - x^3$$

$$-\cancel{x^5} - \cancel{x^4} - x^2$$

$$+ \quad + \quad +$$

$$-\cancel{x^3} + x^2$$

$$-\cancel{x^3} - x^2 - 1$$

$$+ \quad + \quad +$$

$$x^2(1 \oplus 1) + 1 = 1$$

Therefore,

$$C(x) = x^{n-k} d(x) + R(x)$$

$$C(x) = x^6 + x^4 + 1$$

Code vector $(10\,10001) = [1010001]$

Continue :→

Similarly for data vector $d = [1111]$

$$d(x) = x^3 + x^2 + x + 1$$

$$x^{n-k} d(x) = x^3(x^3 + x^2 + x + 1)$$

$$x^{n-k} d(x) = x^6 + x^5 + x^4 + x^3$$

Hence,

$$\begin{array}{r|l} & x^3 + x - 1 \\ \hline x^3 + x^2 + 1 & \cancel{x^6} + \cancel{x^5} + x^4 + \cancel{x^3} \\ & \cancel{x^6} + \cancel{x^5} + \cancel{x^3} \\ & - \quad - \quad - \\ \hline & \cancel{x^4} \\ & -\cancel{x^4} \pm x^3 \pm x \\ \hline & -\cancel{x^3} - x \\ & -\cancel{x^3} - x^2 - 1 \\ & + \quad + \quad + \\ \hline & x^2 - x + 1 \end{array}$$

$x^3(x^3 + x^2 + x + 1) +$

$x^6 + x^5 + x^4 + x^3 + x^2 - x + 1$

(1 1 1 1 1 1 1)

$1 - \frac{1}{x}$

Therefore, $C(x) = x^{n-k} d(x) + R(x)$

$$C(x) = x^6 + x^5 + x^4 + x^3 + x^2 - x + 1$$

Code vector = (1 1 1 1 1 1 1)

Data vector (d)	Code vector (c)
0000	0000000
0001	0001101
0010	0010111
0011	0011010
0100	0100011
0101	0101110
0110	0110100
0111	0111001
1000	1000110
1001	1001011

1010	1010001
1011	1011100
1100	1100101
1101	1101000
~~1110~~	~~1110010~~
1111	1111111

For data vector d = 1011

$$d(x) = x^3 + x + 1$$

$$x^{n-k} d(x) = x^3 (x^3 + x + 1)$$

$$= x^6 + x^4 + x^3$$

Hence

$$x^3 - x^2$$

$$x^3 + x^2 + 1 \,\big)\, x^6 + x^4 + x^3$$

$$x^6 + x^5 + x^3$$

$$- \quad - \quad -$$

$$-x^5 + x^4$$

$$-x^5 - x^4 - x^2$$

$$+ \quad + \quad +$$

$$x^2$$

$$c(x) = x^{n-k} d(x) + R(x)$$

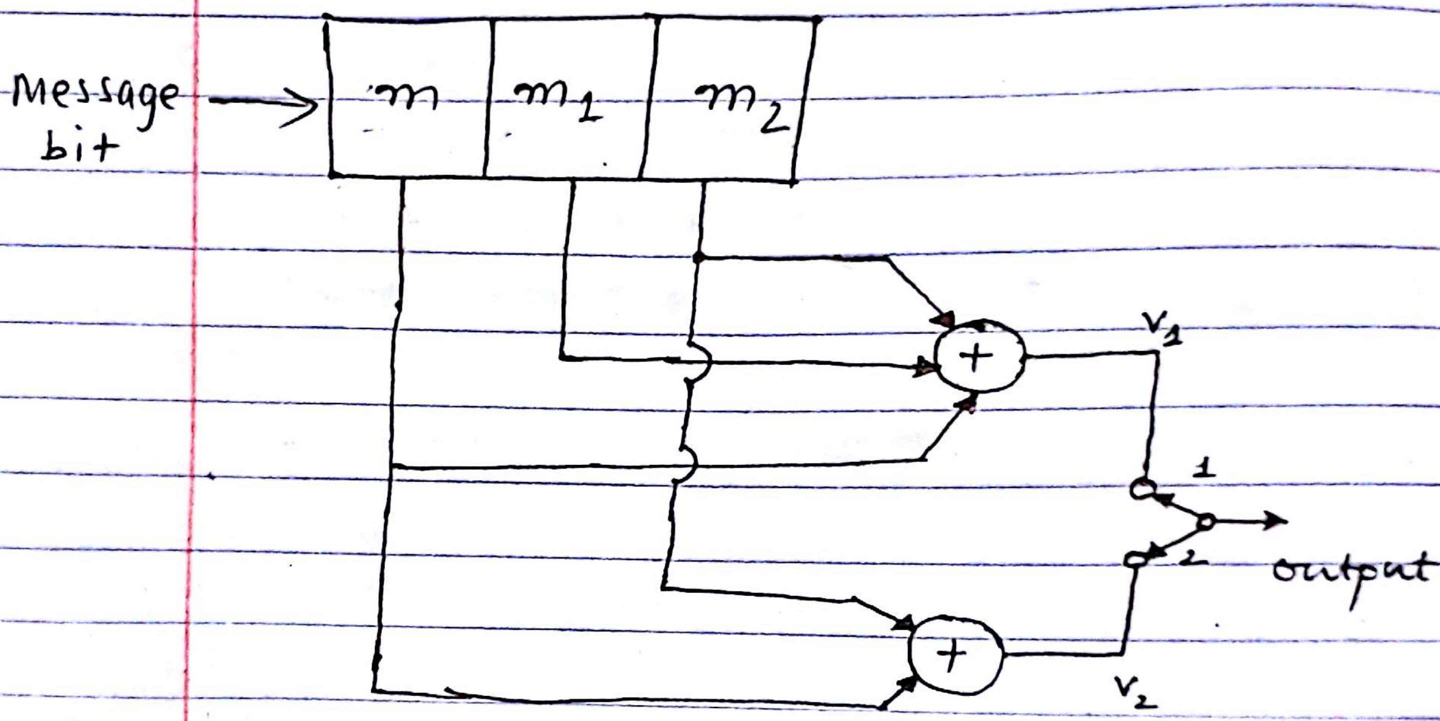
$$= x^6 + x^4 + x^3 + x^2$$

$$c(x) = x^6 + x^4 + x^3 + x^2$$

code vector (1011100)

* Convolutional Codes

In convolutional coding the input bits are stored in the shift register and combined with mod 2 adders. This operation is equal to binary convolution and hence is called convolutional coding.



When message bit is shifted to m, new value of v_1 and v_2 are generated depending upon $m, m_1,$ and m_2. m_1, m_2 stores previous two message bits and current bit is in shift register m.

Therefore, output

$$v_1 = m_1 \oplus m_2 \oplus m$$

$$v_2 = m \oplus m_2$$

output then samples v_1 and v_2. The shift register then shifts contents of m to m_1 and m_1 to m_2. Next input is taken and stored in m.

The output is then again sampled. Thus the output bit stream for successive input

$$x = v_1 v_2 v_1 v_2 v_1 v_2 \dots$$

From above we can determine for single message bit the encoded code word or output is two bits.

Number of message bits $k = 1$.

Number of encoded output bits $n = 2$

Therfore coderate of encoder

$$r = k/n = \frac{1}{2}$$

For input digit = 11010

$m, m_1, m_2 = 000$ [cleared]

1^{st} step,

$m = 1 \quad m_1 = 0 \quad m_2 = 0$

$v_1 = m \oplus m_1 \oplus m_2$

$v_1 = 1 \oplus 0 \oplus 0 = 1$

$v_2 = m \oplus m_2 = 1 \oplus 0 = 1$

The output is sampled and hence coder output is

$v_1 v_2 = 1\,1$

2nd step

$m = 1, \; m_1 = 1 \quad m_2 = 0$

$v_1 = m \oplus m_1 \oplus m_2$

$v_1 = 1 \oplus 1 \oplus 0 = 0$

$v_2 = m \oplus m_2 = 1 \oplus 0 = 1$

The output is sampled and hence coder output is $v_1 v_2 = 0\,1$

3rd step

$m = 0 \quad m_1 = 1 \quad m_2 = 1$

$v_1 = m \oplus m_1 \oplus m_2 = 0 \oplus 1 \oplus 1 = 0$

$v_2 = m \oplus m_2 = 0 \oplus 1 = 1$

The output is sampled and hence coder output is $v_1 v_2 = 0\,1$

4th step

$m = 1 \quad m_1 = 0 \quad m_2 = 1$

$v_1 = m \oplus m_1 \oplus m_2 = 1 \oplus 0 \oplus 1 = 0$

$v_2 = m \oplus m_2 = 1 \oplus 1 = 0$

The output is sampled and hence coder output is $v_1 v_2 = 0\ 0$

5th step

$m = 0 \quad m_1 = 1 \quad m_2 = 0$

$v_1 = m \oplus m_1 \oplus m_2 = 0 \oplus 1 \oplus 0 = 1$

$v_2 = m \oplus m_2 = 0 \oplus 0 = 0$

The output is sampled and hence coder output is

$v_1 v_2 = 1\ 0$

* **Advantage of Cyclic Codes**

1) Error correcting and decoding methods of cyclic codes are simple and easy to implement

2) Encoders and Decoders are simple.

* **Disadvantage**

1) Error detection is simpler but error correction is complicated.

Linear Block codes	Convolutional codes
1) For the block of K message bits, n-k parity or check bits are added	1) In this method n code digits generated by encoder depends on preceding N-1 block of message.
2) used for error detection	2) used for error detection and correction
3) shift registers are not used	3) shift register is used for encoding.

Advantages of convolutional codes

a) Decoding delay is small since they operate on smaller blocks of data.

b) The storage hardware required by convolutional decoder is less since the block size are smaller.

c) synchronization problem doesnot affect performance

* Disadvantage

a) Convolution codes are difficult to analyze.

For the convolution encoding, there are three graphical representation widely used.

1) Code Tree
2) Code Trellis
3) State Diagram.

1) Code Tree :→ Initially we assume the register is cleared so that it contain all zeros when the first bit m_0 arrives.

The encoder output

$$x_1 = m_0 \oplus m_1 \oplus m_2$$

If $m_0 = 0$ $\quad x_1 = 0 \oplus 0 \oplus 0 = 0$

Similarly $x_2 = m_0 \oplus m_2 = 0 \oplus 0 = 0$

$\therefore \; x_1 x_2 = 00$

If $m_0 = 1$, Then we have

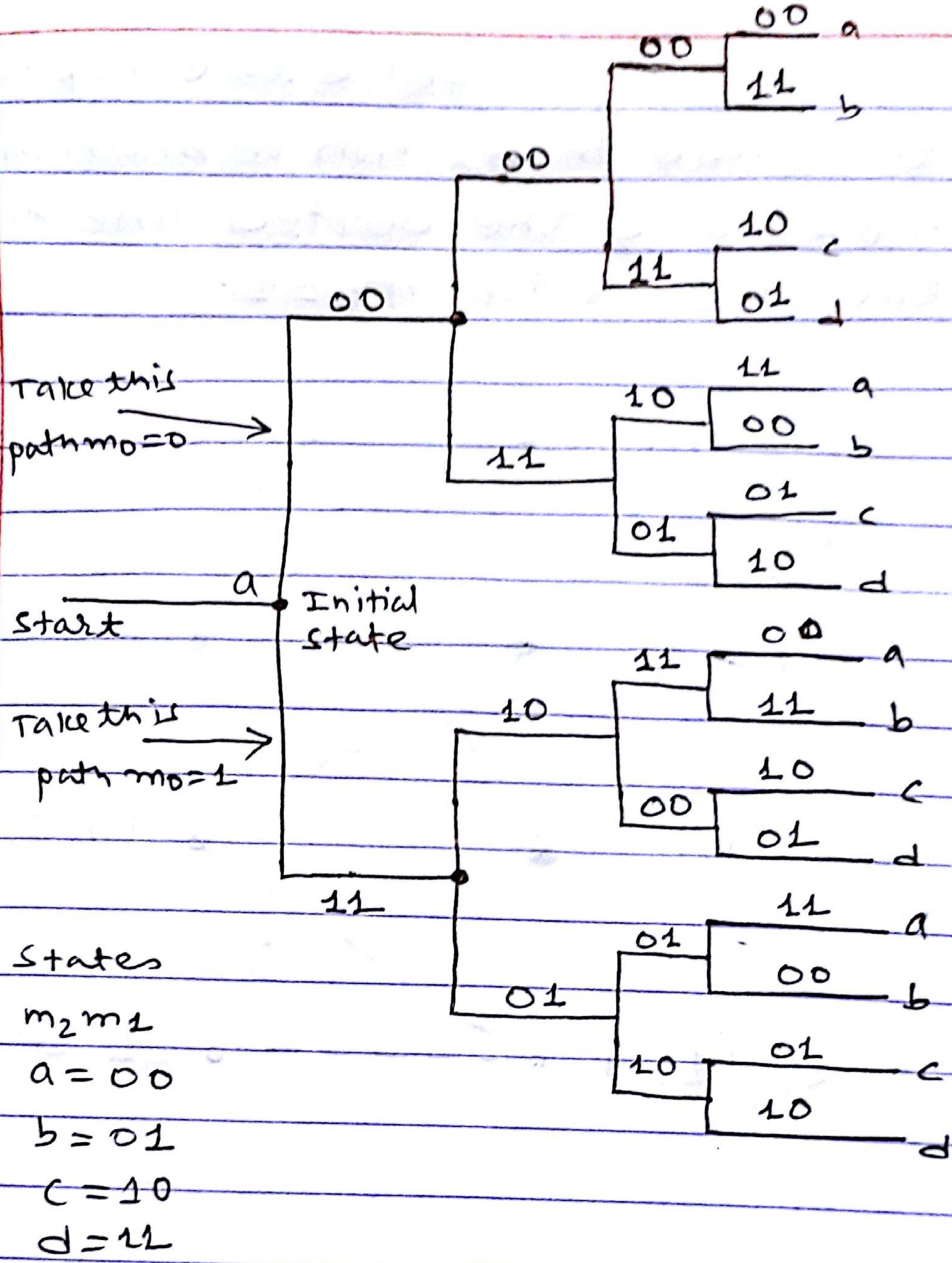
$$x_1 = 1 \oplus 0 \oplus 0 = 1$$

$$x_2 = 1 \oplus 0 = 1$$

$\therefore \; x_1 x_2 = 11$

For $m_0 = 0$, It will take upper branch from node a.

Similarly for $m_0 = 1$, It will take lower branch from node a.



2) **Code Trellis :→** It shows more compact graphical representation. Nodes on the left denote the four possible current states and the nodes on the right are the resulting next state.

solid line represents the state transition or branch for $m_0 = 0$ and the dotted line represents the branch for $m_0 = 1$.

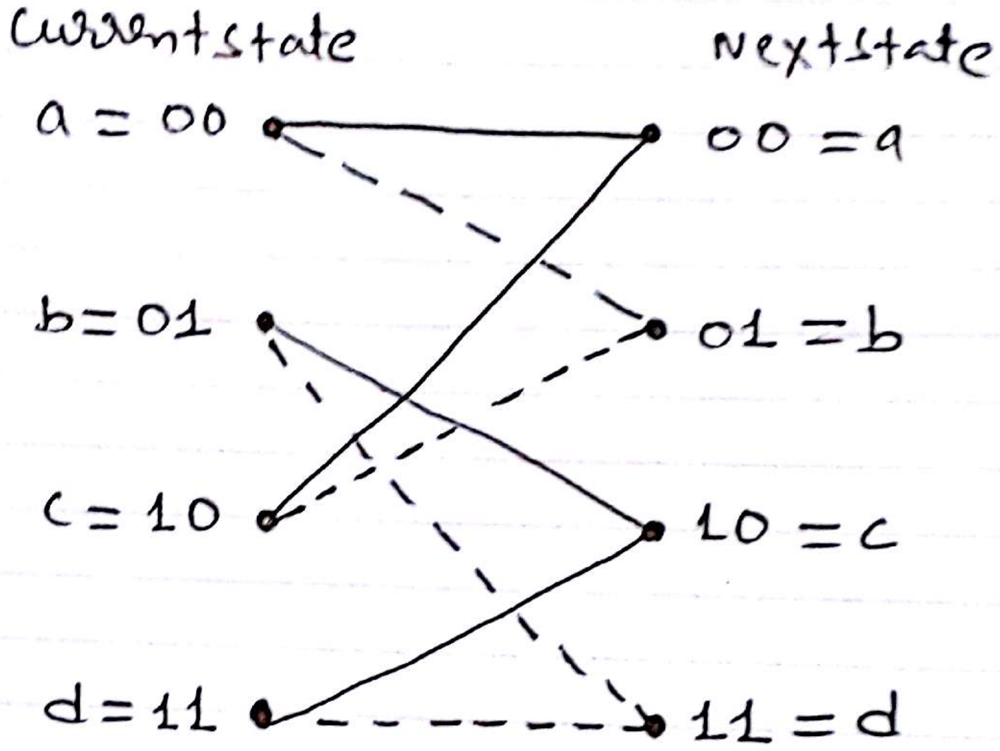


fig:→ Code trellis for the (2, 1, 2) convolutional encoder.