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6. FM & PM

6.1 Basic Definitions :

Angle Modulation :

Let us consider an unmodulated carrier signal,

$$s(t) = A_c \cos[\omega_c t + \phi_0(t)]$$

where,

A_c = amplitude of ~~signal~~ carrier
ω_c = angular velocity
$\phi_0(t)$ = some phase angle

Substituting $\omega_c t + \phi_0(t) = \theta(t)$, we get

$$s(t) = A_c \cos \theta(t)$$

Here, $\theta(t)$ = total phase angle of carrier signal.

Now, if the total phase angle of carrier signal '$\theta(t)$' is varied in accordance with the instantaneous value of modulating or message signal, keeping amplitude of carrier constant, it is known as ANGLE MODULATION.

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So, we have

$$\theta(t) = \omega_c t + \phi(t)$$

Now since the total phase angle '$\theta(t)$' is dependent to time, we define instantaneous phase angle as,

$$\theta_i(t) = \omega_c t + \phi(t)$$

where,

$\theta_i(t)$ = instantaneous phase angle
$\phi(t)$ = phase deviation due to modulating signal.

Now,

Differentiating $\theta_i(t)$ w.r.t. time 't',

$$\frac{d\theta_i(t)}{dt} = \frac{d\omega_c t}{dt} + \frac{d\phi(t)}{dt} = \omega_i(t)$$

where $\omega_i(t)$ = instantaneous angular velocity,

or $\omega_i(t) = \omega_c + \dfrac{d\phi(t)}{dt}$

Here, $d\phi(t)/dt$ = frequency deviation $\left(\frac{rad}{sec}\right)$

Now,

$$\omega_i(t) = \frac{d\,\theta_i(t)}{dt}$$

or $2\pi f_i(t) = \frac{d\,\theta_i(t)}{dt}$

or $f_i(t) = \frac{1}{2\pi} \cdot \frac{d\,\theta_i(t)}{dt}$

$$= \frac{1}{2\pi}\left[\omega_c + \frac{d\,\phi(t)}{dt}\right]$$

Here,

$f_i(t)$ = instantaneous frequency.

$$= f_c + \frac{1}{2\pi}\frac{d\,\phi(t)}{dt}$$

Thus we can see that we can vary the total phase angle $\theta(t)$ in two ways i.e. by altering the phase deviation or by changing the frequency deviation.

The two types of angle modulation can be named as,

i) Phase Modulation (PM)

ii) ~~Fre~~ Frequency Modulation (FM).

Phase modulation (PM).

Phase modulation is one of the variations of angle modulation. In PM, the phase deviation of the carrier signal $\phi(t)$ is directly proportional to the modulating signal $m(t)$.

i.e. $m(t) \propto \phi(t)$

or $\phi(t) = k_p\, m(t)$

where,

k_p is phase sensitivity or phase deviation constant.

Now, we have,

$$s(t) = A_c \cos[\omega_c t + \phi(t)]$$

$$\therefore\ s_{PM}(t) = A_c \cos[\omega_c t + k_p\, m(t)]$$

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Frequency Modulation (FM)

Frequency modulation is another variation of angle modulation. In FM, the frequency deviation $\left[\frac{d\phi(t)}{dt}\right]$ in the carrier signal is directly proportional to the modulating signal $m(t)$, i.e.

$$\frac{d\phi(t)}{dt} \propto \cdot m(t)$$

or $$\frac{d\phi(t)}{dt} = k_d \cdot m(t)$$

or $$\frac{d\phi(t)}{dt} = 2\pi k_f \cdot m(t)$$

where,

$k_d = 2\pi k_f$ is the frequency sensitivity or frequency deviation constant.

Now, we have instantaneous frequency,

$$f_i(t) = \frac{1}{2\pi}\left[\omega_c + \frac{d\phi(t)}{dt}\right]$$

where, $\omega_c = 2\pi f_c$

& $\frac{d\phi(t)}{dt} = 2\pi k_f \cdot m(t)$

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Therefore,

$$f_i(t) = \frac{1}{2\pi}\left[2\pi f_c + 2\pi k_f \cdot m(t)\right]$$

$$= f_c + k_f \cdot m(t)$$

Also, $f_i(t) = \frac{1}{2\pi}\frac{d\theta_i(t)}{dt}$

Therefore,

$$\frac{1}{2\pi}\frac{d\theta_i(t)}{dt} = f_c + k_f \cdot m(t)$$

or $$\frac{d\theta_i(t)}{dt} = 2\pi\left[f_c + k_f \cdot m(t)\right]$$

Therefore

$$\theta_i(t) = \int_0^t \left(2\pi\left[f_c + k_f \cdot m(t)\right]\right) dt$$

$$= 2\pi\int_0^t f_c\, dt + 2\pi\int_0^t k_m \cdot m(t)\, dt$$

$$\theta_i(t) = 2\pi f_c t + 2\pi k_f \int_0^t m(t)\, dt$$

Therefore,

$$s_{FM}(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(t)\, dt\right]$$

$\because$ $s(t) = A_c \cos \theta(t)$

So, we have time domain expression for FM and PM as,

$$s_{PM}(t) = A\cos[2\pi f_c t + k_p m(t)]$$

& $$s_{FM}(t) = A\cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(t)\,dt\right]$$

where,

k_p = phase sensitivity (radian/volt)

&

k_f = frequency sensitivity (Hertz/volt)

~~So,~~ We can thus see that PM and FM are closely related as both include the variation in the total phase angle.

In PM, the phase angle varies linearly with message signal m(t) whereas in FM, the phase angle varies linearly with the integral of ~~b~~ message signal m(t).

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So, FM can be obtained by using PM and conversely, PM can be obtained by using FM.

Generation of FM using PM.

→ To get FM, we first integrate the message signal and then apply to the phase modulator.

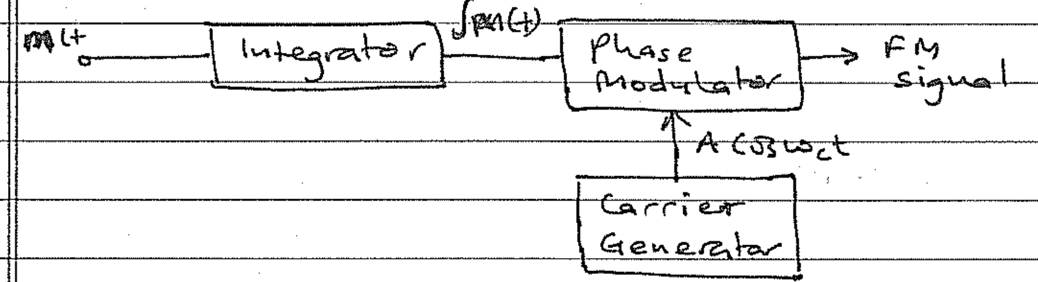
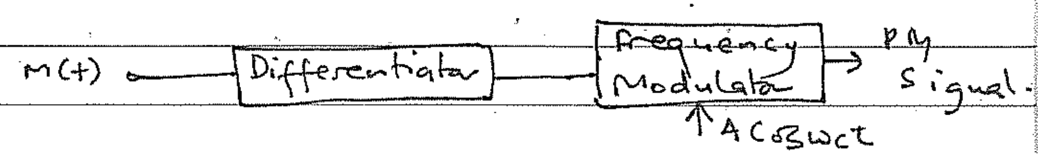


fig. generation of FM.

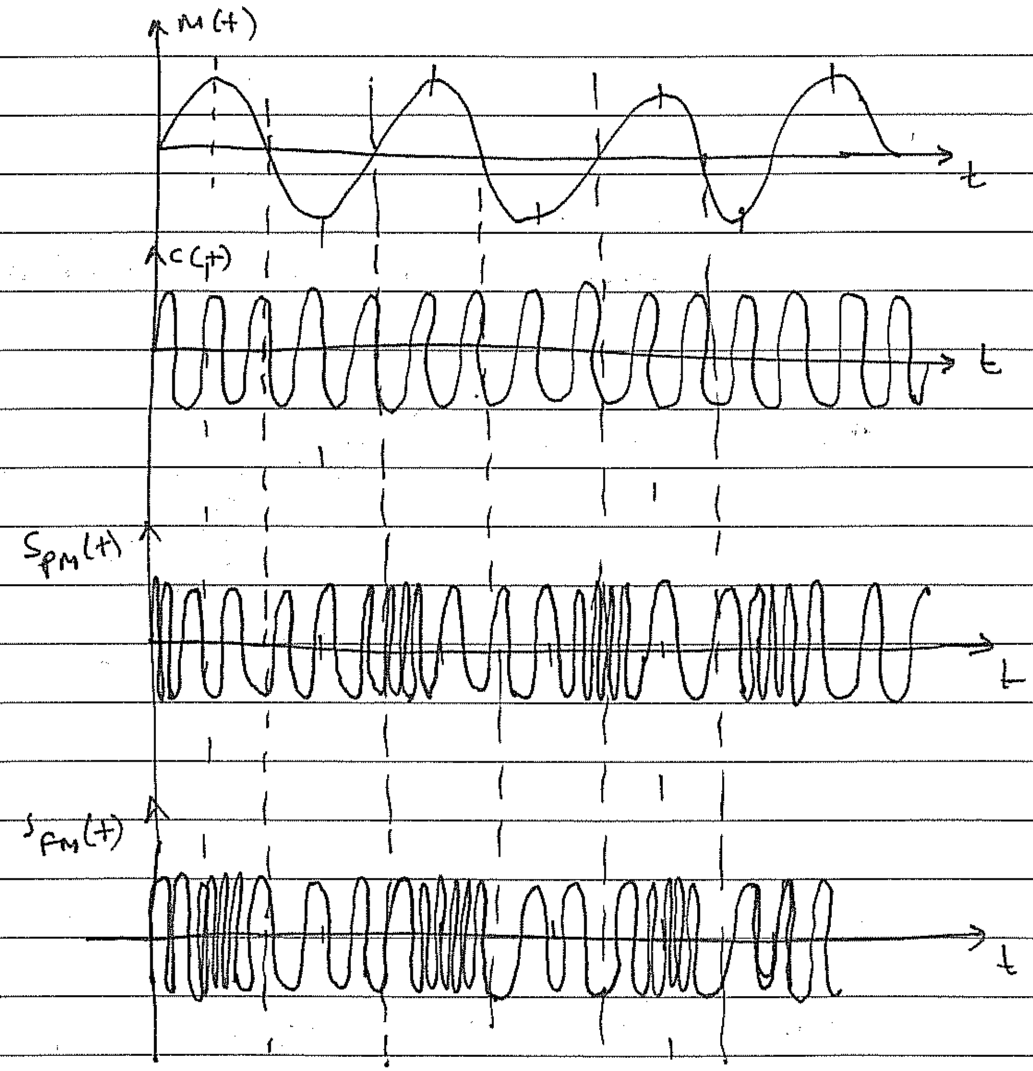
Generation of PM using FM.



→ first, m(t) is differentiated and then applied to frequency modulator.

~~Waveforms~~

General waveforms for PM & FM.



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Frequency Deviation :

We have instantaneous frequency given as,

$$f_i(t) = f_c + \underbrace{2\pi k_f}_{\text{frequency sensitivity}} \cdot m(t)$$

This instantaneous frequency of FM signal varies with time around center carrier frequency f_c.

This means that the value of $f_i(t)$ varies according to the modulating signal.

The maximum change in this instantaneous frequency from the carrier frequency 'f' is called 'frequency deviation,' and is denoted by Δf.

where,

$$\Delta f = |2\pi k_f \cdot m(t)|_{max}$$

And $2 \times \Delta f$ = carrier swing

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Single tone frequency modulation.

Consider a sinusoidal modulating signal defined by,

$$m(t) = A_m \cos 2\pi f_m t$$

Now,

$$f_i(t) = f_c + 2\pi k_f \cdot m(t)$$
$$= f_c + 2\pi k_f \cdot A_m \cos 2\pi f_m t$$

or $f_i(t) = f_c + \Delta f \cdot \cos 2\pi f_m t$

where, Δf = frequency deviation
$= |2\pi k_f \cdot m(t)|_{max}$.
$= 2\pi k_f \cdot A_m$.

A_m = Max. amplitude of $m(t)$

f_m = frequency of $m(t)$.

Also, let carrier wave be,

$$c(t) = A_c \cos 2\pi f_c t$$

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Now, the expression for FM wave is,

$$s_{FM}(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(t)\,dt\right]$$

$$= A_c \cos\left[2\pi f_c t + 2\pi k_f \cdot \int_0^t A_m \cos 2\pi f_m t\,dt\right]$$

$$= A_c \cos\left[2\pi f_c t + 2\pi k_f \cdot A_m \int_0^t \cos 2\pi f_m t\,dt\right]$$

$$= A_c \cos\left[2\pi f_c t + \frac{2\pi k_f \cdot A_m}{2\pi f_m} \Big|\sin 2\pi f_m t\Big|_0^t\right]$$

$$= A_c \cos\left[2\pi f_c t + \frac{\Delta f}{f_m} \cdot \sin 2\pi f_m t\right]$$

$$\because \Delta f = k_f \cdot A_m$$

Now, $\frac{\Delta f}{f_m} = \beta_{FM}$ = modulation index.

Therefore

$$s_{FM}(t) = A_c \cos\left[2\pi f_c t + \beta_{FM} \sin 2\pi f_m t\right]$$

$\beta_{fm} = \frac{\Delta f}{f_m} = \frac{\text{frequency deviation}}{\text{modulating frequency}}$ = Modulation index.

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(II) Single tone phase modulation.

Let $m(t) = A_m \cos 2\pi f_m t$

& $c(t) = A_c \cos 2\pi f_c t$

then for PM, we have,

$$s_{PM}(t) = A_c \cos\left[2\pi f_c t + k_p \cdot m(t)\right]$$

$$= A_c \cos\left[2\pi f_c t + k_p \cdot A_m \cos 2\pi f_m t\right]$$

$$s_{PM}(t) = A_c \cos\left(2\pi f_c t + \beta_{pm} \cos 2\pi f_m t\right)$$

$$= A_c \cos \theta(t)$$

where,

$$\beta_{pm} = k_p \cdot A_m$$

Now,

$$f_i(t) = \frac{1}{2\pi} \frac{d\theta_i(t)}{dt} = \frac{1}{2\pi} \frac{d}{dt}\left[2\pi f_c t + k_p \cdot A_m \cos 2\pi f_m t\right]$$

$$= \frac{1}{2\pi}\left[2\pi f_c + k_p \cdot A_m \cdot \left[-\sin 2\pi f_m\right] \cdot 2\pi f_m\right]$$

$$= f_c - k_p \cdot A_m \cdot f_m \sin 2\pi f_m.$$

$$f_i(t) = f_c - \Delta f \sin 2\pi f_m.$$

where, $\Delta f = k_p \cdot A_m \cdot f_m$

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So, frequency deviation for phase modulation,

$$\Delta f_p = k_p \cdot A_m \cdot f_m.$$

And for FM,

$$\Delta f_f = k_f \cdot A_m$$

So, for an equal bandwidth in FM & AM,

$$k_f = k_p \cdot f_m$$

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(3) spectral representation of single-tone modulated FM signals.

Let the carrier signal be

$$c(t) = A_c \cos 2\pi f_c t$$

&

modulating signal be,

$$m(t) = A_m \cos 2\pi f_m t$$

Then,

$$S_{FM}(t) = A_c \cos\left[2\pi f_c t + \beta \sin 2\pi f_m t\right]$$

In the exponential form, the above expression can be written as,

$$S_{FM}(t) = Re\left[A_c\, e^{j[2\pi f_c t + \beta \sin 2\pi f_m t]}\right]$$

$$= Re\left[A_c\, e^{j 2\pi f_c t} \cdot e^{j\beta \sin 2\pi f_m t}\right]$$

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As $\beta \sin 2\pi f_m t$ is a periodic f^n, with period $T_m = 1/f_m$, $e^{j\beta \sin 2\pi f_m t}$ is also periodic signal with same period.

Therefore $e^{j\beta \sin 2\pi f_m t}$ can be expanded in Fourier series as,

$$e^{j\beta \sin 2\pi f_m t} = \sum_{n=-\infty}^{\infty} C_n\, e^{j 2\pi f_m t n}$$

where,

$$C_n = \frac{1}{T_m} \int_{-\frac{T_m}{2}}^{\frac{T_m}{2}} e^{j\beta \sin 2\pi f_m t} \cdot e^{-j 2\pi n f_m t}\, dt$$

let $2\pi f_m t = x$, then,

as $t = -\frac{T_m}{2}$, $x = 2 \cdot \pi \cdot \frac{1}{T_m} \cdot -\frac{T_m}{2} = -\pi$

as $t = \frac{T_m}{2}$, $x = \pi$

and $\frac{dx}{dt} = 2\pi f_m$, $\therefore$ $dt = \frac{1}{2\pi f_m} dx = \frac{T_m}{2\pi} dx$

Substituting with 'x' in integration,

$$C_n = \frac{1}{T_m} \int_{-\pi}^{\pi} e^{j\beta \sin x} \cdot e^{-jnx} \cdot \frac{T_m}{2\pi} \cdot dx$$

or $C_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j(\beta \sin x - nx)} \, dx.$

The integral on the right hand side is the n^{th} order Bessel f^n of the first kind and argument β. This f^n is represented by $J_n(\beta)$.

Therefore,

$$C_n = J_n(\beta)$$

So, we have,

$$e^{j\beta \sin 2\pi f_m t} = \sum_{n=-\infty}^{\infty} J_n(\beta) e^{jn2\pi f_m t}$$

And,

$$\cancel{S_{FM}(t) =}$$

$$S_{FM}(t) = Re\left[A_c e^{j2\pi f_c t} \cdot \sum_{n=-\infty}^{\infty} J_n(\beta) e^{jn2\pi f_m t}\right]$$

or $S_{FM}(t) = Re\left[A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cdot e^{j(2\pi f_c t + n2\pi f_m t)}\right]$

Therefore,

$$S_{FM}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos[2\pi(f_c + nf_m)t]$$

Now, the discrete spectrum of $S_{FM}(t)$ is obtained by taking F.T. on both sides, i.e.

$$S_{FM}(f) = \frac{A_c}{2} \sum_{n=-\infty}^{\infty} J_n(\beta) \left[\delta(f - f_c - nf_m) + \delta(f + f_c + nf_m)\right]$$

The above expression represents infinite sum of harmonic signals β with frequency sum and difference of f_c and nf_m. Therefore the spectrum of FM signal has infinite number of sidebands whose magnitude depend on Bessel coefficient.

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Properties of Bessel function $(J_n(\beta))$

1. $J_n(\beta)$ is a real valued f^n.
2. $\sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1$ for all β.
3. for $\beta << 1$

$$J_0(\beta) = 1$$
$$J_1(\beta) \simeq \beta/2$$
$$J_n(\beta) = 0 \quad \text{for } n > 1$$

4. $J_{n-1}(\beta) + J_{n+1}(\beta) = \frac{2n}{\beta} J_n(\beta)$
5. $J_{-n}(\beta) = J_n(\beta)$ for n - even

$J_{-n}(\beta) = -J_n(\beta)$ for n - odd.

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from property number 2 of Bessel f^n,

i.e.

$$\sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1$$

Using it to derive power of single-tone FM wave,

$$P = \frac{1}{2} A_c^2 \sum_{n=-\infty}^{\infty} J_n^2(\beta)$$

or $P = \frac{A_c^2}{2}$

Which states that the average power of single-tone FM wave is constant and equal to $\frac{A_c^2}{2}$ dissipated in 1Ω resistor.

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Bandwidth of FM waves.

The spectral of FM wave is given by,

$$S_{FM}(f) = \frac{A_c}{2} \sum_{n=-\infty}^{\infty} J_n(\beta)\left[\delta(f-f_c-nf_m)+\delta(f+f_c+nf_m)\right]$$

which states that theoretically, the bandwidth of FM $= \infty$.

So, the order 'n' of Bessel coefficient is limited such that the number of sidebands are limited in such a way that the radiated power is atleast 98% of the total power.

In such case, for 'n' spectral components with '2n' sidebands, the bandwidth of FM is given by,

$$B = 2nf_m.$$

It is established that for 98% of the total power to transmit, the number 'n' and modulation index β are related by the eq^n,

$$n = \beta + 1$$

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The total bandwidth of the FM therefore can be calculated using the following equation called Carson's rule.

$$B = 2(\beta+1)\cdot f_m$$

Consequently,

$$B = 2\cdot\left(\frac{\Delta f}{f_m}+1\right) f_m$$

$$= 2\cdot\left(\frac{\Delta f + f_m}{f_m}\right)\cdot f_m$$

$$= 2(\Delta f + f_m)$$

Also,

$$B = 2\left(\Delta f + \frac{\Delta f}{\beta}\right)$$

$$B = 2\Delta f\left(1 + 1/\beta\right)$$

$$\therefore \boxed{\begin{aligned} B &= 2(\beta+1) f_m \\ B &= 2(\Delta f + f_m) \\ B &= 2\cdot\Delta f\left(1+\frac{1}{\beta}\right) \end{aligned}}$$ for FM.

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Narrowband FM.

For the values of β much smaller than 1, the bandwidth of FM is almost same as that of DSB-AM. This is the smallest bandwidth available.

In such case, the FM is called narrow-band (NB) FM.

$$B_{NBFM} \approx \cancel{2\beta f_m}\ 2 \cdot f_m \quad \because \beta+1 \approx 1.$$

We have,

$$s_{FM}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos[2\pi(f_c + nf_m)t]$$

So, for $\beta << 1$, using property of Bessel,

$$s_{FM}(t) = A_c\ J_0(\beta) \cos 2\pi f_c t +$$
$$A_c\ J_{-1}(\beta) \cos[2\pi(f_c - f_m)t] +$$
$$A_c\ J_1(\beta) \cos[2\pi(f_c + f_m)t]$$

Now,

$$J_0(\beta) = 1, \quad J_1(\beta) = \beta/2, \quad J_{-1}(\beta) = -J_1(\beta) = -\beta/2$$

$$\therefore s_{NBFM}(t) = A_c \cos 2\pi f_c t + A_c \cdot \beta/2 \cos[2\pi(f_c + f_m)t]$$
$$- A_c \cdot \frac{\beta}{2} \cos[2\pi(f_c - f_m)t].$$

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Wideband FM:

For larger values of β i.e. β is much greater than 1, the bandwidth of FM is large.

In such case the FM wave consists of carrier and infinite number of side-frequency components located symmetrically around the carrier. This type of FM wave is said to be wideband FM, (WBFM).

So, for WBFM,

$$s_{WBFM}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos[2\pi(f_c + nf_m)t]$$

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6.4.

Generation of FM.

FM signals can be generated using two methods

1. Direct method, also known as parameter variation method.
2. Indirect method, also known as Armstrong method.

The FM generation methods can be shown as;

Methods of generation of FM,

- Direct method
  - Reactance Modulator
  - Varactor diode Modulator
- Indirect Method.
  - Armstrong method.

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1. Direct method

In the direct method, the instantaneous frequency is directly related with the modulating signal. i.e. The modulating signal directly modulates the carrier.

This can be accomplished by a voltage controlled oscillator (VCO) whose output frequency is proportional to the voltage of the input signal.

Any oscillator comprising of parallel tuned L-C circuit has the operating frequency given by,

$$f_c = \frac{1}{2\pi\sqrt{LC}}$$

where, L- inductance
C- capacitance

If the reactance of the eq^n above can be altered by making a change in inductance or capacitance, the frequency is altered.

The same concept is used for VCO, where the frequency is controlled by modulating voltage $[m(t)]$.

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The direct method can be subcategorized into,

a) Reactance modulator
b) Varactor diode modulator.

Both of the methods ~~met~~ mentioned above ~~uses~~ use voltage controlled oscillator (VCO) to derive the required frequency which is directly proportional to the input message voltage.

The simplest of the two is varactor diode modulator.

a. Varactor diode modulator.

A varactor diode modulator has a varactor diode shunted to the fixed capacitor of the capacitance ~~x~~ component in a frequency determining n/w [i.e. tuned ~~ckt~~ circuit]

A varactor or varicap is one whose capacitance depends on the voltage applied across its electrodes.

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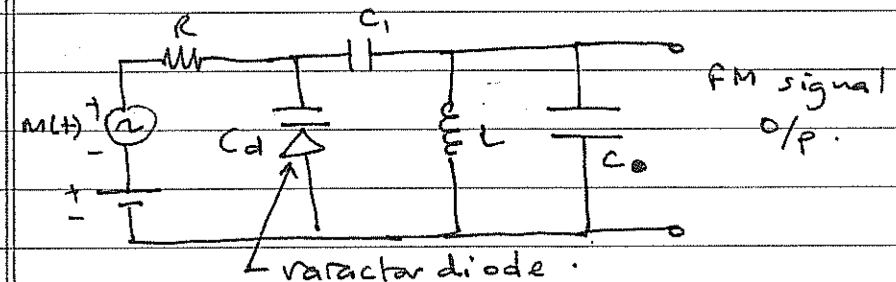
The larger the reverse voltage applied to such a diode, smaller is the transition capacitance of the diode.

i.e. $C_d \propto \frac{1}{\sqrt{V_d}}$ or $C_d = \frac{k_c}{\sqrt{V_d}} = k_c \cdot V_d^{-1/2}$



In absence of m(t) and considering C_1 negligible, the frequency of oscillation of tuned circuit,

$$f_c = \frac{1}{2\pi\sqrt{LC_0}}$$

Now, when m(t) is applied, the total capacitance is the sum of capacitance across varactor diode and capacitor 'C'. Let this capacitance be c(t), then

$$f_i(t) = \frac{1}{2\pi\sqrt{L\,c(t)}}$$

c(t) can be defined as,

$C(t) = C_0 - K_c \cdot m(t)$ [$\therefore C_d = k_c\, m(t)^{-1/2}$ for linearity, $C_d = -k_c\, m(t)$]

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So,

$c(t) = C - k_c\, m(t)$

where,

C = total capacitance in absence of modulation

k_c = varactors sensitivity to voltage change

Now,

$$f_i(t) = \frac{1}{2\pi\sqrt{L\cdot(C - k_c\cdot m(t))}}$$

$$= \frac{1}{2\pi\sqrt{LC - L\cdot k_c m(t)}}$$

$$= \frac{1}{2\pi\sqrt{LC\left(1 - \frac{k_c\cdot m(t)}{C}\right)}}$$

$$= \frac{1}{2\pi\sqrt{LC}\cdot\left(1 - \frac{k_c\cdot m(t)}{C}\right)^{1/2}}$$

$$= f_c\left[1 - \frac{k_c\cdot m(t)}{C}\right]^{-1/2}$$

Expanding the root term, we get,

$$f_i(t) = f_c\left[1 + \frac{k_c\cdot m(t)}{2C}\right]$$

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or $f_i(t) = f_c + k_f\cdot m(t)$

where, $k_f = \frac{k_c\cdot f_c}{2C}$.

Thus, we can see that the deviation produced is directly related to the input message signal. And, the resultant signal for $f_i(t)$ is an FM signal.

b. Reactance Modulator

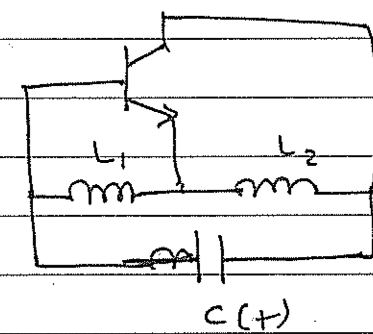

fig. Hartley oscillator

A reactance oscillator generally has transistor that ~~prod~~ provides a capacitance for a given message input voltage.

So, for a message voltage $m(t)$,

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The frequency of oscillation of Hartley oscillator is given by,

$$f_i(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)\,c(t)}}$$

Again, $c(t) = C - k_c\, m(t)$

and $f_i(t) = f_c\left[1 + \frac{k_c \cdot m(t)}{2C_0}\right]$

$$= f_c + k_f \cdot m(t)$$

where $k_f = \frac{f_c \cdot k_c}{2C}$.

This method, i.e. direct method is not suitable for generating wideband FM (WBFM) as the frequency swing from 'fc' will be limited by linear portion of C vs V characteristics of a varactor diode.

To add to that, the frequency thus generated is not stable and L & C components are affected by temperature and other factors.

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2. Indirect method.

The indirect method of generating a wideband FM signal was first proposed by Edwin Howard Armstrong. Thus such frequency modulator are sometimes referred to as Armstrong modulator.

The indirect method consists of two steps.

a) Generate Narrowband FM using modulating wave.

b) Convert NBFM to WBFM.

a) To generate NBFM, we have,

$$s_1(t) = A_1 \cos\left[2\pi f_1(t) + \phi_1(t)\right]$$

where,

$f_1(t)$ = carrier frequency

A_1 = carrier amplitude

$\phi_1(t)$ = angular argument such that

$$\phi_1(t) = 2\pi k_{f1}\int_0^t m(t)\,dt$$

& k_1 = freq. sensitivity of modulator.

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let us assume $\phi_1(t) << 1$ for all t,
then

$$\cos[\phi_1(t)] \simeq 1 \quad \&$$
$$\sin[\phi_1(t)] \simeq \phi_1(t)$$

So, with

$$s_1(t) = A_1 \cos[2\pi f_1 t + \phi_1(t)]$$
$$= A_1 \cancel{\cos} 2\pi f_1(t)$$
$$= A_1 [\cos 2\pi f_1 t \cdot \cos \phi_1(t) - \sin 2\pi f_1 t \cdot \sin \phi_1(t)]$$
$$= A_1 [\cos 2\pi f_1 t - \sin 2\pi f_1 t \cdot \phi_1(t)]$$

$$\boxed{s_1(t) = A_1 \cos 2\pi f_1 t - 2\pi k_1 A_1 \sin 2\pi f_1 t \int_0^t m(t)\,dt}$$

~~For~~ $m(t) = A_m \cos 2\pi f_m t$

$$\int m(t)\,dt = \sin 2\pi f_m t \cdot \frac{A_m}{2\pi f_m}.$$

$$\therefore s_1(t) = A_1 \cos 2\pi f_1 t - A_1 \cdot A_m \cdot \frac{k_1}{f_m} \cdot \sin 2\pi f_1 t \cdot \sin 2\pi f_m t$$

Again, Recall, for NBFM,

$$s_{FM(NB)}(t) = A_c \cos 2\pi f_c t + \frac{A_c \beta}{2} \cos 2\pi (f_c + f_m) t - \frac{A_c \cdot \beta}{2} \cos 2\pi (f_c - f_m) t$$

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or

$$s_{FM(NB)}(t) = A_c \cos 2\pi f_c t - A_c \beta \sin 2\pi f_c t \cdot \sin 2\pi f_m t$$

Thus we can conclude that, $s_1(t)$ is also a narrow band fm wave.

So, for,

$$s_1(t) = A_1 \cos 2\pi f_1 t - 2\pi k_1 A_1 \sin 2\pi f_1 t \int_0^t m(t),$$

we design a modulator as,

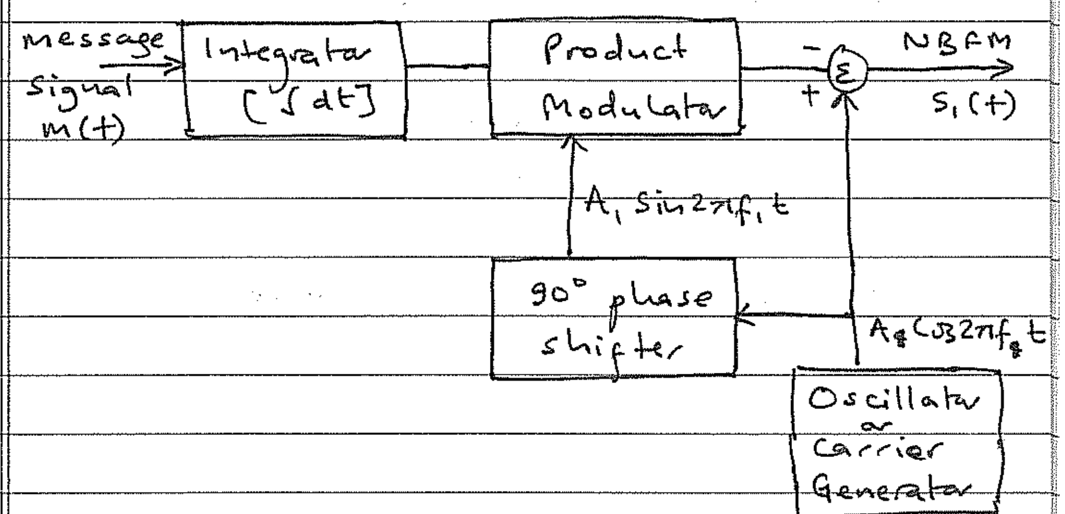

Fig. Block diagram of NBFM generator.

6. Convert NBFM to WBFM.

The next ~~of~~ step in indirect FM method is frequency multiplication.

$$\boxed{NBFM} \xrightarrow{s_1(t)} \boxed{\begin{array}{c}\text{Frequency} \\ \times\ n\end{array}} \xrightarrow{s_2(t)} WBFM.$$

We know that for $s_1(t)$ which is NBFM wave, β modulation index, $\beta_1 << 1$, thus to convert it to $\beta_2 >> 1$, we need to multiply the argument in $s_1(t)$ by some integer 'n' such that,

$$\beta = \beta_1 \times n.$$

We can now consider ~~s_1~~

$$s_1(t) = A_c \cos[2\pi f_{c_1} t + \beta_1 \sin 2\pi f_m t]$$

So, the output of the multiplier will be, 'n' times the argument, i.e.

$$\boxed{s_2(t) = A_c \cos[2\pi \cdot n f_{c_1} t + n \cdot \beta_1 \sin 2\pi f_m t]}$$

freq. multiplier.

A frequency multiplier consists of a memoryless non-linear device with the input-output relationship of the form,

$$s_2(t) = a_1 s_1(t) + a_2 s_1^2(t) + a_2 s_1^2(t) + \dots + a_n s_1^n(t)$$

followed by an appropriate bandpass filter.

~~Substitud~~ substituting $s_1(t) = A_c \cos\theta(t)$ in above eq^n, and expanding, we get carries frequencies $f_1, 2f_1, \dots n f_1$ and freq. deviation $\Delta f_1, 2\Delta f_1, \dots n\Delta f_1$.

For example, $s_1^2(t) = A_c^2 \cos^2\theta(t)$

$$= A_c^2 \left[\frac{1 + \cos 2\theta(t)}{2}\right]$$

$$= \cancel{\frac{A_c^2}{2}} + \frac{A_c^2 \cos 2\theta(t)}{2}$$

(DC component)

Thus we have the argument multiplied by 2. So, for any integer 'n', we have the argument multiplied by 'n'.

Also, we get new modulation index $\beta = n\beta_1$.

And $\beta = \frac{\Delta f_1}{f_m} \Rightarrow \frac{n \cdot \Delta f_1}{f_m}$

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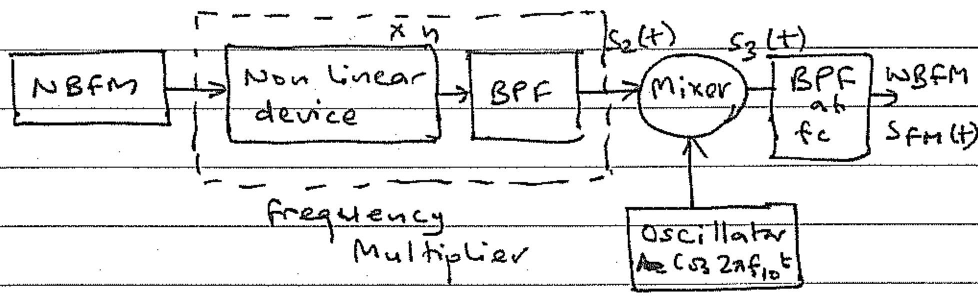
~~Thus~~ # Now, the WBFM wave with new carrier frequency $f_c = n f_1$ ~~is~~ may or may not match the frequency ~~authorized by~~ assigned by the regulatory authority. This is because 'n' is just an integer.

This problem is solved by using a mixer, with oscillation $\cos 2\pi f_{10} t$ i.e.



The output of mixer,

$$S_{FM}(t) = S_3(t) = A_1 \cos[2\pi(nf_{c1} + f_{10})t + n\beta_1 \sin 2\pi f_m t] + A_1 \cos[2\pi(nf_{c1} - f_{10})t + n\beta_1 \sin 2\pi f_m t]$$

This signal is then filtered through BPF having centre frequency at

$nf_{c1} + f_{10}$ &

$nf_{c1} - f_{10}$

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So, the output of BPF through mixer, for $n\beta_1 = \beta$ is,

$$S_{FM}(t) = A_1 \cos[2\pi f_c t + \beta \sin 2\pi f_m t]$$

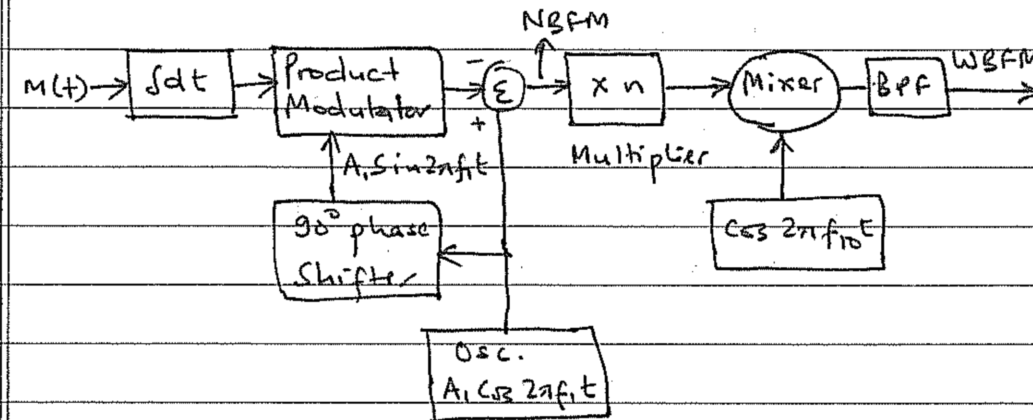


fig. Block diagram of WBFM generator.

Demodulation of FM waves.

The process of retrieving or recovering the original message signal from a frequency modulated (FM) wave is known as the demodulation of FM wave.

The de-modulator thus produces output signal with amplitude (voltage) directly proportional to the instantaneous frequency of FM wave used as input signal.

There are basically two methods of demodulating FM signals -

a. Direct - frequency discriminator or limiter discriminator.

b. Indirect - Phase locked loop (PLL)

a. Frequency Discriminator.

→ it is a non-coherent demodulator i.e. it has FM wave as single input.

The basic idea here is to convert FM to AM and with the use of envelope detector derive the message signal $m(t)$.

It is a system that has linear frequency to voltage transfer characteristics.

A general FM signal is given by,

$$s_{FM}(t) = A_c \cdot \cos[2\pi f_c t + \phi_1(t)]$$

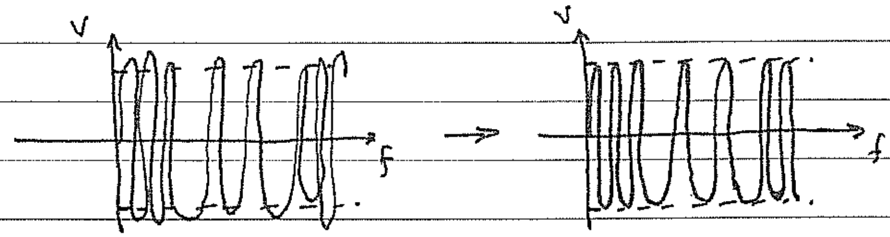
where,

$$\phi_1(t) = 2\pi k_f \int_0^t m(t)\, dt.$$

In order to extract modulating signal $m(t)$ from FM signal, we follow three steps.

i) Amplitude limiter :

- during FM propagation, the amplitude which actually is said to be constant, may change due to fading and noise. So, with limiter, the amplitude of FM signal to actual anticipated level reducing the effect of fading and noise.



distorted FM → Amplitude limited.

ii) Discriminator :

The amplitude limited signal is then fed to discriminator where the FM signal is differentiated such that,

$$\frac{d\, s_{FM}(t)}{dt} = \frac{d}{dt}\left[A_c \cos[2\pi f_c t + \phi_1(t)]\right]$$

$$= d\,[A_c \cos[\cancel{2\pi f_c t}$$

$$= \frac{d}{dt}\left[A_c \cos[\omega_c t + \phi_1(t)]\right]$$

$$= A_c \cdot \frac{d \cos[\omega_c t + \phi_1(t)]}{d[\omega_c t + \phi_1(t)]} \cdot \frac{d[\omega_c t + \phi_1(t)]}{dt}$$

$$= \left[\omega_c + \frac{d\phi_1(t)}{dt}\right] \cdot A_c\,[-\sin\{\omega_c t + \phi_1(t)\}]$$

$$= -A_c \sin[\omega_c t + \phi_1(t)]\left[\omega_c + \frac{d\phi_1(t)}{dt}\right]$$

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Again, $\phi_1(t) = 2\pi k_f \int_0^t m(t)\,dt$

$$\therefore \frac{d\phi_1(t)}{dt} = \cancel{2\pi k_f}\; 2\pi k_f \cdot m(t)$$

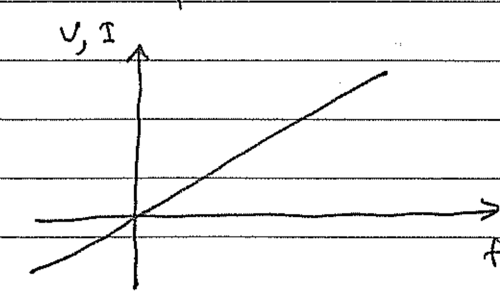
and $\omega_c = 2\pi f_c$.

$$\therefore \frac{d\, s_{FM}(t)}{dt} = -A_c\,[2\pi f_c + 2\pi k_f \cdot m(t)] \cdot \sin[2\pi f_c t + \phi_1(t)]$$

We see that the above equation is almost a standard AM signal with carrier component and side bands containing $m(t)$.

The process of differentiation, thus is also known as FM to AM conversion.

In practice, the differentiator can be approximated by a slope detector that has a linear frequency to voltage or current transfer characteristics. A discriminator must have transfer characteristics as below.



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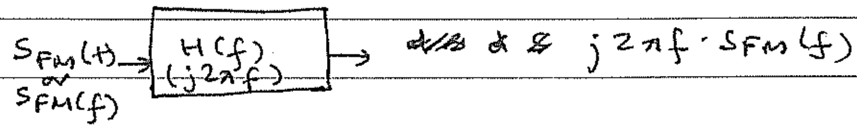
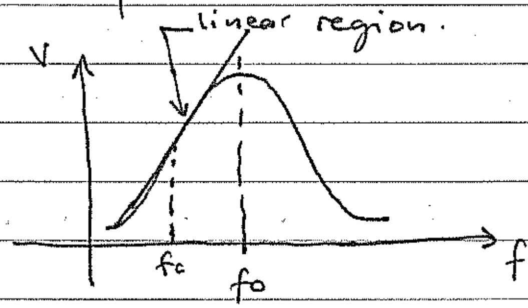
$$\left[\frac{dx(t)}{dt} \longleftrightarrow j\omega X(\omega) \right]$$

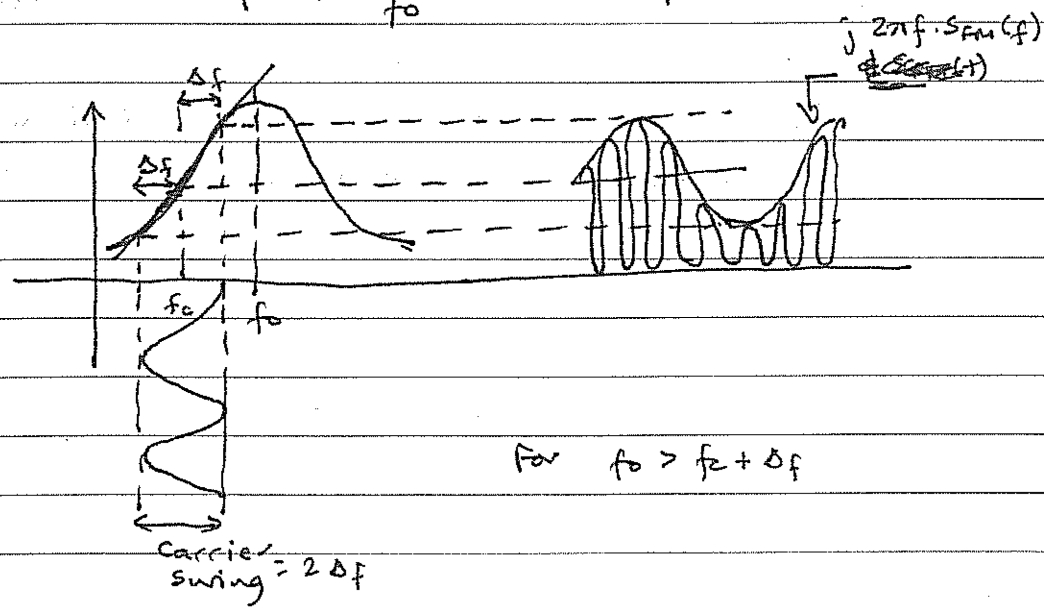
$$v_2(t) = \frac{dv_1(t)}{dt} \Longleftrightarrow V_2(f) = j2\pi f\, V_1(f)$$

Mathematically a differentiator has the transfer function $H(f) = j2\pi f$.



Now a tuned circuit tuned at $f_0 \neq f_c$ where f_0 is the resonance frequency of the tank circuit, have such transfer function at its slope.





For $f_0 > f_c + \Delta f$

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iii) Envelop detection :

As the output of the discriminator is now AM with $m(t)$ within the amplitude of the carrier signal, simple envelop detection will result in a signal,

$$S(t) = A_c \left[2\pi f_c + 2\pi k_f\, m(t) \right]$$

The first component in the above equation is the DC component and the second one is the message signal multiplied by some constant coefficient.

The FM ~~discriminator~~ demodulator based on frequency discriminator can be shown as,

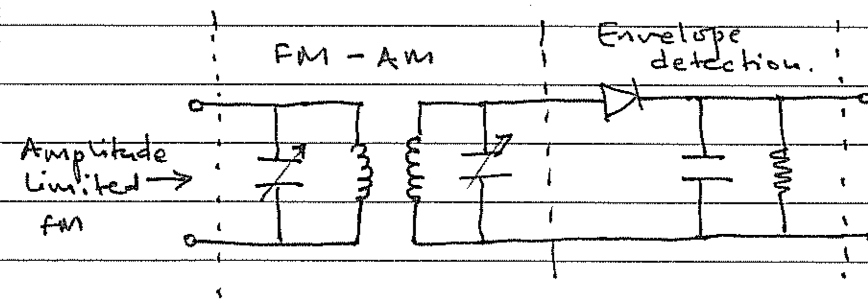

Fig. FM demodulator based on frequency discriminator a limiter discriminator

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Drawback :

The major drawback of such demodulator is the limited linear portion in the slope of tuned circuit response. Because of this, the above circuit is not suitable for WBFM where the frequency deviation is high.

A better solution to that is to implement a balanced slope detector where two tuned circuits tuned at $f_c + \Delta f$ and $f_c - \Delta f$ are used. ~~with~~ Two envelope detectors are used to extend the linear portion of the slope of the tank circuit.

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b. Indirect method :

→ This type of method implements the coherent demodulators. One of the most used indirect method is the phase locked loop (PLL).

PLL as FM detector (demodulator)

A PLL is a coherent detector. It is used in tracking the phase and frequency of carrier component of an incoming FM signal. It is useful for demodulating FM signals in presence of large noise and low signal power.

It can be considered as a negative feedback system.

A PLL consists of ;

i) Phase detector :

Consists of a multiplier followed by a LPF. It produces ~~voltage~~ voltage proportional to the phase difference between incoming carrier signal and the local oscillator signal produced by the local oscillator.

Basically, at the phase detector, if the signal feedback doesn't have phase equal to the input signal, a loop is created until the phase of ~~feedb~~ feedback signal is equal to the input signal.

ii) Loop filter :

A loop filter is assigned to make the phase of feedback signal equal to the phase of input signal. It has an impulse response of $h(t)$.

iii) voltage controlled oscillator (VCO).

A VCO is set to define a local oscillator that has the frequency determined by the output of loop filter.

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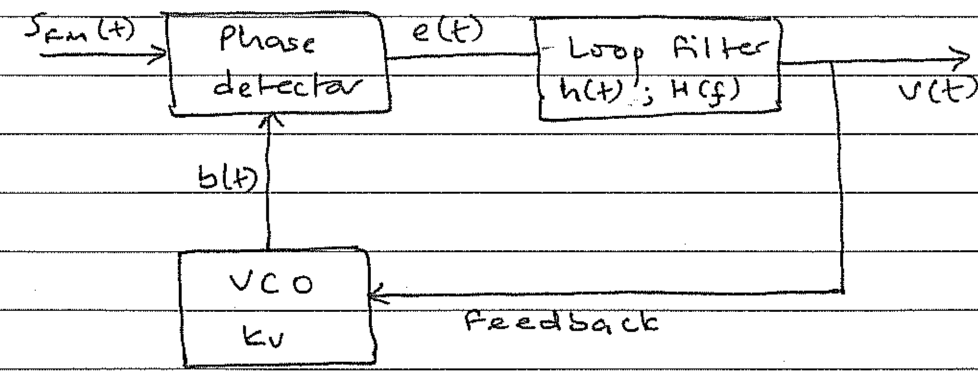

Fig. A basic PLL detector.

Operation:

Initially, VCO is adjusted in such a way that when control voltage $v(t) = 0$, ~~for~~ i.e. when $S_{FM}(t) = 0$, following two conditions are satisfied.

i) Frequency of VCO is precisely set at unmodulate carrier frequency 'f_c'.

ii) VCO output has a 90° phase shift with respect to the carrier.

i.e. for $c(t) = A_v \cos 2\pi f_c t$

$$b(t) = A_v \sin 2\pi f_c t$$

The second condition makes sure that the input carrier signal & VCO signal are in quadrature such that when the frequency

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and phase of these two signals match, the output of phase detector '$e(t)$' will be zero.

Let $S_{FM}(t) = A_c \cos[2\pi f_c t + \phi_i(t)]$

where $\phi_i(t) = 2\pi k_f \int_0^t m(t)\,dt$.

Now,

when $v(t) \neq 0$, the output of VCO,

$b(t) = A_v \sin[2\pi f_c t + \phi_o(t)]$

where $\phi_o(t)$ is proportional to the output of PLL

i.e. $\phi_o(t) = 2\pi k_v \int_0^t v(t)\,dt$

where k_v = VCO sensitivity.

So,

$e(t) = S_{FM}(t) \times b(t)$

$= k_m A \cdot A_v \cdot \cos\underbrace{[2\pi f_c t + \phi_i(t)]}_{A} \cdot \sin\underbrace{[2\pi f_c t + \phi_o(t)]}_{B}$

where, k_m = multiplier gain

Now, $e(t) = k_m A_c A_v \left[\frac{\sin(A+B) + \sin(A-B)}{2}\right]$

$A + B = 4\pi f_c t + \phi_i(t) + \phi_o(t)$

$A - B = \phi_i(t) - \phi_o(t)$

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or $e(t) = \frac{k_m A_c \cdot A_v}{2} \cdot \left[\sin[4\pi f_c t + \phi_i(t) + \phi_o(t)] + \sin[\phi_i(t) - \phi_o(t)]\right]$

After passing through LPF, the phase detector output will be,

$e(t) = \frac{k_m \cdot A_c \cdot A_v}{2} \sin[\phi_i(t) - \phi_o(t)]$

$= \frac{k_m A_c \cdot A_v}{2} \cdot \sin[\phi_e(t)]$

where, $\phi_e(t) = \phi_i(t) - \phi_o(t)$.

Now, the output of PLL will be the convolution of error voltage and the impulse response of loop filter

i.e.

$v(t) = e(t) \otimes h(t)$

or $v(t) = \int_{-\infty}^{\infty} e(\tau)\, h(t-\tau)\, d\tau$

Now, when the PLL enters into the lock mode i.e. the frequency and phase of $S_{FM}(t)$ & $b(t)$ are same, the error voltage $e(t) = 0$.

i.e. $\sin[\phi_e(t)] = 0$

or $\phi_e(t) = 0$

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So,

$$\phi_e(t) = \phi_i(t) - \phi_o(t) = 0$$

or

$$\phi_i(t) = \phi_o(t)$$

or differenting both sides we get,

$$\frac{d\,\phi_i(t)}{dt} = \frac{d\,\phi_o(t)}{dt}$$

$$\text{or } \frac{d}{dt} 2\pi k_f \cdot \int_0^t m(t)\,dt = \frac{d}{dt} 2\pi k_v \int_0^t v(t)\,dt$$

$$= 2\pi k_f \cdot m(t) = 2\pi k_v \cdot v(t)$$

$$\text{or } v(t) = \frac{k_f}{k_v} \cdot m(t)$$

So, we can see that in locked mode, the output of PLL is directly proportional to the message such that if $k_f = k_v$,

$$v(t) = m(t).$$

It should though be considered that the bandwidth of loop filter is not less than the bandwidth of the message signal.

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Stereo FM. broadcasting.

Stereo basically means two or double and in broadcasting stereo means to send or transmit two different elements of a program, for better effect.

For better quality of signal reproduction (specially that of music signal) at the receiving end, the output of group of microphones located at various positions of a concert hall are combined into two groups and the signal from each group is transmitted independently. So, stereo broadcasting can be taken as a frequency division multiplexing designed to transmit two separate signals through the same channel.

The specifications of standards for FM stereo transmission is influenced by two factors;

i) The transmission has to operate within the allocated FM broadcast channels.
ii) It has to be compatible with mono-phonic radio receivers.

In stereo broadcast system, two separate channels; Left (L) & right (R) are combined in the following manner to constitute the base-band FM signal.

i) Sum of L and R i.e. (L+R) is the original baseband form available for monophonic reception.
ii) DSB-AM of difference of L & R ie (L-R) signals for the purpose of separation of L and R signals at receiver end.
iii) A ~~pl~~ pilot tone at 19 KHz as synchronizing signal and an indicator of stereo transmission.
iv) space for subsidiary communication at 59-75 KHz.

The baseband spectrum of FM stereo can be shown as below,

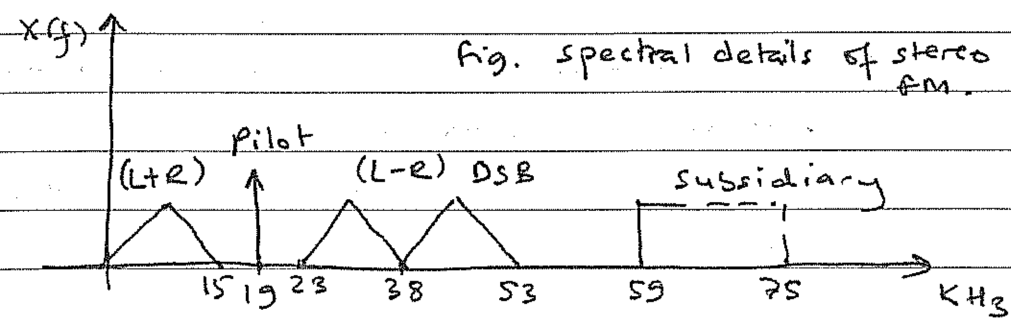

fig. spectral details of stereo FM.

FM stereo encoder / transmitter

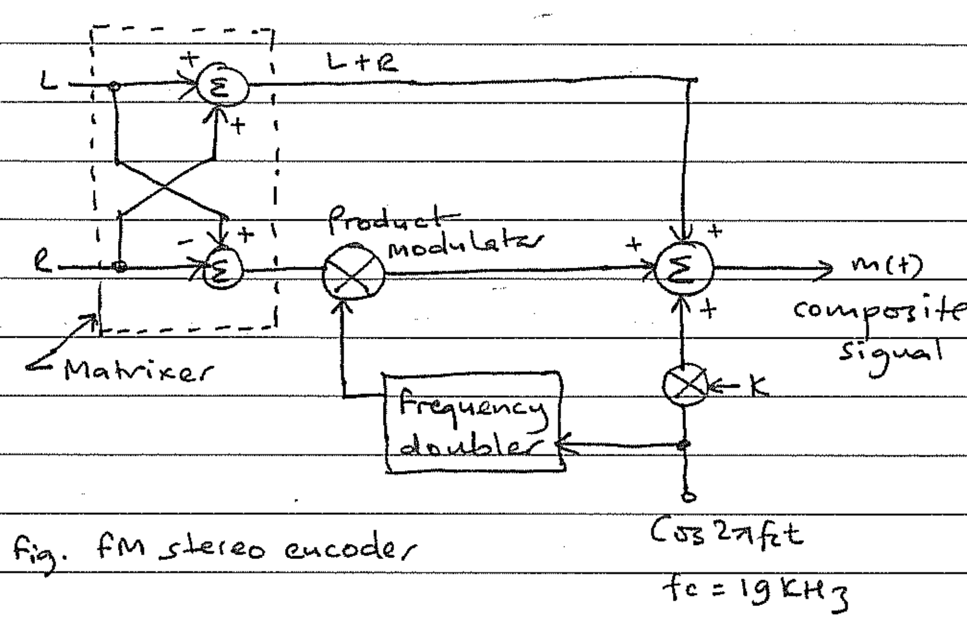

fig. FM stereo encoder

Let L & R denote the signals picked up by left hand and right hand microphones at the transmitting end of the system. They are applied to a simple 'matrixer' that generates the sum signal, (L+R) and difference signal, (L-R). The sum signal is left unprocessed ~~and~~ in its baseband form.

L+R is available for monophonic reception.

(L-R) and a 38 KHz subcarrier which is derived from a 19 KHz crystal oscillator by frequency doubling, are applied to a

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product modulator producing a DSB-SC modulated wave.

Also, a 19 KHz pilot signal is provided as a reference for coherent detection of the difference signal at the stereo receiver.

So,

$$m(t) = [L+R] + [L-R]\cos(4\pi f_c t) + K\cos 2\pi f_c t$$

where, f_c = 19 KHz.

K = amplitude of pilot tone.

This m(t) is then fed to frequency modulator to get the required FM stereo signal, and transmitted through the antenna.

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FM stereo decoder / receiver.

→ At a stereo receiver, first the composite signal m(t) is recovered from the FM wave using demodulation technique.

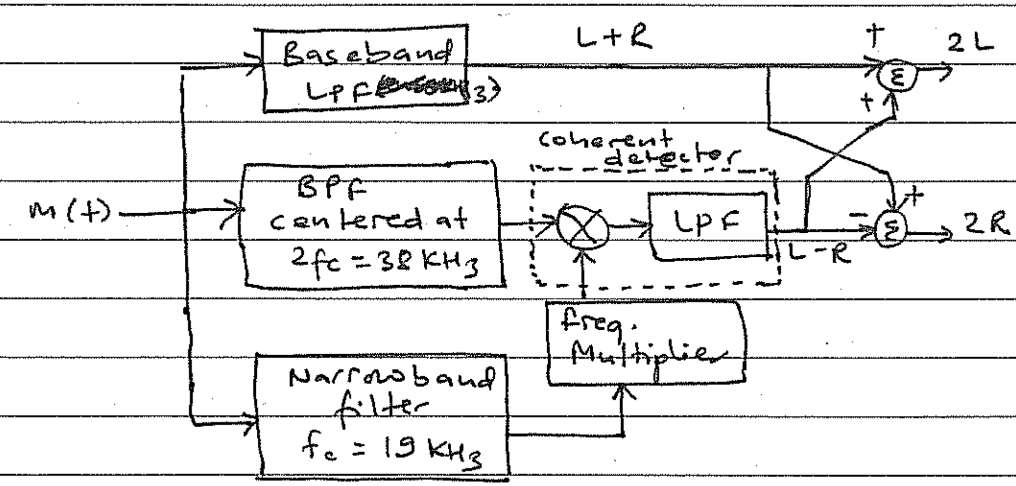

fig. FM stereo decoder.

Now, at the decoder, the composite signal 'm(t)' is divided into the individual components by using appropriate filters.
A baseband lowpass filters is designed to pass the sum signal (L+R) with the baseband frequency 0-15 KHz.

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The recovered pilot (using narrowband filter tuned at 19 KHz) is frequency doubled to produce the desired 38 KHz subcarrier. This 38 KHz subcarrier then helps in the coherent detection of the DSB-SC modulated wave. Hence, the difference signal (L-R) is recovered.

Finally, the simple matrixer reconstructs the original left hand signal 'L' & right hand signal 'R', except for the scaling factor 2 and applies them to their respective speakers.

FM stereophonic reception is thus accomplished.

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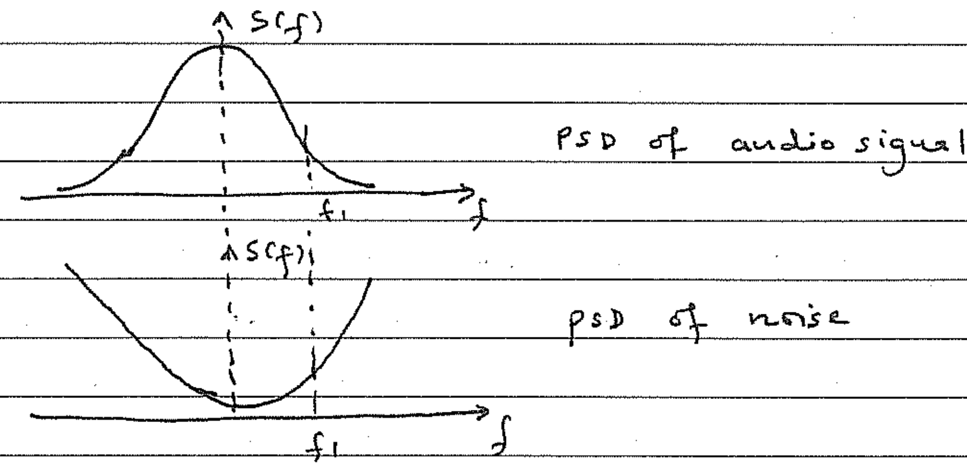
⊕ Pre-emphasis and de-emphasis networks

For any communication system, at the receiver end, SNR should be high or atleast be maintained at a particular value.

Now, if we look at the power spectral density of audio signal and noise as below;



In the above figure we see that the signal to noise ratio is higher for lower frequencies whereas for higher frequencies, signal to noise ratio will be comparatively lower.

In such case these higher frequency components ~~will be~~ are difficult to be reproduced.

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We have human voice around 3.3 KHz but high quality music contains frequencies around 15 KHz.

In stereo FM, we used L+R signals in the range 0-15 KHz and L-R signals in 23-53 KHz. Thus we see that these higher frequency signals get higher interferences.

To cope with this problem we use pre-emphasize and de-emphasize ckts.

a. Pre-emphasize ckt or n/w.

Basically, a ~~prea~~ pre-emphasis is the process of artificial boosting of high frequency components of message signal to increase signal to noise ratio. It can be termed as the increasing of the relative strength or amplitude of the high frequency components of the message signal before modulation.

So, pre-emphasis is nothing but selective amplification of higher frequencies of our audio signal.

Pre-emphasis is thus done at the transmitter before frequency modulation. The pre-emphasis circuit can be derived using a RC filter as below which is a high pass filter,

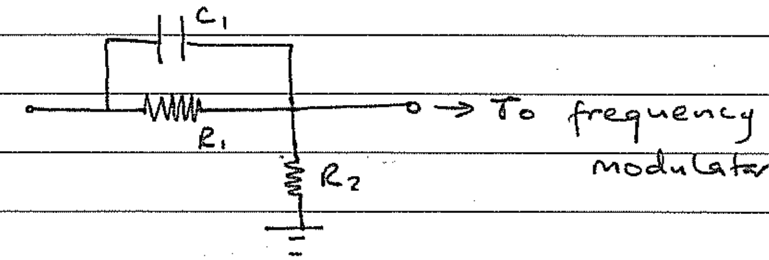

Fig. Pre-emphasis circuit.

The transfer function of this filter is defined as,

$$H(f) = \frac{R_2\,(1 + j\omega/\omega_1)}{(R_1+R_2)\,(1 + j\omega/\omega_2)}$$

where,

$$\omega_1 = \frac{1}{R_1 C_1} \quad \& \quad \omega_2 = \frac{R_1 + R_2}{R_1 R_2 C_1}$$

are cutoff frequencies.

The normalized gain versus frequency characteristics of the pre-emphasis circuit can be see on next ~~pl~~ page.

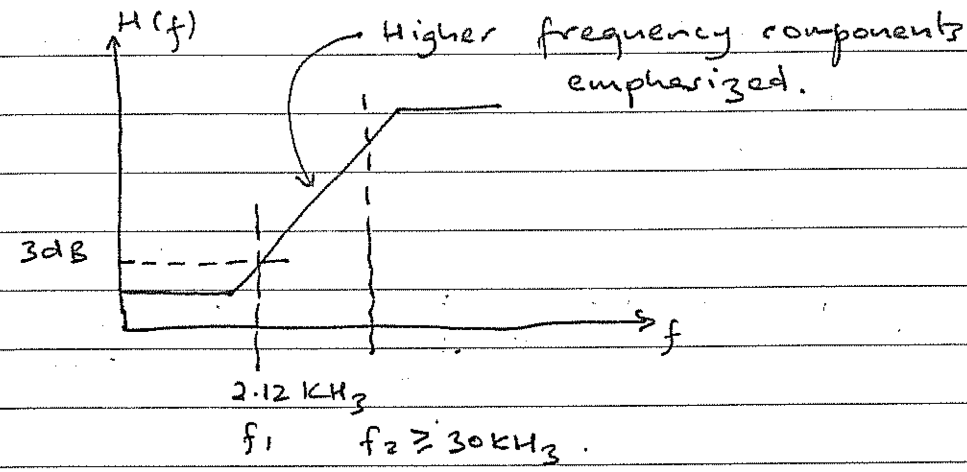

Fig. Pre-emphasis curve.

b. De-emphasis circuit.

The reducing of the amplitude level of the received high ~~signal level~~ frequency signal by the same amount as the increase in the pre-~~em~~ emphasis is known as the de-emphasis.

So, at the receiver end, the received message signal is de-emphasized after demodulation to simulate the original message signal. The de-emphasis circuit is a first order LPF.

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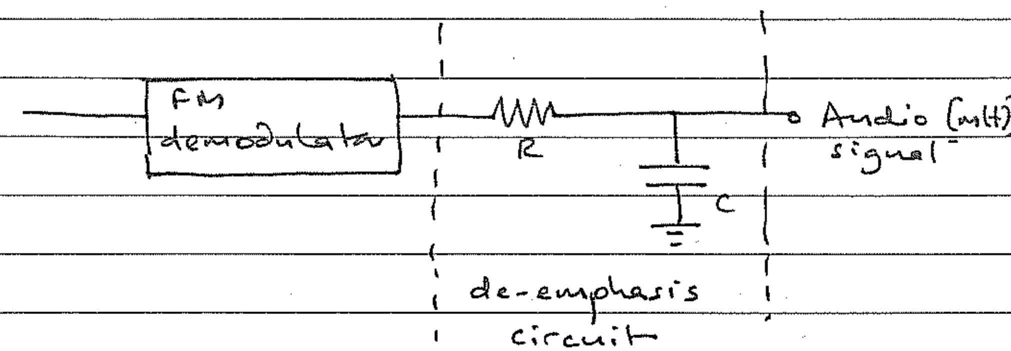


Fig. De-emphasis circuit

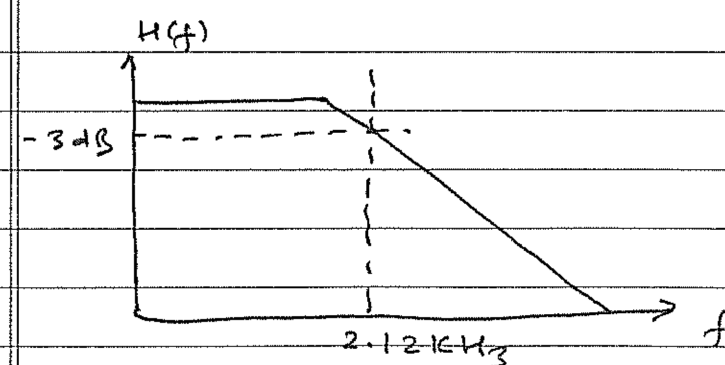
Now, the transfer f^n of the LPF is,

$$H(f) = \frac{1}{1 + j\omega/\omega_1}$$

where,

$$\omega_1 = \frac{1}{RC}$$

such that the cut off frequency is at 2.12 KHz.



Thus if we look at commercial FM radio system, it ~~consists of p~~ comprises of,

At transmitter end,

~~i) Pre-emphasis circuit~~

~~ii)~~

i) FM stereo encoder

ii) Pre-emphasis circuit

iii) FM modulator / transmitter

At receiving end

i) FM demodulator / receiver

ii) De-emphasis circuit

iii) FM stereo decoder

⊕ The superheterodyne radio receivers for AM & FM.

We know that for AM & FM, the source signal is audio. The different sources have diffirent spectrum and hence BWs which can be tabulated as below,

Speech $\simeq$ 4 KHz

High quality music = 15 KHz.

Now, we must know that AM radio limits baseband BW to 5 KHz and FM radio limits baseband BW to 15 KHz.

The baseband signals are then transmitted using RF range such that the radio frequency (RF) range for AM & FM are,

AM - 540 KHz - 1600 KHz

FM - 88 MHz - 108 MHz

For AM, in the spectrum of 540 KHz - 1600 KHz i.e. 1140 KHz, each station occupies a maximum bandwidth of 10 KHz

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with the carrier spacing of 10KHz.

And for FM we have 20MHz of spectrum available for different FM stations so each station can now occupy a bandwidth of 200KHz with carrier spacing of 200KHz.

For the basic design of FM and AM radio receivers, they must be cost effective.

Requirements of AM/FM radio receiver

- The radio has to work with both AM & FM.
- the radio must tune to and amplify desired radio station.
- Filter out all other stations
- demodulator has to work with all radio stations regardless of carrier frequency.

Now for the demodulator to work with any radio signal we convert the carrier frequency of any radio signal to an intermediate frequency (IF).

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design, the radio receiver then can be optimized for the intermediate frequency. Hence we focus on the IF filter and IF demodulator for intermediate frequency.

So, the IF determined for AM & FM are 455KHz & 10.7MHz respectively. And the radio receiver that performs such conversion of RF to IF before demodulating the message signal is known as superheterodyne radio receivers.

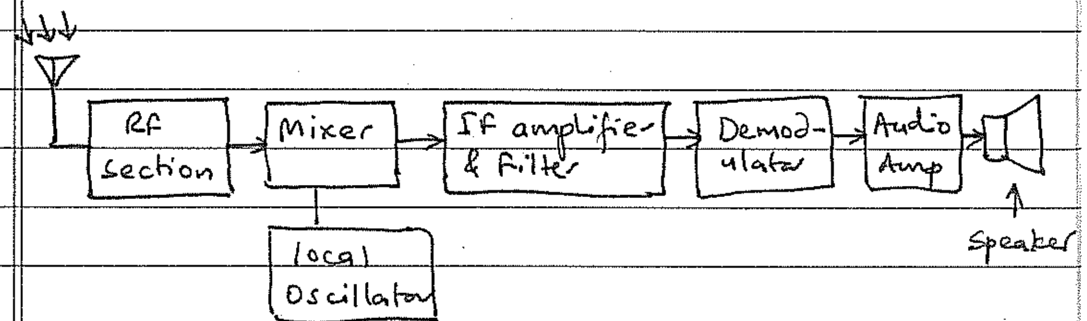

Fig. Block diagram of Superheterodyne Radio Receiver.

In the above figure, the multiplexed signal from different stations are detected at the receiver antenna.

At the RF section, tuning to the desired station is performed i.e. a certain RF frequency 'f_c' is determined.

Then, the RF signal is fed to a RF-IF converter which comprises of a mixer and local oscillator. This local oscillator uses variable capacitor to obtain variable frequency oscillation.

The frequency of local oscillator is synchronized with incoming carrier frequency such that

$$f_{LO} = f_c + f_{IF}$$

where, f_{IF} = Intermediate frequency.

Now, the mixer output will be,

$$f_{LO} + f_c = 2f_c + f_{IF}$$

$$\& \quad f_{LO} - f_c = f_{IF}$$

When passed through IF filter, we obtain the IF component which is then amplified and fed to the demodulator.

Now depending on the type of the received signal, the output of 'IF filter' is demodulated using AM or FM demodulators.

i.e.

For AM – Envelope detector

For FM – frequency discriminator.