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Pulse Modulation Systems

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Modulation Techniques

modulation

- continuous wave modulation (← carrier signal →) Pulse modulation
  - (message signal)
  - Analog ~~(…)~~ CW modulation
    - AM
    - FM
    - PM
  - Digital CW modulation
    - ASK
    - FSK
    - PSK
  - Analog Pulse modulation
    - PAM
    - PWM
    - PPM
  - Digital pulse Modulation
    - PCM
    - DPCM
    - DM

Pulse modulation schemes

→ In pulse modulation, some parameter of a pulse train is varied in accordance with message signal.

Types of Pulse modulation

① Analog pulse modulation

② Digital pulse modulation

1. Analog pulse modulation

a> PAM (Pulse amplitude modulation)

b> PWM (Pulse width modulation)

c> PPM (Pulse position modulation)

2. Digital pulse modulation

a> PCM (Pulse code modulation)

b> DPCM (Differential PCM)

c> DM (Delta modulation)

PAM

→ In pulse amplitude modulation, the amplitude of the pulse is varied in accordance amplitude of information signal.

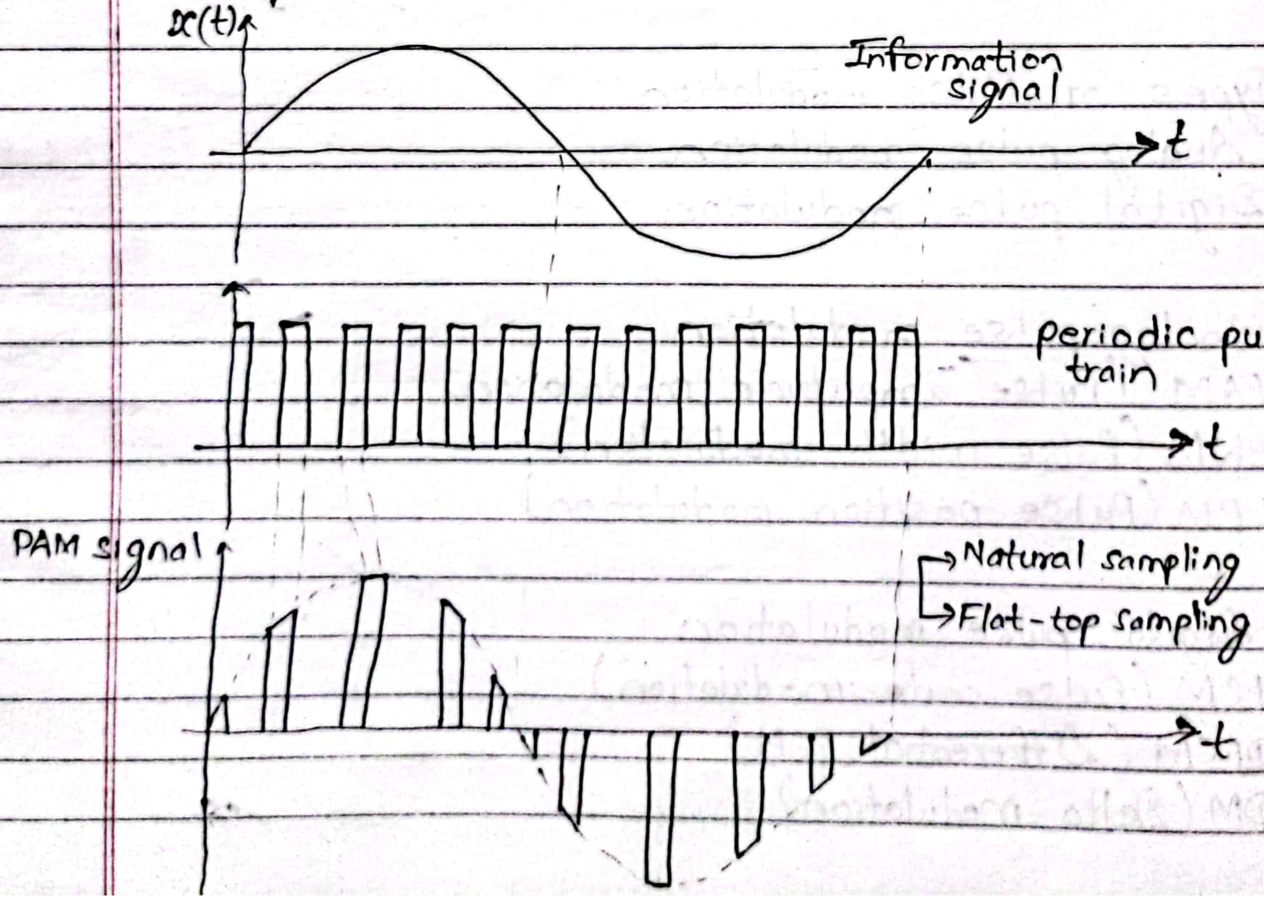
PWM

→ In PWM, width of each pulse is made directly proportional to the amplitude of the information signal.

PPM (PDM / PLM)

duration, length

→ In PPM, constant width pulses are used and position or time of occurance of each pulse from some reference time is made directly proportional to the amplitude of the information signal. (position change according to Amp).



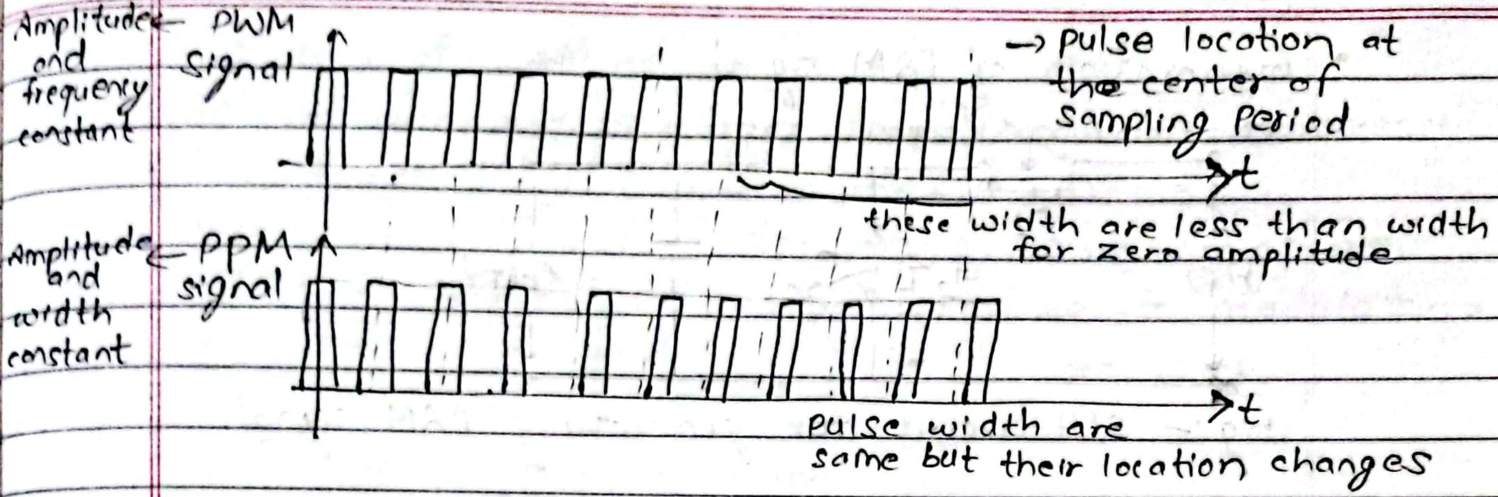


Fig:- waveforms of PAM, PWM, PPM signal

• Pulse Amplitude Modulation (PAM)

→ PAM is used to describe the conversion of the analog signal to a pulse-type signal in which amplitude of the pulse denotes the analog information.

Analog waveform $\xrightarrow{\text{converted to}}$ PAM $\xrightarrow{\text{converted to}}$ PCM

Purpose of PAM

→ Bandwidth of PAM is wider than Analog waveform.

→ Pulses are more practical in digital systems.

Types of PAM

1. PAM using natural sampling
2. PAM using flat top sampling

Flat top PAM is most popular and widely used (because noise interfaces with top of transmitted pulses, so noise can be easily removed).

PAM

- Generation of PAM signal

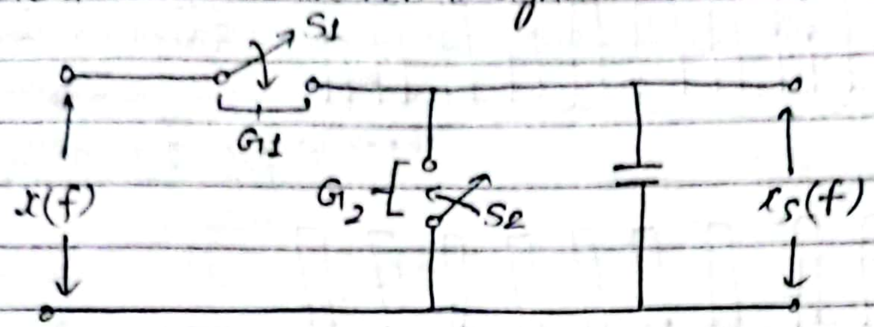


Fig :- S/H circuit for generating PAM signal.

S1 : Sampling switch

S2 : Discharging switch

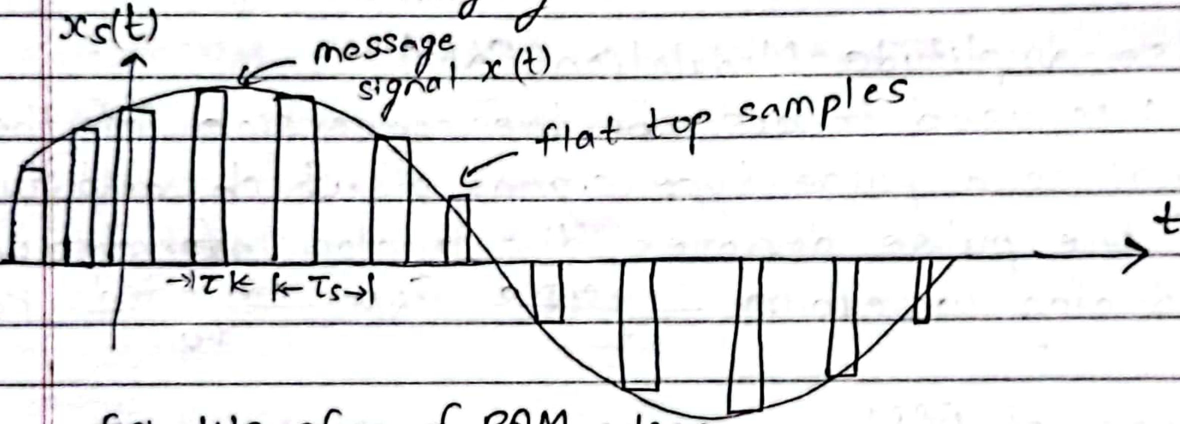


fig:- Waveform of PAM pulses

$\tau \rightarrow$ Sample duration

$T_s \rightarrow$ Sampling Time

PAM demodulator

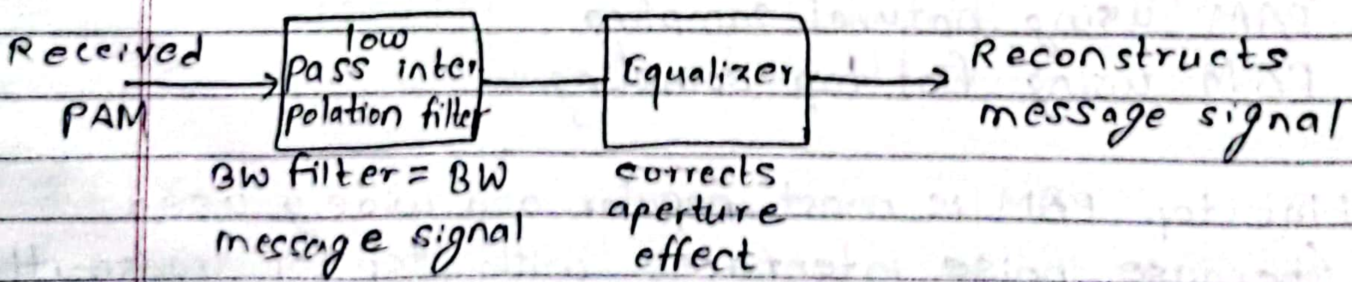


Fig :- Reconstruction of PAM signal

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Drawback of PAM

- → BW requirement is larger than maximum frequency.
- → Interference of Noise is maximum.
- → Peak power required for transmission varies as amplitude of pulses vary according to modulating signal.

Transmission BW in PAM

→ In the PAM signal, the pulse duration is considered to be very small in comparison to time period i.e sampling period.

i.e $\tau << T_s$

let f_x be maximum frequency component of $x(t)$.

By sampling theorem:

$$f_s \geq 2f_x$$

or, $$T_s \leq \frac{1}{2f_x}$$

→ If 'on' and 'off' time of PAM is same then max^m freq. is

$$f_{max} = \frac{1}{\tau+\tau} = \frac{1}{2\tau}$$

$$BW \geq f_{max}$$

or, $$BW \geq \frac{1}{2\tau}$$

But $\tau << \frac{1}{2f_x} \Rightarrow \frac{1}{2\tau} >> f_x$

i.e $BW \geq \frac{1}{2\tau} >> f_x$

Hence $\boxed{BW >> f_x}$

Q. For a PAM transmission of voice signal having maximum freq. $f_x = 3$ KHz. Calculate the BW. Given $f_s = 8$ KHz and $\tau = 0.1 T_s$

Sol^n:- We have,

$$BW \geq \frac{1}{2\tau}$$

$$T_s = \frac{1}{f_s} =$$

$$\tau = 0.1 T_s =$$

$$B.W \geq \frac{1}{2 \times}$$

$$B.W \geq \frac{1}{2 * 0.1} f_s$$

$$B.W \geq \frac{8 \text{ KHz}}{2 * 0.1}$$

- PWM / PDM / PLM and PPM

Generation of PWM signal and PPM signal

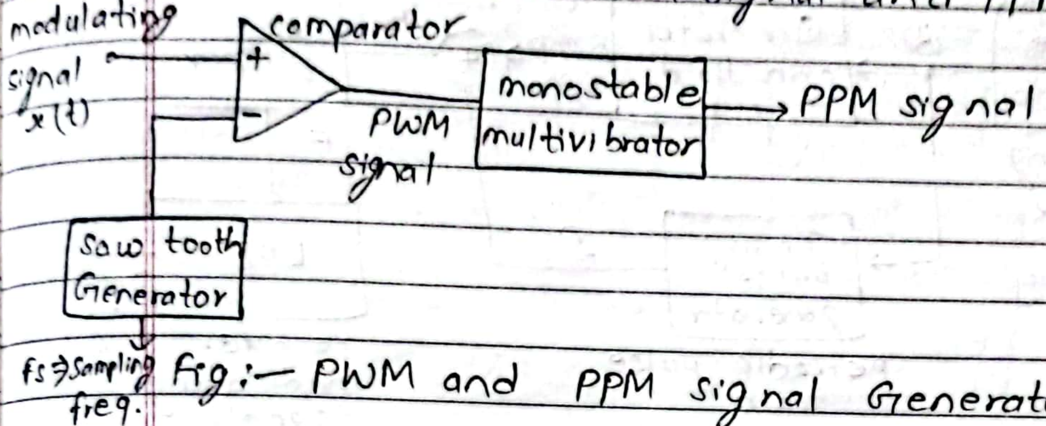


Fig :— PWM and PPM signal Generator

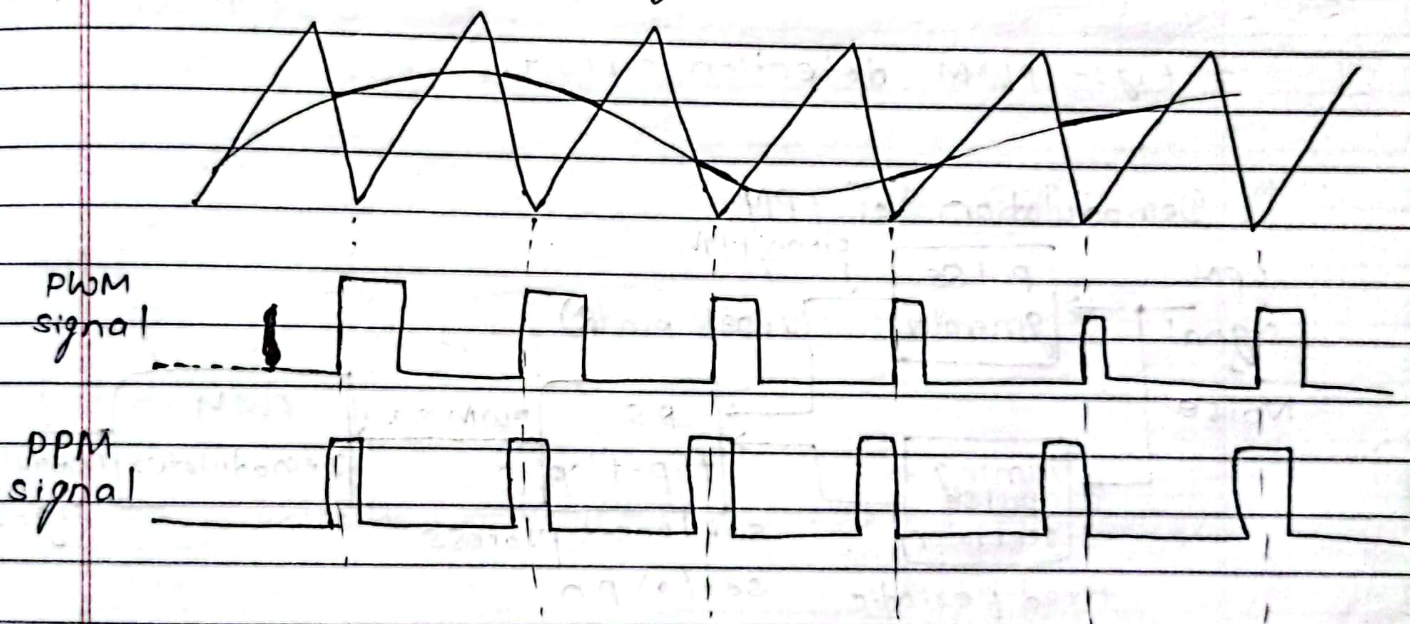


Fig :- waveform of PWM and PPM signal

PPM signal
- $x(t) = 0$ (centre of reference signal)
- $x(t) = +ve$ (Right shift)
- $x(t) = -ve$ (Left shift)

- Detection of PWM signal

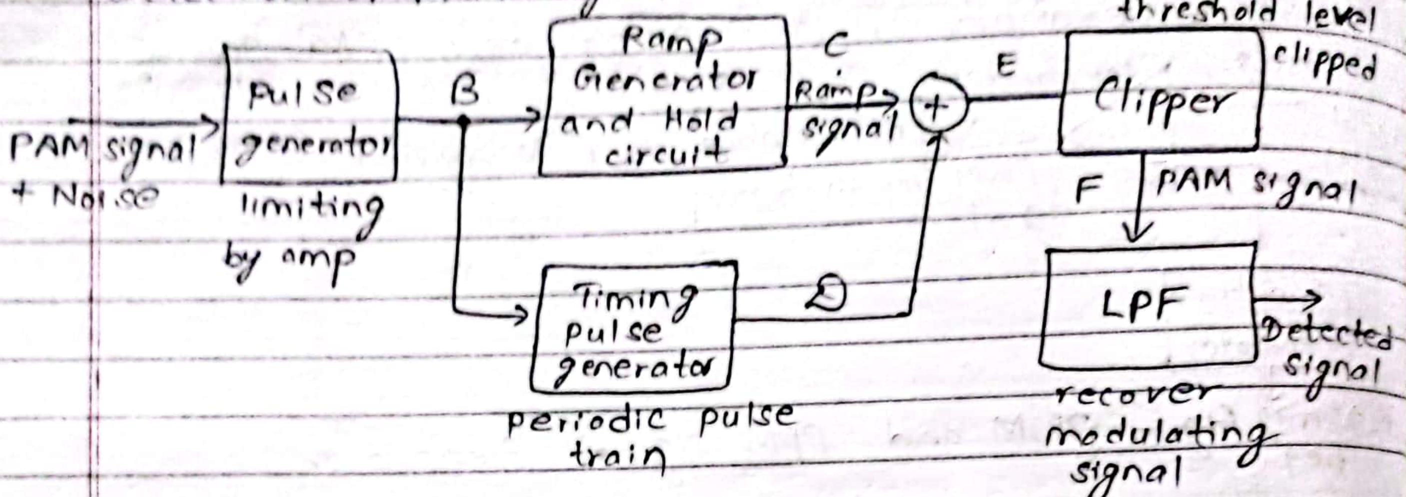


Fig :- PWM detection circuit

Demodulation of PPM

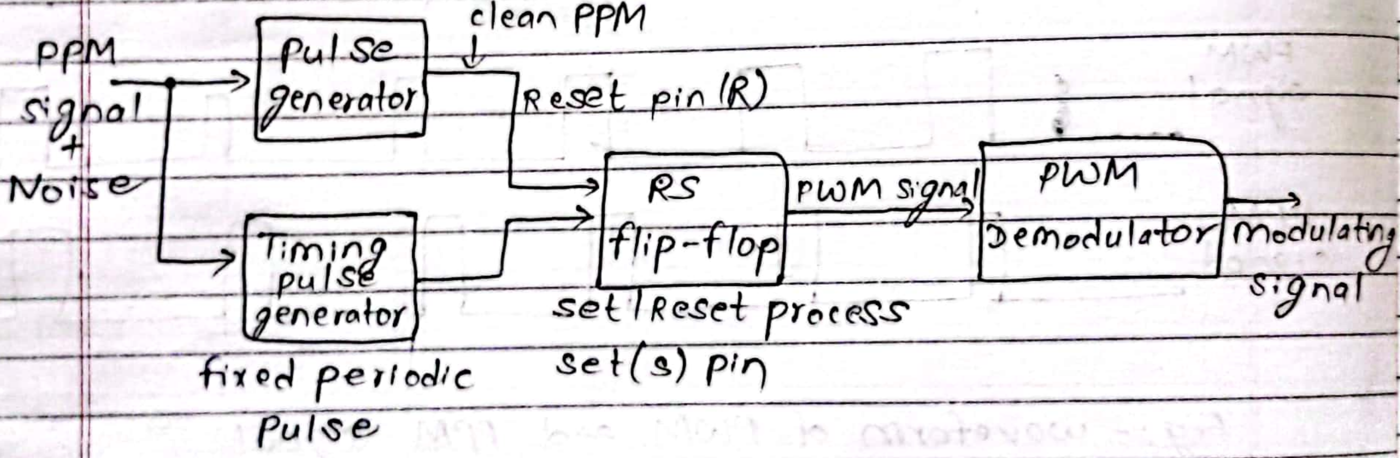


Fig :- PPM demodulator circuit

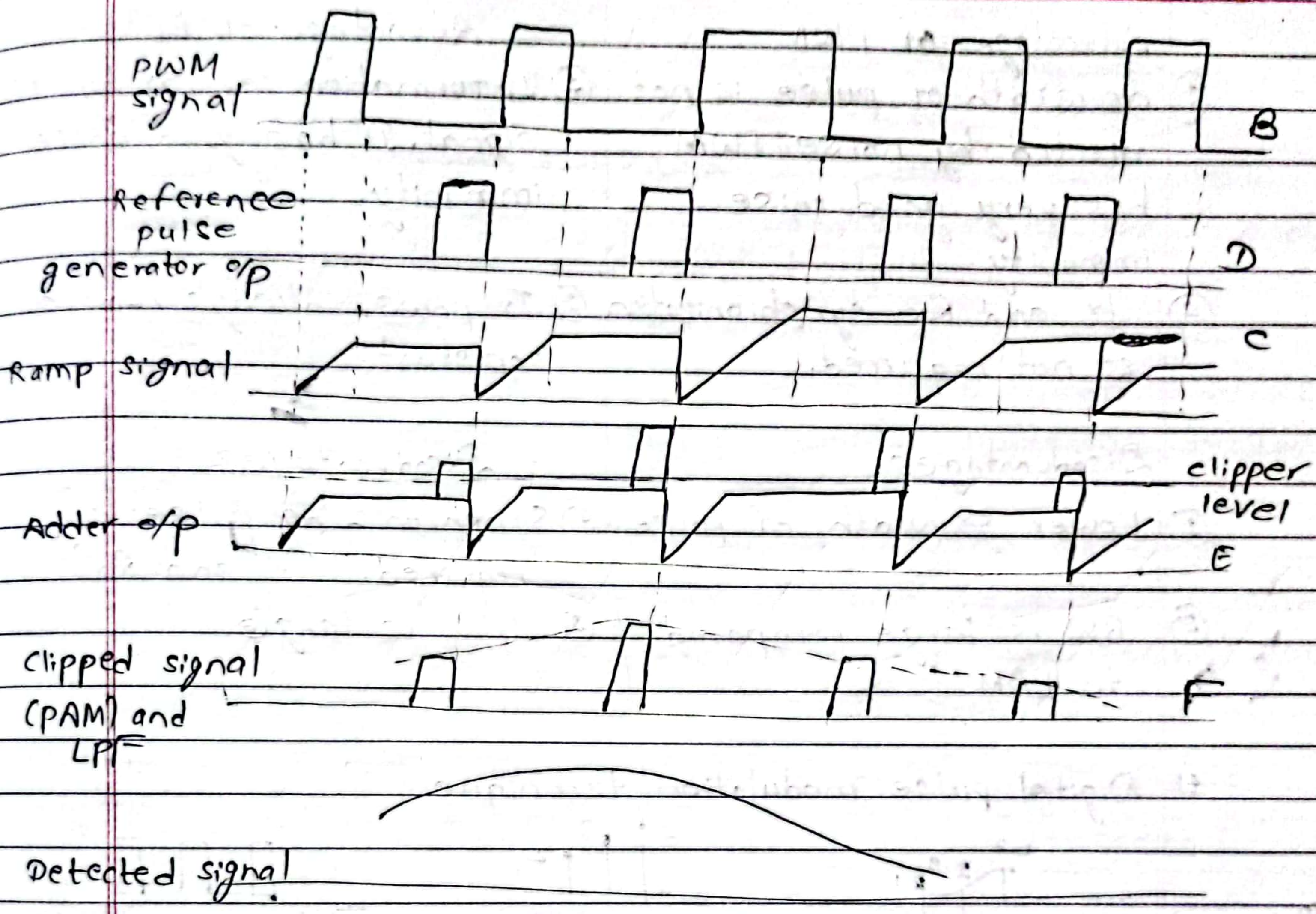


Fig :- Waveforms for PWM detection circuit.

Advantage of PWM

1. As width of pulse is not affected by noise, PWM has very good noise immunity.
2. Tx and Rx synchronization is not required.

Disadvantages

1. Power $\propto$ width of pulse.
2. BW is large compared to PAM.

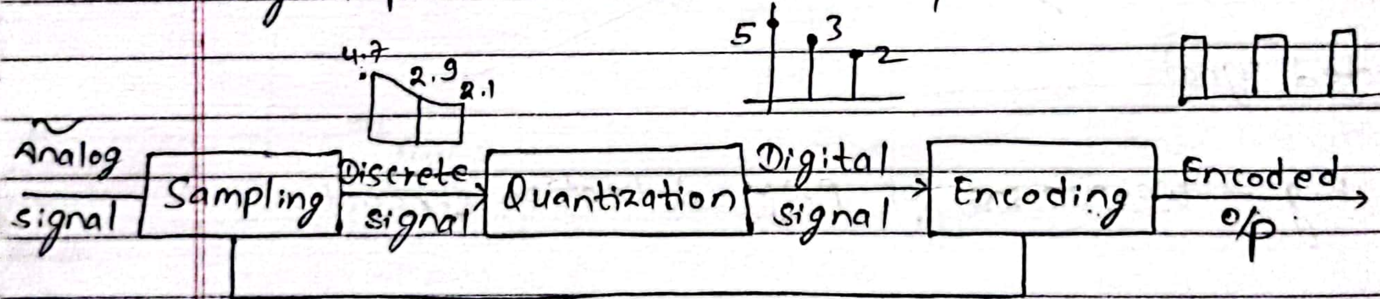
Advantage of PPM

1. Information carried by PPM signal. It has good noise immunity.
2. Tx power always remain constant.

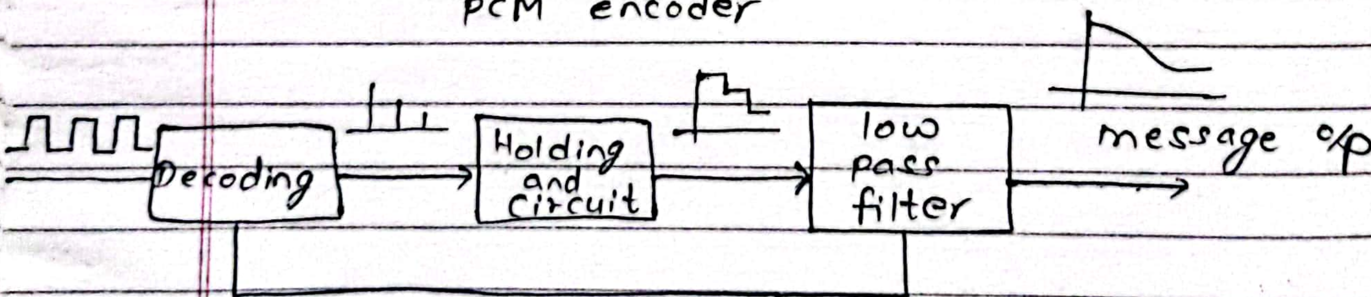
Disadvantages

1. Synchronizing pulse is required. (Tx and Rx)
2. BW is large.

Digital pulse modulation technique



PCM encoder



PCM decoder

- The PCM is not a modulation is conventional sense.
- In PCM, the only section in which this happens in

while sampling.

- **Pulse code modulation (PCM)**
- Pulse code Modulation is a technique by which analog signals are converted into digitally encoded signal.
- first method for digital coding of waveforms.
- In PAM, PWM and PPM information is transmitted in an analog pulses of constant amplitude, width and position.
- Basic operations: Sampling, quantizing, encoding
- Analog to digital conversion and reverse operation for recovering message signal by D/A conversion.

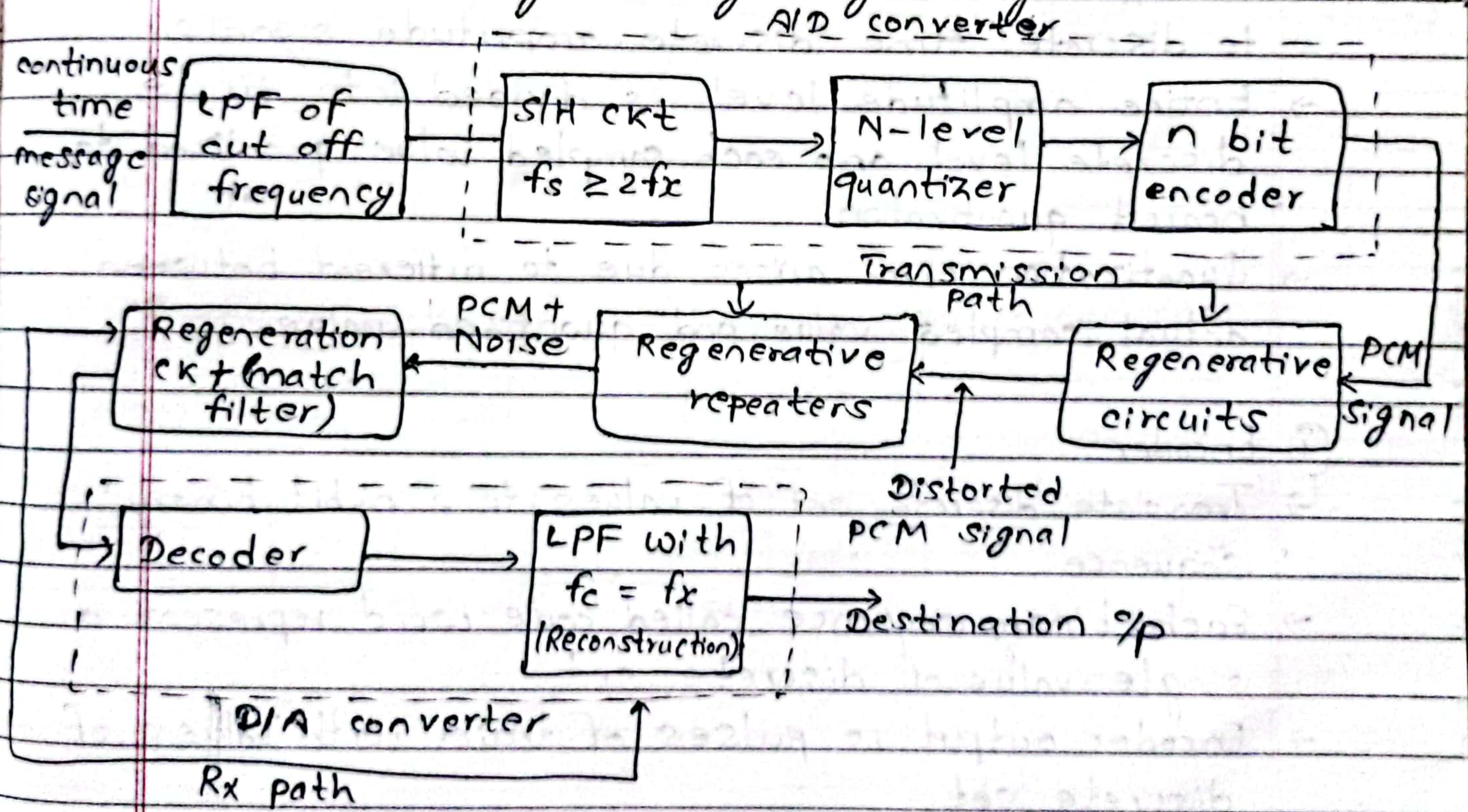


Fig:- Basic elements of PCM system

① Low Pass filter

→ pre or antialising filter

→ used to block frequency greater than f_x Hz before sampling i.e band limiting the analog signal.

② Sample and hold circuit

→ Sampling is carried out to produce flat top sampling.

→ It must follows Nyquist criteria

$$f_s \geq 2f_x.$$

③ Quantizer and quantization

→ converts discrete time continuous amplitude signal to discrete time discrete amplitude signals.

→ Entire amplitude level is divided into finite discrete level and each sampled value quantized to nearest quantization.

→ Quantization error arises due to different between actual sampled value and quantized value.

④ Encoder

→ Translate discrete set of values to a n-bit binary sequence.

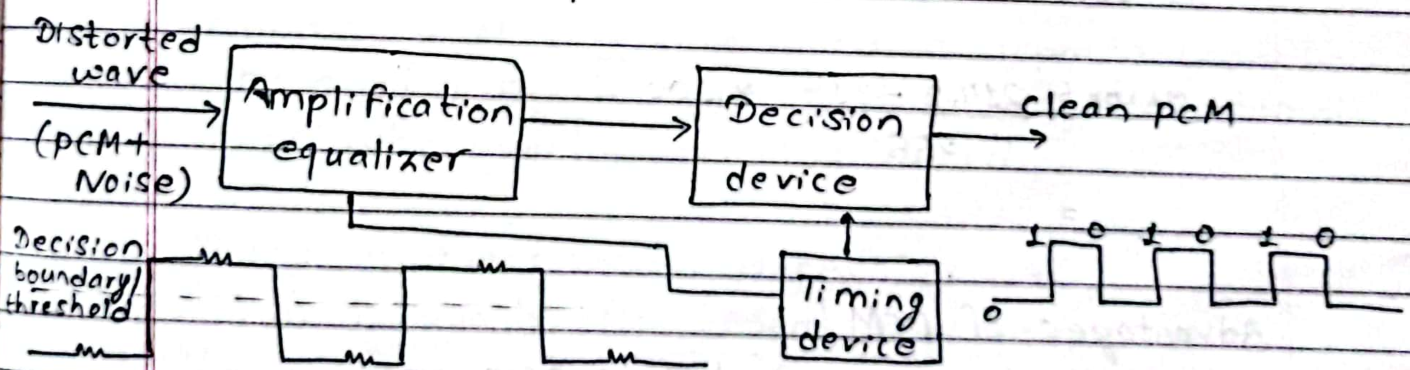
→ Each binary sequence called code word represent a single value of discrete set.

→ Encoder output is pulses of train with values of discrete set.

⑤ Regenerative repeaters (ckts)
→ The result of encoder is digitally encoded signal is transmitted over a communication channel to the receiver.
→ For elimination of noise and distortion a chain of regenerative repeaters are placed along the channel path and regenerative ckt at the receiver for the reconstruction of PCM data.

⑥ Decoder and Decoding
→ Regroup the received bits into binary code word and decode them into quantized PAM signal.
→ This signal or level is proportional to the weighted sum of binary word/code word.

Regenerative Repeater



(2-10) Km range using Regenerative Circuit

Q. For 8 bit (PCM) ADC converter whose peak voltage is 5V. Calculate the binary value of 4 and 4.2 V. Calculate quantization error.

Solution: 5 volt $= (1111\ 1111)_2$

$= (255)_{10}$

$(1\ V)_{10} = \frac{5\ V}{255} = \frac{1}{51}$ ⇒ resolution

For 4 volt

1 volt $= \frac{255}{5} = 51$

4 volts $= 51 \times 4 = (204)_{10}$

$= (11001100)_2$ error $= 0$

4.2 volts $= 214.2 \approx 214$

error $= \frac{214.2 - 214}{255} * 100\ \%$

$=$

Advantages of PCM

- Robustness to noise and interference
- Efficient regeneration
- Efficient SNR and BW trade off.
- uniform format
- secure
- multiplexing easy

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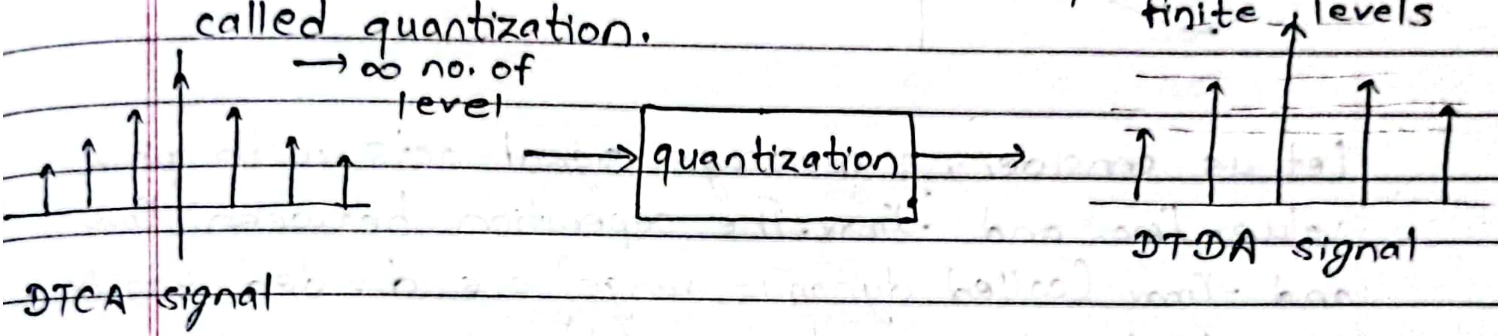
Disadvantages

- BW requirement is high.
- Synchronization problem.

Quantization

- The process of representing the analog sampled values (infinite amplitudes) into finite amplitude level is called quantization.



- If n is the no. of bits per sample, the maximum distinct possible levels are $N = 2^n$.

$$N = \infty \quad \text{so} \quad n \to \infty$$

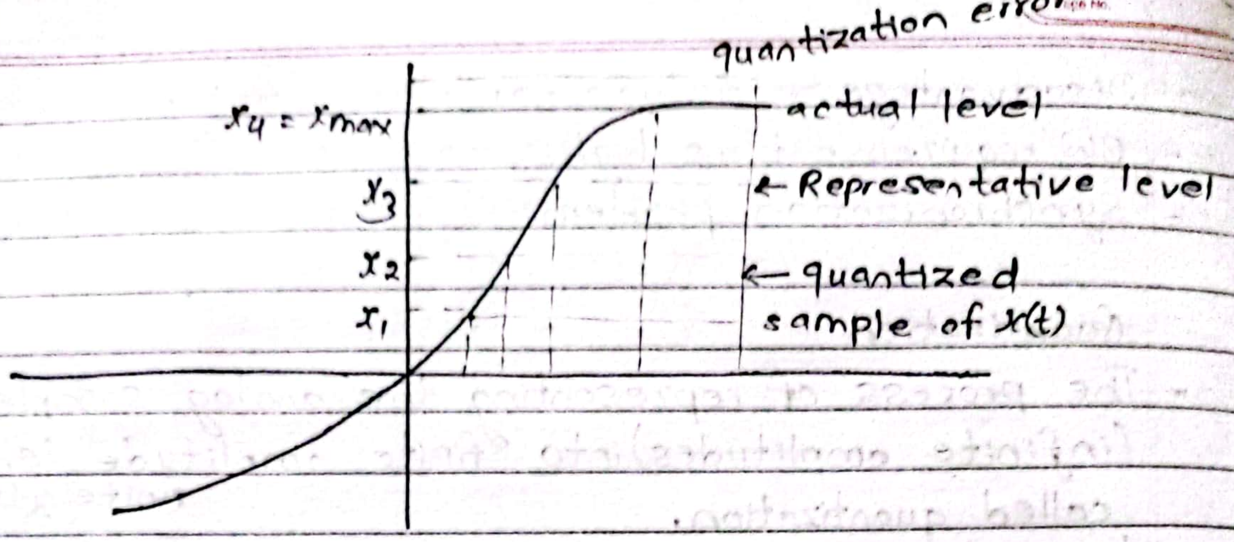
- Infinite no. of bits are used to represent each sample of infinite time to transmit each sample thats why quantization is required.

Types of quantization process

a) Uniform quantization (equal stepsize)
b) Non Uniform quantization (non equal step size)

+ Uniform Quantization

In uniform quantization, total amplitude range of signal is divided into finite no. of levels with constant step size (Δ).



Let us consider, a message signal acts with peak value X_{max} and $-X_{max}$. The seperation between X_{max} and $-X_{max}$ (called dynamic range) are divided into N equal interval of each size (Δ).

where, Δ is step size (quantum)

$$x_2 - x_1 = x_3 - x_2 = x_4 - x_3 = \Delta$$

$$\Delta = \frac{X_{max} - (-X_{max})}{N} = \frac{2X_{max}}{N}$$

If N is total no. of quantization level then

$$N = 2^n$$

$$\Delta = \frac{2X_{max}}{N} = \frac{2X_{max}}{2^n} = \frac{X_{max}}{2^{n-1}}$$

Types of uniform quantization

a) Mid Trade Type → Representation level $(0, \pm\Delta, \pm 2\Delta, \pm 3\Delta, ---)$

→ Decision level $\left(\pm \frac{\Delta}{2}, \pm \frac{3\Delta}{2}, \pm \frac{5\Delta}{2}, ----\right)$

b) Mid riser type → representation level $\left(\pm\frac{\Delta}{2}, \pm\frac{3\Delta}{2}, \pm\frac{5\Delta}{2}, \pm\ldots\right)$

Decision level $(0, \pm\Delta, \pm 2\Delta, \pm 3\Delta \ldots)$

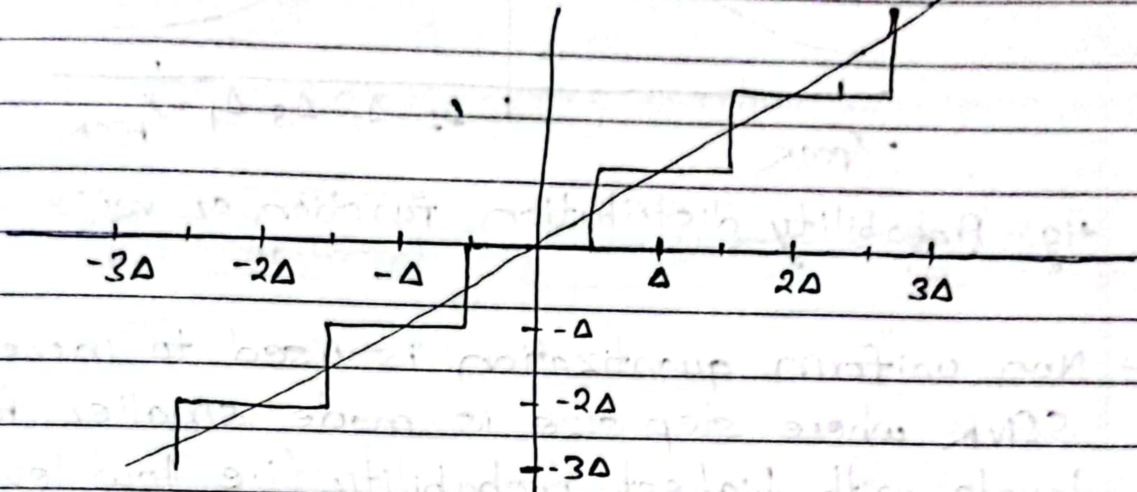
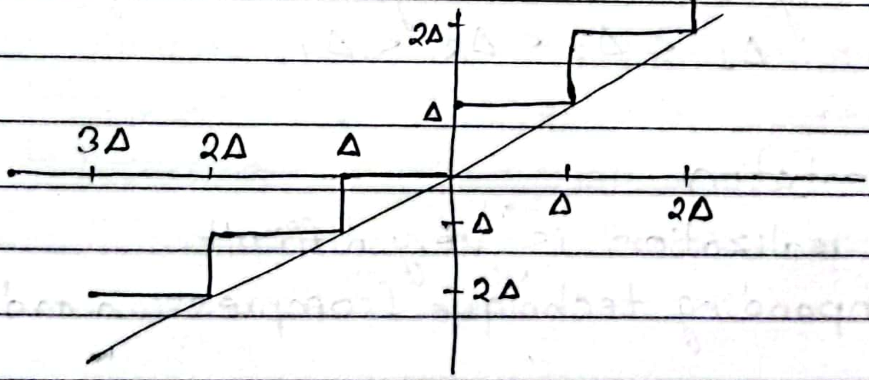


Fig:- Symmetrical (mid ~~trend~~ trade type quantizer like star care characteristics)



Imp
+ Non Uniform Quantization
→ In uniform quantization, step size is uniform throughout dynamic range of signal so average quantization noise power.

$$P_{qE} = \frac{\Delta^2}{12}$$

But for some P_{qE} at x_{peak} (high) → high signal power, → high SQNR

at x_{peak} (low) → low signal power, → low SQNR

Quantization steps making them small near zero and large at extremes.

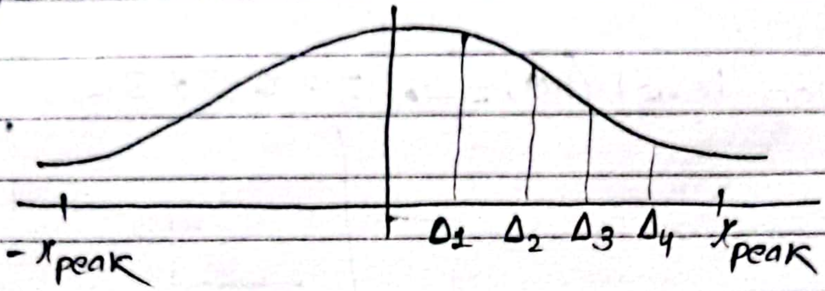


fig:- Probability distribution function of voice signals

→ Non uniform quantization is used to increase average SQNR where step size is made smaller for signal levels with highest probability (i.e low level signals in audio/voice) and step size is increased as probability decreases (high level signal).

So, $\Delta_1 < \Delta_2 < \Delta_3 < \Delta_4$.

Implementation

(a) Direct realization is very difficult.

(b) Use companding technique (compression and expansion)

→ In companding process, weak signals are amplified and strong signals are attenuates before applying them to a uniform quantizer. The process is called compression.

→ At the receiver excatly opposite process is followed called expansion.

→ The process of compressing and expanding of message signal (baseband signal) is called companding technique.

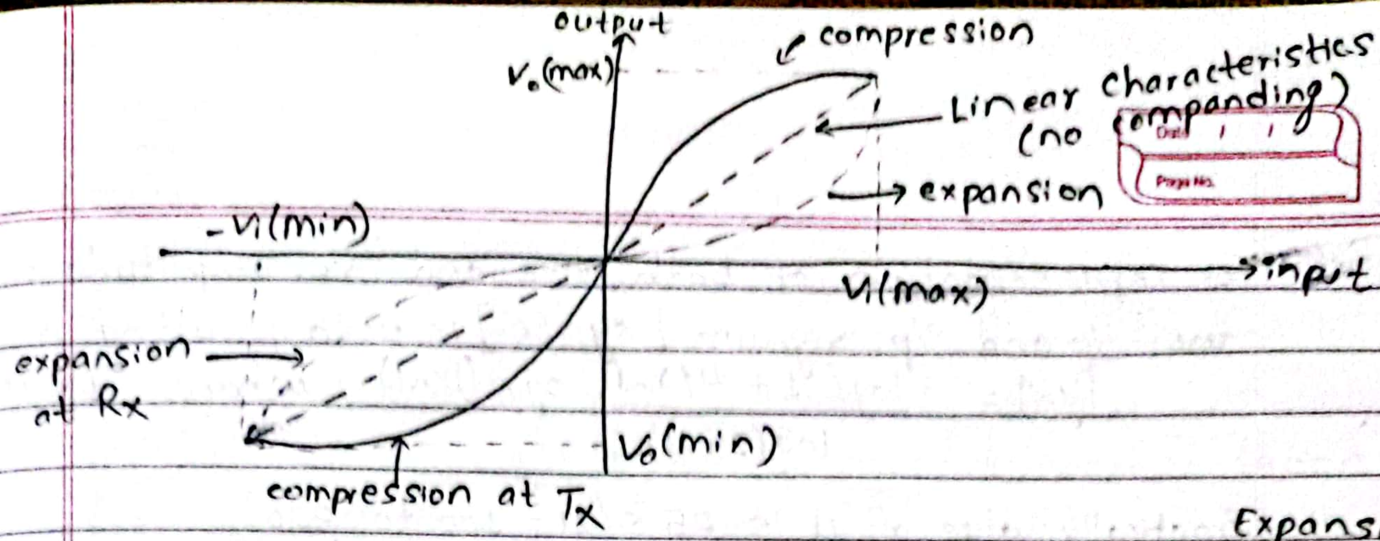


Fig:- Transfer characteristics of compressor and ~~Expansion~~ Expanser

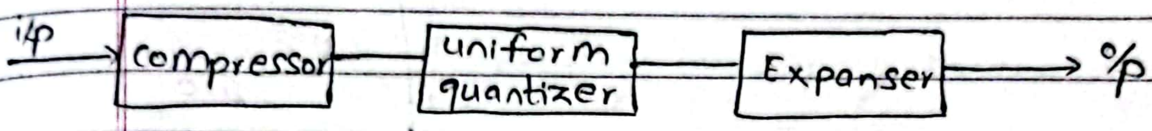


Fig :- Companding model

+ Types of compression laws or companding technique
  ↳ companding characteristics

⇒ Two most commonly used compression laws are :

① μ-law → American standard → US, Canada, Japan
  ↳ used in PCM telephone system
  ↳ used for music signal

② A-law → European standard → Europe, Nepal and rest in world

μ-law expanding

The o/p of μ-law compressor is defined as

$$|y_n| = \frac{\log\left[1+\mu\left|\frac{x}{X_{max}}\right|\right]}{\log(1+\mu)} = \frac{\log[1+\mu|x_n|]}{\log(1+\mu)}$$

(high level of x → logarithmic)
(low level of signal linear characteristics) where $0 \le |x_n| \le 1$

The compressor characteristics of μ-law compressor is continuous μ is +ve constant (+ve amplitude)

$\mu = 0$ (uniform quantization no companding)

For representation of both +ve and -ve amplitude of the i/p and o/p, signum [sgn(x)] is used resulting in

$$|y_n| = \frac{\log(1+\mu|x_n|)}{\log(1+\mu)}\,sgn(|x_n|) \text{ , where } -1 \le |x_n| \le 1$$

Practically, value of μ is 255 (μ = 100 to 300)

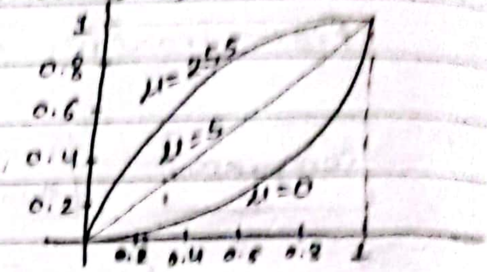


Fig:- μ-law characteristics

② A-law of companding

The input-output relationship of A-law compressor.

$$|y_n| = \frac{A|x_n|}{1+\log A} \quad \text{for } 0 \le |x_n| \le \frac{1}{A}$$

$$= \frac{1+\log(A|x_n|)}{1+\log A} \quad \text{for } \frac{1}{A} \le |x_n| < 1$$

A-law compressor characteristics is piecewise model of linear segment for low level inputs and logarithm segment for high level inputs.

→ Both the μ-law and A-law curve have odd symmetry about vertical axis.

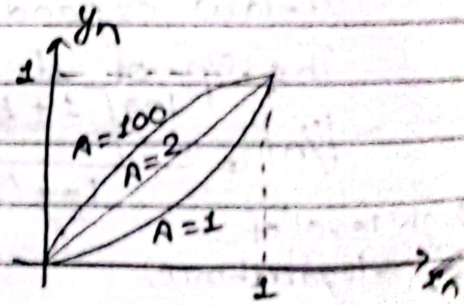
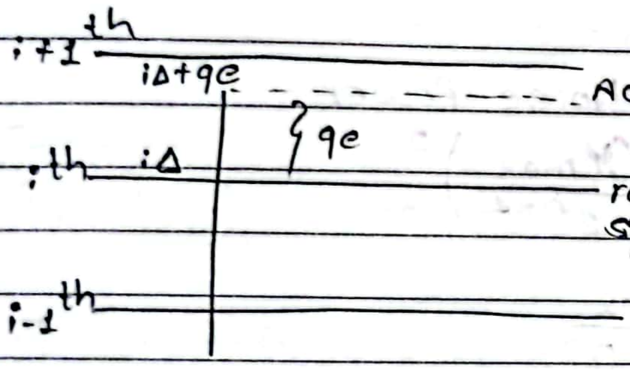


Fig:- A-law characteristics

Practical value of A=100

Quantization error/noise

- It is produced during the process of quantization because of rounding off of sampled values of the continuous message signal to nearest representation level.
- It is difference between input signal level and quantized version.
- It can be decreased by simply increasing the number of level N.



$$Q_{error} = i\Delta + q_e - i\Delta$$
$$= q_e$$

- Quantization error q_e lies between $-\Delta/2$ and $\Delta/2$ randomly with zero mean. Quantization noise is due to quantization error.
- Average quantization noise power is

$$P_{qe} = \frac{1}{\Delta} \int_{-\Delta/2}^{\Delta/2} q_e^2 \, dq_e$$

$$P_{qe} = \frac{1}{\Delta} \int_{-\Delta/2}^{\Delta/2} q_e^2 \, dq_e$$

$$= \frac{1}{3\Delta} * \frac{\Delta^3}{4}$$

$$= \frac{\Delta^2}{12}$$

→ Quantization noise for uniform quantization depend only on step size.
So $N \uparrow \Rightarrow \Delta \downarrow \Rightarrow P_{qe} \downarrow$ for given range of signal.

SQNR (Signal quantization Noise ratio) in uniform quantization
↳ SQNR in PCM.

In PCM system

$$SQNR = \frac{\text{Average signal power}}{\text{Average quantization noise power}}$$

So, average quantization Noise Power

$$P_{qe} = \frac{\Delta^2}{12} = \frac{1}{2} * \left(\frac{x_{max}}{2^{n-1}}\right)^2$$

$$= \frac{x_{max}^2}{3 * 4^n}$$

Let $x(t)$ be the denoted signal. Average power of signal $x(t)$ is $\overline{x^2}$.

$$\text{So, } SQNR = \frac{\overline{x^2}}{x_{max}^2} \times 3 \times 4^2$$

$$\tilde{x}^2 = \frac{\overline{x^2}}{x_{max}^2}$$ ⇒ normalized signal power which lying 0 to 1.

$$SQNR = \tilde{x}^2 \times 3 \times 4^n$$

$$\boxed{SQNR_{dB}} = 10 \log_{10}\left[\tilde{x}^2 \times 3 \times 4^n\right]$$

$$= 10 \log_{10} \tilde{x}^2 + 10 \log_{10} 3 + 10 \log_{10} 4^n$$

$$= 10 \log_{10} \tilde{x}^2 + 4.8 + 6.02\, n$$

CamScanner

Upper limit of SQNR
↳ max^m

$$[SQNR]_{max} = 3 \times 4^n \quad [\because \overline{x^2} = 1]$$

In dB

$$[SQNR_{dB}]_{max} = 10 \log_{10} [3 \times 4^n]$$

$$= \cancel{10 \log 10^3} + 10 \log_{10} 4^n$$
$$10 \log_{10} 3$$

$$= 4.8 + 6.02n$$

For $n=1$

$$(SQNR_{dB})_{max} = 10.82 \text{ dB}$$

For $n=2$

$$(SQNR_{dB})_{max} = 16.84 \text{ dB}$$

So for each extra bit n used for representing each quantization level SQNR is increased by 6 dB (4 times)

In terms of level of quantization

$$[SQNR]_{max} = 3 \times 4^n$$
$$= 3 \times (2^n)^2$$
$$= 3 \times N^2$$

In dB

$$[SQNR_{max}]_{dB} = 10 \log_{10} (3 \times N^2)$$
$$= 10 \log_{10} 3 + 10 \log_{10} N^2$$
$$= 4.8 + 20 \log_{10} N$$

Num **Q.** Derive an expression for SQNR for a PCM system using uniform quantization technique. Given that input to the PCM system is sinusoidal signal.

sol^n:-
$$SQNR = \frac{\text{Average Signal Power}}{\text{Average quantization noise Power}}$$

For sinusoidal signal
$$x(t) = x_m \sin(\omega t).$$

$$\text{Average signal power} = \frac{x_m^2}{2}$$

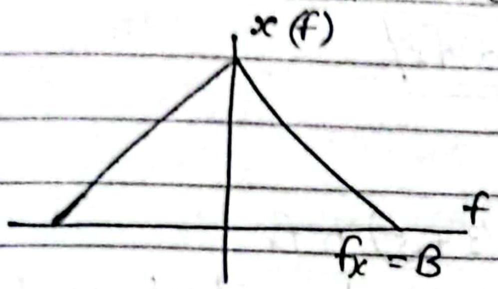
$$\text{Average quantization noise power } (P_{qe}) = \frac{x_{max}^2}{3 \times 4^n}$$

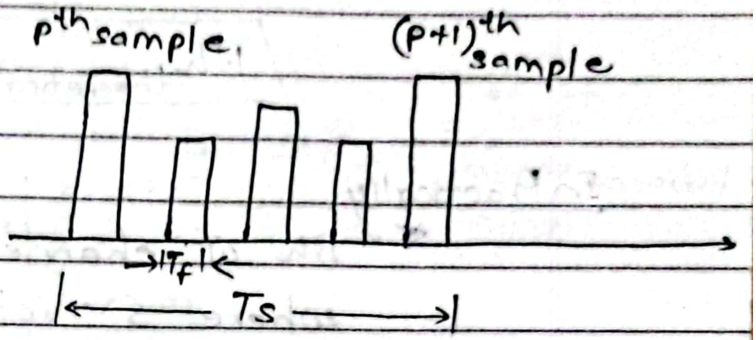
Bandwidth requirement for PCM

→ Consider a band limited signal with upper maximum frequency f_x and Bandwidth B.

max^m sampling frequency is

$$f_s \geq 2f_x = 2B.$$





within a time duration of successive sample.

$$T_s = \frac{1}{f_s} = \frac{1}{2f_x} = \frac{1}{2B}$$

→ n pulses of T_f duration has to transmitted.

→ maximum possible pulse duration.

$$T_{f\,max} = \frac{T_s}{n} = \frac{1}{2f_x n}$$

→ No. of bits per second is known as signalling Rate (R).

No. of bits per second = no. of sample per sec * no. of bits per sample

$$= f_s * n$$

$$= n f_s$$

Signalling rate $(R) = n f_s = 2 n f_x \quad [\because f_s \geq f_x]$

→ maximum of 2B information per second can be transmitted error freely through a channel having B Hz bandwidth

→ In other words, we can transmit at most two pieces of information per second per hertz Bandwidth.

Theoretically

minimum $BW_{PCM} = \frac{1}{2} \times$ Signalling rate (R)

$$\boxed{BW_{theoretical} = n f_x}$$

In Practically

BW of channel $\geq (1+\rho) n f_x$

where ρ = roll-off factor which is lying 0 to 1.

$$\boxed{BW_{practical} = 2n f_x = \text{signalling rate}}$$

Num Q. Telephone voice channel 300 Hz - 3400 Hz standard at 4 KHZ = f_x and n = 8 bits. Calculate BW in PCM and SQNR.

Sol^n:- Given: n = 8 bits

f_x = 4 KHZ

$BW_{PCM} = n f_x$

$\therefore$ $BW_{theoretical} = 8 \times 4$

$= 32$ ~~dB~~ kHz

$BW_{practical} = 2n f_x = 2 \times 32$

$= 64$ kHz

$[SQNR]_{dB} = 10 \log_{10} (3 \times N^2)$

$= 10 \log_{10} 3 + 10 \log_{10} N^2$

$= 4.8 + 20 \log_{10} N$
$= 4.8 + 6.02 \times 8$
$= 52.96$ dB

Differential PCM :-

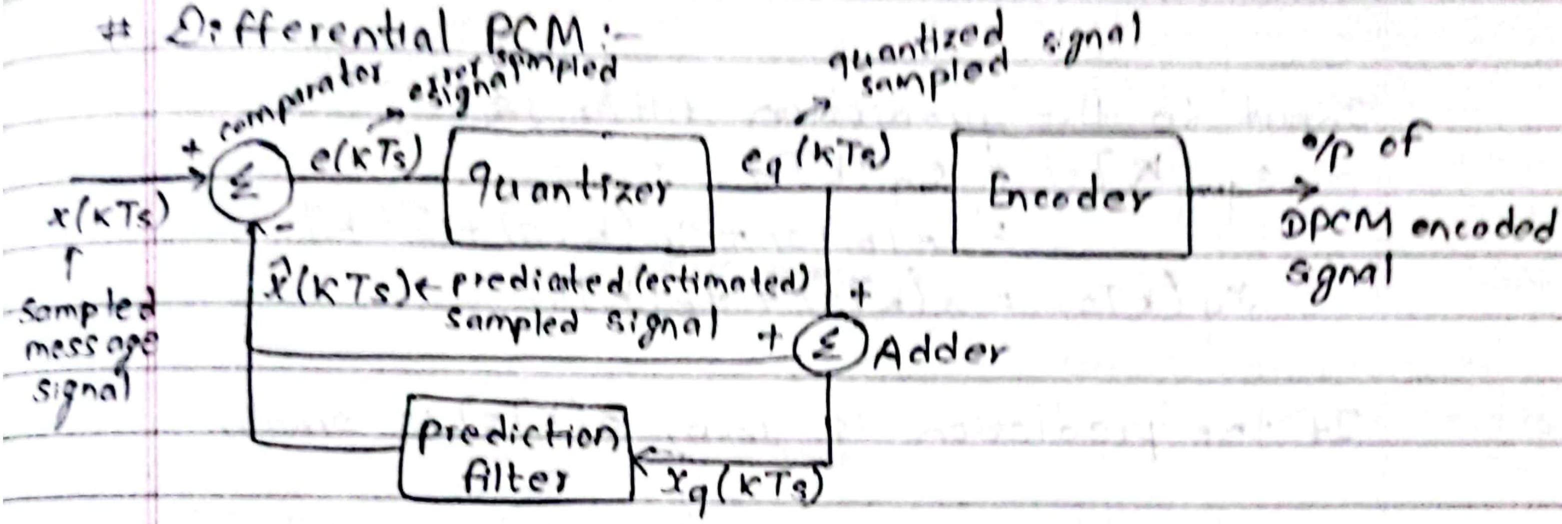


Fig :- Block diagram DPCM transmitter (encoder)

→ If the message signal is sampled at Nyquist rate or higher than that sample to sample corelation is very high.

→ In DPCM instead of ~~quantization~~ quantizing the absolute amplitude of the sample. We quantize the difference between actual (present) sample value and its predicted (previous) value and then transmit.

→ This reduces dynamic range of signal ~~in~~ required quantization levels is reduced. But for same level of quantization DPCM yields lower value of qe than direct quantization.

→ Then the quantized error signal eq(KTs) is very small and can be encoded using smaller number of bits and channel BW is reduced.

For predication error
$$e(kT_s) = x(kT_s) - \hat{x}(kT_s)$$
For Quantizer o/p $e_q(kT_s) = e(kT_s) + q_e(kT_s)$

quantization error

Input to the prediction filter is
$$x_q(kT_s) = e_q(kT_s) + \hat{x}(kT_s)$$
$$= e(kT_s) + q_e(kT_s) + \hat{x}(kT_s)$$
$$x_q(kT_s) = x(kT_s) + q_e(kT_s)$$

If the prediction is good, $e(kT_s)$ is small.

- DPCM receiver

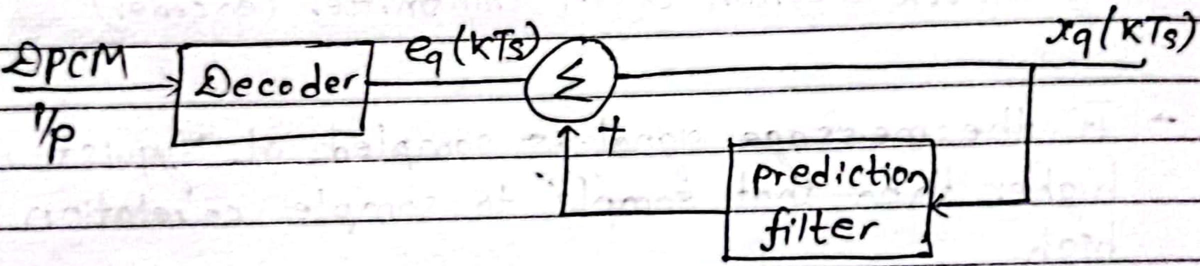


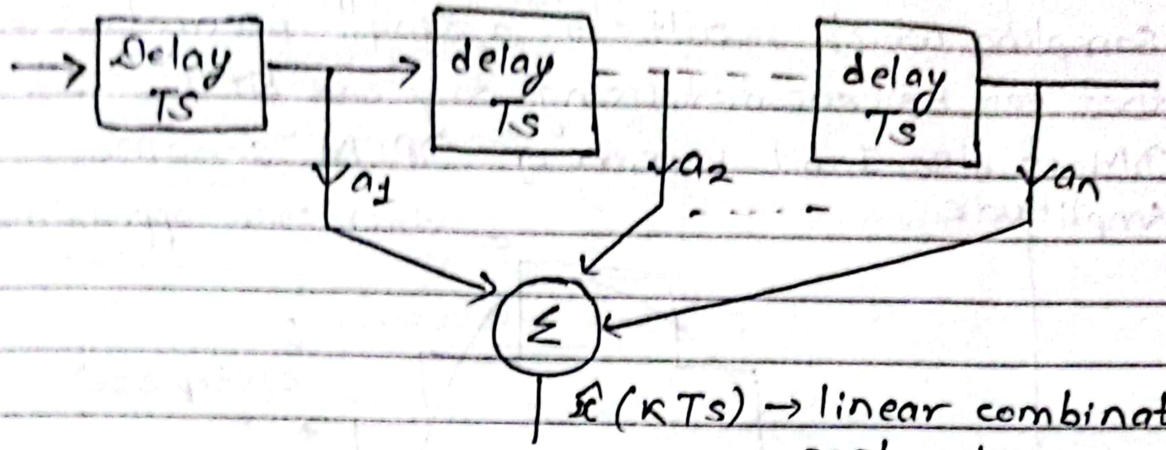
Fig:- DPCM receiver (Decoder)

→ Quantized version of prediction error is reconstructed by decoder which is added to output of prediction filter. Output of DPCM receiver is quantized version of the original signal.

→ Output ~~filter~~ differ from original signal by quantization error of $q_e(kT_s)$.

Prediction filter (PF)

→ It is tapped delay line network whose i/p is quantized version of



$\hat{x}(KTs)$ → linear combination of past values quantize i/ps

$$\hat{x}(KTs) = \sum_{m=1}^{N} a_m x_n[(K-m)Ts]$$

N = order of PF

→ o/p of PF is linear sum of n previous value of sampled scaled by coefficients $(a_1, a_2 -- a_n)$.

- Delta modulation (DM)

→ DM by oversampling (typically 4-times Nyquist rate)

→ This increases the co-relation between adjacent sampling which results in a small prediction error that can be encoded using only one bit ($L = 2$).

→ DM is also 1-bit version of DPCM scheme.

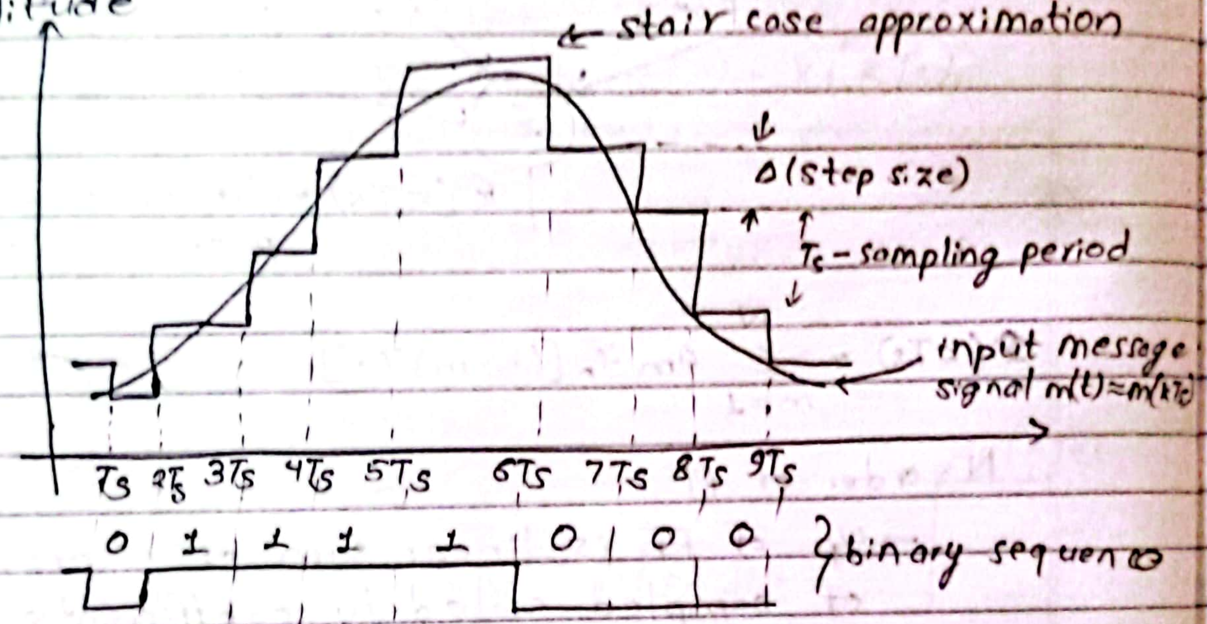


Fig:- Illustration of Delta modulation

→ If approximate falls below the original sample it is level increased by Δ and if it lies above then it is decreased by Δ.

→ When steps is reduced, '0' is transmitted and if step is increased, '1' is transmitted.

[ ] and ( ) is same.

- Delta transmitter

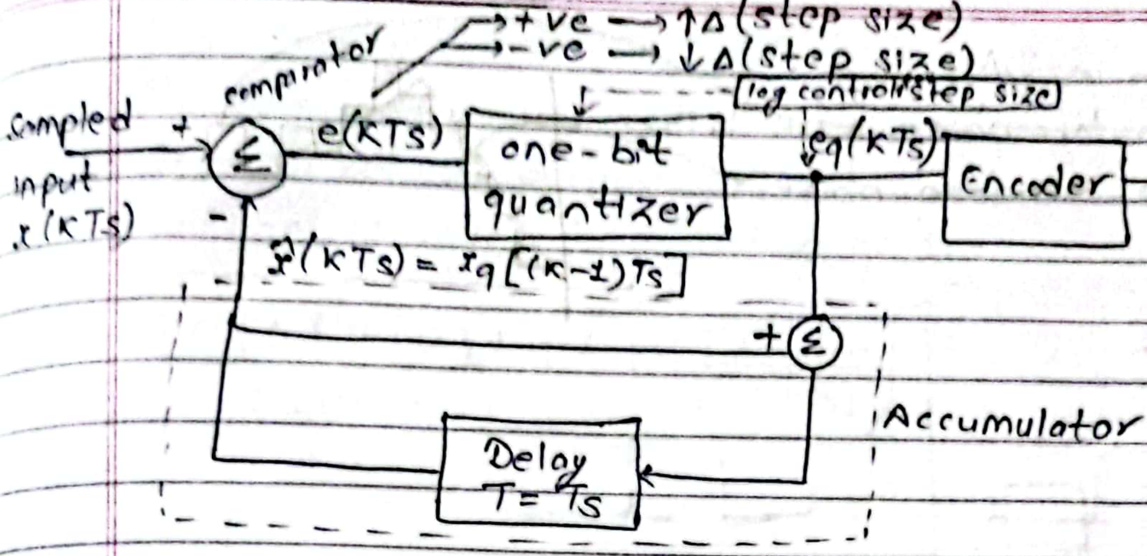


Fig:- DM transmitter

$$e_q[KT_s] = \begin{cases} +\Delta & \text{if } x[KT_s] \geq \hat{x}(KT_s) \\ -\Delta & \text{if } x[KT_s] < \hat{x}(KT_s) \end{cases}$$

$+\Delta \rightarrow$ binary '1' is transmitted

$-\Delta \rightarrow$ binary '0' is transmitted

- Delta modulation receiver

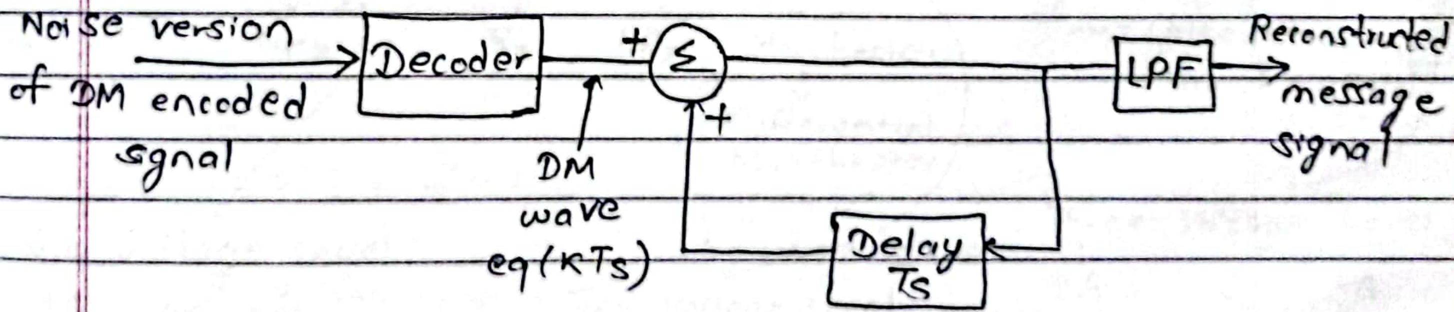


Fig:- DM receiver

→ Staircase Approximation is regenerated by passing the DM sequence through accumulator.
if i/p DM is '1' → add Δ to previous value.
i/p DM is '0' → subtract Δ from previous value.

Noise in DM

(a) Granular noise

(b) Slope-overload noise