CHAPTER 1: ANTENNA THEOREMS

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CHAPTERR 1 : ANTENNA THEOREMS

[A] Superposition Theorem

The statement of the superposition theorem is as follows.

"In any linear and bilateral network consisting linear and bilateral impedances and energy sources (e.g. generators), the current flowing at any point in the network is the algebraic sum of the currents those flow with each source were considered separately, with all the remaining sources replaced at a time by their internal impedances."

The theorem is clearly based on the linearity property and Ohm's law. The fundamental concept is that in linear impedance, for an increase in voltage across it, current through it increases which is independent of the magnitude of the original current flowing through it.

[B] Thevenin's Theorem

The statement of the Thevenin's theorem is as follows.

"In any linear, bilateral network consisting one or more energy sources along with the linear impedances, the current flowing the load impedance ZR is same as obtained by replacing an original network across ZR with an equivalent network consisting single voltage source Voc and impedance Zeq in series with it, where Voc is the open circuit voltage measured across terminal by removing ZR and Zeq is the impedance measured at the same open terminals looking back into the network replacing all the energy sources by their respective internal impedances.

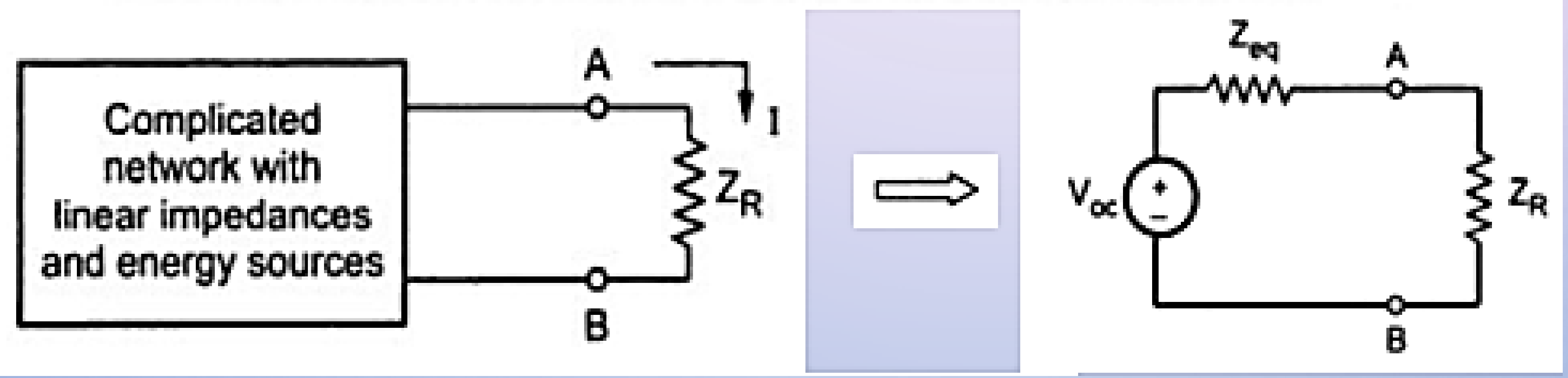


Fig. 2.12 Thevenin's theorem representation

The equivalent circuit consisting the voltage source Voc and the impedance Zeq is called Thevenin's equivalent circuit or voltage source equivalent circuit.

An antenna system can be resolved into its Thevenin's equivalent circuit. Figure 1.19 illustrates Thevenin's equivalent circuit for a transmitting antenna.

Where,

R_L = loss resistance of the antenna

R_r = radiation resistance of the antenna

X_A = reactance of the antenna

R_g = resistance of generator,

X_g = reactance of generator, and

V_g = voltage of generator

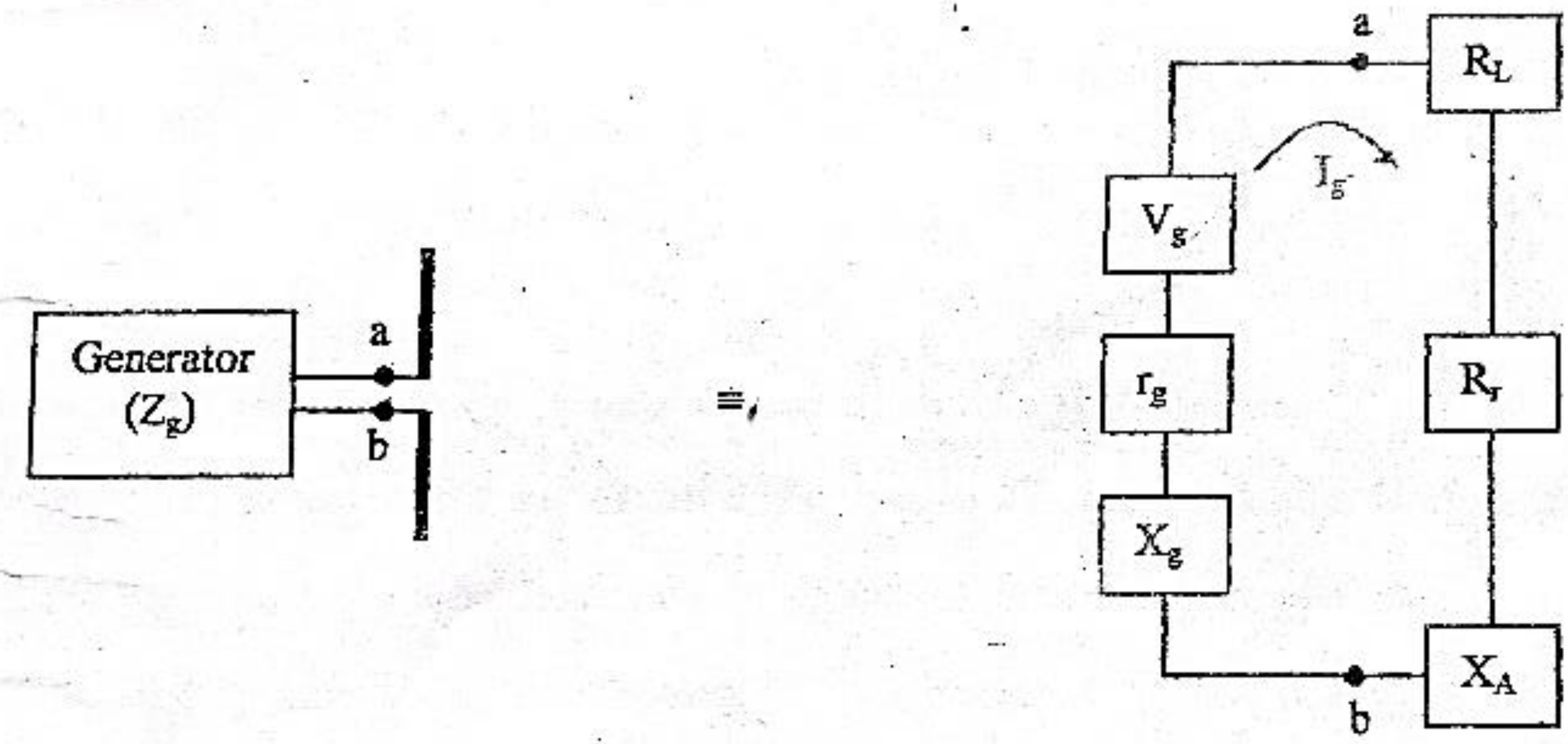


Figure 1.19 (a) A transmitting antenna system, and (b) its Thevenin's equivalent

[C] Compensation Theorem

Any impedance in a network may be replaced by a generator of zero internal impedance, whose generated voltage at every instant is equal to the instantaneous potential difference that existed across the impedance because of current flowing through it.

[D] Maximum Power Transfer Theorem

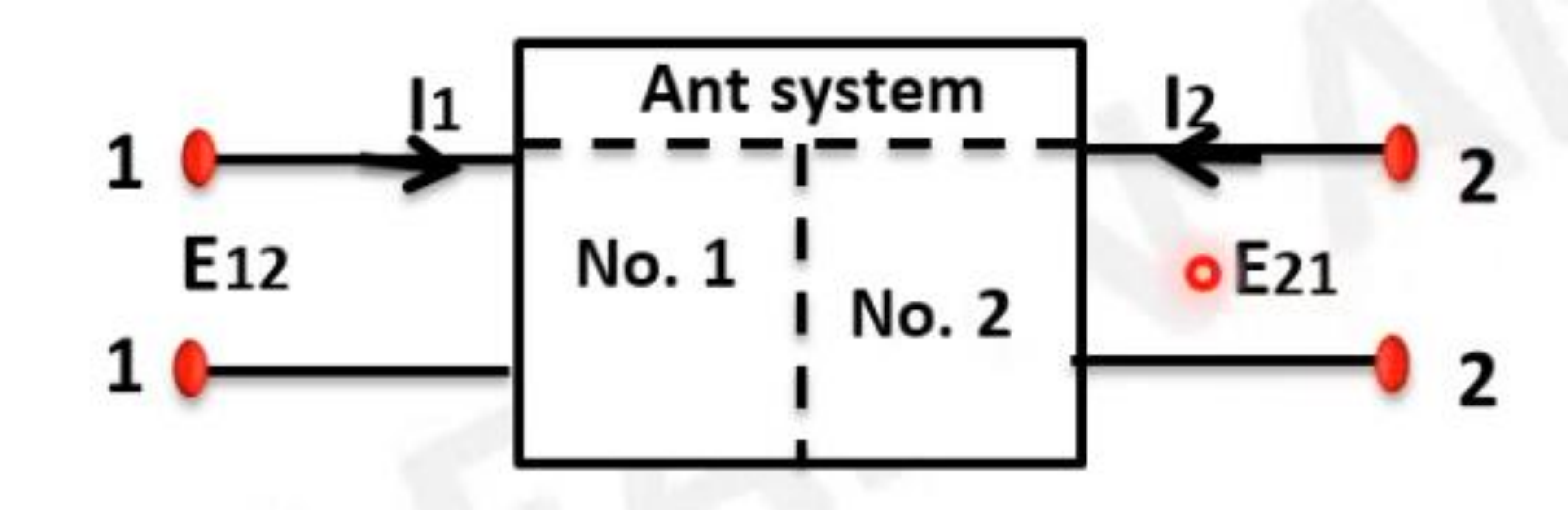
An impedance connected to two terminals of a network will absorb maximum power from the network when the impedance is equal to the conjugate of the impedance seen looking back into the network from the two terminals.

The maximum power that can be absorbed from a network equals $\frac{{V_{oc}}^2}{4R}$ where $V_{oc}$is is the open circuit voltage at the output terminals and R is the resistive component of the impedance looking back from the output terminals.

[E] RECIPROCITY THEOREM

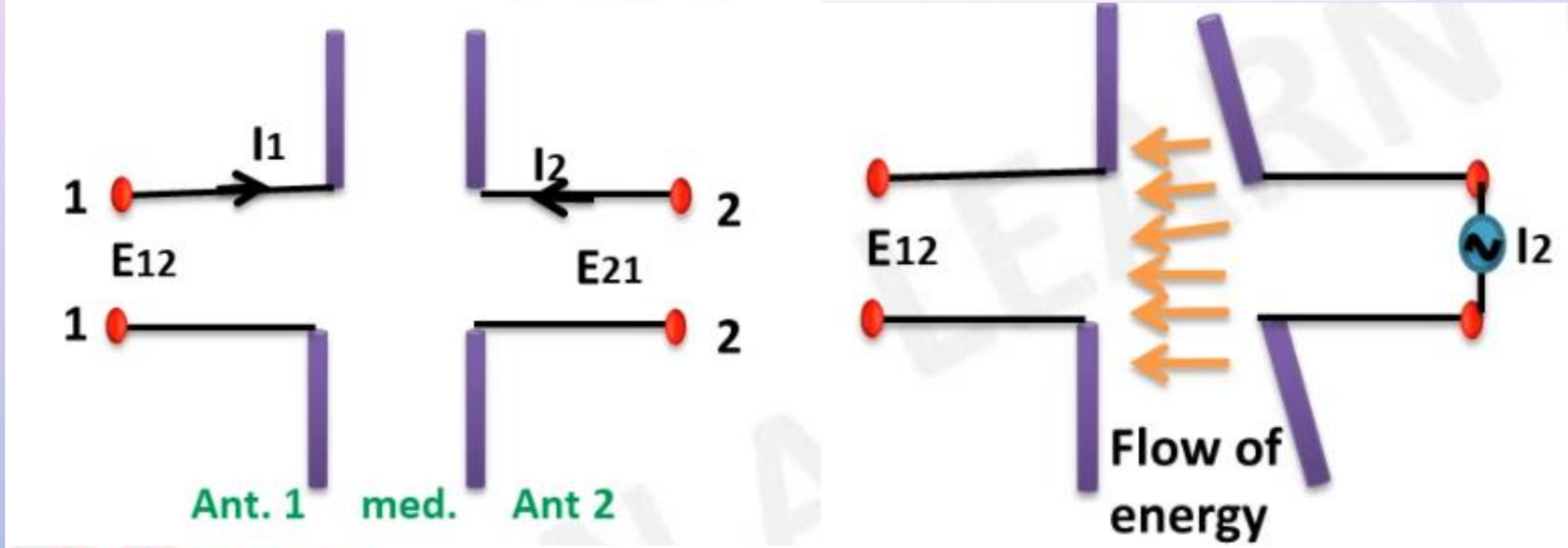
Reciprocity theorem statement show the

If emf apply to the terminal of an antenna 1 and the current measure at the terminal of another antenna No.2 then an equal current both in amplitude and phase obtained at terminal or antenna no. 1 If same emf applied to the terminal of antenna no. 2



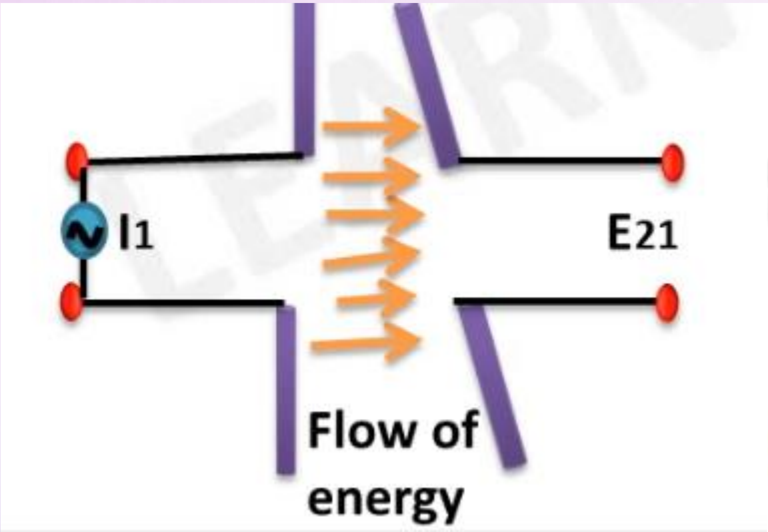
Reciprocity Theorem

emf are of same frequency
Medium b/w two antenna are linear , passive and Isotopic



Explanation

Transmitter of frequency f and 0 impedance connected to Ant. 2 , current I2 generate and induced emf E_{12} at open terminal ant. 1



According to reciprocity theorem

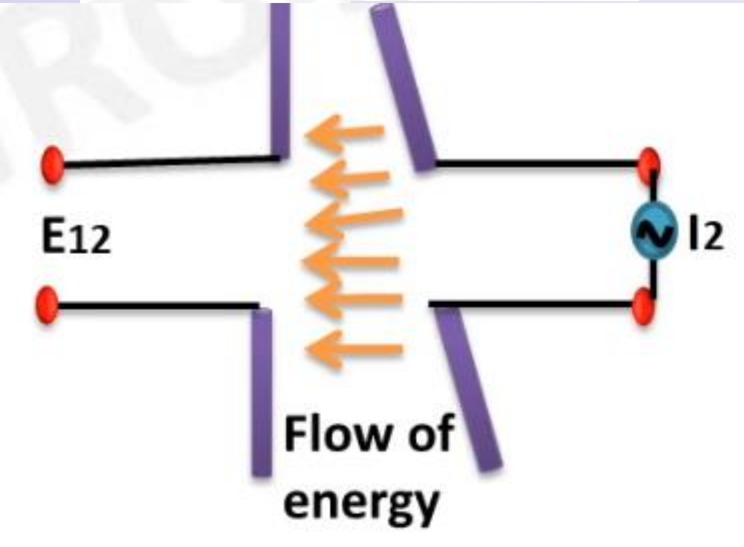
$I1 = I2$ produce $E12 = E21$

Ratio of emf and current is impedance $= \frac{E12}{I2}$

Transfer impedance Z12 in case 1

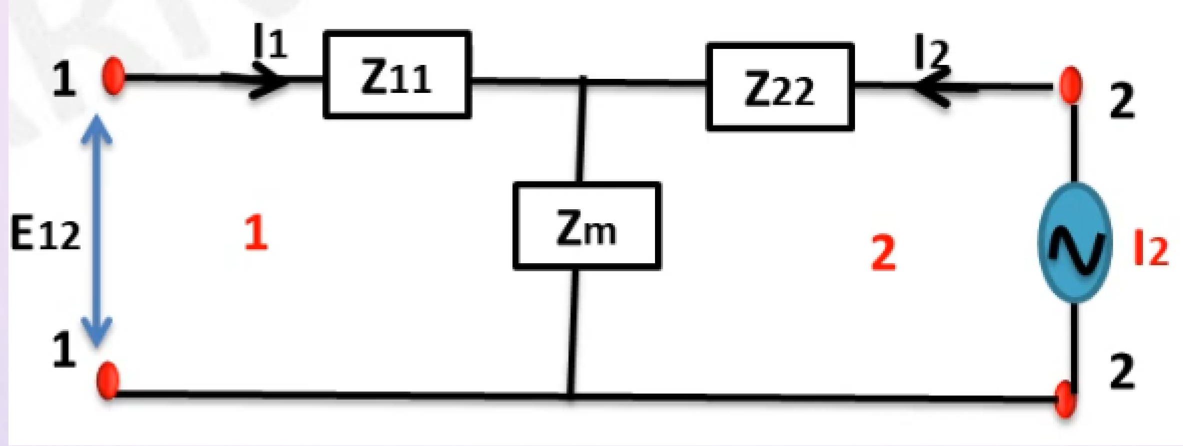
$= \frac{E21}{I1}$ transfer impedance Z21

where $Z12 = \frac{E12}{I2}$ $Z21 = \frac{E21}{I1}$



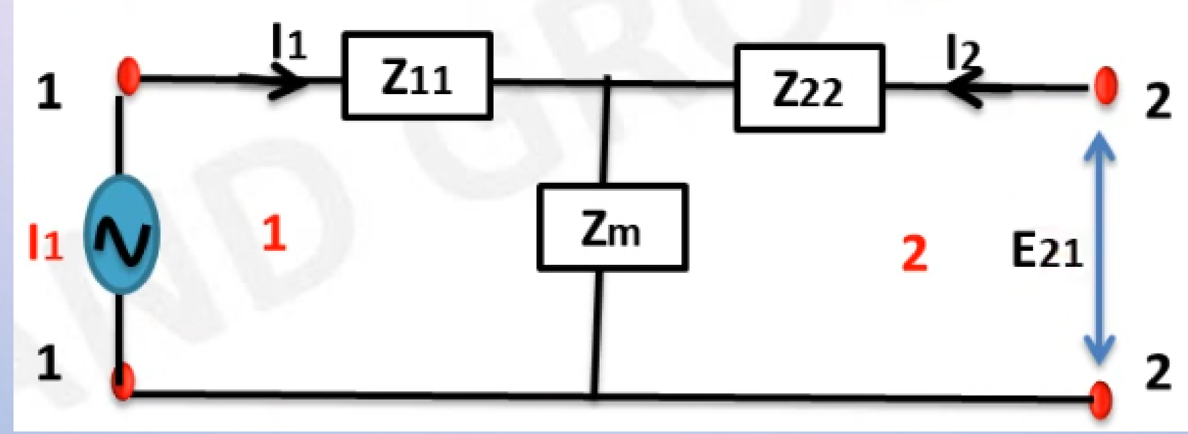
Voltage E1 1st cct and current I2 2nd cct define transfer impedance Z_T or Z_{12}

Terminal of antenna No 1



Terminal of antenna No 2

Terminal of antenna No 2



Terminal of antenna No 1

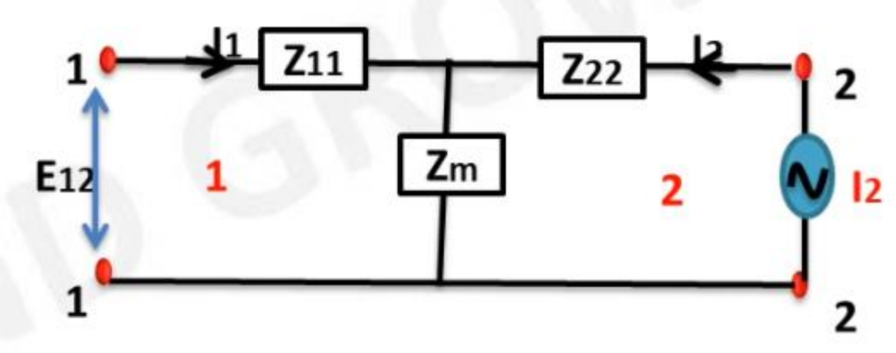
From reciprocity

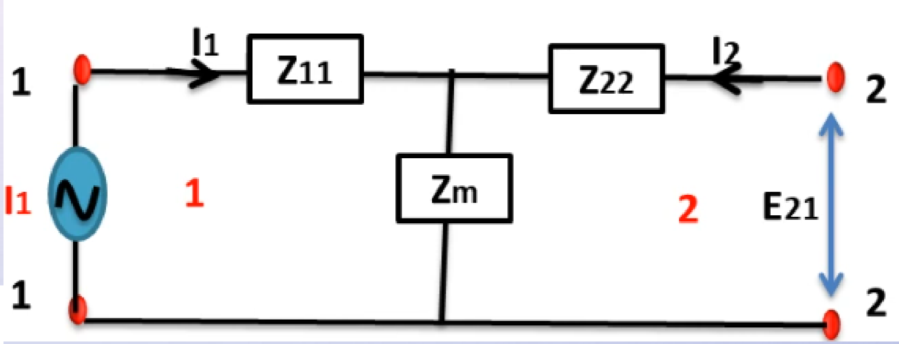
$Z_{12} = Z_{21}$

$$Z_m = Z_{12} = Z_{21} = \frac{E12}{I2} = \frac{E21}{I1}$$

Reciprocity Theorem Proof

Z_{11} and Z_{22} self impedance ant. No 1 and 2





Kirchhoff's mesh law from loop 2

$$(Z_{22} + Z_m)\, I_2 - Z_m\, I_1 = 0$$

$$I_2 = I_1 \frac{Z_m}{(Z_{22} + Z_m)}$$

$$(Z_{11} + Z_m)\, I_1 - Z_m\, I_2 = E_{12}$$

$$(Z_{11} + Z_m)\, I_1 - \frac{Z_m^2\, I_1}{(Z_{22} + Z_m)} = E_{12}$$

$$I_1 \left[\frac{(Z_{11} + Z_m)\,(Z_{22} + Z_m) - Z_m^2}{(Z_{22} + Z_m)} \right] = E_{12}$$

$$I_1 \left[\frac{Z_{11}\, Z_{22} + Z_{11}\, Z_m + Z_{22}\, Z_m + Z_m^2 - Z_m^2}{(Z_{22} + Z_m)} \right] = E_{12}$$

$$I_1 = \frac{E_{12}\,(Z_{22} + Z_m)}{[\,Z_{11}\, Z_{22} + Z_m\,(Z_{11} + Z_{22})\,]}$$

$$I_2 = \frac{E12\,(Z22+Zm).Zm}{[\,Z11\ \ Z22 + Zm\,(Z11 + Z22)]\,(Z22 + Zm)}$$

$$I_2 = \frac{E12\,.Zm}{Z11\ \ Z22 + Zm\,(Z11 + Z22)}$$

The current I1 obtained by

$$I1 = \frac{E21\ \ .Zm}{Z22\ \ Z11 + Zm\ \ (Z22 + Z11)}$$

$$I1 = \frac{E21\ \ .Zm}{Z22\ \ Z11 + Zm\ \ (Z22 + Z11)} \qquad I_2 = \frac{E12\,.Zm}{Z11\ \ Z22 + Zm\,(Z11 + Z22)}$$

Except the value of emf , according to theorem statement , the theorem is provide

$E_{12} = E_{21}$ if $I_1 = I_2$

Apply the condition $I_1 = I_2$

$$\frac{E12\,.Zm}{Z11\ \ Z22 + Zm\,(Z11 + Z22)} = \frac{E21\ \ .Zm}{Z22\ \ Z_{11} + Zm\ \ (Z22 + Z11)}$$

$E_{12} = E_{21}$

It provide directivity pattern , aperture and terminal impedance of an antenna

Limitation of reciprocity theorem

This theorem applicable to radio communication but it fail when radio wave effected by Earth magnetic field

It hold good all practical radio work but for long distance communication .

APPLICATION OF RECIPROCITY THEOREM

1. EQUALITY OF DIRECTIONAL PATTERNS

> The directional pattern of a receiving antenna is identical with its directional pattern as a transmitting antenna.

Proof: This theorem is an application of reciprocity theorem. The directional pattern of a transmitting antenna indicates the strength of the radiated field at a fixed distance in different directions in the space. The directional pattern of a receiving antenna indicates the response of the antenna to unit field strength from different directions. Thus, the directional pattern is the polar plot for both transmitting and receiving antenna. The pattern as a transmitting antenna could be measured as indicated in Fig. 2.3, by means of radiation pattern, for transmitting and receiving antenna will be same.

The antenna no. 1 is oriented to transmit maximum radiation while antenna oriented to receive the maximum radiation. So, using KVL

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

where Z_{11} and Z_{22} are the self impedances of antenna no. 1 and 2, respectively. Z_{12} and Z_{21} are the mutual impedances between antenna 1 and 2. Now if current I_1 is applied at antenna no. 1 and voltage V_{2oc} is measured at the terminal of antenna no. 2 with open circuit ($I_2 = 0$). Then an equal current I_2 is applied at the terminal of antenna no. 2 and measured V_{1oc} at the open terminal ($I_1 = 0$) of antenna no. 1. Thus

$$Z_{21} = \left.\frac{V_{2oc}}{I_1}\right|_{I_2=0}$$

$$Z_{12} = \left.\frac{V_{1oc}}{I_2}\right|_{I_1=0}$$

If medium between two antennas is linear, passive, isotropic, and the waves monochromatic. Then

$$Z_{21} = \left.\frac{V_{2oc}}{I_1}\right|_{I_2=0} = \left.\frac{V_{1oc}}{I_2}\right|_{I_1=0} = Z_{12}$$

Using reciprocity theorem, i.e. $I_1 = I_2$ then

$$\boxed{V_{1oc} = V_{2oc}}$$

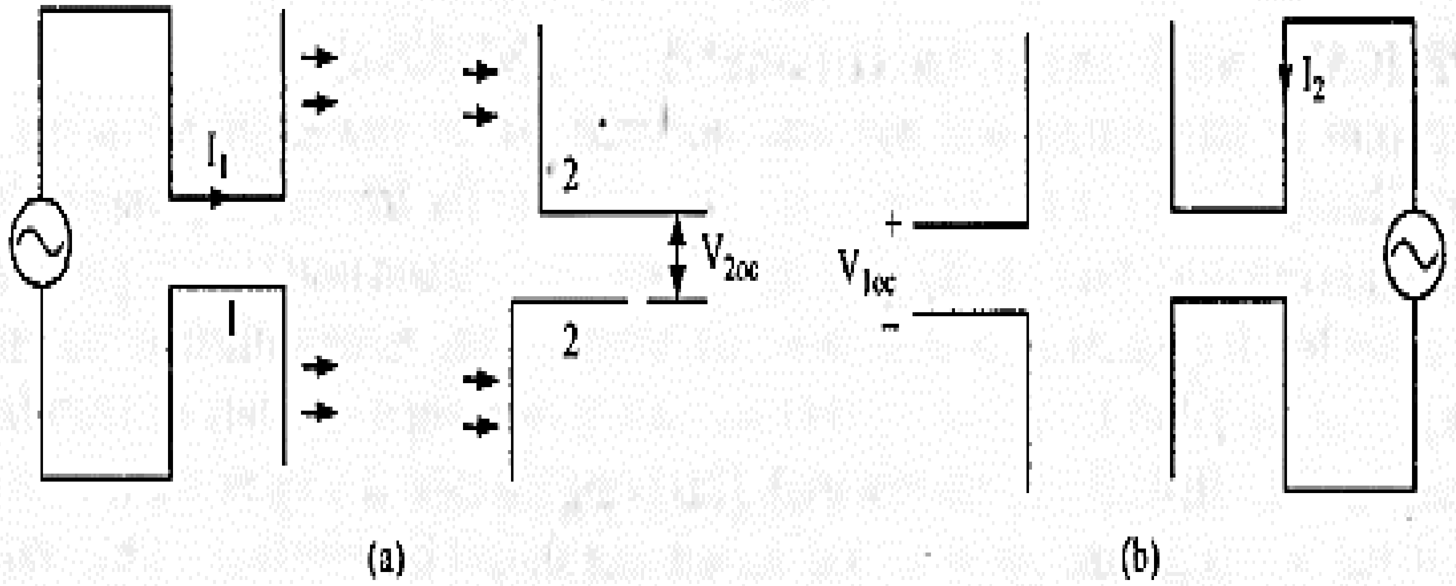


Fig. 2.3 *Antenna arrangement for pattern measurement*

2. EQUIVALENCE OF TRANSMITTING AND RECEIVING ANTENNA IMPEDANCE

> The impedance of an isolated antenna when used for receiving is the same as when used for transmitting.

Proof: This theorem is particularly easy to prove for the case of two antennas which are widely separated. If antenna no. 2 is far from antenna no. 1, the self-impedance Z_{S1} of antenna no. 1 is given by

$$Z_{S1} = \frac{V_1}{I_1} = Z_{11} \qquad ...(2.7)$$

If antenna no. 1 is used for transmitting then the mutual impedance Z_{12} may ignored, due to the wide separation of two antennas. Since mutual impedance provides coupling between the two antennas, therefore, Z_{12} cannot ignored for the receiving purpose. For this case, one load impedance Z_L attached to antenna 1 and may represent the voltage $Z_{12}\,I_2$ as a generator as shown in the equivalent circuit of figure 2.4. Since the two antennas are far apart, so varying Z_L cannot cause I_2 to vary. Therefore, the generator behaves as an ideal zero-impedance constant voltage generator. Under these conditions, antenna no. 1 exhibits the terminal behaviour of a generator with internal impedance Z_{11} and thus the receiving impedance is equal to the transmitting impedance.

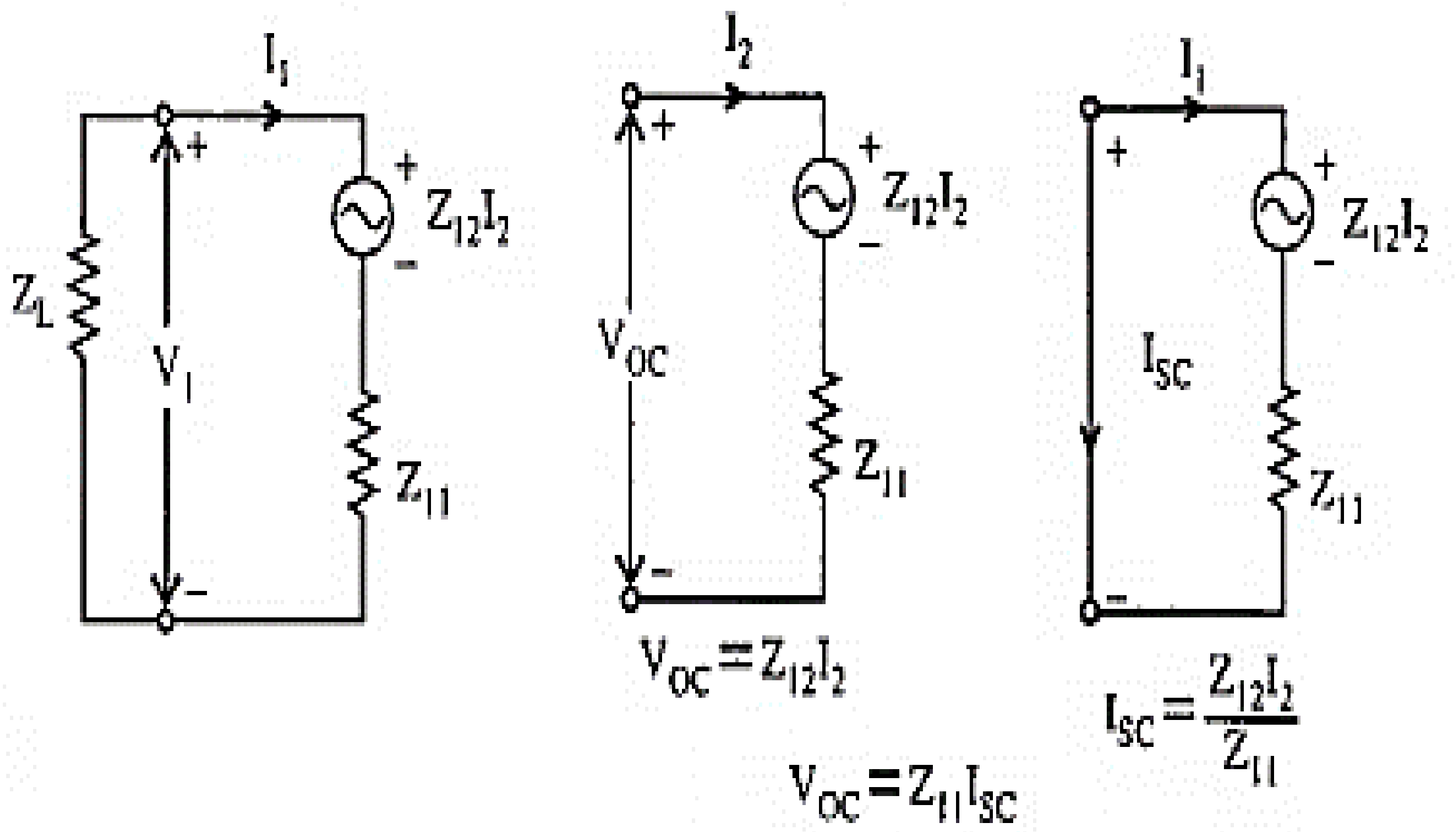


Fig. 2.4 *Equivalent circuit of a receiving antenna under loaded, open-circuit and short-circuit conditions.*

3. EQUALITY OF EFFECTIVE LENGTH

The effective Length, L_{eff}, of an antenna is a term used to indicate the effectiveness of the antenna as a radiator or collector of electromagnetic energy. That means how effectively a transmitting antenna is radiating and receiving antenna is collecting the electromagnetic energy.

The significance of the term as applied to transmitting antennas is shown in figure 2.5. The effective length of a transmitting antenna is that length of an equivalent linear antenna which has a current I (0) at all points along its length and that radiates the same field strength as the actual antenna in the direction perpendicular to its length. I (0) is the current at the terminals of the actual antenna. That is, for transmitting antenna.

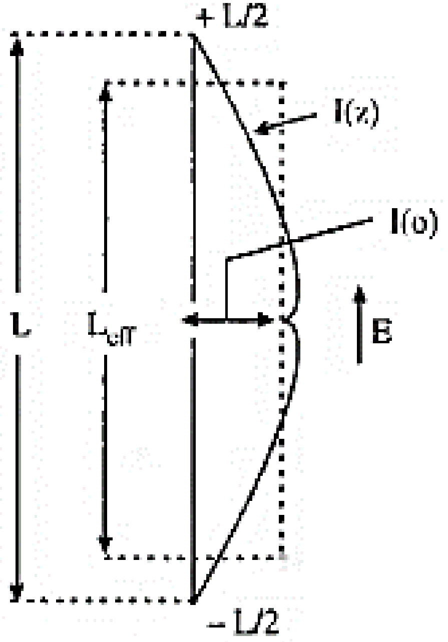


Fig. 2.5

$$I(0)\, L_{eff}\,(\text{trans}) = \int_{-\frac{L}{2}}^{+\frac{L}{2}} I(z)\, dz \qquad \text{...(2.8)}$$

or

$$L_{eff}\,(\text{trans}) = \frac{1}{I(0)} \int_{-\frac{L}{2}}^{+\frac{L}{2}} I(z)\, dz \qquad \text{...(2.9)}$$

The effective length of a receiving antenna is defined in terms of the open-circuit voltage developed at the terminals of the antenna for a given received field strength.

For receiving antenna

$$L_{eff} \cdot (\text{rec}) = -\frac{V_{oc}}{E}$$

where V_{oc} is the open-circuit voltage at the antenna terminals produced by a uniform excitating field E volts per meter parallel to the antenna.

THANK YOU