4. Baseband Data communication system.

4.1 Introduction to information theory, measure of information, entropy, symbol rates and data (bit) rates.

In digital system, a ~~messg~~ message is measured as the amount of information provided by the message. So, the messages differ as the information provided by each message differ.

Consider you are planning to visit Biratnagar and you check the summer weather forecast and you find the following information,

i) It would be very hot and sunny
ii) There would be heavy rain
iii) There would be snow.

It is certain that the third reading will surely strike you first. The first reading is too obvious as the city being in terai will be hot. The second contains some information as in dry season, chances of rain occurs rarely. Whereas, the chance of snow is rarest as we have not yet heard of snow in terai.

As we term these readings in term of probability, reading 'i' will have probability almost equal to '1' whereas reading (iii) will have probability equal to '0'.

Thus we can see that as the probability is high, the information content is low and vice versa.

i.e. as P tends to 1, I tends to 0

& as $P \to 0$, $I \to 1$.

P = probability

I = information.

So, the relation can be expressed mathematically as,

$$I = \log_n (1/p) \quad \text{i.e. } I \propto 1/p$$

The unit of 'I' depends on the value of 'n', i.e.

If $n = e$, the unit is nat

$n = 2$, the unit is bit

$n = 10$, the unit is decit or Hartley

Measure of information.

So, in digital system it is important that we know the information content in any message.

Now we know that the information received from the knowledge of occurence of an event is inversely proportional to the probability of the occurence of the same event.

It is therefore important that we ~~can~~ have a ~~the~~ clear idea of the probability ~~of~~ or the likelihood of occurence of any event.

In the previous example, the three set of conditions ~~the~~ carry different information for a person from Nepal, but if a person of any other nationality had viewed the weather forecast, it would have bore no information as he or she wouldnot be familiar with the weather conditions trends of Biratnagar.

Entropy.

Entropy is the average information per message emitted by the source.

Let us consider a discrete memoryless source (DMS) denoted by X having 'M' symbols $\{x_1, x_2, x_3, \ldots, x_m\}$ such that probability of occurence of any one symbol doesn't depend on other symbols of the source.

So, we have probabilities for each symbol given as $\{P(x_1), P(x_2), P(x_3), \ldots, P(x_m)\}$.

Now, the information content of ~~the~~ ~~any s~~ any ~~signal~~ symbol (x_i) is given as,

$$I(x_i) = \log_n \frac{1}{P(x_i)}$$

$$= \log_2 \frac{1}{P(x_i)} \quad [\text{Taking } n=2]$$

$$\therefore I(x_i) = -\log_2 P(x_i)$$

where, $P(x_i)$ is the probability of occurence of any symbol x_i in the DMS.

Now, entropy is given as the average information per message emitted and is equated as,

$$H(X) = \sum_{i=1}^{m} P(X_i) \cdot I(X_i)$$

where, m = number of message symbols.

or $$H(X) = \sum_{i=1}^{m} P(X_i) \cdot \log_2 \left(\frac{1}{P(X_i)} \right)$$

$$H(X) = - \sum_{i=1}^{m} P(X_i) \cdot \log_2 [P(X_i)] \text{ bits/symbol}$$

Now, if the symbol rate is R_s, then the information rate is given as,

$$R_{inf} = R_s \times H(X) \ [\text{bits/sec}]$$

\# A source produces one of four possible symbols during each interval having probabilities $P(X_1) = 1/2$, $P(X_2) = 1/4$, $P(X_3) = P(X_4) = 1/8$. obtain the information content of each of these symbols and hence find the entropy.

Given,

$P(X_1) = 1/2$, $P(X_2) = 1/4$, $P(X_3) = P(X_4) = 1/8$

Now,

$$I(X_i) = \log_2 \frac{1}{P(X_i)}$$

Therefore,

$$I(X_1) = \log_2 \frac{1}{1/2} = \log_2 2 = 1 \text{ bit}$$

$$I(X_2) = \log_2 \frac{1}{1/4} = \log_2 4 = 2 \text{ bit}$$

$$I(X_3) = \log_2 \frac{1}{1/8} = \log_2 8 = 3 \text{ bit}$$

$$I(X_4) = \log_2 \frac{1}{1/8} = \log_2 8 = 3 \text{ bit}$$

P.T.O.

Also,

$$H(x) = \sum_{i=1}^{M} P(x_i) \cdot I(x_i) \qquad [m = 4]$$

$$= P(x_1) \cdot I(x_1) + P(x_2) \cdot I(x_2) + P(x_3) \cdot I(x_3) + P(x_4) \cdot I(x_4)$$

$$= \frac{1}{2} \cdot 1 + \frac{1}{4} \cdot 2 + \frac{1}{8} \cdot 3 + \frac{1}{8} \cdot 3$$

$$= \frac{1}{2} + \frac{1}{2} + \frac{3}{8} + \frac{3}{8}$$

$$= \frac{14}{8} = \frac{7}{4} = 1.75 \text{ bits / symbol}$$

OR, We can have,

$P(x_1) = 0.5$, $P(x_2) = 0.25$, $P(x_3) = P(x_4) = 0.125$

Now,

$$H(x) = -\sum_{i=1}^{4} P(x_i) \cdot \log_2 [P(x_i)]$$

$$= -[0.5 \times \log_2(0.5) + 0.25 \times \log_2(0.25) + 0.125 \times \log_2(0.125) + 0.125 \times \log_2(0.125)]$$

$$= 1.75 \text{ bits / symbol}$$

Arrah &

⊕ An analog signal bandlimited to 10KHz is quantized in 8 levels of PCM system with probabilities of 1/4, 1/5, 1/5, 1/10, 1/10, 1/20, 1/20 & 1/20 respectively. Find the entropy and rate of information

Given,

$f_m = 10 \text{ KHz}$

so, $f_s = 2 \times 10 \text{ KHz} = 20 \text{ KHz}$

$R_s = f_s = 20000$ messages / sec

Considering each of the eight quantized levels as a message, the entropy of the source is given by,

$$H(x) = \frac{1}{4} \cdot \log_2 4 + \frac{1}{5} \log_2 5 + \frac{1}{5} \log_2 5 + \frac{1}{10} \log_2 10 + \frac{1}{10} \log_2 10 + \frac{1}{20} \log_2 20 + \frac{1}{20} \log_2 20 + \frac{1}{20} \log_2 20$$

$$= \frac{1}{4} \log_2 4 + \frac{2}{5} \log_2 5 + \frac{2}{10} \log_2 10 + \frac{3}{20} \log_2 20$$

$\therefore H(x) = 2.84$ bits / message

And, $R_{inf} = R_s \times H(x)$

$= 20000 \times 2.84$

$= 56800$ bits / sec

Shannon Hartley Channel capacity theorem.

For a channel with additive white Gaussian noise, the relation between ~~cap~~ channel capacity 'C', channel bandwidth 'B' and the received signal to noise ratio 'SNR' is given by,

$$C = B \log_2 (1 + SNR) \text{ bits/sec}$$

This relationship is called Shannon-Hartley Channel capacity theorem.

Implication of theorem:

1. A designer can estimate 'C' for required SNR and bandwidth 'B' specifications needed for reliable communication. Thus can indicate the upper limit of data transmission for reliable communication.

2. Trade off between B and SNR for given 'C'

From the relation, we see that for a limited channel capacity, one can alter the values of the channel bandwidth or the signal power or the noise power (i.e. B or SNR).

We have, $C = B \log_2 (1 + SNR)$

or $\frac{C}{B} = \log_2 (1 + SNR)$

or $1 + SNR = 2^{C/B}$

or $SNR = 2^{C/B} - 1$.

So, we see that for a constant 'C', the bandwidth and SNR are inversely related. Thus there is a trade off between B and SNR for given C.

A signal with data rate R = 10000 bps is required to transmit over a channel with bandwidth of B = 3000 Hz. Find SNR. Also, if you change B to 1000 Hz, find the new SNR for same data rate.

Given,

$R = 10000$ bps

We have required transmission bandwidth $B.W = \frac{R}{2} = 5000$ Hz

But the question says a bandwidth,

$B = 3000$ Hz.

Now,

the minimum channel capacity,

$C_{min} = R = 10000$ bps

We have,

$C = B \log_2 (1 + SNR)$

or

$SNR = 2^{C/B} - 1$

$= 2^{10000/3000} - 1$

$\therefore SNR \simeq 9$

i.e. The signal power must be 9 times greater than noise power to achieve channel capacity of 10000bps with channel bandwidth equal to 3000 Hz.

If $B = 10000$ Hz,

then,

$SNR = 2^{10000/10000} - 1$

$= 2 - 1$

$= 1$

Which implies that signal power is equal to noise power resulting in capacity of the channel equal to 10000 bps and channel bandwidth equal to 10000 Hz.

Thus we can see ~~can~~ that we can achieve bandwidth compression from 10000 Hz to 3000 Hz but at the cost of increasing signal power by 9 times.

3. Bandwidth compression.

Shannon's capacity theorem indicates that it is possible to transmit signal with upper frequency f_{max}, through a channel having bandwidth less than f_{max}.

let f_{max} be the highest frequency content of a signal $x(t)$.

let us sample $x(t)$ at $f_s = 3 f_{max}$

i.e. 1.5 times greater than Nyquist rate.

Then data rate $= 3 \cdot b \cdot f_{max} = R$

where, $b =$ no. of bits per sample

Now if channel bandwidth $B = \frac{f_{max}}{2}$

then for $C_{min} \geq R$, $n = 6$, then required SNR is,

$$SNR = 2^{C/B} - 1$$

$$= 2^{3 \cdot b f_{max} / f_{max}/2} - 1$$

$$= 2^{3 \times 2 \times b} - 1$$

$$= 2^{3 \times 2 \times 6} - 1$$

$$= 6.8 \times 10^{10}$$

$\therefore$ SNR $\simeq 7 \times 10^{10}$

i.e. If we increase signal power by 7×10^{10} times that of noise power then we can transfer a signal through a channel having bandwidth equal to half of the signal bandwidth i.e. bandwidth compression is achieved.

Limitation of Shannon's channel theorem.

$$C = B \log_2 (1 + SNR) = B \log_2 \left(1 + \frac{S}{N}\right)$$

1. As the noise in the channel tends to zero, the value of SNR will tend to infinity. this will make channel capacity reach infinity. Thus a noise-less channel has infinite capacity. This type of channel is referred to as 'Ideal channel.'

But in general, noise is bound to occur and thus the channel capacity is reduced to some finite value.

2. As the bandwidth of the channel 'B', tends to infinity, rather than the capacity tending to infinity, it attains an upper limit as the noise power is proportional to the channel bandwidth. Such that for a power spectral density $N_0/2$, we have noise power $N = N_0 B$.

So,

$$C = B \log_2 \left(1 + S/N_0 B\right)$$

where, $S =$ signal power

$N_0 =$ normalized spectral density (power) or $C = \frac{S}{N_0} \log_2 \left(1 + \frac{S}{N_0 B}\right) \frac{N_0 \cdot B}{S}$

taking $\frac{S}{N_0 B} = x$,

$$C = \frac{S}{N_0} \frac{1}{x} \log_2 (1+x)$$

Now,

as $B \to \alpha$, $x \to 0$

i.e.

$$\underset{B\to\alpha}{C} = \lim_{x\to 0} \frac{S}{N_0} \frac{1}{x} \log_2 (1+x)$$

$$= \lim_{x\to 0} \frac{S}{N_0} \log_2 (1+x)^{1/x}$$

$$\underset{B\to\alpha}{C} = \frac{S}{N_0} \lim_{x\to 0} \log_2 (1+x)^{1/x}$$

Now,

$$\lim_{x\to 0} \log_2 (1+x)^{1/x} = \log_2 \cdot e$$

because, $\lim_{x\to 0} (1+x)^{1/x} = e$

$$\therefore \underset{B\to\alpha}{C} = C_{max} = \frac{S}{N_0} \log_2 e = 1.44 \times \frac{S}{N_0}$$

Given an AWGN channel with 4 KHz bandwidth and noise power spectral density $N_0/2 = 10^{-12}$ W/Hz. The signal power required at the receiver is 0.1 mW. Calculate the capacity of the channel.

Given,

$B = 4000$ Hz

$\frac{N_0}{2} = 10^{-12} \Rightarrow N_0 = 2 \times 10^{-12}$ W/Hz

Therefore noise power $N = N_0 \cdot B$

$= 2 \times 10^{-12} \times 4 \times 10^3$

or $N = 8 \times 10^{-9}$ W

And,

$$C = B \log_2 \left(1 + \frac{S}{N}\right)$$

$$= \cancel{B \log_2}\ 4000 \times \log_2 \left(1 + \frac{0.1 \times 10^{-3}}{8 \times 10^{-9}}\right)$$

$$\therefore C = 54.43 \times 10^3 \text{ bps}.$$

Also,

$$C = 1.44 \times \frac{S}{N_0} = 1.44 \times \frac{0.1 \cancel{mW} \times 10^{-3}}{2 \times 10^{-12}}$$

$$= 1.44 \times 0.05 \times 10^9$$

$$= 1.44 \times 5 \times 10^{-2} \times 10^9 = 7.2 \times 10^7 \text{ bps}$$

for $B \to \alpha$.

Recall Shannon-Fano & Huffman coding.
Both of the coding produce codewards for a source symbol, characterized by,

i) Codeword length (n)

It is the number of bits required to represent a symbol.

ii) Average codeword length (L)

$$L = \sum_{i=1}^{m} n_i \, P(X_i)$$

It is the average of number of bits required to represent every symbol.

n_i = no. of bits required for individual symbols.

$P(X_i)$ = probabilities for individual symbol.

iii) Code efficiency, $\eta = \frac{H(X)}{L}$

$= \frac{\text{entropy}}{\text{avg. codeward length}}$

iv) Code redundancy

$\gamma = 1 - \eta$

Given, probabilities of 6 symbols are, 0.12, 0.08, 0.05, 0.25, 0.20 and 0.30. Find the codewards using Shannon-Fano and Huffman technique. Hence derive the entropy, average codeword length and code efficiency.

X_i	$P(X_i)$					Shannon-Fano-Codeward
X_1	0.30	0	0			00
X_2	0.25	0	1			01
X_3	0.20	1	0			10
X_4	0.12	1	1	0		110
X_5	0.08	1	1	1	0	1110
X_6	0.05	1	1	1	1	1111

Now,

$$H(X) = -\sum_{i=1}^{6} P(X_i)\log_2[P(X_i)]$$

$= -[0.30\times\log_2(0.30) + 0.25\times\log_2(0.25) + 0.20\times\log_2(0.20)$
$+ 0.12\times\log_2(0.12) + 0.08\times\log_2(0.08) + 0.05\times\log_2(0.05)]$

$= 2.36$ bits / symbol

$$L = \sum_{i=1}^{6} n_i P(x_i)$$

$$= 2\times0.30 + 2\times0.25 + 2\times0.20 + 3\times0.12 + 4\times0.08 + 4\times0.05$$

or $L = 2.38$ bits / symbol

Therefore, code efficiency,

$$\eta = \frac{H(X)}{L} = \frac{2.36}{2.38} = 0.99$$

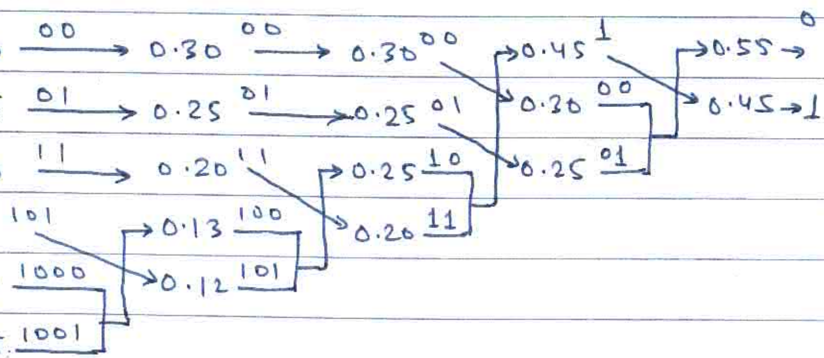
or $\eta\,\% = 99\%$

and

$$\gamma = 1-\eta = 1-0.99 = 0.01 = 1\,\%.$$

Now, for Huffman coding,

Huffman Codewords	x_i	$P(x_i)$
00	x_1	0.30
01	x_2	0.25
11	x_3	0.20
101	x_4	0.12
1000	x_5	0.08
1001	x_6	0.05



$H(X) = 2.36$ bits / symbol
$L(X)$ $L = 2.38$ bits / symbol

$\eta = 0.99$
$\gamma = 0.01$

Electrical representation of binary data or Line codes. [line coding]

Line coding is the process of converting binary data or a sequence of bits to the electrically ~~representation~~ represented digital signal.

There are different methods for converting binary data to these digital signals which we term as line codes.

The line codes now must possess following properties,

1. Transmission bandwidth :
   It should be as small as possible.

2. Power efficiency :
   For given bandwidth and specified error probability detection, the transmitted power should be as small as possible.

3. Error detection and correction capability.
   It must be possible to detect and preferably correct the detected error.

4. Favourable power spectral density.

It is desirable to have zero power spectral density (PSD) at $f=0$ i.e. dc.

5. Adequate timing content:

It must be possible to extract timing or clock information from the signal.

6. Transparency.

It must be possible to transmit a digital signal correctly regardless of the pattern of 1's and 0's.

There are several types of line codes used. They are,

i) Polar line codes → Return to zero (RZ)
  → Non-return to zero (NRZ)

ii) Unipolar line codes → RZ
  → NRZ

iii) Bipolar or AMI → RZ
  → NRZ

iv) Manchester (split phase).

i) Polar line codes.

Polar line codes have symbol '1' represented by a positive voltage whereas symbol '0' is represented by negative voltage polarity.

These polarities are maintained over the complete pulse duration for non-return to zero (Polar NRZ) line codes.

For Polar RZ (return to zero), the pulse is transmitted only for half duration.

i.e. for pulse duration T_b, we have,

Polar RZ as

$$\left.\begin{aligned} x(t) &= +A/2 \quad \text{for } 0 \leq t \leq T_b/2 \\ &= 0 \quad \text{for } T_b/2 \leq t < T_b \end{aligned}\right\} \text{for symbol '1'}$$

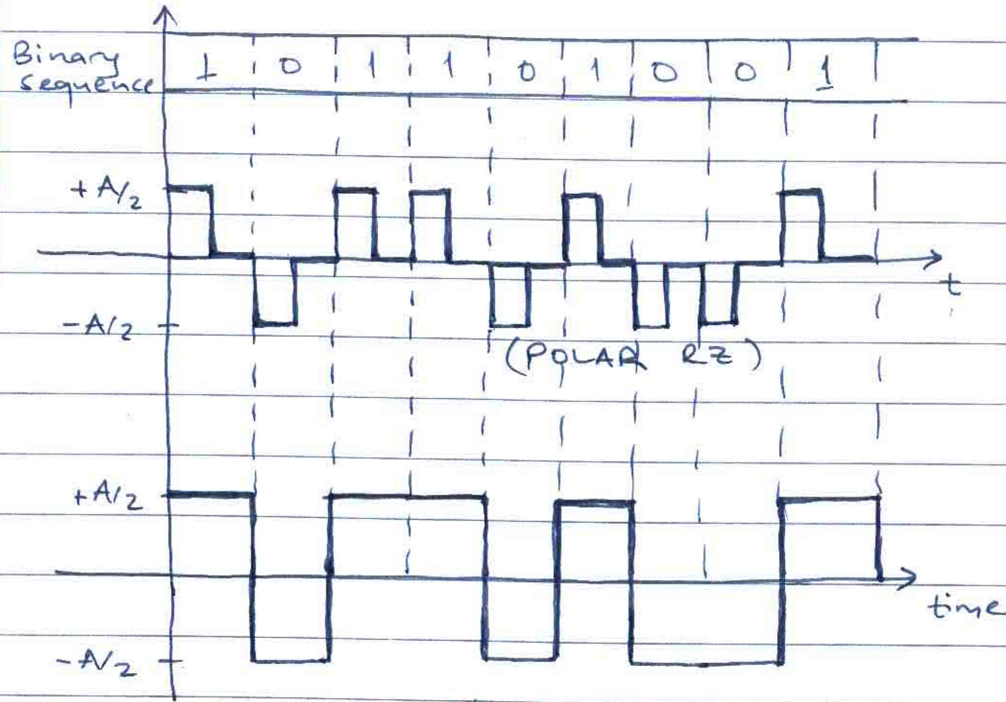
$$\left.\begin{aligned} x(t) &= -A/2 \quad \text{for } 0 \leq t < T_b/2 \\ &= 0 \quad \text{for } T_b/2 \leq t < T_b \end{aligned}\right\} \text{for symbol '0'}$$

whereas,

Polar NRZ is,

$$x(t) = +A/2 \quad \text{for } 0 \leq t < T_b \quad \text{for symbol '1'}$$

$$= -A/2 \quad \text{for } 0 \leq t < T_b \quad \text{for symbol '0'}.$$



(POLAR NRZ)

ii) Unipolar line codes.

In unipolar line codes, symbol '1' is represented by a positive voltage whereas symbol '0' is represented by a zero volts (i.e. no signal).

So, in unipolar RZ, the pulse is transmitted only for half the pulse duration and in unipolar NRZ, the pulse is transmitted through whole of pulse duration.

So, the relation can be equated as,

for unipolar RZ,

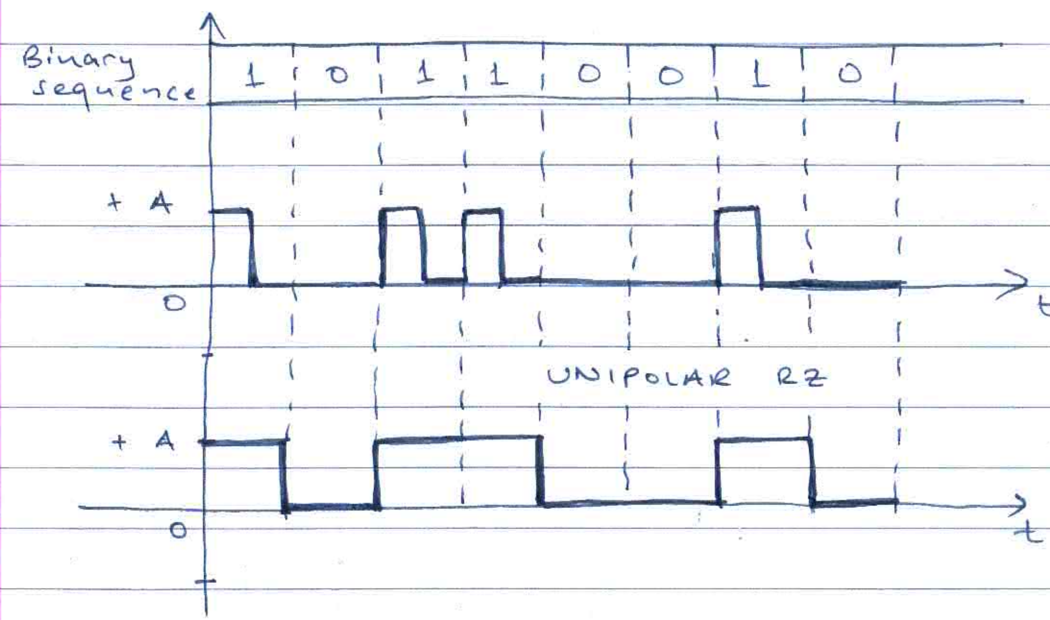
$$\left.\begin{aligned} x(t) &= A \quad \text{for } 0 \le t < T_b/2 \\ &= 0 \quad \text{for } T_b/2 \le t < T_b \end{aligned}\right\} \text{ for symbol '1'}$$

$$x(t) = 0 \quad \text{for } 0 \le t \le T_b \;\} \text{ for symbol '0'}$$

for unipolar NRZ,

$$x(t) = A \quad \text{for } 0 \le t < T_b \rightarrow \text{for symbol '1'}$$

$$x(t) = 0 \quad \text{for } 0 \le t < T_b \rightarrow \text{for symbol '0'}.$$



UNIPOLAR NRZ

iii) Bipolar or alternate mark inversion (AMI)

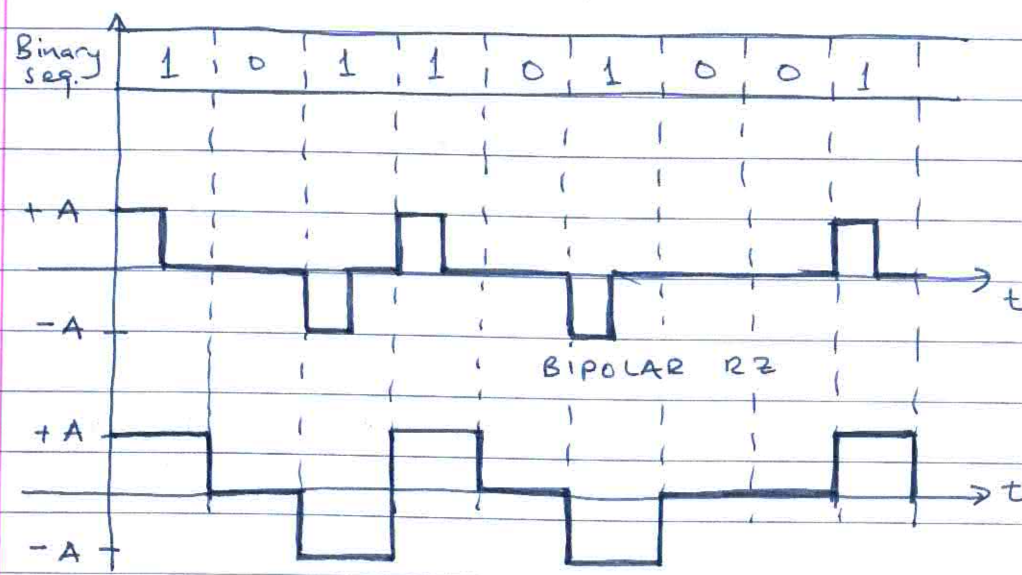
In this line code format, the successive '1's are represented by pulses with alternate polarity and '0's are represented by no pulses.

(AMI) Bipolar RZ :

→ Here the pulses are transmitted half the duration of pulse.

(AMI) Bipolar NRZ.

→ Here, the pulses are transmitted for the whole of the pulse duration.



BIPOLAR NRZ

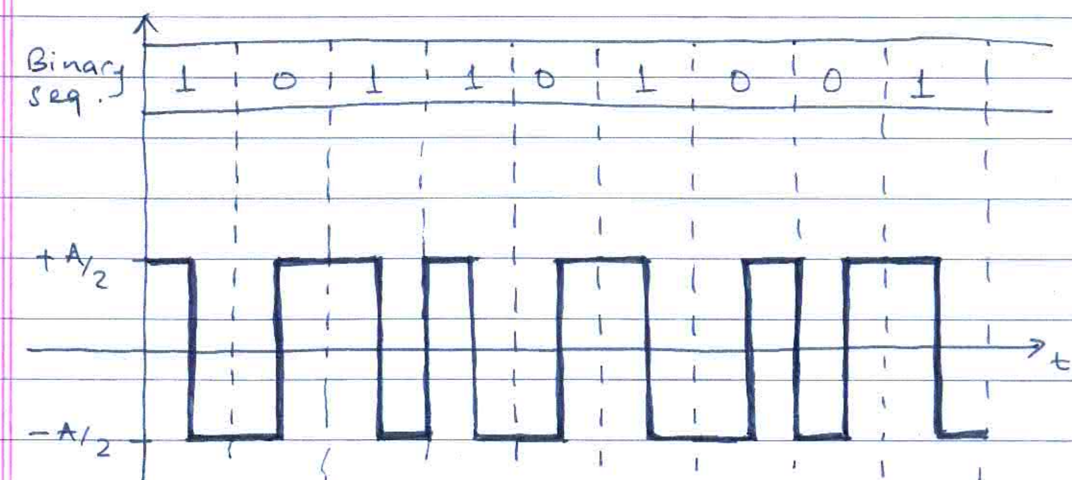
iv) Manchester or split phase

In Manchester coding, if symbol '1' is to be transmitted then a positive half interval pulse is followed by a negative half interval pulse. And if symbol '0' is to be transmitted then a negative half pulse is followed by a positive half pulse.

i.e.

$$x(t) = +\frac{A}{2} \text{ for } 0 \le t < T_b/2, \quad = -\frac{A}{2} \text{ for } T_b/2 \le t < T_b \quad \text{for symbol '1'}$$

$$x(t) = -A/2 \text{ for } 0 \le t < T_b/2, \quad = +A/2 \text{ for } T_b/2 \le t < T_b \quad \text{for symbol '0'}$$



Manchester coding

④ Baseband data communication system.

A signal whose spectrum extends from $0Hz$ (dc) to some finite frequency (usually a few MHz) is called a baseband signal.

And a baseband data communication system refers to a system where the transmission and reception of signals over a bandlimited channel (i.e. wired line) is accomplished without employing modulation (bandpass) technique.

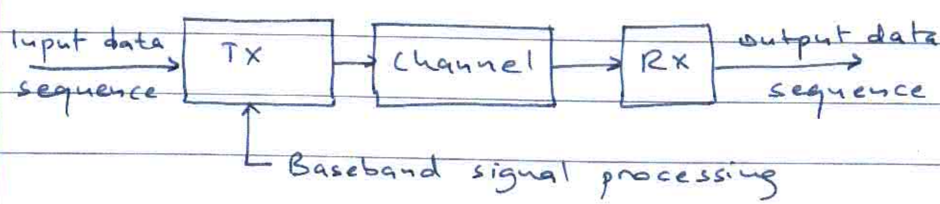


Fig. Baseband data comm^n system.

With the help of ~~Kisresodes~~ encoders, we have the input data sequence ~~as~~ in the form of voltage or current with limited amplitude values. Now, the channel incorporates transmission losses thus the input signal's amplitude level must be increased to compensate the transmission losses. This can be accomplished by pulse modulation where PAM is considered to be simplest and efficient.

So, the baseband data communication system using PAM can be shown using following functional blocks.

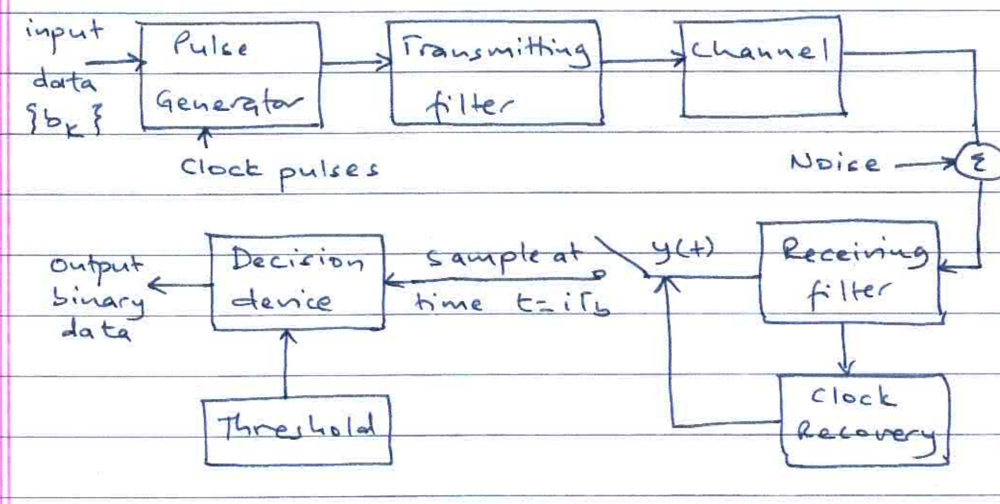


Fig. Baseband binary data system (using PAM).

The input to the pulse generator clocked at the bit rate $(1/T_b)$, is a sequence of binary data.

[for PAM
$x(t) = \sum_{n=-\infty}^{\infty} x[nT_s]\, h(t-nT_s)$]

The output of the pulse generator is PAM signal represented as,

$$x(t) = \sum_{k=-\infty}^{\infty} a_k\, p_x(t - kT_b)$$

where, $p_x(t)$ denotes basic pulse normalized such that $p_x(0) = 1$.

The pulse generator converts the input binary sequence to polar form in a way that,

if

$b_k = 1$ then $a_k = 1$

$b_k = 0$ then $a_k = -1$

The o/p signal from the pulse generator '$x(t)$' is then passed through a transmitting filter, the channel and the receiving filter having impulse responces $h_T(t)$, $h_c(t)$ & $h_R(t)$ respectively. So the transfer f^n for them will be $H_T(f)$, $H_c(f)$ & $H_R(f)$ respectively.

Furthermore, noise is added to the signal and some delay in signal is anticipated.

Thus, the output of receiving filter can be written as,

$$y(t) = \mu \sum_{k=-\infty}^{\infty} a_k\, p(t - kT_b - \tau_d) + n(t)$$

where,

μ = scaling factor

$n(t)$ = noise added

τ_d = delay introduced by the system

~~and~~ T_b = bit time

and, $\mu \cdot p(t - kT_b - \tau_d)$ represents combined impulse response of the receiving filter, channel and transmission filter.

The receiving filter output is sampled synchronously with the transmitter. The sampling instants are determined by a clock or timing signal which is extracted from receiver filter.

Each of these samples are then compared to a threshold level in the decision making device. such that,

if amplitude > threshold, '1' is received

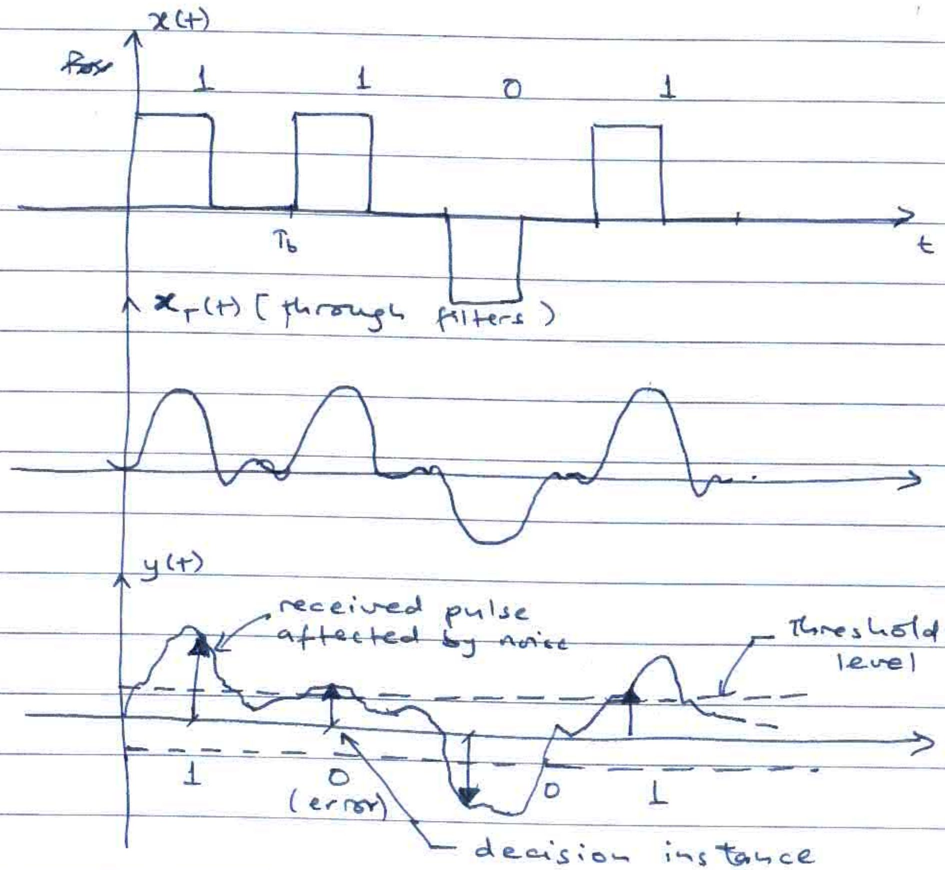
amplitude < threshold, '0' is received.

The amplitude of received pulse is,

$a_k = \mu a_k$

and shape of received pulse

$= \mu p_x(t)$



let $\mu p_x(t) = p_y(t)$, the $\tau_d = 0$, and the channel is noiseless, i.e. $n(t) = 0$, then,

$$y(t) = \mu \sum_{k=-\infty}^{\infty} a_k \, p_y(t - kT_b)$$

Now, if the output signal is sampled at time, $t = iT_b$, then,

$$y(t = iT_b) = \mu \sum_{k=-\infty}^{\infty} a_k \, p_y(iT_b - kT_b)$$

where, i = any integer

or, $$y(t = iT_b) = \mu a_i + \mu \sum_{\substack{k=-\infty \\ k \neq i}}^{\infty} a_k \, p_y(iT_b - kT_b)$$

$\because$ for $k = i$, $p_y(iT_b - iT_b) = p_y(0) = 1$.

or $$y(t = iT_b) = \mu a_i + \mu \sum_{\substack{k=-\infty \\ k \neq i}}^{\infty} a_k \, p_y[(i-k)T_b]$$

or $$y(t = iT_b) = \mu a_i + \mu \sum_{\substack{k=-\infty \\ k \neq -i}}^{\infty} a_k \, p_y[(i-k)T_b]$$

so, for receiver output $y(t)$ at $t = iT_b$,
we can see that there are two terms,

i) μa_i → it is produced by i^{th} transmitted bit and thus represents the original message bit.

ii) $\mu \sum_{\substack{k=-\infty \\ k \neq i}}^{\infty} a_k \, p_y[(i-k)T_b]$

→ This is the residual effect of all the transmitted bit obtained at the time of sampling of i^{th} bit.

Now, this residual effect is known as intersymbol interference (ISI). This ISI arises due to the dispersion of pulse shapes by filters and channels.

We know that a pulse ~~an~~ has sinc f^n as its transfer function and the sinc f^n ~~th~~ has frequencies oscillating through out, thus a pulse always disperses as the frequencies tend to overlap for adjacent pulses.

Thus optimal designing of transmitting and receiving filter as well as the shape of basic pulse lead to minimized ISI.

Nyquist pulse shaping criteria.
(for zero ISI)

We have ~~for~~ the receiver's output ~~at~~ for baseband digital communication as,

$$y(t) = \mu \sum_{k=-\infty}^{\infty} a_k \, p_y(t - kT_b)$$

and at $t = iT_b$,

$$y(t = iT_b) = \mu a_i + \mu \sum_{\substack{k=-\infty \\ k \neq i}}^{\infty} a_k \, p_y[(i-k)T_b]$$

Therefore for zero ISI, the residual term should be zero,

i.e.

$$\mu \sum_{\substack{k=-\infty \\ k \neq i}}^{\infty} a_k \, p_y[(i-k)T_b] = 0$$

or $\sum_{\substack{k=-\infty \\ k \neq i}}^{\infty} p_y[(i-k)T_b] = 0$

i.e. $p_y[(i-k)T_b] = 1$ for $i = k$ $[p_y(0) = 1]$

$p_y[(i-k)T_b] = 0$ for $i \neq k$

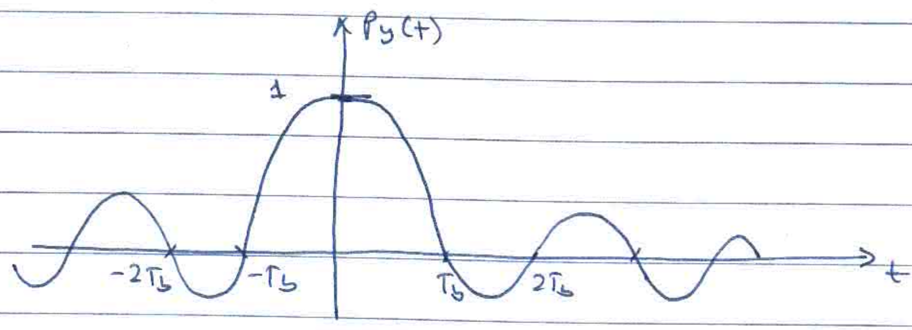
let $i-k = n$, then, if $i = k \rightarrow n = 0$

$p_y(nT_b) = 1$ for $n = 0$

$p_y(nT_b) = 0$ for $n \neq 0$

or $p_y(t) = 1$ for $t = nT_b = 0$

$p_y(t) = 0$ $t = \pm nT_b \neq 0$



Thus $p_y(nT_b)$ is nothing but a sinc function with bit rate $R_b = \frac{1}{T_b}$

So the minimum bandwidth, $B_0 = \frac{R_b}{2}$

Therefore, or $R_b = 2B_0$

$p_y(t) = \text{sinc}\left(\frac{t}{T_b}\right)$

$= \text{sinc}\,(\cancel{f}\ t \cdot R_b)$

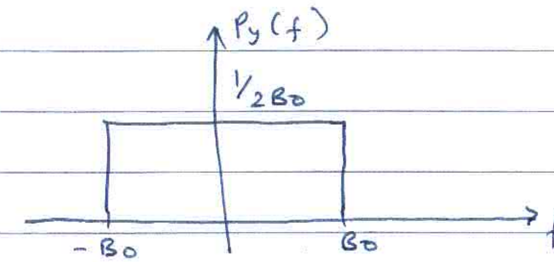
$= \text{sinc}\,(2B_0 t)$

And the fourier transform is,

$P_y(f) = \frac{1}{2B_0} \text{rect}\left(\frac{f}{2B_0}\right)$

i.e. $P_y(f) = \begin{cases} \frac{1}{2B_0} & 0 \leq |f| \leq B_0 \\ 0 & \text{elsewhere,} \end{cases}$

i.e.



So, instead of sending pulses as the binary form, if we send the sinc version of the input binary sequency which has bandwidth, $B_0 = \frac{R_b}{2}$, there will be no ISI.

⊕ Raised cosine pulse.

We have seen that if we use sinc function as the transmitted message, we get zero ISI. But the ideal sinc function is non-causal as well as the function relates to the infinite time, thus they are physically unrealizable.

So, the best we can do is to make the shape the pulse look nearer to a sinc function.

For this, the minimum bandwidth $B_0 = \frac{R_b}{2}$ is extended to an adjustable value between B_0 and $2B_0$. Such approximation is known as raised cosine pulses.

Now, for zero ISI, we have,

$$p(nT_b) = 1 \quad \text{for } n = 0$$
$$= 0 \quad n \neq 0$$

such, that,

$$P(f) = F[p(nT_b)] = \frac{1}{T_b} \sum_{n=-\infty}^{\infty} P(f - nf_b)$$

$$= R_b \sum_{n=-\infty}^{\infty} P(f - nR_b)$$

Also,

$$P(f) = \int_{-\infty}^{\infty} \sum_{m=-\infty}^{\infty} [p(mT_b)\delta(t - mT_b)] e^{-j2\pi ft} dt$$

where, $m = i - k$,

$\therefore$ for $m = 0$,

$$P(f) = \int_{-\infty}^{\infty} p(0)\,\delta(t)\, e^{-j2\pi ft} dt$$

$$= p(0)$$

$$P(f) = 1 \quad \because p(0) = 1.$$

$\therefore \quad 1 = R_b \sum_{n=-\infty}^{\infty} P(f - nR_b)$

$$\text{or} \quad \sum_{n=-\infty}^{\infty} P(f-nR_b) = \frac{1}{R_b} = T_b .$$

Expanding the above f^n,

$$+\ldots+ p(f+2R_b) + p(f+R_b) + p(f) + P(f-R_b) + p(f-2R_b) + \ldots = \frac{1}{R_b}.$$

Now, $R_b = 2B_0$

$$\text{or} \ldots + p(f+2B_0) + p(f) + p(f-2B_0) + \ldots = \frac{1}{R_b}$$

We will take only the three terms which correspond to $n=-1$, $n=0$ & $n=1$, thus the bandwidth is extended from B_0 to $2B_0$.

i.e.

$$p(f+2B_0) + p(f) + p(f-2B_0) = \frac{1}{2B_0} \quad -B_0 \le f \le B_0 .$$

There are several functions that will satisfy the above equation. Among the raised cosine function is mostly used.

The raised cosine function has a spectrum that consists of a flat portion and a roll off portion.

This raised cosine spectrum can be expressed as,

$$P(f) = \begin{cases} 1/2B_0 & \rightarrow 0 \le |f| < f_1 \\ \dfrac{1}{4B_0}\left[1 + \cos\left[\dfrac{\pi(|f| - f_1)}{2B_0 - 2f_1}\right]\right] & \rightarrow f_1 < |f| \le 2B_0 - f_1 \\ 0 & \rightarrow |f| \ge 2B_0 - f_1 \end{cases}$$

The frequency parameter f_1 and B_0 are related by,

$\alpha = 1 - \dfrac{f_1}{B_0}$ = roll off factor which indicates excess bandwidth required over the ideal Nyquist criterion.

And,

the transmission bandwidth is given by,

$$B_T = 2B_0 - f_1 = B_0(1+\alpha)$$

The time response p(t) ~~of~~ can be made by taking inverse fourier transform of P(f), such that,

$$p(t) = \text{sinc}(2B_0t) \cdot \left[\frac{\cos 2\pi\alpha B_0 t}{1 - 16\alpha^2 B_0^2 t^2}\right]$$

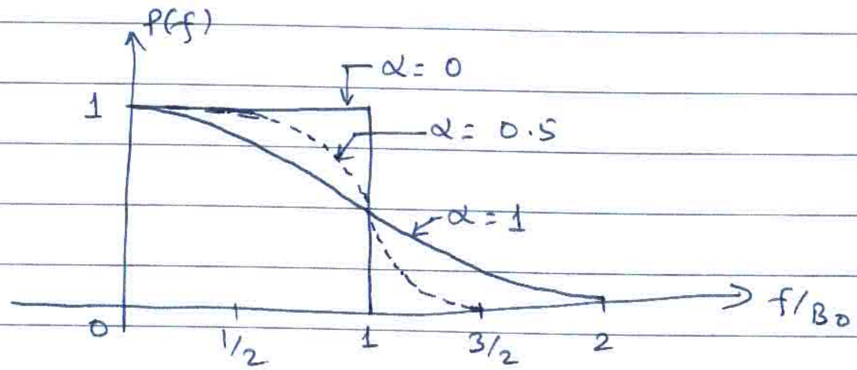


fig. frequency response of raised cosine f^n.

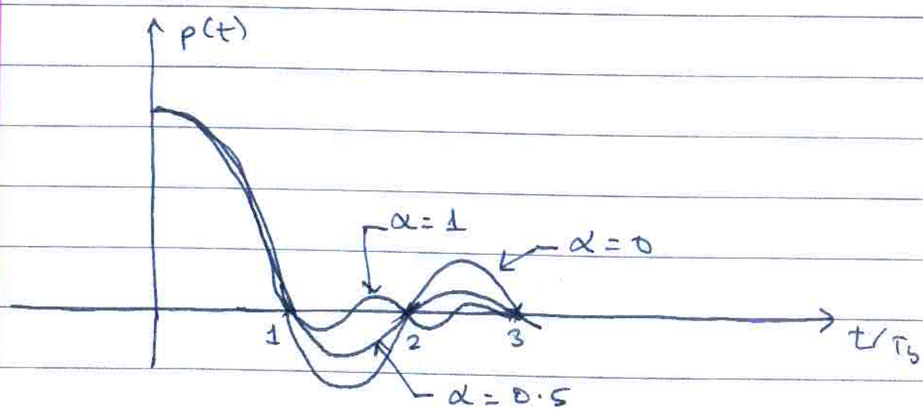


fig. Time response for raised cosine f^n.

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Characteristics of P(f) :-

i) For $\alpha = 0.5$ & 1, P(f) rolls of gradually with respect to 'f', therefore is physically realizable.

ii) P(f) has odd symmetry with respect to cut off frequency of ideal filter (B_0).

iii) With $\alpha = 1$, the bandwidth required is maximum i.e. $2B_0$.

iv) P(f) is non-negative and,

$$\int_{-\infty}^{\infty} P(f)\, df = 1.$$

Characteristics of p(t)

i) At $t = \pm T_b/2$, $p(t) = 0.5$, i.e. pulse width measured at half amplitude is exactly $T_b/2$.

ii) For $\alpha = 1$, there is additional zero crossing of p(t) at,

$$t = \pm \frac{3T_b}{2}, \pm \frac{5T_b}{2}$$

These additional zero crossing is useful for generating timing signal for synchronization from

Transmission bandwidth consideration.

As seen from the curve $P(f)$, the absolute minimum bandwidth,

$$B_T = B_0(1+\alpha)$$
$$= B_0 \quad [\text{for } \alpha = 0].$$

where,

$$B_0 = \frac{R_b}{2} = \frac{1}{2T_b}$$

Because of raised cosine spectrum, the bandwidth increases as α increases, therefore for $\alpha = 1$,

$$B_T = B_0(1+\alpha) = B_0(1+1)$$
$$= 2B_0.$$
$$= R_b. \quad [\because R_b = 2 \cdot B_0]$$
$$= \frac{1}{T_b}.$$

$\therefore R_b = \frac{1}{T_b} = 2B_0$ for zero ISI.

So, for practically realizable pulse shaping filter, the bandwidth of the channel for given signalling rate 'R_b', $B_T = 2B_0$.

And if channel bandwidth is B_0, then signalling rate should be $\frac{R_b}{2}$.

Both the cases are for $\alpha = 1$.

Correlative coding techniques.

As per the Nyquist criteria for zero ISI, for the ideal case $(\alpha = 0)$, for R_b data rate, the absolute minimum bandwidth equals to $R_b/2$ and with raised cosine pulse shaping method, bandwidth would be R_b $(\alpha = 1)$.

So,

Data rate	Bandwidth	ISI	Condition
R_b	$R_b/2$	zero	Ideal
R_b	R_b	zero	realizable (raised cosine)

So, we see that to transmit R_b data rate over a channel with bandwidth $B_0 = \frac{R_b}{2}$, we had to implement ideal Nyquist pulse shaping which is unrealizable whereas using raised cosine function, to transmit R_b data rate we need channel bandwidth equal to $2B_0$.

Now it is practically possible to transmit R_b data rate over the channel with $B_0 = R_b/2$ bandwidth by introducing a controlled amount of ISI.

This method of adding ISI in a controlled manner to the transmitted signal to achieve a bitrate of $R_b = 2B_0$ bits per second in a channel of $B_0 = \frac{R_b}{2}$ Hertz.

In correlative coding, a known ISI is introduced to the transmitted signal such that its effect can be compensated at the receiver.

Correlative coding is also termed as partial response signalling.

⊕ Duo-binary encoding (DBE).

DBE is one of the correlative coding method. In duo-binary encoding, the transmission capacity of a straight binary system is doubled.

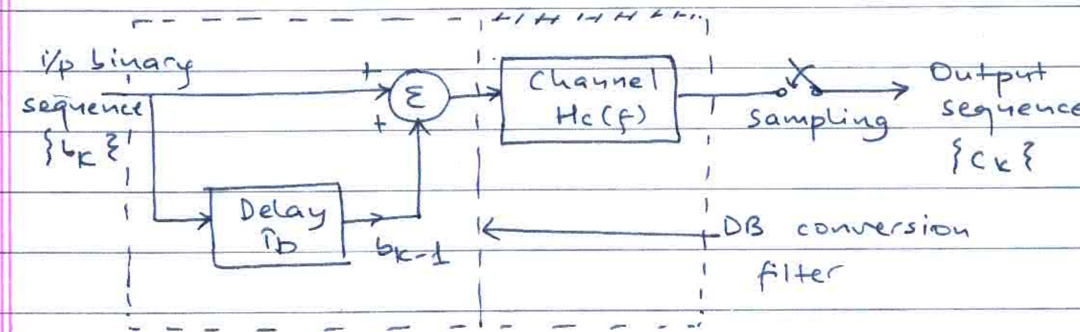


Fig. Duo-binary encoder

Let us have an input binary sequence $\{b_k\}$ of uncorrelated binary digits having duration T_b.

Let b_k be represented as,

$b_k = +1$ for binary digit '1'

$b_k = -1$ for binary digit '0'.

Now, this b_k is converted into three level bit sequence c_k having three levels, +2, 0 & -2, by employing the following technique,

$c_k = b_k + b_{k-1}$

So,

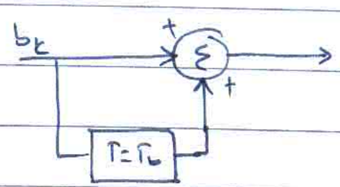
$$c_k = \begin{cases} +2 & \text{if } b_k \,\&\, b_{k-1} \text{ are } +1 \;('1') \\ 0 & \text{if } b_k \,\&\, b_{k-1} \text{ are different} \\ -2 & \text{if } b_k \,\&\, b_{k-1} \text{ are } -1 \;('0') \end{cases}$$

Now the frequency response of a filter consisting of DB conversion and channel

$$H(f) = H_{DB}(f) \cdot H_c(f).$$

where,

$H_{DB}(f)$ is the frequency responsing consisting of an adder and delay network,



So, $H_{DB}(f) = 1 + e^{-j2\pi f T_b}$

$$\therefore H(f) = H_c(f) \cdot [1 + e^{-j2\pi f T_b}]$$

or $H(f) = H_c(f)\left[\cancel{ex}\, e^{(j\pi f T_b)} + e^{(-j\pi f T_b)}\right] \cdot e^{(-j\pi f T_b)}$

$$= H_c(f) \cdot 2\cos(\pi f T_b) \cdot e^{(-j\pi f T_b)}$$

$$\therefore H(f) = 2 H_c(f) \cos(\pi f T_b)\, e^{(-j\pi f T_b)}$$

Assuming that $H_c(f)$ is ideal,

$$H_c(f) = \begin{cases} 1 & \text{for } |f| \le \frac{1}{2T_b} = B_0 \\ 0 & \text{for } |f| > \frac{1}{2T_b} \end{cases}$$

Therefore,

$$H(f) = \begin{cases} 2\cos(\pi f T_b) \cdot e^{(-j\pi f T_b)} & \text{for } |f| \le \frac{1}{2T_b} \\ 0 & \text{elsewhere} \end{cases}$$

The advantage of this transfer function $H(f)$ is that for transmission of R_b rate, the required bandwidth is only B_0 (Nyquist bandwidth). Now, $H(f)$ can easily be approximated and implemented.

The ~~ee~~ impulse response of DBE with transfer function $H(f)$ is,

$$h(t) = \frac{\sin(\pi t/T_b)}{\pi t/T_b} - \frac{\sin(\pi t/T_b)}{\pi(t-T_b)/T_b}$$

$$= \frac{T_b^2 \sin(\pi t/T_b)}{\pi t(T_b - t)}$$

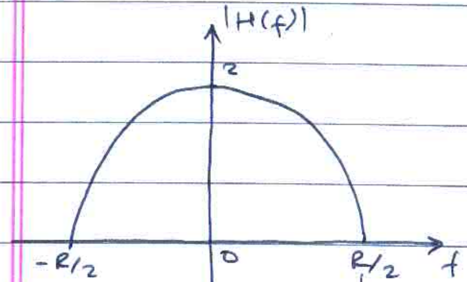
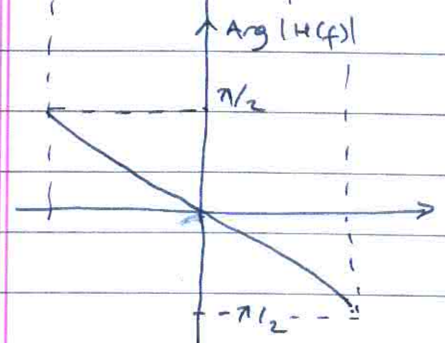


Fig Amplitude Response



Phase response

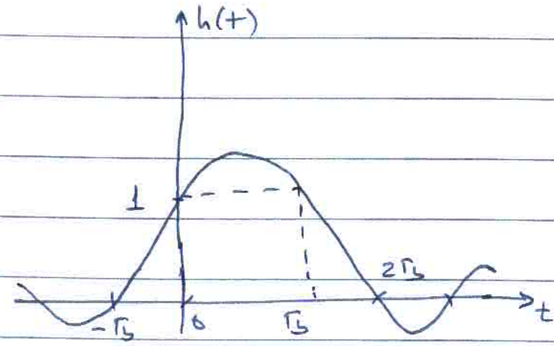


Fig. Impulse responce of duobinary filter

The figure shows that ~~h(t)~~ $h(t)$ has two distinguished values at sampling instances $+T_b$ & $-T_b$

Now, at the receiver end, the original bit b_k can be recovered from c_k by subtracting previously decoded bit from the received bit c_k.

i.e.

$$\hat{b}_k = c_k - \hat{b}_{k-1}$$

Now, $\hat{b}_k = b_k$ if the previous bit b_{k-1} was correctly decoded at sampling instance $t = (k-1)T_b$, i.e. $\hat{b}_{k-1} = b_{k-1}$

So, any error in decoding previous data will affect the present decoding resulting in an error. The ~~propb~~ problem of DBE is that, this error tend to propagate.

To overcome this problem we use a precoder preceeding the DBE.

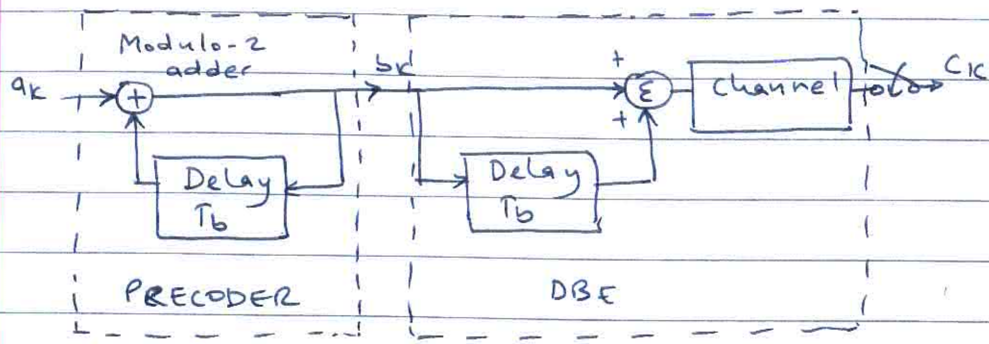


Fig. DBE with precoder.

let a_k be a binary sequence then

from figure, precoder output,

modulo-2 adder / Exclusive-OR operator.

$$b_k = a_k \oplus b_{k-1}$$

i.e.

o/p is 1 if any one input is 1.

i.e. 1+1=0.

Also

the output of DBE,

$$c_k = b_k + b_{k-1}$$

Now, let $b_k = +1$ if precoder output is '1'

$b_k = -1$ if precoder o/p is '0'.

Then, if $a_k = 0$, then, $b_k = b_{k-1}$

So, ~~a_k~~ b_k & b_{k-1} could be +1 or -1.

if $a_k = 1$, then, $\overline{b_k} = b_{k-1}$ ~~$b_k = 1 = +1$. i.e., & $b_{k-1} = \mp 1$.~~

Now, we have,

$$c_k = b_k + b_{k-1}$$

Therefore,

$$c_k = \begin{cases} +2 & \text{if } b_k = b_{k-1} = +1 \\ 0 & \text{if } b_k \text{ and } b_{k-1} \text{ are different} \\ -2 & \text{if } b_k = b_{k-1} = -1 \end{cases}$$

Now, at the receiver end, the threshold level is set to ~~±A~~ ±1 and ~~a_k~~ b_k can be decoded as,

$$b_k = 0 \quad \text{if} \quad c_k \geq \pm A$$

$$b_k = 1 \quad \text{if} \quad c_k < \pm A$$

for example,

let $\{a_k\}$: 0 0 1 0 1 0

a_1 a_2 a_3 a_4 a_5 a_6

Now,

$b_k = a_k \oplus b_{k-1}$

let $b_{k-1} = 1$, then, for $k = 1$ to 6,

$\cancel{b_{-1} = 1}$ $b_0 = 1$

$b_1 = a_1 \oplus b_0 = 0 \oplus 1 = 1$

$b_2 = a_2 \oplus b_1 = 0 \oplus 1 = 1$

$b_3 = a_3 \oplus b_2 = 1 \oplus 1 = 0$

$b_4 = a_4 \oplus b_3 = 0 \oplus 0 = 0$

$b_5 = a_5 \oplus b_4 = 1 \oplus 0 = 1$

$b_6 = a_6 \oplus b_5 = 0 \oplus 1 = 1$.

Or, the above can be shown as,

	k=0	k=1	k=2	k=3	k=4	k=5	k=6
a_k		0	0	1	0	1	0
b_k	1	1	1	0	0	1	1
b_k	+1	+1	+1	-1	-1	+1	+1
c_k		+2	+2	0	-2	0	+2
O/P $\hat{b}_k$		0	0	1	0	1	0

we have, $c_k = b_k + b_{k-1}$

so,

$c_1 = b_1 + b_0 = +1 + 1 = +2$

$c_2 = b_2 + b_1 = +1 + 1 = +2$

$c_3 = b_3 + b_2 = -1 + 1 = 0$

$c_4 = b_4 + b_3 = -1 - 1 = -2$

$c_5 = b_5 + b_4 = +1 - 1 = 0$

$c_6 = b_6 + b_5 = +1 + 1 = +2$.

and at the receiver end, b_k is decoded

as, if $c_k \geq \pm 2$, $b_k = 0$

if $c_k < \pm 2$, $b_k = 1$.

Now, assuming $b_{k-1} = 0$, then,

$\{k\}$	0	1	2	3	4	5	6
a_k		0	0	1	0	1	0
b_k	0	0	0	1	1	0	0
b_k	-1	-1	-1	+1	+1	-1	-1
c_k		-2	-2	0	+2	0	-2
O/P $\hat{b}_k$		0	0	1	0	1	0

So, for $b_{k-1} = 1$ or 0, we get the decoded output $\hat{b}_k$ as the original input b_k.

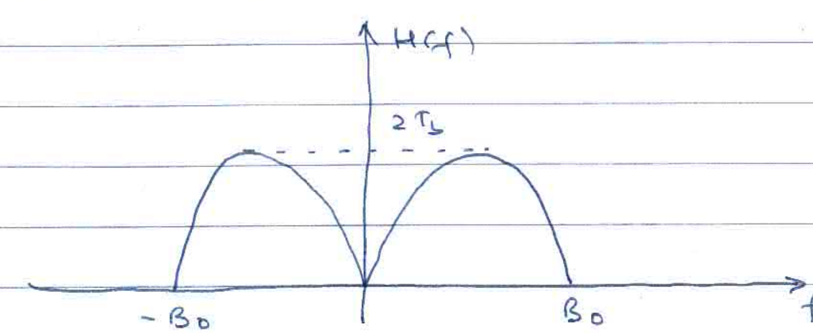
Thus we see that the decoding of present symbol 'b_k' is not affected by correctiveness of decoding of immediate past symbol 'b_{k-1}'.

⊕ Modified duo-binary encoding. (MDB)

A modified duobinary encoding involves a correlation span of two binary digits i.e. in MDB instead of one T_b delay, $2T_b$ delay is employed.

In DB encoding it can ~~see~~ be seen that the frequency responce at $f=0$ is a non-zero value thus it is not suitable for circuitry with no dc path. As most of the circuitry uses non-dc path, DB encoding is not used, rather MDB is preferred.

So, making $2T_b$ delay, the frequency response becomes symmetrical with respect to the origin $(f=0)$ with zero magnitude at the origin $(f=0)$.



A modified duobinary encoding with precoder can be shown as,

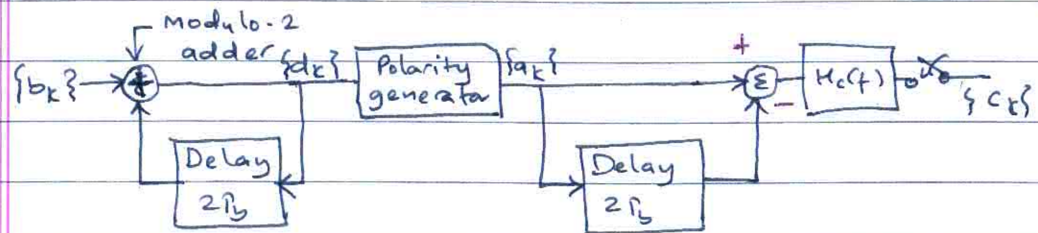


Fig. MDB encoder with a precoder.

Here,

$$d_k = b_k \oplus d_{k-2} \qquad \begin{bmatrix} 0+0=0 \\ 0+1=1 \\ 1+0=1 \\ 1+1=0 \end{bmatrix}$$

and $a_k = +1$ if $d_k = 1$

$a_k = -1$ if $d_k = 0$

Now,

the output,

$$c_k = a_k - a_{k-2}$$

The decoding of MDB encoding is done as,

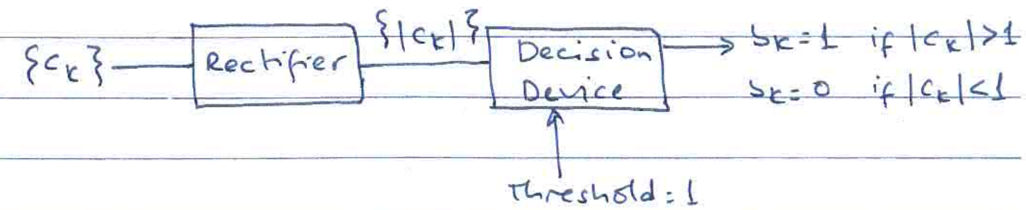


Fig. Decoder for MDB encoding.

For, $\{b_k\}$ = 0 1 1 1 0 0 1 0 1

b_k			0	1	1	1	0	0	1	0	1
d_{k-2}	1	1	1	0	0	1	0	1	1	1	0
d_k			1	0	0	1	0	1	1	1	0
a_k			+1	−1	−1	+1	−1	+1	+1	+1	−1
a_{k-2}	+1	+1	+1	−1	−1	+1	−1	+1	+1	+1	−1
c_k			0	−2	−2	+2	0	0	+2	0	−2
$\lvert c_k \rvert$			0	2	2	2	0	0	2	0	2

At decoder,

b_k 0 1 1 1 0 0 1 0 1

M-ary signalling

In binary signalling, the output of pulse generator can have one of two possible levels. Where as in M-ary signalling, the output can have one of 'M' possible levels. So in M-ary system, the input source emits one of the 'M' distinct symbol and each symbol is assigned a distinct level of M possible levels.

for, binary we had $n = 1$, ie number of bits $= 1$ either 0 or 1.

And signal levels, $2^n = 2^1 = 2$.

Therefore we could represent binary input with '+1 and 0', or '+1 and -1'.

For M-ary system, let number of bits $= 2$,

then $M = 2^2 = 4$.

So, for $M = 4$, the signal representation can be,

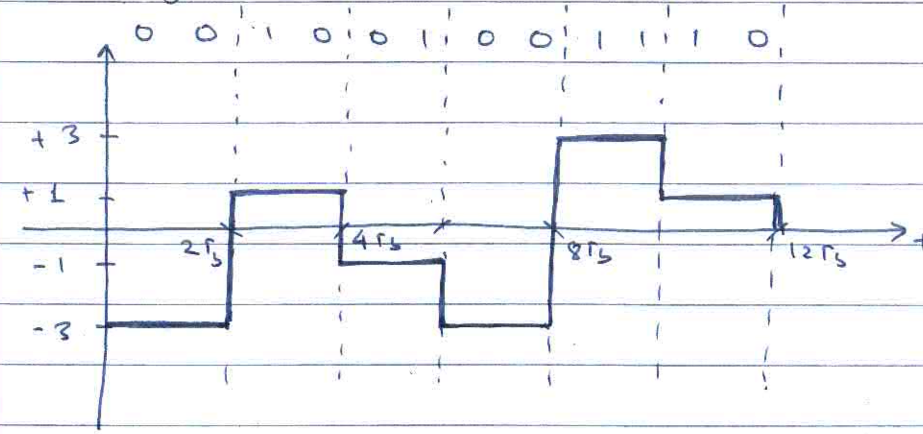
Input symbol	Representation level
B (11)	+3 V
C (10)	+1 V
D (01)	−1 V
E (00)	−3 V

Possible combination of 2 bits are,

00, 01, 10 & 11.

so, these combinations can be line coded using polar quaternary NRZ as,

say for 001001001110



Signaling rate and bandwidth requirement for M-ary signalling.

Let R_s (symbols per second or Baud) be the rate of symbols emitted by the source.

Now, if these 'M' symbols are equi-probable and statistically independent then ~~source~~ entropy will be

$$H = \sum_{i=1}^{M} P(x_i) \cdot I(x_i)$$

$$H = \sum_{i=1}^{M} P(x_i) \cdot \log_2 \left(\frac{1}{P(x_i)} \right)$$

Since, $P(x_i)$ are equiprobable for $i \to M$,

$$\therefore P(x_i) = 1/M$$

$$H = \sum_{i=1}^{M} \left(\frac{1}{M} \right) \log_2 \left(\frac{1}{1/M} \right)$$

$$= \sum_{i=1}^{M} \frac{1}{M} \cdot \log_2 (M)$$

$$\therefore H = \log_2 M \qquad \because \sum_{i=1}^{M} P(x_i) = 1.$$

Also, information rate,

$$R_M = R_s \cdot \log_2 M \quad \text{(bits / sec)}$$

The signal interval duration $T_s = \frac{1}{R_s}$ is same for both binary and M-ary systems.

So, absolute minimum bandwidth to transmit $R_s \log_2 M$ bits/sec of information,

$$B_0 = \frac{R_s}{2} \ Hz.$$

i.e.

$$R_M = R_s \log_2 M$$

$$B_0 = \frac{R_s}{2} \qquad \boxed{R_s = n f_s}$$

So, if we take binary form,

$$R_b = R_s \log_2 2 = R_s$$

but bandwidth $B_0 = \frac{R_s}{2}$.

It means M-ary signaling can transmit data $\log_2 M$ times faster than binary system.

i.e. $R_b = R_s$ & $R_M = R_s \cdot \log_2 M$.

Also, bandwidth for binary signaling,

$n = 1,$

$$B_0 = \frac{R_s}{2} = \frac{\cancel{n} f_s}{2} = \frac{f_s}{2} = \frac{1}{2nT_s} = \frac{1}{2nT_b} = \frac{1}{2T_b}$$

for M-ary signaling,

$$B_0' = \frac{1}{2nT_s} = \frac{1}{2nT_b}$$

∴ we see that the bandwidth decreases by a factor 'n' where,

$n = \log_2 M.$

So, the price paid for higher data rate with lesser bandwidth in M-ary signaling is the higher power required to transmit M-ary signal.

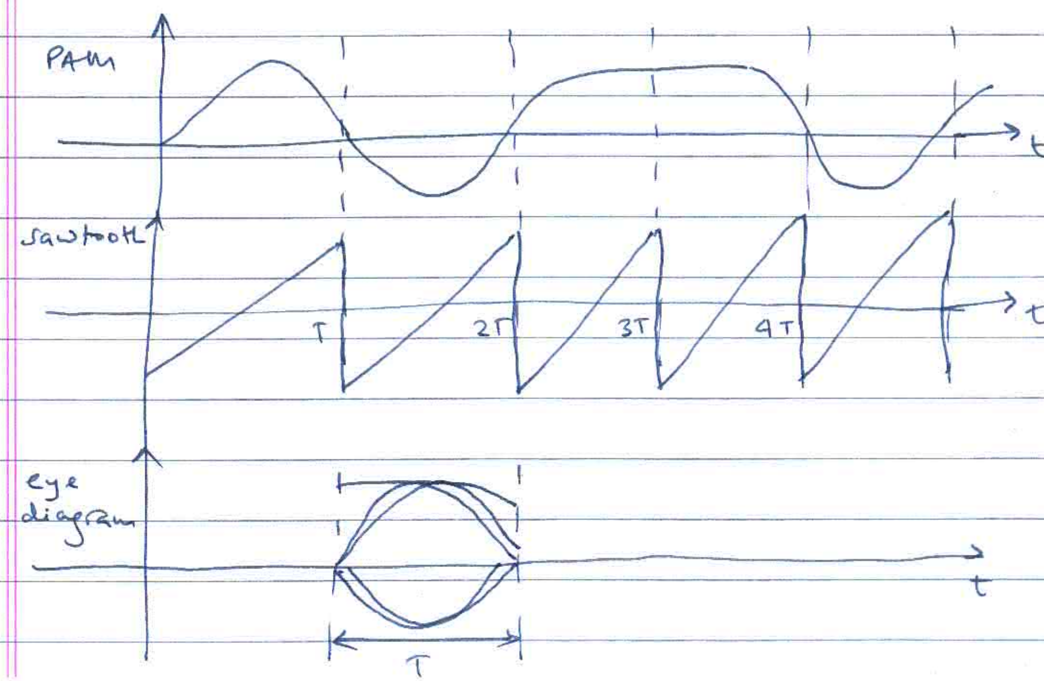
In general for $M >> 1$ and ~~error~~ error probability $P_e << 1$, the power required is M-ary is nearly $M^3/3\log_2 M$ times greater than in binary.

Also, M-ary systems are more complex than binary systems.

The eye ~~dig~~ diagram. (Eye pattern).

The eye diagram is the study of ISI experimentally. It can be seen as a performance analysis of baseband PAM signal where the obtained signal is applied to the vertical input (deflection plates) of oscilloscope and a sawtooth signal with symbol rate $R = 1/T$ to the horizontal input.

The waveforms in successive symbol interval are thereby ~~translated~~ translated into one interval on the oscilloscope display.



The resulting display resembles the human eye and is thus is called an eye diagram or eye pattern.

The eye diagram now can be generalized as,

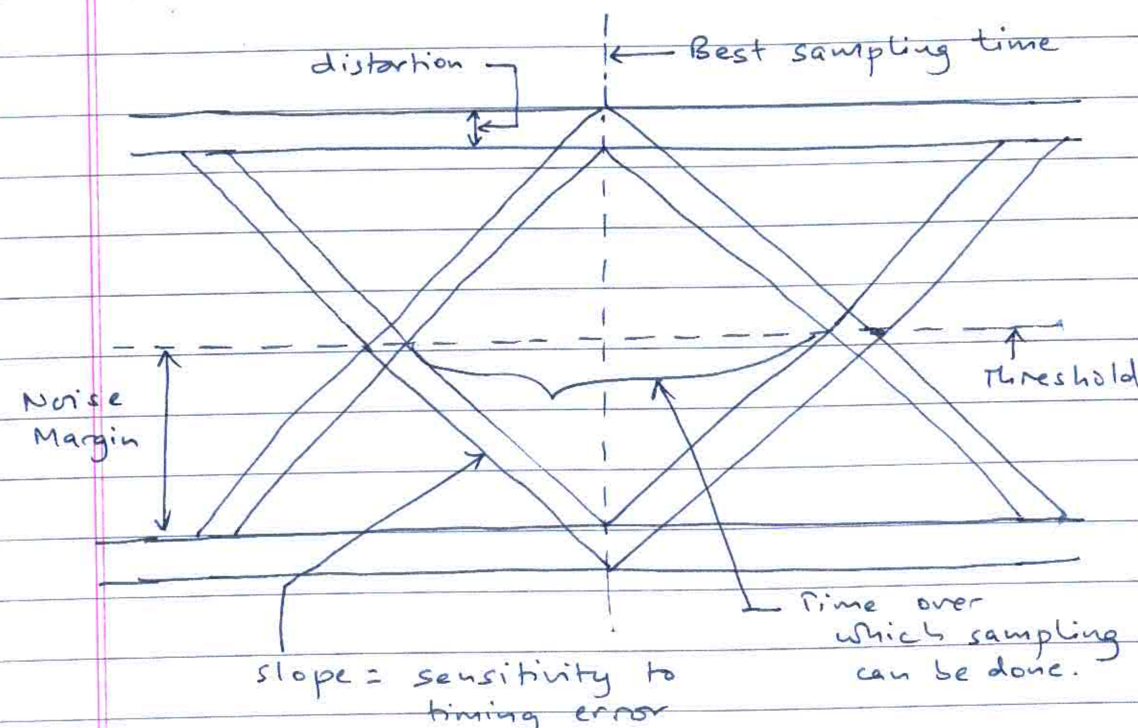


fig. Interpretation of eye diagram.

The interior region of the eye pattern is called the eye opening.

An eye pattern provides following information on the basis of their form.

i. The width of the eye opening defines the time interval over which the received wave can be sampled without error from ISI. i.e. the best time to sample is the instance when the eye opening is largest.

ii) Sensitivity of the system to timing error is determined by the rate of closure of eyes (slope) as the sampling time is varied.

iii) Noise margin (immunity to noise) is indicated by the ~~wid~~ vertical width of eye opening w.r.t. threshold.

iv) The maximum distortion and ISI are indicated by vertical width of the two branches at sampling time.

(11) A communication channel of bandwidth 75 KHz is required to transmit binary data at a rate 0.1 Mbp's using raised-cosine pulses. Determine the roll-off factor α.

Given,

$R_b = 0.1$ Mbps

$B_T = 75$ KHz

Now,

$$B_0 = \frac{R_b}{2} = 0.05 \times 10^6$$

And, $B_T = B_0 (1+\alpha)$

$$\text{or } 1+\alpha = \frac{B_T}{B_0} = 2 \cdot \frac{B_T}{R_b} = 2\, B_T \cdot T_b$$

$$\text{or } \alpha = \frac{75 \times 10^3}{0.05 \times 10^6} - 1$$

$$= 1.5 - 1$$

$$= 0.5$$