CHAPTER 2: POINT SOURCES AND ARRAYS OF POINT SOURCES

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INTRODUCTION

- THE FIELD RADIATED BY SOME ANTENNAS ARE NOT DISTRIBUTED UNIFORMLY IN THE PARTICULAR DIRECTION, HOWEVER SUCH A CHARACTERISTICS ARE NOT PREFERRED IN **POINT TO POINT COMMUNICATION**.

- IN POINT TO POINT COMMUNICATION, IT IS DESIRED TO HAVE MOST OF THE ENERGY RADIATED IN ONE PARTICULAR DIRECTION. I.E. IT IS DESIRED TO HAVE **GREATER DIRECTIVITY** IN A DESIRED DIRECTION PARTICULARLY WHICH IS NOT POSSIBLE WITH SINGLE DIPOLE

- HENCE TO INCREASE THE FIELD STRENGTH IN THE DESIRED DIRECTION BY USING **GROUP OF ANTENNAS** EXCITED SIMULTANEOUSLY. SUCH A GROUP OF ANTENNAS IS CALLED ARRAY OF ANTENNAS

- IN GENERAL, ANTENNA ARRAY IS THE RADIATING SYSTEM IN WHICH SEVERAL ANTENNAS ARE SPACED PROPERLY SO AS TO GET GREATER FIELD STRENGTH AT A FAR DISTANCE.

- THE TOTAL FIELD STRENGTH OR INTENSITY PRODUCED BY THE ANTENNA ARRAY AT A FAR DISTANCE IS THE **VECTOR SUM OF THE FIELDS PRODUCED BY THE INDIVIDUAL ANTENNAS OF THE ARRAY.**

- THE ANTENNA ARRAY IS SAID TO BE **LINEAR** IF THE ELEMENTS OF THE ANTENNA ARRAY ARE EQUALLY SPACED ALONG A STRAIGHT LINE.

DIFFERENT TYPES OF SOURCES

I.) SOURCE WITH HEMISPHERE POWER PATTERN:

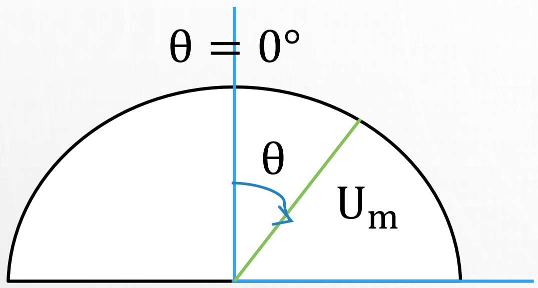


Fig -1 a

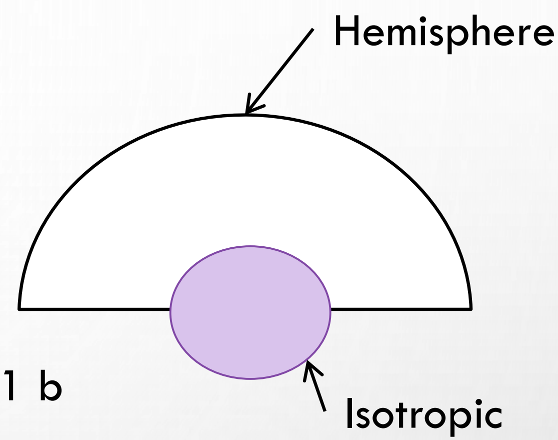


Fig -1 b

TOTAL POWER RADIATED

$$P = \iint U\, D\Omega \;=\; \int_0^{2\pi}\int_0^{\pi/2} U_M \mathrm{SIN}\Theta\, D\Theta\, D\emptyset \;=\; 2\,\Pi\, U_M$$

NOW SUPPOSE IF WE COMPARE THIS POWER RADIATED BY HEMISPHERE WITH AN REFERENCE ANTENNA(ISOTROPIC) THEN WE CAN DETERMINE THE DIRECTIVITY OF THE SOURCE ANTENNA AS WE KNOW

DIRECTIVITY ALSO DEFINE AS = $\dfrac{Radiation\ intensity\ of\ an\ source\ antenna}{Radiation\ intensity\ of\ an\ isotropic\ antenna}$

SO HENCE HERE

$$2\,\Pi\, U_M \;=\; 4\,\Pi\, U_O$$

$$\boxed{\boldsymbol{\frac{U_m}{U_o} = 2 \;=\; Directivity\ \mathrm{D}}}$$

VARIOUS FORM OF ANTENNA ARRAY

1. BROADSIDE ARRAYS
2. END-FIRE ARRAY

BROADSIDE ARRAYS

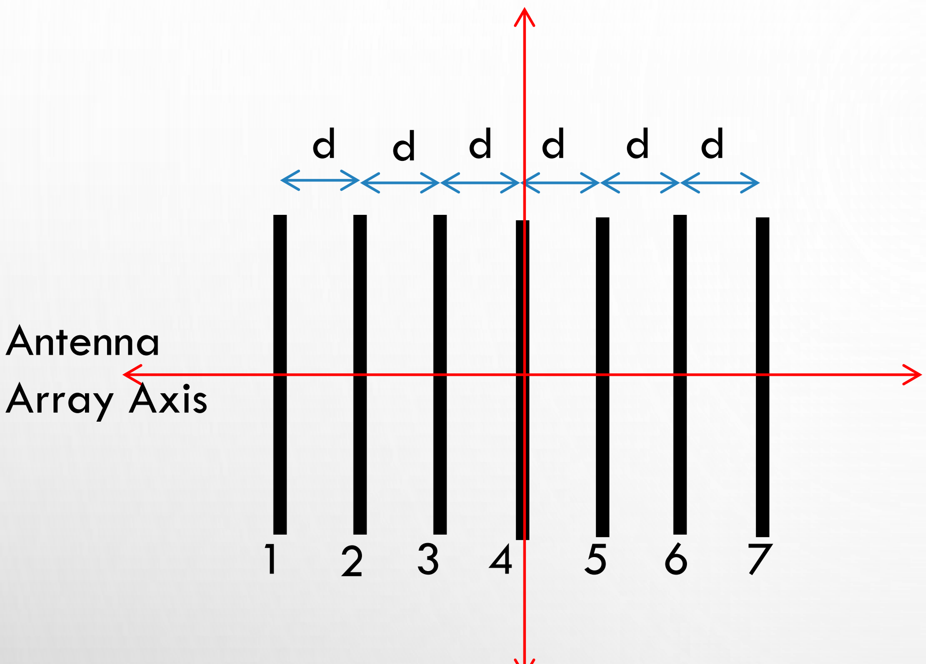
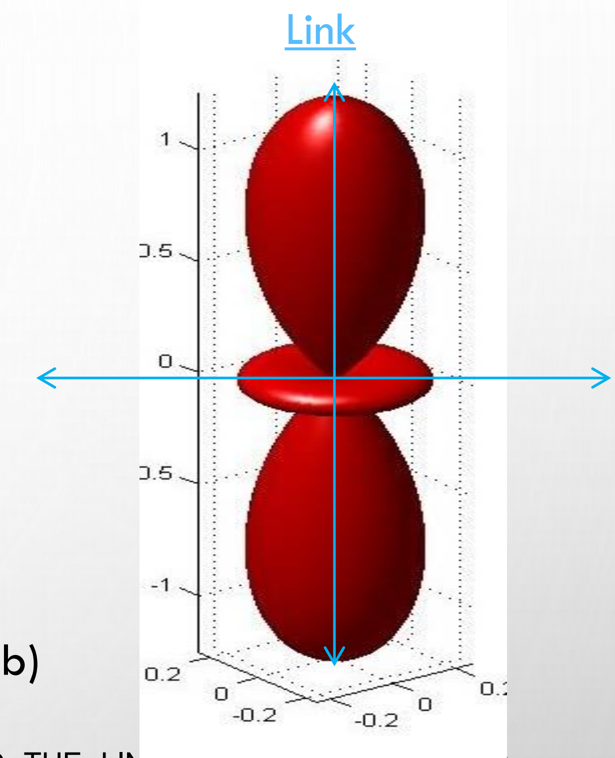


Fig 1(a)

Fig 1(b)



- THE RADIATION PATTERN OF BROADSIDE ARRAY IS PERPENDICULAR TO THE LINE OF ARRAY AXIS AND BIDIRECTIONAL.

- THE BROADSIDE ARRAY IS BIDIRECTIONAL WHICH RADIATES EQUALLY IN BOTH DIRECTION OF MAXIMUM RADIATION.

- THE BROADSIDE ARRAY MAY BE DEFINED AS “IT IS AN ARRANGEMENT IN WHICH THE PRINCIPLE DIRECTION IS PERPENDICULAR TO THE ARRAY AXIS AND ALSO THE PLANE CONTAINING THE ARRAY ELEMENT.

END-FIRE ARRAYS

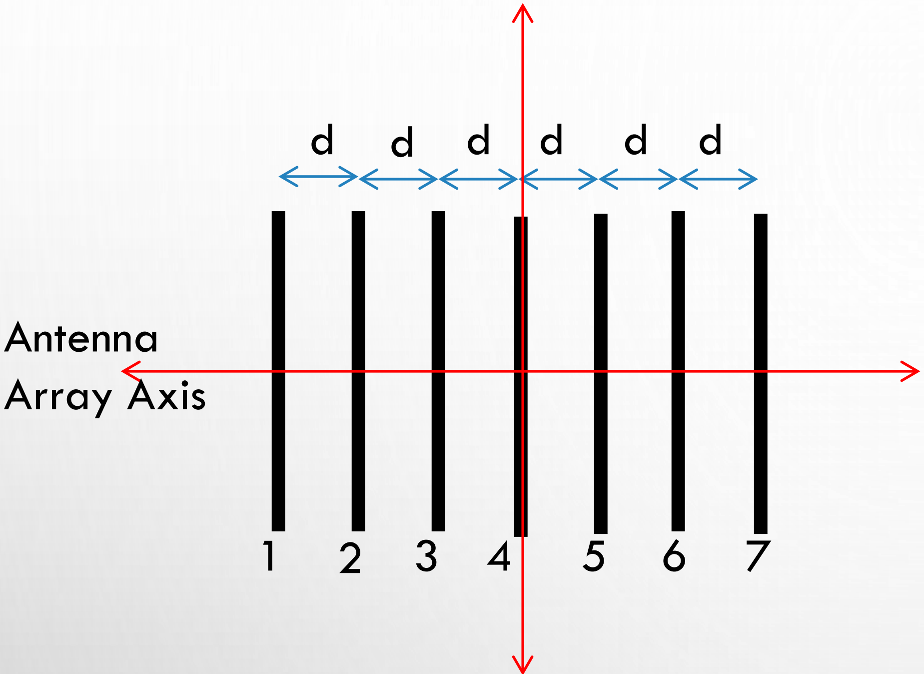


Fig 2(a)

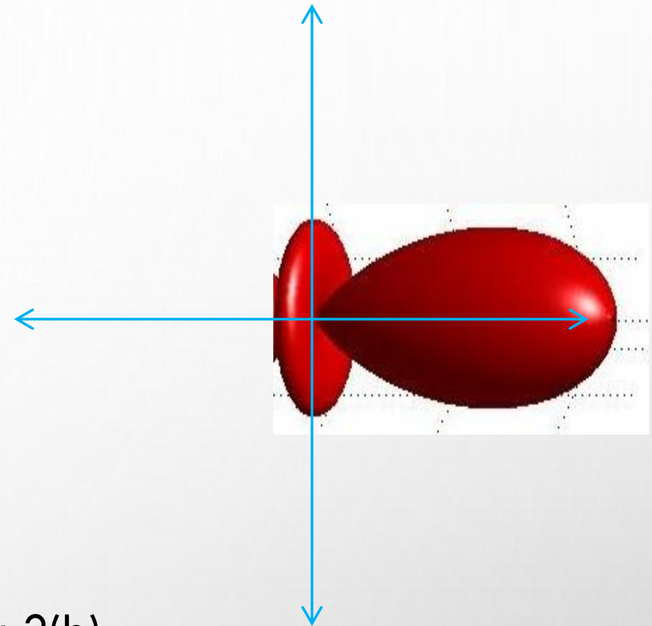


Fig 2(b)

- INSTEAD OF HAVING THE MAXIMUM RADIATION NORMAL TO THE AXIS OF THE ARRAYS IT MAY BE DESIRABLE TO DIRECT IT ALONG THE AXIS OF THE ARRAY.

- HOWEVER, THE END FIRE ARRAYS IS SAME AS THE BROADSIDE ARRAY BUT THE INDIVIDUAL ELEMENT ARE FED IN OUT OF PHASE(I.E. 180°)

ARRAYS OF TWO POINT SOURCE

- IF TWO ISOTROPIC POINT SOURCES ARE SEPARATED BY A DISTANCE ***D*** AND HAVING THE SAME POLARIZATION, SO THIS SITUATION IS A SIMPLEST FORM OF ARRAYS.

- IN THE THEORY OF ARRAYS, THE SUPERPOSITION OR ADDITION OF FIELD ARE DEPEND UPON THE PHASES OF THE POINT SOURCES.

1. EQUAL AMPLITUDE AND PHASE.
2. EQUAL AMPLITUDE AND OPPOSITE PHASE.
3. UNEQUAL AMPLITUDE AND ANY PHASE.

CASE-1:-Arrays of two Point Source with Equal amplitude and phase

- TWO ISOTROPIC SOURCES SYMMETRICALLY SITUATED W.R.T THE ORIGIN IS SHOWN IN BELOW FIGURE.

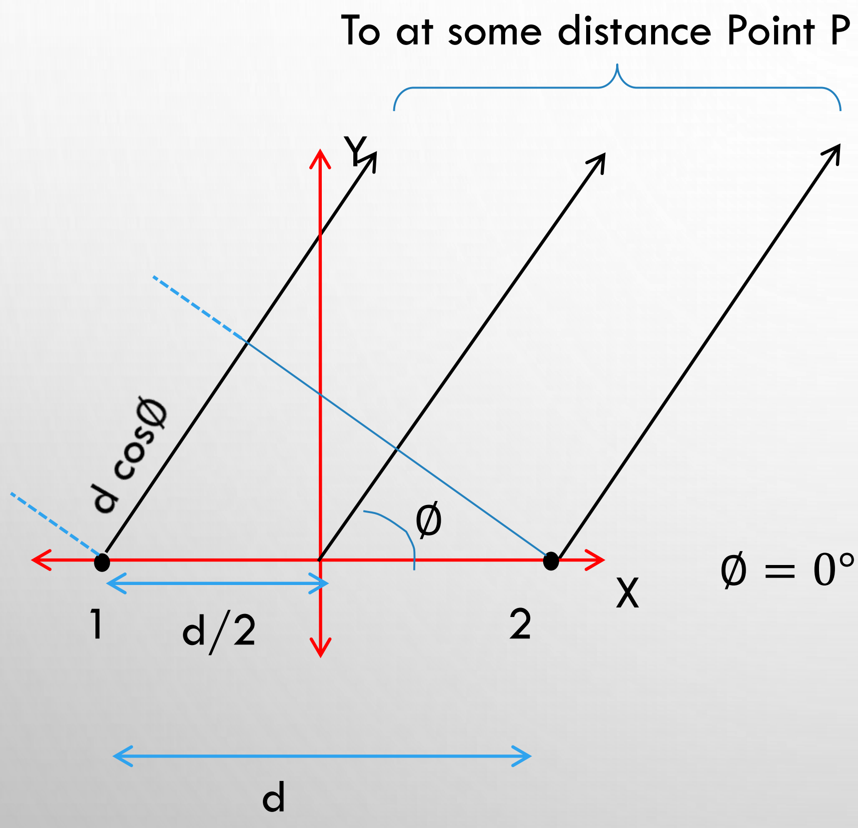


Fig 5(a)

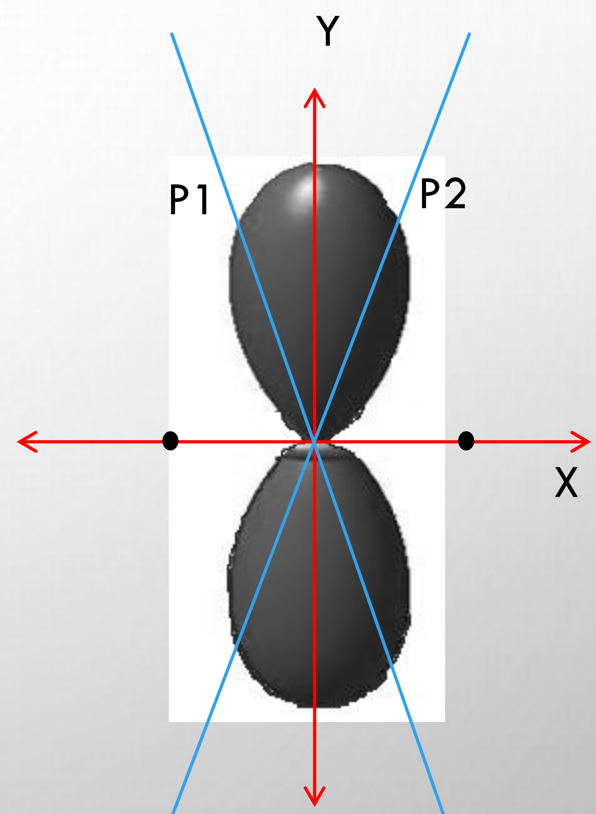


Fig 5(b)

- THE RADIATION FROM POINT SOURCE 2 WILL REACH EARLIER AT SOME DISTANCE POINT P THAN THAT FROM POINT SOURCE 1 BECAUSE OF THE PATH DIFFERENCE.
- THE EXTRA DISTANCE IS TRAVELLED BY THE RADIATED WAVE FROM POINT SOURCE 1 THAN THAT BY THE WAVE RADIATED FROM POINT SOURCE 2.

Hence path difference is given by,

$$\boxed{\text{Path difference} = d \cos \phi} \qquad \ldots(1)$$

The path difference can be expressed interms of wavelength as,

$$\boxed{\text{Path difference} = \frac{d \cos\phi}{\lambda}} \qquad \ldots(2)$$

Hence the phase angle ψ is given by,

$$\text{Phase angle} = \psi = 2\pi\,(\text{Path difference})$$

$$\therefore \qquad \psi = 2\pi\left(\frac{d\cos\phi}{\lambda}\right)$$

$$\therefore \qquad \boxed{\psi = \frac{2\pi}{\lambda} d\cos\phi \ \text{ rad}} \qquad \ldots(3)$$

But phase shift $= \beta = \frac{2\pi}{\lambda}$, thus equation (3) becomes,

$$\therefore \qquad \boxed{\psi = \beta d\cos\phi \ \text{ rad}} \qquad \ldots(4)$$

- LET E_1 BE THE FAR FIELD AT A DISTANT POINT P DUE TO POINT SOURCE 1. SIMILARLY LET E_2 BE THE FAR FIELD AT POINT P DUE TO POINT SOURCE 2.

- THEN THE TOTAL FIELD AT POINT P BE THE ADDITION OF THE TWO FIELD COMPONENT DUE TO THE POINT SOURCE 1 AND 2.

- IF THE PHASE ANGLE BETWEEN THE TWO FIELD IS $\boldsymbol{\psi = \beta \mathrm{d} \cos\phi}$ THEN THE FAR FIELD COMPONENT AT POINT P DUE TO POINT SOURCE 1 IS GIVEN BY

$$\mathrm{E}_1 = \mathrm{E}_0\, \mathrm{E}^{-\frac{j\psi}{2}} \qquad \ldots\ (5)$$

- SIMILARLY THE FAR FIELD COMPONENT AT POINT P DUE TO THE POINT SOURCE 2 IS GIVEN BY,

$$\mathrm{E}_2 = \mathrm{E}_0\, \mathrm{E}^{+\frac{j\psi}{2}} \qquad \ldots\ (6)$$

- NOTE THAT THE AMPLITUDE OF BOTH THE FIELD COMPONENTS IN E_0 AS CURRENTS SAME AND THE POINT SOURCES ARE IDENTICAL.

The total field at point P is given by,

$$E_T = E_1 + E_2 = E_0 \cdot e^{-j\frac{\psi}{2}} + E_0 \cdot e^{j\frac{\psi}{2}}$$

$$\therefore \quad E_T = E_0\left(e^{-j\frac{\psi}{2}} + e^{j\frac{\psi}{2}}\right)$$

Rearranging the terms on R.H.S., we get,

$$\therefore \quad E_T = 2E_0\left(\frac{e^{j\frac{\psi}{2}} + e^{-j\frac{\psi}{2}}}{2}\right) \qquad \ldots(7)$$

By trigonometric identity, $\frac{e^{j\theta} + e^{-j\theta}}{2} = \cos\theta$.

Hence equation (7) can be written as,

$$E_T = 2E_0 \cos\left(\frac{\psi}{2}\right) \qquad \ldots(8)$$

Substituting value of ψ from equation (4), we get,

$$\therefore \quad \boxed{E_T = 2E_0 \cos\left(\frac{\beta d \cos\phi}{2}\right)} \qquad \ldots(9)$$

The array factor is the ratio of the magnitude of the resultant field to the magnitude of the maximum field.

$$\therefore \qquad \text{A.F.} = \frac{|E_T|}{|E_{max}|}$$

But maximum field is $E_{max} = 2\,E_0$.

$$\therefore \qquad \boxed{\text{A.F.} = \frac{|E_T|}{|2\,E_0|} = \cos\left(\pi \frac{d}{\lambda} \cos\phi\right)}$$

The array factor represents the relative value of the field as a function of ϕ. It defines the radiation pattern in a plane containing the line of the array.

Maxima direction

From equation (9), the total field is maximum when $\cos\left(\frac{\beta d \cos\phi}{2}\right)$ is maximum. As we know, the variation of cosine of a angle is ± 1. Hence the condition for maxima is given by,

$$\boxed{\cos\left(\frac{\beta d \cos\phi}{2}\right) = \pm 1}$$

Let spacing between the two point sources be $\frac{\lambda}{2}$. Then we can write,

$$\cos\left[\frac{\beta(\lambda/2)\cos\phi}{2}\right] = \pm 1 \qquad \ldots(10)$$

$$\text{i.e. } \cos\left[\frac{\frac{2\pi}{\lambda}\cdot\frac{\lambda}{2}\cos\phi}{2}\right] = \pm 1 \qquad \ldots \because \beta = \frac{2\pi}{\lambda}$$

$$\text{i.e. } \cos\left(\frac{\pi}{2}\cos\phi\right) = \pm 1$$

$$\text{i.e. } \frac{\pi}{2}\cos\phi_{max} = \cos^{-1}(\pm 1) = \pm n\pi, \text{ where } n = 0, 1, 2, \ldots\ldots$$

If $n = 0$, then

$$\frac{\pi}{2}\cos\phi_{max} = 0$$

$$\text{i.e. } \cos\phi_{max} = 0$$

$$\text{i.e. } \boxed{\phi_{max} = 90^\circ \text{ or } 270^\circ} \qquad \ldots(11)$$

Minima direction

Again from equation (9), total field strength is minimum when $\cos\left(\frac{\beta d\cos\phi}{2}\right)$ is minimum i.e. 0 as cosine of angle has minimum value 0. Hence the condition for minima is given by,

$$\therefore \quad \boxed{\cos\left(\frac{\beta d\cos\phi}{2}\right) = 0} \qquad \ldots(12)$$

Again assuming $d = \frac{\lambda}{2}$ and $\beta = \frac{2\pi}{\lambda}$, we can write

$$\cos\left(\frac{\pi}{2}\cos\phi_{min}\right) = 0$$

$$\therefore \quad \frac{\pi}{2}\cos\phi_{min} = \cos^{-1} 0 = \pm(2n+1)\frac{\pi}{2}, \text{ where } n = 0, 1, 2, \ldots\ldots\ldots$$

If $n = 0$, then,

$$\frac{\pi}{2}\cos\phi_{min} = \pm\frac{\pi}{2}$$

$$\text{i.e. } \cos\phi_{min} = \pm 1$$

$$\text{i.e.} \quad \boxed{\phi_{min} = 0° \text{ or } 180°} \qquad \ldots(13)$$

Half power point directions

When the power is half, the voltage or current is $\frac{1}{\sqrt{2}}$ times the maximum value.

Hence the condition for half power point is given by,

$$\boxed{\cos\left(\frac{\beta d \cos\phi}{2}\right) = \pm \frac{1}{\sqrt{2}}} \quad \text{...(14)}$$

Let $d = \frac{\lambda}{2}$ and $\beta = \frac{2\pi}{\lambda}$, then we can write,

$$\cos\left(\frac{\pi}{2}\cos\phi\right) = \pm \frac{1}{\sqrt{2}}$$

$$\text{i.e. } \frac{\pi}{2}\cos\phi = \cos^{-1}\left(\pm\frac{1}{\sqrt{2}}\right) = \pm(2n+1)\frac{\pi}{4}, \text{ where } n = 0, 1, 2, ...$$

If $n = 0$, then

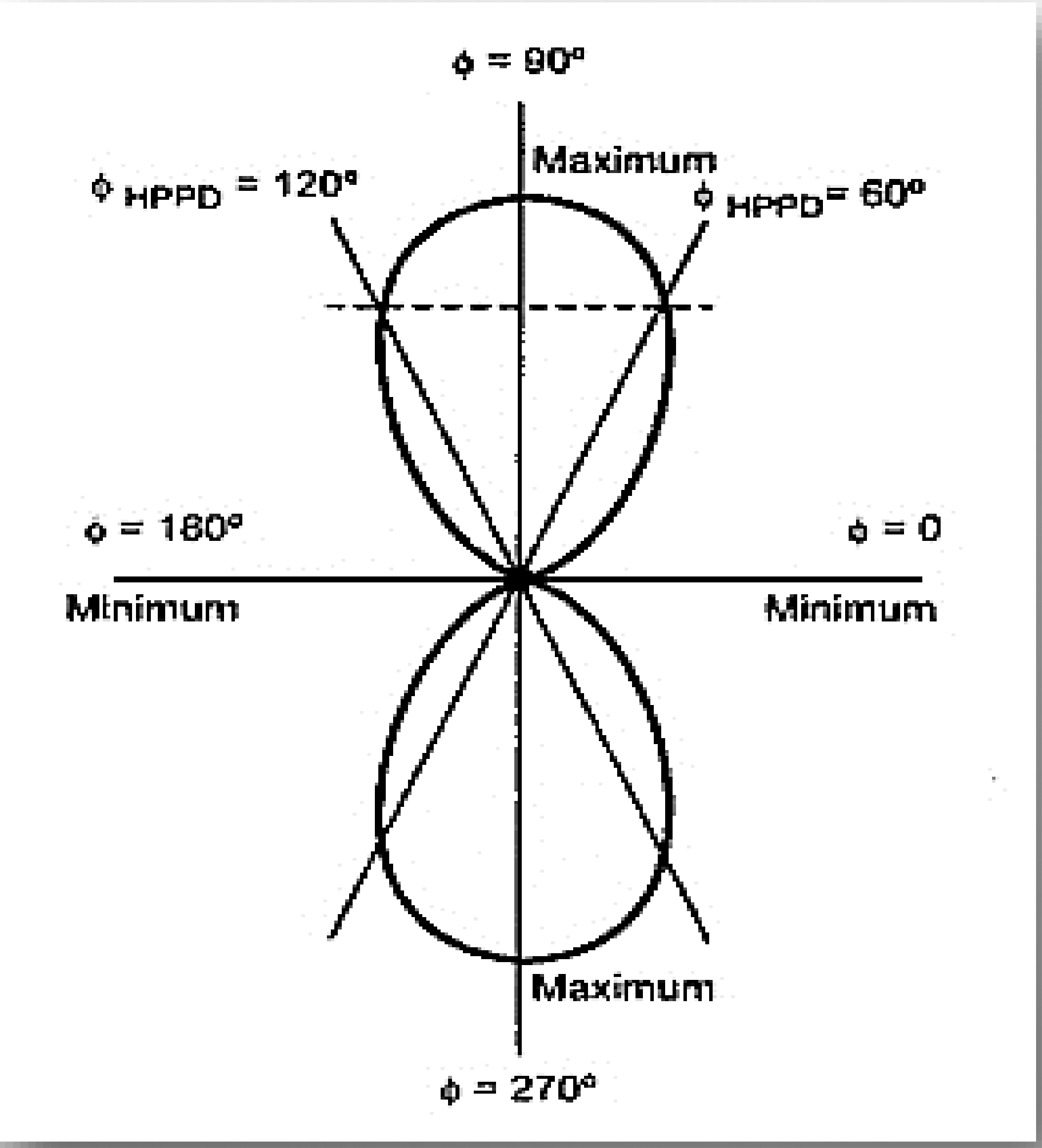
$$\frac{\pi}{2}\cos\phi_{HPPD} = \pm\frac{\pi}{4}$$

$$\text{i.e. } \cos\phi_{HPPD} = \pm\frac{1}{2}$$

$$\text{i.e. } \phi_{HPPD} = \cos^{-1}\left(\pm\frac{1}{2}\right)$$

$$\therefore \quad \boxed{\phi_{HPPD} = 60^\circ \text{ or } 120^\circ} \quad \text{...(15)}$$

The field pattern drawn with E_T against ϕ for $d = \frac{\lambda}{2}$, then the pattern is bidirectional as shown in the Fig. 4.6. **The field pattern obtained is bidirectional and it is a figure of eight (8).** If this patterns is rotated by 360° about axis, it will represent three dimensional **doughnut shaped space pattern. This is the simplest type of broadside array of two point sources and it is called Broadside couplet** as two radiations of point sources are in phase.



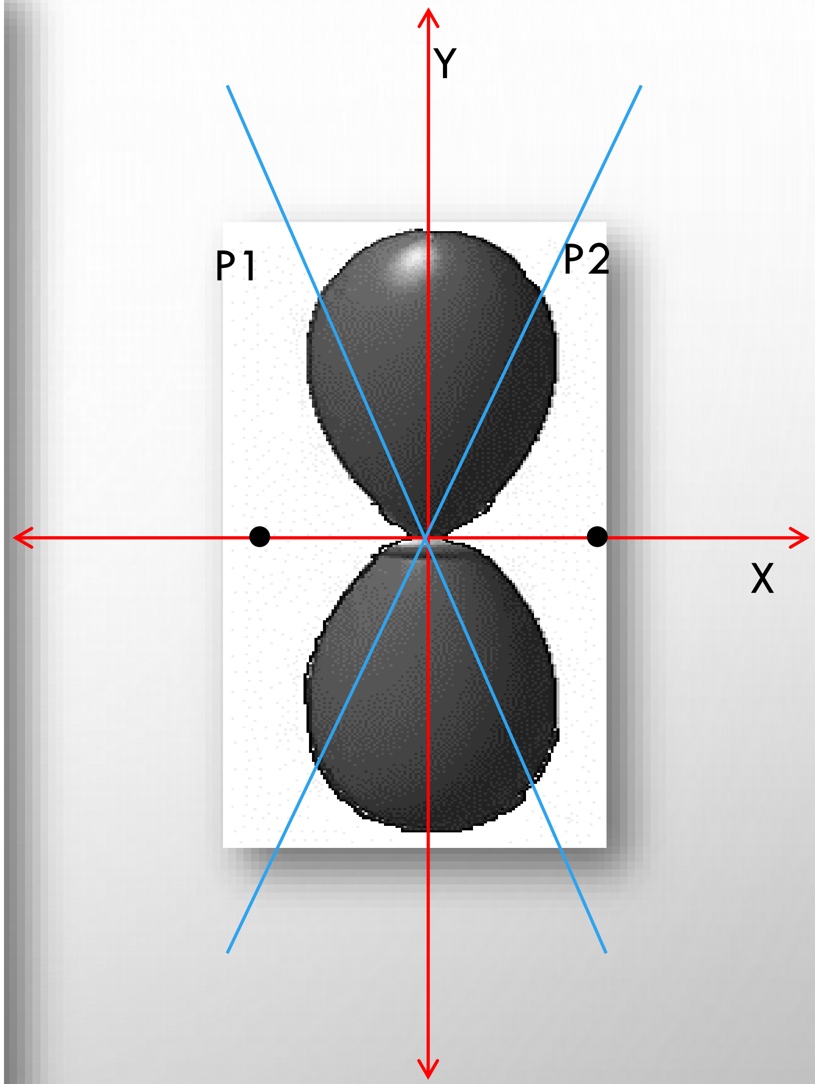


Fig 5(c) Field pattern for two point source with spacing d=$\lambda/2$ and fed with currents equal in magnitude and phase.

CASE 2:- Arrays of two Point Source with Equal amplitude and opposite in phase

- CONSIDER TWO POINT SOURCES SEPARATED BY DISTANCE **D** AND SUPPLIED WITH CURRENTS EQUAL IN MAGNITUDE BUT OPPOSITE IN PHASE.

- ALL CONDITIONS ARE EXACTLY SAME EXCEPT THE PHASE OF THE CURRENTS IS OPPOSITE I.E. 180°. WITH THIS CONDITION, THE TOTAL FIELD AT FAR POINT P IS GIVEN BY,

$$\mathrm{E_T} = (-\mathrm{E_1}) + \mathrm{E_2} \qquad \ldots(1)$$

- ASSUMING EQUAL MAGNITUDE OF CURRENTS, THE FIELD AT POINT P DUE TO THE POINT SOURCE 1 AND 2 CAN BE WRITTEN AS,

$$\mathrm{E_1} = \mathrm{E_0}\,\mathrm{E}^{-\frac{j\psi}{2}} \qquad \ldots(2)$$

AND $\mathrm{E_2} = \mathrm{E_0}\,\mathrm{E}^{+\frac{j\psi}{2}}$ ...(3)

Substituting values of E_1 and E_2 in equation (1), we get,

$$E_T = -E_0 \cdot e^{-j\frac{\psi}{2}} + E_0 \cdot e^{j\frac{\psi}{2}}$$

$$\therefore \qquad E_T = E_0\left(-e^{-j\frac{\psi}{2}} + e^{j\frac{\psi}{2}}\right)$$

Rearranging the terms in above equation, we get,

$$\therefore \qquad E_T = (j2)\, E_0\left(\frac{e^{j\frac{\psi}{2}} - e^{-j\frac{\psi}{2}}}{j2}\right) \qquad \ldots(4)$$

By trigonometric identity, $\frac{e^{j\theta} - e^{-j\theta}}{2} = \sin\frac{\theta}{2}$.

Hence equation (4) can be written as,

$$E_T = j2E_0 \sin\left(\frac{\psi}{2}\right) \qquad \ldots(5)$$

Now as the condition for two point sources with currents in phase and out of phase is exactly same, the phase angle can be written as previous case,

$$\text{Phase angle} = \psi = \beta d \cos\phi \qquad \ldots(6)$$

Substituting value of phase angle in equation (5), we get,

$$\boxed{E_T = j(2E_0)\sin\left(\frac{\beta d \cos\phi}{2}\right)} \qquad \ldots(7)$$

Maxima direction

From equation (7), the total field is maximum when $\sin\left(\frac{\beta d\cos\phi}{2}\right)$ is maximum i.e. ± 1 as the maximum value of sine of angle is ± 1. Hence condition for maxima is given by,

$$\boxed{\sin\left(\frac{\beta d\cos\phi}{2}\right) = \pm 1} \qquad \ldots(8)$$

Let the spacing between two isotropic point sources be equal to $\frac{\lambda}{2}$, i.e. $d = \frac{\lambda}{2}$. Substituting $d = \frac{\lambda}{2}$ and $\beta = \frac{2\pi}{\lambda}$ in equation (8), we get,

$$\sin\left(\frac{\pi}{2}\cos\phi\right) = \pm 1$$

i.e. $$\frac{\pi}{2}\cos\phi = \pm(2n+1)\frac{\pi}{2}, \text{ where } n = 0, 1, 2, \ldots\ldots$$

If $n = 0$, then

$$\frac{\pi}{2}\cos\phi_{max} = \pm\frac{\pi}{2}$$

i.e. $$\cos\phi_{max} = \pm 1$$

i.e. $$\boxed{\phi_{max} = 0° \text{ and } 180°} \qquad \ldots(9)$$

Minima direction

Again from equation (7), total field strength is minimum when $\sin\left(\frac{\beta d \cos\phi}{2}\right)$ is minimum i.e. 0.

Hence the condition for minima is given by,

$$\boxed{\sin\left(\frac{\beta d \cos\phi}{2}\right) = 0} \qquad \ldots(10)$$

Assuming $d = \frac{\lambda}{2}$ and $\beta = \frac{2\pi}{\lambda}$ in equation (10), we get,

$$\sin\left(\frac{\pi}{2}\cos\phi\right) = 0$$

$$\text{i.e.} \quad \frac{\pi}{2}\cos\phi = \pm\, n\,\pi, \quad \text{where } n = 0, 1, 2, \ldots\ldots$$

If $n = 0$, then

$$\frac{\pi}{2}\cos\phi_{min} = 0$$

$$\text{i.e.} \quad \cos\phi_{min} = 0$$

$$\text{i.e.} \quad \boxed{\phi_{min} = +90° \text{ or } -90°} \qquad \ldots(11)$$

Half Power Point Direction (HPPD)

When the power is half of maximum value, the voltage or current equals to $\frac{1}{\sqrt{2}}$ times the respective maximum value. Hence the condition for the half power point can be obtained from equation (7) as,

$$\boxed{\sin\left(\frac{\beta d \cos\phi}{2}\right) = \pm\frac{1}{\sqrt{2}}} \qquad \text{...(12)}$$

Let $d = \frac{\lambda}{2}$ and $\beta = \frac{2\pi}{\lambda}$, we can write,

$$\sin\left(\frac{\pi}{2}\cos\phi\right) = \pm\frac{1}{\sqrt{2}}$$

$$\text{i.e. } \frac{\pi}{2}\cos\phi = \pm(2n+1)\frac{\pi}{4}, \quad \text{where } n = 0, 1, 2.$$

If $n = 0$, we can write,

$$\frac{\pi}{2}\cos\phi_{HPPD} = \pm\frac{\pi}{4}$$

$$\text{i.e. } \cos\phi_{HPPD} = \pm\frac{1}{2}$$

$$\therefore \quad \boxed{\phi_{HPPD} = 60° \text{ or } 120°} \qquad \text{...(13)}$$

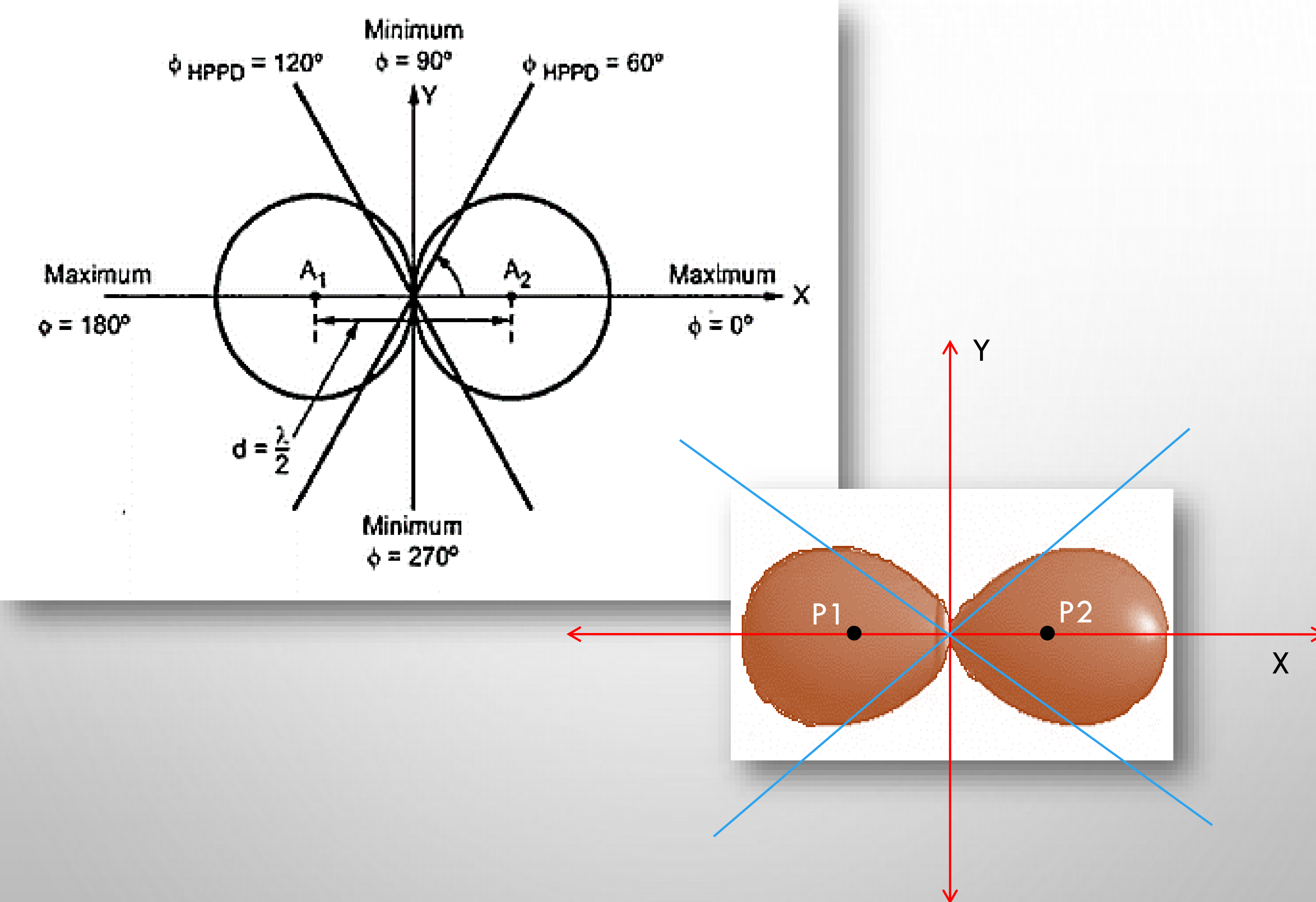


Fig 6 Field pattern for two point source with spacing d=$\lambda/2$ and fed with currents equal in magnitude but out of phase by 180.

CASE 3 (a) :- Arrays of two Point Source with Unequal amplitude and any phase

Assume that the two point sources are separated by distance d and supplied with currents which are different in magnitudes and with any phase difference say α. Consider that source 1 is assumed to be reference for phase and amplitude of the fields E_1 and E_2, which are due to source 1 and source 2 respectively at the distant point P. Let us assume that E_1 is greater than E_2 in magnitude as shown in the vector diagram in Fig. 4.8.

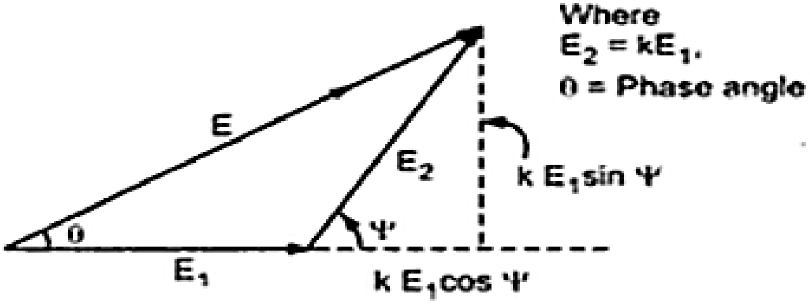


Fig. 4.8 Vector diagram of fields E_1 and E_2

Now the total phase difference between the radiations by the two point sources at any far point P is given by,

$$\boxed{\Psi = \frac{2\pi}{\lambda}\cos\phi + \alpha} \quad \dots (1)$$

where α is the phase angle with which current I_2 leads current I_1. Now if $\alpha = 0$, then the condition is similar to the two point sources with currents equal in magnitude and phase. Similarly if $\alpha = 180°$, then the condition is similar to the two point source with currents equal in magnitude but opposite in phase. Assume value of phase difference α as $0 < \alpha < 180°$. Then the resultant field at point P is given by,

$$E_T = E_1 e^{j0} + E_2 e^{j\Psi} \quad \text{...(source 1 is assumed to be}$$

$$\therefore \quad E_T = E_1 + E_2 e^{j\Psi} \quad \text{reference hence phase angle is 0)}$$

$$\therefore \quad E_T = E_1\left(1 + \frac{E_2}{E_1} e^{j\Psi}\right)$$

Let $\frac{E_2}{E_1} = k$ ...(2)

Note that $E_1 > E_2$, the value of k is less than unity. Moreover the value of k is given by, $0 \leq k \leq 1$.

$$\therefore \quad \boxed{E_T = E_1 [1 + k(\cos\psi + j\sin\psi)]} \quad \text{... (3)}$$

The magnitude of the resultant field at point P is given by,

$$|E_T| = |E_1 [1 + k\cos\psi + jk\sin\psi]|$$

$$\therefore \quad \boxed{|E_T| = E_1 \sqrt{(1 + k\cos\psi)^2 + (k\sin\psi)^2}} \quad \text{... (4)}$$

The phase angle between two fields at the far point P is given by,

$$\therefore \quad \boxed{\theta = \tan^{-1} \frac{k\sin\psi}{1 + k\cos\psi}} \quad \text{... (5)}$$

CASE 3 (b) :- Arrays of two Point Sources with equal amplitude and Quadrature phase

- Let two point sources be placed at d distance . Let source 1 be retarded by 45 deg. and source 2 advanced by 45 deg.

$\psi = \beta d\ cos(\phi) + \alpha$ ……………………1

- So for a quadrature phase $\boldsymbol{\alpha = \frac{\pi}{4}}$.

- Then total field in the direction Ø at a large distance r is given by

$$E = E_o\ e^{(j\psi)} + E_o\ e^{(-j\psi)}$$ …………………… 2

$$\boxed{E = E_0 \exp\left[+j\left(\frac{d_r \cos\phi}{2} + \frac{\pi}{4}\right)\right] + E_0 \exp\left[-j\left(\frac{d_r \cos\phi}{2} + \frac{\pi}{4}\right)\right]}$$ …………………… 3

From (Eq3), we have $$E = 2E_0 \cos\left(\frac{\pi}{4} + \frac{d_r}{2}\cos\phi\right)$$

Let $2E_0 = 1$ and $d = \lambda/2$, then (Eq3) will be

$$E = \cos\left(\frac{\pi}{4} + \frac{\pi}{2}\cos\phi\right) \qquad \text{Eq 4}$$

The directions ϕ_m of maximum field are found by equating the argument of equation Eq 4 to $k\pi$ just as

$$\frac{\pi}{4} + \frac{\pi}{2}\cos\phi_m = k\pi, \qquad k = 0, 1, 2\ldots\ldots\ldots \qquad \ldots (\text{Eq 5})$$

for $k = 0$, $\frac{\pi}{2}\cos\phi_m = -\frac{\pi}{4}$

and $\phi_m = 120°$ and $240°$

The complete field pattern corresponding to equation Eq 4 is shown in Fig

It is observed that most of the radiation is in the second and third quadrants, and the field in the direction $\phi = 0°$ is the same as in the direction $\phi = 180°$.

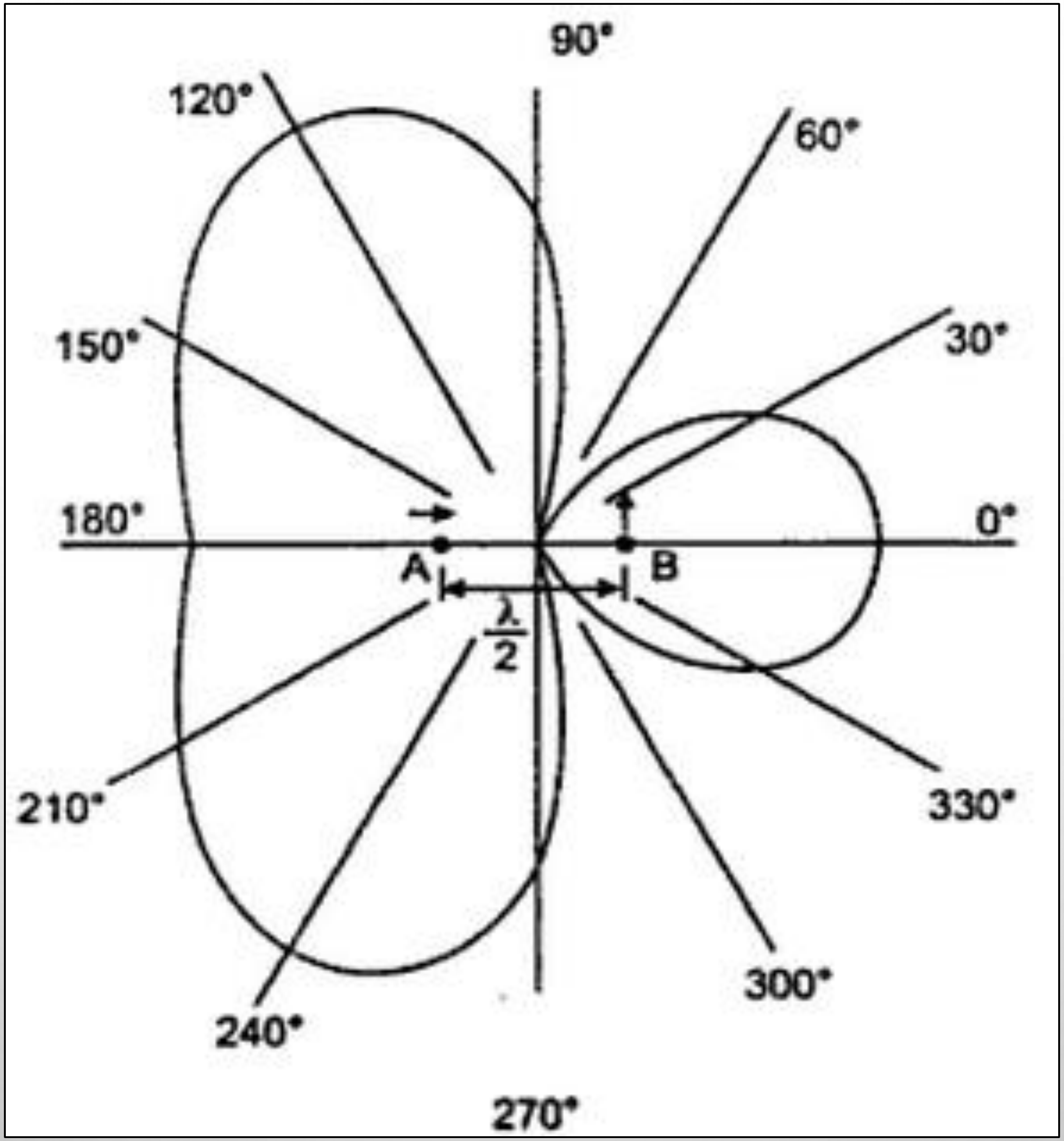


Fig 7 Field pattern for two point source with spacing $d=\lambda/2$ and fed with currents equal in magnitude and in phase quadrature.

- **Now let the spacing between the sources is reduced to $\boldsymbol{\lambda/4}$**

$$E = 2E_0 \cos\left(\frac{\pi}{4} + \frac{\pi}{4}\cos\phi\right)$$

- As Maxima is at $\cos\left(\frac{\pi}{4} + \frac{\pi}{4}\cos\phi\right) = 1$

So,

$$\frac{\pi}{4} + \frac{\pi}{4}\cos\phi = n\pi$$

For n=0 $\cos\phi_{max} = -1$

$$\boxed{\phi_{max} = 180°}$$

- As Minima is at $\cos\left(\frac{\pi}{4} + \frac{\pi}{4}\cos\phi\right) = 0$

So,

$$\frac{\pi}{4} + \frac{\pi}{4}\cos\phi = (2n+1)\frac{\pi}{2}$$

For n=0 $\cos\phi_{max} = 1$

$$\boxed{\phi_{max} = 0°}$$

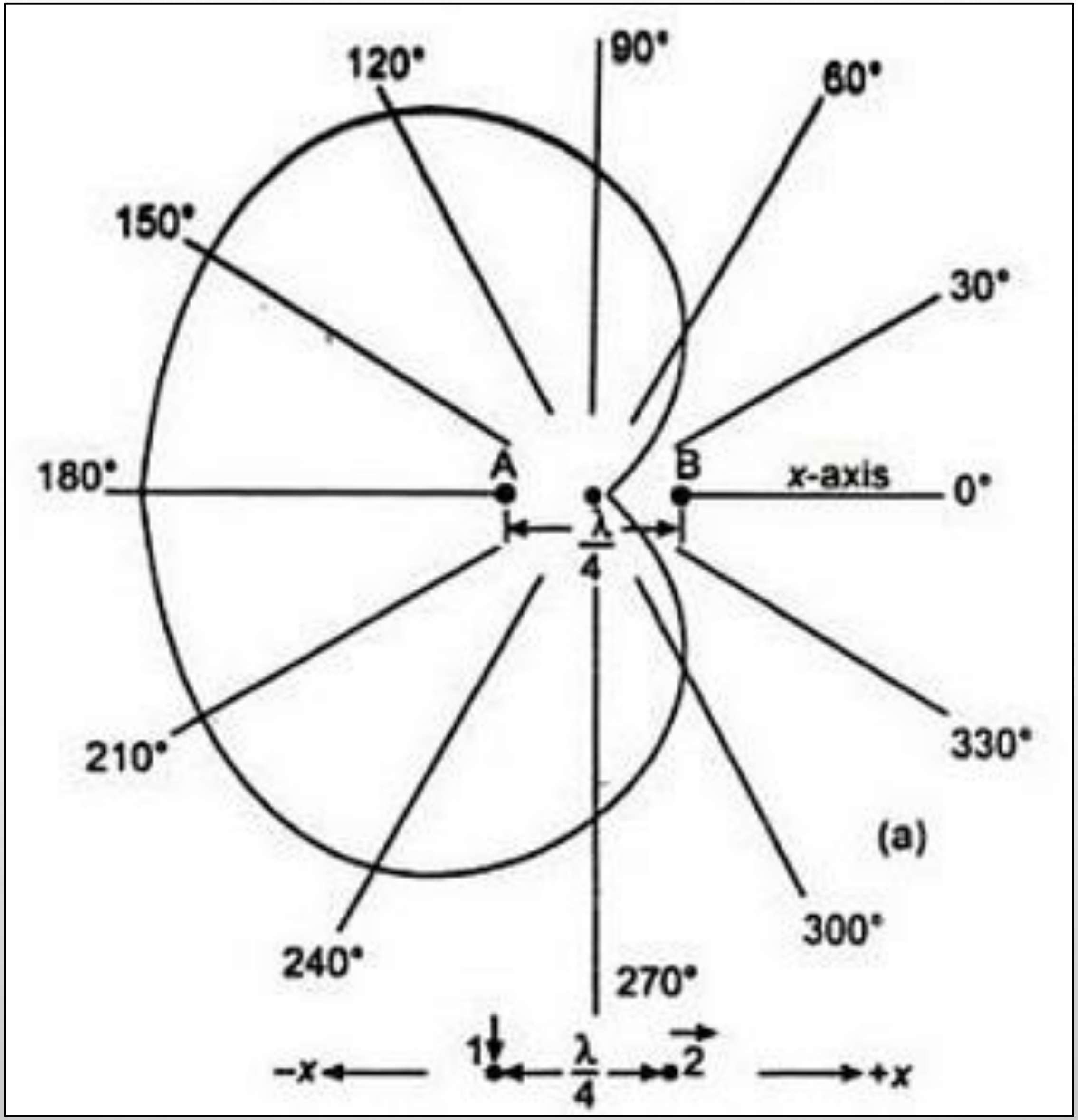


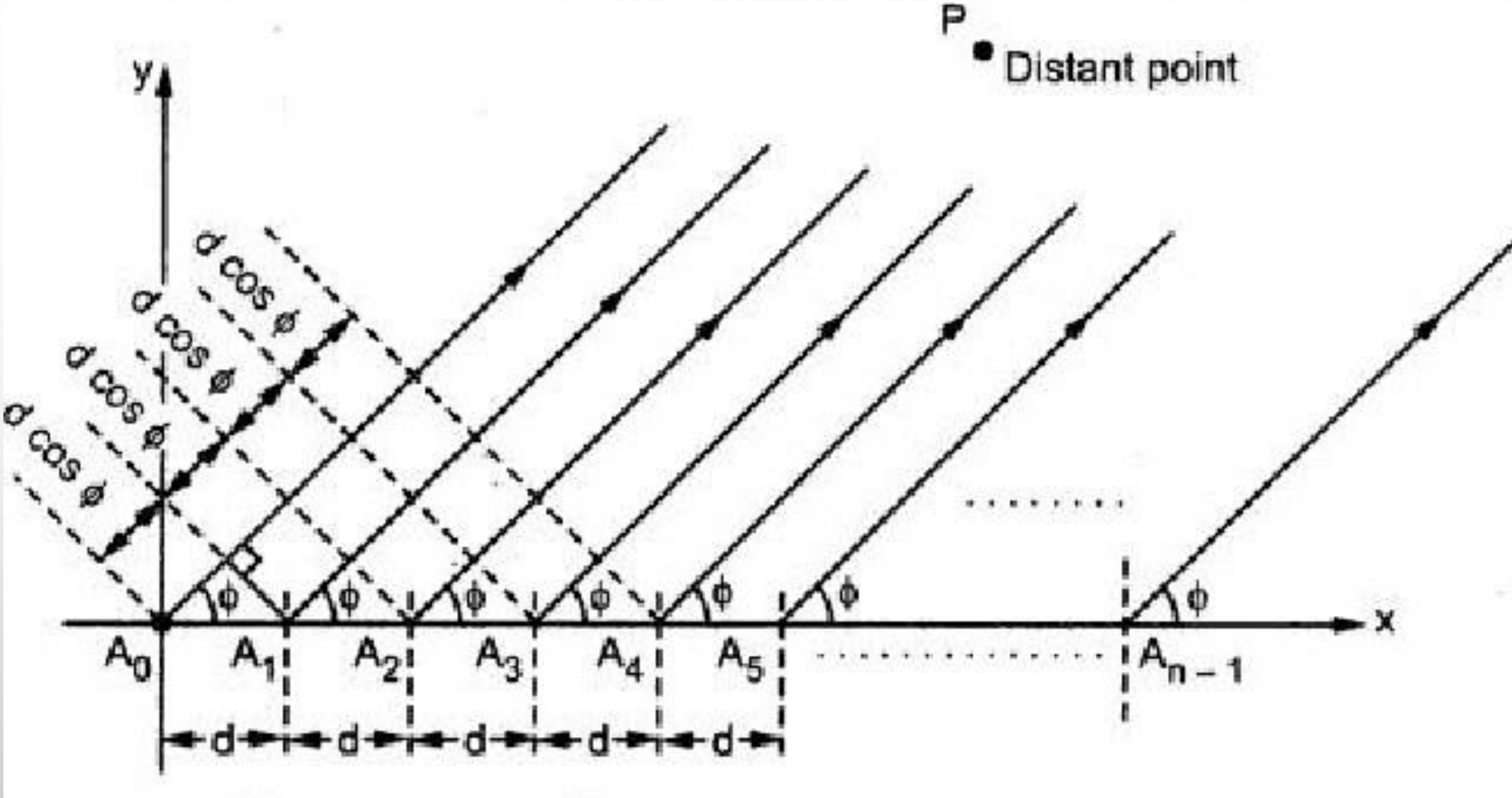
Fig 8 **Field pattern for two point source with spacing $d = \lambda/4$ and fed with currents equal in magnitude and in phase quadrature.**

N ELEMENT UNIFORM LINEAR ARRAYS

- AT HIGHER FREQUENCIES, FOR POINT TO POINT COMMUNICATIONS IT IS NECESSARY TO HAVE A PATTERN WITH SINGLE BEAM RADIATION, SUCH HIGHLY DIRECTIVE SINGLE BEAM PATTERN CAN BE OBTAINED BY INCREASING THE POINT SOURCES IN THE ARROW FROM 2 TO N SAY.

- AN ARRAY OF N ELEMENTS IS SAID TO BE LINEAR ARRAY IF ALL THE INDIVIDUAL ELEMENTS ARE SPACED EQUALLY ALONG A LINE, AN ARRAY IS SAID TO BE UNIFORM ARRAY IF THE ELEMENTS IN THE ARRAY ARE FED WITH CURRENTS WITH EQUAL MAGNITUDES AND WITH UNIFORM PROGRESSIVE PHASE SHIFT ALONG THE LINE.

- CONSIDER A GENERAL N ELEMENT LINEAR AND UNIFORM ARRAY WITH ALL THE INDIVIDUAL ELEMENTS SPACED EQUALLY AT DISTANCE D FROM EACH OTHER AND ALL ELEMENTS ARE FED WITH CURRENTS EQUAL IN MAGNITUDE AND UNIFORM PROGRESSIVE PHASE SHIFT ALONG LINE AS SHOWN IN THE FIG 4.9.



Fig, 4.9 Uniform, linear array of n elements

The total resultant field at the distant point P is obtained by adding the fields due to n individual sources vectorically. Hence we can write,

$$E_T = E_0 \cdot e^{j0} + E_0 e^{j\psi} + E_0 e^{2j\psi} + \dots + E_0 e^{j(n-1)\psi}$$

$$\boxed{E_T = E_0 [1 + e^{j\psi} + e^{j2\psi} + \ldots + e^{j(n-1)\psi}]} \quad \ldots (1)$$

Note that $\psi = (\beta\, d \cos\phi + \alpha)$ indicates the total phase difference of the fields from adjacent sources calculated at point P. Simlarly α is the progressive phase shift between two adjacent point sources. The value of α may lie between 0° and 180°. If α **= 0°**, we get **n element uniform linear broadside array**. If α **= 180°**, we get **n element uniform linear end fire array**.

Multiplying equation (1) by $e^{j\psi}$, we get,

$$E_T\, e^{j\psi} = E_0 [e^{j\psi} + e^{j2\psi} + e^{j3\psi} + \ldots + e^{jn\psi}] \quad \ldots (2)$$

Subtracting equation (2) from (1), we get,

$$E_T - E_T\, e^{j\psi} = E_0 \left\{[1 + e^{j\psi} + e^{j2\psi} + \ldots + e^{j(n-1)\psi}] - [e^{j\psi} + e^{j2\psi} + \ldots + e^{jn\psi}]\right\}$$

$$E_T (1 - e^{j\psi}) = E_0 (1 - e^{jn\psi})$$

$$\therefore \quad \boxed{E_T = E_0 \left[\frac{1 - e^{jn\psi}}{1 - e^{j\psi}}\right]} \quad \ldots (3)$$

Simplying mathematically, we get

$$E_T = E_0\left[\frac{e^{j\frac{n\psi}{2}}\left(e^{-j\frac{n\psi}{2}} - e^{j\frac{n\psi}{2}}\right)}{e^{j\frac{\psi}{2}}\left(e^{-j\frac{\psi}{2}} - e^{j\frac{\psi}{2}}\right)}\right]$$

According to trigonomeric identity,

$$e^{-j\theta} - e^{j\theta} = -2j\sin\theta,$$

The resultant field is given by,

$$E_T = E_0\left[\frac{\left(-j2\sin\frac{n\psi}{2}\right)e^{j\frac{n\psi}{2}}}{\left(-j2\sin\frac{\psi}{2}\right)e^{j\frac{\psi}{2}}}\right]$$

$$\therefore \quad \boxed{E_T = E_0\left[\frac{\sin n\frac{\psi}{2}}{\sin\frac{\psi}{2}}\right]e^{j\left(\frac{n-1}{2}\right)\psi}} \qquad \ldots (4)$$

This equation (4) indicates the resultant field due to n element array at distant point P.

The magnitude of the resultant field is given by,

$$\therefore \quad \boxed{E_T = E_0 \left[\frac{\sin n \frac{\psi}{2}}{\sin \frac{\psi}{2}} \right]} \quad \ldots (5)$$

The phase angle θ of the resultant field at point P is given by,

$$\therefore \quad \boxed{\theta = \frac{(n-1)}{2} \psi = \frac{(n-1)}{2} \beta d \cos \phi + \alpha} \quad \ldots (6)$$

Array of n elements with Equal Spacing and Currents Equal in Magnitude and Phase • Broadside Array

Consider the 'n' number of identical radiators carry currents which are equal in magnitude and in phase. The identical radiators are equi-spaced. Hence the maximum radiation occurs in the directions normal to the line of array. Hence such an array is known as Uniform broadside array.

Consider a broadside array with n identical radiators as shown in the Fig. 4.10.

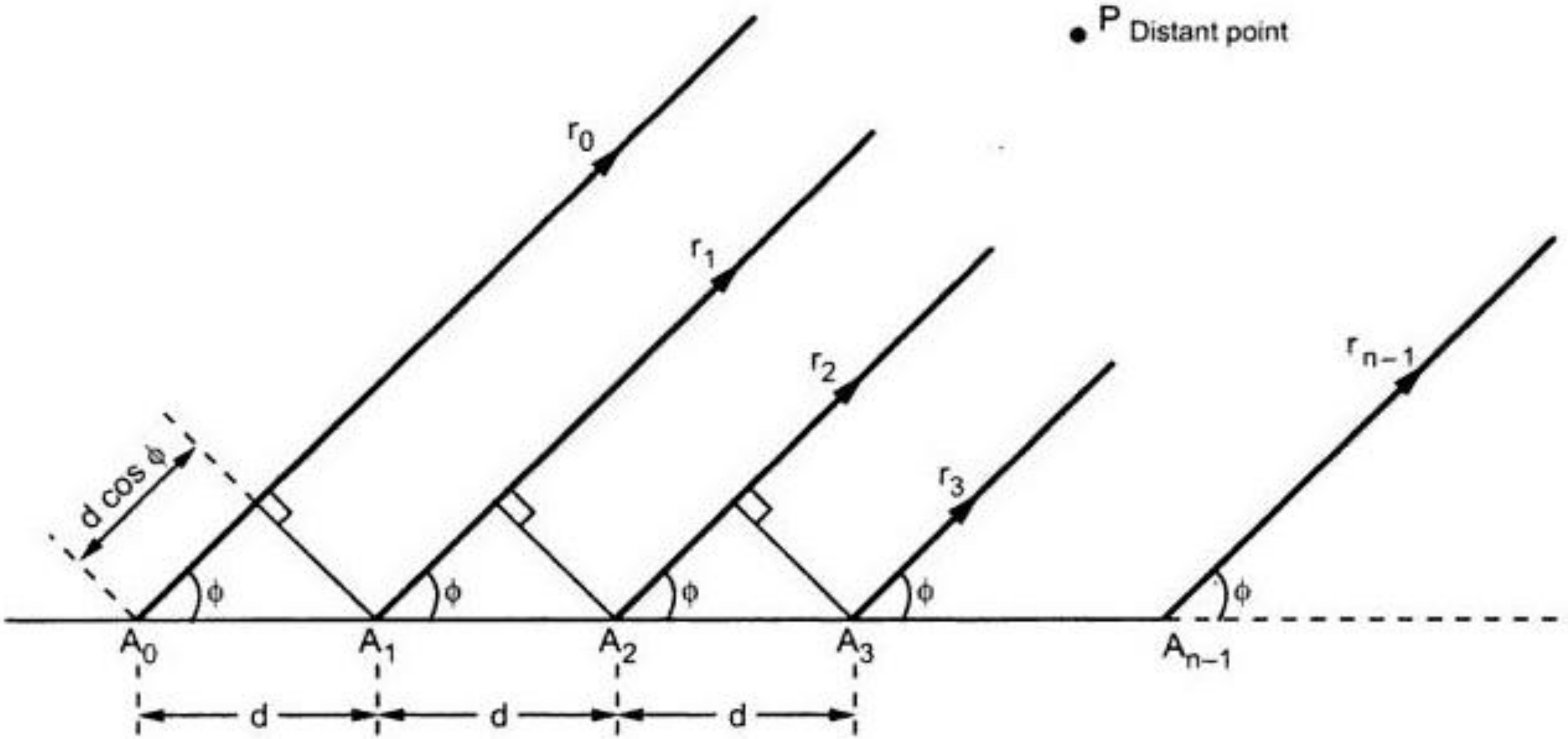


Fig. 4.10

The electric field produced at point P due to an element A_0 is given by,

$$E_0 = \frac{I\,dL \sin\theta}{4\pi\omega\varepsilon_0}\left[j\frac{\beta^2}{r_0}\right] e^{-j\beta r_0} \quad \text{... (1)}$$

As the distance of separation d between any two array elements is very small as compared to the radial distances of point P from $A_0, A_1, \ldots A_{n-1}$, we can assume $r_0, r_1, r_2, \ldots. r_{n-1}$ are approximately same.

Now the electric field produced at point P due to an element A_1 will differ in phase as r_0 and r_1 are not actually same. Hence the electric field due to A_1 is given by,

$$E_1 = \frac{I\,dL\sin\theta}{4\pi\omega\varepsilon_0}\left[j\frac{\beta^2}{r_1}\right]e^{-j\beta r_1}$$

But $$r_1 = r_0 - d\cos\phi$$

$$\therefore \quad E_1 = \frac{I\,dL\sin\theta}{4\pi\omega\varepsilon_0}\left[j\frac{\beta^2}{r_0}\right]e^{-j\beta(r_0-d\cos\phi)} \qquad \ldots r_1 \approx r_0$$

$$\therefore \quad E_1 = \frac{I\,dL\sin\theta}{4\pi\omega\varepsilon_0}\left[j\frac{\beta^2}{r_0}\right]e^{-j\beta r_0}\,e^{j\beta d\cos\phi}$$

$$\therefore \quad E_1 = E_0 \cdot e^{j\beta d\cos\phi} \qquad \ldots (2)$$

Exactly on the similar lines we can write the electric field produced at point P due to an element A_2 as,

$$E_2 = \frac{I\,dL\sin\theta}{4\pi\omega\varepsilon_0}\left[j\frac{\beta^2}{r_2}\right]e^{-j\beta r_2}$$

$$\therefore \quad E_2 = \frac{I\,dL\sin\theta}{4\pi\omega\varepsilon_0}\left[j\frac{\beta^2}{r_1}\right]e^{-j\beta(r_1-d\cos\phi)} \qquad \ldots r_2 = r_1 - d\cos\phi$$

$$\therefore \qquad E_2 = \left\{ \frac{I\,dL \sin\theta}{4\pi\omega\varepsilon_0} \left[j\frac{\beta^2}{r_1} \right] e^{-j\beta r_1} \right\} e^{j\beta d\cos\phi}$$

But the term inside the bracket represents E_1

$$\therefore \qquad E_2 = E_1\, e^{j\beta d\cos\phi}$$

From equation (2), substituting value of E_1, we get,

$$E_2 = \left[E_0\, e^{j\beta d\cos\phi} \right] e^{j\beta d\cos\phi}$$

$$\therefore \qquad E_2 = E_0 \cdot e^{j2\beta d\cos\phi} \qquad \ldots (3)$$

Similarly the electric field produced at point P due to an element A_{n-1} is given by,

$$E_{n-1} = E_0 \cdot e^{j(n-1)\beta d\cos\phi} \qquad \ldots (4)$$

The total electric field at point P is given by,

$$E_T = E_0 + E_1 + E_2 + \ldots + E_{n-1}$$

$$\therefore \qquad E_T = E_0 + E_0\, e^{j\beta d\cos\phi} + E_0\, e^{j2\beta d\cos\phi} + \ldots + E_0\, e^{j(n-1)\beta d\cos\phi}$$

Let $\beta d\cos\phi = \psi$, then rewritting above equation,

$$E_T = E_0 + E_0\, e^{j\psi} + E_0\, e^{j2\psi} + \ldots + E_0\, e^{j(n-1)\psi}$$

$$\therefore \qquad E_T = E_0 \left[1 + e^{j\psi} + e^{j2\psi} + \ldots + e^{j(n-1)\psi} \right] \qquad \ldots (5)$$

Consider a series given by

$$s = 1 + r + r^2 + r^3 + \dots r^{n-1} \qquad \dots \text{(i)}$$

where $\quad r = e^{j\psi}$

Multiplying both the sides of the equation (i) by r,

$$s.r = r + r^2 + r^3 + r^4 + \dots r^n \qquad \dots \text{(ii)}$$

Subtracting equation (ii) from (i), we get,

$$s - sr = 1 - r^n$$

$$\therefore \quad s(1-r) = 1 - r^n$$

$$\therefore \quad s = \frac{1-r^n}{1-r} \qquad \dots \text{(iii)}$$

Using equation (iii), equation (5) can be modified as,

$$E_T = E_0\left[\frac{1-e^{jn\psi}}{1-e^{j\psi}}\right]$$

$$\therefore \quad \frac{E_T}{E_0} = \frac{e^{jn\frac{\psi}{2}}\left[e^{-jn\frac{\psi}{2}} - e^{jn\frac{\psi}{2}}\right]}{e^{j\frac{\psi}{2}}\left[e^{-j\frac{\psi}{2}} - e^{j\frac{\psi}{2}}\right]} \qquad \dots (6)$$

From the trigonometric identities

$$e^{-j\theta} = \cos\theta - j\sin\theta$$

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$\text{and} \quad e^{-j\theta} - e^{j\theta} = -j\,2\sin\theta$$

Using above trigonometric identities, equation (6) can be written as,

$$\frac{E_T}{E_0} = \frac{e^{jn\frac{\psi}{2}}\left[-j\,2\sin\left(\frac{n\psi}{2}\right)\right]}{e^{j\frac{\psi}{2}}\left[-j\,2\sin\left(\frac{\psi}{2}\right)\right]}$$

$$\therefore \quad \frac{E_T}{E_0} = e^{j\frac{(n-1)\psi}{2}}\left[\frac{\sin\left(\frac{n\psi}{2}\right)}{\sin\left(\frac{\psi}{2}\right)}\right] \quad \ldots (7)$$

The exponential term in equation (7) represents the phase shift, Now considering magnitudes of the electric fields, we can write

$$\left|\frac{E_T}{E_0}\right| = \frac{\sin\frac{n\psi}{2}}{\sin\frac{\psi}{2}} \quad ..(8)$$

• **PROPERTIES OF BROADSIDE ARRAY**

1. Major lobe

In case of broadside array, the field is maximum in the direction normal to the axis of the array. Thus the condition for the maximum field at point P is given by,

$$\psi = 0 \quad \text{i.e.} \quad \beta d \cos\phi = 0 \qquad \ldots(9)$$

$$\text{i.e.} \quad \cos\phi = 0$$

$$\text{i.e.} \quad \boxed{\phi = 90^\circ \text{ or } 270^\circ} \qquad \ldots(10)$$

Thus $\phi = 90^\circ$ and $\phi = 270^\circ$ are called **directions of principle maxima.**

2. Magnitude of major lobe

The maximum radiation occurs when $\psi = 0$. Hence we can write,

$$|\text{Major lobe}| = \left|\frac{E_T}{E_0}\right| = \lim_{\psi \to 0} \left\{ \frac{\frac{d}{d\psi}\left(\sin n\frac{\psi}{2}\right)}{\frac{d}{d\psi}\left(\sin\frac{\psi}{2}\right)} \right\}$$

$$= \lim_{\psi \to 0} \left\{ \frac{\left(\cos n\frac{\psi}{2}\right)\left(n\frac{\psi}{2}\right)}{\left(\cos\frac{\psi}{2}\right)\left(\frac{\psi}{2}\right)} \right\}$$

$$\therefore \quad \boxed{|\text{Major lobe}| = n} \qquad \ldots(11)$$

where, n is the number of elements in the array.

Thus from equation (10) and (11) it is clear that, all the field components add up together to give total field which is 'n' times the individual field when $\phi = 90°$ or $270°$.

3. Nulls

The ratio of total electric field to an individual electric field is given by,

$$\left|\frac{E_T}{E_0}\right| = \frac{\sin n\frac{\psi}{2}}{\sin\frac{\psi}{2}}$$

The find direction of minima, equating ratio of magnitudes of the fields to zero.

$$\therefore \quad \left|\frac{E_T}{E_0}\right| = \frac{\sin n\frac{\psi}{2}}{\sin\frac{\psi}{2}} = 0$$

Thus condition of minima is given by,

$$\therefore \quad \boxed{\sin n\frac{\psi}{2} = 0; \quad \text{but } \sin\frac{\psi}{2} \neq 0} \qquad \ldots(12)$$

Hence we can write,

$$\sin n\frac{\psi}{2} = 0$$

$$\text{i.e.} \quad n\frac{\psi}{2} = \sin^{-1}(0) = \pm m\,\pi, \text{ where } m = 1, 2, 3, \ldots\ldots$$

Now $\psi = \beta d \cos\phi = \frac{2\pi}{\lambda}(d)\cos\phi$

$$\therefore \quad \frac{n}{2}\left(\frac{2\pi}{\lambda}d\right)\cos\phi_{min} = \pm m\pi$$

$$\text{i.e.} \quad \frac{nd}{\lambda}\cos\phi_{min} = \pm m$$

$$\therefore \quad \boxed{\phi_{min} = \cos^{-1}\left(\pm\frac{m\lambda}{nd}\right)} \quad \ldots(13)$$

where n = Number of elements in array

d = Spacing between elements in meter

λ = Wavelength in meter

m = Constant = 1, 2, 3, ...

Thus equation (13) gives the directions of nulls.

4. Subsidary maxima (or side lobes)

The directions of the subsidary maxima or side lobes can be obtained if in equation (8),

$$\sin\left(n\frac{\psi}{2}\right) = \pm 1$$

$$\therefore \qquad \boxed{n\frac{\psi}{2} = \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2}, \ldots\ldots} \qquad \ldots(14)$$

Hence $\sin\left(n\frac{\psi}{2}\right) = \pm 1$ is not considered. Because if $n\frac{\psi}{2} = \frac{\pi}{2}$ then $\sin n\frac{\psi}{2} = 1$ which is the **direction of principle maxima.**

Hence we can skip $n\frac{\psi}{2} = \pm\frac{\pi}{2}$ value

Thus, we get

$$\psi = \pm\frac{3\pi}{n}, \pm\frac{5\pi}{n}, \pm\frac{7\pi}{n}, \ldots\ldots\ldots\ldots$$

Now $\qquad \psi = \beta d \cos\phi = \left(\frac{2\pi}{\lambda}\right) d \cos\phi$

Hence equation for ψ can be written as,

$$\frac{2\pi}{\lambda} d\cos\phi = \pm\frac{3\pi}{n}, \pm\frac{5\pi}{n}, \pm\frac{7\pi}{n}, \ldots\ldots\ldots$$

$$\therefore \qquad \cos\phi = \frac{\lambda}{2\pi d}\left[\pm\frac{(2m+1)}{n}\pi\right] \text{ where } m = 1, 2, 3, \ldots\ldots$$

$$\therefore \qquad \boxed{\phi = \cos^{-1}\left[\pm\frac{\lambda(2m+1)}{2nd}\right]} \qquad \ldots(15)$$

The equation (15) represents the directions where certain radiation which is not maximum. Hence it represents **directions of subsidary maxima or side lobes.**

5. Beamwidth of major lobe

The **beamwidth** is defined as the angle between first nulls. Alternatively beamwidth is the angle equal to twice the angle between first null and the major lobe maximum direction.

Hence the beamwidth between first nulls is given by,

$$\therefore \qquad \boxed{\text{BWFN} = 2\times\gamma, \text{ where } \gamma = 90-\phi} \qquad \ldots(16)$$

But $\qquad \phi_{min} = \cos^{-1}\left(\pm\frac{m\lambda}{nd}\right), \quad$ where m = 1, 2, 3,……

Non-iosotropic sources but similar point sources with PATTERN MULTIPLICATION

- THE WORD *SIMILAR* MEANS THAT THE VARIATION WITH ABSOLUTE ANGLE ϕ OF BOTH THE AMPLITUDE AND PHASE OF THE FIELD IS THE SAME.

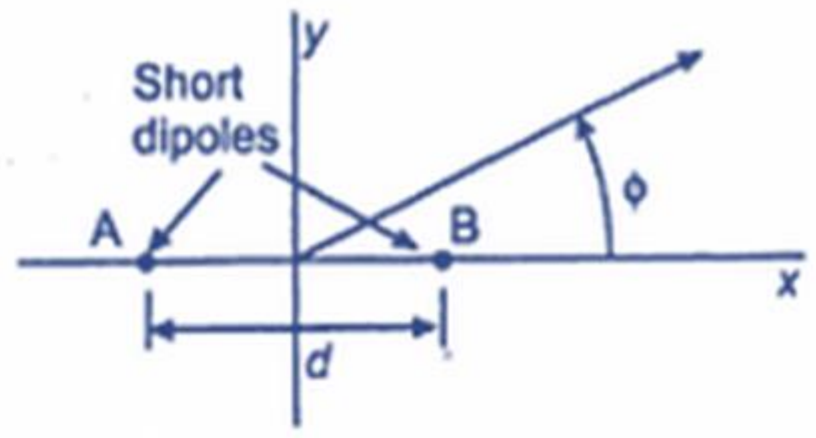


Figure *Two nonisotropic sources.*

- LET US CONSIDER TWO NON-ISOTROPIC SOURCE (SHORT DIPOLE) SAY A AND B HAVE FIELD PATTERN IS

$$E_o = E'_o \, \mathrm{SIN}\, \phi$$

- AND HENCE TOTAL FIELD PATTERN IS JUST REPLACING E_o BY ABOVE EQUATION WE HAVE

$$E = \mathbf{2}E'_o \, \mathrm{SIN}\, \phi \, \mathrm{COS}\frac{\psi}{2}$$

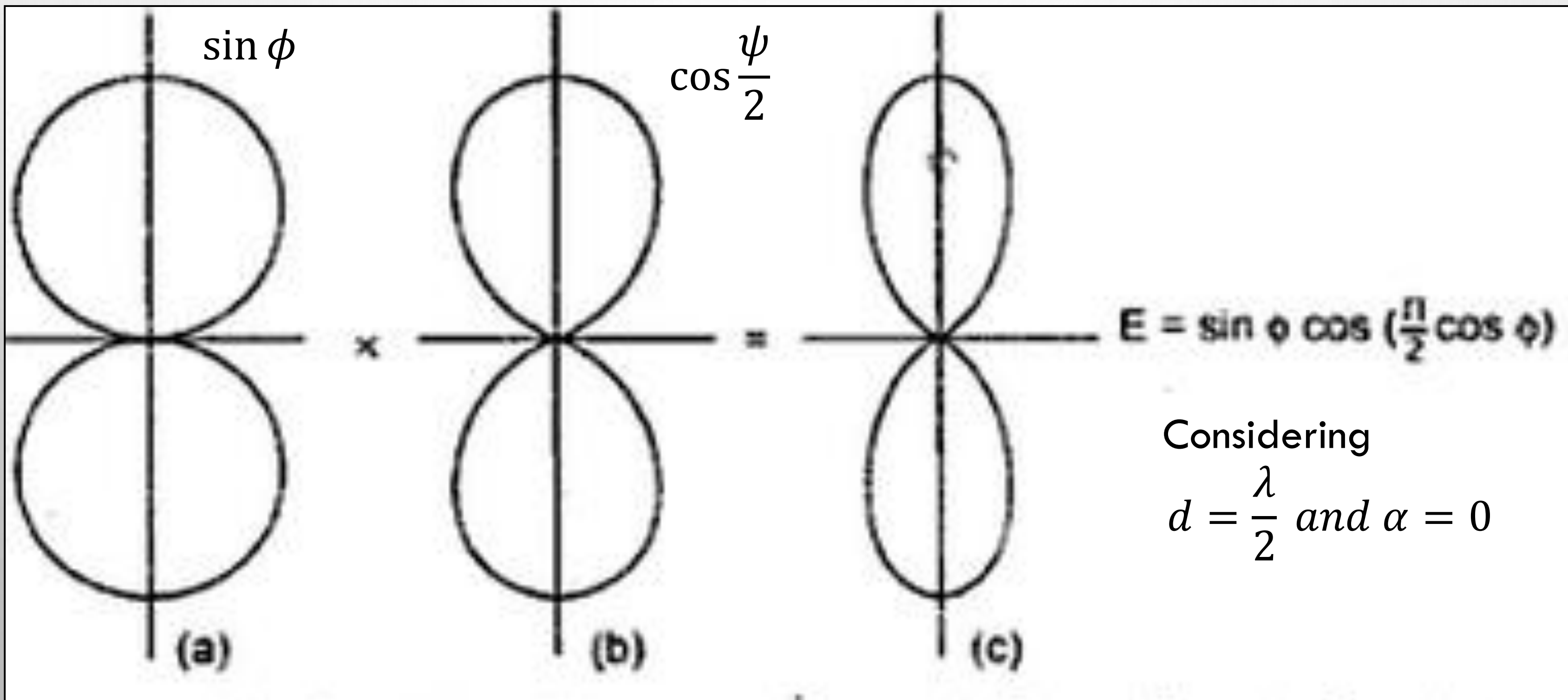
WHERE $\psi = \beta d \; cos\phi + \alpha$

And hence total field pattern is just replacing E_o by above equation we have

$$\boxed{E_T = \mathbf{2}E'_o \sin\phi \cos\frac{\psi}{2}} \quad \text{..................1}$$

where $\psi = \beta d \ cos\phi + \alpha$

From above equation we can say that the total pattern is obtained by multiplying the pattern of the individual source ($\sin\phi$) by the pattern of an array two isotropic point sources $\cos\left(\frac{\psi}{2}\right)$



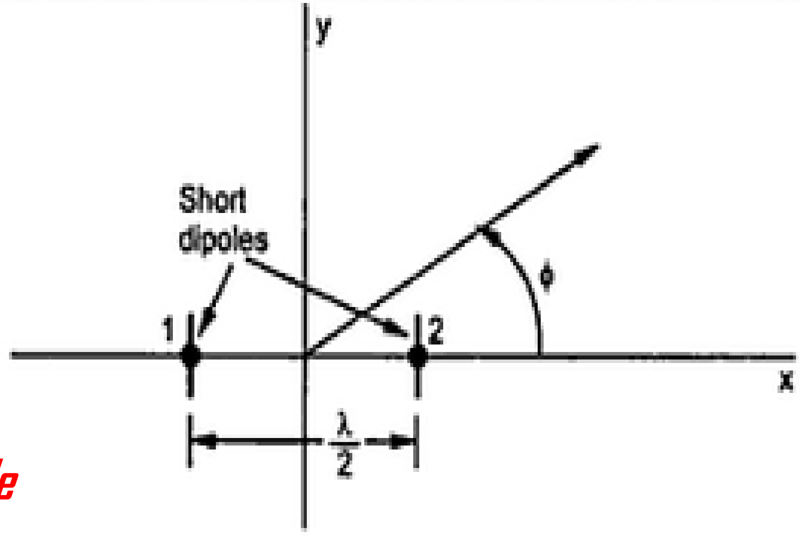
Let us consider two non-isotropic source (short dipole) say 1 and 2 have field pattern is

$$E_o = E'_o \cos\phi$$

And hence total field pattern is just replacing E_o by above equation we have

$$E = \mathbf{2}E'_o \cos\phi \cos\frac{\psi}{2}$$

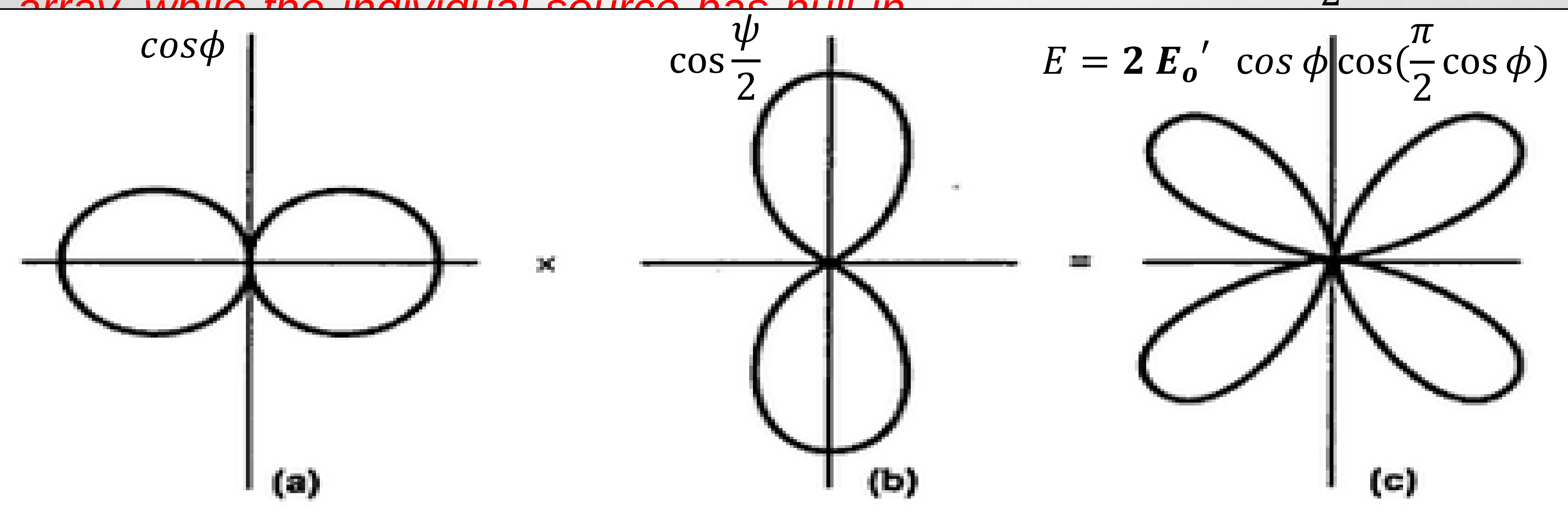
where $\psi = \beta d \cos\phi$



Such a pattern is possible by ***short dipole oriented parallel to the y-axis*** *as in below figure. The maximum field of the individual source is in the direction* ($\phi = 0°$) *of a null form the array, while the individual source has null in*

Considering

$$d = \frac{\lambda}{2} \; and \; \alpha = 0$$



THERE THE PRINCIPLE OF PATTERN MULTIPLICATION , MAY BE EXPRESSED AS:

THE FIELD PATTERN OF AN ARRAY OF NON-ISOTROPIC BUT SIMILAR POINT SOURCES IS THE MULTIPLICATION OF THE PATTERN OF THE INDIVIDUAL SOURCE AND THE PATTERN OF AN ARRAY OF ISOTROPIC POINT SOURCES AND HAVING THE SAME LOCATIONS, RELATIVE AMPLITUDES AND PHASES AS THE NON-ISOTROPIC POINT SOURCES.

CASE 2:- n Elements Uniform Linear Arrays of point sources – END-FIRE ARRAY

Same Equation of Normalize resultant Field Pattern

$$\left|\frac{E_T}{E_0}\right| = \frac{\sin\frac{n\psi}{2}}{\sin\frac{\psi}{2}} \qquad \dots (8)$$

For an array to be end fire, the phase angles is such that makes the maximum radiation in the of array i.e. $\phi = 0° \; or \; 180°$. Thus for an array to be $\psi = 0$ and $\phi = 0°$ or 180°

$$\psi = \beta d \; cos\phi + \alpha$$

$$0 = \beta d \; cos\phi + \alpha$$

$$\alpha = -\beta d = -\frac{\boldsymbol{2\pi d}}{\boldsymbol{\lambda}}$$

4.9.1 Properties of End Fire Array

1. Major lobe

For the end fire array where currents supplied to the antennas are equal in amplitude but the phase changes progressively through array, the phase angle is given by,

$$\psi = \beta d(\cos\phi - 1) \quad \text{...(9)}$$

In case of the end fire array, the condition of principle maxima is given by,

$$\psi = 0 \text{ i.e. } \boxed{\beta d(\cos\phi - 1) = 0} \quad \text{...(10)}$$

$$\text{i.e. } \cos\phi = 1$$

$$\text{i.e. } \boxed{\phi = 0^\circ} \quad \text{...(11)}$$

Thus $\phi = 0^\circ$ indicates the **direction of principle maxima.** Also it indicates that the maximum radiation is along the axis of array or line of array.

2. Magnitude of the major lobe

The maximum radiation occurs when $\psi = 0$. Thus we can write,

$$|\text{Major lobe}| = \lim_{\psi \to 0} \left\{ \frac{\frac{d}{d\psi}\left(\sin n\frac{\psi}{2}\right)}{\frac{d}{d\psi}\left(\sin\frac{\psi}{2}\right)} \right\}$$

$$= \lim_{\psi \to 0} \left\{ \frac{\left(\cos n\frac{\psi}{2}\right)\left(n\frac{\psi}{2}\right)}{\left(\cos\frac{\psi}{2}\right)\left(\frac{\psi}{2}\right)} \right\}$$

$$\therefore \quad \boxed{|\text{Major lobe}| = n} \qquad \text{...(12)}$$

where, n is the number of elements in the array.

Thus from above equations it is clear that the field component at point P is 'n' times the individual field in the direction $\phi = 0°$.

3. Nulls

The ratio of the total field to the individual field is given by,

$$\left|\frac{E_T}{E_0}\right| = \frac{\sin n\frac{\psi}{2}}{\sin\frac{\psi}{2}}$$

To find direction of minima, equating ratio of magnitudes to zero,

$$\therefore \qquad \left|\frac{E_T}{E_0}\right| = \frac{\sin n\frac{\psi}{2}}{\sin\frac{\psi}{2}} = 0$$

Thus condition of minima is given by,

$$\boxed{\sin n\frac{\psi}{2} = 0, \quad \text{but } \sin\frac{\psi}{2} \neq 0} \qquad \ldots(13)$$

Hence we can write,

$$\sin n\frac{\psi}{2} = 0$$

$$\text{i.e.} \quad n\frac{\psi}{2} = \pm m\pi, \text{ where } m = 1, 2, 3, \ldots\ldots$$

Substituting value of ψ from equation (9), we get,

$$\therefore \quad \frac{n\beta d(\cos\phi - 1)}{2} = \pm m\pi$$

Put $\beta = \frac{2\pi}{\lambda}$, we get

$$\therefore \quad \frac{nd}{\lambda}(\cos\phi - 1) = \pm m \qquad \text{...(14)}$$

Note that value of $(\cos\phi - 1)$ is always less than 1. Hence it is always negative. Hence only considering –ve values, R.H.S., we get

$$\frac{nd}{\lambda}(\cos\phi - 1) = -m$$

$$\text{i.e. } \cos\phi - 1 = -\frac{m\lambda}{nd}$$

$$\text{i.e. } \quad \cos\phi = 1 - \frac{m\lambda}{nd}$$

i.e. $$\boxed{\phi_{min} = \cos^{-1}\left[1 - \frac{m\lambda}{nd}\right]} \quad ...(15)$$

where

m = Constant = 1, 2, 3,.........

n = Number of elements in array

d = Spacing between element in meter

λ = Wavelength in meter

Thus equation (15) gives direction of nulls.

Consider equation (14).

$$\cos\phi_{min} - 1 = \pm\frac{m\lambda}{nd}$$

Expressing term on L.H.S. interms of half angles, we get

$$2\sin^2\frac{\phi_{min}}{2} = \pm\frac{m\lambda}{nd} \quad ...\left(\cos\theta - 1 = 2\sin^2\frac{\theta}{2}\right)$$

$$\therefore \quad \sin^2\frac{\phi_{min}}{2} = \pm\frac{m\lambda}{2nd}$$

$$\therefore \quad \sin\frac{\phi_{min}}{2} = \pm\sqrt{\frac{m\lambda}{2nd}}$$

$$\therefore \quad \frac{\phi_{min}}{2} = \sin^{-1}\left[\pm\sqrt{\frac{m\lambda}{2nd}}\right]$$

$$\therefore \quad \boxed{\phi_{min} = 2\sin^{-1}\left[\pm\sqrt{\frac{m\lambda}{2nd}}\right]} \quad ...(16)$$

4. Subsidary maxima (or side lobes)

The directions of the subsidary maxima or side lobes can be obtained if in equation (8),

$$\sin\left(n\frac{\psi}{2}\right) = \pm 1$$

$$\therefore \qquad \boxed{n\frac{\psi}{2} = \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2}, \ldots\ldots} \qquad \ldots(17)$$

Hence $n\frac{\psi}{2} = \pm\frac{\pi}{2}$ is skipped because with this value of $n\frac{\psi}{2}$ we get $\sin n\frac{\psi}{2} = 1$ which is the **direction of principle maxima.**

Thus we can write,

$$\frac{n\psi}{2} = \pm(2m+1)\frac{\pi}{2}, \text{ where } m = 1, 2, 3, \ldots\ldots$$

Putting value of ψ from equation (9), we get,

$$\frac{n\beta d(\cos\phi - 1)}{2} = \pm(2m+1)\frac{\pi}{2}$$

$$\therefore \quad n\beta d(\cos\phi - 1) = \pm(2m+1)\pi$$

Put $\beta = \frac{2\pi}{\lambda}$, we get,

$$n\left(\frac{2\pi}{\lambda}\right)d(\cos\phi - 1) = \pm(2m+1)\pi$$

$$\text{i.e. } \cos\phi - 1 = \pm(2m+1)\frac{\lambda}{2nd}$$

Similar to previous case value of ($\cos\phi - 1$) is always less than zero i.e. it is always –ve. Hence only considering negative values on the R.H.S. of above equation, we get,

$$\cos\phi - 1 = -(2m+1)\frac{\lambda}{2nd}$$

i.e.

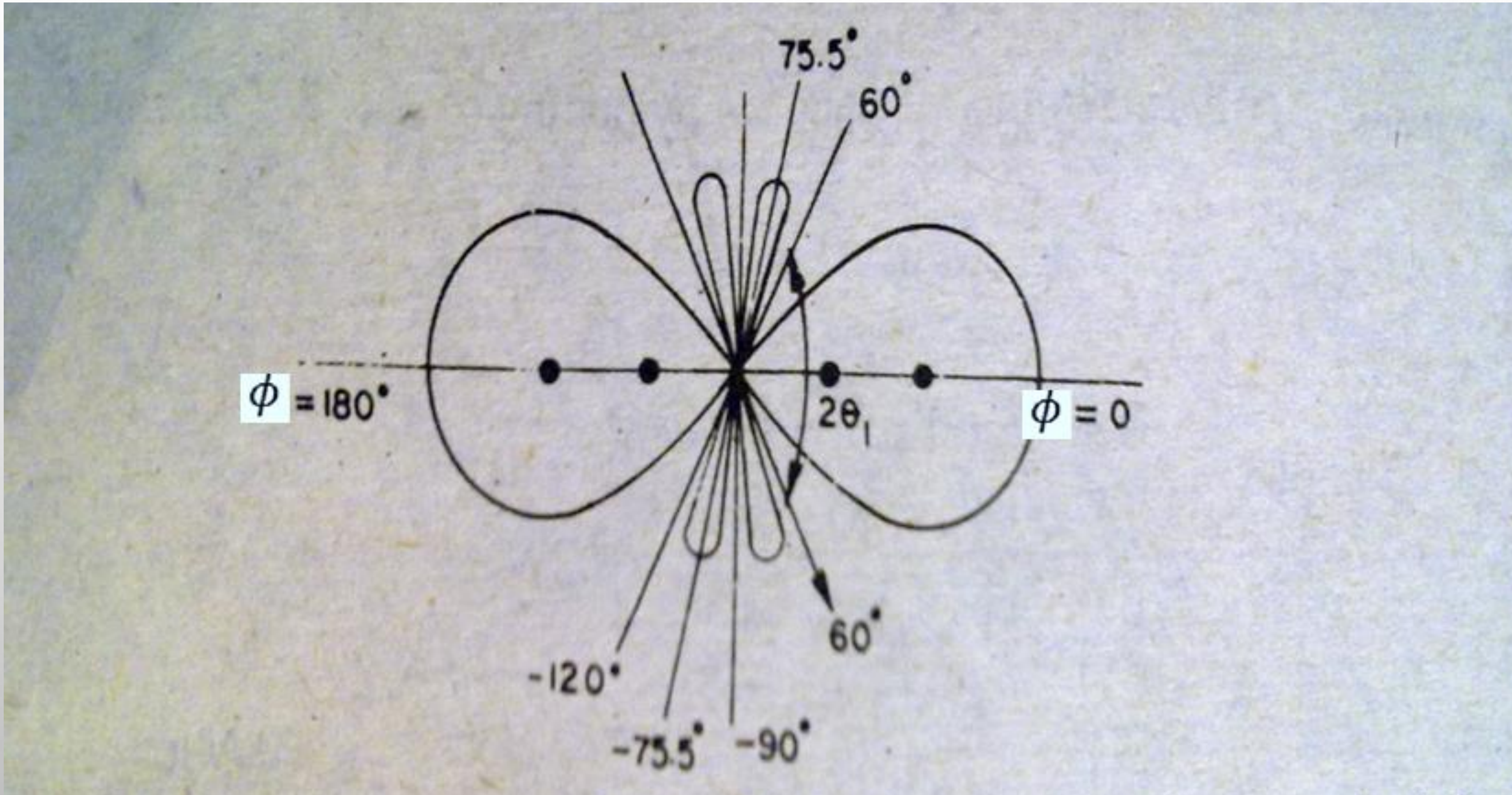
$$\cos\phi = 1 - (2m+1)\frac{\lambda}{2nd}$$

i.e.

$$\boxed{\phi = \cos^{-1}\left[1 - \frac{(2m+1)\lambda}{2nd}\right]} \qquad \ldots(18)$$

Equation (18) represents directions of the subsidary maxima or sides lobes.

For example if
n=4, d=$\boldsymbol{\lambda/2}$, $\alpha = -\pi$



Thus +75.5°& -75.5° are the 4 minor lobe maxima of the array of 4 Isotropic sources, 180 deg out of phase, spaced $\boldsymbol{d = \lambda/2}$ **apart.**

5. Beamwidth of major lobe

The beamwidth of the end fire array is greater than that of broadside array.

$\therefore$ Beamwidth = 2 × Angle between first nulls and maximum of the major lobe i.e. θ_{min}.

From equation (16),

$$\phi_{min} = 2\sin^{-1}\left[\pm\sqrt{\frac{m\lambda}{2nd}}\right] \qquad \ldots(19)$$

$$\therefore \quad \sin\frac{\phi_{min}}{2} = \pm\sqrt{\frac{m\lambda}{2nd}}$$

NOTE:- Here the complementary angle θ, as has been used in broadside case, is not required. Because the beam width of a end fire array is larger than broadside.

If ϕ_{min} is very low, then we can write $\sin\frac{\phi_{min}}{2} = \frac{\phi_{min}}{2}$. Using this property in above equation we get,

$$\frac{\phi_{min}}{2} = \pm\sqrt{\frac{m\lambda}{2nd}}$$

$$\therefore \qquad \phi_{min} = \pm\sqrt{\frac{4m\lambda}{2nd}} = \pm\sqrt{\frac{2m\lambda}{nd}} \qquad \ldots(20)$$

But nd = L i.e. length of the antenna array, so equation (20) becomes,

$$\phi_{min} = \pm\sqrt{\frac{2m\lambda}{L}} = \pm\sqrt{\frac{2m}{L/\lambda}} \qquad \ldots(21)$$

The beamwidth between first nulls is given by,

$$\boxed{BWFN = 2\phi_{min} = \pm 2\sqrt{\frac{2m}{L/\lambda}}} \qquad \ldots(22)$$

Expressing BWFN in degrees, we get

$$BWFN = \pm 2\sqrt{\frac{2m}{L/\lambda}} \times 57.3 = \pm 114.6\sqrt{\frac{2m}{L/\lambda}} \text{ degree}$$

For m = 1,

$$\boxed{BWFN = \pm 2\sqrt{\frac{2}{L/\lambda}} \text{ rad} = 114.6\sqrt{\frac{2}{L/\lambda}} \text{ degree}} \qquad \ldots(23)$$

THANK YOU