# CHAPTER 2: ANTENNA PARAMETERS AND ARRAYS

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# COURSE STRUCTURE

- BASIC ANTENNA PARAMETERS
- PATTERN MULTIPLICATION:  LINEAR AND TWO-DIMENSIONAL ANTENNA ARRAYS, END FIRE AND BROADSIDE ARRAYS.

# ANTENNA PARAMETERS

- An antenna is the basic fundamental component of the communication system.
- Basically antennas are nothing but links between the transmitter-to-free space or free space-to-receiver.
- The characteristics of the communication system are mainly dependent on the characteristics of the antennas used in the system.
- Antennas used in different systems are of different types.
- In some systems, the operational characteristics of the system are designed around the directional properties of an antenna.
- Or in some other type of system, the antennas are used only to radiate energy in all directions equally i.e. omnidirectional property.
- Now the applications of antennas are different in different systems. But in spite of various variety and application areas, the antennas possess certain common basic properties.
- Such important properties of antennas include radiation pattern, radiation intensity, directive gain, beam width, band width etc.
- In this chapter we will discuss various fundamental parameters of antenna along with review of antenna theorems. Before starting discussion of antenna parameters, let us start with the concept of isotropic radiator.

# ISOTROPIC RADIATOR

- In general, isotropic radiator is a hypothetical or fictitious radiator.
- The isotropic radiator is defined as a radiator which radiates energy in all directions uniformly.
- It is also called isotropic source.
- As it radiates uniformly in all directions, it is also called omnidirectional radiator or unipole.
- Basically isotropic radiator is a lossless ideal radiator or antenna.
- Generally all the practical antennas are compared with the characteristics of the isotropic radiator.
- The isotropic antenna or radiator is used as reference antenna.
- Practically all antennas show directional properties i.e. directivity property. That means none of the antennas radiate energy in all directions uniformly. Hence practically isotropic radiator can not exist.
- Consider that an isotropic radiator is placed at the center of sphere of radius r.
- Then all the power radiated by the isotropic radiator passes over the surface area of the sphere given by $4\pi r^2$, assuming zero absorption of the power. Then at any point

on the surface, the poynting vector $\vec{p}$ gives the power radiated per unit area in any direction. But radiated power travels in the radial direction. Thus the magnitude of the poynting vector $\vec{p}$ will be equal to radial component as the components in θ & ϕ and - ϕ directions are zero i.e. $p_{\theta =} p_{\phi} = 0$. Hence we can write,

$|\vec{p}| = p_r$

The total power radiated is given by,

$$P_{rad} = \iint \overline{P} \bullet d\overline{s}$$

$$= \iint P_r \, ds$$

$$= P_r \iint ds \qquad \ldots \because P_\theta = P_\phi = 0$$

Now this radial component $P_r$ is the average power density component which can be denoted as $P_{avg}$.

$$\therefore \quad P_{rad} = P_{avg}(4\pi r^2) \qquad \text{where } \iint ds = \text{surface area} = 4\pi r^2$$

$$\therefore \quad \boxed{P_{avg} = \frac{P_{rad}}{4\pi r^2} \text{ W/m}^2} \qquad \ldots(2)$$

where $P_{rad}$ = Total power radiated in watts

$P_{avg}$ = Radial component of average power density in $W/m^2$

$r$ = Radius of sphere in meters

As we have discussed earlier that irrespective of antenna types and applications, all the antennas possess certain fundamental properties as listed below.

1) Radiation pattern - i) Field radiation pattern
   ii) Power radiation pattern
2) Radiation intensity
3) Directive gain and directivity
4) Power gain
5) Antenna beamwidth
6) Antenna bandwidth
7) Antenna input impedance
8) Effective length
9) Effective aperture
10) Antenna temperature
11) Antenna polarization

Let us discuss these basic properties one by one.

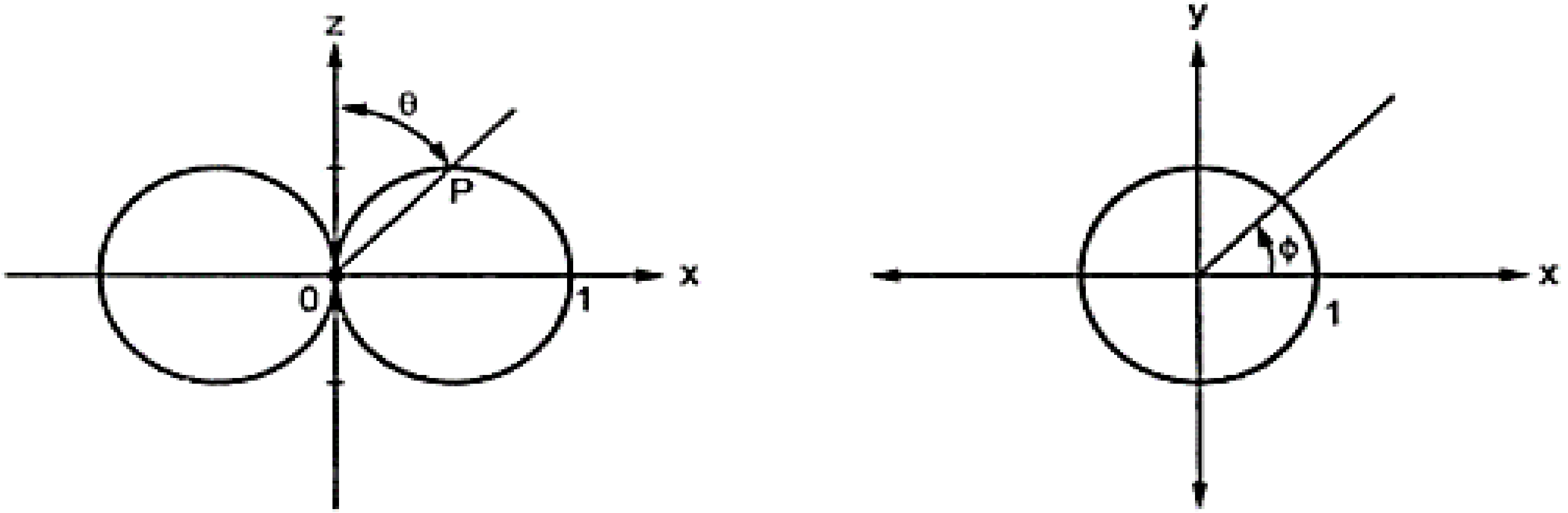
# RADIATION PATTERN

Practically any antenna can not radiate energy with same strength uniformly, in all directions. It is found that the radiation is large in one direction while zero or minimum in other directions. The radiation from the antenna in any direction is measured in terms of **field strength** at a point located at a particular distance from an antenna. The field strength can be calculated by measuring voltages at two points on an electric lines of force and then dividing by distance between two points. Hence unit of the radiation pattern is volt per meter. Generally the field strength is expressed in millivolts per meter. The radiation pattern of an antenna is the important characteristic of antenna because it indicates the distribution of the energy radiated by an antenna in the space.

In general, the radiation pattern is nothing but a graph which shows the variation of actual field strength of electromagnetic field at all the points equidistant from the antenna. Hence it is a three dimensional graph. There are two basic radiation patterns. If the radiation of the antenna is represented graphically as a function of direction it is called **radiation pattern.** But if the radiation of the antenna is expressed in terms of the field strength E(in V/m), then the graphical representation is called **Field Strength Pattern** or **Field Radiation Pattern.** Similarly if the radiation of the antenna is expressed in terms of the power per unit solid angle, then the graphical representation is called **Power Radiation Pattern** or simply **Power Pattern.**

### 2.4.1 Field Radiation Pattern

In general, the complete field radiation pattern is a 3-dimension pattern. It requires three dimensional representation to represent the radiation for all angles of $\phi$ and $\theta$. As the three dimensional pattern can not be plotted in a plane, three dimensional representation is avoided. Instead of this, the polar plots of the relative magnitude of the field in any desired plane are sketched. These polar plots are plotted in two planes; namely one containing the antenna and the other normal to it. These planes are called **principle planes** and the two plots or patterns are called **principle plane patterns.** These patterns are obtained by plotting the magnitude of the normalized field strengths. When the magnitude the normalized field strength is plotted versus $\theta$ with constant $\phi$, the pattern is called **E-plane pattern or vertical pattern.** The E-plane pattern for the Hertzian dipole is as shown in the Fig. 2.1 (a). When the normalized field strength is plotted versus $\phi$ for $\theta = \frac{\pi}{2}$, the pattern is called **H-plane pattern** or **horizontal pattern.** The H-plane pattern for the hertzian dipole is as shown in the Fig. 2.1.




(a) E - plane pattern

(b) H - plane pattern

**Fig. 2.1 E-plane pattern and H-plane pattern for the hertzian dipole**

The radiation pattern gets changed if the length of the dipole is increased. For a centre-fed half wave vertical dipole the radiation pattern is as shown in the Fig. 2.2 (a). Similarly the radiation pattern for a one wavelength long vertical dipole is as shown in the Fig. 2.2 (b).

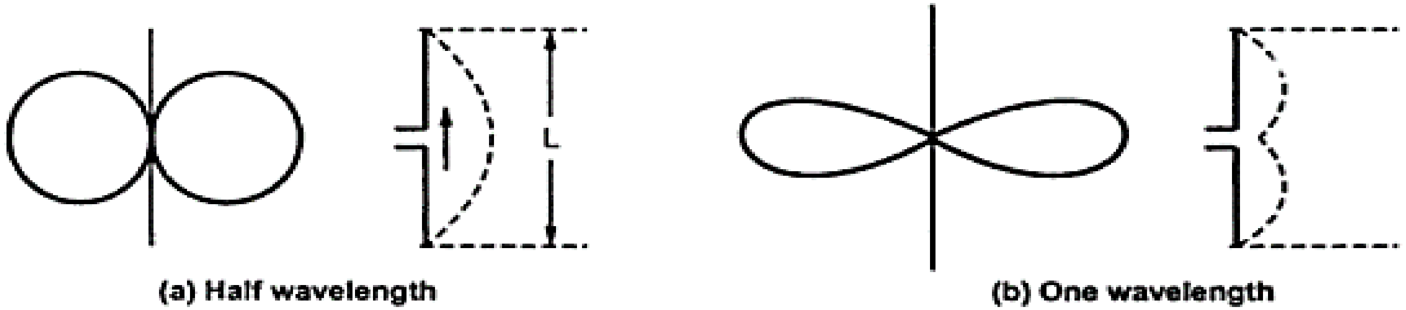



**Fig. 2.2 Field radiation patterns for vertical dipoles of half and one wavelength**

The field radiation patterns for the $1\frac{1}{2}$ wavelength and 2 wavelength vertical dipoles are as shown in the Fig. 2.3 (a) and (b) respectively.

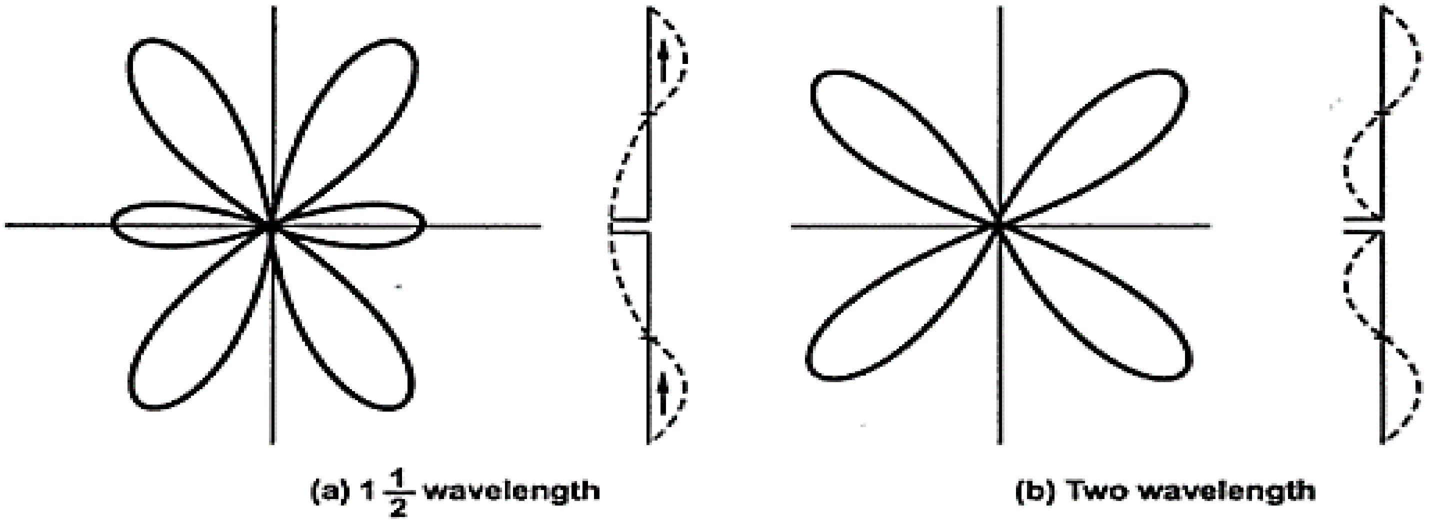



**Fig. 2.3 Field radiation patterns for vertical dipoles of one and half wavelength and two wavelength**

### 2.4.2 Power Radiation Pattern

When the radiation in a given direction is expressed in terms of power per unit solid angle, the pattern is called **power radiation pattern.** Again consider a spherical surface with radius r centred at the point source representing the antenna. At a point on this surface, the radiated power flows radially outwards. The corresponding electric and magnetic fields are normal to this direction and are also mutually perpendicular with $\frac{E}{H} = 120\pi$ for free space.

The **power density** $P_d(\theta, \phi)$ is defined as power flow per unit area and is a function of the direction $(\theta, \phi)$. The power density can be expressed in terms of the magnitude of the electric field intensity as

$$P_d(\theta, \phi) = \frac{1}{2} \frac{|E(\theta, \phi)|^2}{\eta_0} = \frac{1}{2} \frac{|E(\theta, \phi)|^2}{120\pi}$$

In the direction in which $E(\theta,\phi)$ is maximum, $P_d(\theta,\phi)$ is also maximum. In this direction, the maximum value of power density is denoted by $P_{d(max)}$. Then the relative power flow per unit area in the direction of $(\theta,\phi)$ is given by

$$G(\theta,\phi) = \frac{P_d(\theta,\phi)}{P_{d(max)}} = \frac{|E(\theta,\phi)|^2}{E_{max}^2} = f^2(\theta,\phi)$$

The ratio $G(\theta,\phi)$ is called **power radiation pattern** and it is independent of the distance r since both $P_d(\theta,\phi)$ and $P_{d\,max}$, vary inversely with r. The power radiation pattern is plotted similarly to the field radiation pattern. For example, for the Hertzian dipole, the field radiation pattern is given by

$$f(\theta,\phi) = \sin\theta$$

Hence the power radiation pattern is given by

$$G(\theta,\phi) = f^2(\theta,\phi) = \sin^2\theta$$

The sketches of the power radiation pattern for the Hertzian dipole with $\phi$ constant are as shown the Fig. 2.1 (a) and (b) respectively.

# RADIAN AND STERADIAN

The basic difference between radian and streadian is that the radian is the measure of a plane angle; while the steradian is the measure of a solid angle.

One radian is defined as the plane angle with its vertex at centre of a circle with radius 'r' that is subtended by an arc whose length is also r. It is represented as unit rad.

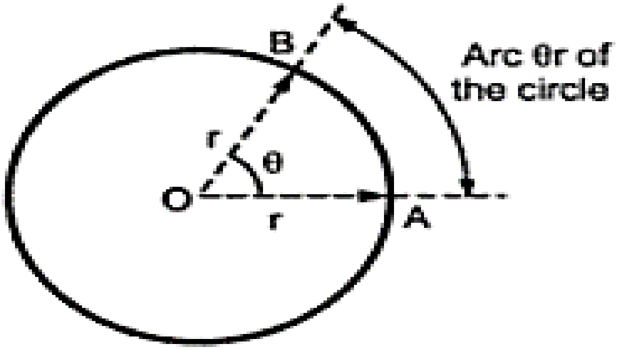



**Fig. 2.4 Representation of 1 radian plane angle**

Refer Fig. 2.4. The total circumference C of a circle with radius r is given by,

$$C = 2\pi r$$

Thus over a complete circle there are $2\pi$ radian.

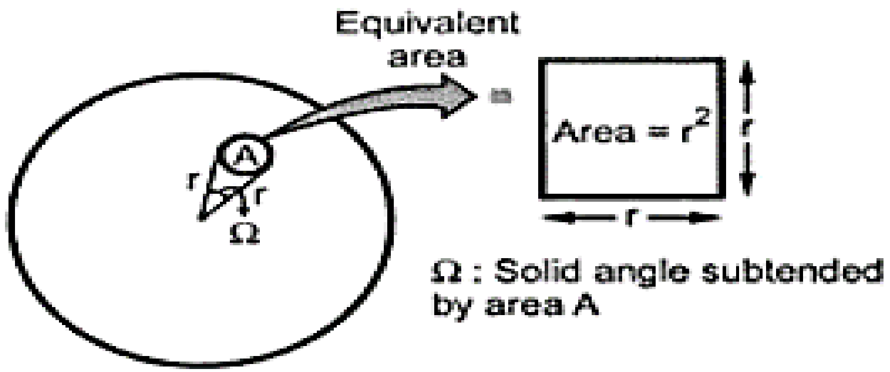



**Fig. 2.5 Representation of 1 steradian solid angle**

In the similar way, the measure of a solid angle is defined as steradian. One steradian is defined as a solid angle with its vertex at the centre of the sphere with radius r that is subtended by a spherical surface area equal to that of a square with each side equal to r. The angle in steradian is expressed in Sr.

Refer Fig. 2.5. The area of a complete sphere with radius r is given by,

$$A = 4\pi r^2$$

Thus over a close sphere with radius r there are $4\pi$ steradian.

The infinitesimal area ds on the surface of a sphere with radius r is given by,

$$ds = r^2 \sin\theta \, d\theta \, d\phi \ m^2$$

Hence the element of solid angle $d\Omega$ of a sphere is given by,

$$\boxed{d\Omega = \frac{ds}{r^2} = \sin\theta \, d\theta \, d\phi \text{ steradian}}$$

## 2.6 Radiation Intensity

The radiation intensity of an antenna does not depend on the distance from the radiator or antenna. It is denoted by U. The radiation intensity is defined as power per unit solid angle. It is expressed in W/Sr (i.e. watts/steradian).

The radiation intensity of an antenna is defined as

$$U(\theta, \phi) = r^2 P_d(\theta, \phi)$$

Then the total power radiated can be expressed in terms of the radiation intensity as

$$P_{rad} = \oint_S P_d(\theta, \phi)\, ds = \oint_S P_d(\theta, \phi)\left[r^2 \sin\theta \, d\theta \, d\phi\right]$$

$$= \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \left[P_{d(\theta,\phi)}\, r^2\right] \sin\theta \, d\theta \, d\phi$$

Let $d\Omega = \sin\theta \, d\theta \, d\phi$ be the differential solid angle in steradian (Sr). Then we can write

$$\boxed{P_{rad} = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} U(\theta, \phi) \, d\Omega}$$

Thus the **radiation intensity** $U(\theta, \phi)$ is expressed in watts per steradian (W/Sr) and it is defined as time average power per unit solid angle. The average value of the radiation intensity is given by

$$\boxed{U_{avg} = \frac{P_{rad}}{4\pi}}$$

## 2.7 Directive Gain and Directivity

An isotropic antenna is the omnidirectional antenna. The meaning of the omnidirectional antenna is the antenna acting as a point radiator which radiates equally in all directions. But practically an isotropic antenna does not exist in practice. Practically antenna does not radiate equally in all directions.

If the antenna were isotropic i.e. if it were to radiate uniformly in all directions, then the power density at all the points on the surface of a sphere will be same. The average power can be expressed in terms of the radiated power as

$$\boxed{P_{avg} = \frac{P_{rad}}{4\pi r^2} \ W/m^2}$$

The directive gain is defined as the ratio of the power density $P_{d(\theta,\phi)}$ to the average power radiated. For isotropic antenna, the value of the directive gain is unity.

$$G_D(\theta,\phi) = \frac{P_{d(\theta,\phi)}}{P_{avg}} = \frac{P_{d(\theta,\phi)}}{\frac{P_{rad}}{4\pi r^2}}$$

Rearranging the terms,

$$G_D(\theta,\phi) = \frac{P_{d(\theta,\phi)} \cdot r^2}{\left(\dfrac{P_{rad}}{4\pi}\right)}$$

The numerator in the above ratio is the radiation intensity while the denominator is the average value of the radiation intensity. Hence the **directive gain** can be written as,

$$\boxed{G_D(\theta,\phi) = \frac{U(\theta,\phi)}{U_{avg}} = \frac{4\pi\, U(\theta,\phi)}{P_{rad}}}$$

Thus the **directive gain** can be defined as a measure of the concentration of the radiated power in a particular direction $(\theta,\phi)$.

The ratio of the maximum power density to the average power radiated is called **maximum directive gain or directivity** of the antenna.

$$\therefore \quad \boxed{G_{D\,max} = \frac{P_{d\,max}}{\dfrac{P_{rad}}{4\pi r^2}}}$$

The directivity can alternatively defined as,

$$\boxed{G_{D\,max} = \frac{U_{max}}{U_{avg}} = \frac{4\pi\, U_{max}}{P_{rad}}}$$

The directivity of an antenna is dimensionless quantity. The directivity can also be expressed in terms of the electric field intensity as

$$\boxed{G_{D\,max} = \frac{4\pi \left|E_{max}\right|^2}{\int_0^{2\pi}\int_0^{\pi} \left|E(\theta, \phi)\right|^2 \sin\theta \, d\theta \, d\phi}}$$

## 2.8 Power Gain and Radiation Efficiency (Antenna Efficiency)

The practical antenna is made up of a conductor having finite conductivity. Hence we must consider the ohmic power loss of the antenna. If the practical antenna has ohmic losses $(I^2 R)$ represented by $P_{loss}$, then the power radiated $P_{rad}$ is less than the input power $P_{in}$. Then we can express the $P_{rad}$ in terms of the $P_{in}$ as

$$P_{rad} = \eta_r \, P_{in}$$

where $\eta_r$ is called **radiation efficiency** of an antenna. Thus the radiation efficiency of an antenna can be written as,

$$\eta_r = \frac{P_{rad}}{P_{in}}$$

But the total input power to the antenna can be written as,

$$P_{in} = P_{rad} + P_{loss}$$

Hence the radiation efficiency can be written as,

$$\boxed{\eta_r = \frac{P_{rad}}{P_{rad} + P_{loss}}}$$

The power radiated and the ohmic power loss can be expressed interms of r.m.s. current as,

$$P_{rad} = I^2_{r.m.s.} \; R_{rad}$$

and

$$P_{loss} = I^2_{r.m.s.} \; R_{loss}$$

Then the radiation efficiency is given by,

$$\boxed{\eta_r = \frac{R_{rad}}{R_{rad} + R_{loss}}}$$

The ratio of the power radiated in a particular direction $(\theta, \phi)$ to the actual power input to the antenna is called **power gain of antenna**. The power gain of the antenna is denoted by $G_p(\theta, \phi)$ and it is given by

$$\boxed{G_P(\theta,\phi) = \frac{P_d(\theta,\phi)}{P_{in}}}$$

The maximum power gain can be defined as the ratio of the maximum radiation intensity to the radiation intensity due to isotropic lossless antenna.

$$\therefore \qquad \boxed{G_{P\,max} = \frac{\text{Maximum radiation intensity}}{\text{Radiation intensity due to isotropic lossless antenna}}}$$

$$\therefore \qquad G_{P\,max} = \frac{U_{max}}{\left(\frac{P_{in}}{4\pi}\right)}$$

But the maximum radiation intensity is given by

$$U_{max} = \frac{P_{rad}}{4\pi}\, G_{D\,max} = \frac{(\eta_r \times P_{in})}{4\pi}\, G_{D\,max}$$

Substituting value of $U_{max}$ in the expression for maximum power gain, we get,

$$G_{P\,max} = \frac{\eta_r \left(\frac{P_{in}}{4\pi}\right) G_{D\,max}}{\left(\frac{P_{in}}{4\pi}\right)}$$

$$\therefore \qquad \boxed{G_{P\,max} = \eta_r \; G_{D\,max}}$$

For many practical antennas, the radiation efficiency $\eta_r$ is 100%. Then the maximum power gain is approximately same as the directivity or the maximum directional gain of the antenna. Generally, both power gain and the directional gain are expressed in decibels (dB).

## 2.9 Front to Back Ratio (FBR)

It is the ratio of the power radiated in the desired direction to the power radiated in the opposite direction.

$$\therefore \qquad \boxed{\text{FBR} = \frac{\text{Power radiated in desired direction}}{\text{Power radiated in opposite direction}}}$$

Obviously, the front-to-back ratio value desired is very high as it is expected to have large radiation in the front or desired direction rather than that in the back or opposite direction.

The FBR depends on frequency of operation. So when frequency of antenna changes, the FBR also changes. Similarly the FBR depends on the spacing between the antenna elements. If the spacing between antenna elements increases the FBR decreases. The FBR also depends on the electrical length of the parasitic elements of the antenna.

The FBR can be raised by divertng the gain of backward direction response of the antenna to the front or forward or desired drection by adjusting the length of the parasitic elements. The method of adjusting the electrical length of the parasitic element is called **tuning.** Thus higher FBR is obtained at the cost of gain from the opposite direction.

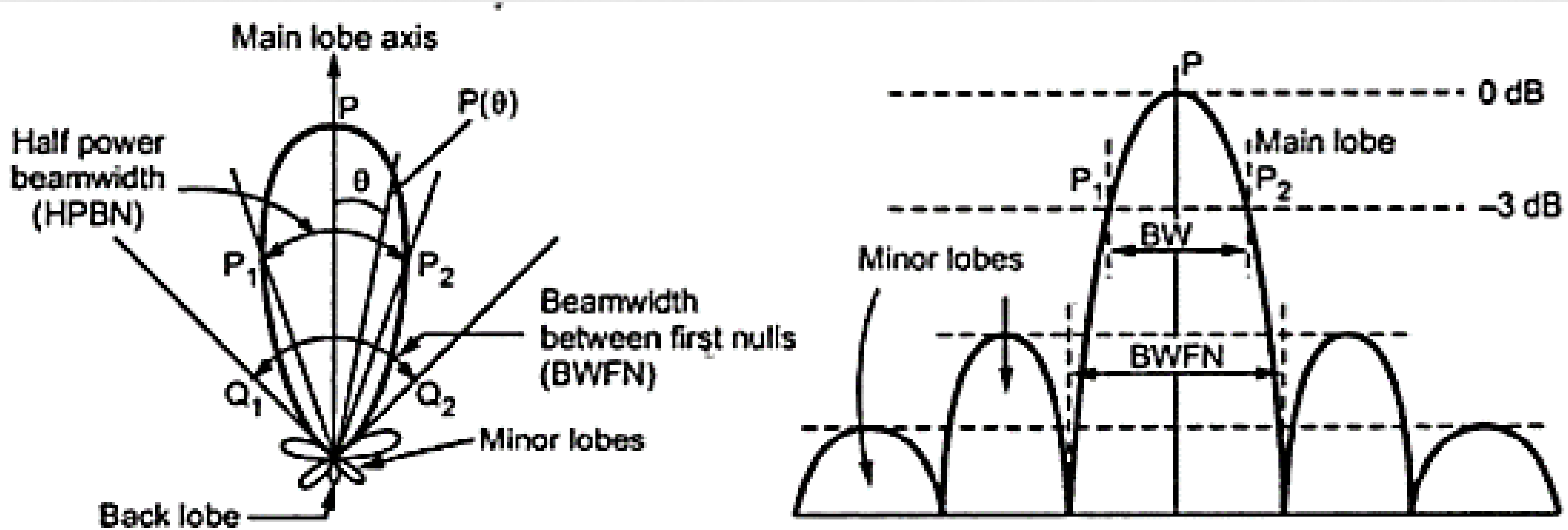
Practically the FBR is important in case of the receiving antennas rather than transmitting antennas. At the receiving antenna, adjustments are made in such a way to obtain maximum FBR rather than maximum gain.

## 2.10 Antenna Beamwidth

Basically antenna beamwidth is the measure of the directivity of the antenna. The antenna beamwidth is an angular width in degrees. It is measured on a radiation pattern on major lobe.

The **antenna beamwidth** is defined as the angular width in degrees between the two points on a major lobe of a radiation pattern where the radiated power decreases to half of its maximum value. The measurement of the antenna beamwidth is illustrated in the Fig. 2.6 (a) and (b).

In the Fig. 2.6 (a), antenna power pattern is represented on linear scale in polar co-ordinates. The same pattern is represented on logarithmic scale in rectangular co-ordinates in the Fig. 2.6 (b).




(a) Antenna power pattern in polar co-ordinates

(b) Antenna power pattern in rectangular co-ordinates and logarithmic scale

**Fig. 2.6 Measurement of antenna beamwidth**

The beamwidth is also called Half Power Beamwidth (HPBW) because it is measured between two points on the major lobe where the power is half of its maximum power, From Fig. 2.6 (a) it is clear that the power is maximum at point P, while it is half at points $P_1$ and $P_2$ both. Hence the angular width between points $P_1$ and $P_2$ is nothing but antenna **beamwidth** or **half power beamwidth (HPBW).** The beamwidth is also called **3-dB beamwidth** as reduction of power to half of its maximum value corresponds to the reduction of power (expressed in dB) by 3 dB. From Fig. 2.6 (b) it is clear power is maximum at point P, but at points $P_1$ and $P_2$ the power is 3 dB down the maximum power.

Many times, the antenna radiation pattern is described interms of the angular width between first nulls or first side lobes. Then such an angular beamwidth is called **Beamwidth between First Nulls (BWFN).**

The directivity (D) of the antenna is related with beam solid angle $\Omega_A$ or beam area B through expression.

$$\boxed{D = \frac{4\pi}{\Omega_A} = \frac{4\pi}{B}} \quad \ldots(1)$$

where $\quad B = $ Beam area

$\approx$ (HPBW) in horizontal plane × (HPBW) in vertical plane.

$\approx$ (HPBW) in E-plane × (HPBW) in H-plane

or $\quad B \approx \theta_E \times \theta_H \quad$ ... where $\theta_E$ and $\theta_H$ in radians.

$$\therefore \quad \boxed{D = \frac{4\pi}{\theta_E \theta_H} \text{ ... if } \theta_E \text{ and } \theta_H \text{ are in radians}} \quad \text{...(2)}$$

We can convert angles expressed in radians into angles in degrees by using relation,

$$1 \text{ rad} = \frac{180^\circ}{\pi} = 57.295^\circ \approx 57.3^\circ$$

Then,

$$\therefore \quad \boxed{D = \frac{4\pi(57.3)^2}{\theta_E^\circ \theta_H^\circ} = \frac{41257}{\theta_E^\circ \theta_H^\circ}} \quad \text{...(3)}$$

Above formula is approximate formula and it is applicable only to the antennas with narrow beamwidth (about 20°) with no minor lobes in the radiation pattern.

The beamwidth of the antenna is affected by the shape of the radiation pattern, wavelength and dimensions.

## 2.11 Antenna Beam Efficiency

To examine the quality of the transmitting and receiving antennas, the antenna beam efficiency parmeter is important. For the antenna with major lobe coincident with z-axis the **beam efficiency** is defined as,

$$\boxed{BE = \frac{\text{Power transmitted (or received) within the cone angle } \theta_1}{\text{Power transmitted (or received) by antenna}}} \quad \ldots (1)$$

Where $\theta_1$ is the half angle of the cone within which the percentage of total power is found.

Mathematically, the beam efficiency is given by,

$$\boxed{BE = \frac{\int_{\phi=0}^{2\pi} \int_{\theta=0}^{\theta_1} U(\theta, \phi) \sin\theta \, d\theta \, d\phi}{\int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} U(\theta, \phi) \sin\theta \, d\theta \, d\phi}} \quad \ldots (2)$$

The beam efficiency can be expressed interms of the mean beam area ($\Omega_M$) and total beam area ($\Omega_A$). Then the **beam efficiency can also be defined as the ratio of the main beam area to the total beam area.** It is also denoted by $\varepsilon_M$.

$$\therefore \quad \boxed{BE = \varepsilon_M = \frac{\Omega_M}{\Omega_A}} \quad \ldots (3)$$

Note that the total beam area ($\Omega_A$) is the combination of the main beam area ($\Omega_M$) and the minor lobe area ($\Omega_m$) i.e.

$$\boxed{\Omega_A = \Omega_M + \Omega_m} \qquad \ldots(4)$$

Dividing equation (4) by $\Omega_A$ on both the sides, we get

$$1 = \frac{\Omega_M}{\Omega_A} + \frac{\Omega_m}{\Omega_A}$$

where $\dfrac{\Omega_M}{\Omega_A} = \dfrac{\text{Main beam area}}{\text{Total beam area}} = \varepsilon_M$ = **Beam efficiency**

And $\dfrac{\Omega_m}{\Omega_A} = \dfrac{\text{Minor lobe area}}{\text{Total beam area}} = \varepsilon_m$ = **Stray factor**

Thus $$\boxed{\varepsilon_M + \varepsilon_m = 1} \qquad \ldots(5)$$

## 2.12 Antenna Bandwidth ($\Delta\omega$)

In general, the performance of antenna depends on various characteristics such as antenna gain, side lobe level, standing wave ratio (SWR), antenna impedance, radiation patterns antenna polarization, front-to-back (FBR) ratio etc. During the operation of antenna these requirements may change. Thus there is no unique definition for antenna bandwidth. The functional bandwidth of the antenna is generally limited by one or more factors mentioned above. So the antenna bandwidth can be specified in many ways such as

i) bandwidth over which the gain of the antenna is higher than the acceptable value, or

ii) bandwidth over which the standing wave ratio of transmission line feeding antenna is below acceptable value, or

iii) bandwidth over which the FBR is minimum equal to the specified value.

Thus in general we can define the bandwidth of antenna as the band of frequencies over which the antenna maintains required characteristics to the specified value. But as the requirements of antenna change during the operation, the specifications are set depending upon the application for which that antenna is used. That means for certain antenna where due to the increase in side lobe level, antenna gain decreases and resistance value changes, then the lower frequency limit is obtained by considering one of the parameters like pattern, gain or impdeance. While the other parameters decide higher freqency limit.

In general, the antenna bandwidth mainly depends on impedance and pattern of antenna. At low frequency, generally impedance variation decides the bandwidth as pattern characteristics are frequency insensitive. Under such condition, bandwidth of the antenna is inversely proportional to Q factor of antenna.

Thus bandwidth can be expressed mathematically as,

$$\text{Bandwidth} = (\text{B.W.}) = \Delta\omega = \omega_2 - \omega_1 = \frac{\omega_0}{Q}$$

$$\therefore \qquad \boxed{\Delta\omega = \frac{\omega_0}{Q}} \qquad \ldots(1)$$

$$\therefore \qquad \boxed{\Delta f = f_2 - f_1 = \frac{f_0}{Q}\,\text{Hz}} \qquad \ldots(2)$$

Where $f_0$ is the **centre frequency** or **design frequency** or **resonant frequency**, while Q factor of antenna is given by,

$$\boxed{Q = 2\pi \frac{\text{Total energy stored by antenna}}{\text{Energy radiated per cycle}}} \qquad \ldots(3)$$

Thus for lower Q antennas, the antenna bandwidth is very high and vice a versa.

## 2.13 Effective Length

The effective length of an antenna carrying peak current $I_m$ is defined as the length of an imaginary linear antenna with a uniformly distributed current, such that both the antennas have the same far field in $\theta = \frac{\pi}{2}$ plane. It is represented by $L_{eff}$.

For practical antenna,

$$E_\theta = j \frac{\eta_0 \beta}{4\pi r} e^{-j\beta r} \int_{-l/2}^{l/2} I(z)\, dz \qquad \text{... (1)}$$

For practical antenna, variation of current can have any distribution. But for an imaginary antenna, current is assumed to be uniformly distributed over the length. Hence for imaginary antenna,

$$E_\theta = j \frac{\eta_0 \beta}{4\pi r} e^{-j\beta r} I_m \int_{-\frac{l_{eff}}{2}}^{\frac{l_{eff}}{2}} dz$$

$$\therefore \quad E_\theta = j \frac{\eta_0 \beta}{4\pi r} e^{-j\beta r} I_m \left[ L_{eff} \right] \qquad \text{... (2)}$$

But practical and imaginary antenna should produce same electric field at far point. So equating equations (1) and (2), we can write,

$$L_{eff} = \frac{1}{I_m} \int_{-l/2}^{l/2} I(z)\, dz \quad \text{... (3)}$$

Equation (3) represents the effective length of a transmitting antenna. The effective length of a receiving antenna is defined as the ratio of the open circuit voltage $V_{oc}$ induced at the open terminals of an antenna to the incident electric field intensity $E_i$ producing $V_{oc}$.

$$\therefore \quad \boxed{L_{eff} = \frac{V_{oc}}{E_i}} \quad \text{... (4)}$$

## 2.14 Effective Aperture or Effective Area

In general, this term is used in relation with the receiving antenna. The **effective aperture** is the ability of antenna to extract energy from the electromagnetic wave. It is also called **effective area.** Effective aperture is defined as the ratio of power received in the load to the average power density produced at the point.

$$\therefore \qquad \boxed{A_e = \frac{P_{received}}{\overline{P}_{avg}} \ m^2} \qquad ...(1)$$

Power received by the antenna may be denoted by $P_R$. It is measured in watts. The power density is measured in watts/$m^2$. Hence effective aperture is measured in $m^2$.

In other words we can explain effective aperture as an area which extracts energy from the electromagnetic wave, out of the total area of antenna. Obviously an antenna should have maximum useful area to extract energy. Thus the maximum effective aperture is obtained when power received is maximum. It is denoted by $A_{em}$. In general, antenna has certain impedance which is made up of resistive and reactive part. The resistive part of the impedance is nothing but the radiation resistance $R_{rad}$. This condition is true because same antenna can be used for transmitting and receiving signal. Now the power transfered will be maximum if the load connected to the antenna is complex conjugate of the antenna impedance.

- **UNDER MAXIMUM POWER TRANSFER CONDITION THE POWER RECEIVED IS MAXIMUM AND HENCE THE EFFECTIVE APERTURE IS MAXIMUM.**

$$\therefore \quad \boxed{A_{em} = 1.5 \frac{\lambda^2}{4\pi}} \quad \dots (6)$$

Above equation represents the maximum effective aperture of the Hertzian dipole. But the directivity of the Hertzian dipole is 1.5.

Hence we can rewrite the expression for the maximum effective aperture as,

$$\boxed{A_{em} = (G_{D\,max}) \frac{\lambda^2}{4\pi}} \quad \dots (7)$$

### 2.14.1 Relationship between Directive Gain, Radiation Resistance and Effective Length

Consider two antennas as antenna 1 and antenna 2. Let the directivities of these antennas be denoted by $D_1$ and $D_2$. Assume that their maximum effective areas are denoted by $(A_{e_1})_{max}$ and $(A_{e_2})_{max}$. As we have discussed, the directivity of an antenna is proportional to the maximum effective area. Hence we can write.

$$D_1 \propto (A_{e_1})_{max} \qquad \text{...(8)}$$

And

$$D_2 \propto (A_{e_2})_{max} \qquad \text{...(9)}$$

$$\therefore \quad \frac{D_1}{D_2} = \frac{(A_{e_1})_{max}}{(A_{e_2})_{max}} \qquad \text{...(10)}$$

Let antenna 1 be the isotropic radiator for which the directivity is unity. i.e. $D_1 = 1$. Hence we can write,

$$\frac{1}{D_2} = \frac{(A_{e_1})_{max}}{(A_{e_2})_{max}}$$

$$\therefore \qquad D_2 = \frac{(A_{e_2})_{max}}{(A_{e_1})_{max}} \qquad \text{...(11)}$$

and
$$(A_{e_1})_{max} = \frac{(A_{e_2})_{max}}{D_2} \qquad \text{...(12)}$$

Let us assume that antenna 2 be the test antenna which is a short dipole. As we know for the short dipole antenna, maximum effective aperture is $\left(\frac{3}{8\pi}\right)\lambda^2$ and the directivity is $\frac{3}{2}$. Hence we can write,

$$(A_{e_1})_{max} = \frac{\left(\frac{3}{8\pi}\right)\lambda^2}{\left(\frac{3}{2}\right)} = \frac{\lambda^2}{4\pi} \qquad \text{...(13)}$$

Putting value of $(A_{e_1})_{max}$ in the expression for $D_2$, we get,

$$D_2 = \frac{(A_{e_2})_{max}}{\left(\frac{\lambda^2}{4\pi}\right)} = \frac{4\pi}{\lambda^2}(A_{e_2})_{max} \qquad \text{...(14)}$$

Hence in general we can write.

$$\boxed{D = \frac{4\pi}{\lambda^2}(A_e)_{max}}$$

But maximum effective aperture can be expressed interms of the effective length as,

$$(A_e)_{max} = \frac{L_{eff}^2 \eta_0}{4 R_{rad}} \quad \ldots(15)$$

Hence putting value of $(A_e)_{max}$ in the general expression of D given above, we get, $D = \left(\frac{4\pi}{\lambda^2}\right)\left(\frac{L_{eff}^2 \eta_0}{4 R_{rad}}\right)$.

$$\therefore \quad \boxed{D = \left(\frac{\pi}{\lambda^2}\right) L_{eff}^2 \left(\frac{\eta_0}{R_{rad}}\right)} \quad \ldots(16)$$

## 2.15 Antenna Input Impedance

The input impedance of an antenna is a function of frequency. At a single frequency antenna impedance can be represented as a complex impedance with a resistance in series with a reactance. Eventhough this representation is used for single frequency, we can extend this for a small band of frequencies also. But for the better approximation, the antenna is represented as a series R-L-C resonant circuit where the resonant frequency of the antenna becomes centre frequency in the band of frequencies defined. But again this approximation fails if the frequency band extends further. For such extension in the frequency band, we can add elements in the equivalent circuit of the antenna. But if frequency band extends further, the number of additional elements in the equivalent circuit becomes very large which is not possible practically. So for better approximation, the lumped network (i.e. series R-L-C circuit) is replaced by a distributed network such as transmission line. Such open circuited transmission line represents the input impedance of the antenna over a wide range of frequencies.

## 2.17 Radiation Resistance

In general, an antenna radiates power into free space in the form of electromagnetic waves. So the power dissipated is given by,

$$W' = I^2 \cdot R \qquad \ldots (1)$$

Assuming all the power dissipated in the form of electromagnetic waves, then we can write,

$$R = \frac{W'}{I^2} \qquad \ldots (2)$$

The resistance which relates power radiated by radiating antenna and the current flowing through the antenna is a fictitious resistance. Such resistance is called radiation resistance of antenna and it is denoted by $R_{rad}$ or $R_a$ or $R_0$.

**Key Point** : *The radiation resistance is a fictitious resistance such that when it is connected in series with antenna dissipates same power as the antenna actually radiates.*

But practically the energy supplied to the antenna is not completely radiated in the form of electromagnetic waves, but there are certain radiation losses due to the loss resistance denoted by $R_{loss}$. Thus the total power is given by,

$$W = W' + W'' = \text{Ohmic loss} + \text{Radiation loss}$$

$$\therefore \quad W = I^2 R_{rad} + I^2 R_{loss}$$

$$\therefore \quad \boxed{W = I^2 (R_{rad} + R_{loss})} \qquad \dots (3)$$

**Key Point** : *The radiation resistance of antenna depends on antenna configuration, ratio of length and diameter of conductor used, location of the antenna with respect to ground and other objects.*

## 2.20 Antenna Polarization

Polarization is nothing but the physical orientation of the electromagnetic wave in the free space. The antenna polarization in a given direction refers to the polarization of an electromagnetic wave radiated or transmitted by the antenna. When the direction is not specified, the polarization is then conveniently considered to be the polarization in the direction of maximum gain. The different parts of the antenna pattern may have different polarization because the polarization varies with the distance from the centre of the antenna.

Polarization of an electromagnetic wave describes the time varying direction and relative magnitude of the electric field vector. Thus conventionally, the polarization is described in terms of the electric field vector $\mathbf{E}$. The polarization of the electric field $\mathbf{E}$ can be obtained by observing the field along the direction of propagation.

# THANK YOU