Chapter 02: Representation of signals and systems in Communication

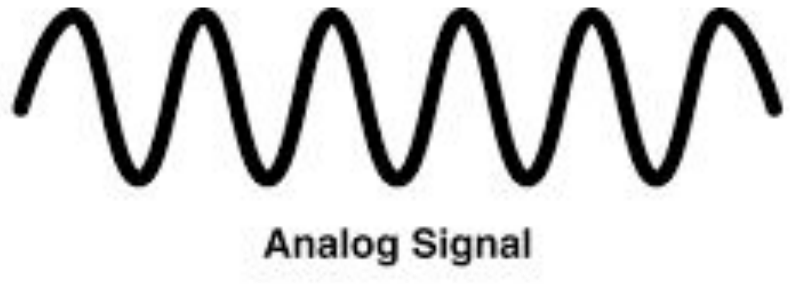
Review of signals:

Signal:

The physical quantity that contains the information. The signal may be one dimensional or multidimensional

Example:

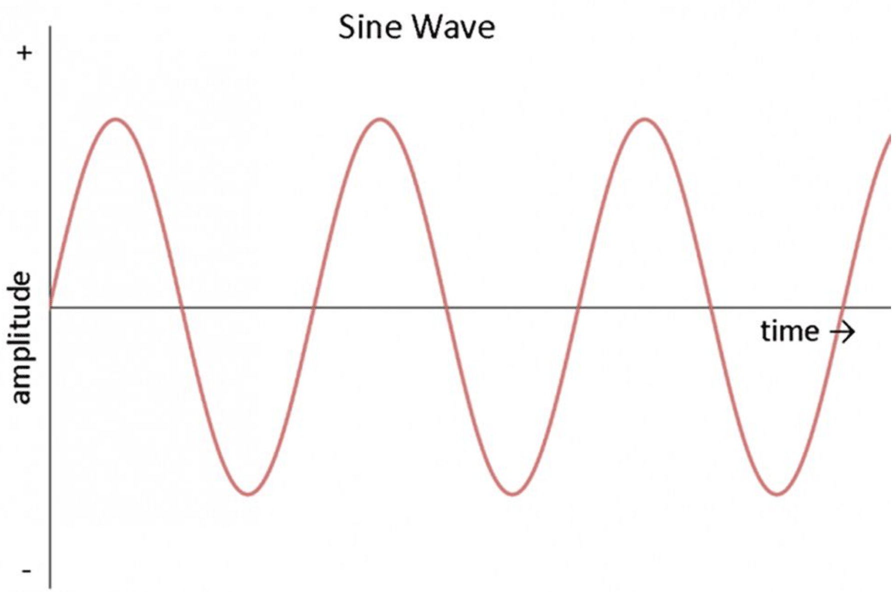
I(t) , V(t) , X(t1,t2,t3)



Types of Signal or Classification of Signals

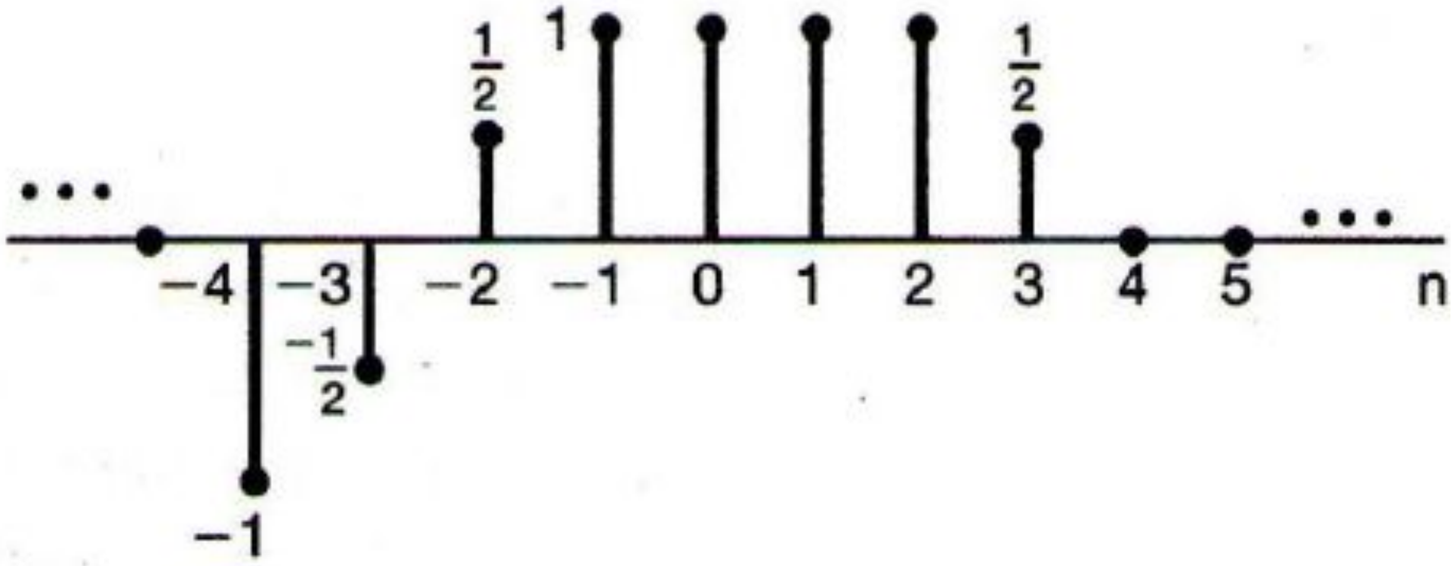
1. Continuous time signal and discrete time signal

A continuous time signal is an infinite and uncountable set of numbers i.e. between a start and end time, there are infinite possible values for time ‘t’ and instantaneous amplitude, x(t).



In discrete time signal:

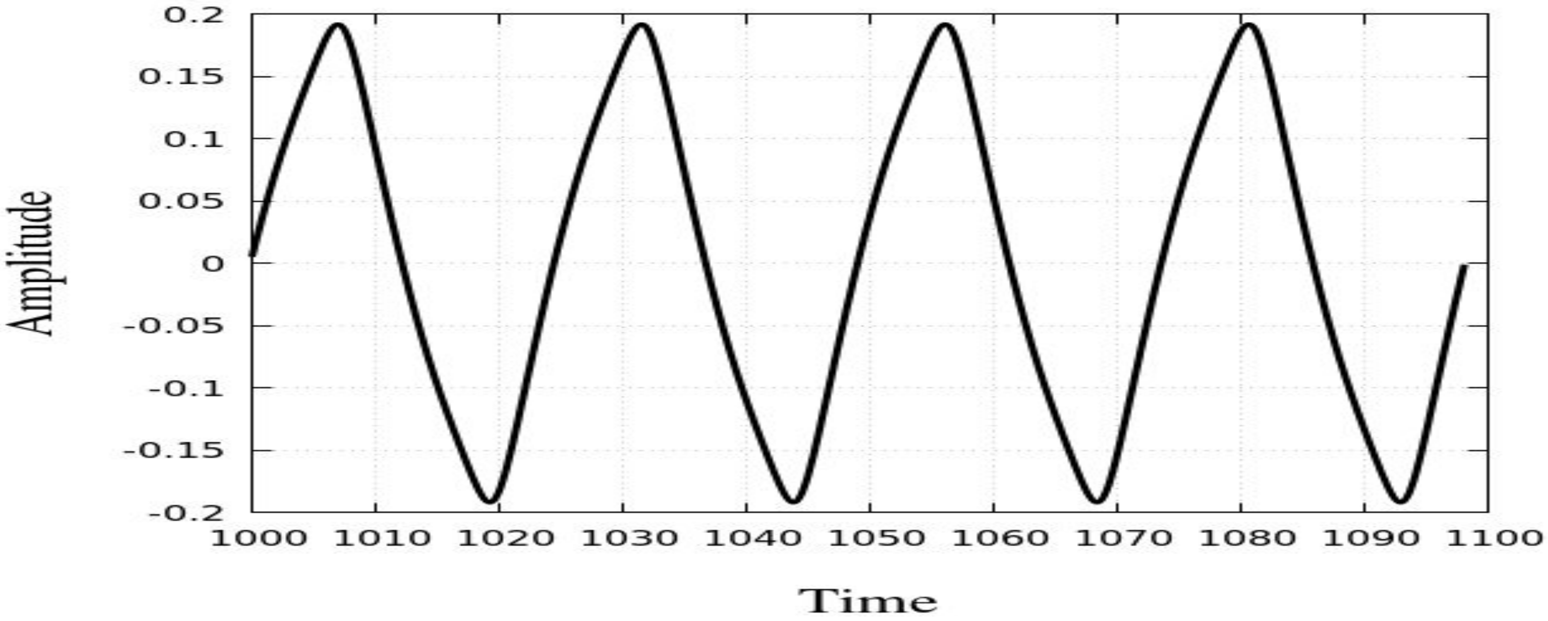
- The number of elements in the set as well as the possible values of each element is finite and countable and represented with bits.



2. Periodic and non-periodic signal

A signal which repeats itself after a specific interval of time is called periodic signal. They are deterministic signal.

Examples: sine, cosine, square wave



For Continuous:

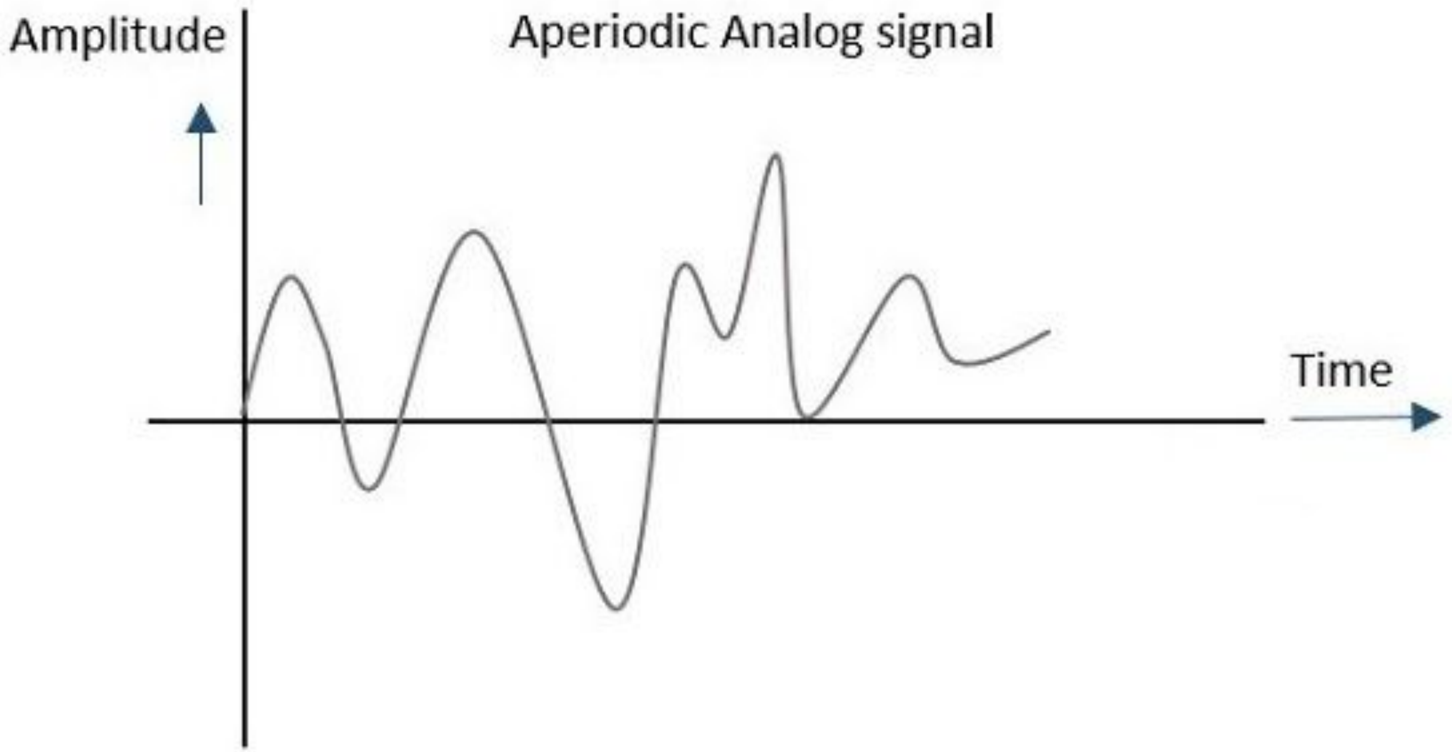
$X(t) = X(t + KT_0)$
for all integer K
and positive value T_0

For Discrete:

$X[n] = X(n + KN_0)$

Non Periodic signals are the signals which does not repeat itself after a specific interval of time. They are random signals. They cannot be represented in form of mathematical equations.

Examples: sound signals from radio noise signals.

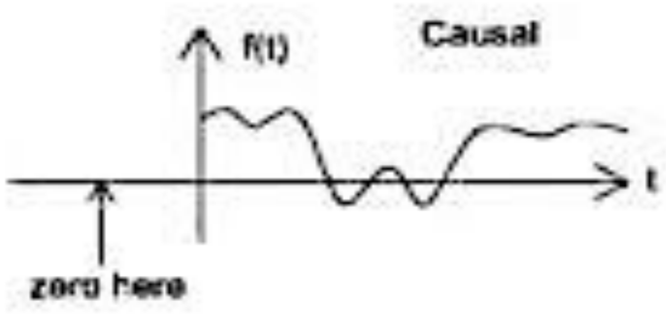


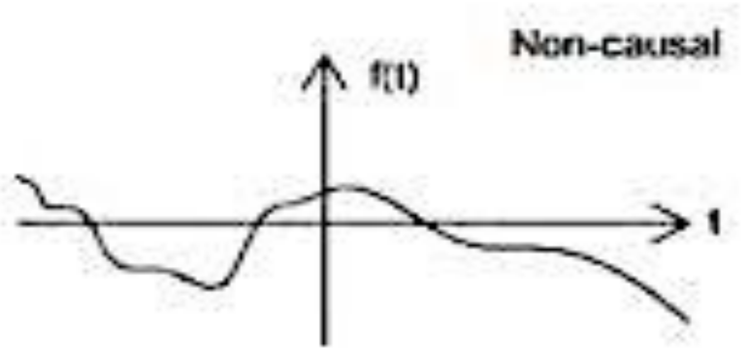
3. Casual and Non Casual Signals

Casual signals are signals that are zero for all negative time.

here **x(t) = 0 , t<0**

Non casual signals are signals that have non zero values in both negative and positive time.





<u>Example :</u> Causal and Noncausal.

$$y(n) = \frac{1}{3}\left(x[n+1] + x[n] + x[n-1]\right)$$

Causal or non-causal?

Solution:

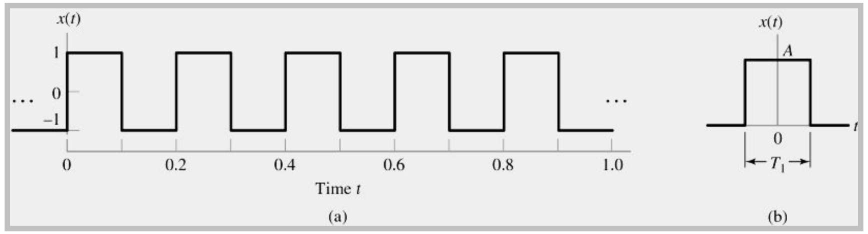
Non-causal; the output signal ***y*[*n*]** depends on a future value of the input signal, **x[n+1]**

$$y(n) = \frac{1}{3}\left(x[n+1] + x[n] + x[n-1]\right)$$

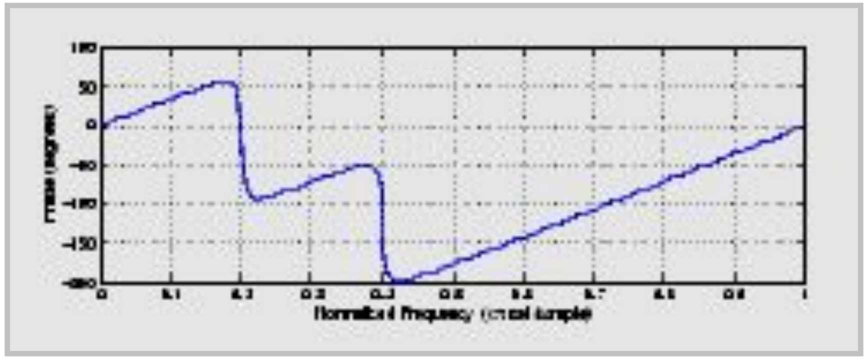
Causality is required for a system to be capable of operating in real time.

4. Deterministic and Random Signal

- **Deterministic signals** are those signals whose values are completely specified for any given time. Thus a deterministic signal can be modeled by a known function of time.

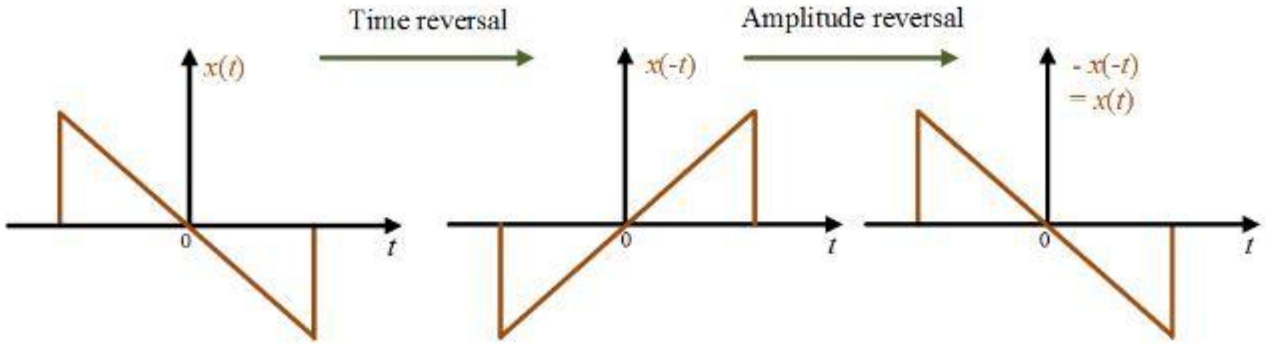


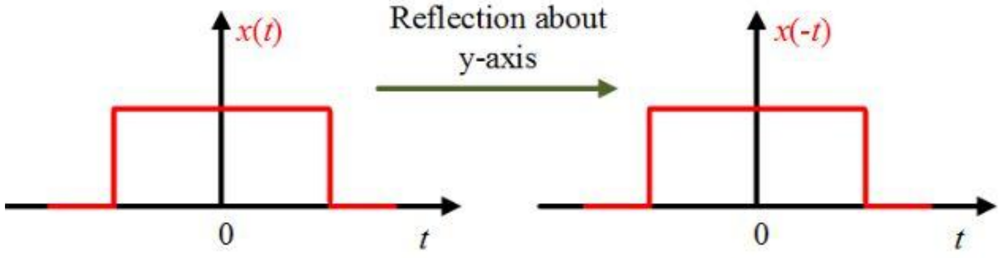
- **Random signals** are also called non-deterministic signals are those signals that take random values at any given time and must be characterized statistically.

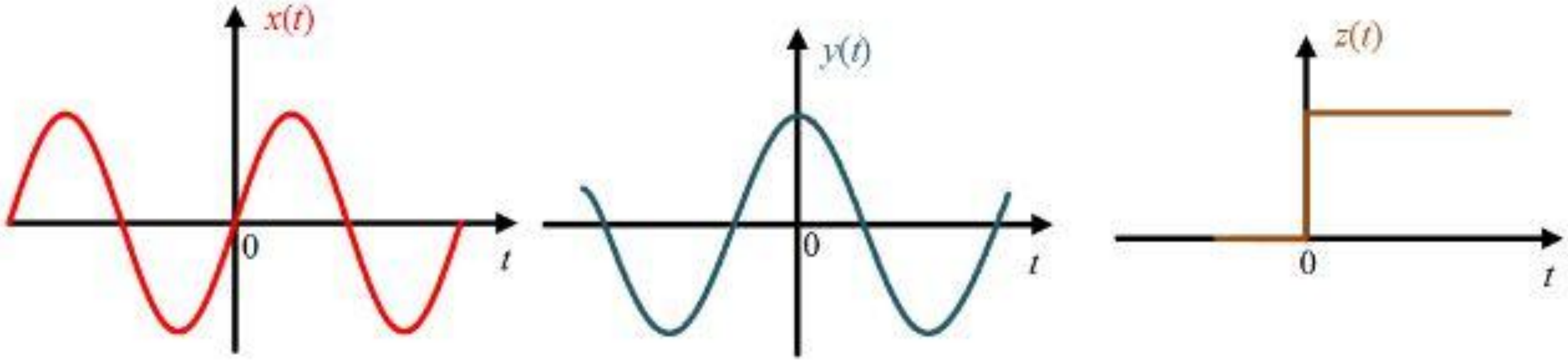


5. Even and odd Signals:

- If **x(-t)= x(t)** then the signal is even
- And if **x(-t)= - x(t)** then the signal is odd







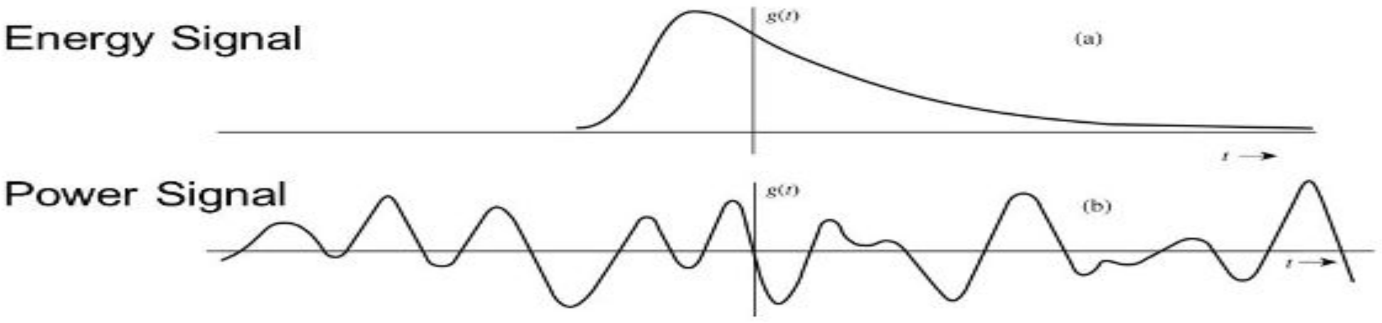
6. Energy and Power signal

- A signal is **energy signal** if total energy of signal satisfies the condition

  $0 < E < \infty$ And $P_{av} = 0$

- A signal is a **power signal** if and only if average power of signal satisfies the condition

  $0 < P < \infty$ and $E = \infty$



Energy $E_x = \int_{-\infty}^{\infty} x^2(t)\,dt$ $E_x = \int_{-\infty}^{\infty} |x(t)|^2\,dt$

Power $P_x = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\,dt$ $P_x = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2\,dt$

Special types of Signals

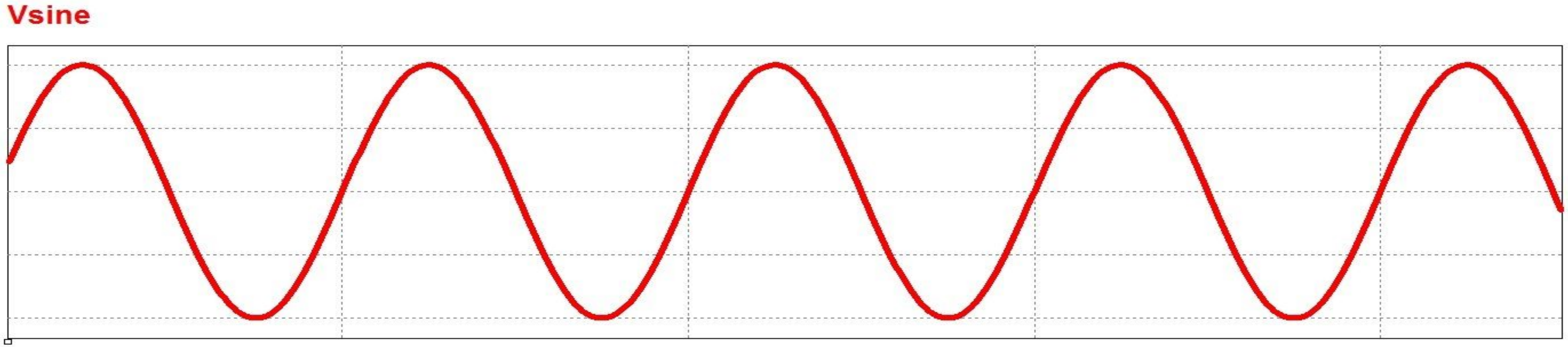
1. Harmonic signal

A periodic signal defined for:

$$- \infty \leq t \leq \infty$$

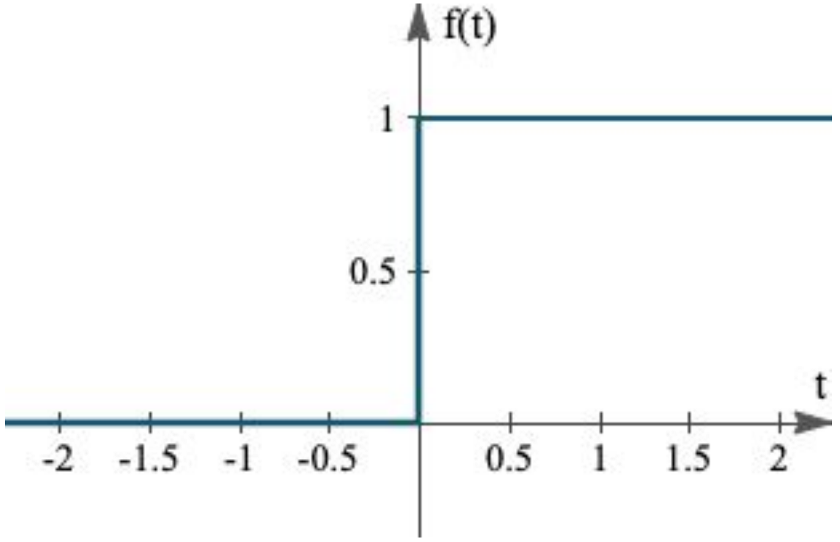
And expressed in terms of sinusoidal function

$x(t) = A \cos (2\pi f t + \theta)$



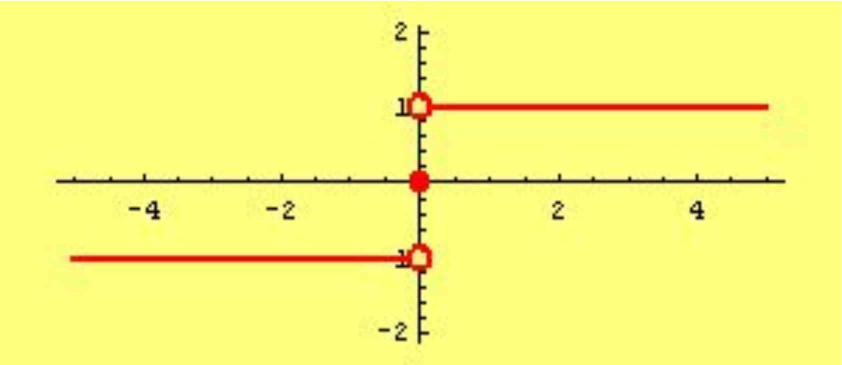
2. Unit step function, signum function and Impulse function

- A signal which exist only for positive side and is zero for negative side. The unit step signal is denoted by u(t).
- $u(t) = \begin{cases} 1 \text{ for } t \geq 0 \\ 0 \text{ for } t < 0 \end{cases}$

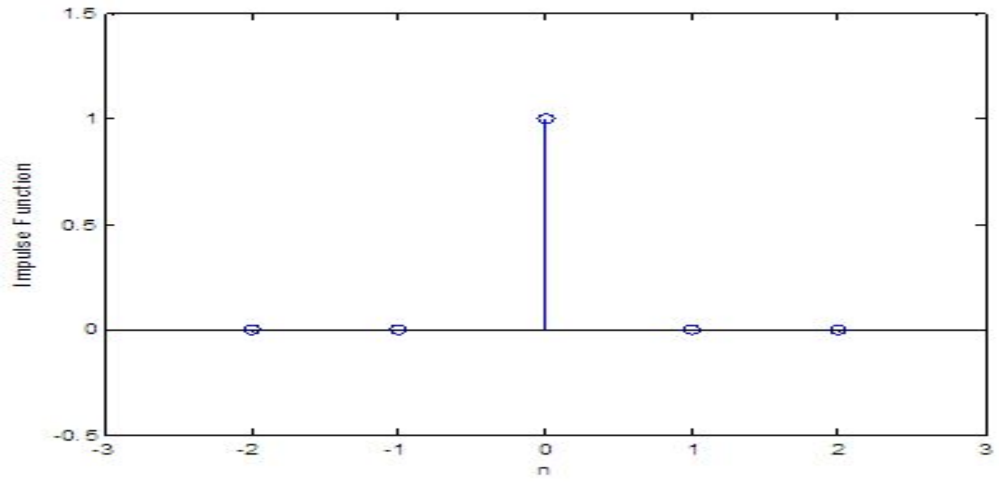


- **Signum function** is the signal used to define the sign of the signal

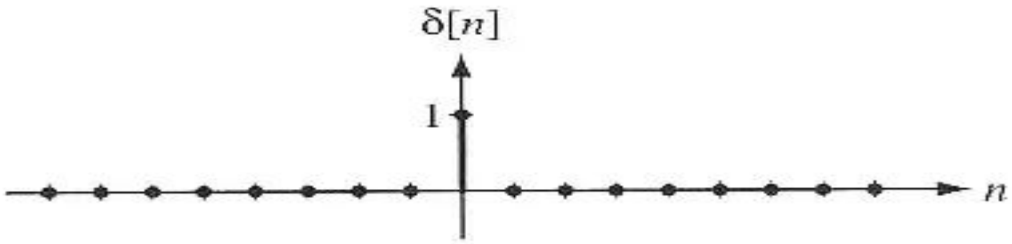
$$\mathrm{Sgn}(t) = \begin{cases} 1 \text{ for } t > 0 \\ -1 \text{ for } t < 0 \\ 0 \text{ for } t = 0 \end{cases}$$



Impulse function is the mathematical model to represent the physical phenomenon that takes place in very short period. It is also known as **delta signal**.



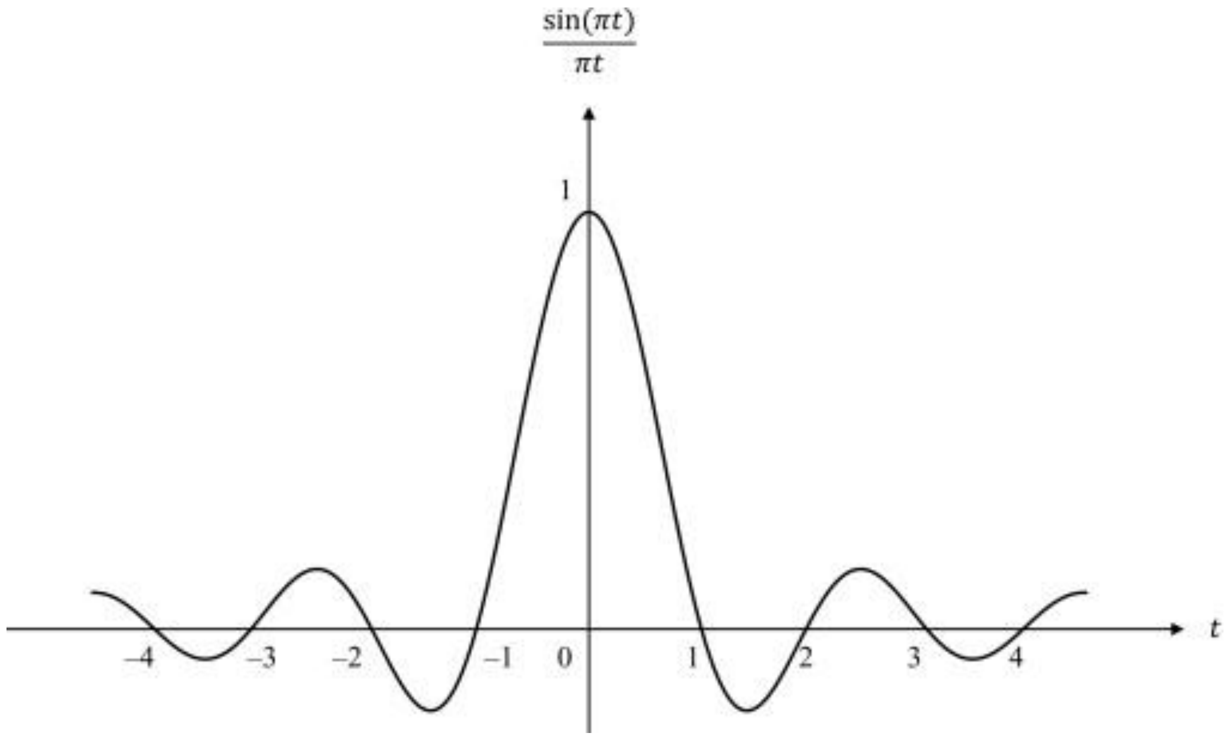
$$\delta[n] = \begin{cases} 1 \ , \ n = 0 \\ 0 \ , \ n \neq 0 \end{cases}$$



3. Sinc Signal:

- A **sinc** signal has its maximum value equal to 1 at t=0 and gradually tends to zero for t tending to infinite.

$$\mathrm{Sinc}(t) = \begin{cases} \dfrac{\sin \pi t}{\pi t} & \text{for } t \neq 0 \\ 1 & \text{for } t = 0 \end{cases}$$



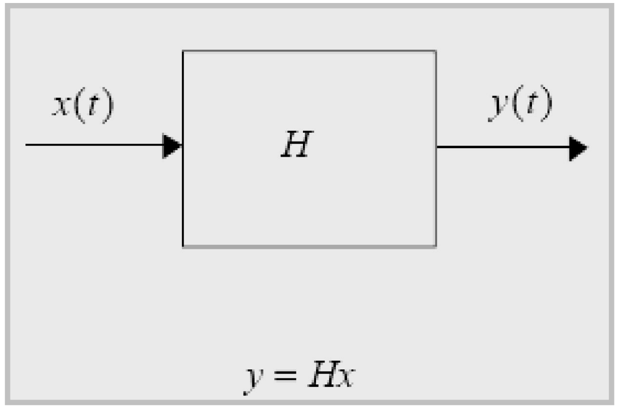
System:

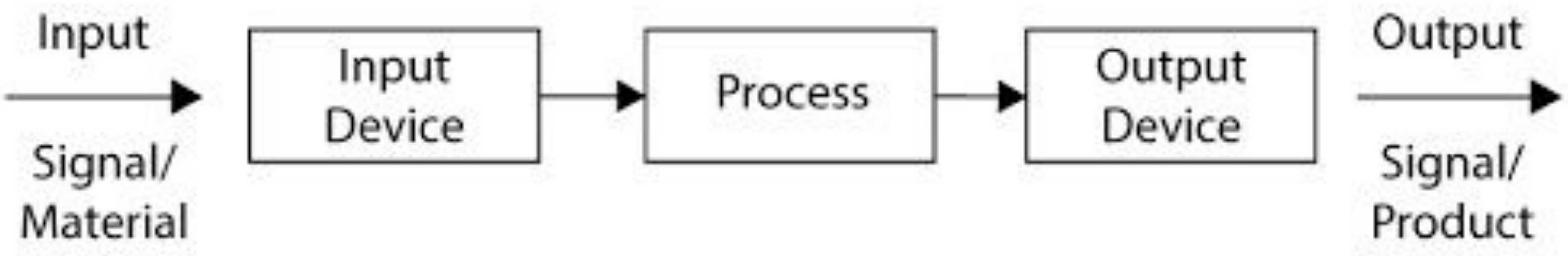
A system refers to any physical device that produces output signal in response to an input signal.

Input x(t)

Process h(t)

Output y(t)





Example of system

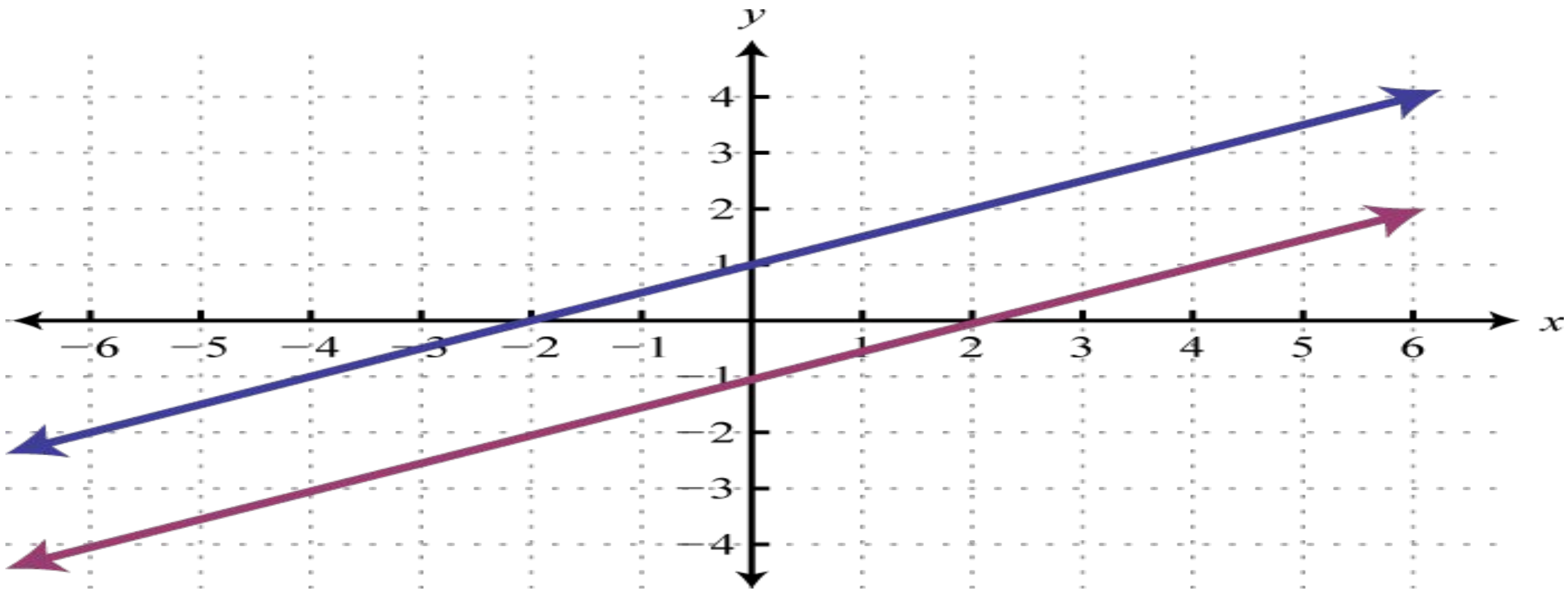
- In **automatic speaker recognition system**; the system is to extract the information from an incoming speech signal for the purpose of recognizing and identifying the speaker.
- In **communication system**; the system will transport the the information contained in the message over a communication channel and deliver that information to the destination.

Figure: Elements of a communication system.

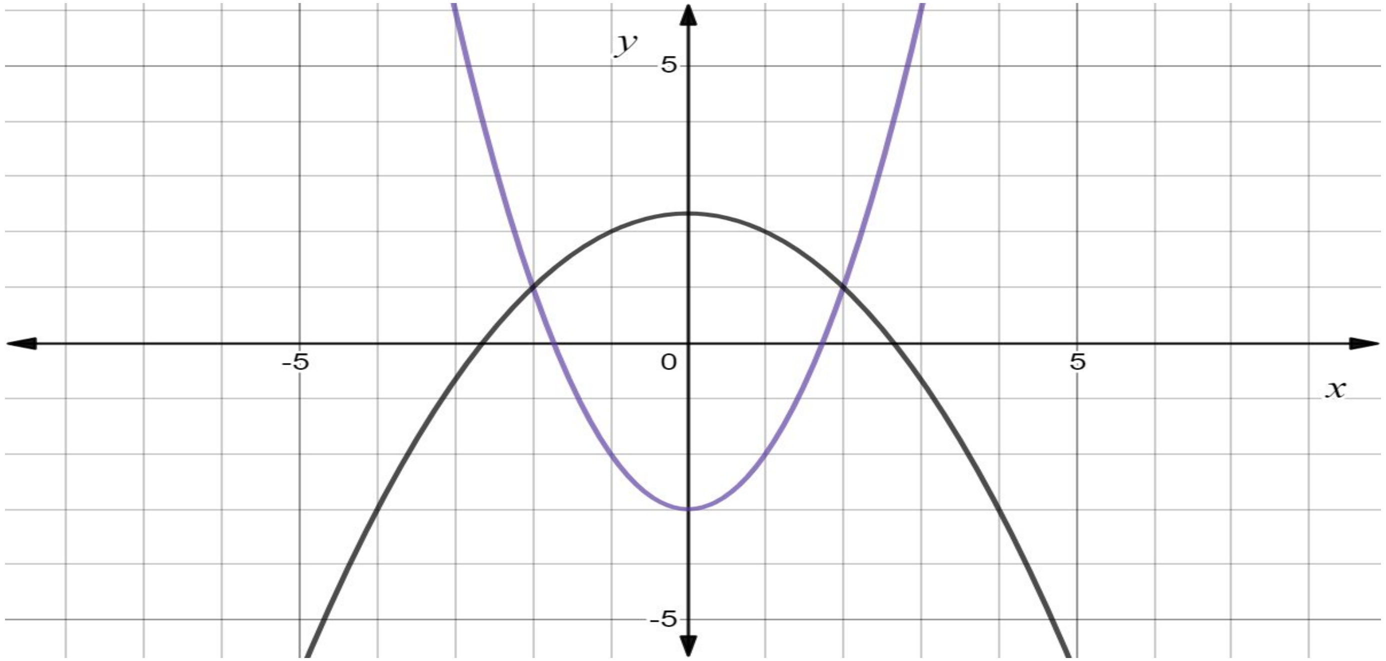
Figure : Block diagram representation of a system.

Linear and non linear system

- A linear system is any system that obeys the property of scaling and superposition. i.e. output is linearly proportional to input.



- Non linear system do not posses linearity.



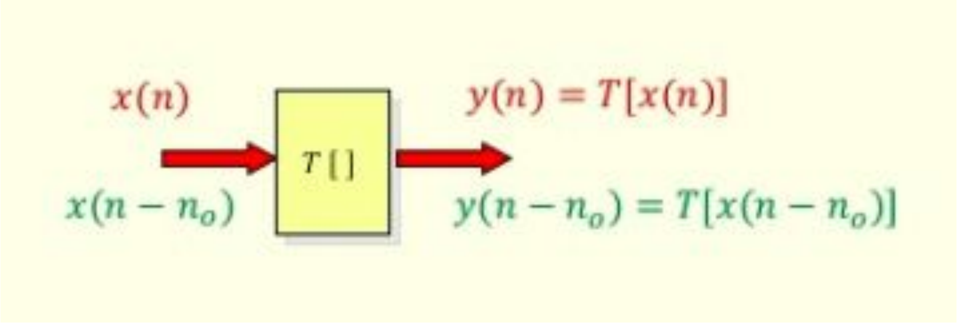
Time variant and Time invariant system

- A system is said to be time variant system if input output characteristics changes with time.

$X(n-n_0)$ □ $Y(n+n_0)$

- A system is said to be time invariant system if its input output characteristics do not changes with time.

$X(n-n_0)$ □ $Y(n-n_0)$



Properties of Linear time invariant system

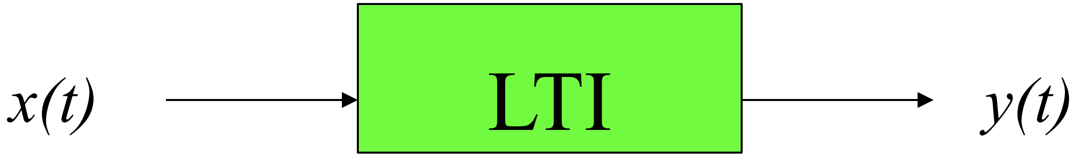
1. The output of the LTI system is convolution sum of its input and the impulse response (IR)
2. Commutative property: $\mathbf{x(t) * h(t) = h(t) * x(t)}$
3. Distributive property: $\mathbf{x(t) *(h_1(t) + h_2(t)) = x(t) *h_1(t) +x(t) *h_2(t)}$
4. Associative property: $\mathbf{x(t) *\{h1(t) *h2(t) \} = \{ x(t) * h1(t) \}*h2(t)}$
5. Memory and Memoryless LTI system: In memory LTI system, the output depends on present and past input and output value.

$$\mathbf{Y(t) = X(t) + Y(t-1)}$$

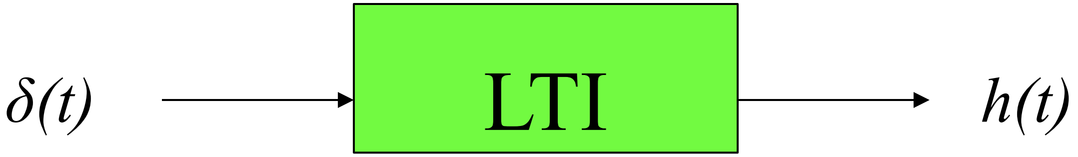
In memoryless LTI system, the output depends only on present input.

$$\mathbf{Y(t) = X(t)}$$

IMPULSE RESPONSE *h(t)*



$y(t)$ is the output of the continuous-time LTI system with input $x(t)$ and no initial energy.



With the unit impulse as an input [i.e., $x(t)=\delta(t)$], the output is defined as the **IMPULSE RESPONSE** and is represented by $h(t)$.

A conceptual view of convolution

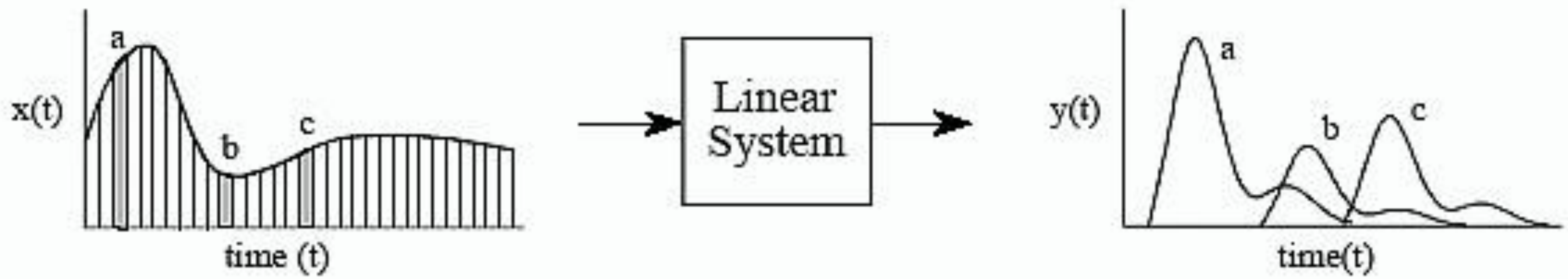


FIGURE 13-2
Convolution viewed from the input side. The input signal, $x(t)$, is divided into narrow segments, each acting as an impulse to the system. The output signal, $y(t)$, is the sum of the resulting scaled and shifted impulse responses. This illustration shows how three points in the input signal contribute to the output signal.

Convolution is a mathematical way of combining two **signals** to form a third **signal**.

Convolution is a mathematical operation used to express the relation between input and output of an **LTI** system. It relates input, output and impulse response of an LTI system as:

$$y(t)=x(t)*h(t)$$

CT CONVOLUTION INTEGRAL

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

$$= \int_{-\infty}^{\infty} x(t-\tau)h(\tau)d\tau$$

DT CONVOLUTION SUM

$$y[n] = x[n] * h[n] = \sum_{i=-\infty}^{\infty} x[i]h[n-i]$$

Commutative

$$x(t) * h(t) = h(t) * x(t)$$

$$x(t) \longrightarrow \boxed{h_1(t)} \longrightarrow \boxed{h_2(t)} \longrightarrow y(t)$$

Same output!

$$x(t) \longrightarrow \boxed{h_2(t)} \longrightarrow \boxed{h_1(t)} \longrightarrow y(t)$$

Distributive

$$x(t) * [h_1(t) + h_2(t)] = x(t) * h_1(t) + x(t) * h_2(t)$$

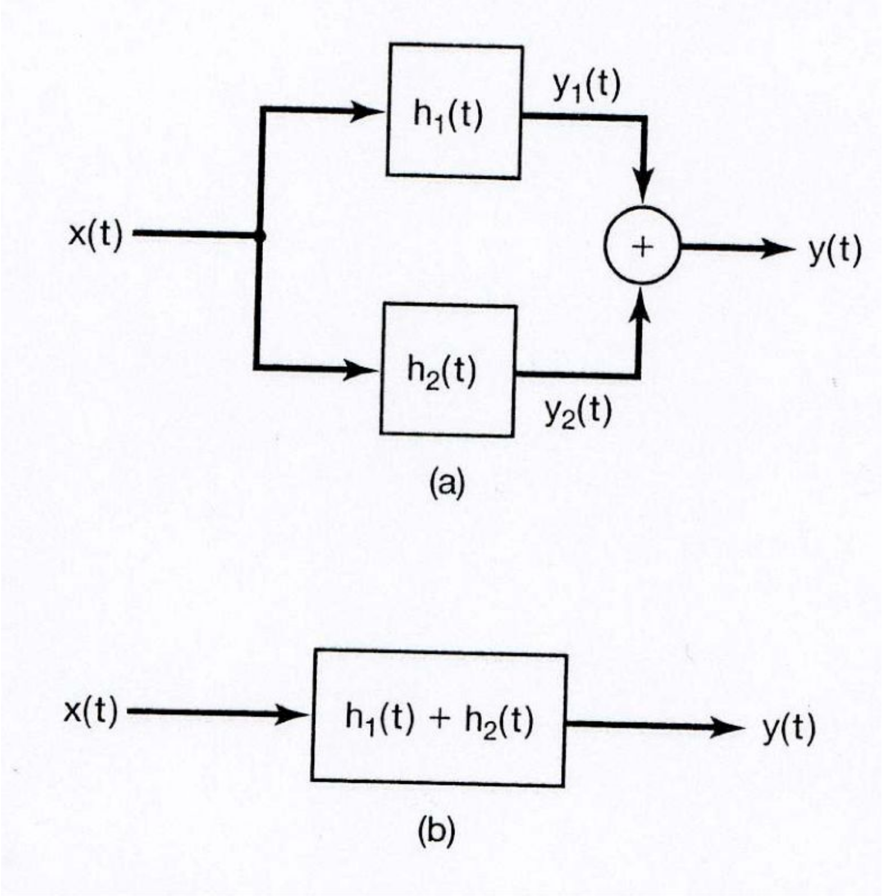
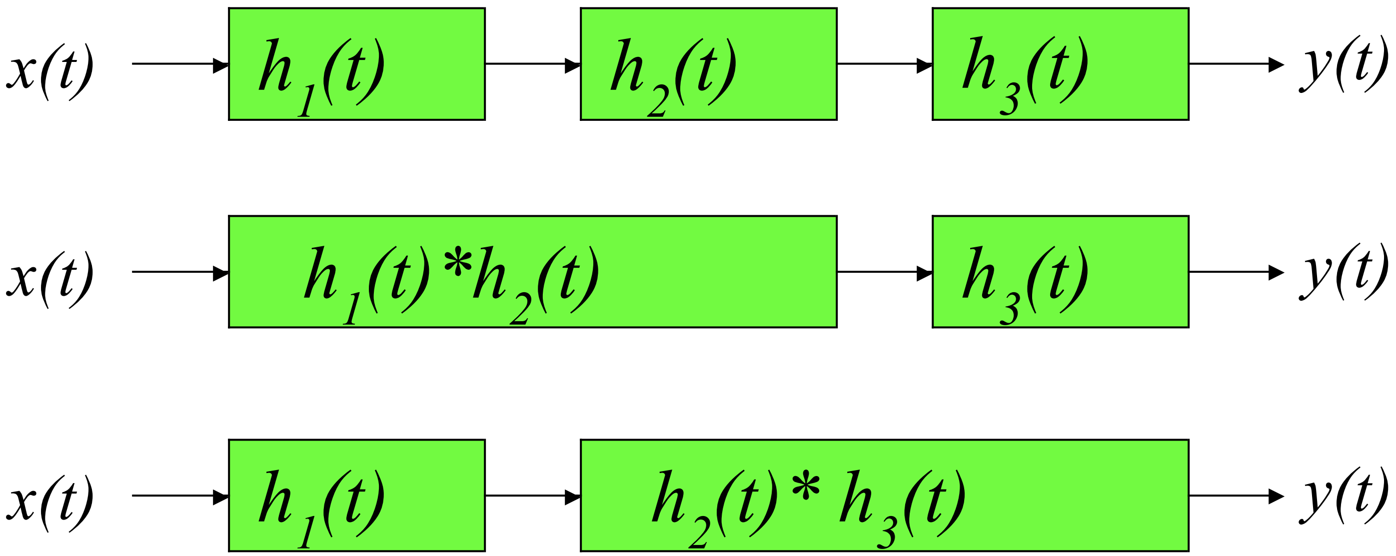


Figure 2.23 Interpretation of the distributive property of convolution for a parallel interconnection of LTI systems.

Associative

$$x(t) * [h_1(t) * h_2(t)] = [x(t) * h_1(t)] * h_2(t)$$



Same output!

Convolution with the unit impulse

$$x(t) * \delta(t) = \delta(t) * x(t) = \int_{-\infty}^{\infty} \delta(\lambda) x(t-\lambda) d\lambda$$

$$x(t) * \delta(t) = \int_{-\infty}^{\infty} \delta(\lambda) x(t) d\lambda = x(t) \int_{-\infty}^{\infty} \delta(\lambda) d\lambda = x(t)$$

Assignment

Determine v(t)*w_1 (t) and also v(t)*w_2(t). Show your work graphically and mathematically.

$$v(t) = e^{-t}u(t)$$

$$w_1(t) = \delta(t)$$

$$w_2(t) = \delta(t-10)$$

SYSTEM MEMORY

A system is said to possess **memory** if its output signal depend on **past** or **future** values of the input signal.

A system is **memoryless** if for any time $t=t_1$, the value of the output at time t_1 depends only on the value of the input at time $t=t_1$. In other words, the value of the output signal depends only on the **present value** of the input signal.

Example : Memory and Memory-less System.

Below is the moving-average system described by the input-output relation. Does it has memory or not?

(a) $\boxed{y[n]=\frac{1}{3}(x[n]+x[n-1]+x[n-2])}$

(b) $\boxed{y[n]=x^2[n]}$

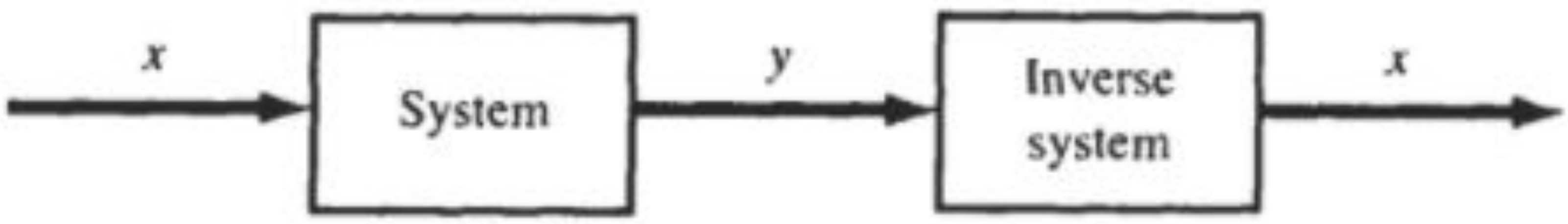
Solution:

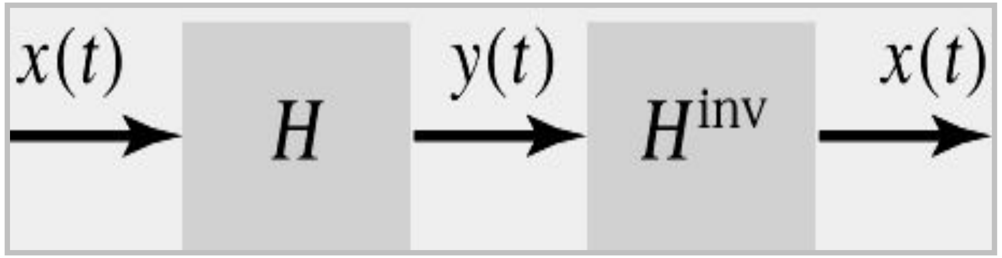
(a) It has memory, the value of the output signal ***y[n]*** at time n depends on the present and two pass values of ***x[n]***.

(b) It is memoryless, because the value of the output signal ***y[n]*** depends only on the present value of the input signal ***x[n]***.

Inverse Systems

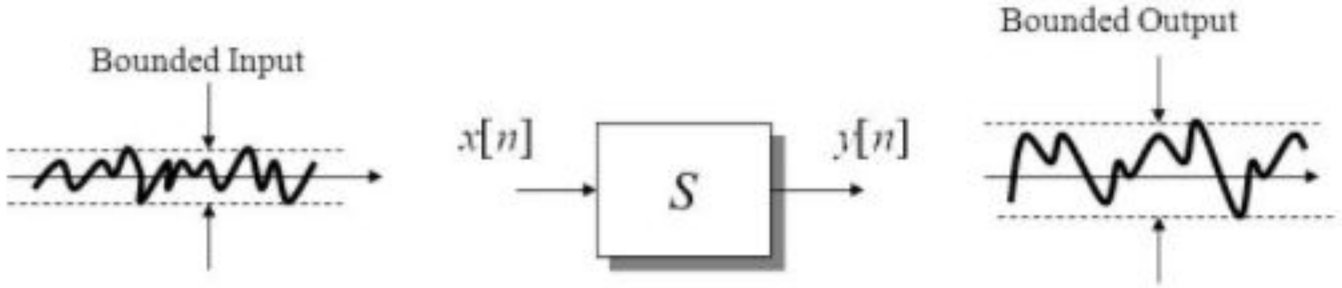
A system is **invertible** if the input of the system can be recovered from the output of the system. For example, this concept is important in communication applications. We will focus on this property with our echo cancellation lab.





Stable LTI system

- A system is said to be **BIBO** stable, i.e. bounded input and bounded output if every bounded input produces a bounded output.
- The output doesn't diverge until and unless input is diverged.



Signal transfer in LTI system

Prove that Impulse response of a system is the output of the system when the input is a delta function

- The impulse response **h(t)** of a system is defined as response of the system to a **delta function** (impulse function).

$$\mathbf{h(t) = f(\delta(t))}$$

By convolution of **x(t)** and ***δ(t)***, *it gives the original signal* **x(t)**. *So , we express*

$$x(t) = x(t) * \delta(t)$$

$$= \int_{-\infty}^{\infty} x(\tau)\, \delta(t-\tau)\, d\tau$$

Now if ***y(t)*** *is the output response then:*

$$y(t) = f(x(t))$$

$$= f\left[\int_{-\infty}^{\infty} x(\tau)\, \delta(t-\tau)\, d\tau\right]$$

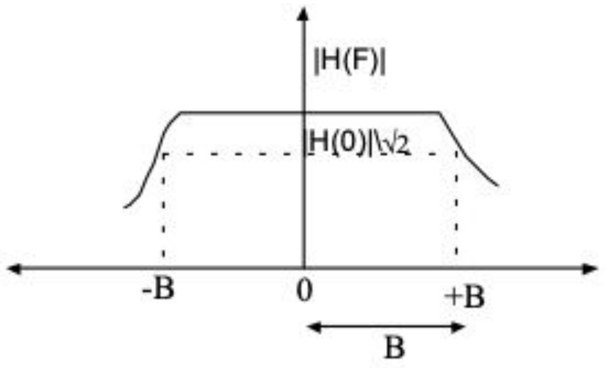
$$= \int_{-\infty}^{\infty} x(\tau)\, f[\delta(t-\tau)\, d\tau]$$

$$= \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

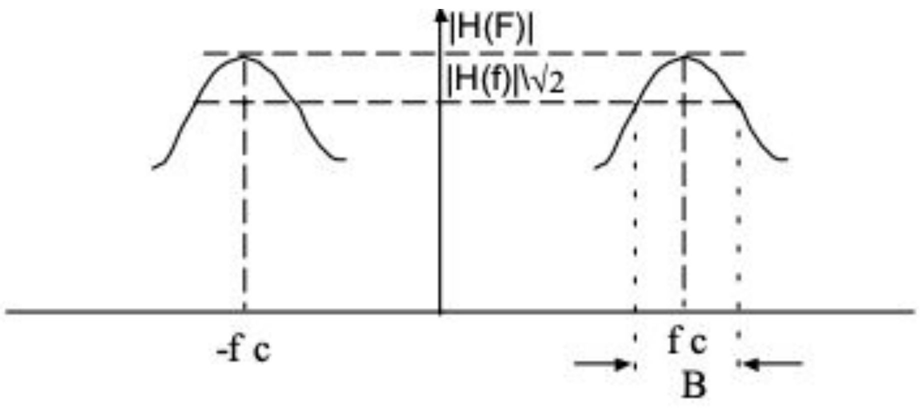
$$= x(t) * h(t)$$

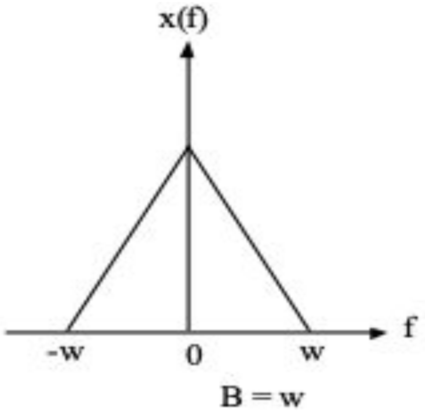
System Bandwidth:

In the case of a low pass system , the 3-dB bandwidth is defined as the difference between zero frequency at which the amplitude response attains its peak value *H*(0) and the frequency at which the amplitude response drops to a value equal to *H*(0)/√2 .



In the case of band-pass system the 3-dB bandwidth is defined as the difference between the frequencies at which the amplitude response drops to a value equal to 1/√2 time the peak value *H*(*fc*) at mid band frequency fc .

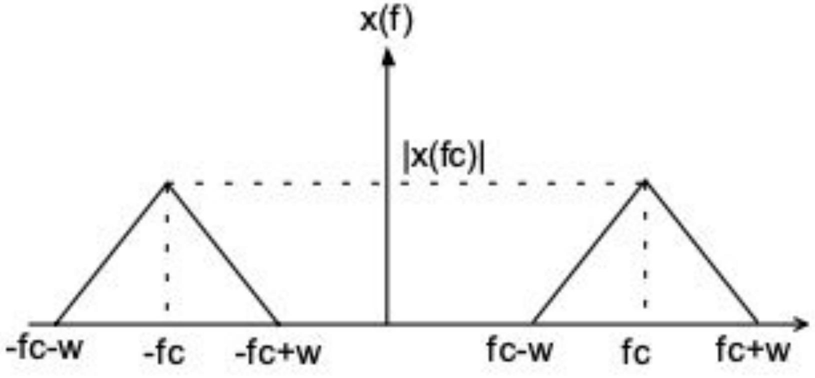




Signal bandwidth: The bandwidth of a signal provides a measure of the extent of significant spectral content of the signal for positive frequencies .

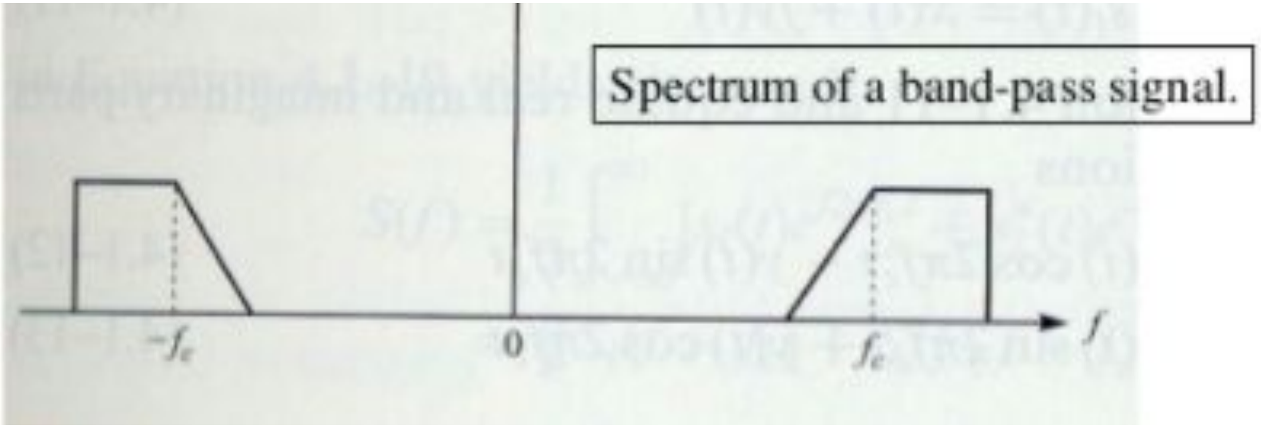
A signal is said to be low pass if its significant spectral content is centered around the origin, and bandwidth is defined as one half total width of main spectral lope.

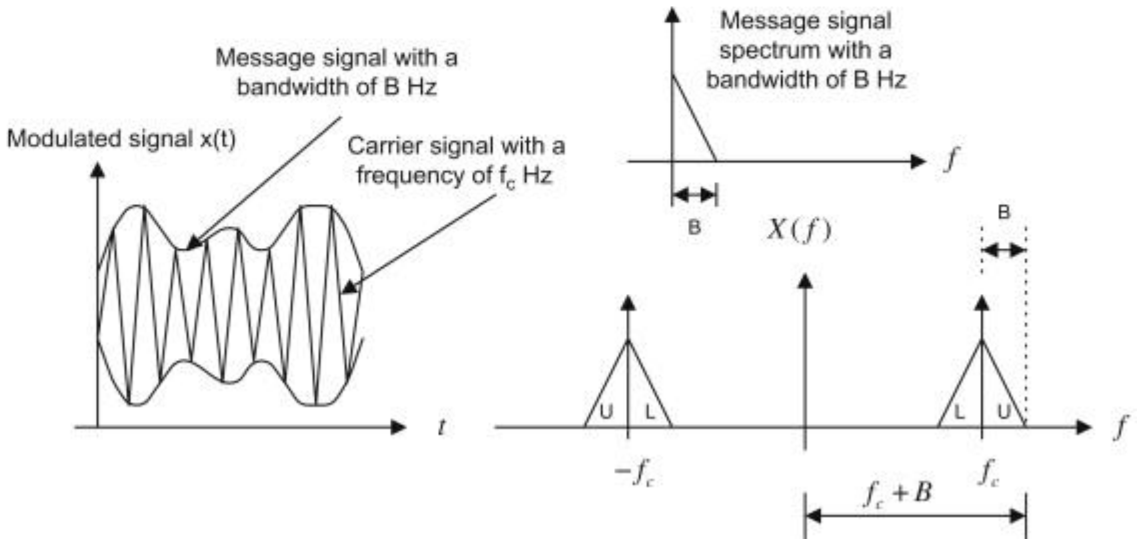
A signal is said to be band pass if its significant spectral content is central around ±fc where fc is a non zero frequency and the bandwidth is defined as the width of main lope for positive frequencies.



Band pass signals and system

Band pass signal are the signal that posses the pass band frequencies.



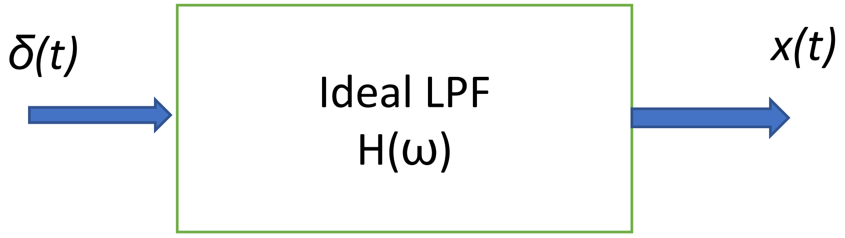


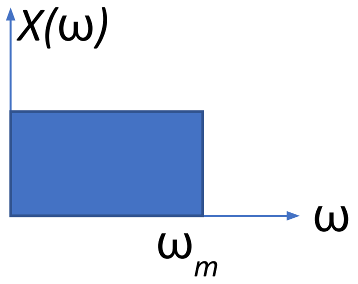
It can be represented by **inverse Fourier transform**

For **example**, a radio receiver contains a **band-pass filter** to select the frequency of the desired radio **signal** out of all the radio waves picked up by its antenna.

Ideal low pass filter is non casual

- An **ideal low pass filter** is a filter that passes pass band frequency without any **distortion** and complete attenuate stop band frequencies.

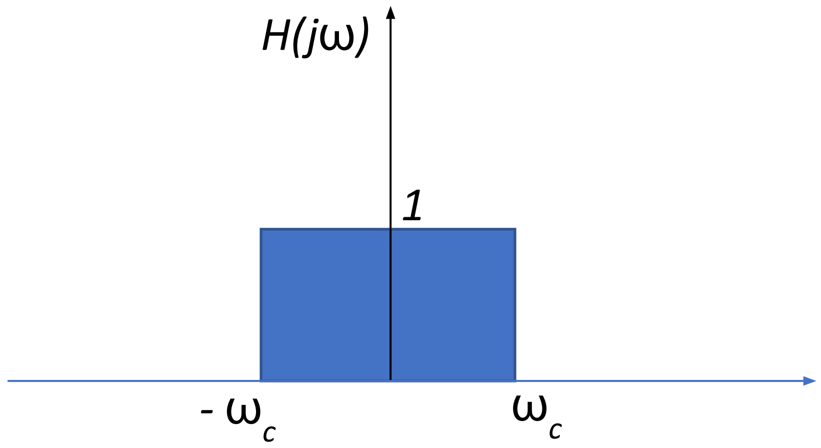




- An ideal low pass filter should pass without any **attenuation or distortion** , all signal frequencies below a certain frequency **'ω_m'** in rad/sec . Whereas signal frequencies above **ω_m** are completely attenuated.
- Thus, the frequencies response (magnitude response) of ideal LPF is a **gate function** and phase response is linear and equal to $-\omega t_d$.

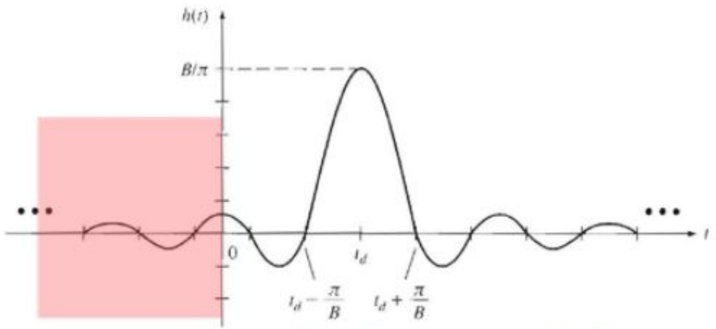
Let us assume we have a desired frequency response of

$$H(j\omega) = \begin{cases} 1 & |\omega| < \omega_c \\ 0 & |\omega| \geq \omega_c \end{cases}$$



This filter will need the following impulse response

$$\begin{aligned} h(t) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} H(j\omega)\, e^{j\omega t}\, \mathrm{d}\omega \\ &= \int_{-\omega_c}^{\omega_c} e^{j\omega t}\, \mathrm{d}\omega \\ &= \frac{\omega_c}{\pi} sinc(\omega_c t) \end{aligned}$$



Where, $$sinc(x) = \sin(x)/x.$$

Unfortunately, the **sinc** function has infinite support. That is, it has non-zero values all the way from $-\infty$ to ∞. In other words, our ideal filter requires infinite memory which makes it non-realizable.

Distortionless transmission

- A transmission of signal through a system is said to be **distortionless** if output signal is an exact replica of input signal.
- However a constant change in magnitude and constant time delay in output replica are not distortion.
- A signal **x(t)** is transmitted through the system without distortion of the output signal **y(t)** is defined by

$$\mathbf{y(t) = K\,x(t-t_d)\ \ldots\ldots\ldots\ldots\ (i)}$$

where **K** is a constant that accounts for the change in amplitude and t_d for the constant time delay in transmission.

Applying F.T. to equation (i) and using time shifting property, we have,

$$\mathbf{Y(F) = K\,X(F)e^{-j2\pi f t_d}\ \ldots\ldots\ldots\ldots\ (ii)}$$

The transfer function of a distortion less system is $-j2\pi f t_d$

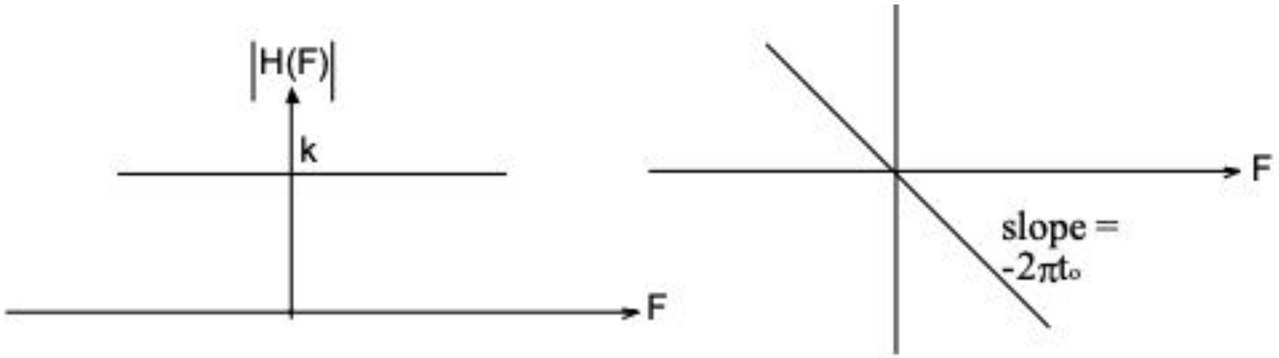
$$\mathbf{Y(f)= H(f).X(f)}$$

$$\mathbf{H(F) = K\,e^{-j2\pi f t_o}\ \ldots\ldots\ldots\ldots\ldots\ (iii)}$$

Thus, distortionless transmission of a signal x(t) is achieved when:

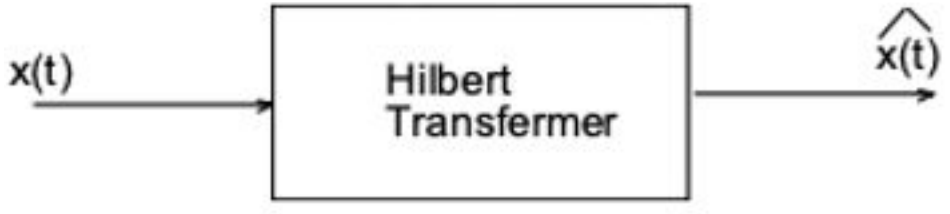
1. The amplitude response **|H(F)|** is constant for the frequency i.e. **|H(F)| = K**
2. The phase **θ(F)** is linear with frequency passing through origin , i.e. **θ(F) = -2πft$_d$**

θ(F)



Hilbert Transformation

- Hilbert transform (HT) is an operator which adds **-90˚** phase shift to all **positive frequencies** and **+90˚** to all **negative frequencies** of the input signal spectrum.



The amplitude of all frequency component of the input signal are unaffected by the transmission through the Hilbert transformer if x(t) is input signal then the output signal of the Hilbert transformer is denoted by $\hat{x}$(t).

Assume that x(t) is real and has no DC component : $X(f)|_{f=0} = 0$,

then

$$F[\hat{x}(t)] = -j\,\mathrm{sgn}(f)X(f)$$

$$F^{-1}[-j\,\mathrm{sgn}(f)] = \frac{1}{\pi t}$$

$$\hat{x}(t) = \frac{1}{\pi t} * x(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{x(\tau)}{t-\tau} d\tau$$

The operation of the Hilbert transform is equivalent to a convolution, i.e., filtering

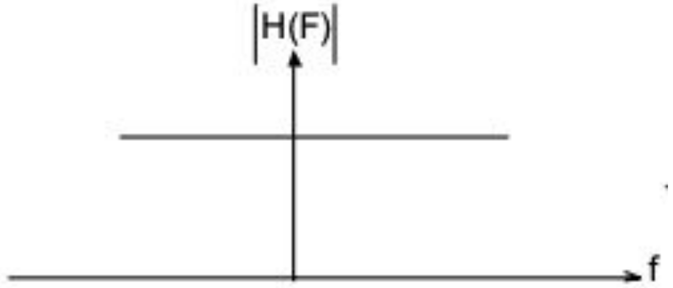
The impulse response of HT is the F.T of its frequency response H(f) and is equal to h(t) = 1/πt

The **frequency response** of the HT is denoted as **H(f)** and defined by:

$$H(f) = -j\ sgn(f) \ = \begin{cases} -j & if \quad f > 0 \\ 0 & if \quad f = 0 \\ j & if \quad f < 0 \end{cases}$$

Where **sgn(f)** is the signum function. And the phase response will be:

$$\theta(f) = \angle H(f) \ = \begin{cases} -\frac{\pi}{2} & if \quad f > 0 \\ 0 & if \quad f = 0 \\ \frac{\pi}{2} & if \quad f < 0 \end{cases}$$



Properties of Hilbert transform

1. The energy content in x(t) and its HT $\hat{x}$(t) are same i.e.

$$|x(t)^2| = |\hat{x}(t)^2|$$

2. x(t) and $\hat{x}$(t) are orthogonal to each other therefore cross correlation between them is zero.
3. The energy spectral density is same for both x(t) and .
4. x(t) and $\hat{x}$(t) are orthogonal to each other therefore Hilbert transform of $\hat{x}$(t) is -x(t)
5. If Fourier transform exist then Hilbert transform also exists for energy and power signals.
6. If c(t) is a high frequency sinusoidal signal and m(t) a low pass signal then HT of product of c(t) and m(t) is equal to product of m(t) and $\hat{c}$(t).

Assignment

1. Describe in short about Complex envelopes rectangular and Polar representation of band pass band limited signals.

Thank You.