$f_s \Rightarrow$ Sampling Rate or frequency

Chapter-2: Sampling Theory (8 marks)

* Sampling :→

The process of converting analog or continuous signal into a discrete signal is called sampling. In other words the analog input voltage is converted to a series of constant amplitude pulse.

* Sampling Theorem/Nyquist Criteria

Sampling Theorem are explained into two parts.

1. For transmitting end :→

A band limited signal which has no frequency component higher than fmax Hz is completely described by specifying the values of signal at instant of time seperated by $\frac{1}{2f_{max}}$ seconds, the sampling Rate or Frequency is

$$\boxed{f_s \geq 2f_{max}}$$

2. For Receiving end :→

A band limited signal which has no frequency component higher than fmax Hz may be completely recovered from a knowledge of its sample taken at the rate of 2fmax per second, the sampling rate or frequency is

$$\boxed{f_s \geq 2f_{max}}$$

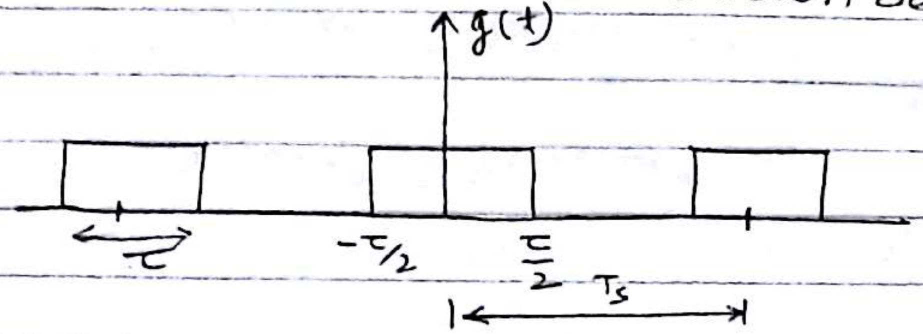
$f_x \Rightarrow$ Maximum Analog Input Frequency

proof :→

Let $x(t)$ be the band limited signal, then its sampled signal can be represented as

$$\boxed{x_s(t) = x(t)\cdot g(t)} - ① \text{ where } g(t) \text{ is the sampling function.}$$

shown below



$T_s \Rightarrow$ Sampling period

$\tau \Rightarrow$ duration of sampling pulse

The sampling function $g(t)$ is given by

$$g(t) = C_0 + 2\sum_{n=1}^{\infty} C_n \cos 2\pi n f_s t \quad - ②$$

As

$$x_s(t) = x(t)\, g(t)$$

$$x_s(t) = x(t)\left[C_0 + 2\sum_{n=1}^{\infty} C_n \cos 2\pi n f_s t\right]$$

$$x_s(t) = C_0 x(t) + 2C_1 x(t) \cos 2\pi f_s t + 2C_2 \, x(t) \cos 4\pi f_s t + \dots \quad - ③$$

$$\cancel{C_0 = \frac{1}{T_s}\int_{T_s} x(t)\, dt} = \frac{1}{T_s}\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} 1\cdot dt = \frac{\tau}{T_s} \quad - ④$$

Similarly,

$$C_n = \frac{1}{T_s}\int_{T_s} x(t)\, e^{-jn2\pi f_s t}\, dt = \frac{1}{T_s}\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} e^{-jn2\pi f_s t}\, dt$$

$$= \frac{1}{T_s} \left. \frac{e^{-jn2\pi f_s t}}{-jn2\pi f_s} \right|_{-\tau/2}^{\tau/2}$$

$$= -\frac{1}{jn2\pi f_s T_s} \left[e^{-jn2\pi f_s \tau/2} - e^{-jn2\pi f_s \cdot -\tau/2} \right]$$

$$= -\frac{1}{jn2\pi f_s T_s} \left[e^{-jn2\pi f_s \tau/2} - e^{jn2\pi f_s \tau/2} \right]$$

$$= \frac{1}{n\pi f_s T_s} \left[\frac{e^{jn2\pi f_s \tau/2} - e^{-jn2\pi f_s \tau/2}}{2j} \right]$$

$$= \frac{1}{n\pi f_s T_s} \sin(n 2\pi f_s \tau/2)$$

$$= \frac{\tau}{T_s} \frac{\sin(n\pi f_s \tau)}{n \cdot \pi f_s \tau}$$

$$= \frac{\tau}{T_s} \operatorname{sinc}(n\pi f_s \tau)$$

$$\boxed{C_n = \tau f_s \operatorname{sinc}(n\pi f_s \tau)} \quad - (5)$$

Taking Fourier Transform (F.T) of eq^n (5) we get

$$x(t) \xleftrightarrow{F.T} X(f)$$

$$\cos w_0 t \xleftrightarrow{F.T} \frac{1}{2}\left[\delta(f-f_0) + \delta(f+f_0)\right]$$

$$x(t)\cos w_0 t \xleftrightarrow{F.T} \frac{1}{2}\left[X(f-f_0) + X(f+f_0)\right]$$

$$X_s(f) = C_0 X(f) + C_1 X(f-f_s) + C_1 X(f+f_s) + C_2 X(f-2f_s) + C_2 X(f+2f_s) + \dots \quad (6)$$

Above eq^n can be graphically represented as

$f_x \Rightarrow$ Maximum frequency of message signal

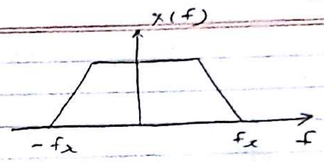


fig (a)

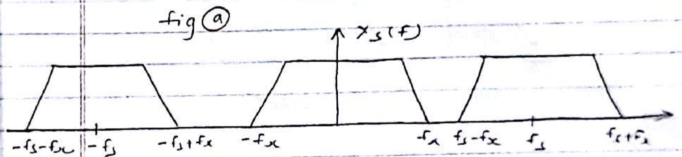


Fig (a) represents the Fourier transform of the original signal $x(t)$ and fig (b) represents the spectrum of the signal at the output of the sampler.

For errorless recovery of message signal from the spectrum of the sampled signal the sampling rate or frequency is equal to or more than the twice of the maximum frequency of message signal

$$\boxed{f_s \geq 2f_x}$$

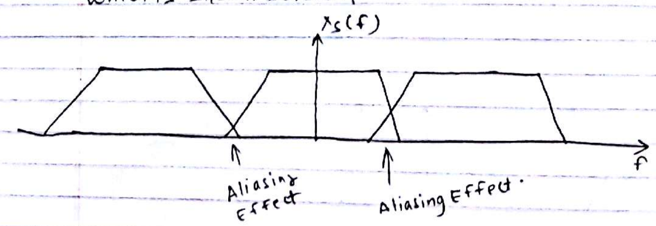
which is clearly shown in above fig (b)

Similarly distortion will occur while recovering the message signal when sampling rate or frequency (f_s) is less than the twice of the maximum frequency of message signal

$$\boxed{f_s < 2f_x}$$

In this condition the sample

gets overlapped and known as Aliasing effect which is shown below.



* Instantaneous Sampling / Ideal Sampling.

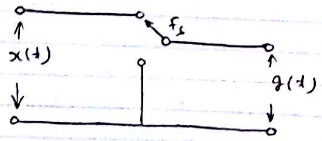


fig :→ switching Sampler

Above figure is a switching sampler which performs ideal sampling. The circuit simply consists of a switch and sampling function is the train of impulses.

Assume closing time 't' approaches to '0', the output $g(t)$ of the circuit will contain only the instantaneous value of input signal $x(t)$.

Instantaneous sampling gives train of impulses equal to the instantaneous value of input signal at sampling instant.

* For instantaneous sampling samples width (τ) is almost equal to zero. Therefore the power content is negligible and thus not suitable for transmission.

The sampling function is represented by train of impulses as

dirac delta function

$$\delta T_s(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s) \quad — (1)$$

output $g(t)$ is expressed as

$$g(t) = x(t) \cdot \delta T_s(t)$$

$$g(t) = x(t) \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

$$g(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\, \delta(t - nT_s) \quad — (2)$$

Taking Fourier Transform (FT) we get,

$$\boxed{G(f) = f_s \sum_{n=-\infty}^{\infty} X(f - nf_s)} \quad — (3)$$

exponential fourier series

$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn2\pi f_s t}$

where $c_n = 1/T_s$

$\delta_{T_s}(t) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} e^{jn2\pi f_s t}$

$g(t) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} x(t) e^{jn2\pi f_s t}$

$x(t) \xrightarrow{FT} X(f)$

$x(t) e^{jn2\pi f_s t} \xrightarrow{FT} X(f - nf_s)$

* Reconstruction of Sampled Signal (Interpolation Formula)

The process of reconstructing a original signal $x(t)$ from its sampled signal is called Reconstruction.

Let signal $x(t)$ be bandlimited to 'f_m' Hz. it can be reconstructed from its sample by passing the sampled signal through Ideal low pass filter (LPF) at cut-off frequency 'f_m' Hz.

Impulse response of Ideal low pass Filter (LPF) $h_{LPF}(t) = 2BT_s \, \text{sinc}[2BT_s]$

The expression for sampled signal is

$$g(t) = x(t)\,\delta_{T_s}(t) \quad -(1)$$

$$g(t) = \frac{1}{T_s}[x(t) + 2x(t)\cos 2\pi f_s t + 2x(t)\cos 4\pi f_s t \ldots] \quad -(2)$$

$$g(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\,\delta(t - nT_s) \quad -(3)$$

To recover the original signal, the sampled signal is passed through the ideal low pass filter (LPF) of bandwidth f_m Hz. Therefore transfer function of LPF is

$$H(\omega) = T_s \times \text{rect}\left(\frac{\omega}{2\pi f_m}\right) \quad -(4)$$

The impulse response of the filter is

$$\boxed{h(t) = 2f_m T_s \,\text{sinc}(2\pi f_m t)} \quad -(5)$$

Assuming that sampling is done at Nyquist rate

Then, $T_s = \frac{1}{2f_m} \Rightarrow 2f_m T_s = 1$

$\therefore h(t) = 1 \, \text{sinc}(2\pi f_m t)$

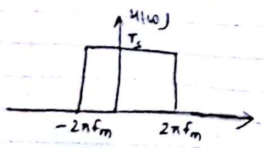


fig:→ Transfer function of LPF

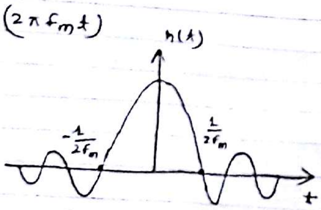


fig:→ Impulse response of LPF

Therefore, the output of filter is

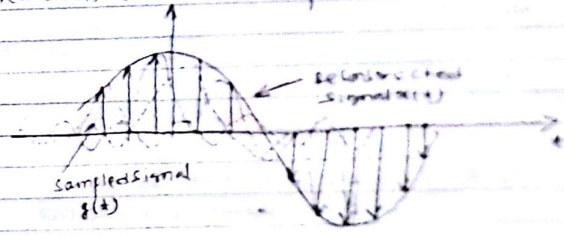
$$x(t) = \sum_n x(nT_s)\,h(t - nT_s)$$

$$= \sum_n x(nT_s)\,\text{sinc}[2\pi f_m(t - nT_s)]$$

$$= \sum_n x(nT_s)\,\text{sinc}[2\pi f_m t - 2\pi f_m nT_s]$$

$$\boxed{x(t) = \sum_n x(nT_s)\,\text{sinc}[2\pi f_m t - n\pi]}$$

is known as Interpolation formula which provides the value of message signal $x(t)$ between samples as a weighted sum of all the sample values.



Sampling period (T_s) ⇒ During sampling, the time elapsed between sample to sample or time taken by the next sample to occur is known as sampling period (T_s)

* Effect of Under Sampling : Aliasing

When a continuous-time band limited signal is sampled at a rate lower than Nyquist rate $f_s < 2f_m$, the successive cycle of spectrum $G(\omega)$ of the sampled signal $g(t)$ overlap each other. This phenomenon is called Aliasing where higher frequency component overlaps or undertakes lower frequency component in the spectrum of sampled signal. The signal gets distorted and original signal cannot be recovered.

To prevent Aliasing we use

1. Low Pass Filter (LPF) called prealias filter to limit band of frequency of the signal to 'f_m' Hz.
2. Sampling Frequency must be selected for $f_s > 2f_m$.

(previous section we discussed sampling theorem for low pass signal)

* Sampling of Band pass Signal :→ (sub Sampling Theorem)

Sampling Theorem of band pass signal $x(t)$ whose maximum bandwidth is '$2f_m$' can be completely represented into and recovered from its sample, if it is sampled at the minimum rate of twice the bandwidth.

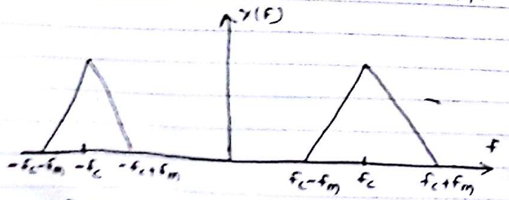


fig : Spectrum of Band pass Signal

Sampling Rate (f_s) is reciprocal of sampling period

$\boxed{f_s = 1/T_s}$

Above figure shows the spectrum of Band pass signal. If the bandwidth is $2f_m$, then the sampling rate for band pass signal must be $4 f_m$ samples per second.

Let $x_I(t)$ = Inphase Component of $x(t)$.

$x_Q(t)$ = Quadrature Component of $x(t)$.

Therefore the signal $x(t)$ in terms of Inphase and Quadrature component will be expressed as

$$x(t) = x_I(t)\cos(2\pi f_c t) - x_Q(t)\sin(2\pi f_c t) \quad -(1)$$

By solving we get

$$x(t) = \sum_{n=-\infty}^{\infty} x\left(\frac{n}{4f_m}\right) \operatorname{sinc}\left(2f_m t - \frac{n}{2}\right) \cos\left[2\pi f_c\left(t - \frac{n}{4f_m}\right)\right]$$

Comparing this Reconstruction formula with low pass signals

$$x(t) = \sum x(nT_s)\operatorname{sinc}[2\pi f_m t - n\pi]$$

$x\left(\frac{n}{4f_m}\right) = x(nT_s)$ = Sampled version of bandpass signal

$$T_s = \frac{1}{4f_m}$$

$\therefore$ $\boxed{\text{Minimum sampling rate} = \text{Twice of bandwidth} = 4f_m}$

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$A \Rightarrow$ Amplitude of $c(t)$

* Natural sampling :→

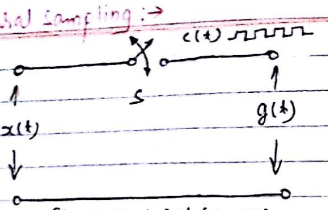


fig:→ Natural sampler

Natural sampling is obtained using Natural sampler. It is a practical method where periodic pulse of sampling function $c(t)$ is of width 'τ'.

Let us consider continuous time signal $x(t)$ to be sampled at rate of 'f_s' Hz assuming higher than the Nyquist rate satisfying the sampling theorem.

The sampled signal $g(t)$ is obtained by

$$g(t) = x(t)\, c(t) \quad -(1)$$

where $c(t)$ is the periodic train of pulse of width τ and frequency 'f_s' and is expressed as

$$c(t) = \sum_{n=-\infty}^{\infty} C_n e^{j2\pi n t/T_0}$$

$$T_0 = T_s = \frac{1}{f_s}$$

$$c(t) = \sum_{n=-\infty}^{\infty} C_n e^{j2\pi f_s n t} \quad -(2)$$

Since $c(t)$ is a rectangular pulse train

$$C_n = \frac{TA}{T_0}\, \mathrm{sinc}(f_n \cdot T)$$

$$e^{j2\pi f_s n t} x(t) \overset{F.T}{\longleftrightarrow} X(f - f_s n)$$

T = pulse width = τ $\quad f_n \Rightarrow$ Harmonic frequency $= n f_s$

$$C_n = \frac{\tau A}{T_s} \sum_{n=-\infty}^{\infty} \mathrm{sinc}(f_n \tau) \quad -(3)$$

putting value of C_n in eq.^{n} (2)

we get

$$\boxed{g(t) = \frac{\tau A}{T_s} \sum_{n=-\infty}^{\infty} \mathrm{sinc}(f_n \tau)\, e^{j2\pi f_s n t}\, x(t)} \quad -(4)$$

is the required time domain representation for naturally sampled signal $g(t)$.

Again, Taking Fourier Transform (F.T) of eq.^{n} (4)

we get,

$$\boxed{G(f) = \frac{\tau A}{T_s} \sum_{n=-\infty}^{\infty} \mathrm{sinc}(n f_s \tau)\, X(f - n f_s)} \quad (\because f_n = n f_s)$$

is the spectrum of Naturally sampled signal.

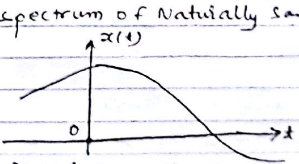


fig:→ Continuous time signal

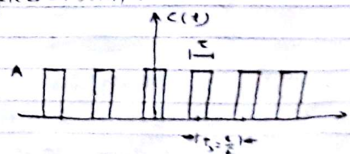


fig → sampling function waveform (periodic pulse train)

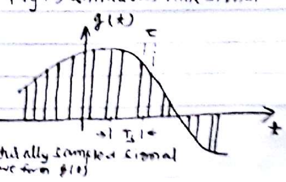
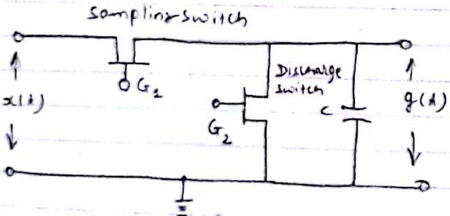


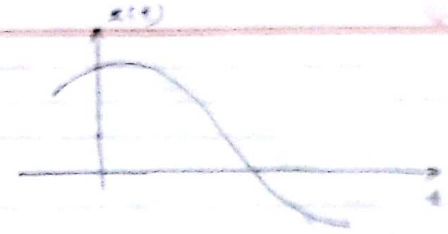
fig:→ Naturally sampled signal waveform $g(t)$

Natural Sampling has the output sample of varying top in accordance with continuous time Analog signal.
Due to this reason, it is difficult to determine the shape of top of pulse and amplitude detection is not exact. It is more susceptible to be contaminated by noise.

* Flat Top Sampling →
Sampling method which has flat top is called Flat top Sampling. In Flat Top Sampling or Rectangular pulse sampling the top of the samples remain constant and is equal to the instantaneous value of the base band signal. (message signal)



fig(a): Sample and Hold Circuit
Fig (a) shows the functional diagram of a Sample and Hold Circuit which generates flat top sample.



fig(b) Base band signal

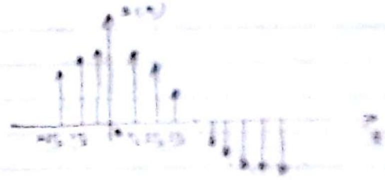


fig: Instantaneous sample signal

fig(c): Constant width function

fig(e): Flat Top Sampled Signal $g(t)$

Let the duration or width of each sample is 'τ' and sampling rate is $f_s = \frac{1}{T_s}$. If $x(t)$ is a band band signal which instantaneous sample signal is $s(t)$. $h(t)$ is the constant width function

Therefore Flat top sampled signal $g(t)$ is obtained by

$$g(t) = s(t) \otimes h(t) \quad — (1)$$

As train of impulses is represented as

$$\delta_{T_s}(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s) \quad — (2)$$

The signal $s(t)$ is obtained by multiplication of base band signal $x(t)$ and $\delta_{T_s}(t)$

$$\therefore \; s(t) = x(t)\, \delta_{T_s}(t)$$

$$s(t) = x(t) \sum_{n=-\infty}^{\infty} \delta(t-nT_s) = \sum_{n=-\infty}^{\infty} x(nT_s)\, \delta(t-nT_s)$$

From eq^n (1) putting value of $s(t)$ we get

$$g(t) = s(t) \otimes h(t) = \int_{-\infty}^{\infty} s(t)\, h(t-\tau)\, d\tau$$

$$g(t) = \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} x(nT_s)\, \delta(t-nT_s)\, h(t-\tau)\, d\tau$$

$$g(t) = \sum_{n=-\infty}^{\infty} x(nT_s) \int_{-\infty}^{\infty} h(t-\tau)\, \delta(t-nT_s)\, d\tau$$

Taking shifting property of delta function

$$\int_{-\infty}^{\infty} f(t)\, \delta(t-t_0)\, dt = f(t_0)$$

$$\boxed{g(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\, h(t-nT_s)} \quad — (3)$$

Again, Taking Fourier Transform (F.T) of eq^n (1) we get

$$G(f) = S(f)\, H(f)$$

As we know,

$$S(f) = f_s \sum_{n=-\infty}^{\infty} X(f - nf_s)$$

Therefore

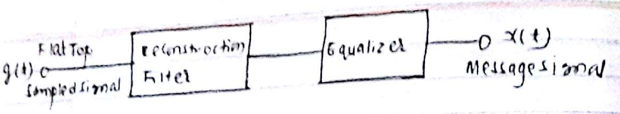
$$\boxed{G(f) = f_s \sum_{n=-\infty}^{\infty} X(f-nf_s)\, H(f)}$$ is the spectrum of flat top sampled signal.

* Aperture Effect :→

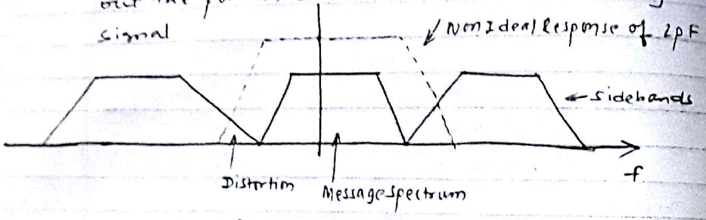
The attenuation of the upper portion of message signal spectrum affecting the high frequency component is called Aperture Effect. This effect introduces Amplitude distortion. Aperture Effect depends upon the duration or width of each sample 'τ'. Higher value of 'τ' are more prominent to Aperture Effect.

* Solution for Aperture Effect

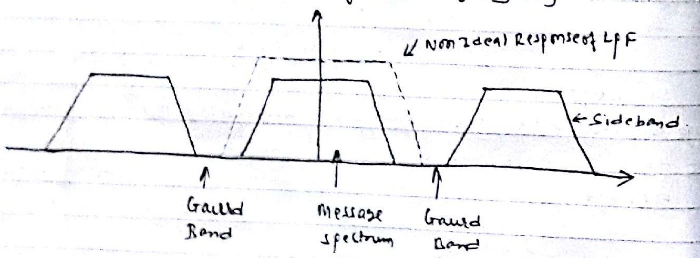
1) Using the duration or sampling pulse 'τ' as narrow as possible
2) By the use of Equalizer during Reconstruction. It compensates the attenuation caused by Low pass Reconstruction Filter.



3) Effect of Non Ideal Reconstruction Filter may filter out the portion of sidebands with message signal



Good Filter Design eliminates the above Non Ideal effect of LPF by introducing Guard band by selecting 'f_s' slightly higher than '$2f_x$'.



* Practical Considerations

In Real practice, due to various implementation constraints the expected result may differ from Ideal Sampling method.

The difference between Ideal Sampling and practical sampling are

1. The sampled waveform consist of finite Amplitude and duration pulses rather than Ideal Impulses.

2. Reconstruction Filters are not Ideal.

3. The input waveforms are time limited rather than band limited.

1. An Analog signal is expressed by the equation $x(t) = 3\cos(50\pi t) + 10\sin(300\pi t) - \cos(100\pi t)$. Calculate the Nyquist rate for this signal.

Given, $x(t) = 3\cos(50\pi t) + 10\sin(300\pi t) - \cos(100\pi t)$ — ①

$x(t) = 3\cos(2\pi f_1 t) + 10\sin(2\pi f_2 t) - \cos(2\pi f_3 t)$ — ②

Compairing eqn ① and ②

we get

$f_1 = 25\,Hz$ $f_2 = 150\,Hz$ $f_3 = 50\,Hz$

Therefore Maximum Frequency present in signal $x(t)$ is

f_x or $f_m = 150\,Hz$

Nyquist rate or sampling Rate or sampling frequency is

$f_s = 2f_m$ or $2f_x = 2 \times 150\,Hz = 300\,Hz$.

2) Find the Nyquist rate and the Nyquist interval for the signal $x(t) = \frac{1}{2\pi}\cos(4000\pi t)\cos(1000\pi t)$

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$$x(t) = \frac{1}{2\pi} \cos(4000\pi t) \cdot \cos(1000\pi t)$$

$$x(t) = \frac{1}{4\pi} \left[2\cos(4000\pi t) \cdot \cos(1000\pi t)\right]$$

$$x(t) = \frac{1}{4\pi} \left[\cos(4000\pi t + 1000\pi t) + \cos(4000\pi t - 1000\pi t)\right]$$

$$x(t) = \frac{1}{4\pi} \left[\cos 5000\pi t + \cos 3000\pi t\right]$$

$$x(t) = \frac{1}{4\pi} \cos 5000\pi t + \frac{1}{4\pi} \cos 3000\pi t \quad -(1)$$

$$x(t) = \frac{1}{4\pi} \cos 2\pi f_1 t + \frac{1}{4\pi} \cos 2\pi f_2 t \quad -(2)$$

on comparing eqn (1) and (2) we get

$f_1 = 2500HZ \quad f_2 = 1500HZ$

Therefore Maximum Frequency present in signal $x(t)$ is f_x or $f_m = 2500HZ$

Nyquist rate or Sampling Rate or Sampling frequency is $f_s = 2f_x = 2 \times 2500HZ = 5000HZ$

Nyquist Interval or Sampling period $T_s = \frac{1}{2f_x} = \frac{1}{2 \times 2500} = 0.2 \times 10^{-3}$ sec

3) Find the Nyquist rate and the Nyquist interval for the following signal

$$m(t) = \frac{\sin(500\pi t)}{\pi t}$$

$$m(t) = \text{sinc}(500\pi t)$$

Comparing with $m(t) = \text{sinc}(2\pi f_m t)$

f_m or $f_x = 250HZ$

Nyquist rate or Sampling Rate or Sampling frequency

$f_s = 2f_m = 500HZ$

Nyquist Interval $(T_s) = \frac{1}{2f_m} = \frac{1}{2 \times 500} = \frac{1}{1000} = 10^{-3}s$
$= 1ms$

4) Find the Nyquist rate and the Nyquist interval for the following signal

$$m(t) = \text{sinc}^2(200\pi t)$$

$$m(t) = \text{sinc}(200\pi t) \cdot \text{sinc}(200\pi t)$$

on comparing f_1 or f_m or $f_x = 100HZ$

Nyquist or Sampling Rate or Sampling Frequency $f_s = 2f_m = 200HZ$

Nyquist Interval $(T_s) = \frac{1}{2f_m} = \frac{1}{200} = 5 \times 10^{-3}s = 5ms$

5) A continuous time signal is given below

$$x(t) = 8\cos 200\pi t$$

Determine

a. Minimum sampling Rate i.e Nyquist rate required to avoid Aliasing

b. If sampling frequency $f_s = 400HZ$. What is discrete time signal $x(n)$ or $x(nT_s)$ obtained after sampling.

c. If sampling frequency $f_s = 150HZ$. What is discrete time signal $x(n)$ or $x(nT_s)$ obtained after sampling.

d. What is frequency $0 < f < f_s/2$ of sinusoidal that yields sample identical to those obtained in part (c).

a) Given $x(t) = 8\cos 200\pi t$

f_m or $f_x = 100 HZ$

Nyquist Rate $f_s = 2f_x = 200 HZ$

b) $f_s = 400 HZ$

The frequency of the discrete time signal will be

$$F = \frac{\text{Frequency of Continuous time signal } (f)}{\text{sampling frequency } (f_s)}$$

$$F = \frac{100}{400} = \frac{1}{4}$$

Then the discrete time signal will be given as

$$x(n) = 8\cos 2\pi Fn = 8\cos 2\pi \times \frac{1}{4}n = 8\cos \frac{\pi n}{2}$$

c) If sampling frequency $f_s = 150 HZ$

The frequency of discrete time signal will be

$$F = \frac{f}{f_s} = \frac{100}{150} = \frac{2}{3}$$

Discrete time signal

$$x(n) = 8\cos 2\pi Fn = 8\cos 2\pi \times \frac{2}{3}n$$

$$x(n) = 8\cos \frac{4\pi}{3}n = 8\cos\left(2\pi - \frac{2\pi}{3}\right)n = 8\cos \frac{2\pi n}{3}$$

d) Sampling rate $f_s = 150 HZ$. $F = 1/3$ from c

Discrete time signal $F = \frac{f}{f_s}$ $\quad f = F \times f_s = \frac{1}{3} \times 150 = 50$

Sinusoidal signal

$$x(t) = 8\cos 2\pi f t = 8\cos 2\pi 50 t = 8\cos 100\pi t$$

6. A bandpass signal has the spectral range that extends from 20 KHZ to 82 KHZ. Find the acceptable range of sampling frequency f_s.

As spectral range of band pass signal is 20 KHZ to 82 KHZ.

Bandwidth $= 82 - 20 = 62$ KHZ

Minimum sampling Rate for bandpass $f_s = 2 \times$ Bandwidth $= 2 \times 62 = 124$ KHZ

Range of Minimum sampling frequency is

$2 \times$ Bandwidth to $4 \times$ Bandwidth

$2 \times 62 = 124$ to $4 \times 62 = 248$ KHZ

7. A band pass signal with the spectrum in the range of (80-115) KHZ is to be digitized. Calculate minimum sampling frequency required for the signal.

spectral Range of bandpass signal 80 KHZ to 115 KHZ

Bandwidth $= 115 - 80 = 35$ KHZ

Minimum sampling Rate for Band pass signal $(f_s) = 2 \times$ Bandwidth

$f_s = 2 \times 35 = 70$ KHZ