CHAPTER 5: PROPAGATION BETWEEN ANTENNAS

ER.HOM NATH TIWARI

PASHCHIMANCHAL CAMPUS,

LAMACHAUR, POKHARA

HOMNATH@WRC.EDU.NP

DEPARTMENT OF ELECTRONICS AND COMPUTER ENGINEERING

COURSE STRUCTURE

- FREE SPACE PROPAGATION: POWER DENSITY OF THE RECEIVING ANTENNA, PATH LOSS
- PLANE EARTH PROPAGATION: THE GROUND REFLECTION, EFFECTIVE ANTENNA HEIGHTS, THE TWO RAY
- PROPAGATION MODEL, PATH LOSS
- FRESNEL ZONES AND KNIFE EDGE DIFFRACTION

2.6 FRIIS FREE SPACE EQUATION

The Friis free space equation is used to find out the transmission loss during the transmission of radio (or EM) wave. Hence, many times it is referred as *Friis transmission equation*. The transmission loss occurs due to the absorption of radiated wave by objects and it follows the inverse square law. The transmission loss is defined as the ratio of radiated power to the received power.

To obtain Friis free space equation, let us assume that there is an isotropic antenna at the transmitter side and the receiver antenna is at distance (d) away from transmitter as shown in Figure 2.6.

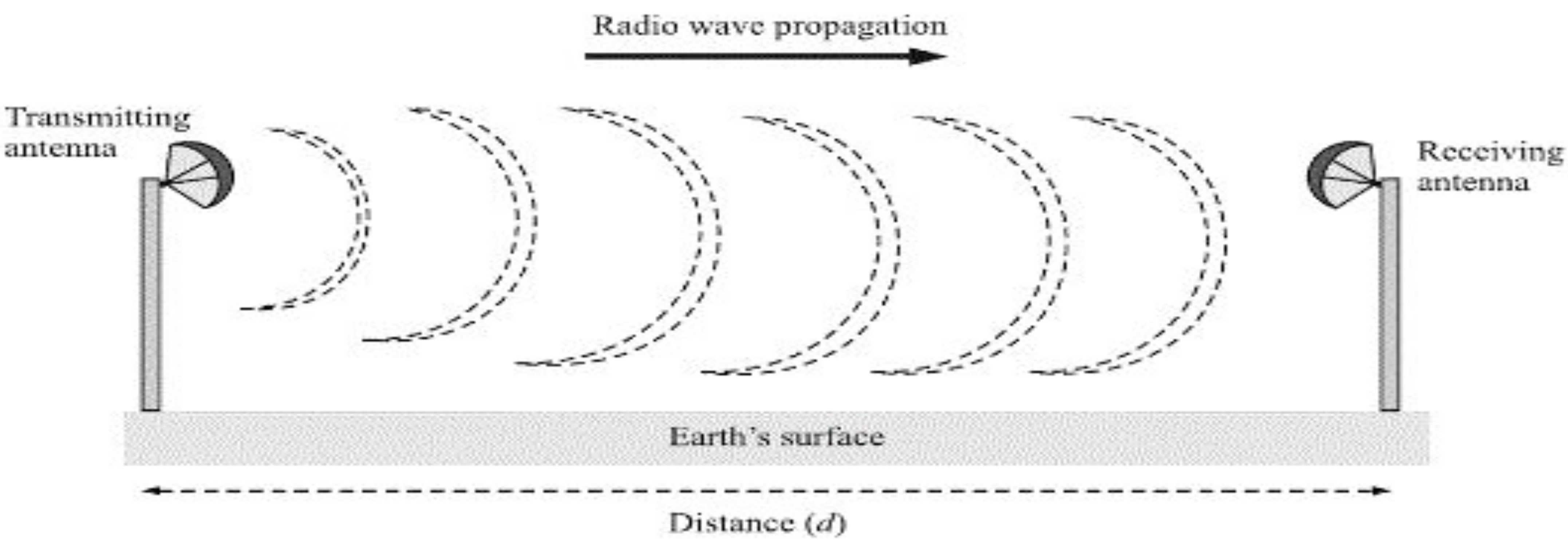


FIGURE 2.6 Radio wave propagation in free space.

The isotropic antenna radiates the signal in the form of EM energy and the radiation is uniform in all the directions. Hence, the power density remains same at all the points lying on the surface of a sphere. If the isotropic antenna radiates power P_T then the average power at any point at distance d from the transmitter is defined as:

$$P_{avg} = \frac{P_T}{4\pi d^2} \text{ W/m}^2 \quad \ldots(2.32)$$

In practical, isotropic antenna does not exist and all the practical antennas radiate well in a particular direction only. This property of antennas is defined as *antenna gain* (G) and defined as the ratio of the maximum power radiated to the average power radiated by an antenna. In case of transmitting antenna, if the gain of the antenna is G_T then G_T is defined as:

$$G_T = \frac{P_{max}}{P_{avg}} \quad \ldots(2.33)$$

or

$$P_{\max} = P_{\text{avg}}\, G_{\text{T}} = \frac{P_T G_T}{4\pi d^2} = \text{power density} \qquad \ldots(2.34)$$

The received power at the receiving antenna depends upon the *effective aperture* (A_e) of the receiving antenna; hence the received power (P_R) is defined as:

$$P_R = P_{\max} \cdot A_e = \frac{P_T\, G_T\, A_e}{4\pi d^2} \qquad \ldots(2.35)$$

The effective aperture of any antenna depends upon its gain and wavelength. For receiving antenna, it is defined as:

$$G_R = \frac{4\pi A_e}{\lambda^2} \qquad \ldots(2.36)$$

or

$$A_e = \frac{\lambda^2 G_R}{4\pi} \qquad \ldots(2.37)$$

By substituting the value of effective aperture (A_e) from Eqs. (2.37) to (2.35), the received power (P_R) comes out as:

$$P_R = P_T\, G_T\, G_R \left(\frac{\lambda}{4\pi d}\right)^2 \qquad \ldots(2.38)$$

The product of power transmitted (P_T) and gain of transmitting antenna (G_T) is known as *effective radiated power* (ERP) and is given as:

$$\text{ERP} = P_T\, G_T \qquad \ldots(2.39)$$

or $$\text{ERP (dB)} = 10 \log P_T + 10 \log G_T$$

or $$\text{ERP (dB)} = P_T\text{(dB)} + G_T\text{(dB)} \qquad \text{...(2.40)}$$

The *path loss* (L_s) or *spatial attenuation coefficient* is defined as:

$$L_s = \left(\frac{4\pi d}{\lambda}\right)^2 \qquad \text{...(2.41)}$$

or

$$L_s\text{(dB)} = 20 \log\left(\frac{4\pi d}{\lambda}\right) \qquad \text{...(2.42)}$$

$$L_s\text{ (dB)} = 32.42 + 20 \log d \text{ (in km)} + 20 \log f \text{ (in MHz)} \qquad \text{...(2.43)}$$

Hence, the received power (P_R) can be written as:

$$P_R\text{ (dB)} = \text{ERP (dB)} + G_R\text{ (dB)} - L_s\text{ (dB)} \qquad \text{...(2.44)}$$

PROBLEM 8: The power transmitted from a transmitter is 10 kW and the antenna gain is 30 dB. Find out the power density at any point which is located at the distance of 2×10^3 km away from the transmitter.

Solution: Given:

$$P_T = 10\text{ kW};\ G_T = 30\text{ dB} = 10^{(30/10)} = 1000;\ d = 2\times10^3\text{ km} = 2\times10^6\text{ m}$$

The power density at any point located d distance away from transmitter is given as:

$$P_{\max} = \frac{P_T G_T}{4\pi d^2}$$

By substituting the values,

$$P_{\max} = \frac{(10\times10^3)\times1000}{4\pi\times(2\times10^6)^2}$$

$$P_{\max} = 2\times10^{-7}\text{ W}$$

PROBLEM 9: If the power transmitted from a transmitter is 10 kW and gains of transmitting and receiving antennas are 30 dB and 20 dB respectively then calculate the maximum power received at a distance of 10 km over free space for 2 GHz transmission frequency.

Solution: Given:

$P_T = 10\text{ kW} = 10^4\text{ W}$; $G_T = 30\text{ dB} = 10^{(30/10)} = 1000$; $G_R = 20\text{ dB} = 10^{(20/10)} = 100$; $d = 10\text{ km} = 10^4\text{ m}$; $f = 2\text{ GHz} = 2\times10^3\text{ MHz}$

From Friis free space equation,

$$P_R(\text{dB}) = \text{ERP (dB)} + G_R(\text{dB}) - L_s(\text{dB})$$

where,

$$\text{ERP (dB)} = 10 \log P_T + 10 \log G_T$$

$$\text{ERP (dB)} = 10 \log 10^4 + 30 = 70 \text{ dB}$$

and

$$L_s(\text{dB}) = 32.42 + 20 \log d \text{ (in km)} + 20 \log f \text{ (in MHz)}$$

$$L_s(\text{dB}) = 32.42 + 20 \log 10 + 20 \log(2 \times 10^3)$$

$$L_s(\text{dB}) = 118.44 \text{ dB}$$

Hence, the received power comes out as:

$$P_R(\text{dB}) = 70 + 20 - 118.44$$

$$P_R(\text{dB}) = -28.44 \text{ dB}$$

In terms of watt,

$$P_R = 10^{\left(\frac{-28.44}{10}\right)} \text{ W}$$

$$P_R = 1.43 \text{ mW}$$

PROBLEM 10: If the power transmitted from a transmitter is 20 W and gains of transmitting and receiving antennas are 30 dB then calculate the maximum power received at a distance of 100 km over free space for 0.04 m transmission wavelength.

Solution: Given:

$$P_T = 20\text{ W};\ G_T = 30\text{ dB} = G_R;\ d = 100\text{ km};\ f = \frac{c}{\lambda} = \frac{3\times 10^8}{0.04} = 7.5\text{ GHz} = 7500\text{ MHz}$$

From Friis free space equation,

$$P_R(\text{dB}) = \text{ERP (dB)} + G_R(\text{dB}) - L_s(\text{dB})$$

where,

$$\text{ERP (dB)} = 10\log P_T + 10\log G_T$$
$$\text{ERP (dB)} = 10\log 20 + 30 = 43\text{ dB}$$

and

$$L_s(\text{dB}) = 32.42 + 20\log d\ (\text{in km}) + 20\log f\ (\text{in MHz})$$
$$L_s(\text{dB}) = 32.42 + 20\log 100 + 20\log(7500)$$
$$L_s(\text{dB}) = 149.92\text{ dB}$$

Hence, the received power comes out as:

$$P_R(\text{dB}) = 43 + 30 - 149.92$$
$$P_R(\text{dB}) = -76.92\text{ dB}$$

In terms of watt,

$$P_R = 10^{\left(\frac{-76.92}{10}\right)}\text{ W}$$
$$P_R = 0.02\ \mu\text{W}$$

GROUND REFLECTION(TWO RAY) MODEL

- IN MOBILE RADIO CHANNEL, SINGLE DIRECT PATH BETWEEN BASE STATION AND MOBILE AND IS SELDOM ONLY PHYSICAL MEANS FOR PROPAGATION
- FREE SPACE MODEL AS A STAND ALONE IS INACCURATE
- TWO RAY GROUND REFLECTION MODEL IS USEFUL
  - BASED ON GEOMETRIC OPTICS
  - CONSIDERS BOTH DIRECT AND GROUND REFLECTED PATH
- REASONABLY ACCURATE FOR PREDICTING LARGE SCALE SIGNAL STRENGTH OVER SEVERAL KMS THAT USE TALL TOWER HEIGHT
- ASSUMPTION: THE HEIGHT OF TRANSMITTER >50 METERS

GROUND REFLECTION(TWO RAY) MODEL

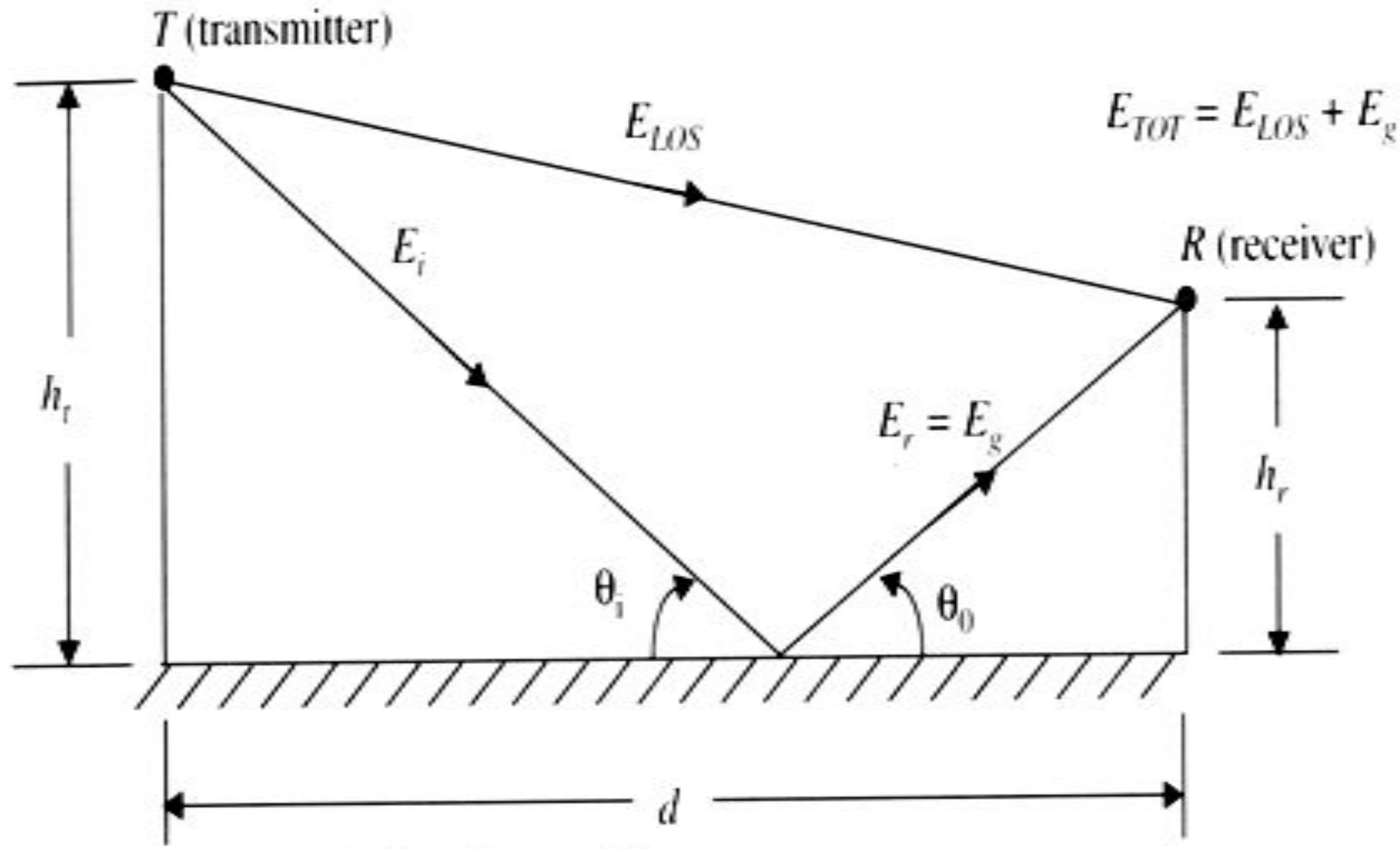


Figure 4.7 Two-ray ground reflection model.

GROUND REFLECTION(TWO RAY) MODEL

$$\vec{E}_{TOT} = \vec{E}_{LOS} + \vec{E}_g$$

let E_o be $|\ \vec{E}\ |$ at reference point d_o then

$$\vec{E}(d,t) = \left(\frac{E_0 d_0}{d}\right)\cos\left(\omega_c\left(t - \frac{d}{c}\right)\right) \quad d > d_0$$

$$E_{LOS}(d',t) = \frac{E_0 d_0}{d'}\cos\left(\omega_c\left(t - \frac{d'}{c}\right)\right)$$

$$E_g(d'',t) = \Gamma\frac{E_0 d_0}{d''}\cos\left(\omega_c\left(t - \frac{d''}{c}\right)\right)$$

$$\vec{E}_{TOT}(d,t) = \left(\frac{E_0 d_0}{d'}\right)\cos\left(\omega_c\left(t - \frac{d'}{c}\right)\right) + \Gamma\left(\frac{E_0 d_0}{d''}\right)\cos\left(\omega_c\left(t - \frac{d''}{c}\right)\right)$$

$$E_{TOT}(d,t) = \frac{E_0 d_0}{d'}\cos\left(\omega_c\left(t - \frac{d'}{c}\right)\right) + (-1)\frac{E_0 d_0}{d''}\cos\left(\omega_c\left(t - \frac{d''}{c}\right)\right)$$

GROUND REFLECTION(TWO RAY) MODEL

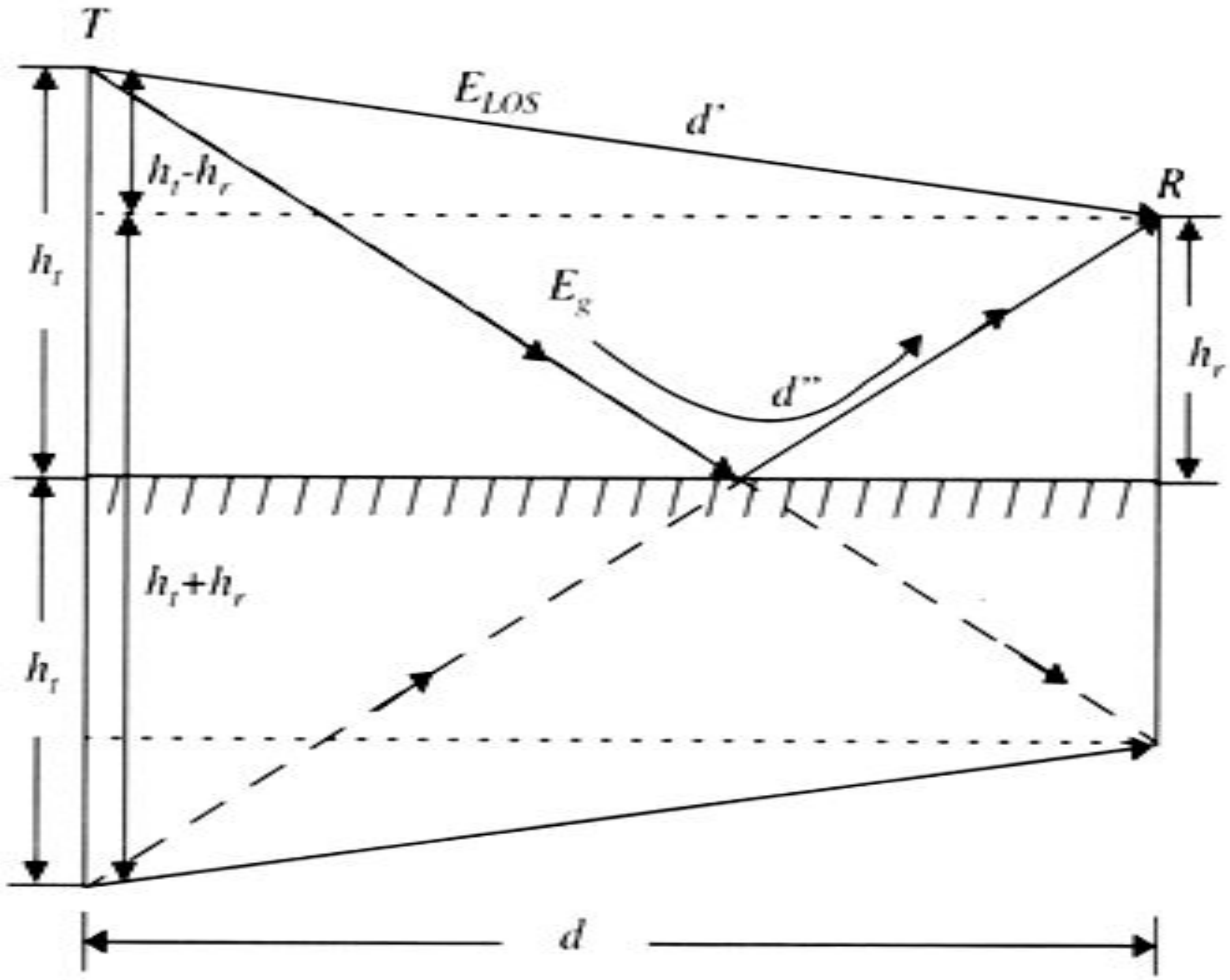


Figure 4.8 The method of images is used to find the path difference between the line-of-sight and the ground reflected paths.

PATH DIFFERENCE

$$\Delta = d'' - d' = \sqrt{(h_t + h_r)^2 + d^2} - \sqrt{(h_t - h_r)^2 + d^2}$$

$$= d\sqrt{\left(\left(\frac{h_t + h_r}{d}\right)^2 + 1\right)} - d\sqrt{\left(\left(\frac{h_t - h_r}{d}\right)^2 + 1\right)}$$

$$\approx d\left(1 + \frac{1}{2}\left(\frac{h_t + h_r}{d}\right)^2\right) - d\left(1 + \frac{1}{2}\left(\frac{h_t - h_r}{d}\right)^2\right)$$

$$\approx \frac{1}{2d}\left((h_t + h_r)^2 - (h_t - h_r)^2\right)$$

$$\approx \frac{1}{2d}\left(\left(h_t^2 + 2h_t h_r + h_r^2\right) - \left(h_t^2 - 2h_t h_r + h_r^2\right)\right)$$

$$\approx \frac{2h_t h_r}{d}$$

PHASE DIFFERENCE

$$\theta_\Delta \text{ radians} = \frac{2\pi\Delta}{\lambda} = \frac{2\pi\Delta}{\left(c/f_c\right)} = \frac{\omega_c\Delta}{c}$$

$$\left|E_{TOT}(t)\right| = 2\frac{E_0 d_0}{d}\sin\left(\frac{\theta_\Delta}{2}\right)$$

$$\frac{\theta_\Delta}{2} \approx \frac{2\pi h_r h_t}{\lambda d} < 0.3 \text{ rad}$$

$$E_{TOT}(t) \approx 2\frac{E_0 d_0}{d}\frac{2\pi h_r h_t}{\lambda d} \approx \frac{k}{d^2} \text{ V/m}$$

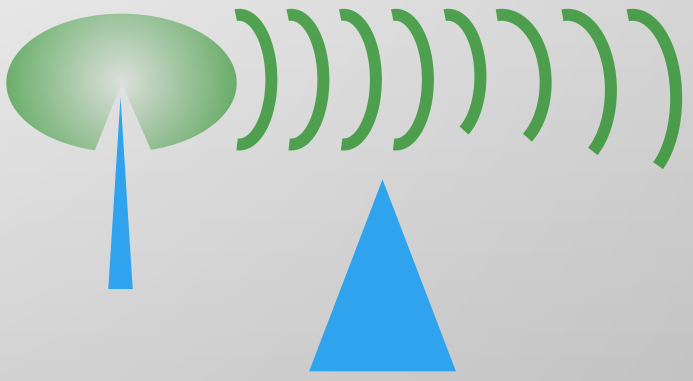
$$P_r = P_t G_t G_r \frac{h_t^2 h_r^2}{d^4}$$

DIFFRACTION

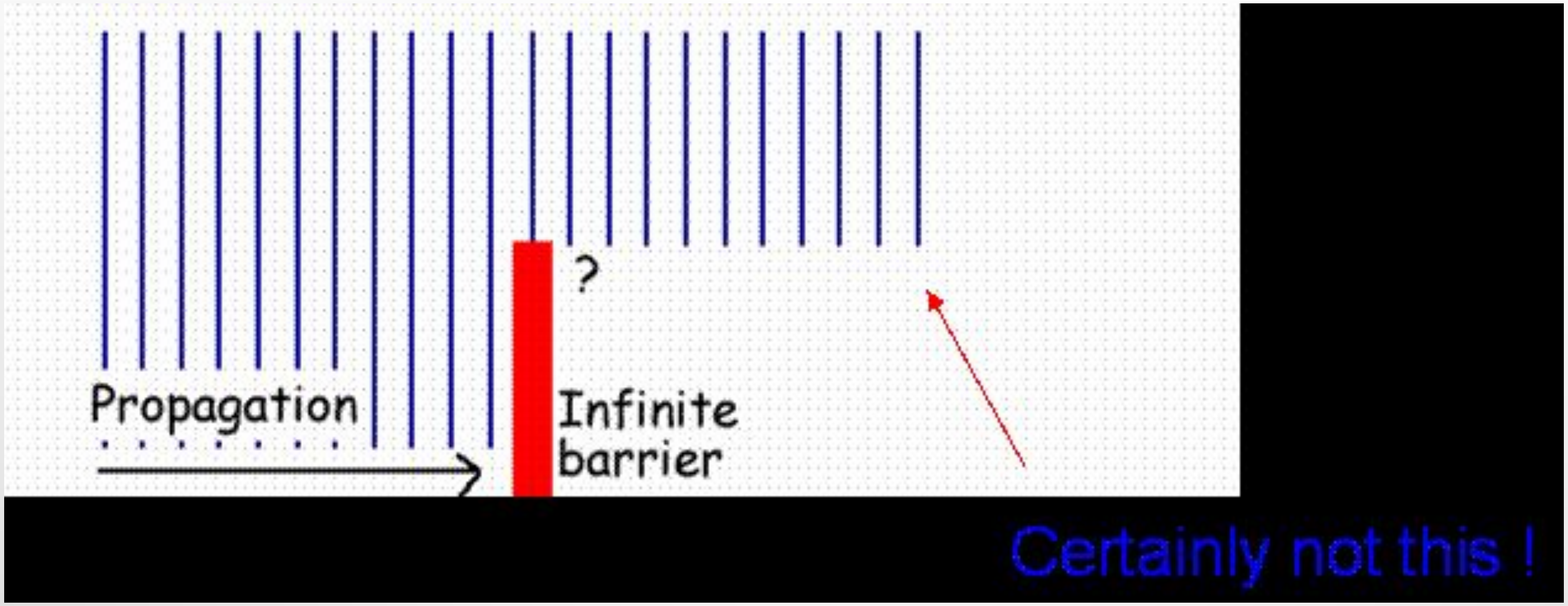
- DIFFRACTION IS THE BENDING OF WAVE FRONTS AROUND OBSTACLES.

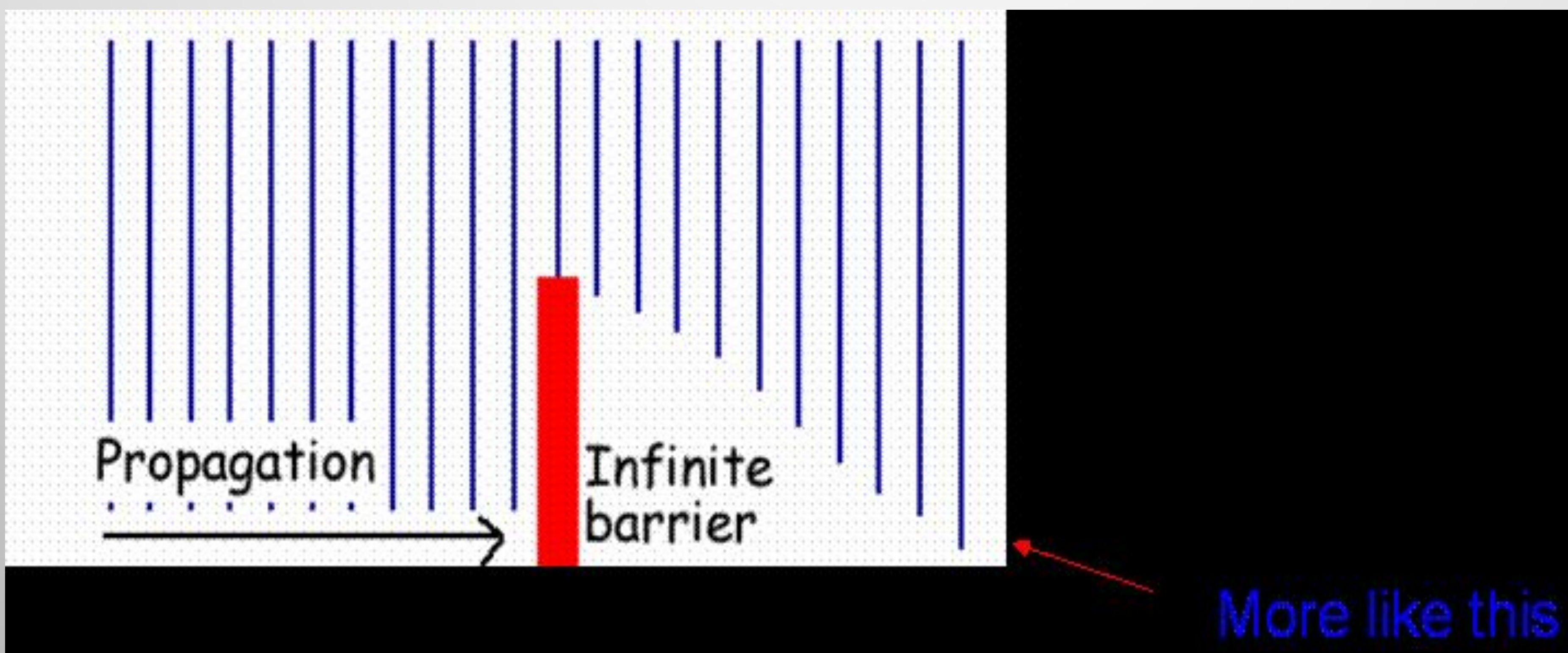
- DIFFRACTION ALLOWS RADIO SIGNALS TO PROPAGATE BEHIND OBSTRUCTIONS AND IS THUS ONE OF THE FACTORS WHY WE RECEIVE SIGNALS AT LOCATIONS WHERE THERE IS NO LINE-OF-SIGHT FROM BASE STATIONS

- ALTHOUGH THE RECEIVED FIELD STRENGTH DECREASES RAPIDLY AS A RECEIVER MOVES DEEPER INTO AN OBSTRUCTED (SHADOWED) REGION, THE DIFFRACTION FIELD STILL EXISTS AND OFTEN HAS SUFFICIENT SIGNAL STRENGTH TO PRODUCE A USEFUL SIGNAL.



DIFFRACTION





KNIFE-EDGE DIFFRACTION MODEL

- ESTIMATING THE SIGNAL ATTENUATION CAUSED BY DIFFRACTION OF RADIO WAVES OVER HILLS AND BUILDINGS IS ESSENTIAL IN PREDICTING THE FIELD STRENGTH IN A GIVEN SERVICE AREA.

- AS A STARTING POINT, THE LIMITING CASE OF PROPAGATION OVER A KNIFE EDGE GIVES GOOD IN SIGHT INTO THE ORDER OF MAGNITUDE DIFFRACTION LOSS.

- WHEN SHADOWING IS CAUSED BY A SINGLE OBJECT SUCH AS A BUILDING, THE ATTENUATION CAUSED BY DIFFRACTION CAN BE ESTIMATED BY TREATING THE OBSTRUCTION AS A DIFFRACTING KNIFE EDGE

KNIFE-EDGE DIFFRACTION MODEL

Consider a receiver at point R located in the shadowed region. The field strength at point R is a vector sum of the fields due to all of the secondary Huygens sources in the plane above the knife edge.

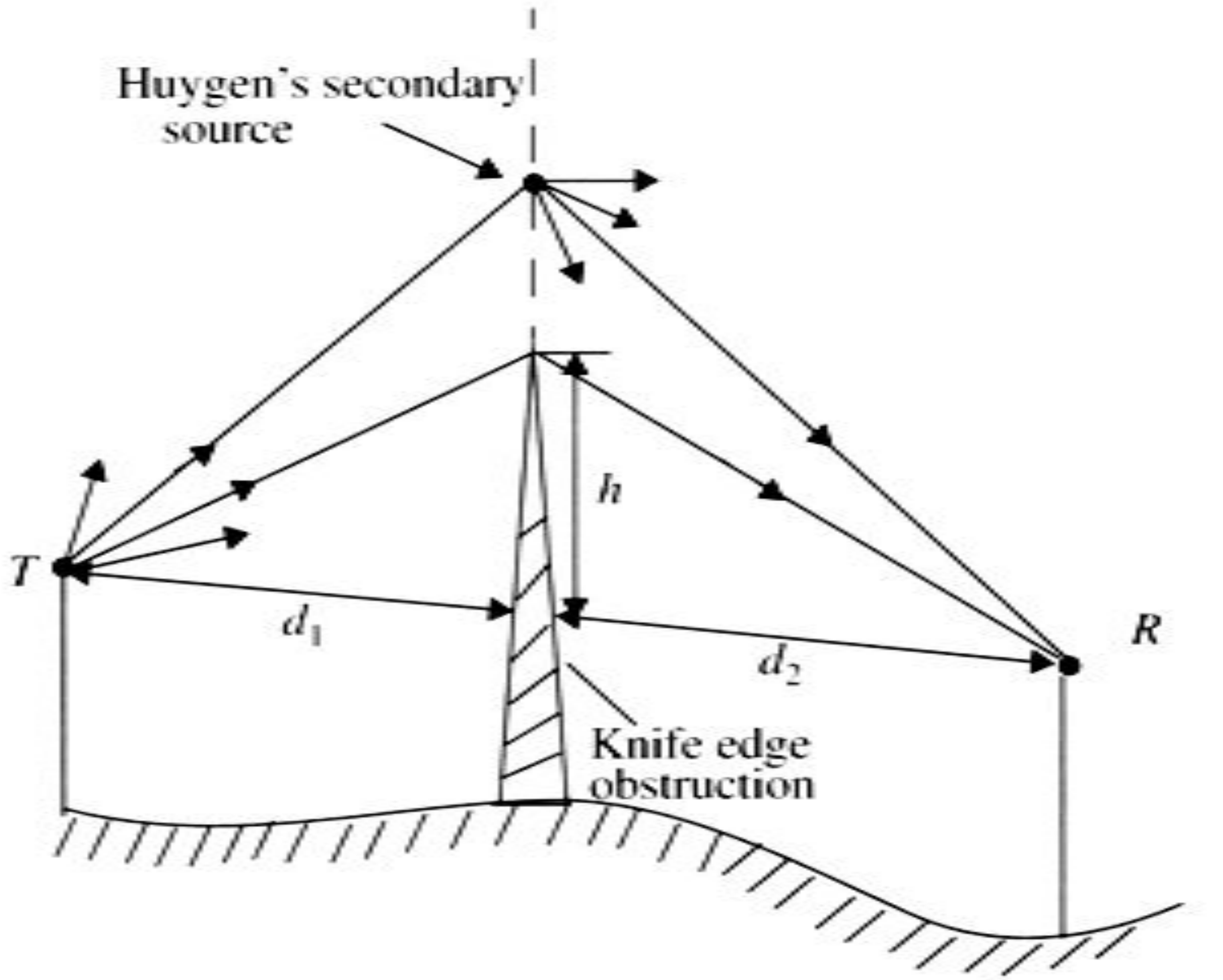


Figure 4.13 Illustration of knife-edge diffraction geometry. The receiver R is located in the shadow region.

KNIFE-EDGE DIFFRACTION MODEL

- THE DIFFERENCE BETWEEN THE DIRECT PATH AND DIFFRACTED PATH, CALL *EXCESS PATH LENGTH*

$$\Delta \approx \frac{h^2}{2}\frac{(d_1+d_2)}{d_1 d_2}$$

- THE CORRESPONDING PHASE DIFFERENCE

$$\phi = \frac{2\pi\Delta}{\lambda} \approx \frac{2\pi}{\lambda}\frac{h^2}{2}\frac{(d_1+d_2)}{d_1 d_2}$$

- *FRESNEL-KIRCHOFF* DIFFRACTION PARAMETER IS USED TO NORMALIZE THE PHASED TERM AND GIVEN AS

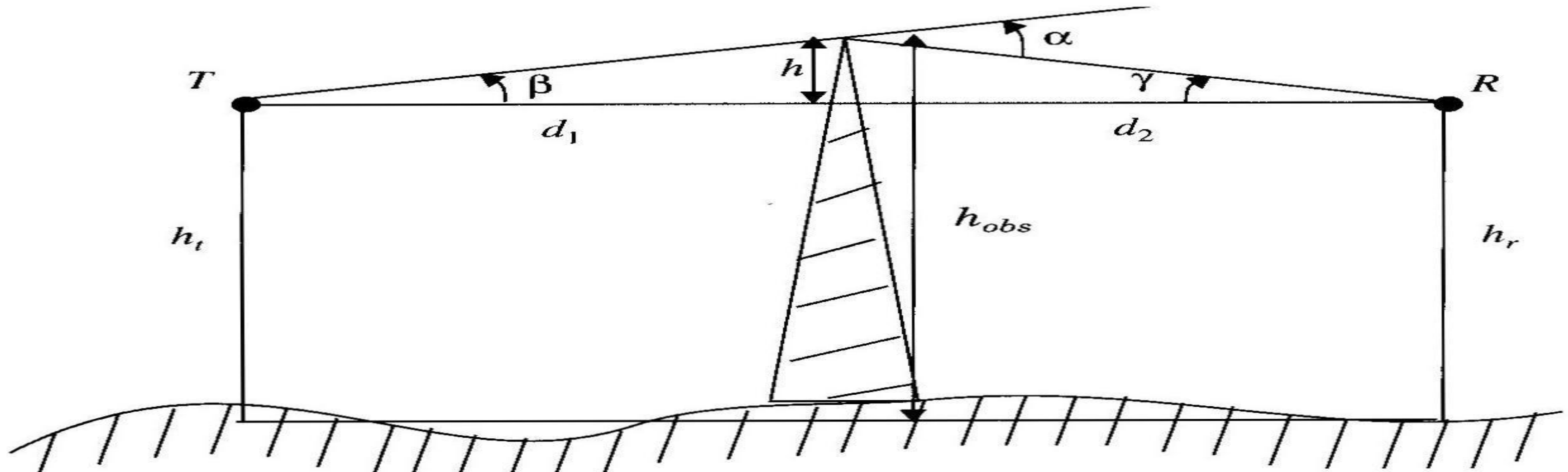
$$v = h\sqrt{\frac{2(d_1+d_2)}{\lambda d_1 d_2}} = \alpha\sqrt{\frac{2 d_1 d_2}{\lambda(d_1+d_2)}}$$

Which gives

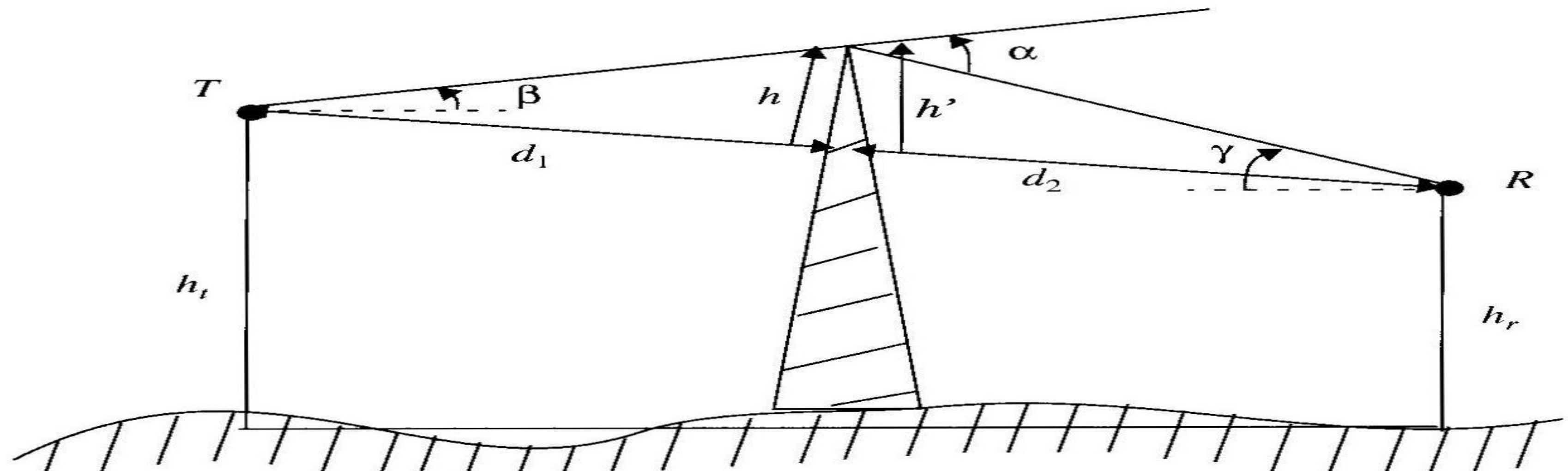
$$\phi = \frac{\pi}{2}v^2$$

- WHERE $\alpha = h(\frac{d_1+d_2}{d_1 d_2})$

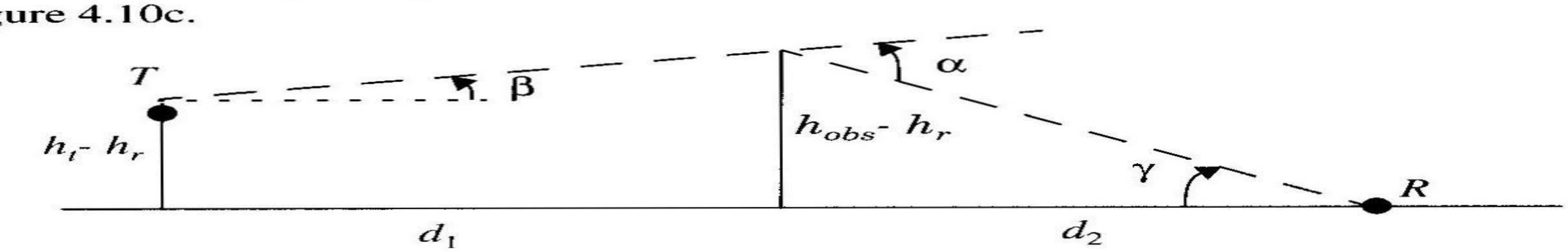
KNIFE-EDGE DIFFRACTION MODEL



(a) Knife-edge diffraction geometry. The point T denotes the transmitter and R denotes the receiver, with an infinite knife-edge obstruction blocking the line-of-sight path.



(b) Knife-edge diffraction geometry when the transmitter and receiver are not at the same height. Note that if α and β are small and $h << d_1$ and d_2, then h and h' are virtually identical and the geometry may be redrawn as shown in Figure 4.10c.



(c) Equivalent knife-edge geometry where the smallest height (in this case h_r) is subtracted from all other heights.

Figure 4.10 Diagrams of knife-edge geometry.

FRESNEL ZONES

- FRESNEL ZONES REPRESENT SUCCESSIVE REGIONS WHERE SECONDARY WAVES HAVE A PATH LENGTH FROM THE TX TO THE RX WHICH ARE NΛ/2 GREATER IN PATH LENGTH THAN OF THE LOS PATH. THE PLANE BELOW ILLUSTRATES SUCCESSIVE FRESNEL ZONES.

$$r_n = \sqrt{\frac{n\lambda d_1 d_2}{d_1 + d_2}}$$

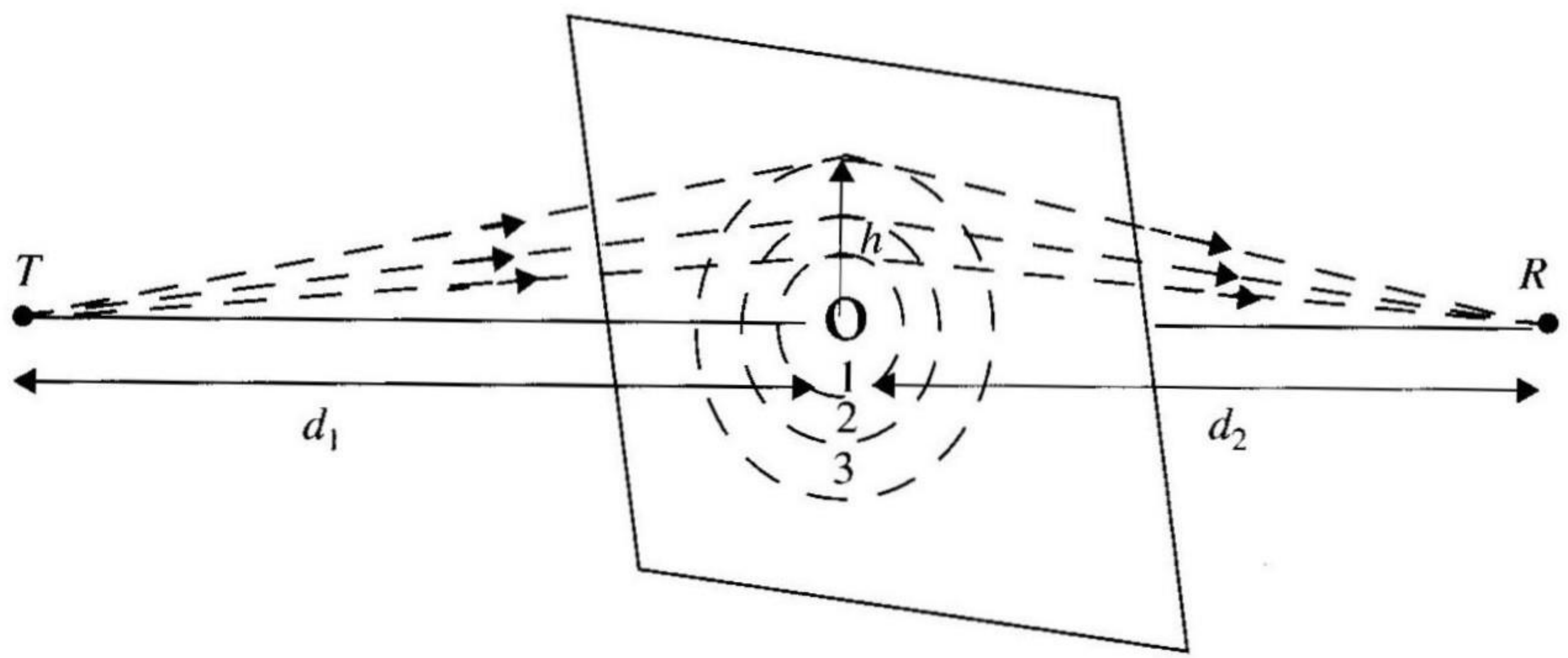
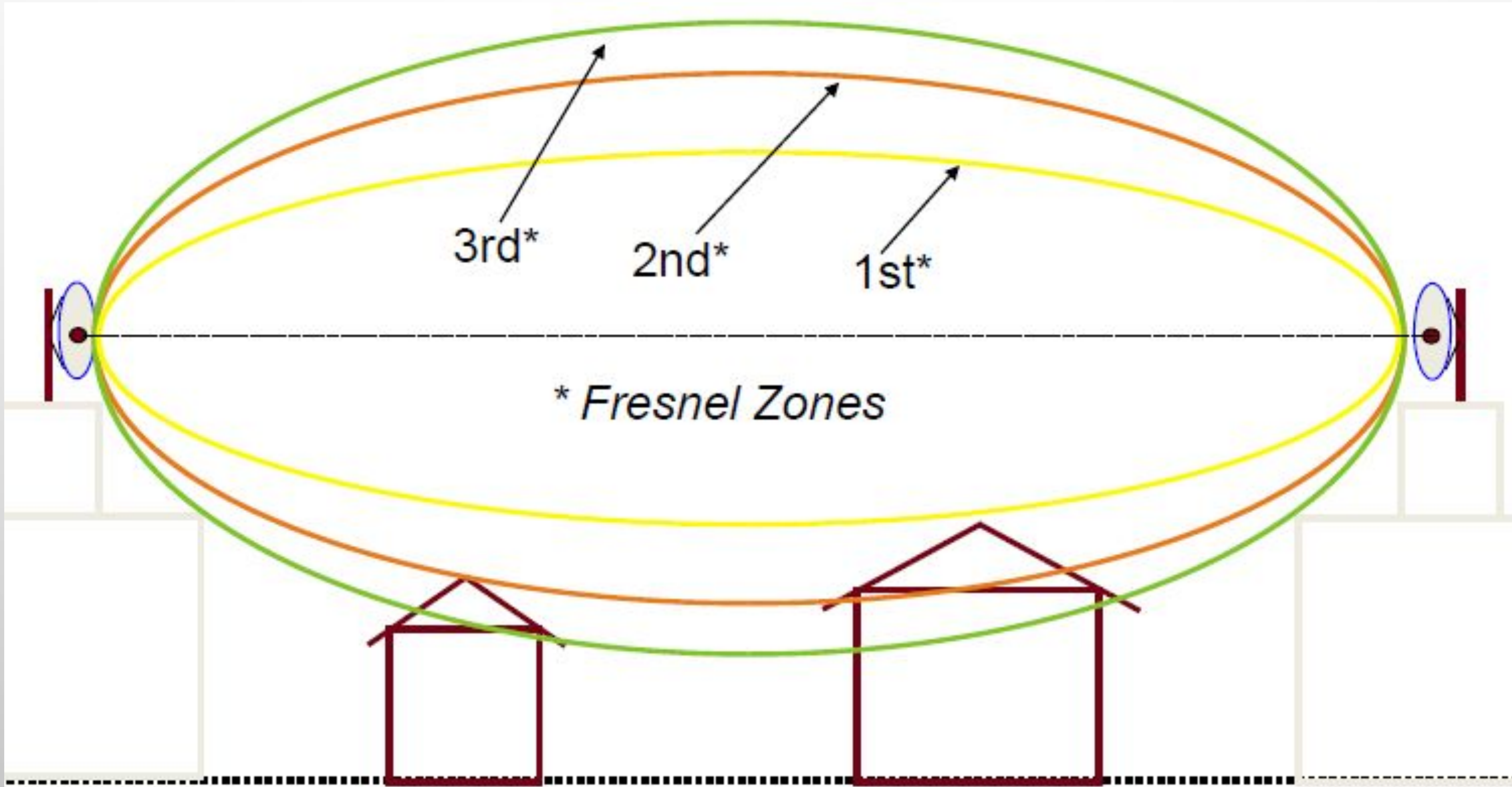


Figure 4.11 Concentric circles which define the boundaries of successive Fresnel zones.

FRESNEL ZONES



DIFFRACTION GAIN

- THE DIFFRACTION GAIN DUE TO THE PRESENCE OF A KNIFE EDGE, AS COMPARED TO THE FREE SPACE E-FIELD

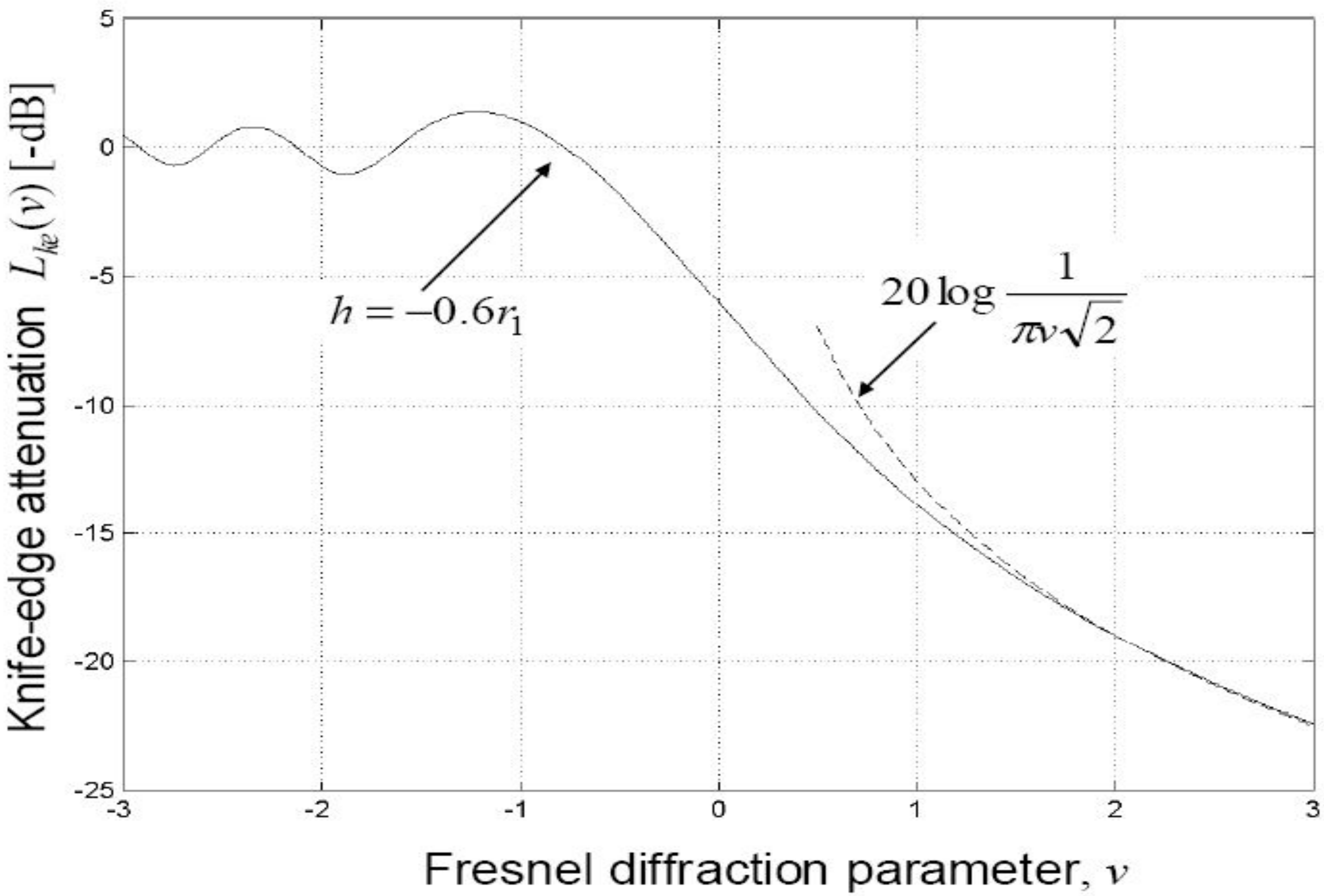
$$G_d(\mathrm{dB}) = 20\log|F(v)|$$

- THE ELECTRIC FIELD STRENGTH, *ED, OF A KNIFE EDGE DIFFRACTED WAVE IS GIVEN BY*

$$\frac{E_d}{E_o} = F(v) = \frac{(1+j)}{2}\int_v^\infty \exp((-j\pi t^2)/2)dt$$

- *EO : IS THE FREE SPACE FIELD STRENGTH IN THE ABSENCE OF BOTH THE GROUND AND THE KNIFE EDGE.*
- *F(V): IS THE COMPLEX FRESNEL INTEGRAL.*
- *V: IS THE FRESNEL-KIRCHOFF DIFFRACTION PARAMETER*

GRAPHICAL CALCULATION OF DIFFRACTION ATTENUATION



THANK YOU