Modulation

- Continuous wave Modulation
  - Analog C.W Modulation
    eg :→ A.M, F.M, P.M
  - Digital C.W Modulation
    eg :→ ASK, FSK, PSK
- Pulse Modulation
  - Analog pulse Modulation
    eg :→ PAM, PWM, PPM
  - Digital pulse Modulation
    eg :→ PCM, DPCM, DM.

Chapter-3 :→ Pulse Modulation Systems (8hrs)

Pulse modulation is a technique to sample analog information signal. In pulse modulation system the carrier is a pulse train instead of sinusoidal carrier as in Analog modulation. Some characteristics or parameter of carrier signal (Amplitude, width, position) is changed with respect to instantaneous value of modulating signal generating Pulse Amplitude Modulation (PAM), Pulse width/Length/Duration Modulation (PWM or PLM or PDM) and pulse position Modulation (PPM).

Advantage of Pulse modulation system is that it permits the simultaneous transmission of several signals on time sharing basis. i.e Time Division Multiplexing (TDM)

3.1 Pulse Amplitude Modulation (PAM)

Pulse Amplitude Modulation (PAM) may be defined as the type of modulation in which amplitude of carrier of periodic train of rectangular pulse is changed in proportion to sample value of message signal.

Pulse Amplitude Modulation (PAM) may be flat-top or natural. Flat-top is most popular and widely used because it has high immunity to noise.

A Sample and Hold Circuit is used to produce Flat top sampled PAM.

Sample and Hold Ckt

Consist of two switch: sample switch (G_1), Discharge switch (G_2) and a capacitor (c).

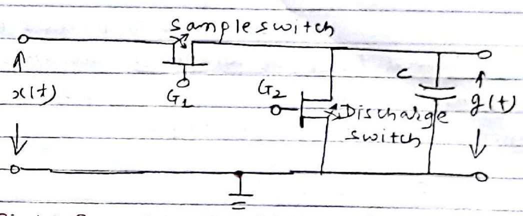


fig:→ Sample and Hold Ckt

Above figure shows the Sample and Hold Ckt. It Consist of two Field Effect transistor (FET) switches and a capacitor. The sampling switch (G_1) is closed for short duration by a short pulse applied to the Gate (G_1) of the transistor. During this period the capacitor 'c' is quickly charged up to a voltage equal to the instantaneous sample value of the incoming signal $x(t)$.

Now sampling switch (G_1) is opened and the capacitor holds the charge. The Discharge switch is then closed by a pulse applied to Gate (G_2) of the other transistor. The capacitor discharges up to '0' volt. Similarly, Discharge switch (G_2) is open and thus capacitor has no voltage. Therefore Flat top sample is produced in the output of sample and Hold circuit.

(copy all from Flat Top Sample)

* Transmission Bandwidth of Pulse Amplitude Modulation (PAM)

In Pulse Amplitude Modulated (PAM) signal the pulse width or duration 'τ' is very small in comparision to time period (sampling period) 'T_s' between two samples.

$$\tau << T_s \quad — (1)$$

If the maximum frequency of the modulating or message signal $x(t)$ is 'f_m'. According to sampling theorem, the sampling frequency 'f_s'

$$f_s \geq 2f_m \quad — (2)$$

$$\frac{1}{T_s} \geq 2f_m \quad \left[\because f_s = \frac{1}{T_s}\right]$$

$$T_s \leq \frac{1}{2f_m} \quad — (3)$$

From eq^n (1) & (3), we get

$$\tau << T_s \leq \frac{1}{2f_m}$$

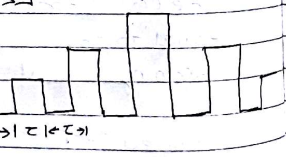


fig:→ PAM signal Illustration

For both ON and OFF time of PAM Signal, maximum frequency of pulse Amplitude modulated Signal is

$$f_{max} = \frac{1}{\tau + \tau} = \frac{1}{2\tau} \quad — (4)$$

Therefore Bandwidth requirement for transmission of PAM signal is given by

Telephone modem, QAM used in telephone modem with data rate more than 300kbps.

Transmission Bandwidth $BW \geq f_{max}$ — (5)

$$BW \geq \frac{1}{2\tau} \quad \text{From eq}^n \text{ (4)}$$

Therefore

$$\boxed{BW \geq \frac{1}{2\tau} >> f_m}$$

$$\boxed{BW >> f_m}$$

* Demodulation (Reconstruction) of PAM signals

Recovering of the modulating signal from the modulated signal is called Demodulation. For PAM signal, the demodulation is done by Holding ckt.

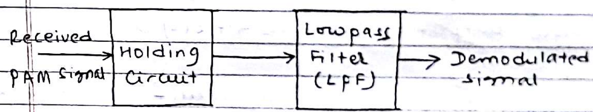


fig:→ Block Diagram of PAM Demodulator

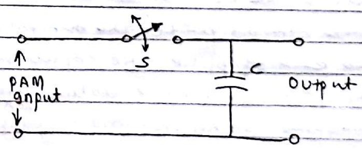
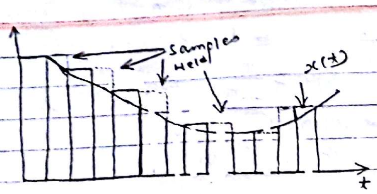
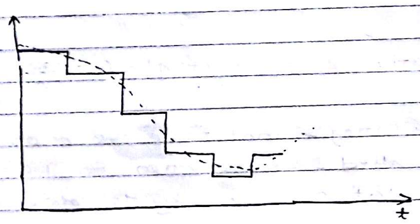


fig (a) Holding circuit (zero-order Holding ckt)

(PDNC 545)



fig(b) output of Holding circuit

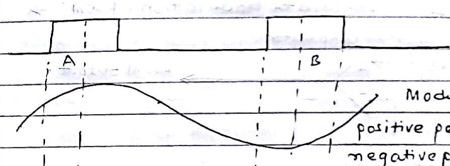


fig(c) output of Low pass Filter (LPF)

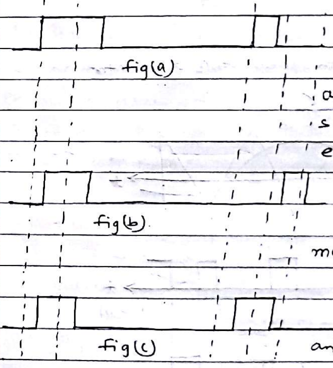
fig(a) shows the simple Holding circuit (Also known as zero-order Holding ckt because it considers previous sample to decide the value between the two pulses). Here, the switch 'S' is closed after the arrival of pulse and opened at the end of pulse. The capacitor 'C' gets charged to the pulse amplitude value and holds for the interval between two pulses. This is then passed through the low pass Filter (LPF) to get reconstructed modulating signal (message signal) which is clearly illustrated by fig(b) and fig(c).

* Pulse width/Length/Duration Modulation (PWM or PLM or PDM)

In pulse width/Length/Duration Modulation the pulse width is changed in proportion to the amplitude of the modulating signal. There are three variation of pulse width/Length/Duration modulation.



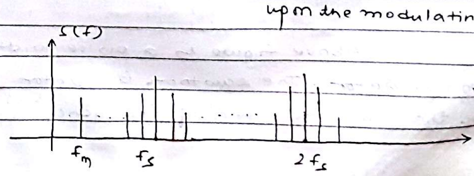
Modulating signal is at its positive peak at point 'A' and its negative peak at point 'B'.



fig(a) Leading edge of the pulse is held constant and change in pulse width with signal is measured w.r.t the leading edge.

fig(b) Tail edge of the pulse is kept constant and pulse width is measured from the tail end of the pulse.

fig(c) Center of the pulse is kept constant and pulse extends on either side of the center of the pulse depending upon the modulating signal.



Above figure shows the frequency spectrum of PWM wave. With a sinusoidal modulating signal at frequency fm, the spectrum of PWM signal consist the modulating frequency (fm) along with several harmonics.

* Generation of Pulse width Modulation (PWM):→

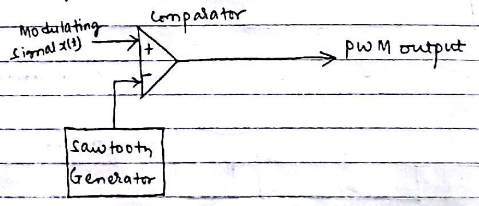
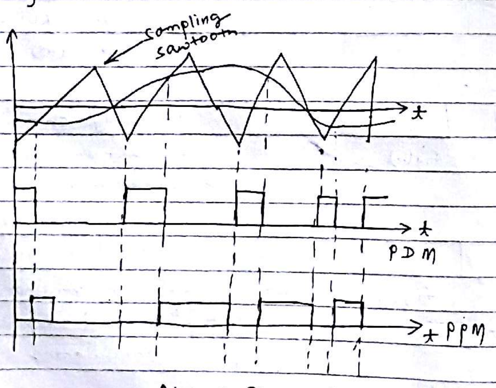


fig:→ pulse width Modulation Generator



Above figure is a pulse width Modulation Generator. The sawtooth Generator generates the sawtooth signal of frequency 'f_s', is a sampling signal and is applied to the inverting terminal of the comparator. The message or modulating signal x(t) is applied to the Non-Inverting Input. Output of the comparator will be high as long as instantaneous amplitude of x(t) is higher than that of sampling signal producing pulse width Modulated signal. Likewise output is zero when amplitude of sampling signal is greater than the instantaneous amplitude of modulating signal x(t).

* Advantages of PWM

1. Better Noise Immunity.
2. Synchronization between transmitter (TX) and Reciver (R) is not required. (Needed in PPM)
3. Possible to reconstruct the signal from noise. (Not possible in PPM).

* Disadvantages

1. It requires large B/W compared to PAM signal in order to avoid distortion.

2. Due to variable pulse width, transmitter must be able to handle the power content of pulse with maximum width.

* pulse position Modulation (PPM)

In pulse position modulation (PPM) the position of each pulse is changed with respect to the amplitude of sampled value of modulating signal. In PPM the amplitude and width of the pulse are kept constant.

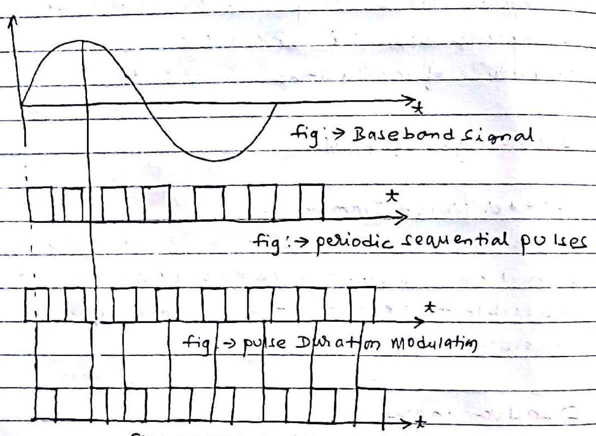


fig:→ Baseband signal

fig:→ periodic sequential pulses

fig:→ pulse Duration Modulation

fig:→ pulse position Modulation

* Generation of PPM signal

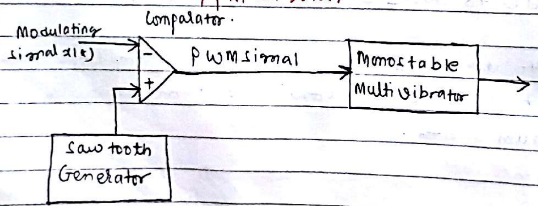


fig → pulse position Modulation Generator

Remote Controlled aircraft (RC)
R.F Commⁿ

Above figure shows the pulse position Modulation (PPM) Generator. The pulse width modulation signal is obtained at the output of the comparator which is fed to Monostable Multivibrator. The multivibrator is negative edge triggered which means monostable o/p remains '0' until it is triggered.

Triggering occurs at the trailing (falling) edge of PDM signal which generates pulse position Modulated signal with same or constant width and amplitude of modulating signal.

* Advantages

1. Like PWM, amplitude held constant in PPM results less interference of noise.
2. Signal and Noise seperation are easy.
3. Since pulse width and amplitude is constant, transmission power for each pulse is same.

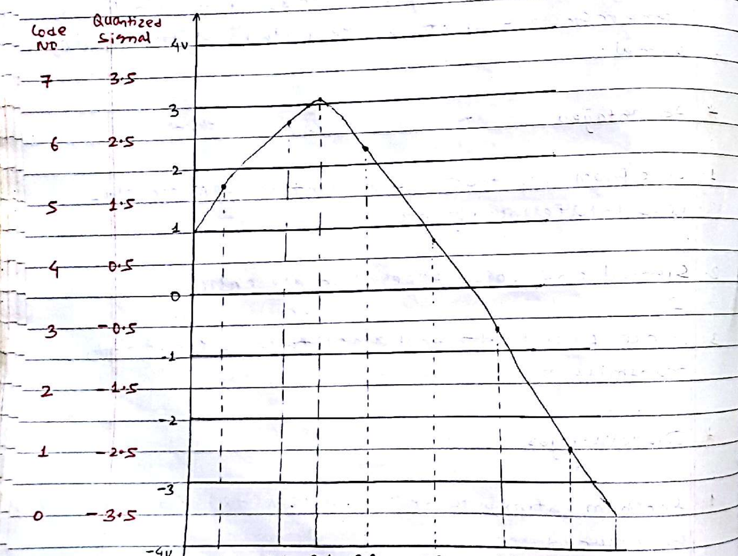
* Disadvantages

1. Synchronization between transmitter (tx) and Receiver (Rx) is required.
2. Large B/w is required compared to Pulse Amplitude Modulation (PAM).

* Pulse Code Modulation (PCM)

Till now we have studied Analog pulse modulation technique (i.e PAM, PWM and PPM).

Pulse Code Modulation (PCM) is a digital pulse modulation technique in which analog signal is sampled and then converted into digital encoded signal which is represented by n-bit binary code.



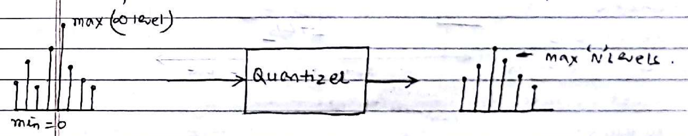
Sampled value →	1.6	2.6	3.1	2.3	0.7	-0.7	-2.5	-3.5
Near of Quantization level →	1.5	2.5	3.5	2.5	0.5	-0.5	-2.5	-3.5
Equivalent Code →	5	6	7	6	4	3	1	0
Binary Code Equivalent →	101	110	111	110	100	011	001	000

Three basic and essential operation in pulse code Modulation (PCM) is

a. Sampling
b. Quantization
c. Encoding

b. Quantization :→

It is the process of representing analog sampled values by a finite set of levels. The sampling process converts a continuous time signal to a discrete time signal with a amplitude that can take any values from 0 to maximum level. It converts continuous amplitude sample to a finite set (discrete) amplitude values.



There are two types of Quantization. They are

i. Uniform Quantization.
ii. Non Uniform Quantization.

i. Uniform Quantization :→

In uniform Quantization the range of input sample is $[-x_{max}, +x_{max}]$ and the number of Quantization level (Q-level) is $N = 2^n$, where n is the number of bits per source sample. Then the step size (Δ) or the length

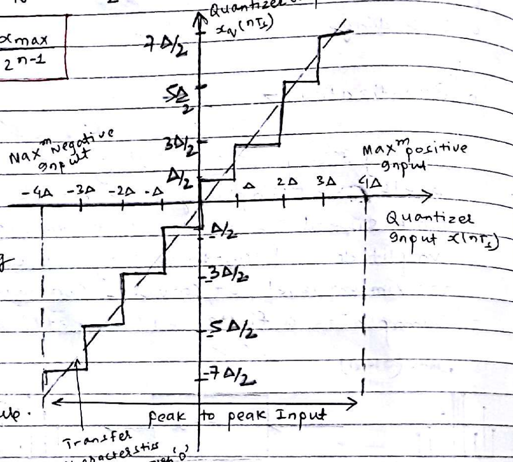
Quantization Error (q_e)

of the Q-level.

$$\Delta = \frac{2x_{max}}{N} = \frac{2x_{max}}{2^n} = \frac{x_{max}}{2^{n-1}}$$

$$\boxed{\Delta = \frac{x_{max}}{2^{n-1}}}$$

* working principle of Quantizer.



Considering midrise type uniform Quantizer. The transfer characteristics is shown in above figure.

Let us take input to quantizer $x(nT_s)$ from -4Δ to 4Δ, where Δ is stepsize. The fixed digital level or Quantizer output is given at

$x(nT_s) = \pm\Delta$ $\quad x_q(nT_s) = \pm\Delta/2$

$x(nT_s) = \pm 2\Delta$ $\quad x_q(nT_s) = \pm 3\Delta/2$

$x(nT_s) = \pm 3\Delta$ $\quad x_q(nT_s) = \pm 5\Delta/2$

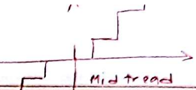
$x(nT_s) = \pm 4\Delta$ $\quad x_q(nT_s) = \pm 7\Delta/2$

Therefore Quantization Error is expressed as

$$E = x_q(nT_s) - x(nT_s)$$

$\boxed{E = \pm\Delta/2}$. Maximum Quantization Error is given by

$$\boxed{E_{max} = \left|\frac{\Delta}{2}\right|}$$



Therefore Quantization Error (E) is the difference between the input signal level $x(nT_s)$ and the level of quantized version $x_q(nT_s)$.

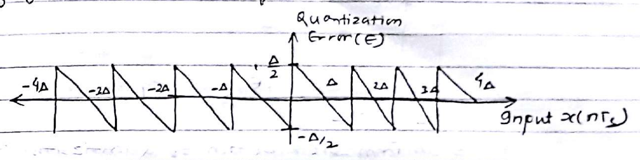


fig:→ Variation of Quantization Error.

From above illustration, we can know that the Quantization Error/Noise are reduced with the increase of level 'N' which in turn reduces the step size.

* Quantization Noise power and signal to Quantization Noise Ratio (SQNR) in PCM

As we know, Quantization Error $\boxed{E = x_q(nT_s) - x(nT_s)}$. If $x(nT_s)$ is continuous amplitude in range $-x_{max}$ to x_{max} mapped to 'N or q' level. Then stepsize is given by

$$\boxed{\Delta = \frac{x_{max} - (-x_{max})}{N \text{ or } q} = \frac{2x_{max}}{N \text{ or } q}} \quad \text{—(1)}$$

Again, If $x(t)$ is normalized to maximum and minimum value equal to 1. $x_{max} = 1$, $x_{max} = -1$

$$\boxed{\text{Stepsize } \Delta = \frac{2}{N \text{ or } q}} \quad \text{—(2)}$$

If step size 'Δ' is sufficiently small, we assume the Quantization error 'E' will be uniformly

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distributed random variable. Quantization error is distributed over the interval $(-\Delta/2, \Delta/2)$

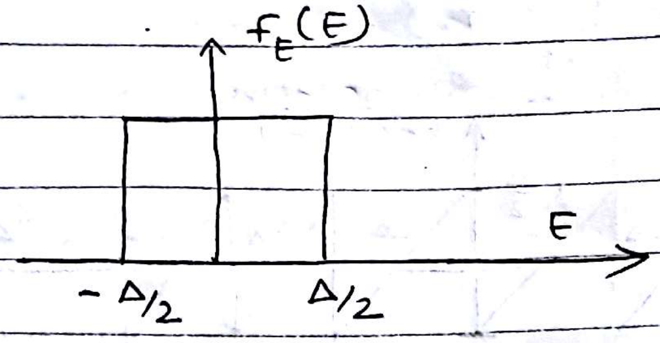

fig :→ Uniform Distribution of Quantization Error.

Therefore probability Density function (PDF) for Quantization Error (E) is defined by

$$f_E(E) = 0 \quad \text{for } E \le -\frac{\Delta}{2}$$

$$= \frac{1}{\Delta} \quad \text{for } -\Delta/2 < E \le \Delta/2 \qquad -③$$

$$= 0 \quad \text{for } E > \Delta/2$$

Again Noise power (V^2_{noise}) is a mean square value of noise voltage. Since Noise is defined by random variable (E) and PDF $f_E(E)$, Mean square value given as

$$\boxed{\text{Noise power} = \frac{V^2_{Noise}}{R}}$$

$$E(\epsilon^2) = \overline{E^2} = V^2_{noise}$$

$$E(E^2) = \int_{-\infty}^{\infty} E^2 f_E(E)\, dE$$

$$E(E^2) = \int_{-\Delta/2}^{\Delta/2} \frac{1}{\Delta} E^2 \, dE = \frac{1}{\Delta}\int_{-\Delta/2}^{\Delta/2} E^2 dE = \frac{1}{\Delta} \left. \frac{E^3}{3} \right|_{-\Delta/2}^{\Delta/2}$$

$$E(E^2) = \frac{1}{3\Delta}\left[\frac{\Delta^3}{8} - \left(-\frac{\Delta^3}{8}\right)\right] = \frac{1}{3\cancel{\Delta}} \frac{\cancel{2}\Delta^{\cancel{3}}}{\cancel{8}_4} = \frac{\Delta^2}{12}$$

$$\therefore \boxed{E(\epsilon^2) = \frac{\Delta^2}{12}} \quad \text{--- (4)}$$

Noise power is normalized for load resitance $R_L = 1\Omega$

$\therefore$ Normalized Noise power or Quantization noise power $= \dfrac{\Delta^2}{12}$

Again, signal to Quantization Noise Ratio for Linear Quantization is given as

$$SQNR = \frac{\text{Normalized signal power}}{\text{Normalized Noise power}}$$

since number of bits given as v or n, the quantization level is

$$N \text{ or } q = 2^v \text{ or } 2^n$$

From eqn (1)

(1) step size $\Delta = \dfrac{2x_{max}}{N \text{ or } q} = \dfrac{2x_{max}}{2^v \text{ or } 2^n}$

$$\therefore SQNR = \frac{P}{\frac{\Delta^2}{12}} = \frac{P}{\left(\frac{2x_{max}}{2^v}\right)^2 / 12} = \frac{P}{\frac{4x_{max}^2}{2^{2v}} \times \frac{1}{\cancel{12}\,3}}$$

$$\boxed{SQNR = \frac{3P\,2^{2v}}{x_{max}^2}}$$

which shows signal to Noise power of Quantized increases exponentially with increase of bits/sample

If $x(t)$ is Normalized than $x_{max} = 1$ and if we consider destination signal power is normalized $P \leq 1$ Then

$$SQNR = \frac{3 \cdot 2^{2V}}{(1)^2} = 3 \cdot 2^{2V}$$

In dB.

$$(SQNR)_{dB} = 10 \log_{10}(SQNR) = 10 \log_{10}(3 \cdot 2^{2V})$$

$$= 10 \log_{10} 3 + 20V \log_{10}(2)$$

$$\boxed{(SQNR)_{dB} = 4 \cdot 8 + 6V}$$

* Transmission Bandwidth in PCM.

Let 'n or v' be the number of binary digits to represent each level. Therefore number of level will be

N or $q = 2^n$ or 2^v.

As each sample in PCM is converted to 'v' or 'n' binary bits.

Number of bits per sample = n or v

As number of samples per second is 'f_s'.

Number of bits per second is expressed as

Number of bits per second = Number of bits per sample x Number of samples per second

signalling rate of PCM is

$r = v f_s$ or $r = n f_s$

As Nyquist criteria the sampling frequency $f_s \geq 2 f_m$

$r \geq v \, 2 f_m \geq 2 v f_m$

Bandwidth for transmission is given by half of the PCM signalling rate

$BW = \frac{1}{2} r \geq \frac{1}{\cancel{2}} \cancel{2} v f_m$

$\boxed{\text{Bandwidth (BW)} \geq v f_m}$

ii) Non uniform Quantization

In Non uniform Quantization the quantizer characteristics is non linear and the step size is not remain fixed. In Non uniform Quantization, the step size is small or reduced for small amplitude or weak signal and the step size is increased or big for large amplitude or strong signal level to improve SQNR. The non-uniform Quantization is practically achieved through a process called Companding.

* Companding

Companding is the process of Compressing signal at the transmitter and expansion at the receiver side. Companding improves the SQNR of weak signal. There are two compression laws or companding technique that are commonly in practice

a. μ-law Companding :→

The Compressor characteristics is continuous in μ-law Companding. It is approximately linear for smaller value of input level and logarithmic for high input level.

Mathematically Compressed output is given as

$$\boxed{|v| = \frac{\log(1 + \mu |x|/x_{max})}{\log(1+\mu)}}$$

v = Normalized Compressed O/P Voltage.

μ = parameter used to define amount of Compression.

x = Input to the Compressor $|x|/x_{max}$ ⇒ Normalized value of input w.r.t maximum value.

x_{max} = maximum value

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u-law ⇒ American standard · used in U.S and Japan

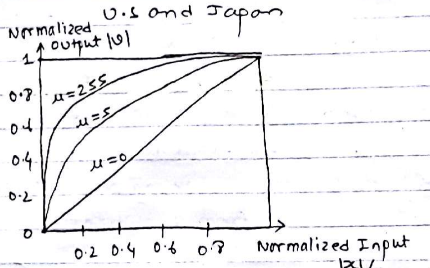


fig: → Compressor characteristics of a u-law compressor

Compressor characteristics of a u-law compressor is neither strictly linear nor strictly logarithmic. The curve is almost linear for $\mu << 1$ and gets logarithmic for greater values. Practical value of 'u' is 255 and used in U.S, Japan, Canada.

b. A-Law Companding :→

In A-law companding, the compressor characteristics is piecewise, made up of linear segment for low level input and logarithmic curve for high level input which is clearly illustrated in the figure below.

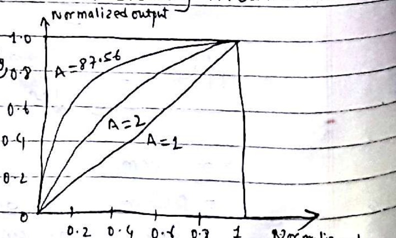


fig: → Compressor characteristics of A-Law Compressor

Mathematically, compressor output of A-Law companding is

[u-law companding is better than A-law in terms of signal quality.]

$$|v| = \frac{A|x|/x_{max}}{1+\log A} \quad \text{for } 0 \le \frac{|x|}{x_{max}} \le \frac{1}{A}$$

$$\frac{1+\log(A|x|/x_{max})}{1+\log A} \quad \text{for } \frac{1}{A} \le \frac{|x|}{x_{max}} \le 1$$

A-law ⇒ European standard used in Europe & rest of the world and is used in PCM telephones system.

* Necessity of Non Uniform Quantization for speech signal

Crest factor is defined as the ratio of peak value to the rms value of signal.

Mathematically,

$$\text{Crest factor} = \frac{\text{Peak value of signal}}{\text{rms value of signal}}$$

If $x(t)$ is the input to quantizer with its amplitude range of $-x_{max}$ to x_{max}.

peak value of signal $= x_{max}$

rms value of signal $= \sqrt{x^2(t)}$

As power is defined as $P = \frac{v^2_{signal}}{R} = \frac{x^2(t)}{R}$

For normalized power $R = 1$ will be

$$P = x^2(t)$$

$$\text{Crest factor} = \frac{x_{max}}{\sqrt{x^2(t)}}$$

~~For normalized the signal $x(t)$ has $x_{max} = 1$~~

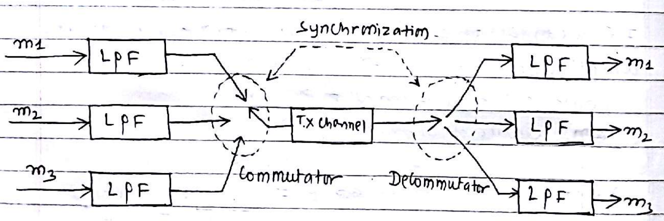
$$\boxed{\text{crest factor} = \frac{1}{\sqrt{p}}}$$

which clearly illustrates that for large crest factor 'p' should be very less.

i.e $p << 1$ which will decrease the SQNR.

To overcome this effect, Non uniform Quantization is appropirate. For large crest factor, signal (i.e speech, music) of low level are easily effected by the noise which means noise increases. To mitigate this effect, step size is kept small for low signal level to maintain optimum SQNR.

* Time Division Multiplexing with PCM



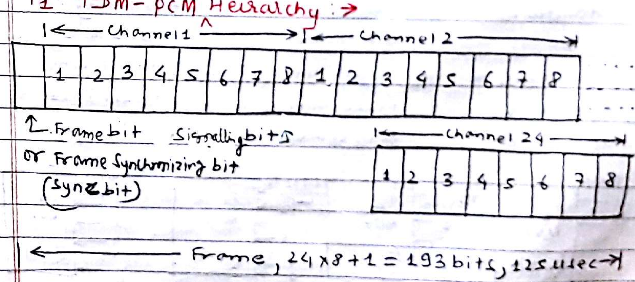
In Time Division Multiplexing with PCM, the samples of message are transmitted at some fixed interval of time. Thus over same communication channel, samples of number of message signal can be transmitted serially and then recovered and seperated at the receiving end.

In other words time interval between two adjacent samples of one message can be used to transmit samples of other message and such technique is known as Time Division Multiplexing.

The signals to be multiplexed are first individually bandlimited by Low pass Filter (LPF). The commutator takes sample of each signal sequentially at fixed interval of time. These samples are then transmitted through the common channel using digital transmission technique.

The decommutator at the receiving end seperates sequentially transmitted signal into individual signals. The commutator and decommutator are synchronized using timing signal. The low pass Filter (LPF) at the receiving end converts the sample of the message signal into original signal.

* T1 TDM-PCM Telephone Heirarchy :→



AT&T Heirarchy

framing, Common signalling information in Sep. Overhead channel

T_1 TDM-PCM Telephone Heirarchy is 24 channel TDM Multiplexer which is also known as T_1-System. T_1 system is North American Digital multiplexing standard recognized by ITU-T.

In T_1-System 24 voice channel, each voice channel are bandlimited to 300-3400 HZ voice signal, sampled at $f_s = 8$ KHz are multiplexed. Each sample are quantized and converted into 7-bit code word with additional one extra bit for synchronization resulting 8 bits per voice channel. Here the 8th bit of each channel is reserved for signalling (call initiated, terminated, address of calling party)

$$\text{Bit rate} = [(24 \times 8) + 1] \text{ bits/frame} \times 8000 \text{ samples/sec}$$

$$\text{Bit rate} = 1.544 \text{ mbps}$$

Similarly, there are other standards which are

Standards	Channel	Signalling Rate
$T_2 = 4 \times T_1$	96 channel	6.312 mbps
$T_3 = 7 \times T_2$	672 channel	44.736 mbps
$T_4 = 6 \times T_3$	4032 channel	274.176 mbps
$T_5 = 2 \times T_4$	8064 channel	560.16 mbps

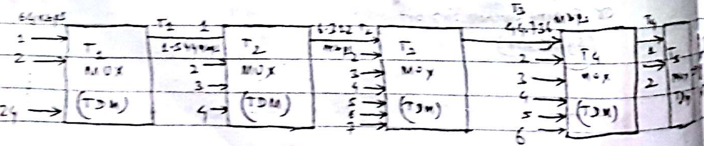


fig:→ T_1 Digital Heirarchy (AT&T Heirarchy)

= 64 kbps

* E1 TDM-PCM Telephone Heirarchy

Timeslot 0	Timeslot 1	Timeslot 2-16	Timeslot 17	Timeslot 18-30	Timeslot 32
Framing and Alarm channel	voice channel	voice channel 2-15	common signalling channel	voice channel 16-29	voice channel 30
8 bits	8 bits	112 bits	8 bits	112 bits	8 bits

fig:→ Frame Allignment for E_1 standard

E1 TDM-PCM Telephone Heirarchy is 32 channel TDM Multiplexer which is also known as E1-system. E1 system is European Digital Multiplexing standard recognized by ITU-T.

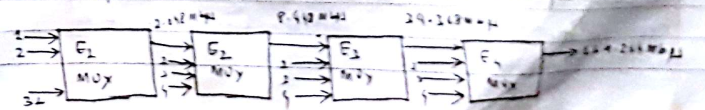
In E1 system 32 voice channel, each voice channel are bandlimited to 300-3400 HZ voice signal, sampled at $f_s = 8$ KHz are multiplexed. Here, each frame is divided into 32 equal time slots where two time slots are reserved for signalling and controlling. Each time slot has 8 bits and bit rate can be calculated as

$$\text{Bit Rate} = \underbrace{[32 \times 8]}_{\text{bits/frame}} \times 8000 \text{ samples/sec}$$

$$\text{Bit Rate} = 2.048 \text{ mbps}$$

Similarly there are other standards which are

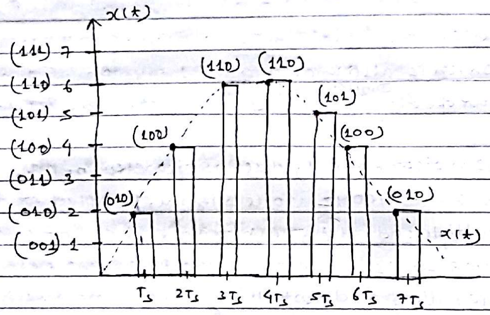
Standard	Channel	Signalling Rate
$E_2 = 4 \times E_1$	120	8.448 mbps
$E_3 = 4 \times E_2$	480	34.368 mbps
$E_4 = 4 \times E_3$	1920	139.264 mbps



E_1 Digital Heirarchy

1.7 Diffrential pulse Code Modulation (DPCM)

In ordinary or standard pulse code Modulation (PCM), after sampling the message signal each sample is quantized in independent manner. It means previous sample value have no effect on quantization of new sample.



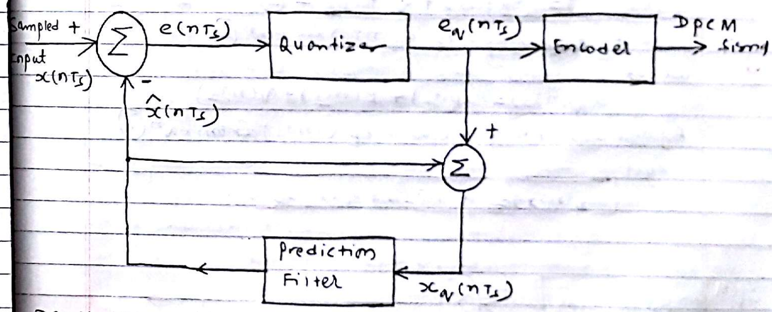
Let $x(t)$ be the continuous time signal which is flat top sampled at interval $T_s, 2T_s, 3T_s, 4T_s, 5T_s, 6T_s$ & $7T_s$. The sampling frequency is selected higher than the Nyquist rate. The samples are then quantized to the nearest level shown by dotted point. These quantized samples are encoded by using 3-bit (8 level) PCM.

We can observe from the figure that samples taken at $3T_s$ and $4T_s$ are encoded to same value (110). In PCM this information is carried by two samples which can be done by single sample instead.

CCITT → ITU-T

Similarly, sample at $5T_s$ and $6T_s$. The difference between two sample is last bit only. The first two bit are redundant.

If this redundancy is reduced, the overall bit rate will decrease and number of bits required to transmit. This type of digital pulse modulation technique is called Differential pulse Code Modulation (DPCM). Infact DPCM works on the principle of prediction. The value of the present sample is predicted from the past sample. The prediction may not be exact but is very close to actual sample value.



DPCM Transmitter

Above figure shows the DPCM transmitter. Let $x(nT_s)$ is the sampled signal and $\hat{x}(nT_s)$ is the predicted signal. The comparator finds the difference between the sampled and predicted sample value $\hat{x}(nT_s)$ given by the prediction error denoted by $e(nT_s)$ which is

$$\boxed{e(nT_s) = x(nT_s) - \hat{x}(nT_s)} \quad \text{—}(1)$$

Here, predicted sample value is produced by prediction filter. If '$e_q(nT_s)$' is the Quantizer output which is fed back to prediction filter as $x_q(nT_s)$ with the sum of prediction filter. This process or mechanism will make prediction more closer to the original sampled value. $\therefore$ $\boxed{x_q(nT_s) = \hat{x}(nT_s) + e_q(nT_s)}$ —(2)

If $q(nT_s)$ is the Quantization error. Then Quantizer output is

$$\boxed{e_q(nT_s) = e(nT_s) + q(nT_s)} \quad \text{—}(3)$$

substituting value of $e_q(nT_s)$ in eqn (2)

we get

$$x_q(nT_s) = \hat{x}(nT_s) + e(nT_s) + q(nT_s)$$

Again substituting value of $e(nT_s)$ from eqn (1)

we get

$$\boxed{x_q(nT_s) = x(nT_s) + q(nT_s)}$$

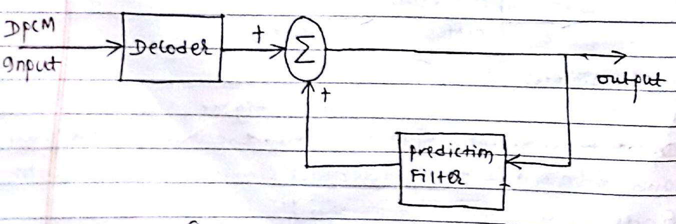
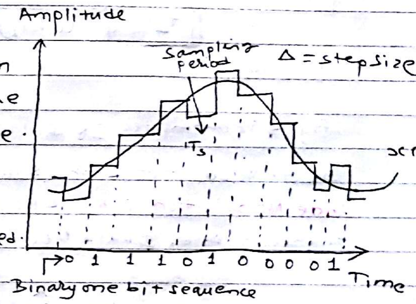


fig:→ DPCM Receiver

DPCM Receiver is shown in above figure. Decoder reconstructs the quantized error signal. The quantized version of the input is recovered from the decoder output using the same predictor circuit used in transmitter.

3.8 Delta Modulation :→



Pulse Code Modulation (PCM) transmits all the bits to code a sample. Thus High channel Bandwidth and signalling rate are required.

Unlike Delta Modulation transmits only one bit per sample. In Delta modulation present sample value is compared with the previous sample value. If the current sample is smaller than previous sample, it is defined by $-\Delta$ level and 0 is transmitted. If the current sample is large, it is defined by $+\Delta$ level and '1' is transmitted. Hence for each sample only one binary bit is transmitted.

Technically speaking Delta modulation is defined as the staircase approximation of the input waveform in which each step is represented by 1 or 0 depending upon the rise or fall of the steps of the stair.

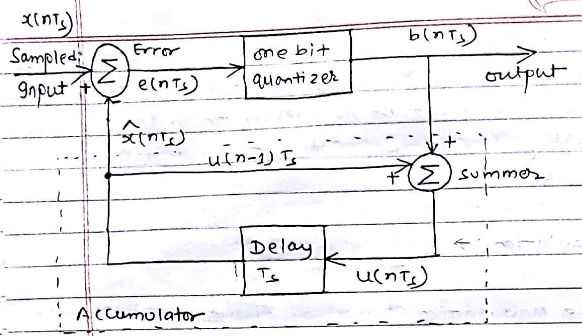


fig: → Transmitter of Delta Modulation

Delta Modulation is the simpler version of pulse code modulation (PCM). Above figure shows the transmitter of Delta Modulation.

Let $x(nT_s)$ is sampled signal and $\hat{x}(nT_s)$ is the approximation of the stair case waveform of last sample. The error can be calculated as

$$\boxed{e(nT_s) = x(nT_s) - \hat{x}(nT_s)} \quad -(1)$$

where

$e(nT_s) \Rightarrow$ error at present sample.

Assume $u(nT_s)$ is the present sample approximation. Then

$$u(n-1)T_s = \hat{x}(nT_s) \quad -(2)$$

If $b(nT_s)$ is the output of the one bit quantizer. It is defined as

$$\boxed{b(nT_s) = \Delta \, \text{sgn}[e(nT_s)]} \quad -(3)$$

Therefore output is defined as

$b(nT_s) = +\Delta$ If $x(nT_s) \geq \hat{x}(nT_s)$ then binary '1' is transmitted.

$\quad\quad = -\Delta$ If $x(nT_s) < \hat{x}(nT_s)$ then binary '0' is transmitted.

Again, for next approximation the summer in the accumulator adds quantizer output ($\pm\Delta$) with previous sample approximation giving

$$\boxed{u(nT_s) = u(n-1)T_s \pm \Delta}$$

$$\boxed{u(nT_s) = u(n-1)T_s + b(nT_s)}$$

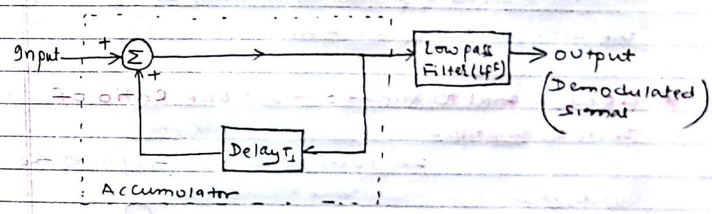


fig: → Delta Modulation Receiver

Above figure shows the Delta modulation Receiver. It consist of Accumulator and Low pass Filter (LPF).

The accumulator generates the staircase approximated signal output and is delayed by one sampling period (T_s). If the input (Delta modulated PCM) signal is binary '1', $+\Delta$ step is added to the previous output and if the input signal is binary '0', $-\Delta$ step is subtracted from the previous output. Output is then passed through Low pass Filter (LPF) to reconstruct the original message signal.

* Bit Rate (signaling Rate) of Delta Modulation

bit rate (r) = Number of samples/sec × Number of bit/sample

$$r = f_s \times 1 = f_s$$

$$\boxed{r = f_s}$$

Bit rate of PCM is $\boxed{r = n f_s}$

Therefore bit rate of Delta modulation is $1/n$ times the bit rate of PCM system.

* (SQNR) signal to Quantization Noise Ratio of Delta Modulation.

In Delta Modulation, output of the quantizer is

$$e_q(nT_s) = b(nT_s) = \Delta \, sgn[e(nT_s)] \quad -(1)$$

output value may be $+\Delta$ or $-\Delta$ and total swing is 2Δ.

The total Noise power is given as

$$P_q = \frac{1}{2\Delta}\int_{-\Delta}^{\Delta} e_q^2 \, de_q = \frac{1}{2\Delta}\left.\frac{e_q^3}{3}\right|_{-\Delta}^{\Delta}$$

$$P_q = \frac{1}{2\Delta}\left[\frac{\Delta^3}{3} + \frac{\Delta^3}{3}\right] = \frac{1}{\cancel{2\Delta}} \times \frac{\cancel{2}\Delta^{\cancel{3}}}{3} = \frac{\Delta^2}{3} \quad -(2)$$

For normalized power of $e_q(nT_s)$ is uniformly distributed over interval of $(0, f_s)$ then

PDF of $e_q(nT_s)$ is

$$G_{e_q}(f) = \frac{P_q}{2f_s'} = \frac{\Delta^2}{6f_s'} \quad -(3)$$

If the output of Receiver filter is ideal over frequency range $(0, f_x)$ then average power at the output of the filter is

$$P_q(LPF) = \int_{-f_x}^{f_x} G_{e_q}(f) \, df$$

$$P_q(LPF) = \frac{\Delta^2}{6f_s'}(f_x + f_x) = \frac{\Delta^2 f_x}{3f_s'} \quad -(4)$$

Let us consider the case when all signal power is concentrated at the upper end of the bandwidth of message signal i.e

$$x(t) = A\cos 2\pi f_x t$$

output of Ideal LPF

$$x_{LPF}(t) = A\cos 2\pi f_x t$$

$\Rightarrow$ Signal power $P_x = \frac{A^2}{2} \quad -(5)$

To Avoid slope overload distortion, the maximum value of A should be less or equal to

$$A \leq \frac{\Delta f_s'}{2\pi f_x} \quad -(6)$$

putting this value in eq^n (5) we get

$$\boxed{P_x = \frac{\Delta^2}{8\pi^2}\left(\frac{f_s'}{f_x}\right)^2}$$

Therefore, signal to Quantization Noise Ratio (SQNR) is

$$SQNR = \frac{P_x}{P_q(LPF)} = \frac{\cancel{\Delta^2}}{8\pi^2}\left(\frac{f_s'}{f_x}\right)^2 \times \frac{3 f_s'}{\cancel{\Delta^2} f_x}$$

$$\boxed{SQNR = \frac{3}{8\pi^2}\left(\frac{f_s'}{f_x}\right)^3}$$

since $f_s = 2 f_x$

If $f_s' = n f_s = 2n f_x$ then for Delta Modulation

$$SQNR = \frac{3}{8\pi^2}\left(\frac{2n\cancel{f_x}}{\cancel{f_x}}\right)^3$$

$$= \frac{3}{\cancel{8}\pi^2} \times \cancel{8} n^3$$

$$SQNR = 0.3 n^3$$

In dB, $\boxed{SQNR = 10\log(0.3) + 30\log n}$

* Comparing PCM and Delta Modulation

For $n = 8$

$(SQNR)_{PCM} = 52.8\,dB$

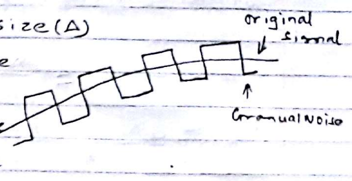
$(SQNR)_{DM} = 21.86\,dB$

For similar condition pulse Code Modulation (PCM) has better SQNR and performance than Delta Modulation.

* Quantization Noise in Delta Modulation

1. Granual Noise :→

If the stepsize (Δ) is too large compared to the input signal. Small variation in input signal, the staircase signal is changed by large amount because of large step size. It starts swinging from $-\Delta$ to Δ causing high noise levels. It can be minimized by reducing the step size.



2. Slope overload Distortion :→

If the input signal slope is high then the given step size. It may not be sufficient to follow the rate of change of the signal. In this case condition called slope overload is present and the noise produced by such overload is called slope overload Distortion.

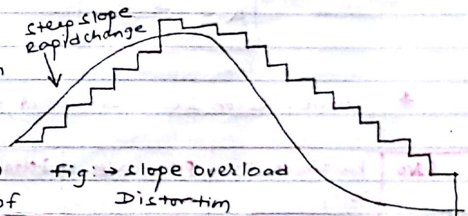


fig : → slope overload Distortion

To minimize this effect increase the magnitude of the minimum stepsize.

let us consider sine wave as

$$x(t) = A\sin(2\pi f_m t)$$

$$\text{Maximum slope of Delta Modulator} = \frac{\text{Step size}}{\text{Sampling period}} = \frac{\Delta}{T_s} \quad -(1)$$

slope overload distortion will not occur if slope of delta modulator is

$$\max\left|\frac{d\,x(t)}{dt}\right| \le \frac{\Delta}{T_s}$$

$$\max\left|\frac{d\,A\sin 2\pi f_m t}{dt}\right| \le \frac{\Delta}{T_s}$$

$$\max\left|A\,2\pi f_m \cos 2\pi f_m t\right| \le \frac{\Delta}{T_s}$$

Maximum value of cosine term is 1.

Therefore, $A\,2\pi f_m \le \frac{\Delta}{T_s}$

$$\boxed{A \le \frac{\Delta f_s}{2\pi f_m}}$$

* Comparision between PCM, Delta Modulation and DPCM

S.No	Parameter of Comparision	Pulse Code Modulation (PCM)	Delta Modulation	Differential PCM
1	Number of Bits	It uses 4, 8 or 16 bits per sample	It uses only one bit for one sample	Bits can be more than one but are less than PCM
2	Level and step size	Number of levels depend on number of bits	step size is kept fixed and cannot be changed	Fixed number of levels are used
3	Transmission Bandwidth	Highest bandwidth is required since number of bits are high	Lowest bandwidth is required	Bandwidth required is lower than PCM.
4	Quantization Error and Distortion	Depends on number of level used	slope overload distortion and Granual noise are present	slope overload distortion and quantization noise is present.
5.	Complexity	System Complex	simple	simple.
6	Feedback	No Feedback	Feedback exist	Feedback Exist.

3.9 parametric speech coding, Vocoders - Linear prediction coding.

Digital speech coders can be classified into two categories: waveform coders and vocoders.

Waveform coders use algorithm to encode and decode so that the system output is an approximation of the input waveform eg: → PCM and DPCM. This coder provides high quality of signal but requires relatively high bit rates.

Vocoders operates significantly at lower bit rates. Vocoders encode speech signal by modeling the signal and extracting set of parameters. These parameters are then digitized and transmitted to the receiver.

At the Receiver end the original voice signal is predicted using the parameters of model. Such technique of coding of speech signal is called Linear prediction Coding.

*

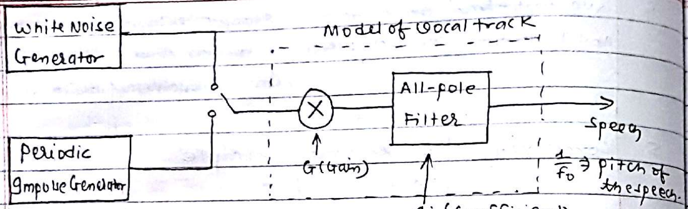


fig: → Speech Model

A speech signal can be modeled as a system consisting of short pulses of repetition frequency 'f_0' which is fundamental frequency of vibration of the vocal chord with all pole filters with coefficient 'a_i' and the gain parameter 'G'. The unvoiced sounds are simulated by white Noise source.

A property of human speech is that it is stationary for a period of 20-30ms i.e within this period its statical property remain constant so filter coefficient can be assumed constant for this period. we analyze values of f_0, a_i and G for these period and transmit. At receiving end we use same prediction filter to reconstruct original speech signal.

* Bit Representation each parameter

Voiced/unvoiced information - 1 bit

pitch - 6 bit

Gain - 5 bit

Filter Coefficient - 8 - 10 bits / coefficient

(Normally first order filter is used)

eg: → voice - 4KHz $f_s = 8000$ samples/sec

Rate in PCM = 8000 × 8 = 64 kbps.

In Linear prediction Coding 20ms - 1 frame - 1 sec - 50 frame

1 frame contain 22 bits.

total no of bits = 22 × 50 = 1100 bits/sec - cellular telephone system uses LPC at 4800 bps = 4.8 kbps for good quality of speech.

Surat Tulachan

* Numerical

1) For a pulse - Amplitude Modulated (PAM) transmission of voice signal having maximum frequency equal to $f_m = 3$ KHZ. calculate the transmission bandwidth given $f_s = 8$ KHZ and pulse width $\tau = 0.1 T_s$

Given, $f_m = 3$ KHZ, sampling frequency $f_s = 8$ KHZ

pulse width $\tau = 0.1 T_s$

Sampling period $T_s = \frac{1}{f_s} = \frac{1}{8KHz} = \frac{1}{8 \times 10^3} = 125 \times 10^{-6}$

pulse width $\tau = 0.1 \times 125 \times 10^{-6} = 12.5\mu s$

Therefore,

Transmission Bandwidth $\geq \frac{1}{2\tau}$

Transmission Bandwidth $\geq \frac{1}{2 \times 12.5 \times 10^{-6}}$

Transmission Bandwidth $\geq$ 40KHZ

2) A television signal having a bandwidth of 4.2 MHZ is transmitted using Binary PCM system. Given that the number of quantization level is 512.

0.2

Determine

1) Code word Length
ii. Transmission Bandwidth
iii. Final bit rate.
iv. Output Signal to Quantization Noise Ratio.

Given,
Maxm^m frequency $f_m = 4.2$ MHZ, Quantization level N or $q = 512$

i) Code word length.

$N \text{ or } q = 2^V = 512$

$2^V = 512$

$\log 2^V = \log(512)$

$V \log 2 = \log(512)$

$V = 9$

Number of bit or Codeword length = 9

2) Transmission Bandwidth $= V f_m$

Transmission Bandwidth $= 9 \times 4.2 = 37.8$ MHZ

3) Final Bit Rate $r = V f_s = 9 \times 8.4 \times 10^6$

$r = 75.6 \times 10^6$ bits/sec

$f_s \geq 2 f_m$

$f_s \geq 2 \times 4.2$

$f_s = 8.4$

4) $SQNR = 4.8 + 6V = 4.8 + 6 \times 9 = 58.8$ dB.

3) The information in an Analog waveform with maximum frequency $f_m = 3$ KHZ is to be transmitted over an M-level PCM system where the number of quantization level is $M = 16$. The quantized distortion is specified not exceed 1% of peak to peak analog signal

i. What would be the maximum number of bits per sample that should be used in this PCM system.

ii. What is the minimum sampling rate and what is the resulting bit transmission rate.

Given, $f_m = 3$ KHZ, N or $q = 16$

i) $N \text{ or } q = 2^V \text{ or } 2^n$

$16 = 2^V \text{ or } 2^n$

$\log 16 = V \log 2$

$n \text{ or } V = 4$

ii) $f_s \geq 2 f_m$

$f_s = 2 \times 3$ KHZ

$f_s = 6$ KHZ

Sampling Rate or Frequency = 6 KHZ

$r = V f_s = 4 \times 6 \times 10^3$

$r = 24 \times 10^3$ bits/sec

Rate$(r) = 24$ Kbps

4) Given an audio signal consisting of the sinusoidal term given as $x(t) = 3\cos(500\pi t)$

i. Determine the Signal to Quantization Noise Ratio (SQNR) when this is quantized using 10 bit PCM.

ii. How many bits of quantization are needed to achieve a signal to quantization noise ratio of at least 40 dB.

i) $A_m = 3V$ covers complete range

$SQNR = 4.8 + 6V$

$SQNR = 4.8 + 6 \times 10$ (for $V = 10$)

$SQNR = 64.8$ dB #

ii) Again $SQNR = 40$ dB

$SQNR = 4.8 + 6V$

$40 - 4.8 = 6V$

$V = 5.86 = 6$ #

5) A PCM system uses a uniform quantizer followed by a 7-bit binary encoder. The bit rate of the system is equal to 50×10^6 bits/sec.

i) What is the maximum message signal bandwidth for which the system operates satisfactorily.

ii) Calculate the output signal to Quantization Noise Ratio (SQNR) when a full load sinusoidal modulating wave of frequency 1 MHZ is applied to the input?

n or $V = 7$ bit

Bitrate $r = 50 \times 10^6$ bits/sec

Signalling rate is given as

$r \geq V f_s$

$r = V f_s = V 2 f_m$ [From Nyquist Sampling Rate]

$r = V 2 f_m$

$50 \times 10^6 = 7 \times 2 \times f_m$

$f_m = 3.57 \times 10^{6} = 3.57$ MHZ #

$SQNR = 1.8 + 6V = 1.8 + 6 \times 7 = 43.8$ dB #

6) The bandwidth of an input signal to the PCM is restricted to 4 KHZ. The input signal varies in amplitude from -3.8V to $+3.8$V and has the average power of 30mW. The required signal to Noise Ratio is given as 20dB. The PCM modulator produces binary output. Assuming uniform quantization

i) Find the number of bits required per sample.

ii) Outputs of 30 such PCM coders are time multiplexed what would be the minimum required transmission bandwidth for this multiplexed signal.

Given, $f_m = 4$ KHZ, $x_{max} = 3.8$V, $-x_{max} = -3.8$V

$P = 30$ mW $= 30 \times 10^{-3}$ W

SQNR = 20dB

i) Number of bits V or $n = 2$.

$(SQNR)_{dB} = 10 \log_{10}(SQNR)$

$20 = (SQNR)_{dB} = 10 \log_{10}(SQNR)$

$\log_{10}(SQNR) = 2$

$SQNR = 10^2 = 100$

$SQNR = \frac{3P\, 2^{2V}}{x^2_{max}}$

$100 = \frac{3 \times 30 \times 10^{-3} \times 2^{2V}}{(3.8)^2}$

$16044.45 = 2^{2V}$

$\log(16044.45) = 2V \log(2)$ $V = 6.98 = 7$ #

ii) $BW \geq V f_m$

$BW \geq 7 \times 4 \times 30 \geq 840$ KHZ $BW = 840$ KHZ.

Signalling rate $(r) = 2 \times BW$

$= 2 \times 840 = 1680$ bits/sec

7) A signal having bandwidth equal to 3.5 KHZ is sampled, quantized and coded by a PCM system. The coded signal is then transmitted over a transmission channel of supporting a transmission rate of 50 kbps. Determine the maximum signal to Noise Ratio that can be obtained by this system. The input signal has peak to peak value of 4 volt and r.m.s value of 0.2V.

Given, $f_m = 3.5\,KHz$ Signalling Rate $r = 50$ kbps.

$x_{max} = 2$ volt. $v_{rms} = 0.2V$

SQNR = ?

$$SQNR = \frac{3P\,2^{2v}}{x_{max}^2}$$

Signaling Rate $r = v f_s$ $f_s \geq 2f_m$ $f_s \geq 2 \times 3.5 = 7\,KHz$

$50 \times 10^3 = v \times 7 \times 10^3$

$v = 7.142 = 8$ bits

As given $v_{rms} = 0.2V$

$$\text{Power}(P) = \frac{v_{rms}^2}{R} \quad [\because R = 1 \text{ for normalize power}]$$

$$P = (0.2)^2 = 0.04W$$

$10\log(1966.08)$

$$\therefore SQNR = \frac{3 \times 0.04 \times 2^{2\times8}}{(2)^2} = 1966.08 = 32.93\,dB$$

8) The information in an analog signal voltage waveform is to be transmitted over a PCM system with an accuracy of ±0.1% (full scale). The analog voltage waveform has a bandwidth of 100 HZ and an amplitude range of -10 to +10 volt.

i) Find the minimum sampling rate required.

ii) Find the number of bits in each PCM word.

iii) Find the minimum bit rate required in the PCM signal.

iv) Find the minimum absolute channel bandwidth required for the transmission of the PCM signal.

Given: $\Rightarrow \pm x_{max} = \pm 10$ volt, $f_m = 100\,HZ$

$E_{max} = \pm 0.1\% = \pm 0.001$

$$E_{max} = \left|\frac{\Delta}{2}\right| = 0.001$$

step size $\Delta = 0.002$

We know step size $\Delta = \frac{2x_{max}}{q}$

$$q = \frac{2x_{max}}{\Delta} = \frac{20}{0.002} = 10000$$

Number of levels $(q) = 10,000$

i) Minimum sampling Rate $f_s \geq 2f_m$ $f_s \geq 2 \times 100 = 200\,HZ$

ii) As we know $q = 2^v$ $10,000 = 2^v$ $v = 13.288 = 14$ bits

iii) Bit Rate or signaling Rate $r \geq v f_s \geq 14 \times 200 = 2800$ bps

iv) Bandwidth $(BW) \geq v f_m = 14 \times 100 = 1400\,HZ$

9) A delta modulator system is designed to operate at five times the Nyquist rate for a signal having a bandwidth equal to 3KHZ bandwidth. Calculate the maximum amplitude of a 2KHZ input sinusoid for which the delta modulator does not have slope over load. Given that the quantizing step size is 250mV.

Given, $f_m = 3\,KHZ$,

Nyquist rate $= 2 f_m = 6\,KHZ$

sampling frequency $f_s = 5 \times$ Nyquist Rate $= 5 \times 6\,KHZ = 30\,KHZ$

Therefore slope overload will not occur

$$A_m \leq \frac{\Delta f_s}{2\pi f_m}$$

$f_m = 2\,KHZ \quad \Delta = 250\,mV$

$$A_m \leq \frac{250 \times 10^{-3} \times 30 \times 10^3}{2 \times \pi \times 2 \times 10^3}$$

~~$A_m \leq 0.6$ volt.~~

10) A sinusoidal voice signal $x(t) = \cos(6000\pi t)$ is to be transmitted using either PCM or DM. The sampling rate for PCM system is 8KHZ and for transmission with D.M, the step size (Δ) is decided to be of 31.25 mV. The slope overload distortion is to be avoided. Assume that the number of quantization level for a PCM system is 64. Determine the signalling rate of both these systems and also comment on the result.

Given $x(t) = \cos(6000\pi t) = \cos(2\pi f_m t)$

$f_m = 3000\,HZ \quad f_s = 8\,KHZ \quad \Delta = 31.25\,mV$

q or $N = 64$ signalling Rate (r) for both PCM and D.M.

Signaling rate $r = v f_s = n f_s$

$$q = 2^v \qquad 64 = 2^v \Rightarrow v = 6 \text{ bits}$$

Signalling Rate for PCM $r = v f_s = 6 \times 8 \times 10^3 = 48\,kbps$

For slope overload distortion

$$A \leq \frac{\Delta f_s}{2\pi f_m}$$

$$\frac{A \times 2\pi f_m}{\Delta} \leq f_s$$

$$f_s \geq \frac{1 \times 2\pi \times 3000}{31.25 \times 10^{-3}}$$

$$f_s \geq 603.18\,KHZ$$

signaling rate for D.M $= 603.18$ kbps

Therefore to transmit the same voice information, the Delta Modulator requires large signaling rate than PCM, which is biggest drawback and makes it an impractical system.

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11) A message signal $x(t) = 6\cos(5000\pi t)$ is quantized in 128 levels using Nyquist sampling rate

a) Find SQNR of the PCM signal.

b) Find the sampling frequency required when same signal uses delta modulation for same SQNR

c) If the system uses DM using Nyquist sampling rate, find SQNR degradation in D.M as compared to PCM.

Given,

$$x(t) = 6\cos(5000\pi t)$$
$$= 6\cos(2\pi f_m t)$$

Maximum frequency $f_m = 2500\,Hz$

$$N \text{ or } q = 128$$
$$q = 2^V$$
$$128 = 2^V$$
$$\log(128) = V \log 2$$
$$V = 7$$

a) $(SQNR)_{PCM} = 4.8 + 6V = 4.8 + 6\times7 = 46.8\,dB$

b)
$$SQNR = \frac{3}{8\pi^2}\left(\frac{f_s'}{f_x}\right)^3$$
$$46.8 = \frac{3}{8\pi^2}\left(\frac{f_s'}{2500}\right)^3$$
$$f_s' = 26798.55\,Hz$$
$$f_s' = 26.798\,kHz$$

c) $(SQNR)_{DM} = 10\log(0.3) + 30\log(7)$

$SQNR = 20.124\,dB$

12) A speech signal with maximum frequency of 4 kHz and maximum amplitude of $\pm 1.1V$ is applied to a PCM system with its bit rate of 32 kbps. Calculate the SQNR and number of bits per sample.

f_m or $f_x = 4\,kHz$ $\quad x_{max} = 1.1V$

$r = 32\,kbps.$

$r = V f_s \qquad f_s \geq 2f_m = 8\,kHz$

$32\times\cancel{10^3}\,bps = V\times 8 \times \cancel{10^3}$

$V = 4$ bits

$SQNR = 4.8 + 6V$

$SQNR = 4.8 + 6\times4 = 28.8\,dB$