Communication System (I)

Bex III/II

## Chapter 1 : Introduction

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### Communication system.

Communication is the process of sending information from source to a receiver.

Communication system is an interconnected system designed to carry information from one place to another, by performing necessary operations on the signal to facilitate optimum transfer.

### Basic communication system.

The purpose of the communication system is to transmit information bearing signal from source to destination.

The block diagram of communication system :-

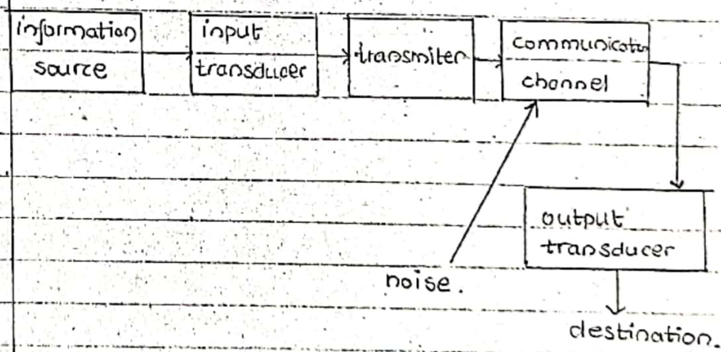



Fig:- Block diagram of communication system.

information source

The source generates the information to be transmitted. The information could be speech, image, video, text etc. and also known as message signal.

The message signal could be

(i) analog : speech, video.

(ii) digital signals : text, multimedia etc.

Input transducer.

The message from the information source may or may not be in electrical form so the purpose of the input transducer is to convert non electrical form of signal into electrical form.

Transmitter

The transmitter modifies the message signal into a suitable form for transmission over channel. This process is known as modulation. In modulation the message signal is superimposed upon the high frequency carrier signal.

channel

The function of the channel is to provide a physical connection between the transmitter & receiver.

It should be noted that there is always the additi of noise signal in the channel which tries to dist the information carried by channel. The channel co be :-

- Point to point :- wire lines, microwave links, optical
- broadcast :- satellite, FM broadcasting etc.

receiver

The receiver has the function opposite to the transi This means the receiver extracts the message or information from the received signal. This process is known as demodulation.

destination

Finally the information or message is received b the desired destination in original form.

digital communication system

In digital communication system the signal to be transmitted from the source to destination is in discrete form, with maximum possible rate & accur

The block diagram of digital communication system is shown below.

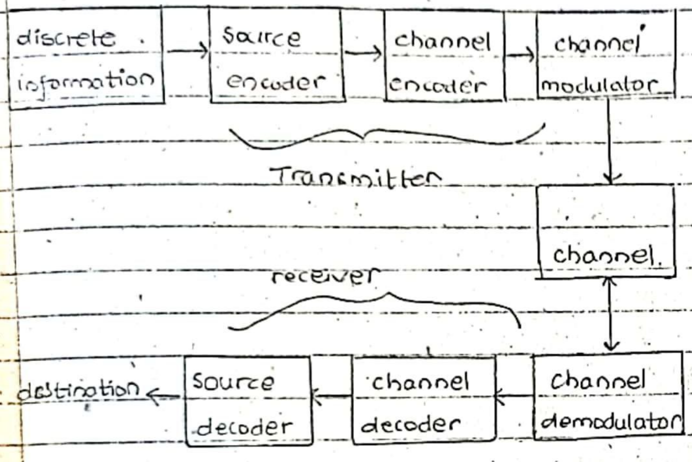



Fig: block diagram of Digital communication system

1. Source encoder

It converts the sequence of symbols occuring at fixed interval of time into binary sequence of 0's & 1's by assigning code word to each symbol.

ii. channel encoder

It adds some error control bits, to the output of source encoder. These donot carry information but makes the receiver to detects errors & correct them.

channel modulator & demodulator.

The channel modulator converts the bit streams from channel encoder into electrical form for transmission over the communication channel. In modulation some of the parameters of carrier wave is varied with the message signal. Basic digital modulation techniques are ASK, PSK, FSK.

* channel demodulator converts electrical signal into sequence of bits.

iii. channel decoder.

It recovers information bearing bit stream from coded bit stream with minimum error & maximum efficiency.

iv. Source decoder.

It performs the reverse operation of source encoder & converts binary output of channel decoder into sequence of symbols.

advantages of digital communication system.

1. Long distance transmission with greater accuracy
2. Increased immunity to channel noise & external interference.
3. Common format for the transmission of different kinds of message signals
4. error at the receiver may be detected & corrected by channel

mathematically by Linear time voriant with filter with impulse response $c(\tau,t)$

for an input signal $s(t)$ the channel output signal is

$$r(t) = s(t) * c(\tau,t) + n(t)$$
$$= \int_{-\infty}^{\infty} c(\tau,t)\, s(t-\tau)\, d\tau + n(t).$$

SNR = Signal noise Ratio

Shannon Hartley channel capacity.

The shannon channel capacity theorem states that for a given channel bandwidth B and required level of SNR the maximum speed of data transmission c is given as;

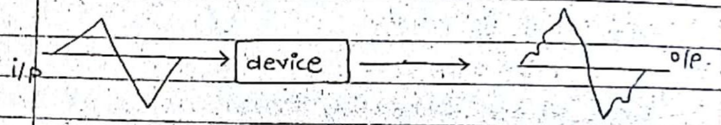
$$C = B \log_2 (1 + SNR) \quad \text{------} \quad (i)$$

- For given B and SNR the channel capacity is fixed or constant.

- To increase the channel capacity
  (i) Increase SNR (but high power transmission required is limited by system power handling capacity.
  (ii) increase bandwidth (increases noise power & hence decreases SNR.

Distortion.

The unwanted changes in the output waveform of the device in comparision to the input waveform is known as distortion.




nature of distortion in audio & video signals.

Human ear can readily (quickly & without difficulty) sense amplitude distortion although it is relatively insensitive to phase distortion. For video signals situation is exactly opposite. The human eye is sensitive to phase distortion & relatively insensitive to amplitude distortion.

Types of distortion

1. Linear distortion.

Linear distortion is produced by Linear devices due to non-uniform frequency and phase responses. In this different frequency components of input signal experiance different gain

- A Linear device is distortionless if magnitude response transfer function / impulse response is constant & the phase is linearly proportional to frequency for $f \leq f_{max}$.

example:

$y(t) = K_1 A_1 \cos(\omega_1 t + \theta_1) + K_2 A_2 \cos(\omega_2 t + \theta_2)$ – Linear disn

$y(t) = K \cdot A_1 \cos(\omega_1 t + \theta_1) + K \cdot A_2 \cos(\omega_2 t + \theta_1)$ - distortionless system.

2. Non linear distortion.

- Produced due to non linear I/O characteristics of the system or device (eg diode)
- Non linear distortion produces new frequency component at the o/p which is not present at i/p.
- I/O characteristics of system can be approximated as;

$$y(t) = a_1 x(t) + a_2 x^2(t) + a_3 x^3(t) + \ldots$$

- Spectral dispersion causes interference with other signals which is a serious problem in FDM.
- because of spectral dispersion original spectrum may overlap & called as crosstalk or intermodulation distortion.

* Generation of harmonics of fundamental frequency of i/p signal due to non-linearity of device is evaluated in terms of harmonic distortion

% $n^{th}$ harmonic distortion = % $D_n$

$$= \frac{|A_n|}{|A_1|} \times 100\%$$

Where,

$A_n$ = amplitude of $n^{th}$ harmonic component

$A_1$ = amplitude of fundamental frequency component.

Total harmonic distortion = $\sqrt{D_2^2 + D_3^2 + D_4^2 + \ldots + D_n^2}$

Noise.

- Noise is an unwanted signal that adds to the receive signal & degrades the perfomance of the communication system.
- It can be categorized as
  - Internal noise
  - External noise

1) Internal noise:

The noise which is generated within the system

a) Thermal/white/Johnson noise.

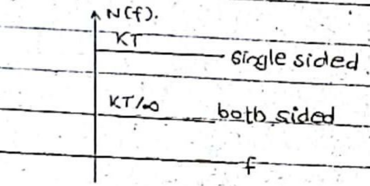
- It is due to the random movement of free electrons within the system.
- Temperature ↑es, internal KE ↑es means energy of the e ↑es thus randomness ↑es.

Thermal noise power;

$$P_n = KTB \quad \text{-----} (*)$$

where, K = Boltzman constant.
T = temp of conductor in K.
B = bandwidth of noise spectrum, Hz.

- Power spectral density function of thermal noise
$S_n = KT$ (watt/Hz)




b. Shot noise.
- due to the random fluctuation in electron emission from cathode in vaccum tube.
- e emission variation means change in current.
- In s/c due to variation in no of electrons crossing the potential barrier.

c. Partition noise.
- due to random fluctuation in division of current into two or more path.
- In s/c transistor, it is due to the emitter current divided into base & collector currents.

- partition noise is less in diode than transistor.

d) flicker or low frequency noise.
- due to the fluctuation in carrier density
$N(f) = 1/f$.

- Non uniform distribution of carrier psdf of flicker noise is inversly proportional to frequency.

e) transit time / high frequency noise.
In s/c at high frequency some of the carriers may diffuse back to the source before crossing the junction that produces the noise.
- Psdf of such noise increases with frequency.

f) Generation-recombination noise.
due to the random process of recombination & generatio[n] of charge carriers.

2) External noise.
Noise which is generated external to the communication system.
Example: Human made noise.
- interference from signals transmitted or nearby channels,

fluroscent light, industrial noise by motors, smoke of automobiles.

Natural noise: lightning, solar radiation, cosmic noise
* can be minimized or even eliminated when proper care is taken.

Interference.

Interference may be defined as the non uniform distribution of the / contamination of received signal by unwanted signals.

- It may due to the man made as well as natural disturbances.

Natural: Lightining.

Manmade: from various broadcasting & communication system.

Modulation.

Modulation is the process of combining low frequency signal with very high frequency radio wave called carrier wave. It is performed at transmitting end.

- The wave resulting from modulation is called modulated wave & the wave that is being modulated is called message signal, modulating signal or baseband signal.

- carrier wave is the high frequency undamped radio waves that carry the signal from transmitting station to receiving station.

Need of modulation

(i) Long range transmission.

To transmit audio signal (have relatively short range of transmission) over a longer distance it is necessary to modulate the signal before transmission

- when frequency is increased energy is increased thus long range transmission is possible.

(ii) Practical antenna size.

For the wireless communication the size of the receiving & transmitting antenna depends on the frequency of signal to be transmitted. If the low frequency are to be transmitted then size of the antenna would be very large thus by the use of modulation technique the antenna size can be greatly reduced.

Example:

frequency of audio signal ($f$) = 1000 Hz
for efficient radiation of signal, the length of antenna should be about one quarter of its wavelength.

$$\lambda = \frac{3 \times 10^8}{1000} = 300\,km$$

ie $\lambda/4 = 75\,km$ (practically impossible)

(iii) reduction of noise & interference.

The noise effect cannot be completely eliminated but with the help of several modulation schemes the noise & interference effect can be minimized.

after modulation frequency of modulated wave is 100 MHz.

Thus,

$$\lambda = c/f = \frac{3 \times 10^8}{100 \times 10^6} = 3m$$

thus, $\lambda/4 = 3/4\,m$.

(iv) multiplexing.

Broadcasting of audio baseband signals directly without modulation would interfere with each other because the spectra of all signals more or less occupy same bandwidth.

Multiplexing is the technique in which several message signals are combined to composite signal for transmission over common channel. Inorder to transmit no of these signals over the same channel the signal must be kept apart so they do not interfere with each other & can be separated easily at the receiver end.

Types of modulation.

1. Continuous or analog modulation.

- carrier wave is continuous in nature.

(i) amplitude modulation.

- amplitude of carrier wave is varied in accordance with message signal or baseband signal.

(ii) angle modulation.

- angle of carrier wave (frequency phase) is varied in accordance with modulating signal.

a) frequency modulation.
b) Phase modulation.

2. Pulse modulation (analog/digital)

when carrier wave is pulse type waveform (periodic sequence of rectangular pulses or digital pulses) the modulation process is known as pulse modulation.

chapter: 2

## Representation of signal & systems in communication sys.

### Signal

A signal may be defined as the function of one or more independent variables which convey information about the behaviour and nature of some phenomena.

### Types of signal.

(i) continuous time signal & discrete time signal.
(ii) deterministic and random signal.
(iii) causal & non causal signal.
(iv) energy & power signal.
(v) periodic & non periodic signal.
(vi) even & odd signal.

### continuous and discrete time signal.

- The signal is said to be continuous time signal if it can be defined for all possible real no values of its independent variables (such as time t). It is also called as analog signal.
- The signal is said to be discrete time signal if it can be defined for only for set of discrete (or integer) value of independent variable.

It may arise in 2 ways

- selecting values of analog signal at discrete time instants called sampling.

Scanned with CamScanner

- by accumulating a variable over a time period.

Example:- counting the no of cars using a given street every hour.

(ii) Deterministic & random signals.

- Those signals which can be completely specified in the time are called deterministic signals.
- This type of signal have regular pattern & can be characterized mathematically.
- The nature of this type of signal at any time can be predicated

&

- Random signals are those signals about which there is uncertainty before its actual occurance.
- In real world all the signals are of random nature but for simplicity we consider all signals to be deterministic
- Ex: noise generated in the amplifier of radio.

(iii) Causal and non causal signal.

The signal $x(t)$ and $x[n]$ is said to be causal if it satisfies the condition

$x(t) = 0$ for $t < 0$

and $x[n] = 0$ for $n < 0$.

&

The signal is said to be non causal if

$x(t) \neq 0$ for $t < 0$

$x[n] \neq 0$ for $n < 0$.

(iv) Energy & power signals.

A signal having finite value of total energy and 0 average power is known as energy signal.

&

A signal having finite value of average power and has finite energy ie $E = \infty$ is known as power signals.

(v) periodic & non periodic signals.

A periodic signal is that which satisfies the condition

$x(t) = x(t+T)$ for all $t$ ----- (i)

where $T$ is +ve. constant.

- If this condition satisfies for $T = T_0$ then it is also satisfied for $T = 2T_0, 3T_0, 4T_0$ ....
- The smallest value of $T$ for which eqn (i) is satisfied is called fundamental period

&

A signal is said to be aperiodic if it doesn't repeat.

Sometimes aperiodic signals are said to have period equal to infinity.

(vi) Even & odd signal

A signal is said to be even signal if it satisfies the condition

$$\left.\begin{array}{l} x(t) = x(-t) \text{ for all } t \\ x[n] = x[n] \end{array}\right\} \text{for real signal.}$$

- even signal are symmetric about vertical axis.

&

A signal $x(t)$ is said to be odd signal if it satisfies the condition.

$x(t) = -x(-t)$ for all $t$

$x[n] = -x[-n]$ for all $n$

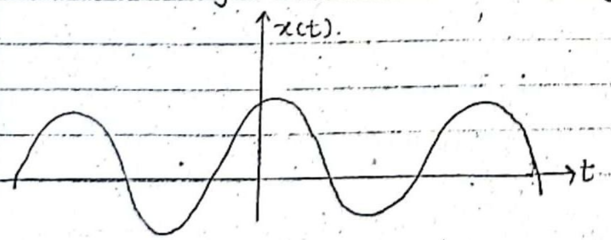
Example: $t^3$, $\sin t$, $t\cos t$ etc.

Other types of signals.

1) Harmonic signal.

A periodic signal defined for $-\infty \le t \le \infty$ and expressed in terms of sinusoidal function,

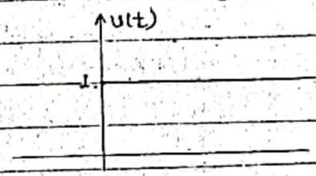
$x(t) = A\cos(2\pi f t + \theta)$ is called harmonic signal.




2. Unit step signal.

A signal which exist only for +ve side and is zero for -ve side.

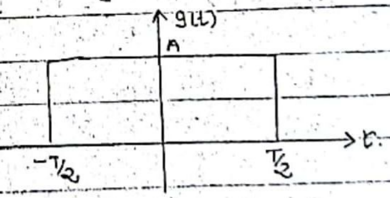
$$u(t) = \begin{cases} 1 & \text{for } t \ge 0 \\ 0 & \text{for } t < 0 \end{cases}$$




3. Rectangular pulse signal.

A rectangular pulse signal $g(t)$ is defined by

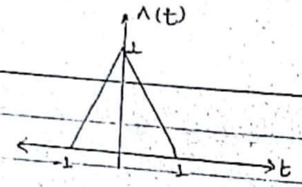
$$g(t) = A\,\text{rect}\left(t/T\right) = \begin{cases} A & -T/2 \le t \le T/2 \\ 0 & |t| > T/2 \end{cases}$$




4. Triangular pulse signal.

Triangular pulse signal is defined by

$$A(t) = \begin{cases} t+1 & \text{for } -1 \le t \le 0 \\ -t+1 & \text{for } 0 \le t \le 1 \\ 0 & \text{otherwise.} \end{cases}$$

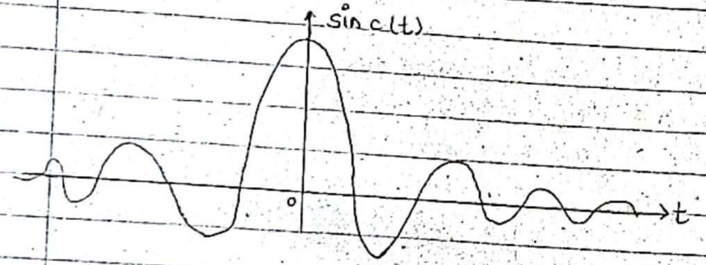




6. <u>sinc signal</u>

A sinc signal has maximum value equal to 1 at $t=0$ & gradually tends to zero for $t \to \infty$.

$$\text{sinc}(t) = \begin{cases} \dfrac{\sin \pi t}{\pi t} & t \neq 0 \\ 1 & t = 0 \end{cases}$$

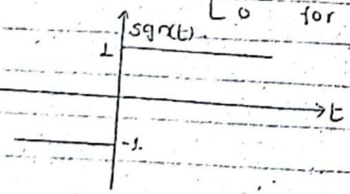




6. <u>signum or sign signal</u>

It is mathematically defined as

$$\text{sgn}(t) = \begin{cases} 1 & \text{for } t > 0 \\ -1 & \text{for } t < 0 \\ 0 & \text{for } t = 0. \end{cases}$$

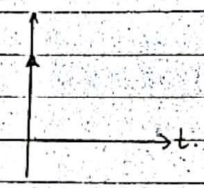




7. <u>Delta or impulse signal.</u>

It is the mathematical model to represent the physical phenomena that takes place in very short period.

$$\delta(t) = \begin{cases} 1 & \text{for } t = 0 \\ 0 & \text{otherwise.} \end{cases}$$





The convolution of any function with delta function gives the value of that function.

$$x(t) * \delta(t) = x(t)$$

$$x(t) * \delta(t - t_0) = x(t - t_0)$$

also

$$\delta(at) = \frac{1}{|a|} \delta(t) \text{ for all } a \neq 0.$$

<u>Proof</u>.

$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1$$

put $t = at$ ; $dt = a\,dt$.

Thus,

$$\int_{-\infty}^{\infty} \delta(t)\, a\,dt = a \int_{-\infty}^{\infty} \delta(at)\, dt.$$

$$= 1.$$

$$\int_{-\infty}^{\infty} \delta(at)\,dt = 1/a \; ; \; a>0$$

or $$\int_{-\infty}^{\infty} \delta(at)\,dt = -1/a \; ; \; a<0$$

Thus.

$$\int_{-\infty}^{\infty} \delta(at)\,dt = 1/|a|$$

$$\therefore \; \delta(at) = \frac{1}{|a|}\,\delta(t) \; = \frac{1}{|a|}\int_{-\infty}^{\infty} \delta(t)\,dt$$

Systems.

- A system may be defined as the set of elements or functional blocks connected together & produces an output in response to an input signal.
- The response or output of the system depends on the transfer function of the system.

Types of systems.

(i) Linear system.
(ii) non-linear system.
(iii) Time variant system.
(iv) Time invariant system.

* The system is linear if principle of superposition can be applied. If $x_1(t)$ and $x_2(t)$ be the inputs to the system with $y_1(t)$ and $y_2(t)$ be corresponding outputs. Then the output of the system would be the linear combination of the inputs.

$$y(t) = \alpha\phi[x_1(t)] + \beta\phi[x_2(t)] \; \ldots\ldots (i)$$

* all the other system not satisfying the above conditions are called non linear systems.

* a system is called time invariant if for all $t_0$ and for all $x(t)$, the response of the system does not depend upon the shift in time of application of $x(t)$

It Means $y(t) = \phi[x(t)]$ then,

$y(t-t_0) = \phi[x(t-t_0)]$.

Examples: integrators, differentiators.

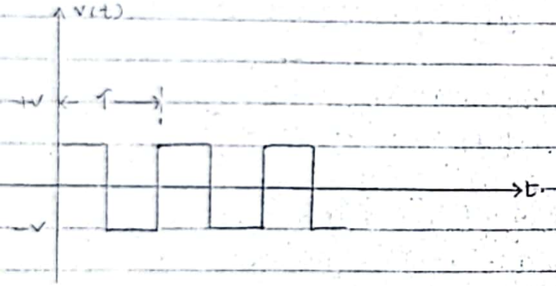
* all the system not invariant to shift in time are called time variant systems.

Example: amplitude modulator.

## Representation of signals.

(i) Time domain representation.

In time domain representation, a signal is time varying quantity.

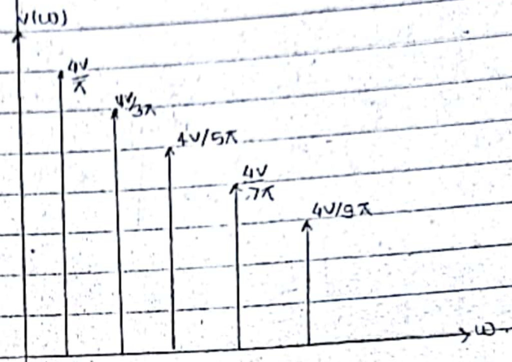



(ii) frequency domain representation.

In frequency domain, a signal is represented by its frequency spectrum. To obtain frequency spectrum of a signal, fourier series (for periodic continuous signals) and fourier transform (for non period) are used.

when fourier series is applied to above signal v(t) then we get

$$v(t) = \frac{4V}{\pi}\left(\sin w_0 t + \frac{1}{3}\sin 3w_0 t + \frac{1}{5}\sin 5w_0 t + \dots\right)$$



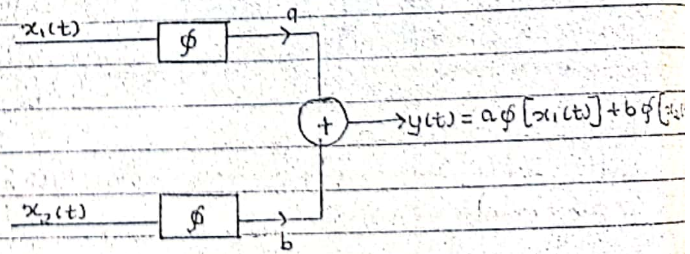

## LTI systems

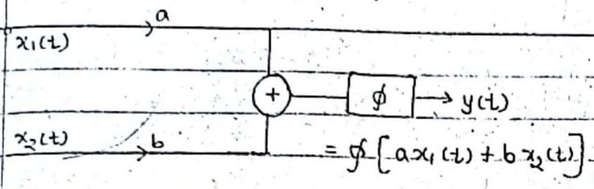
The system which is both linear and time invariation (if i/p is delayed by $t_0$ then o/p is delayed by $t_0$) is called as LTI systems.

- a LTI system can be characterized by its impulse response (IR)

The system is linear if

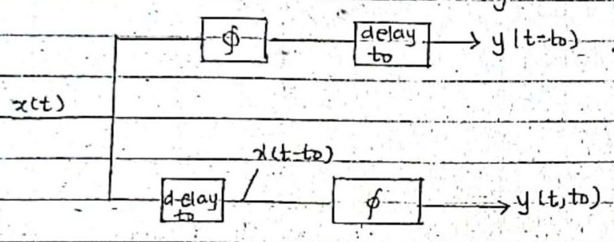
$$\phi[a x_1(t)] + \phi[b x_2(t)] = \phi[a x_1(t) + b x_2(t)]$$






$$= \phi\left[a x_1(t) + b x_2(t)\right]$$

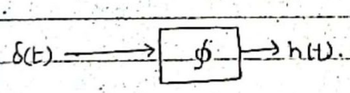
The system is said to be time invariant if.




$$y(t-t_0) = \phi\left[x(t-t_0)\right] = y(t, t_0).$$

impulse response.

The output of a system when input is impulse or delta function is called impulse response of the system.




$$h(t) = \phi\left[\delta(t)\right].$$

Transfer function.

- The fourier transform of impulse response $h(t)$ is called transfer function.

- The fourier transform function $H(f)$ and impulse response $h(t)$ form fourier transform pair shown by.

$$H(f) = \int_{-\infty}^{\infty} h(t)\, e^{-j2\pi f t}\, dt$$

$$h(t) = \int_{-\infty}^{\infty} H(f)\, e^{j2\pi f t}\, dt$$

$$\therefore H(f) \rightleftharpoons h(t)$$

convolution.

The convolution between two signals $x_1(t)$ and $x_2(t)$ is defined as:

$$y(t) = \int_{-\infty}^{\infty} x_1(\tau) \cdot x_2(t-\tau)\, d\tau. \quad \text{-----} (1)$$

where $y(t)$ = result of convolution

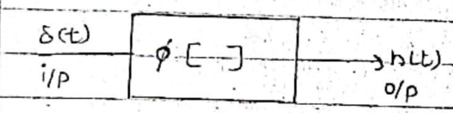
Then we can write.

$$y(t) = x_1(t) * x_2(t)$$

$*$ = symbol of convolution.

## Signal transfer in LTI systems

The impulse response $h(t)$ is defined as the response or output of a system to the impulse or delta function $\delta(t)$




We can express $x(t)$ as

$$x(t) = \int_{-\infty}^{\infty} x(\tau)\cdot\delta(t-\tau)\,d\tau \;\;\ldots\ldots(i)$$

combination of weighted & shifted impulse

If $y(t)$ is the output response of system to input $x(t)$ then;

$$y(t) = \phi\,[x(t)]$$

$$= \phi \int_{-\infty}^{\infty} x(\tau)\cdot\delta(t-\tau)$$

$$= \int_{-\infty}^{\infty} x(\tau)\cdot\phi[\delta(t-\tau)]\,d\tau.$$

$$= \int_{-\infty}^{\infty} x(\tau)\cdot h(t-\tau)\,d\tau.$$

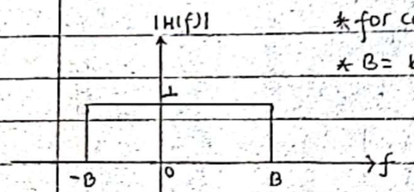
$$\therefore y(t) = x(t) * h(t)$$

## Ideal low pass filter

A filter is a frequency selective network which permits a band of frequency to pass through it with little attenuation. A low pass filter is one that allows only low frequency signal to pass through it.

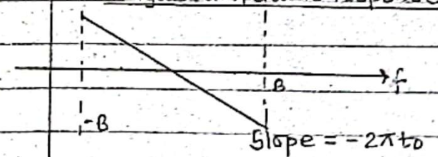
- The allowed band of frequency are known as pass band & attenuated & non passed frequency called as stop band.
- An ideal low pass filter transmits all frequencies in the passband without any distortion & completely rejects all frequencies in the stop band.
- The transfer function of ideal low pass filter is given by

$$H(f) = \begin{cases} \exp(-j2\pi f t_0) & ; -B \le f \le B \\ 0 & ; |f| > B. \end{cases}$$




* for convenience we set $k=1$.
* $B$ = bandwidth.

Fig(a): amplitude response




Fig(b): phase response

- for a finite $t_0$ the ideal LPF is non causal.
- The impulse response of ideal LPF is given by

$$h(t) = \int_{-\infty}^{\infty} H(f)\cdot \exp(+j2\pi f t)\, df$$

$$= \int_{-B}^{B} \exp(-j2\pi f t_0)\cdot \exp(j2\pi f t)\, df$$

$$= \int_{-B}^{B} \exp[j2\pi f(t-t_0)]\, df$$

$$= \left[\frac{e^{j2\pi f(t-t_0)}}{j2\pi(t-t_0)}\right]_{-B}^{B}$$

$$= \left[\frac{e^{j2\pi B(t-t_0)} - e^{-j2\pi B(t-t_0)}}{2j}\right]\cdot \frac{1}{\pi(t-t_0)}$$

$$= \frac{\sin[2\pi B(t-t_0)]}{\pi(t-t_0)}$$

$$= 2B\,\frac{\sin[2\pi B(t-t_0)]}{2\pi B(t-t_0)}$$

$$= 2B\,\mathrm{sinc}[2\pi B(t-t_0)].$$

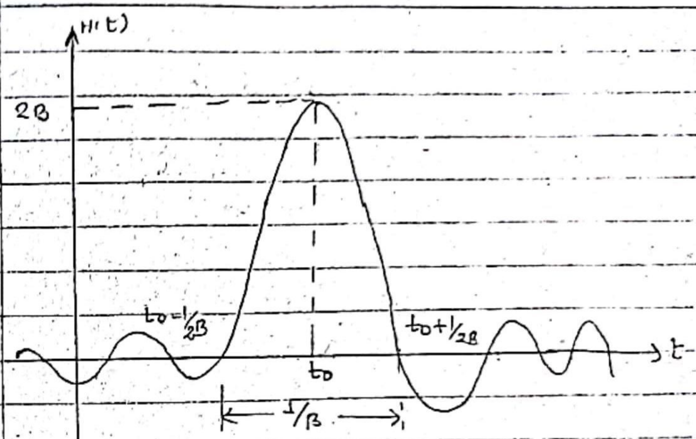



Fig: impulse response of ideal LPF

→ The impulse response has peak amplitude of $2B$ centred at time $t_0$

→ The duration of main lobe of impulse response is $1/B$ and build up time from 0 at the begining of main lobe to the peak value $1/2B$

→ For any finite value of $t_0$ there is some response from the filter before the time $t=0$ confirming that ideal LPF is non causal.

System bandwidth

In case of low pass system, the 3dB bandwidth is defined as the difference between zero frequency at which the amplitude response attains its peak value $|H_0|$ and the frequency at which the amplitude response drops to value equal to $|H(0)|/\sqrt{2}$.

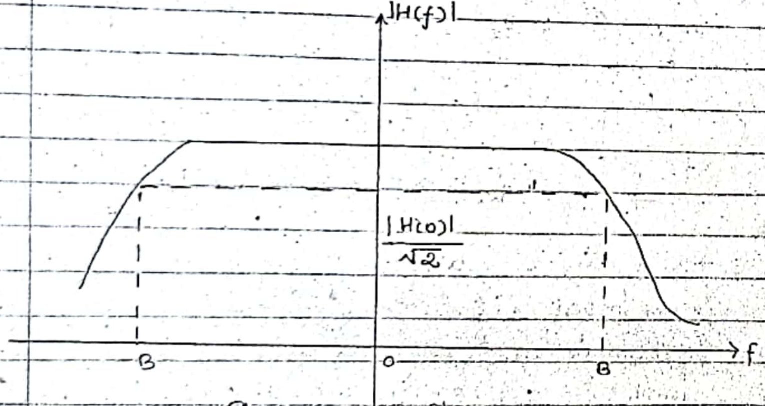
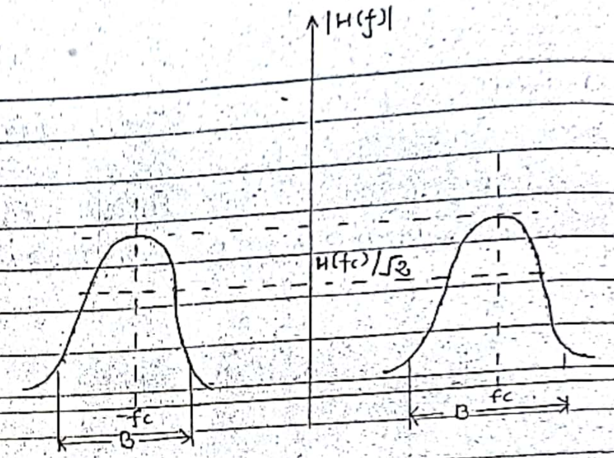



Fig: Low pass system

In the case of bandpass system the 3dB bandwidth is defined as the difference between the frequencies at which the amplitude response drops to value equal to $1/\sqrt{2}$ times the peak value $|H(f_0)|$ at mid band frequency $f_c$.

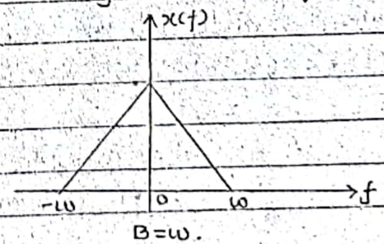



Fig(b): band pass system.

Signal bandwidth.

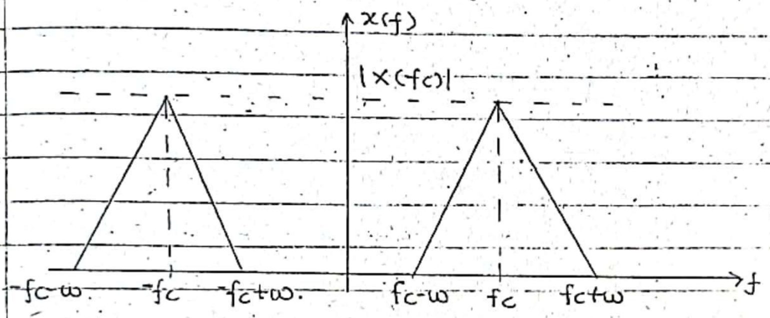
The bandwidth of the signal provides a measure the extent of significant spectral content of signal for positive frequencies.

* A signal is said to be lowpass if its significant content is centred around the origin & bandwidth i defined as one half total width of main spectral




B=ω.

A signal is said to be bandpass if its significant spectral content around $\pm f_c$ where $f_c$ is non zero frequency and the bandwidth is defined as the width of main lobe for +ve frequencies.




distortionless transmission.

The transmission of a signal through a system is said to be distortionless if the output signal is an exact replica of the input signal.

If the $x(t)$ be the input signal then the system without distortion be $y(t)$ defined by

$$y(t) = Kx(t-t_0) \text{ - - - - - (i).}$$

where,

$K$ = change in amplitude

$t_0$ = delay in transmission.

Now taking the fourier transform of eqn (i)

$$Y(f) = KX(f)\exp(-j2\pi f t_0)$$ { time shifting property

then, the transfer function of distortionless system is :

$$H(f) = \frac{Y(f)}{X(f)}$$

or, $H(f) = K\exp(-j2\pi f t_0)$ - - - - - - - (ii)

correspondingly, the impulse response of the system is

$$h(t) = K\delta(t-t_0) \text{ - - - - - - (iii).}$$

From eqn (ii) we see that in order to achieve the distortionless transmission through system, the transfer function of system must satisfy two conditions

(i) The amplitude response $|H(f)|$ is constant for all frequencies.

ie $|H(f)| = K$

(ii) The phase $\angle H(f)$ is linear with frequencies passing through zero.

$\angle H(f) = -2\pi f t_0 = -\omega t_0$.

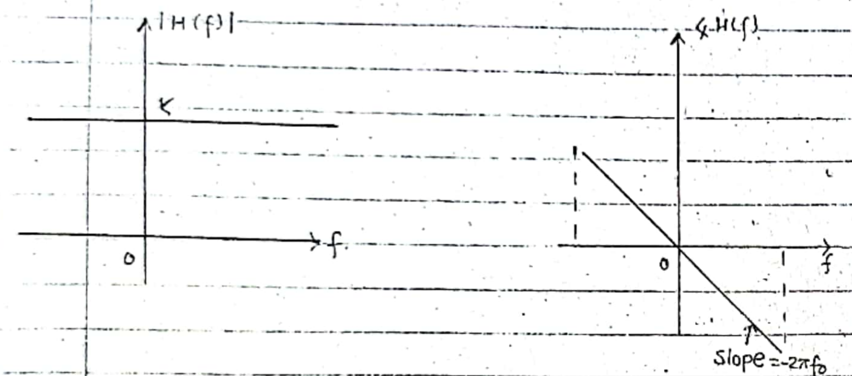
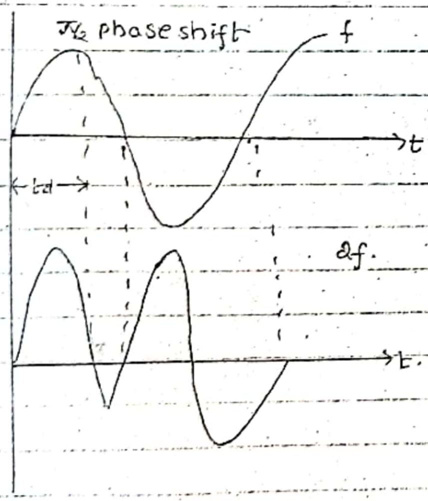



Fig: Magnitude & phase response of distortionless transmission




* The same time delay to accounts to phase shift of $\pi/2$ in upper sinusoid & $\pi$ in lower. ie to achieve same time delay higher sinusoid must undergo proportionately higher phase shift.

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## Hilbert transformation and its property

Hilbert transform (HT) is an operator which adds -90° phase shift to all positive frequencies and 90° to all negative frequencies of the input signal.

The amplitude response of the system is constant for entire frequency range of interest. If $x(t)$ is the input signal then output signal of Hilbert transformer denoted by $\hat{x}(t)$.

$$x(t) \xrightarrow{H.T.} \hat{x}(t).$$

* The frequency response of H.T. is given by.

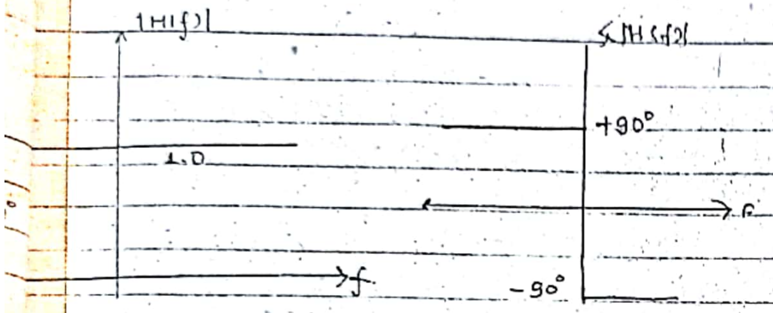
$$H(f) = -j\,\mathrm{sgn}(f)$$

$$= \begin{cases} -j & ; f > 0 \\ 0 & ; f = 0 \\ j & ; f < 0. \end{cases}$$

sgn(f) = signum function.

* Phase response will be given by.

$$\theta(f) = \begin{cases} -\pi/2 & ; f > 0 \\ 0 & ; f = 0 \\ \pi/2 & ; f < 0. \end{cases}$$




Fig(a) : amplitude response        fig (b) : phase response.

NOTE :-

The device that produces a phase shift of -90° for all +ve frequencies and -90° for all +ve frequencies. The amplitude of all frequency components of the input signal are unaffected by transmission through the device such an ideal device is called Hilbert Transformer.

The impulse response of Hilbert Transformer is

$$h(t) = \frac{1}{\pi t}.$$

hence, the convolution of this impulse response with a signal $x(t)$ applied to the input of Hilbert transformer yields resulting output $\hat{x}(t)$ as,

$$\hat{x}(t) = x(t) * h(t).$$

or, $\hat{x}(t) = x(t) * \frac{1}{\pi t}$

$$\therefore \hat{x}(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{x(\tau)}{(t-\tau)}\, d\tau.$$

$\therefore$ $\hat{x}(t)$ is the Hilbert transform of $x(t)$

The inverse Hilbert transform defining $x(t)$ in terms of $\hat{x}(t)$ is given by.

$$x(t) = -\frac{1}{\pi}\int_{-\infty}^{\infty} \frac{\hat{x}(\tau)}{t-\tau}\, d\tau.$$

Properties of H.T.

(i) The energy content in $x(t)$ and $\hat{x}(t)$ are same.

ie $|\hat{x}(f)|^2 = |x(f)|^2$

(ii) A signal $x(t)$ and its HT $\hat{x}(t)$ are mutually orthogonal so, cross correlation between them is zero.

ie $\int_{-\infty}^{\infty} x(t) \cdot \hat{x}(t)\,dt = 0.$

(iii) A signal $x(t)$ and its Hilbert transform $\hat{x}(t)$ have the same correlation function.

(iv) If $c(t)$ is high frequency sinusoidal signal & $m(t)$ is low frequency signal then Hilbert transform of product of $c(t)$ & $m(t)$ is equal to the product of $m(t)$ and $\hat{c}(t)$

ie,

$$H.T.[c(t)\cdot m(t)] = m(t)\cdot H[c(t)]$$
$$= m(t)\cdot \hat{c}(t).$$

(v) If $\hat{x}(t)$ is the Hilbert transform of $x(t)$, then Hilbert transform of $\hat{x}(t)$ is $-x(t)$

If $H.T.[x(t)] = \hat{x}(t)$

then,

$$H.T.[\hat{x}(t)] = -x(t)$$

Uses of H.T.

* Generation of SSB signals
* designing minimum phase type filters
* representation of band pass signals.

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# chapter 3 : Spectral analysis

## Review of fourier series

Fourier series is a tool which is used to express a periodic signal as infinite sum of sine wave components.

If $g_p(t)$ be the periodic signal then the fourier series representation of $g_p(t)$ is given by

$$g_p(t) = \sum_{n=-\infty}^{\infty} c_n \exp\left(\frac{j2\pi nt}{T_0}\right)$$

where,

$$c_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} g_p(t)\exp\left(\frac{-j2\pi nt}{T_0}\right)dt.$$

- The series is referred to as complex exponential fourier series.
- $c_n$ is called as complex exponential fourier coefficient

Drichlet's condition for existence of fourier series.

(i) The function $g_p(t)$ is single valued.

(ii) $g_p(t)$ has finite no of discontinuties.

(iii) It has finite no of maxima & minima.

(iv) It is absolutely integrable.

$$\int_{-T_0/2}^{T_0/2} |g_p(t)|\,dt < \infty$$

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## Trigonometric fourier series.

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + b_n \sin n\omega_0 t.$$

where;

$$a_n = \frac{2}{T}\int_{t_0}^{t_0+T} x(t) \cos n\omega_0 t\, dt.$$

$$b_n = \frac{2}{T}\int_{t_0}^{t_0+T} x(t) \sin n\omega_0 t\, dt.$$

* odd function : $a_n = 0$
* even function : $b_n = 0$.

## Review of fourier transform.

As fourier series exist for continuous periodic signal, Fourier transform is applicable for non periodic signals. It is given by.

$$G(f) = \int_{-\infty}^{\infty} g(t) \exp(-j2\pi f t)\,dt \quad \text{— synthesis eqn}$$

where.

$$g(t) = \int_{-\infty}^{\infty} G(f) \exp(j2\pi f t)\,df \quad \text{- analysis eqn}$$

$G(f) \rightleftharpoons g(t)$ is known as fourier transform pair.

we can express fourier transform in terms of

$$G(j\omega) \text{ or } G(\omega) = \int_{-\infty}^{\infty} g(t)\, e^{-j\omega t}\, dt$$

&

$$g(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} G(\omega)\, e^{j\omega t}\, d\omega.$$

⇒ In communication system we mostly use $G(f)$ but $G(\omega)$ can also be used.

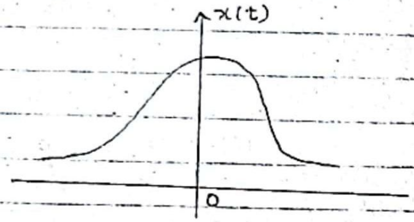
## Energy signal.

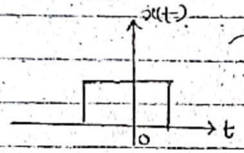
The signal which has finite energy & zero average power $0 < E < \infty$ & $P = 0$ is known as energy signal.

It can be obtained by

* $E = \int_{-\infty}^{\infty} x^2(t)\, dt$ — real signal.
* $E = \int_{-\infty}^{\infty} |x(t)|^2\, dt$ — complex valued signal.
* $E = \sum_{n=-\infty}^{\infty} |x[n]|^2$ — discrete time signal.

→ For energy to be finite the signal amplitude $x(t)$ must be zero ( $x(t) \to 0$) as $|t| \to \infty$.




- Non periodic signals are energy signals
- These signals are time limited.
- Example: Single rectangular pulse

## Power signal.

The signal which has finite average power & infinite energy is termed as power signal

ie $0 < P < \infty$ and $E = \infty$.

The average power can be obtained by

$$* \quad P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\,dt \quad \text{— for real signals.}$$

$$* \quad P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2\,dt \quad \text{— for complex valued signal.}$$

$$* \quad P = \lim_{N\to\infty} \frac{1}{N} \sum_{n=-N/2}^{N/2} |x[n]|^2 \quad \text{— for discrete time signal.}$$

→ For power to be infinite the signal amplitude does not become zero as $|t| \to \infty$

- periodic signals are power signals.
- These signals exists over infinite time.
- example: periodic pulse train.

Question 1:- Prove that $x(t) = A\cos(w_0t + \theta)$ is a power type signal & its rms value.

⇒ soln.

The given signal is periodic signal with period $T_0 = 2\pi/w_0$

hence this is power signal.

The power is defined by

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\,dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} A^2\cos^2(w_0t+\theta)\,dt.$$

$$= \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \frac{A^2}{2} \left[1 + \cos(2\omega_0 t + 2\theta)\right] dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} A^2/2 \, dt + \lim_{T\to\infty} \frac{A^2}{2T} \int_{-T/2}^{T/2} \cos(2\omega_0 + 2\theta) dt$$

$$= \lim_{T\to\infty} \frac{A^2}{2T} . T + \lim_{T\to\infty} \frac{A^2}{2T} \int_{-T/2}^{T/2} \cos(2\omega_0 t + 2\theta) dt$$

$$= \frac{A^2}{2} + 0$$

$$= A^2/2$$

We know that power of signal $= (RMS)^2$

or $\frac{A^2}{2} = (RMS)^2$

or $RMS = \sqrt{A^2/2}$

or $RMS = A/\sqrt{2}$

Parseval's theorem

* Parseval's theorem for energy signal (finite energy continuous time signal)

Statement.

The energy of the signal may be obtained with the help of fourier transform

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = 1/2\pi \int_{-\infty}^{\infty} |x(j\omega)|^2 d\omega.$$

Proof.

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$= \int_{-\infty}^{\infty} x(t) . x^*(t) \, dt.$$

$$= \int_{-\infty}^{\infty} x(t) . \left\{ \frac{1}{2\pi} \int_{-\infty}^{\infty} x^*(j\omega) . e^{-j\omega t} d\omega \right\} dt.$$

$$\left\{ \begin{array}{l} \text{Since } x(t) = 1/2\pi \int_{-\infty}^{\infty} x(j\omega) e^{j\omega t} dt \\ x^*(t) = 1/2\pi \int_{-\infty}^{\infty} x^*(j\omega) e^{-j\omega t} dt. \end{array} \right.$$

changing the order of integration.

$$E = \frac{1}{2\pi}\int_{-\infty}^{\infty} X^*(j\omega)\left[\int_{-\infty}^{\infty} x(t)\cdot e^{-j\omega t}\,dt\right]d\omega$$

or, $$E = \frac{1}{2\pi}\int_{-\infty}^{\infty} X^*(j\omega)\,X(j\omega)\,d\omega$$

or, $$E = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(j\omega)|^2\,d\omega.$$

The term $\psi_x(f)$ or $S(j\omega) = |X(j\omega)|^2$ is known as energy spectral density (ESD). It is the function of $\omega$ which gives average energy of the signal at a instant of $\omega$.

- The above relation is also called Reyleigh energy theorem where energy is given by the area under $|X(j\omega)|$ curve.

** Parseval's theorem for power signal / Parseval's theorem for infinite energy signal

Statement:

Power of the signal is defined in terms of fourier series coefficient.

$$P = \frac{1}{T}\int_0^T |x(t)|^2\,dt = \sum_{k=-\infty}^{\infty} |c_k|^2$$

Proof:-

$$P = \frac{1}{T}\int_0^T x(t)\cdot x^*(t)\,dt$$

we have,

$$x(t) = \sum_{k=-\infty}^{\infty} c_k\, e^{jk\omega_0 t}.$$

&

$$x^*(t) = \sum_{k=-\infty}^{\infty} c_k^*\, e^{-jk\omega_0 t}.$$

so,

$$P = \frac{1}{T}\int_0^T x(t)\left(\sum_{k=-\infty}^{\infty} c_k^*\, e^{-jk\omega_0 t}\right)dt.$$

or, Interchanging the order of integration & summation

$$P = \sum_{k=-\infty}^{\infty} c_k^*\left(\frac{1}{T}\int_0^T x(t)\cdot e^{-jk\omega_0 t}\right)dt$$

or, $$P = \sum_{k=-\infty}^{\infty} c_k^*\cdot c_k, \qquad \left\{ c_k = \frac{1}{T}\int_0^T x(t)\, e^{-jk\omega_0 t}\right.$$

or, $$P = \sum_{k=-\infty}^{\infty} |c_k|^2.$$

The total average power over a period is the sum of the average power contributed by each harmonic

## Power spectral density (Psd)

Let $g(t)$ = power signal (periodic)

$$P = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} g^2(t)\,dt \quad \text{------ (i)}$$

where,

$$g(t) = \sum_{n=-\infty}^{\infty} C_n e^{j2\pi n f_0 t}$$

as $f_0 = 1/T$

To have fourier transform of $g(t)$, consider a truncated version of signal $g(t)$.

$$g_T(t) = g(t)\,\mathrm{rect}\left(t/2T\right)$$

$$= \begin{cases} g(t) & ; -T \le t \le T \\ 0 & ; \text{otherwise} \end{cases}$$

$$g_T(t) \xrightleftharpoons{F.T.} G_T(f)$$

$$P = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} g_T^2(t)\,dt$$

$$= \lim_{T\to\infty} \frac{1}{2T}\int_{-\infty}^{\infty} g_T^2(t)\,dt$$

$$= \lim_{T\to\infty} \frac{1}{2T}\int_{-\infty}^{\infty} |G_T(f)|^2\,df$$

since $\int_{-\infty}^{\infty} g_T^2(t)\,dt = \int_{-\infty}^{\infty} |G_T(f)|^2\,df$

$$P_T = \int_{-\infty}^{\infty} S_g(f)\,df$$

where, $S_g(f) = \lim_{T\to\infty} \frac{1}{2T}|G_T(f)|^2$ called as the power spectral density or power spectrum of a signal and $\frac{1}{2T}|G_T(f)|^2$ is called Periodogram of the signam.

## Energy spectral density.

consider an energy signal $g(t)$ defined over the interval $-\infty<t<\infty$ and its fourier transform is defined by $G(f)$

The total energy of the system is given by

$$E = \int_{-\infty}^{\infty} g^2(t)\,dt \quad \text{------ (i)}$$

&

$$\int_{-\infty}^{\infty} g_1(t)\cdot g_2(t)\,dt = \int_{-\infty}^{\infty} G_1(f)\cdot G_2(f)\,df \quad \text{------ (ii)}$$

where,

$g_1(t)$ and $g_2(t)$ are pair of energy signals with fourier transform $G_1(f)$ & $G_2(f)$ respectively

Let, $g_1(t) = g_2(t) = g(t)$,

correspondingly we may set

$$G_1(f) = G_2(f)$$

for real valued signal, $G(-f) = G^*(f)$

Then eq$^n$ (ii) becomes

$$\int_{-\infty}^{\infty} g^2(t)\,dt = \int_{-\infty}^{\infty} G(f) \cdot G^*(f)\,df$$

$$E = \int_{-\infty}^{\infty} g^2(t)\,dt = \int_{-\infty}^{\infty} |G(f)|^2\,df \quad \text{------ (iii)}$$

where $|G(f)|$ is known as amplitude spectrum of signal and eq$^n$ (iii) is known as Reyleigh energy theorem.

* $|G(f)|^2$ gives the distribution of the energy of the signal $g(t)$ in frequency domain.

The squared of the amplitude spectrum $|G(f)|^2$ is called Energy spectral density.

ie $\psi_g(f) = |G(f)|^2$

correlation.

correlation is the process of matching using two signals to produce the third signal and measure the similarity between the signal & on that basis determining the additional information of the signal.

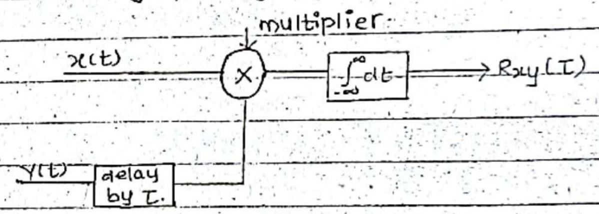
Cross correlation

It is the measure of similarity between signal $x(t)$ and time delayed version of another signal $y(t)$.

The cross correlation between $x(t)$ & $y(t)$ is

$$R_{xy}(\tau) = \int_{-\infty}^{\infty} x(t) \cdot y(t-\tau)\,dt$$

It can be graphically represented as,




autocorrelation function.

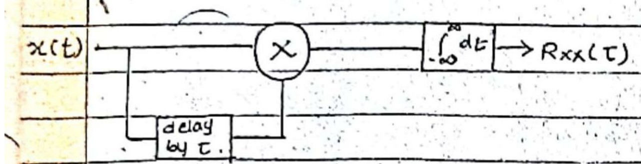
If the cross-correlation is taken between same signals (ie signal itself) then the resulting signal is called auto correlation.

It is given by;

$$R_{xx}(\tau) = \int_{-\infty}^{\infty} x(t) \cdot x(t-\tau)\,dt \quad \text{– for energy signal.}$$

$$R_{xx}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t) \cdot x(t-\tau) \quad \text{– for power signal}$$

It is graphically represented as.




**Properties of auto correlation function.**

1. The autocorrelation function of a real valued energy or power signal g(t) is a real valued even function.

$$R_g(-\tau) = R_g(\tau)$$

ii. The value of the autocorrelation function of a power signal g(t) at the origin is equal to the average power of the signal.

$$R_g(0) = P.$$

iii. The value of the autocorrelation function of a energy signal g(t) at origin is equal to the energy of signal

$$R_g(0) = E.$$

iv. The maximum value of the autocorrelation function of an energy or power signal (g(t)) occurs at origin.

$$[R_g(\tau)] \leq R_g(0) \quad \text{for all } \tau.$$

v. For an energy or power signal g(t) the auto correlation function & ESD/PSD form fourier transform pair.

$$R_g(\tau) \rightleftharpoons \psi_g(f) \text{ — ESD}$$

$$R_g(\tau) \rightleftharpoons S_g(f) \text{ — PSD.}$$

**Question no: 2:**

Show that the time autocorrelation function of a power signal and power spectral density function form a fourier transform pair.

* **Time auto correlation function of power signal.**

The time auto correlation function $R_g(\tau)$ of real power signal g(t) is defined as

$$R_g(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} g(t) \cdot g(t+\tau)\, dt.$$

Now the fourier transform of $R_g(\tau)$ is:

$$F(R_g(\tau)) = \int_{-\infty}^{\infty} e^{-j\omega\tau} \left( \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} g(t) \cdot g(t+\tau)\, dt \right) d\tau.$$

$$= \int_{-\infty}^{\infty} g(t) \cdot \left( \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} g(t+\tau)\, e^{-j\omega\tau}\, d\tau \right) dt$$

The inner integral is the fourier transform of $g(t+\tau)$

$\therefore \mathcal{F}[g(t+\tau)] = G(\omega)\cdot e^{j\omega t}$

$= \lim_{T\to\infty} 1/T \; G(\omega) \int_{-\infty}^{\infty} g(t)\cdot e^{j\omega t}\, dt$

$= \lim_{T\to\infty} 1/T \; G(\omega) . G(-\omega)$

$= \lim_{T\to\infty} 1/T \; |G(\omega)|^2 = S_g(\omega)$

$\therefore R_g(\tau) \xrightleftharpoons{F.T.} \lim_{T\to\infty} 1/T \; |G(\omega)|^2 = S_g(\omega)$

<u>PSDF of harmonic signal.</u>

Let,

$g_p(t)$ = periodic power signal.

$= \sum_{n=-\infty}^{\infty} c_n e^{j2\pi n f_0 t}$

&

$g_p(f) = \sum_{n=-\infty}^{\infty} c_n \,\delta(f-nf_0)$

Now, the power of the real valued signal is

$P = \lim_{T\to\infty} 1/T \int_{-T/2}^{T/2} g_p^2(t)\, dt$

let us consider the truncated version of $g_p(t)$

$g_T(t) = \begin{cases} g_p(t) & ; -T/2 \le t \le T/2 \\ 0 & ; \text{otherwise} \end{cases}$

$\therefore g_T(t) = g_p(t)\, \text{rect}(t/T)$.

Taking fourier transform.

$\{\text{rect}(t/T) \rightleftharpoons T\text{sinc}(fT)\}$

$G_T(f) = G_p(f) * T\text{sinc}(fT)$

$= \sum_{n=-\infty}^{\infty} c_n \,\delta(f-nf_0) * T\text{sinc}(ft)$

$= T \sum_{n=-\infty}^{\infty} c_n \,\text{sinc}\,[(f-nf_0)\cdot T]$

Now, the PSDF of signal is

$S_{gp}(f) = \lim_{T\to\infty} 1/T \; |G_T(f)|^2$

$= \lim_{T\to\infty} 1/T \cdot T^2 \sum_{n=-\infty}^{\infty} |c_n|^2 \cdot \{\text{sinc}[(f-nf_0)\cdot T]\}$

* as $T\to\infty$ sinc function tends delta function at $nf_0$

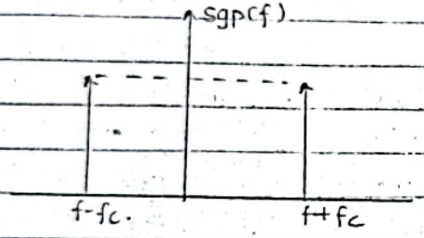
$\therefore S_{gp}(f) = \sum_{n=-\infty}^{\infty} |c_n|^2 \,\delta(f-nf_0)$

If harmonic signal, $g_p(t) = A_c \cos 2\pi f_c t$.

$G_p(f) = \frac{A_c}{2}\,\delta(f+f_c) + \frac{A_c}{2}\,\delta(f-f_c)$

Then,

$$S_{gp}(f) = \frac{A_c^2}{4}\,\delta(f+f_c) + \frac{A_c^2}{4}\,\delta(f-f_c).$$




and the power of harmonic signal is

$$P = \frac{A_c^2}{4} + \frac{A_c^2}{4}$$

$$= \frac{A_c^2}{2}.$$

PSDf of white noise

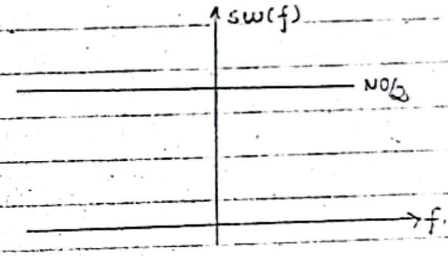
White noise is the idealised form of noise. In white noise there is the presence of all frequencies.

Power spectral density of white noise is independent of frequency and expressed as

$$S_w(f) = N_0/2 \quad ; \quad -\infty < f < \infty$$

The factor 1/2 indicates that half of the power is associated with positive frequency and half with negative frequency.

- The dimension of $N_0$ is watt/Hz measured at the input stage of receiver of communication system.
- PSDf is constant over entire frequency.




- Two samples of noise signals no matter how close they are in time interval are intercorrelated.

- The autocorrelation function of white noise is the inverse fourier transform of the power spectral density.

Then,

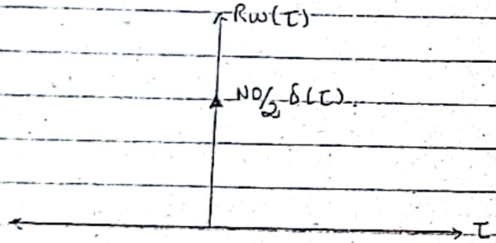
$$R_w(\tau) = \frac{N_0}{2}\,\delta(\tau)$$

also,

$$R_w(\tau) = 0 \quad \text{for } \tau \neq 0.$$

- The autocorrelation function of white noise consists of delta function weighted by the factor $N_0/2$ and occuring at $\tau=0$.

- white noise has infinite average power so it is no physically realizable.




analog spectrum analyser.

- Analog spectrum analyser is an instrument used to measure Psdf of power type signal.

- for periodic signal Psdf is delta function at frequency equal to square of nth harmonic

$$S_{gp}(f) = \sum_{n=-\infty}^{\infty} |c_n|^2 \delta(f - nf_0)$$

- For non periodic signal Psdf ranges from 0 to $\infty$

- So Psdf can be measured by passing the signal to N.B filters whose centre frequency is swept from 0 to infinity and measuring the output of filter at each & even tuned frequency.

- The accuracy of Psdf increases with the decrease in bandwidth of NB filters.

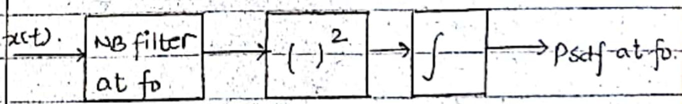
- If $x(t)$ is input applied to NB filter the average power at the output $Y(t)$ of NB filter is.

$$P_Y = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} Y^2(t)\, dt = S_x(f) \cdot 2\Delta f \cdot K^2$$

where $K$ = gain of the filter.

If $K^2 = \frac{1}{2\Delta f}$ then,

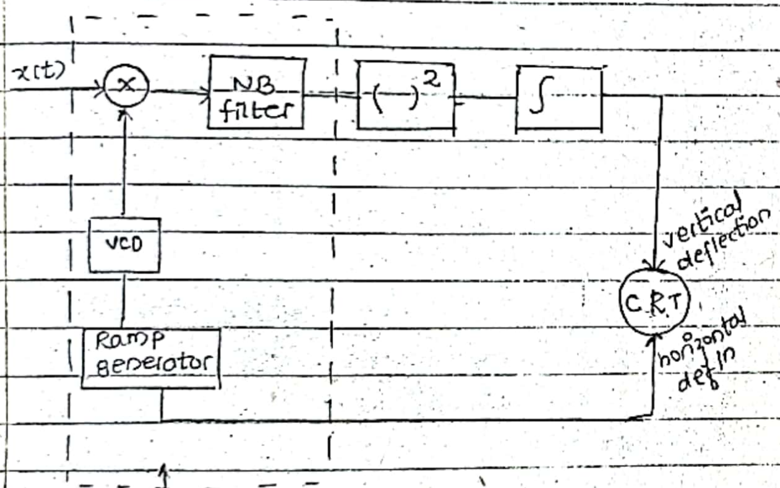
$P_Y = S_X(f)$



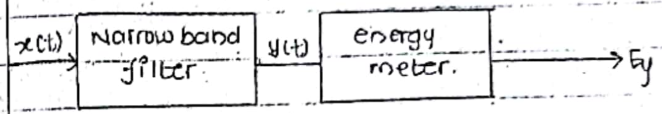

- To obtain the Psdf over an entire frequency range the centre frequency should be swept from 0 to $\infty$

- To sweep centre frequency of NB filter because it is constructed of quartz crystal oscillator.

Thus another method in which NB filter is tuned at precise frequency and $x(t)$ frequency is shifted such that the frequency range falls within the pass band of filter.




## Interpretation of E.S.D.




Let signal $x(t)$ is applied extremely to narrow band (of) of ideal band pass filter centred at $f_c$ with transfer function $H(f)$:

$$|H(f)| = \begin{cases} 1 & ; f_c - \frac{\Delta f}{2} \le |f| \le f_c + \frac{\Delta f}{2} \\ 0 & ; \text{otherwise.} \end{cases}$$

as $\Delta f$ is very small, output response can be assumed to be constant over the frequency interval covered by the pass band of filter.

So,

$$|Y(f)| = |H(f)|\,|X(f)|$$

$$= \begin{cases} |X(f_c)| & ; f_c - \Delta f/2 \le |f| \le f_c + \frac{\Delta f}{2} \\ 0 & ; \text{otherwise} \end{cases}$$

according to Reyleigh energy theorem, energy of filter output is

$$E_y = \int_{-\infty}^{\infty} \psi_y(f) \cdot df$$

$$= \int_{-\infty}^{\infty} |y(f)|^2 df$$

$$= 2\int_{0}^{\infty} |y(f)|^2 df$$

$$= 2 |X(f_c)|^2 \Delta f$$

$$= 2\psi_x(f_c) \cdot \Delta f$$

or, $\psi_x(f_c) = \dfrac{E_y}{2\Delta f}$.

∴ ESD of the filter input at some frequency $f_c$ equals the energy of the filter output divided by $2\Delta f$ so ESD is energy per unit bandwidth.



# Chapter 4. Amplitude modulation

In amplitude modulation the amplitude of hig[h] frequency monochromatic carrier signal $c(t)$ is [varied] according to the rate of change of modulating [signal] $m(t)$.

Types of AM.

(i) DSB-AM/DSB-FC ( Double side band AM - full ca[rrier])

(ii) DSB-SC ( Double side band - supress carrier)

(iii) SSB - SC ( single side band - supressed carrier).

1. DSB-AM or DSB-FC.

- DSB-AM consists of two side band & large ca[rrier] component.
- The standard form of DSB-AM is given by

$$s(t) = [A_c + m(t)] \cos(2\pi f_c t + \phi) \text{ ----}$$

considering $\phi = 0$

$$s(t) = [A_c + m(t)] \cos 2\pi f_c t.$$

where;

$$a = \text{modulation index} = \frac{A_{max}}{A_c} = $$

or, $s(t) = A_c\{1 + a\, m_n(t)\} \cos 2\pi f_c t.$

$m_n(t)$ = normalized message s[ignal]

$= m(t)/\max|m(t)|$

- $A_c[1+a m_n(t)]$ is the envelope of the modulated signal.
- Modulation index expressed in terms of percentage is known as percentage modulation.

For single Tone modulation

Let $c(t) = A_c \cos 2\pi f_c t$.

$m(t) = A_m \cos 2\pi f_m t$.

then,

$$s(t) = [A_c + m(t)].\cos 2\pi f_c t$$

$$= [A_c + A_m \cos 2\pi f_m t].\cos 2\pi f_c t$$

$$= A_c\left[1 + \frac{A_m}{A_c}\cos 2\pi f_m t\right].\cos 2\pi f_c t$$

$$\therefore = A_c[1 + a\cos 2\pi f_m t].\cos 2\pi f_c t \quad \text{----(i)}$$

where,

$a = A_m/A_c$.

Let $A_{max}$ be the maximum value of envelope and $A_{min}$ be the minimum value of envelope.

$$a = \frac{A_{max} - A_{min}}{A_{max} + A_{min}}$$

$$\frac{A_{max}}{A_{min}} = \frac{A_c + A_m}{A_c - A_m}$$

or, $A_c A_{max} - A_m A_{max} = A_{min} A_c + A_{min} A_m$

$$\therefore \frac{A_{max} - A_{min}}{A_{max} + A_{min}} = \frac{A_m}{A_c} = a$$

or, $a = A_c \cos 2\pi f_c t + a.A_c \cos 2\pi f_m t.\cos 2\pi f_c t$

or, $a = A_c \cos 2\pi f_c t + \frac{a.A_c}{2}.2\cos 2\pi f_m t.\cos 2\pi f_c t$

or, $a = A_c \cos 2\pi f_c t + \frac{a.A_c}{2}\{\cos 2\pi(f_c+f_m)t + \cos 2\pi(f_c-f_m)t\}$

$$\therefore s(t) = A_c \cos 2\pi f_c t + \frac{a.A_c}{2}\underbrace{\cos 2\pi(f_c+f_m)t}_{\text{upper side band}} + \frac{a A_c}{2}\underbrace{\cos 2\pi(f_c-f_m)t}_{\text{lower side band}}$$

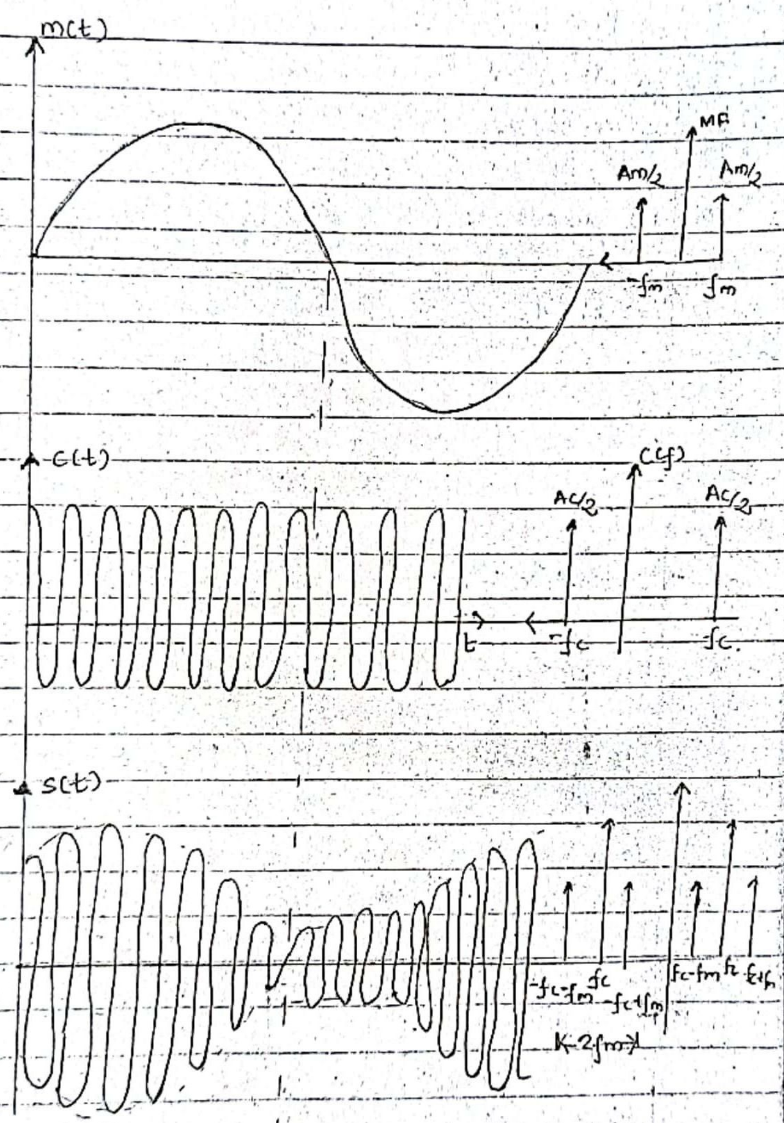
→ The above equation is the Time Domain representation.

Frequency domain representation

$$s(t) = A_c \cos 2\pi f_c t + \frac{a.A_c}{2}\cos 2\pi(f_c+f_m)t + \frac{a A_c}{2}\cos 2\pi(f_c-f_m)t$$

To develop the frequency description of this AM wave we take fourier transform on both sides.

$$S(f) = \frac{A_c}{2}[\delta(f-f_c) + \delta(f+f_c)] + \frac{a A_c}{4}[\delta(f-f_c-f_m) + \delta(f+f_c+f_m)] + \frac{a A_c}{4}[\delta(f-f_c+f_m) + \delta(f+f_c-f_m)].$$




a=1.

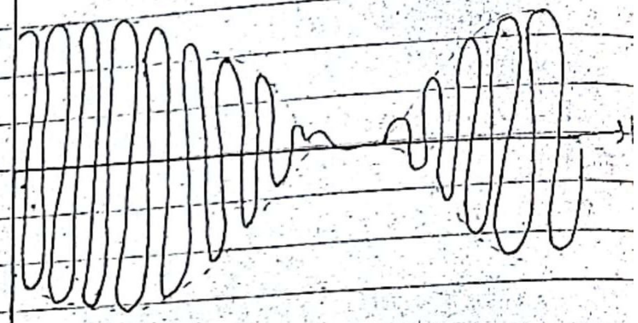


Fig 100% modulation

(a>1)

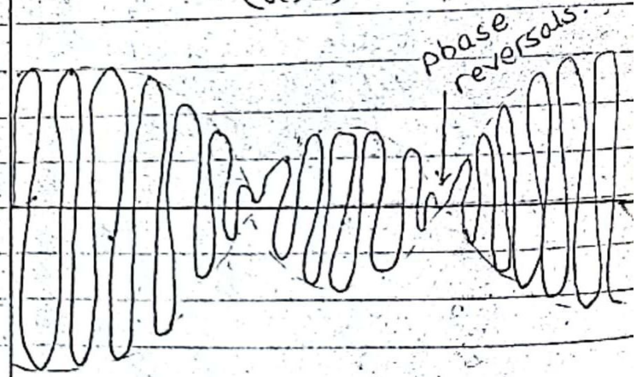
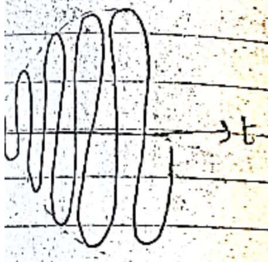
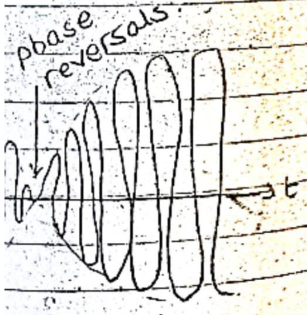



Fig: over modulation




odulation




ulation

## Power of DSB-AM/FC.

We have,

$$\text{Power} = V^2/2R \quad \text{---- (i)}$$

for $R = 1$;

$$\text{Power of carrier} = \frac{A_c^2}{2}$$

Similarly,

$$\text{Power of USB} = \frac{\left(\frac{a \cdot A_c}{2}\right)^2}{2}$$

$$= \frac{a^2 A_c^2}{8}$$

$$\text{Power of LSB} = \frac{a^2 A_c^2}{8}.$$

The total average power of DSB-AM is the sum of carrier power and side band powers.

$$P_t = P_c + P_{USB} + P_{LSB}$$

$$= \frac{A_c^2}{2} + \frac{a^2 A_c^2}{8} + \frac{a^2 A_c^2}{8}$$

$$= \frac{A_c^2}{2} + \frac{a^2 A_c^2}{4}$$

$$\therefore P_t = \frac{A_c^2}{2}\left[1 + \frac{a^2}{2}\right]$$

## efficiency.

The efficiency of any system is defined as the ratio of the useful power at the output to total power consumed.

In DSB-AM the useful power is the power contained in sideband only.

$$\eta = \frac{P_{LSB} + P_{USB}}{P_t} = \frac{\frac{a^2 A_c^2}{8} + \frac{a^2 A_c^2}{8}}{\frac{A_c^2}{2}\left[1 + \frac{a^2}{2}\right]}$$

$$= \frac{a^2}{2\left[1 + a^2/2\right]}$$

$$\therefore \eta = \frac{a^2}{2 + a^2} \quad \text{——— (1)}$$

where $a$ = modulation index.

for $a = 1$;

Maximum efficiency $(\eta_{max}) = 1/3 = 33.33\%$.

## generation of DSB-AM (modulation)

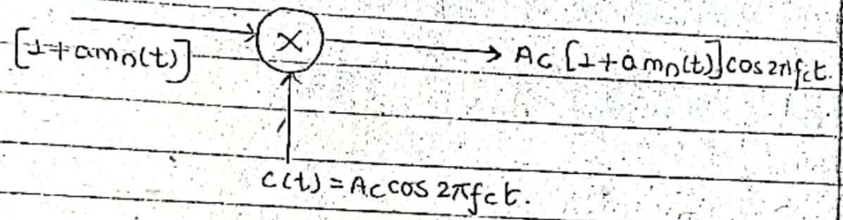
(i) direct method.

(ii) Indirect method.

* square law modulator.
* switching modulator.

(i) Direct method.

In this method, the level shifted modulated signal $[1+am_n(t)]$ is multiplied by carrier signal.




- This is simplest method but not suitable for high frequency applications
- Maintaining linearity is difficult.

(ii) Indirect method

(a) Square law modulator (non linear modulator)

The square law modulator requires three elements.

(i) Means for summing carrier signal $c(t)$ and message signal $m(t)$.

(ii) effect of non linear device operated at definite range.

(iii) Band pass filter tuned at desired o/p frequency.

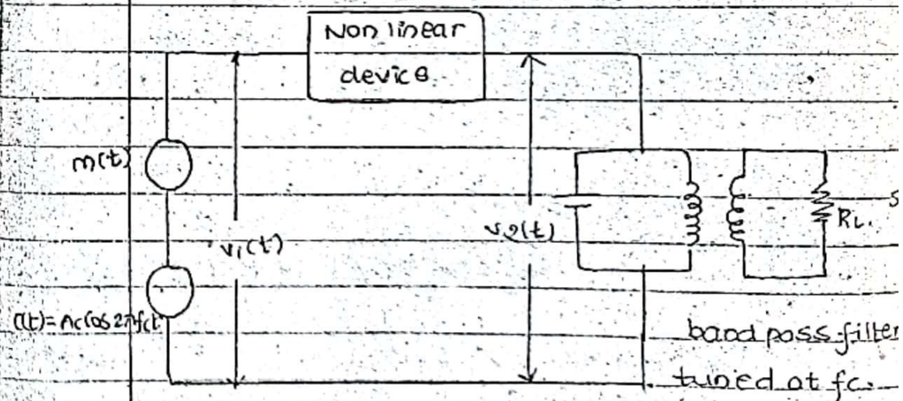
Semiconductor diodes and transistors are the most common non-linear device used for implementing square law modulators. Assuming diode is biased in non-linear region.

I/O characteristics of non linear element is approximated by power series equation.

For small input;

$$v_2(t) = a_1 v_1(t) + a_2 v_1^2(t) \text{ ------ (i)}$$

↑ square law.




we have,

$v_1(t) = m(t) + c(t)$

$= A_c \cos 2\pi f_c t + m(t)$.

Then,

$v_2(t) = a_1[A_c \cos 2\pi f_c t + m(t)] + a_2[A_c \cos 2\pi f_c t + m(t)]^2$

or, $v_2(t) = a_1 A_c \cos 2\pi f_c t + a_1 m(t) + a_2[A_c^2 \cos^2 2\pi f_c t + m^2(t)$

$+ 2m(t) \cdot A_c \cos 2\pi f_c t]$

or, $v_2(t) = a_1 A_c \cos 2\pi f_c t + a_1 m(t) + a_2 A_c^2 \cos^2 2\pi f_c t + a_2 m^2(t)$
$+ 2a_2 m(t) \cdot A_c \cos 2\pi f_c t$.

$= a_1 A_c \cos 2\pi f_c t + 2a_2 A_c m(t) \cos 2\pi f_c t + a_1 m(t)$
$+ a_2 m^2(t) + a_2 A_c^2 \cos^2 2\pi f_c t$.

$= a_1 A_c \left[1 + \frac{2a_2}{a_1} m(t)\right] \cos 2\pi f_c t + a_1 m(t) + a_2 m^2(t) +$

$a_2 A_c^2 \cos^2 2\pi f_c t$.

After passing through bandpass filter at $f_c$, we get

$s(t) = a_1 A_c \left\{1 + \frac{2a_2}{a_1} m(t)\right\} \cos 2\pi f_c t$ ------ (ii)

→ Eqn (ii) is the desired AM wave with amplitude sensitivity of $k_a = \frac{2a_2}{a_1}$ & modulation index of

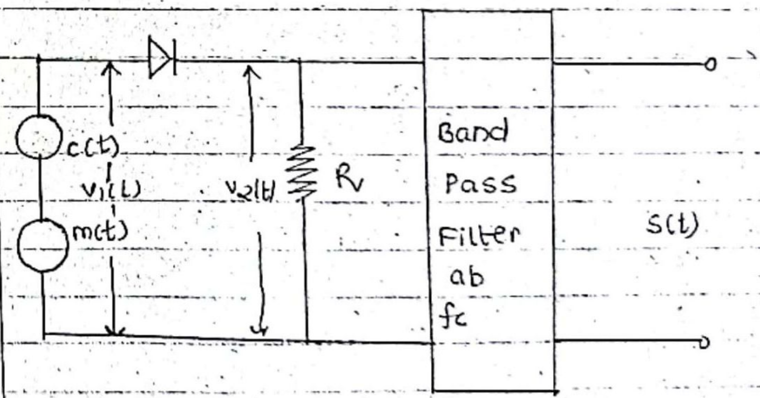
$\mu = \frac{2a_2}{a_1} \times A_m = k_a A_m$

(b) Switching Modulator:

The functional diagram of a switching modulator is given below. Here diode acts as an ideal switch.

following assumptions are made while analysing the modulator.

- The diode is ideal switch ie diode is equivalent to short circuit when $v_1(t) > 0$ the open circuit when $v_1(t) < 0$
- $|m(t)| << |c(t)|$ and $m(t)$ alone can not forward bias the diode (ie turn it on).
- The +ve half cycle of $c(t)$ can forward bias the diode



Then we can write

$$v_2(t) = \begin{cases} v_1(t) = m(t) + c(t) & \text{at } c(t) > 0 \\ 0 & ; \text{ at } c(t) < 0. \end{cases}$$

ie $v_2(t)$ varies periodically between $v_1(t)$ & 0 at the rate equal to carrier wave.

Then;

$$v_2(t) = v_1(t) \cdot g_p(t).$$

where,

$$g_p(t) = \begin{cases} 1 & \text{when } c(t) > 0 \\ 0 & \text{when } c(t) < 0. \end{cases}$$

as $g_p(t)$ is periodic it can be expressed in fourier series as;

$$g_p(t) = \frac{1}{2} + \frac{2}{\pi} \cos 2\pi f_c t + g_{ho}(t).$$

where,

$g_{ho}(t)$ is the components of the series containing higher order harmonics

The output of the diode can be written as;

$$v_2(t) = v_1(t) \cdot g_p(t)$$

$$= \left\{ c(t) + m(t) \right\} \cdot g_p(t).$$

$$= \left\{ A_c \cos 2\pi f_c t + m(t) \right\} \cdot \left\{ \frac{1}{2} + \frac{2}{\pi} \cos 2\pi f_c t + g_{ho}(t) \right\}$$

$$= \frac{A_c}{2} \cos 2\pi f_c t + \frac{1}{2} m(t) + \frac{2A_c}{\pi} \cos^2 2\pi f_c t + \frac{2\, m(t)}{\pi} \cos 2\pi f_c t$$

$$+ A_c \cos 2\pi f_c t * g_{ho}(t) + m(t) \cdot g_{ho}(t).$$

after passing through band pass filter.

$$s(t) = \frac{A_c}{2} \cos 2\pi f_c t + \frac{2}{\pi} m(t) \cos 2\pi f_c t$$

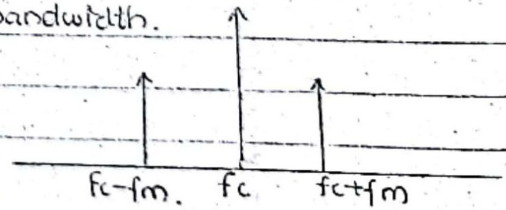
$$= \frac{A_c}{2} \left\{ 1 + \frac{4}{\pi A_c} m(t) \right\} \cos 2\pi f_c t.$$

$$= \frac{A_c}{2} \left\{ 1 + k_a m(t) \right\} \cos 2\pi f_c t.$$

Here $k_a = \frac{4}{\pi A_c}$ is the modulation index.

## features of DSB-AM

(i) The bandwidth requirement is twice the frequency of the message signal. Thus there is waste of the precious bandwidth.



$\cos 2\pi f_c t + \hat{g}(t)\}$

$2\pi f_c t + \frac{2\, m(t)}{\pi} \cos 2\pi f_c$

$) + m(t) - \hat{g}(t).$

filter.

$2\pi f_c t$

$2\pi f_c t.$

$\pi f_c t.$

ition index.

s twice the frequency

here is waste of the

$B.W = (f_c + f_m) - (f_c - f_m)$

$B.W. = 2 f_m$

ii) As the carrier is transmitted, it requires high power for transmission & efficiency is low.

2) DSB-SC (Double side band suppressed carrier modulation)

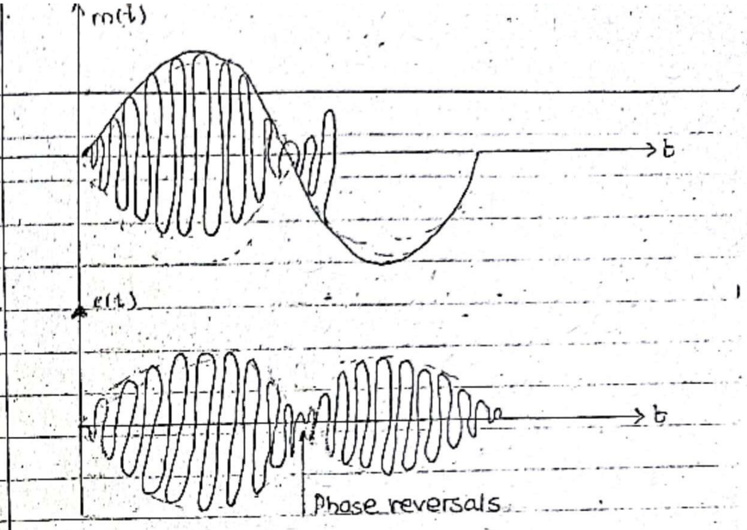
* In DSB-SC modulation the carrier wave is transmitted thus the transmission of carrier wave represents the waste of power.
* To overcome this shortcoming, the carrier component is not transmitted in DSB-SC.
* In DSB-SC the carrier wave is suppressed which increases the efficiency of transmission.

DSB-SC can be represented by

$s(t) = c(t) \cdot m(t)$

$= A_c\, m(t) \cdot \cos 2\pi f_c t$

Time domain representation.




It is obvious from the figure that DSB-SC exists phase reversal at zero crossing ie whenever the baseband signal m(t) crosses zero. Because of this envelope of DSB-SC modulated signal is different from message signal. This is unlike the case of AM wave.

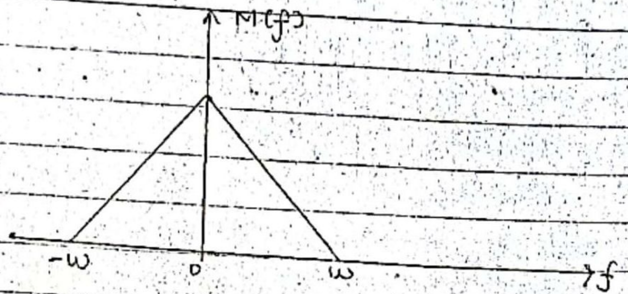
Frequency domain representation

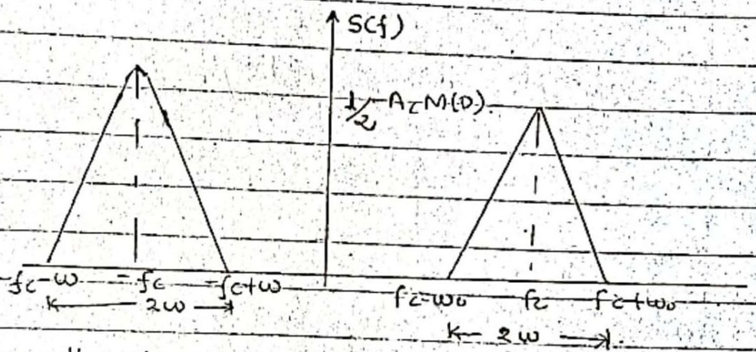
The carrier expression phenomena of DSB-SC can be visualized by examining the spectrum of the waves

$s(t) = A_c\, m(t) \cos 2\pi f_c t.$

Taking fourier transform

$$S(f) = \frac{A_c}{2}\left\{M(f - f_c) + M(f + f_c)\right\}$$




From the above figure,

(i) Except for the scaling factor, the modulation process simply translates the spectrum of the message signal by $\pm f_c$. It is also clear that impulses at $\pm f_c$ are missing which means that carrier term is suppressed in the spectrum & only two side band terms LSB & USB are left.

(ii) considering only positive side.

upper side band frequency $= f_c + W$

lower side band frequeny $= f_c - W$.

Transmission bandwidth of DSB-SC signal.

$$B = (f_c + W) - (f_c - W)$$
$$= 2W$$

ie double the bandwidth of message signal.

Power in DSB-SC.

$$P_t = P_{LSB} + P_{USB}$$
$$= P_{LSB} + P_{LSB} \quad \text{OR} \quad P_{USB} + P_{USB}$$
$$= 2 P_{LSB} \quad \text{OR} \quad 2 P_{USB}.$$

Generation of DSB-SC wave.

(i) Balanced modulator.

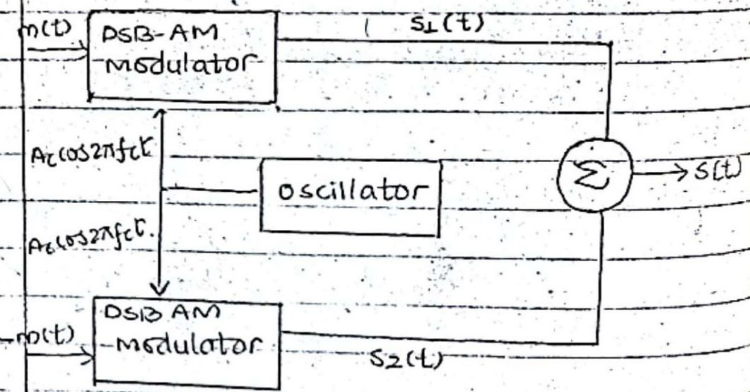



Fig: balanced modulator.

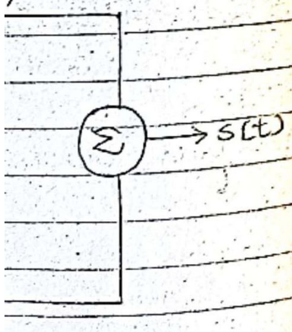
DSB-SC signal.

message signal.

USB

e.

)




It consists of.

- Two DSB-AM modulator in balanced configuration suppress the carrier wave.
- Two modulators are identical except for the sign reversal of modulating signal applied to the input of one of them.
- Oscillator to produce carrier wave.
- Summing network to produce required output.

The output of two modulators are given as

$$s_1(t) = A_c\{1 + k_a m(t)\}\cos 2\pi f_c t.$$

&

$$s_2(t) = A_c\{1 - k_a m(t)\}\cos 2\pi f_c t.$$

Then;

$$s(t) = s_1(t) - s_2(t)$$

$$= 2k_a m(t)\, A_c \cos 2\pi f_c t.$$

Comparing with $s(t) = A_c m(t) \cos 2\pi f_c t$.

Thus the output is the required DSB-SC wave expect for the scaling factor $2k_a$.

(ii) Ring modulator.

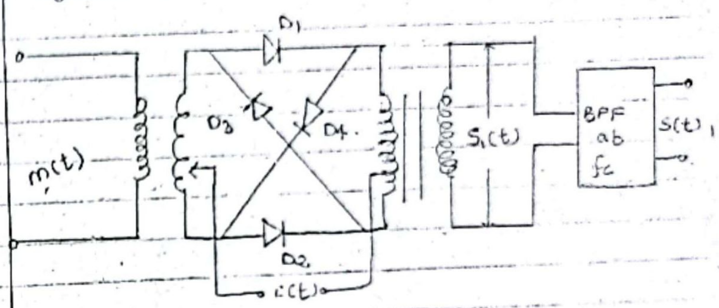



Fig: Ring modulator

- Four diodes are arranged in ring form.
- The operation of diodes are controlled by square wave carrier $c(t)$ of frequency $f_c$ applied by means of two centre tapped transformers.

Following assumptions are made while analysing the operation of the modulator.

- diodes are ideal.
- transformers are perfectly balanced.
- $|m(t)| << |c(t)|$ and $m(t)$ alone can not forward bias the diodes.
- when $c_2$ is +ve $D_1$ & $D_2$ are ON & $D_3$ & $D_4$ are off. then

$$s_1(t) = m(t)$$

- when $c(t)$ is -ve $D_3$ and $D_4$ are ON and $D_1$ & $D_2$ are off then

$$s_1(t) = -m(t)$$

- Then.

$$s_1(t) = \begin{cases} m(t) & ; \text{ when } c(t) > 0 \\ -m(t) & ; \text{ when } c(t) < 0 \end{cases}$$

- we can write.

$$s_1(t) = m(t) \cdot g_p(t)$$

where,

$$g_p(t) = \begin{cases} 1 & ; \text{ for } c(t) > 0 \\ -1 & ; \text{ for } c(t) < 0 \end{cases}$$

- as $g_p(t)$ is periodic its fourier series is expressed as.

$$g_p(t) = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \boxed{\cos 2\pi f_c t (2n-1)}$$

$$= \frac{4}{\pi} \cos 2\pi f_c t + \text{higher order harmonics}$$

Then;

$$s_1(t) = m(t) \cdot g_p(t).$$

$$= m(t) \left[ \frac{4}{\pi} \cos 2\pi f_c t + \text{higher order harmonics} \right]$$

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$$= m(t) \cdot \frac{4}{\pi} \cos 2\pi f_c t + m(t) * \text{higher order harmonics}$$

After passing through BPF at $f_c$

$$s(t) = 4/\pi \; m(t) \cos 2\pi f_c t.$$

Comparing with $s(t) = A_c m(t) \cos 2\pi f_c t$

Thus, the output is the required DSB-SC wave.

<u>Single tone DSB-SC modulation.</u>

$m(t) = A_m \cos 2\pi f_m t$

$c(t) = A_c \cos 2\pi f_c t.$

then,

$$s(t) = m(t) \cdot c(t)$$

$$= A_m \cos 2\pi f_m t \cdot A_c \cos 2\pi f_c t$$

$$= \frac{A_m \cdot A_c}{2} \cdot 2 \cos 2\pi f_m t \cdot \cos 2\pi f_c t$$

$$= \frac{A_m \cdot A_c}{2} \left\{ \cos 2\pi (f_c + f_m) t + \cos 2\pi (f_c - f$$

Taking fourier transform.

$$S(f) = \frac{A_m \cdot A_c}{4} \left[ \delta\{f + (f_c + f_m)\} + \delta\{f - (f_c + f_m)\} + \delta\{f \right.$$

$$+ \delta\{f - (f_c - f_m)\}.$$

3) ...
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s 2πfct

DSB-SC wave.

n.

os 2πfct

nt. cos 2πfct

c+fm)t + cos 2π(fc−fm)t

{f − (fc+fm)} + δ{f+(fc

$$S(f) = \frac{A_m.A_c}{4}\Big[\delta(f+f_c+f_m) + \delta(f-f_c-f_m) + \delta(f+f_c+f_m) + \delta(f-f_c+f_m)\Big].$$

3) Single side band modulation (SSB-SC).

* DSB-AM and DSB-SC modulation are wastefull of bandwidth because they both require a bandwidth equal to twice the message bandwidth.

* Transmission of only one side band either USB or LSB.

* So Transmission bandwidth = Bw of message signal. thus prevents the waste of bandwidth.

* USB & LSB are uniquely related to each other by virtue of their symmetry about the carrier frequency thus amplitude & phase spectra of one side band can be used to generate other.

* So, the transmission of one sideband & if both the carrier & other side band are supressed at the transmitter, no information is lost.

frequency domain analysis.

- Let us consider a message signal m(t) having spectrum M(f) limited to band $-W \le f \le W$ as shown in fig (a)

- The spectrum of DSB-SC modulated wave obtained by multiplying m(t) and c(t) shown in fig (b)

$$S_{DSB-SC} = m(t)\cdot c(t).$$

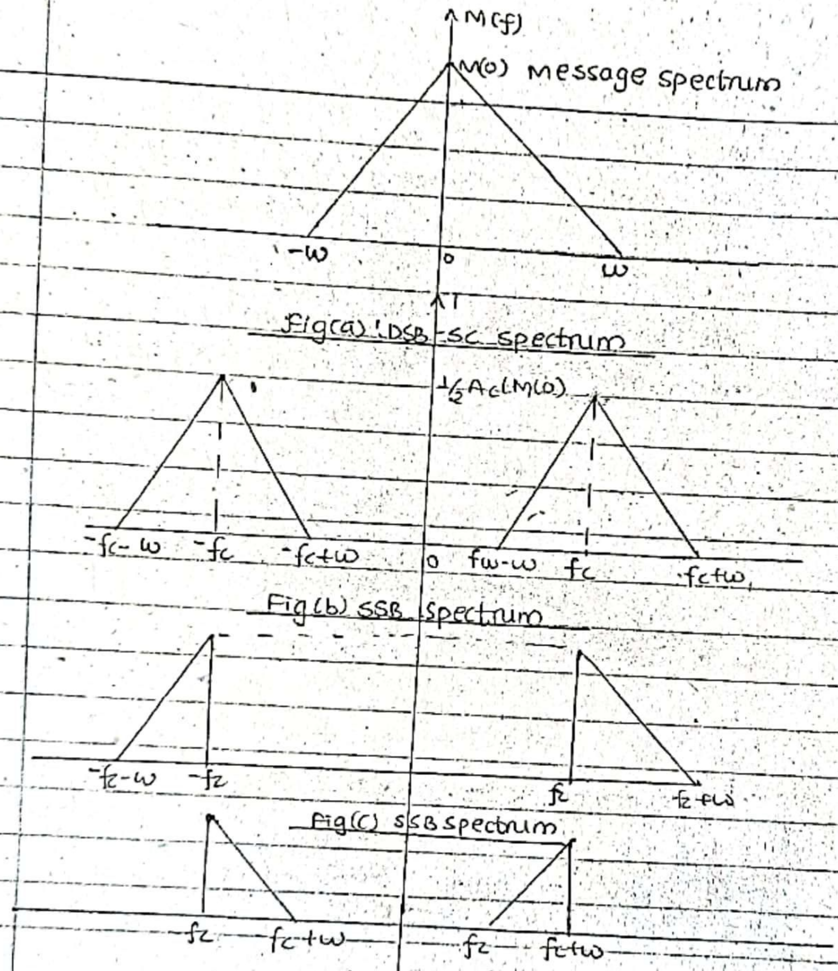
$$= A_m \cos 2\pi f_m t \cdot A_c \cos 2\pi f_c t.$$

$$= \frac{A_m.A_c}{2} \cdot 2\cos 2\pi f_c t \cdot \cos 2\pi f_m t.$$

$$= \frac{A_m.A_c}{2}\left[\cos 2\pi(f_c+f_m)t + \cos 2\pi(f_c-f_m)t\right]$$

- The upper sideband is represented in duplicate by the frequencies above fc and those below −fc & when only upper side band is transmitted the resulting SSB modulated wave has spectrum as shown in fig c.

- Likewise, lower sideband is represented in duplicate by the frequencies below fc and those above −fc when only lower side band is transmitted then resulting SSB modulated wave has the spectrum as in fig (d).




Fig(a): DSB-SC spectrum

Fig(b) SSB spectrum

Fig(c) SSB spectrum

- Translate the spectrum of the modulating signal with or without inversion to new location in the frequency domain.
- The transmission bandwidth requirement of SSB modulation system is one half that of standard AM and DSB-SC.

## Time domain analysis:

In general, a standard AM signal incase of sinusoidal modulating signal can be represented as.

$$u(t) = A_c \cos 2\pi f_c t + \frac{aA_c}{2}\cos 2\pi (f_c - f_m)t + \frac{aA_c}{2}\cos 2\pi (f_c + f_m)t$$

- where the 1st component is the carrier component & the 2nd component represent the LSB & 3rd component is the USB.
- The information about modulating signal is contained in a and frequency of modulating signal $f_m$.
- In real world, modulating signal is not the sinusoidal but complex signal $m(t)$ which can be represented as the sum of large no of sinusoidal signals of various amplitude frequency & phase

$$m(t) = \sum_{i=1}^{n} A_i \cos(2\pi f_i t + \theta_i).$$

for ith component.

$$m_i(t) = A_i \cos(2\pi f_i + \theta_i)$$

* The DSB-SC signal would be

$$u_{DSBi}(t) = \frac{A_i \cdot A_c}{2}\cos\left[2\pi(f_c - f_i)t - \theta_i\right] + \frac{A_i A_c}{2}\cos\left[2\pi(f_c + f_i)t + \theta_i\right]$$

where;

$$U_{LSBi}(t) = \frac{A_i A_c}{2} \cos\left[2\pi(f_c - f_i)t - \theta_i\right]$$

&

$$U_{USBi}(t) = \frac{A_i A_c}{2} \cos\left[2\pi(f_c + f_i)t + \theta_i\right].$$

The overall signal representation for USB will be

$$U_{USBi}(t) = \frac{A_c}{2}\sum_{i=1}^{n} A_i \cos\left[2\pi(f_c + f_i)t + \theta_i\right]$$

$$= \frac{A_c}{2}\sum_{i=1}^{n} A_i \cos\left[\underbrace{(2\pi f_i t + \theta_i)}_{A} + \underbrace{2\pi f_c t}_{B}\right]$$

$$= \frac{A_c}{2}\left\{\left[\sum_{i=1}^{n} A_i \cos(2\pi f_i t + \theta_i)\right]\cos 2\pi f_c t - \left[\sum_{i=1}^{n} A_i \sin(2\pi f_i t + \theta_i)\right]\cdot \sin 2\pi f_c t\right\}.$$

$$= \frac{A_c}{2} m(t) \cos 2\pi f_c t - \frac{A_c}{2}\hat{m}(t) \sin 2\pi f_c t.$$

$\Rightarrow$ where;

$\sum_{i=1}^{n} A_i \cos(2\pi f_i t + \theta_i)$ is the original message signal represented by $m(t)$,

& $\sum_{i=1}^{n} A_i \sin(2\pi f_i t + \theta_i)$ is the Hilbert transformation of the original message signal represented by $\hat{m}(t)$

$\therefore$ In general SSB signal can be represented as

$$U_{SSB}(t) = \frac{A_c}{2} m(t) \cos 2\pi f_c t \pm \frac{A_c}{2}\hat{m}(t) \sin 2\pi f_c t.$$

here the -ve sign represents USB & +ve sign represents LSB

Single tone time domain analysis of SSB wave

let $m(t) = A_m \cos 2\pi f_m t$

$c(t) = A_c \cos 2\pi f_c t$

& $\hat{m}(t) = A_m \sin 2\pi f_m t$ (Hilbert transform of $m(t)$)

Then,

$$S_{DSB-SC}(t) = m(t)\cdot c(t)$$

$$= A_m \cdot A_c \cos 2\pi f_c t \cdot \cos 2\pi f_m t$$

$$= \frac{A_m A_c}{2}\left[\cos 2\pi(f_c + f_m)t + \cos 2\pi(f_c - f_m)\cdot t\right]$$

Taking only upper side band.

$$S_{USB}(t) = \frac{A_m A_c}{2} \cos 2\pi(f_c + f_m)\cdot t.$$

$$= \frac{A_m A_c}{2} \cos\left[2\pi f_c t + 2\pi f_m t\right]$$

$$= \frac{A_m A_c}{2}\left[\cos 2\pi f_c t \cdot \cos 2\pi f_m t - \sin 2\pi f_c t \cdot \sin 2\pi f_m t\right]$$

$$= \frac{A_c}{2}\left[(A_m \cos 2\pi f_m t) \cdot \cos 2\pi f_c t - (A_m \sin 2\pi f_m t) \sin 2\pi f_c t\right]$$

$$= \frac{A_c}{2}\left[m(t) \cos 2\pi f_c t - \hat{m}(t) \sin 2\pi f_c t\right]$$

Now, Taking lower side band

$$S_{LSB}(t) = \frac{A_m A_c}{2} \cos 2\pi (f_c - f_m) \cdot t$$

$$= \frac{A_m A_c}{2} \cos (2\pi f_c t - 2\pi f_m t)$$

$$= \frac{A_m A_c}{2}\left[\cos 2\pi f_c t \cdot \cos 2\pi f_m t + \sin 2\pi f_c t \cdot \sin 2\pi f_m t\right]$$

$$= \frac{A_c}{2}\left[m(t) \cdot \cos 2\pi f_c t + \hat{m}(t) \sin 2\pi f_c t\right]$$

Thus, In general.

$$S_{SSB}(t) = \frac{A_c}{2}\left[m(t) \cos 2\pi f_c t \pm \hat{m}(t) \sin 2\pi f_c t\right]$$

→ The −ve sign represents the USB & +ve sign represents LSB



advantages

* reduced bandwidth requirement.
* elimination of higher power carrier wave.

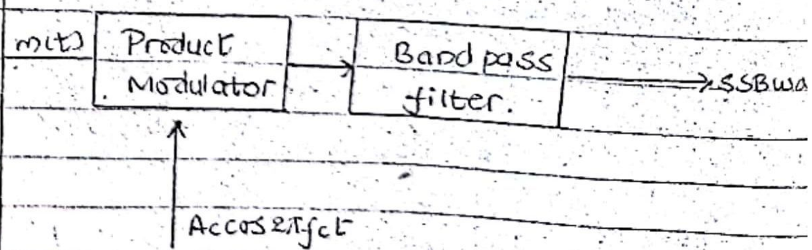
disadvantages

* cost and complexity.

Generation of SSB wave.

(i) Frequency discrimination method (Filtering metho

In frequency discrimination method the message si $m(t)$ and carrier signal $c(t)$ is applied to produce modulator to generate DSB-SC wave which is the passed through band pass filter, out of which one of the sideband is filtered out.

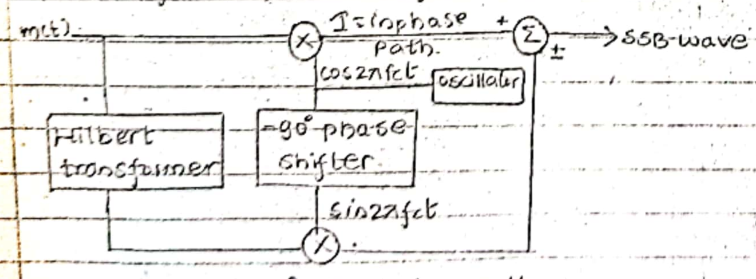


To implement this method following conditions should be satisfied.

(i) The frequency discrimination method is useful only if the baseband signal is restricted c.t its lower edge due to which upper and lower side band is Non-overlapping. However this system is not useful for video communication where the message signal or baseband signal starts from Dc.

(ii) The another restriction is that baseband signal must be appropriately related to carrier frequency ($f_c$). Infact the design of band pass filter (BPF) becomes difficult if carrier frequency is quite higher than bandwidth of the baseband signal.

(ii) Phase shift method (Hartley modulator)




Q = Quadrature path.

- It consists of two product modulator
- The product modulator in the upper path or inphase path receives message signal $m(t)$ & output of oscillator $\cos 2\pi f_c t$ which gives output $m(t)\cos 2\pi f_c t$.
- The phase shifted output of oscillator $\sin 2\pi f_c t$ & Hilbert transform of message signal $\hat{m}(t)$ is fed to another product modulator which gives output $\hat{m}(t)\sin 2\pi f_c t$.
- The output of both the modulator is fed to adder which gives overall output.

$$s(t) = m(t)\cos 2\pi f_c t \pm \hat{m}(t)\sin 2\pi f_c t$$

Which is the SSB wave.

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## Vestigial side band modulation.

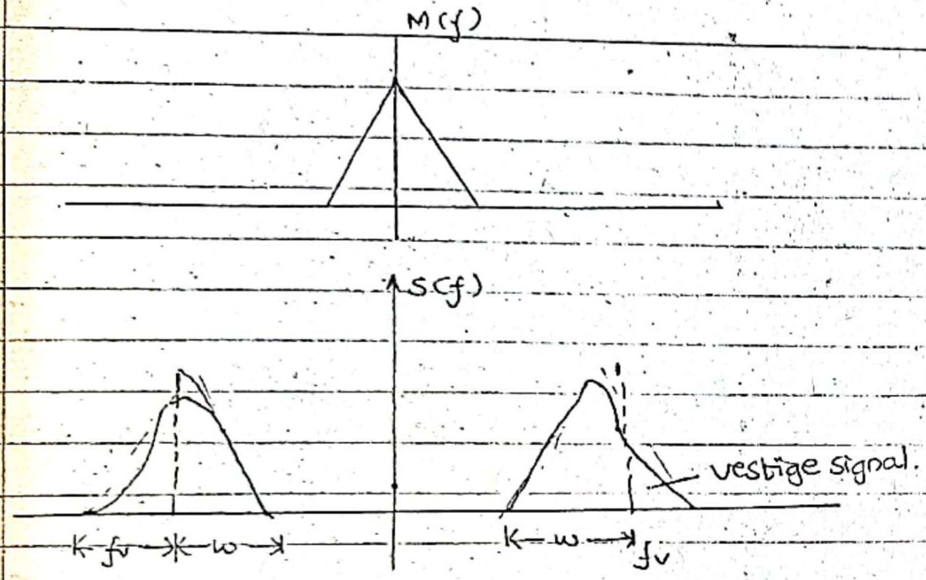
SSB modulation is well suited for the transm of voice signal because of energy gap that ex between zero and a few hundred Hz.

- Generation of SSB wave is difficult due to the of sharp selective filtering
- In SSB modulation, there BPF at the output from there is loss of low frequency components. These frequency components are significant in video sig & their loss leads to picture distortion.
- In VSB modulation one side band is completely pas & just a trace or vestige of another side band is transmitted.
- So, the bandwidth requirement is greater than tha SSB (typically 25%)

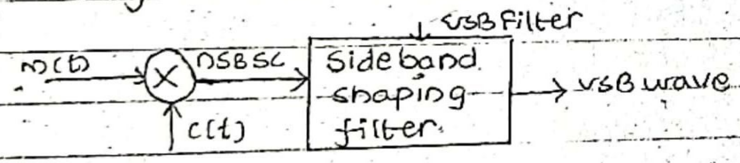
$$B.W. = \underbrace{W}_{\text{BW of message signal}} + f_v \quad \text{------- (a)}$$

where $f_v$ = width of the VSB.

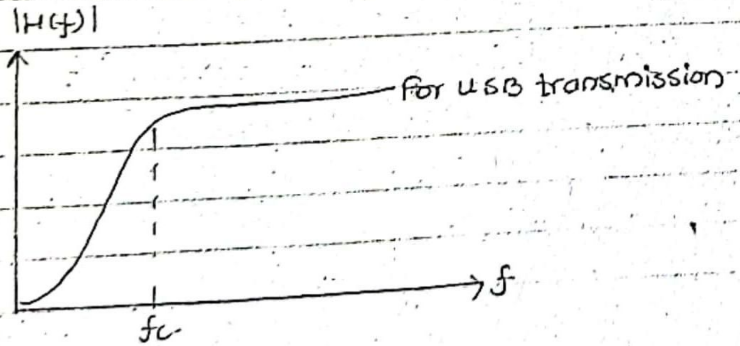
- VSB modulation has become standard for transmission of television & similar signals where good phase characteristics & transmission of low frequency components are important.

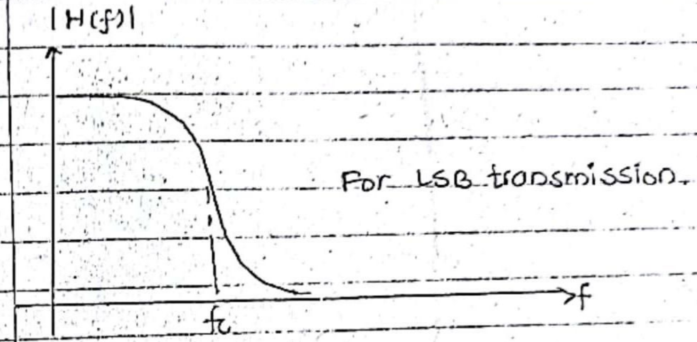



## Generation of VSB modulated wave




Fig(a) VSB modulator




For the;

Generation of VSB modulated wave we provide message signal m(t) and carrier signal c(t) to produce modulator which generates DSB-SC wave. This DSB-SC wave is passed through sideband shaping filter. This filter generates one of the sideband & terace or vestige of another side band.

The output of VSB modulator is

$$S_{VSB}(t) = A_c\, m(t) \cos 2\pi f_c t * h_{VSB}(t) \quad \text{---- (i)}$$

where

$h_{VSB}(t)$ is the impulse response of VSB filter

Frequency domain;

$$S_{VSB}(f) = \frac{A_c}{2}\left\{ M(f-f_c) + M(f+f_c) \right\} \cdot H_{VSB}(f) \quad \text{---- (ii)}$$

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Scanned with CamScanner

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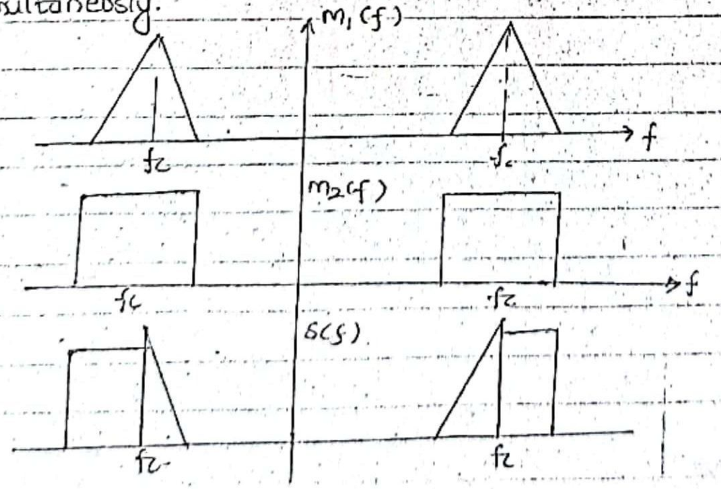
## Independent side band Modulation (ISB mod$^n$)

- For single sideband transmission, the carrier & one side band are removed from modulated signal. It is possible to replace the removed side band with another sideband of information created by modulating a different input signal on the same carrier giving independent sideband or ISB.

- Both input signals have frequencies in the same audio spectrum range but in the transmitted signal each signal occupies a different group of frequencies.

- ISB modulation is the bandwidth conservation scheme which utilizes the existing DSB-SC system to transmit two message signal simultaneously.



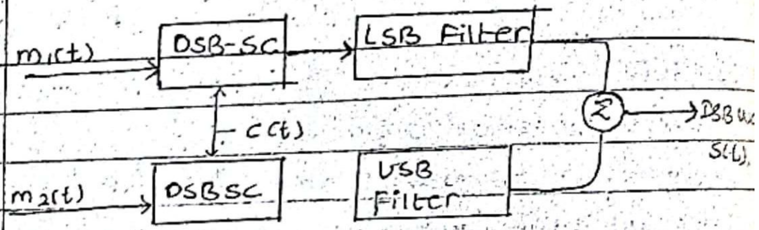
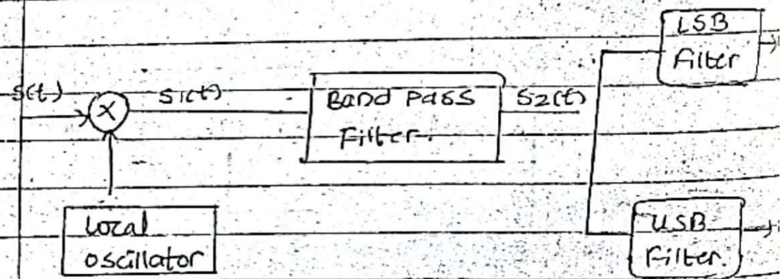




Fig I: ISB modulator.



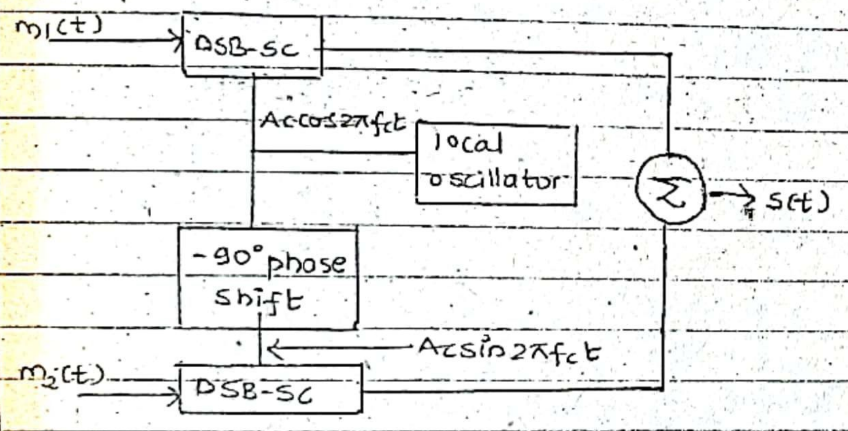

### ISB Modulation.

Message signal are given to two DSB-SC wave genera[tors] followed by SSB filters ie one generator with LSB filte[r] while other with USB filters so as to generate lower sideband and uppersideband respectively.

The output of the SSB filters are summed up t[o] give out ISB wave

## Quadrature amplitude modulation (QAM)

- Bandwidth conservation scheme.
- It allows two DSB-SC modulated waves of two independent message to occupy the same transmission bandwidth & allows separation of two messages at the receiver output.




- QAM has two DSB-SC modulators which are supplied with different message signal and two carrier waves of same frequency but quadrature in phase to each other.

- The output of DSB-SC are fed to adder producing

$$s(t) = A_c m_1(t) \cos 2\pi f_c t + A_c m_2(t) \sin 2\pi f_c t.$$

- Multiplexed signal occupies transmission bandwidth of $2W$, centred at carrier frequency $f_c$.

where,

$W$ = bandwidth of $m_1(t)$ or $m_2(t)$ whichever is the largest.

- Hence, multiplexed signal consists of inphase component $A_c m_1(t) \cos 2\pi f_c t$ and quadrature phase component $A_c m_2(t) \sin 2\pi f_c t$.

# Chapter 5: demodulation of AM signals.

## 1. Demodulation of DSB-AM.

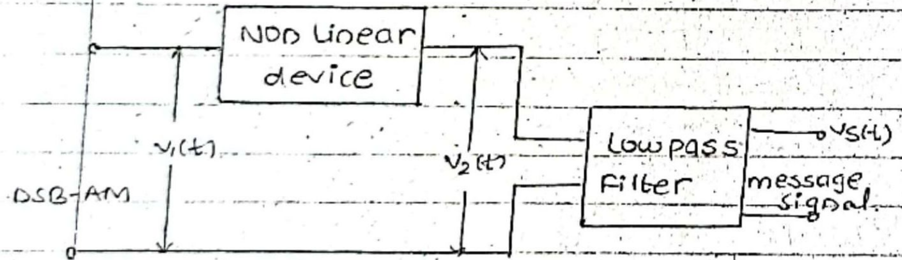
### (i) Square law detector (demodulator).

→ DSB-AM wave is applied to the non-linear device.

→ output of non linear device is given to the low pass filter.

→ output of LPF is the required message signal.

→ In square law modulation bandpass filter is used whereas in demodulation low pass filter is used.




From above figure

$v_1(t) = A_c[1 + k_a m(t)] \cdot \cos 2\pi f_c t.$

where,

$m(t)$ = message signal

$c(t)$ = carrier wave = $A_c \cos 2\pi f_c t.$

$K_a$ = amplitude sensitivity of modulator.

for non-linear device

$$v_2(t) = a_1 v_1(t) + a_2 v_1^2(t)$$

$$= a_1 A_c[1 + K_a m(t)] \cos 2\pi f_c t + a_2 A_c^2 [1 + K_a m(t) \cdot \cos^2 2\pi f_c t.$$

or, $v_2(t) = a_1 A_c [1 + K_a m(t)] \cos 2\pi f_c t + a_2 A_c^2 [1 + 2K_a m(t) + K_a^2 m^2(t)] \cos^2 2\pi f_c$

or, $v_2(t) = a_1 A_c [1 + k_a m(t)] \cos 2\pi f_c t + \frac{a_2 A_c^2}{2} [1 + 2k_a m(t)$

$+ K_a^2 m^2(t)] + \frac{a_2 A_c^2}{2} [1 + 2k_a m(t) + k_a^2 m^2(t)] \cos 4\pi$

After passing through low pass filter we get.

$$v_s(t) = \frac{a_2 A_c^2}{2} (1 + 2k_a m(t) + k_a^2 m^2(t)).$$

$$= \frac{1}{2} a_2 A_c^2 + a_2 A_c^2 k_a m(t) + \frac{1}{2} a_2 A_c^2 k_a^2 m^2(t).$$

Here, $a_2 A_c^2 k_a m(t)$ is the required message signal but there is also unwanted signal $\frac{1}{2} A_c^2 k_a^2 m^2(t)$ noise

→ $\frac{1}{2} a_2 A_c^2$ is a dc signal and has no effect on demodulation (it only shifts message signal) no distortion.

→ The ratio of wanted signal to unwanted signal (noise) is SNR

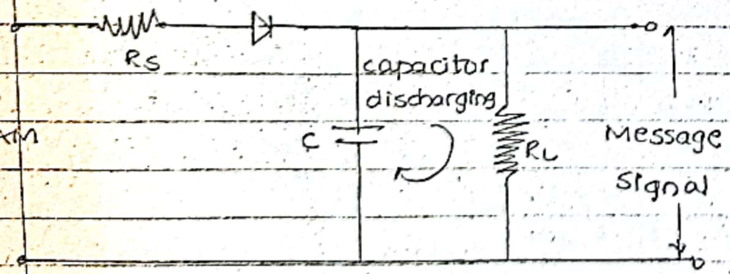
i.e $SNR = S/N = 2/k_a m(t)$

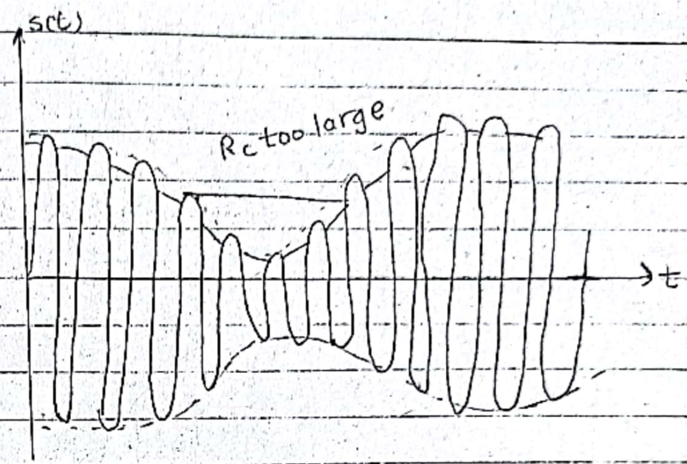
→ To have higher SNR the value of the $k_a m(t)$ or modulation index should be small as compared to unity therefore distortionless recovery of message signal is possible if.

* The DSB AM is weak or small so it satisfies with square law input output relation (higher orders of small signals are negligiable).

* percentage modulation $= |k_a m(t)| \times 100$ is small or low.

(ii) Envelope detector




- Envelope detector produces an output voltage proportional to the envelope which follows modulating or message signal.

- It consists of diode, resistor - capacitor filter.

- Diode acts as rectifier, it is considered to be ideal ie zero impedance to the current flow in forward bias and infinite impedance in the reverse biased condition. thus no envelope detection takes place in -ve half of the AM wave.

- we further assume that AM wave applied to the

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envelope detector supplied by a voltage source of internal impedance $R_s$.

- In +ve half cycle of input voltage (signal), the diode is forward biased.
When the input voltage rises with the modulating cycle, the capacitor c charges up rapidly to the peak value of the input signal.
During the downward swing, when the input signal falls the diode becomes reverse biased. Then capacitor c discharges slowly through the load resistor $R_L$.

- Discharging continuous until the next +ve half cycle when the input voltage becomes greater than the voltage across capacitor, the diode gets forward biased and charging of capacitor c sets in. Thus the process of changing & discharging gets repeated.

- The charging time constant $R_s c$ must be short as compared to the carrier period $1/f_c$

$$\text{ie } R_s c << 1/f_c$$

- The discharging time constant $R_L c$ must be long enough thus capacitor must discharge slowly through $R_L$ between positive peaks of the carrier wave.
But time constant $R_L C$ should not be so long that capacitor voltage will not discharge at the maximum rate of change of modulating wave (or highest frequency of $m(t)$)

- The envelope detector output is $v_c(t) = A + m(t)$ with ripple frequency $w_c$. Dc term A can be blocked by the capacitor or simple RC high pass filter. In order to minimize the distortion & maximize the filteration of high frequency ripple time $R_C$ of filter (cut of frequency) is selected in following manner

$$1/f_c << R_L C \leq 1/w \text{ ------ (a)}$$

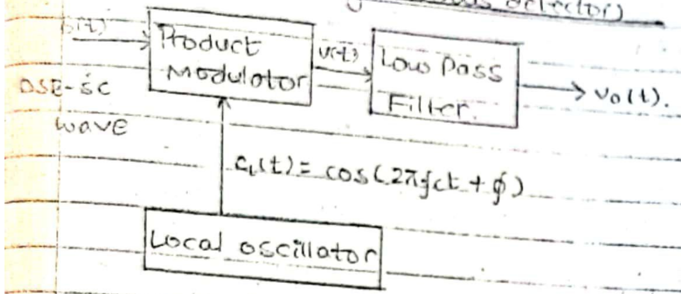
where $w$ = message bandwidth
$f_c$ = carrier frequency.

- Thus the capacitor voltage is nearly same as the envelope of AM wave.

- This detector is simple to implement, highly efficient, & effective cost. therefore this detector circuit is used in almost all commercial AM broadcast receivers.

## Demodulation of DSB-SC modulated wave.

### (i) Coherent detector (Synchronous detector)




- DSB-SC is ($s(t)$) is applied to product modulator where $s(t)$ is multiplied with $c_1(t)$, locally generated sinusoidal wave with same frequency and phase as that of carrier wave therefore called coherent or synchronized.

- However the more general demodulation process is using a local oscillator having same frequency but arbitary phase difference $\phi$ with respect to carrier wave. Thus denoting the local oscillator signal as $c_1(t) = \cos(2\pi f_c t + \phi)$.

- The output of product modulator is.
  $v(t) = c_1(t) \cdot s(t)$.

or, $v(t) = c_1(t) \cdot m(t) \cdot c(t)$

or, $v(t) = \cos(2\pi f_c t + \phi) \cdot m(t) \cdot A_c \cos 2\pi f_c t$.

or, $v(t) = A_c \cos 2\pi f_c t \cdot \cos(2\pi f_c t + \phi)\, m(t)$.

or, $v(t) = \frac{A_c}{2}\left\{ 2\cos 2\pi f_c t \cdot \cos(2\pi f_c t + \phi) \right\} m(t)$

or, $v(t) = \frac{A_c}{2}\left\{ \cos(\phi) + \cos(4\pi f_c t + \phi) \right\} m(t)$.

or, $v(t) = \underbrace{\frac{A_c}{2} m(t) \cos\phi}_{\text{scaled version of message signal}} + \underbrace{\frac{1}{2} m(t) \cdot \cos(4\pi f_c t + \phi)}_{\text{unwanted term}}$

- after passing $v(t)$ through LPF the unwanted term is removed so overall output is given by
  $v_o(t) = \frac{1}{2} A_c m(t) \cdot \cos\phi$.

- The demodulated signal $v_o(t)$ is proportional to message signal $m(t)$ when the phase error $\phi$ is constant.

- The amplitude of demodulated signal is maximum when $\phi = 0°$
  $\therefore v_o(t) = \frac{1}{2} A_c m(t)$

- The amplitude is minimum when $\phi = \pm \pi/2$.

$\therefore v_o(t) = 0.$

- The demodulated signal is 0 for $\phi = \pm\pi/2$ represents the quadrature null effect of the coherent detector.

- Thus the phase error $\phi$ in the local oscillator causes the detector output to be attenuated by factor $\cos\phi$. as long as phase error $\phi$ is constant detector output is undistorted version of message signal $m(t)$.

- but in real practice the multiplying factor $\cos\phi$ at detector output varies randomly with time, which is undesirable.

- Thus there is need of circuitary at receiver that maintains local oscillator in perfect synchronization in both phase & frequency with carrier of transmitter.

- Hence, it is costly & complex.

(ii) Costa's loop (receiver)

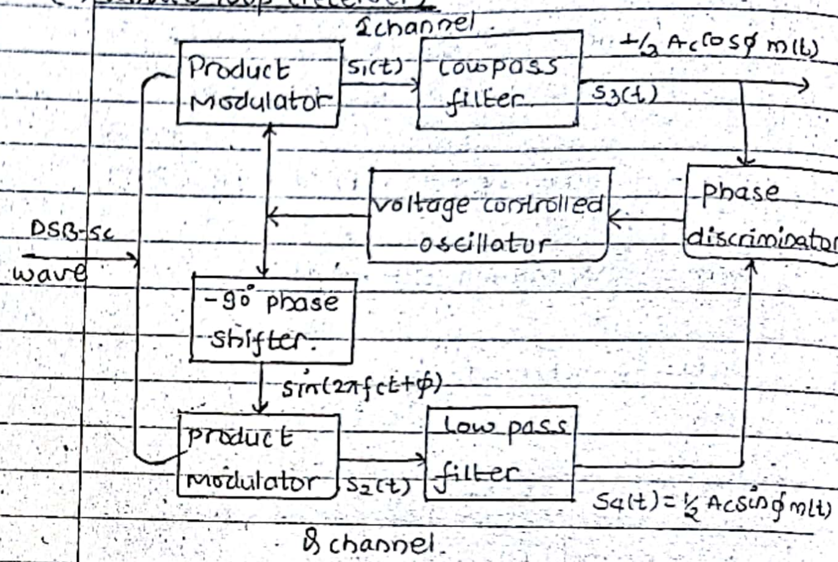



Fig: Costa's loop

- The receiver consists of two coherent detectors supplied with same DSB-SC wave while other input signal to product modulator is in phase quadrature to each other.

- Let the frequency generated by VCO is $f_c$ but has slight phase error $\phi$.

- $S_1(t) = s(t) \cdot \cos(2\pi f_c t + \phi)$

$$= A_c\, m(t) \cdot \cos 2\pi f_c t \cdot \cos(2\pi f_c t + \phi)$$

$$= \frac{A_c\, m(t)}{2} \cdot \left\{ \cos\phi + \cos(4\pi f_c t + \phi) \right\}$$

$$= \frac{A_c\, m(t)\cos\phi}{2} + \frac{A_c\, m(t)}{2}\cos(4\pi f_c t + \phi).$$

after passing through LPF we get:

$$S_3(t) = \frac{A_c}{2} m(t)\cos\phi.$$

- Similarly;

$S_2(t) = s(t) \cdot \sin(2\pi f_c t + \phi)$

$$= A_c \cdot m(t) \cdot \cos 2\pi f_c t \cdot \sin(2\pi f_c t + \phi)$$

$$= \frac{A_c\, m(t)}{2} \left\{ \sin\phi + \sin(4\pi f_c t + \phi) \right\}$$

$$= \frac{A_c}{2} m(t) \cdot \sin\phi + \frac{A_c}{2} m(t) \cdot \sin(4\pi f_c t + \phi)$$

after passing through LPF

$$S_4(t) = \frac{A_c}{2} m(t) \cdot \sin\phi.$$

* when $\phi = 0^\circ$

$S_3(t) = \frac{A_c}{2} m(t).$ — demodulated signal/constant

and $S_4(t) = 0$.

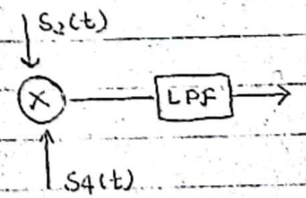
* detector in upper path → In phase coherent detector or I channel.

* detector in lower path → quadrature phase coherent detector or Q channel.

- If the local oscillator phase drifts by small amount $\phi$ radian, then,
  I channel output will remain essentially constant since small variation in $\phi$ doesn't change value of $\cos\phi$ significantly
  but,
  Q channel output will change with $\phi$.
  since for small value of $\phi$, $\sin\phi \approx \phi$

  $\therefore S_4(t) = \frac{A_c}{2} m(t)\,\phi.$

- I & Q channels output are passed to phase discriminator (multiplier followed by LPF).
  ie.



$$S_3(t)\cdot S_4(t) = \frac{A_c}{2} m(t)\cdot\cos\phi \cdot \frac{A_c}{2} m(t)\sin\phi.$$

$$= \frac{A_c^2}{8} m^2(t)\sin 2\phi.$$

after passing through LPF;

$$= \frac{A_c^2}{8} m^2(t)\cdot\sin 2\phi.$$

$$= k\sin 2\phi.$$

where $k$ = dc component of $\frac{A_c^2 m^2(t)}{8}$.

- A dc control signal proportional to the phase error $\phi$ obtained as discriminator output is applied to vco which automatically corrects the oscillators phase errors.

demodulation of SSB wave

(i) coherent detector of SSB wave.

To recover baseband signal $m(t)$ from the SSB signal $s(t)$ we have to shift the spectrum of $s_{LSB}(t)$ or $s_{USB}(t)$ by $\pm f_c$. This can be done by using coherent detection which involves applying SSB wave with locally generated carrier $c_L(t)$ to the product modulator and then to LPF.

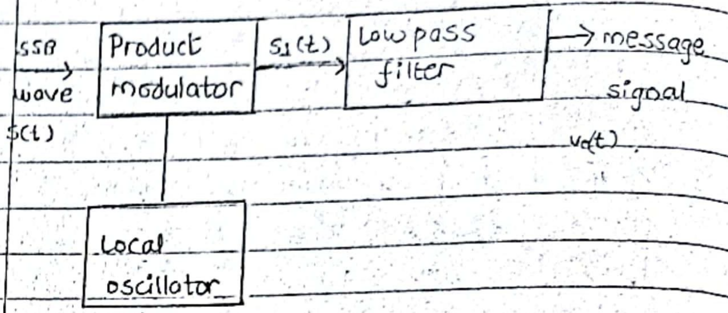



Fig demodulation of SSB wave

We have,

$$s(t) = \frac{A_c}{2}\left\{m(t)\cdot\cos 2\pi f_c t + \hat{m}(t)\sin 2\pi f_c t\right\}$$

Then;

$$s_1(t) = s(t)\cdot c_L(t)$$

or, $$s_1(t) = \frac{A_c}{2}\left\{m(t)\cdot\cos 2\pi f_c t \pm \hat{m}(t)\cdot\sin 2\pi f_c t\right\}\cdot A_c\cos 2\pi f_c t.$$

or, $$s_1(t) = \frac{A_c^2}{2} m(t)\cos^2 2\pi f_c t \pm \frac{A_c^2}{2}\hat{m}(t)\cdot\sin 2\pi f_c t\cdot\cos 2\pi f_c t.$$

or, $$s_1(t) = \frac{A_c^2}{4} m(t)\left\{1+\cos 4\pi f_c t\right\} \pm \frac{A_c^2}{4}\hat{m}(t)\left\{\sin 4\pi f_c t + \sin 0\right\}$$

or, $$s_1(t) = \underbrace{\frac{A_c^2}{4} m(t)}_{\text{desired signal.}} + \underbrace{\frac{A_c^2}{4} m(t)\cdot\cos 4\pi f_c t \pm \frac{A_c^2}{4}\hat{m}(t)\sin 4\pi f_c t}_{\text{unwanted signal.}}$$

after passing through low pass filter.

$v_0(t) = \frac{1}{4} A_c^2 m(t)$.

- This is the case when locally generated carrier and incoming carrier from s(t) are coherent or synchronized in both phase and frequency but practically it is impossible to generate the signal identical in phase & frequency of the incoming carrier wave.

- Let us consider the output oscillator drifts by phase $\phi$ and frequency as $\Delta f$ then;

$$c_L(t) = A_c \cos\{2\pi(f_c+\Delta f)t+\phi\}$$

where;

$\Delta f$ = error in frequency.

$\phi$ = error in phase.

Then,

$S_1(t) = s(t) \cdot c_L(t)$

or, $$s_1(t) = \frac{A_c}{2}\{m(t) \cdot \cos 2\pi f_c t + \hat{m}(t) \sin 2\pi f_c t\} A_c \cos\{2\pi (f_c+\Delta f)t+\phi\}$$

$$= \frac{A_c^2 m(t)}{2} \cos 2\pi f_c t \cdot \cos\{2\pi f_c t + 2\pi\Delta f t + \phi\} + \frac{A_c^2}{2}\hat{m}(t) \sin 2\pi f_c t \cdot \cos(2\pi f_c t + 2\pi\Delta f t + \phi)$$

$$= \frac{A_c^2 m(t)}{4}\{\cos(4\pi f_c t + 2\pi\Delta f t + \phi) + \cos(2\pi\Delta f t + \phi)\}$$

$$\pm \frac{A_c^2 \hat{m}(t)}{4}\{\sin(4\pi f_c t + 2\pi\Delta f t + \phi) + \sin(2\pi\Delta f t + \phi)\}$$

$$= \frac{A_c^2}{4} m(t) \cdot \cos(2\pi f_c t + \phi) \pm \frac{A_c^2}{4}\hat{m}(t) \sin(2\pi\Delta f t + \phi)$$

$$+ \frac{A_c^2}{4} m(t) \cos(4\pi f_c t + 2\pi\Delta f t + \phi) \pm \frac{A_c^2}{4}\hat{m}(t) \sin(4\pi f_c t + 2\pi\Delta f t + \phi).$$

- after passing through low pass filter and supposing $\Delta f = 0$ we get.

$$v_0(t) = \frac{1}{4} A_c^2 m(t) \cos\phi \pm \frac{1}{4} A_c^2 \hat{m}(t) \sin\phi.$$

* The only constraint of this method is the realization of Hilbert transformer over wide range of frequency of message signal.

* Detector output suffers from phase distortion, which is not serious with voice communication.

## demodulation of QAM (quadrature amplitude modulation)

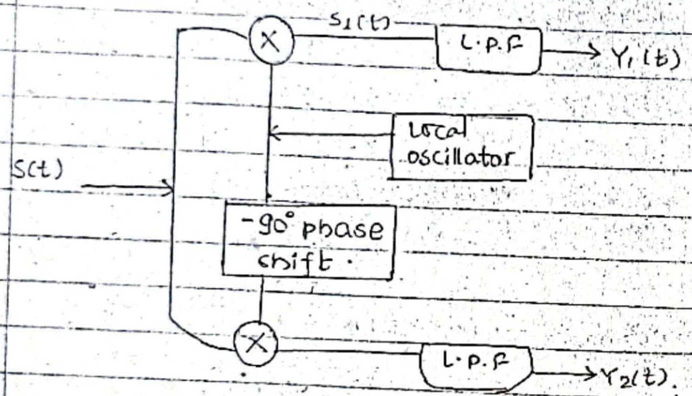



Fig: QAM receiver

$S(t) = A_c m_1(t) \cos 2\pi f_c t + A_c m_2(t) \sin 2\pi f_c t.$

$S_1(t) = S(t) \cdot \cos 2\pi f_c t$

or, $S_1(t) = A_c m_1(t) \cos 2\pi f_c t + A_c m_2(t) \cdot \sin 2\pi f_c t \cdot \cos 2\pi f_c t.$

or, $S_1(t) = \frac{A_c}{2} m_1(t) \left\{1 + \cos 4\pi f_c t\right\} + A_c m_2(t) \cdot \sin 4\pi f_c t.$

or, $S_1(t) = \frac{A_c}{2} m_1(t) + \frac{A_c}{2} m_1(t) \cos 4\pi f_c t + A_c m_2(t) \cdot \sin 4\pi f_c t.$

after passing through LPF.

$Y_1(t) = \frac{A_c}{2} m_1(t).$

Then, $S_2(t) = S(t) \cdot \sin 2\pi f_c t$

or, $S_2(t) = A_c m_1(t) \cdot \cos 2\pi f_c t \cdot \sin 2\pi f_c t + A_c m_2(t) \cdot \sin^2 2\pi f_c t$

or, $S_2(t) = A_c m_1(t) \cdot \sin 4\pi f_c t + \frac{A_c}{2} \left\{1 - \cos 4\pi f_c t\right\} \cdot m_2(t).$

after passing through LPF:

$Y_2(t) = \frac{A_c}{2} m_2(t).$

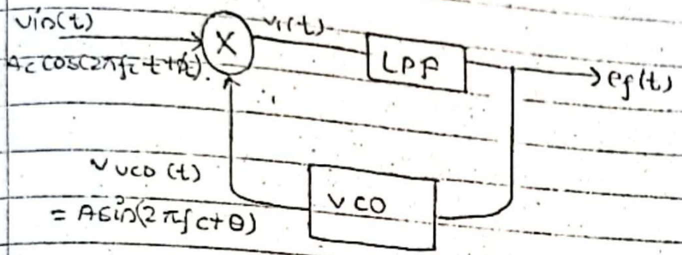
- For satisfactory operation of QAM transmitter & receiver the phase and frequency relationship between oscillator & incoming signal should be synchronized or coherent.

## ISB demodulation.

ISB wave is pass through a synchronous detector BPF follows synchronous detector which selects a band which is identical spectrum.

Then the SSB filters are used to generate the individual side bands of the message.

## PLL as AM demodulator




- PLL consists of multiplier, voltage controlled oscillator and LPF.
- Phase of VCO is controlled by the output voltage of LPF (centre voltage)
- The output of multiplier will be

$$v_1(t) = v_{in}(t) \cdot v_{vco}(t)$$

$$= A_c \cos(2\pi f_c t + \phi_c) \cdot A \sin(2\pi f_c t + \theta)$$

$$= \frac{A \cdot A_c}{2} \left\{ \sin(4\pi f_c t + (\theta + \theta_c) \right\} + \sin(\theta - \theta_c)$$

- The output of LPF will be

$$e_f(t) = \frac{A \cdot A_c}{2} \sin(\theta - \theta_c)$$

when phase difference is small

$$e_f(t) = \frac{A \cdot A_c}{2} (\theta - \theta_c)$$

- Control voltage applied to VCO is proportional to phase difference between two signals.
- When two signals are in same phase ie $\theta = \theta_c$

$$e_f(t) = 0 \text{ — (min)}$$

- When two signals are in quadrature phase ie $\theta - \theta_c = \pm \pi/2$ then

$$e_f(t) = \frac{A \cdot A_c}{2} \text{ — (max)}$$

- The output of VCO will be the exact replica of the input for constant phase difference of 90°.
- If the input equals AM signal with full carrier component ; the PLL will track the carrier component and VCO output will be single tone carrier signal.

PLL used for coherent detection of AM.

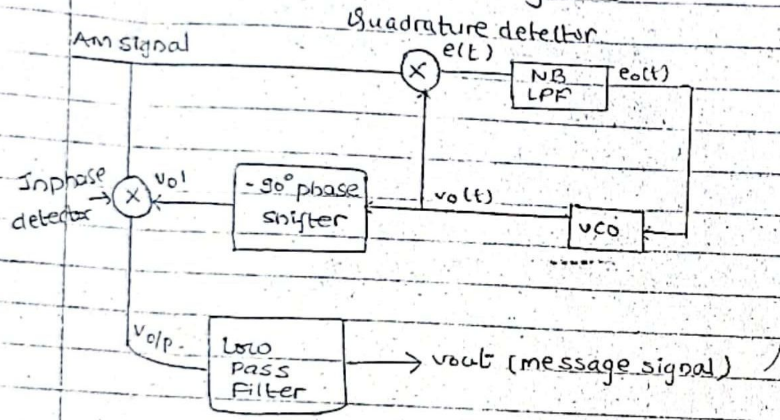



Fig: PLL used for coherent detection of AM.

$$e(t) = A_c\,(1+m(t))\sin(\omega_c t+\theta_i)\cdot\cos(\omega_c t+\theta_o)$$

$$= \{A_c + A_c m(t)\}\cdot \tfrac{1}{2}\{\sin(2\omega_c+\theta_i+\theta_o)+\sin(\theta_i-\theta_o)\}$$

$$= \frac{A_c}{2}\sin\{2\omega_c t+\theta_i+\theta_o\} + \frac{A_c}{2}\sin(\theta_i-\theta_o) +$$

$$\frac{A_c}{2}m(t)\sin(2\omega_c t+\theta_i+\theta_o) + \frac{A_c}{2}m(t)\sin(\theta_i-\theta_o)$$

* Narrow band low pass filter only allows to pass signal centred around origin so,

$$e_o(t) = \frac{A_c}{2}\sin(\theta_i-\theta_o)$$

$$= \frac{A_c}{2}\sin\theta_e$$

where;

$\theta_e = \theta_i - \theta_o$.

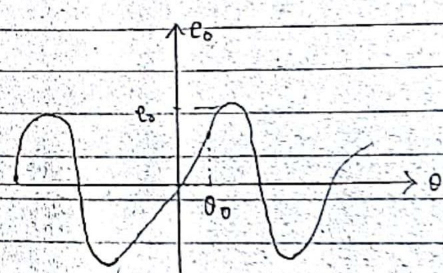



Fig:- Typical operating point 'a' and the corresponding value of $e_o$ and $\theta_e$ on ($e_o$ vs $\theta_e$) plot

Suppose input sinusoidal frequency suddenly increases from $\omega_c$ to $\omega_c+k$. means incoming signal is

$$A_c(1+m(t))\sin(\omega_c t+kt+\theta_i) = A_c(1+m(t))\sin(\omega_c t+\hat{\theta}_i)$$

Thus increase in incoming frequency causes $\theta_i$ to increase to $\theta_i+kt$ thereby incoming $\theta_e$.

The operating point a now shifts upwards along the $e_o$ vs $\theta_e$ characteristics. This increases $e_o$ which in turn increases the frequency of VCO output to match the increase in the input frequency. Similar reason is applicable for decrease in input sinusoidal frequency.

Once the PLL is in lock then;

$$v_o' = \sin(w_c t + \theta_i)$$

or, $$v_{o/p} = A_c(1+m(t))\sin(w_c t+\theta_i)\sin(w_c t+\theta_i)$$

$$= A_c(1+m(t))\left(\frac{1-\cos 2(w_c t+\theta_i)}{2}\right)$$

$$= \frac{A_c}{2} + \frac{A_c}{2}m(t) - \frac{A_c}{2}\left\{2(w_c t+\theta_i)\right\} - \frac{A_c}{2}m(t)\cos\left\{2(w_c t+\theta_i)\right\}$$

after passing through the low pass filter.

$$V_{out}(t) = \frac{A_c}{2} + \frac{A_c}{2}m(t).$$

demodulation of SSB using carrier reinsertion.

consider a SSB signals with an additional carrier (SSB+C)

$$S_{SSB+C} = A\cos w_c t + S_{SSB}$$

$$= A\cos w_c t + \left\{m(t)\cos w_c t + \hat{m}(t)\sin w_c t\right\}$$

although $m(t)$ can be recovered by synchronous detection (multiplying $S_{SSB+C}$ by $\cos w_c t$) if $A$, amplitude is large enough $m(t)$ can also be recovered from $S_{SSB+C}$ by envelop detection.

$$S_{SSB+C} = \left\{A+m(t)\right\}\cos w_c t + \hat{m}(t)\sin w_c t$$

$$= E(t)\cos(w_c t+\theta) \quad \text{-----(a)}$$

where; $E(t)$ is the envelope of $S_{SSB+C}$

$$E(t) = \left\{(A+m(t))^2 + \hat{m}^2(t)\right\}^{1/2}$$

$$= A\left\{1 + \frac{2m(t)}{A} + \frac{m^2(t)}{A^2} + \frac{\hat{m}^2(t)}{A^2}\right\}^{1/2}$$

* If $A \gg |m(t)|$ then, in general $A \gg |\hat{m}(t)|$ so,

$$E(t) = A\left\{1 + \frac{2m(t)}{A}\right\}^{1/2}$$

using binomial expansion.

$$E(t) = A\left(1 + \frac{m(t)}{A}\right)$$

$$= A + m(t).$$

In Am ; envelope detection require $A \geq |m(t)|$
In SSB+C " " " $A >> |m(t)|$ so
higher power is required for SSB+c hence
efficiency of SSB+c is low.

## Carrier recovery circuits.

Two methods of carrier regeneration at the receiver in DSB-SC
(i) signal squaring.
(ii) costa's loop.

## Signal squaring method.

The incoming signal is squared and then passed through a narrow bandpass filter tuned to $2\omega_c$. The output of this filter is the sinusoid $k\cos 2\omega_c t$, with some residual unwanted signal. This signal is applied to PLL to obtain a cleaner sinusoid of twice the carrier frequency which is passed through a 2:1 frequency divider to obtain a local carrier.

carrier in phase and frequency synchronism with the incoming carrier.

Phase locked loop.

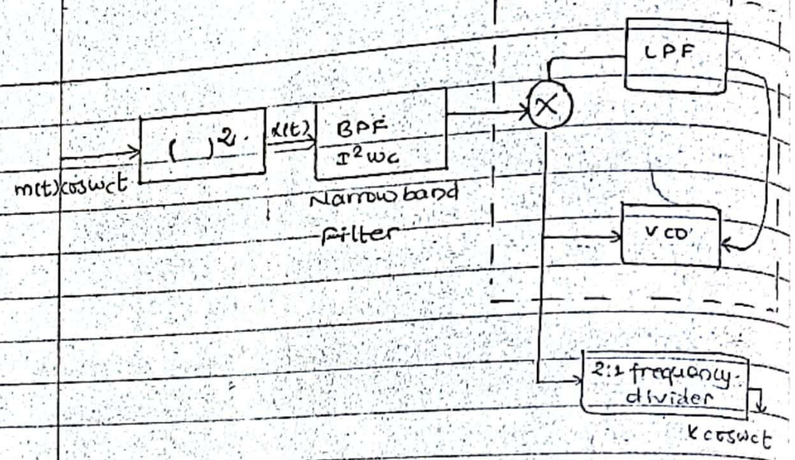



Fig:- squaring ckt for synchronization squarer output.

$$x(t) = \{m(t)\cos\omega_c t\}^2$$

$$= \tfrac{1}{2} m^2(t) + \tfrac{1}{2} m^2(t)\cos 2\omega_c t$$

Let dc component of $\frac{m^2(t)}{2}$ be $k$ thus $\frac{m^2(t)}{2}$ can be expressed as

$$\frac{1}{2} m^2(t) = k + \phi(t)$$

$$x(t) = \tfrac{1}{2} m^2(t) + \tfrac{1}{2} m^2(t)\cos 2\omega_c t$$

$$= \tfrac{1}{2} m^2(t) + k\cos 2\omega_c t + \phi(t)\cos 2\omega_c t.$$

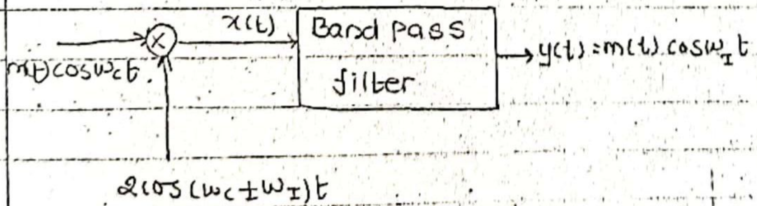
◆ Narrow band filter completely suppresses the signal $m^2(t)$ although $\phi(t)\cos 2\omega_c t$ is entered at $2\omega_c$, it has zero power at $2\omega_c$ so very little of this signal passes through the NB filters.

◆ Spectrum of $k\cos\omega_c t$ consists of impulse located at $\pm 2\omega_c$. Thus filter output is $k\cos 2\omega_c t$ plus small undesired residue from $\phi(t)\cos 2\omega_c t$. This residue can be suppressed by using PLL which tracks $k\cos\omega_c t$

## Frequency converter

In frequency converters the carrier frequency at the modulated signal $m(t)\cos\omega_c t$ from $\omega_c$ changes to some other frequency $\omega_I$.

Let $m(t)\cos\omega_c t$ be modulated or RF signal & $2\cos(\omega_c \pm \omega_I)t$ be the signal generated from local oscillator.




Then;

$x(t) = 2m(t)\cos\omega_c t + \cos(\omega_c+\omega_I)t$

$= m(t).\{\omega_c-(\omega_c\pm\omega_I)t + \cos(\omega_c+(\omega_c\pm\omega_I))t\}$

$= m(t)\cos\omega_I t + m(t)\cos(2\omega_c \pm \omega_I)t$

* The bandpass filter tuned to $\omega_I$ will pass the term $m(t)\cos\omega_I t$ and supress other term

$\therefore y(t) = m(t)\cos\omega_I t$

## Armstrong's super heterodyne AM receiver

In superheterodyne receiver the incoming RF signal is combined with local oscillator signal frequency through a mixed and converted into signal of lower fixed frequency known as intermediate frequency.

It consists of an,
- RF section
- frequency converter.
- IF amplifier.
- detector
- audio amplifier.

RF section.

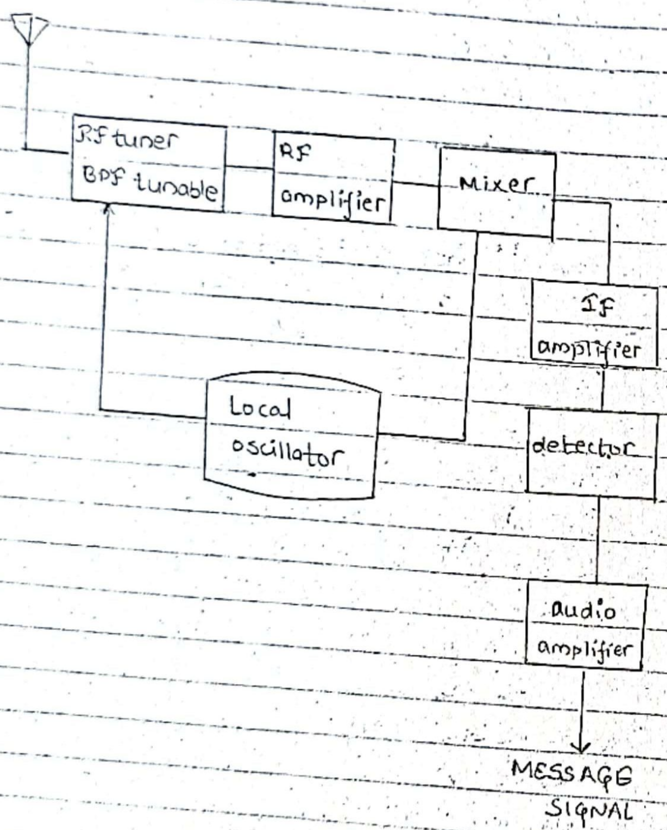



Fig: Block diagram of Heterodyne AM receiver

* RF section.

RF section or system mainly consists of a tunable filter and an amplifier that picks up the the desired station by tuning the filter to exact frequency band.

- The signal at the antenna has lower signal noise found anywhere in the receiver.
- Then, RF amplifier provides gain to increase Signal to noise ratio (SNR).

* Frequency converter.

It converts the carrier frequency $f_c$ to a fixed IF frequency of 455 KHz.

- A constant frequency difference is maintained between the local oscillator signal and frequency and incoming RF signal frequency. (Though capacitor tuning in which the capacitance are together and operated by a common control knob).

- For this purpose it uses a local oscillator whose frequency $f_{co}$ is exactly 455 KHz Hz above the incoming carrier frequency $f_c$

$$f_{co} = f_c + 455 \text{ ------ (i)}$$



R

## IF amplifier

The intermediate frequency generated from the mixer / converter is amplified by IF amplifier after the IF amplifier the signal is applied at the demodulator which extract the original modulated signal ie audio signal

- This signal is further amplified by a power amplifier to get a amplified to get a specified power level so that it may activate the loudspeakers.

* The reason for translating all stations to a fixed carrier frequency of 455 MHz is to obtain adequate selectivity. In other word since the characteristics of the IF amplifier are independent of the incoming frequency to which the receiver is tuned. The selectivity and sensitivity of superheterodyne receiver are quite uniform throughout it tuning range.

* The main function of the RF section is image frequency suppression. The mixer or converter output consists of components of difference between the incoming ($f_c$) and the local oscillator ($f_{co}$)

$$f_{IF} = |f_{co} - f_c|$$

Now if the incoming carrier frequency $f_c = 1000$ KHz then $f_{co} = 1000 + 455 = 1455$ KHz

but another carrier with

$$f_c' = 1455 + 455$$
$$= 1910 \text{ KHz}$$

will also be ~~pitched~~ picked up because the difference $f_c'$ and $f_{co}$ ie $f_c' - f_{co}$ is also 455 KHz if it were not the RF filter at receiver input.

## Chapter 5: Frequency modulation.

In angle modulation, phase & frequency of carrier wave is varied in accordance to modulating signal or message signal.

- amplitude of the carier is kept constant.
- non - linear modulation.
- angle modulated signal can be represented by

$$s(t) = A_c \cos[\theta(t)].$$

### instantaneous phase

The instantaneous phase of angle modulated signal is

$$\theta_i(t) = 2\pi f_c t + \phi(t).$$

phase deviation.

$\phi(t) / \theta_i(t)$ is varied according to instantaneous value of message signal.

### Instantaneous frequency.

The instantaneous frequency is the derivative of the instantaneous phase.

$$\omega_i(t) = \frac{d\theta_i(t)}{dt}$$

$$= \frac{d}{dt}\left[2\pi f_i t + \phi(t)\right]$$

or, $w_i(t) = w_c + \frac{d\phi(t)}{dt}$

↑ frequency daviation (rad/sec)

where $\frac{d\phi(t)}{dt}$ is called frequency daviation.

alternate derivation

If $\theta(t)$ increases monotonically with time; the average frequency in Hertz is given.

$$f_{\Delta t} = \frac{\theta(\Delta t + t) - \theta(t)}{2\pi \Delta t}.$$

Then;
Instantaneous frequency is given by

$$f_i(t) = \lim_{\Delta t \to 0} f_{\Delta t}(t).$$

$$= \lim_{\Delta t \to 0} \frac{\theta(t+\Delta t) - \theta(t)}{2\pi \Delta t}$$

$$= 1/2\pi \frac{d\theta}{dt}$$

also, we have.

$$\theta = 2\pi \int f_i(t)\, dt.$$

Phase modulation (PM)

Phase modulation is the form of angular modulation in which phase variation $\theta(t)$ is varied linearly with the message signal $m(t)$

ie

$$\theta(t) = 2\pi f_c t + K_p m(t) \quad \cdots\cdots (*)$$

here:

$K_p$ = phase sensitivity of the modulator or phase deviation constant (rad/volt)

Then;

$$S_{pm}(t) = A_c \cos\left\{2\pi f_c t + K_p m(t)\right\}.$$

Frequency modulation

Frequency modulation is the form of angle modulation where the instantaneous frequency $f_i(t)$ is varied linearly with message signal $m(t)$.

ie $f_i(t) = f_c + K_f\, m(t). \quad \cdots\cdots (*)$

where;

$f_c$ = frequency of carrier.

$K_f$ = frequency sensitivity of modulator.

we know;

$$w(\theta) = \frac{d\theta(t)}{dt}$$

or, $$2\pi f_i(t) = \frac{d\theta(t)}{dt}.$$

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or, $\theta(t) = 2\pi \int f_i(t)\, dt$

or, $\theta(t) = 2\pi \int_0^t [f_c + k_f m(t)]\, dt$

or, $\theta(t) = 2\pi f_c t + 2\pi k_f \int_0^t m(t)\, dt.$

Then,

Frequency modulated wave can be represented as

$S_{FM}(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(t)\,dt\right]$

Relation between FM and PM.

angle modulated wave can be represented by;

$s(t) = A_c \cos[\theta(t)]$ ------- (i)

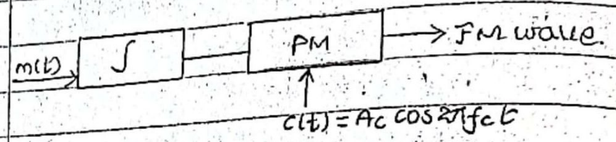
$S_{PM}(t) = A_c \cos[2\pi f_c t + k_p m(t)]$ ----- (ii)

$S_{FM}(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(t)\,dt\right]$ ...... (iii)

In P.M. phase angle varies linearly with $m(t)$

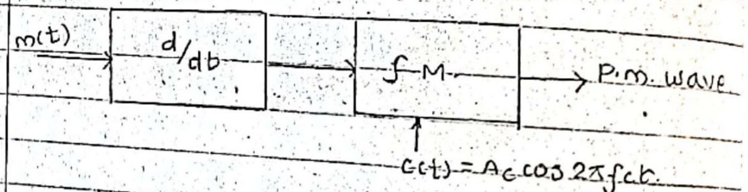
In FM, phase angle varies linearly with $\int_0^t m(t)\,dt$.

FM wave is regarded as PM wave in which modulating signal $\int_0^t m(t)\, dt$ in place $m(t)$. This means that FM wave can be generated by first integrating $m(t)$ and then using the result $(\int_0^t m(t)\,dt)$ as the input to the phase modulator.



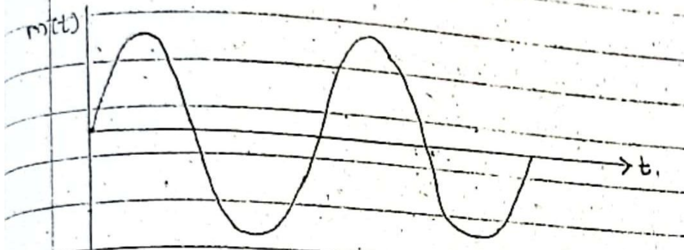

Fig(a): Scheme of generating FM wave by using phase modulation

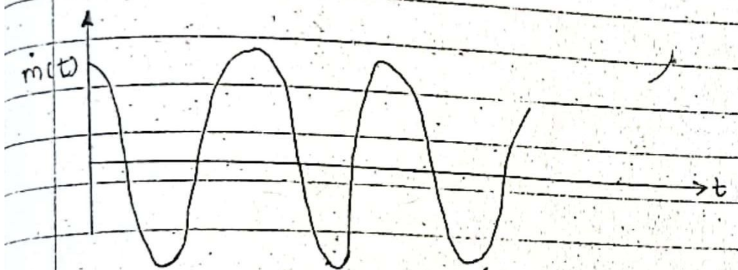
PM wave can be regarded as FM wave in which the modulating signal is the derivative of $m(t)$. $[\dot{m}(t)]$ in place of $m(t)$. This means that PM wave can be generated by first differentiating $m(t)$ and then using this result as the input to frequency modulator.

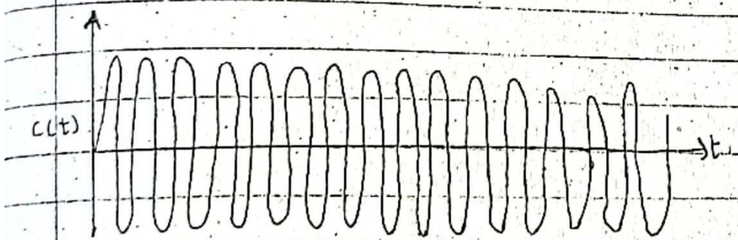


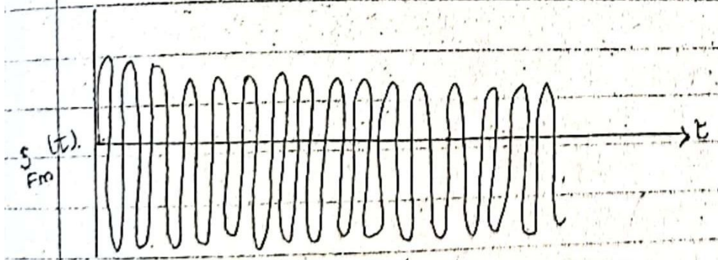

Fig(b): Scheme of generating PM wave by using frequency modulator.

## Sinusoidal Modulation

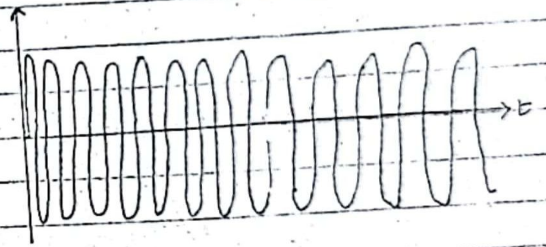


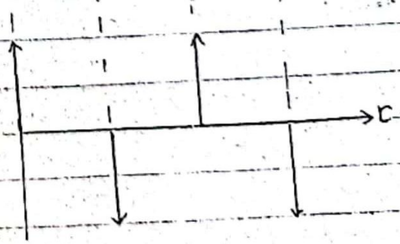



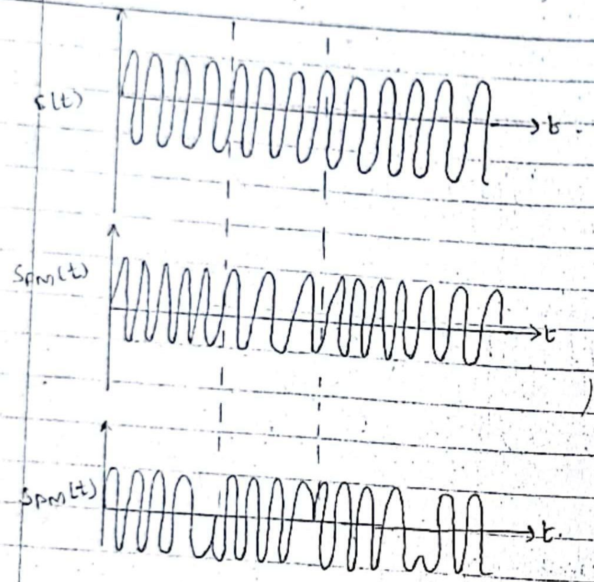




By looking at angle modulated carrier wave there is no way to tell whether it is FM or PM. Distinction between FM and PM wave can be made only by comparing with actual modulating waves.




## SQUARE MODULATION




## Single tone frequency modulation

Consider sinusoidal modulating wave defined by.

$m(t) = A_m \cos(2\pi f_m t)$.

The instantaneous frequency of modulated wave is given by;

$f_i(t) = f_c + k_f\, m(t)$

$f_i(t) = f_c + k_f A_m \cos(2\pi f_m t)$

$f_i(t) = f_c + \Delta f \cos(2\pi f_m t)$.

where;

$\Delta f = k_f A_m$ is called frequency deviation.

- A fundamental characteristics of FM wave is that frequency deviation is proportional to the amplitude of modulating wave and is independent of modulating frequency.

The argument angle $\theta(t)$ of FM is given.

$$\theta(t) = 2\pi \int_0^t f_i(t)\, dt.$$

$$= 2\pi \int_0^t (f_c + \Delta f \cos(2\pi f_m t))\, dt.$$

$$= 2\pi f_c t + \Delta f \cdot 2\pi \frac{\sin 2\pi f_m t}{2\pi f_m}.$$

$$= 2\pi f_c t + \frac{\Delta f}{f_m} \sin 2\pi f_m t.$$

The ratio of frequency deviation to the modulation frequency $f_m$ is called modulation index of FM wave. It is denoted by $\beta$

$$\therefore \beta = \Delta f / f_m.$$

$$\theta(t) = 2\pi f_c t + \beta \sin(2\pi f_m t)$$

Then,

$S_{FM}(t) = A_c \cos[2\pi f_c t + \beta \sin(2\pi f_m t)]$

Spectral analysis of Sinusoidal FM wave.

The fm wave for sinusoidal modulation is given by;

$$s(t) = A_c \cos[2\pi f_c t + \beta \sin(2\pi f_m t)] \quad \text{------(i)}$$

expanding this equation using trigonometric identity

$$s(t) = A_c \cos(2\pi f_c t)\cdot \cos[\beta \sin(2\pi f_m t)] - A_c \sin(2\pi f_c t).$$

$$\sin[\beta \sin(2\pi f_m t)] \quad \text{------(ii)}$$

From this expanded form we see that inphase and quadrature components of FM wave s(t) given as;

$$s_I(t) = A_c \cos[\beta \sin(2\pi f_m t)] \quad \text{------(iii)}$$

$$s_Q(t) = A_c \sin[\beta \sin(2\pi f_m t)] \quad \text{------(v)}$$

Hence, the complex envelope of FM wave is given by;

$$\hat{s}(t) = s_I(t) + j s_Q(t).$$

$$= A_c \cos[\beta \sin(2\pi f_m t)] + j A_c \sin[\beta \sin(2\pi f_m t)]$$

$$= A_c e^{j\beta \sin(2\pi f_m t)}$$

$$= A_c \exp(j\beta \sin(2\pi f_m t)) \quad \text{------(v)}$$

The complex envelope $\hat{s}(t)$ retains all the information related to modulation process indeed we may readily express FM wave s(t) in terms of complex envelope $\hat{s}(t)$.

$$s(t) = Re[A_c \exp(j2\pi f_c t + j\beta \sin(2\pi f_m t))]$$

$$= Re[A_c \exp(j2\pi f_c t)\cdot \exp(j\beta(2\pi f_m t))]$$

$$= Re[A_c \exp(j\beta \sin(2\pi f_m t)).\exp(j2\pi f_c t)]$$

$$= Re[\hat{s}(t) \exp(j2\pi f_c t)] \quad \text{------(vi)}$$

From eqn we visualize that the complex envelope $\hat{s}(t)$ is periodic function, with fundamental frequency equal to modulating frequency $f_m$.

We therefore express $\hat{s}(t)$ in complex fourier series as;

$$\tilde{s}(t) = \sum_{n=-\infty}^{\infty} c_n \exp(j2\pi n f_m t) \quad \text{....(vii)}$$

where;

$$c_n = f_m \int_{-\frac{1}{2}f_m}^{\frac{1}{2}f_m} \tilde{s}(t)\exp(-j2\pi n f_m t)\,dt$$

or, $$c_n = f_m \int_{-\frac{1}{2}f_m}^{\frac{1}{2}f_m} A_c \exp(j\beta\sin(2\pi f_m t)).\exp(-j2\pi n f_m t)\,dt$$

or, $$c_n = A_c f_m \int_{-\frac{1}{2}f_m}^{\frac{1}{2}f_m} \exp[j\beta\sin(2\pi f_m t) - j2\pi n f_m t]\,dt$$

Let us take

$x = 2\pi f_m t$. ; when $t = -\frac{1}{2}f_m$ $x = -\pi$

$dx = 2\pi f_m dt$. ; when $t = \frac{1}{2}f_m$ $x = \pi$

$dt = \frac{dx}{2\pi f_m}$

Now,

$$c_n = \frac{A_c}{2\pi}\int_{-\pi}^{\pi} \exp[j\beta\sin x - jnx]\,dx$$

$$= A_c\left[\frac{1}{2\pi}\int_{-\pi}^{\pi} \exp(\beta\sin x - nx)\,dx\right] \quad \text{------(viii)}$$

The integral on right hand side is recognized as the nth order Bessel function of the first kind and argument $\beta$.

It is denoted by symbol $J_n(\beta)$

$$J_n(\beta) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \exp\{j(\beta\sin x - nx)\}\,dx \quad \text{------(ix).}$$

From (viii) and (ix) we get

$$c_n = A_c J_n(\beta) \quad \text{------(x)}$$

Substituting eqn (x) in (vii) the complex envelope $\tilde{s}(t)$ is given by

$$\tilde{s}(t) = \sum_{n=-\infty}^{\infty} A_c J_n(\beta).\exp(j2\pi n f_m t) \quad \text{----(xi)}$$

now substituting eqn (xi) in (vi) we get

$$s(t) = Re\left\{\sum_{n=-\infty}^{\infty} A_c J_n(\beta).\exp(j2\pi n f_m t).\exp(j2\pi f_c t)\right\}$$

$$s(t) = Re\left\{\sum_{n=-\infty}^{\infty} A_c J_n(\beta).\exp(j2\pi(f_c + n f_m)t)\right\} \quad \text{---(xii)}$$

Interchanging the order of summation and evaluating the real part of eqn (xii) we get

$$s(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cdot \cos\left[2\pi(f_c + nf_m)t\right] \quad \text{....(xiii)}.$$

This is the desired form of fourierseries representation of the single tone FM wave s(t) for an arbitary value of β. The discrete spectrum of s(t) is obtained by taking fourier transform of eqn (xiii) we get.

$$S(f) = \frac{A_c}{2} \sum_{n=-\infty}^{\infty} J_n(\beta) \left\{ \delta(f - f_c - nf_0) + \delta(f + f_c + nf_0) \right\} \quad \text{(xiv)}$$

## Properties of Bessel function ($J_n(\beta)$)

(i) For fixed β, $J_n(\beta) = \begin{cases} J_n(\beta) & ; n = \text{even} \\ -J_n(\beta) & ; n = \text{odd}. \end{cases}$

The plot of $J_n(\beta)$ versus modulation index β for n = 0, 1, 2, 3, 4. These plots show that for fixed n. $J_n(\beta)$ alternates between +ve & -ve values increasing β and $|J_n(\beta)| \to 0$ as $\beta \to \infty$

(2) $\sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1.$

(3) $J_{n-1}(\beta) + J_{n+1}(\beta) = \frac{2n}{\beta} J_n(\beta)$ — recurrance formula.

(4) $J_n(\beta) = (-1)^n J_n(-\beta)$

(5) for small β.

$$J_n(\beta) = \frac{\beta^n}{2^n n!}$$

also,

average power in FM $(P) = \frac{A_c^2 \sum_{n=-\infty}^{\infty} J_n^2(\beta)}{2} = \frac{A_c^2}{2}.$

## Types of fm.

### (i) Narrow band fm (NBFM).

- For smaller value of modulation index ($\beta$) compared to one radian, the fm wave is assumed to be narrow band form consisting carrier, a upper side frequency and lower side frequency.

for $\beta \leq 1$

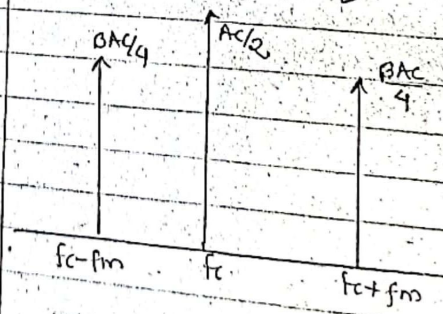
$J_0(\beta) \simeq 1$

$J_1(\beta) \simeq \beta/2$

$J_n(\beta) \simeq 0 \quad , n > 1.$

- The expression for NBFM is given by.

$$S_{NBFM}(t) = A_c \cos 2\pi f_c t + \frac{\beta A_c}{2} \cos\left\{2\pi(f_c + f_m)t\right\} - \frac{\beta A_c}{2} \cos\left\{2\pi(f_c - f_m)t\right\}.$$



ii) Wideband FM.

- For large value of modulation index (β) compared to one radian, the FM wave contains a carrier & an infinite no of side-frequency components located symmetrically about the carrier.

- The expression for WBFM is given by.

$$S_{WBFM}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos\{2\pi(f_c + nf_m)t\}$$

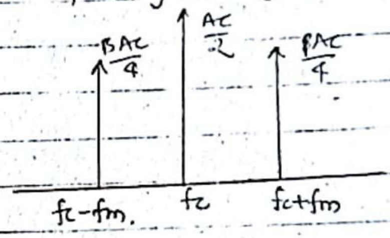
where,

$J_n(\beta)$ = Bessel function.

Transmission bandwidth of FM.

Theoretically FM wave has infinite bandwidth, but in practice FM wave is limited to finite no of significant side frequencies with specified amount of distortion. In FM wave the side frequencies are separated from carrier ($f_c$) by an amount greater than the frequency deviation $\Delta f$, decreases rapidly towards zero.

- For small value of modulation index β, the spectrum of FM wave is limited to the carrier frequency and one pair of side frequencies at $f_c \pm f_m$




So,

The bandwidth = $(f_c + f_m) - (f_c - f_m)$

$= 2f_m$.

Transmission BW can be $(B.W) = 2n_{max} \cdot f_m$ ---(a)

where;

$f_m$ = modulated frequency.

$n_{max}$ = maximum no of integer 'n' that satisfies $|J_n(\beta)| > 0.01$.

The approximate bandwidth of FM is given by

$$B.W(B) \simeq 2\Delta f + 2f_m$$

$$= 2\Delta f\left(1 + \frac{f_m}{\Delta f}\right).$$

$= 2\Delta f \cdot (1 + 1/\beta)$

$= 2(\beta+1) \cdot f_m.$

This relation is called Carson's rule.

- Crason's rule understimates the transmission BW.
- The deviation ratio (D) is defined as the ratio of frequency deviation $\Delta f$ to the highest modulation frequency (ω).
- For sinusoidal modulation.

$$D = \frac{\text{frequency deviation}}{\text{highest frequency of modulation}}$$

$$= \Delta f / \omega.$$

Then CARSON'S B.W. will be.

$$B.W = 2\Delta f(1 + 1/D).$$

## Generation of FM signals.

(i) Direct method

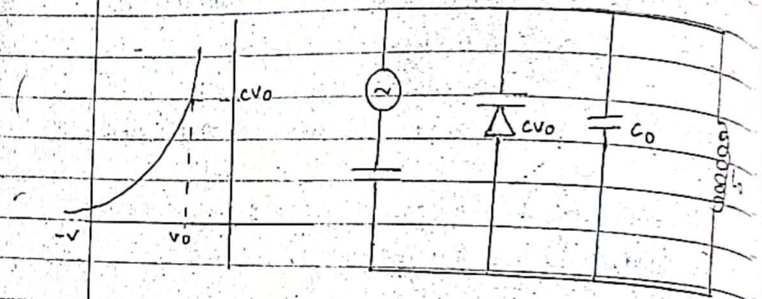



Fig (a). Fig b): Direct method of FM signal genⁿ

In direct method of FM generation the instantaneous frequency of carrier wave is varied directly in accordance with message signal m(t) by means of device known as voltage controlled oscillator (VCO). The simplest type of modulator is oscillator circuit that uses varactor diode in frequency determining section as shown in fig above.

→ Here Hartley oscillator is used.

Before applying m(t), the capacitance of varactor diode be CVo. Then, after applying m(t) the capacitance of the varactor diode will be.

$C_{VD} = C_{V0} + K_c\, m(t)$ ------(a)

where, $K_c$ = Sensitivity of varactor diode

The total capacitance of LC tank circuit will be

$C(t) = C_{fixed} - K_c\, m(t)$ -------(i)

where

$C_{fixed}$ = equivalent cap of $C, C_0$ & $C_{V0}$

for $m(t)=0$, $C(t) = C_{fixed}$ and the frequency of oscillation is the carrier frequency and is given by

$$f_c = \frac{1}{2\pi\sqrt{L_0 C_{fixed}}} \text{ ------(ii)}$$

but,

after applying $m(t)$, the instantaneous frequency will be.

$$f_i(t) = \frac{1}{2\pi\sqrt{L_0(C_{fixed} - K_c m(t))}}$$

or, $$f_i(t) = \frac{1}{2\pi\sqrt{L_0 C_{fixed}\left(1 - \frac{K_c}{C_{fixed}} m(t)\right)}}$$

or, $$f_i(t) = \frac{1}{2\pi\sqrt{L_0 C_{fixed}}}\left(1 - \frac{K_c\, m(t)}{C_{fixed}}\right)^{-1/2}$$

or, $$f_i(t) = f_c\left[1 + \frac{K_c}{2C_{fixed}} \cdot m(t)\right]$$

or, $$f_i(t) = f_c + \frac{K_c f_c}{2C_{fixed}} m(t).$$

or, $f_i(t) = f_c + K_f\, m(t)$

where $K_f = \frac{K_c f_c}{2C_{fixed}}$

which is the basic relation for FM wave.

disadvantage.

1. Non suitable for generation of WBFM.
2. difficult to get higher order stability in carrier frequency.

(ii) Indirect method.

a) Generation of NBFM:

The general equation of FM wave is given by

$$S_{FM} = A_c \cos\left\{2\pi f_c t + 2\pi K_f \int_0^t m(t)\, dt\right\} \text{ ------ (i)}$$

Representing eqn (i) in phasor form we get

$$C_{FM}(t) = A_c e^{j\{2\pi f_c t + 2\pi k_f \int_0^t m(t)\,dt\}} \quad \text{------(ii)}$$

let $\int_0^t m(t)\,dt = Y(t)$

Then eqn (ii) becomes

$$C_{FM}(t) = A_c e^{j(2\pi f_c t + 2\pi k_f Y(t))} \quad \text{-----(iii)}$$

- for narrowband fm
  $|k_f Y(t)| \leq 1.$

expressing $e^{j 2\pi k_f Y(t)}$ in power series form.

$$e^{j2\pi k_f Y(t)} = 1 + j\, 2\pi k_f Y(t) \qquad \left\{ e^x = 1 + x + \frac{x^2}{2!} + \cdots \right.$$

- Then eqn (iii) becomes.

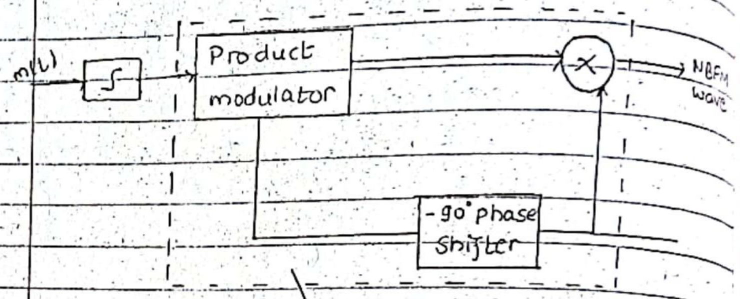
$$C_{FM}(t) = A_c \left\{ 1 + j\, 2\pi k_f Y(t) \right\} e^{j 2\pi f_c t}.$$

Since FM signal is real part of its phasor representation so

$$S_{FM}(t) = Re\,(C_{FM}(t)).$$

or, $S_{FM}(t) = A_c \cos 2\pi f_c t - 2\pi k_f A_c Y(t) \sin 2\pi f_c t.$

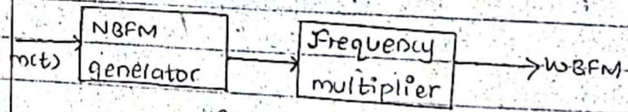
$$\therefore S_{FM}(t) = A_c \cos 2\pi f_c t - 2\pi k_f A_c \left( \int_0^t m(t)\,dt \right) \sin 2\pi f_c t.$$




Narrow band phase modulator.

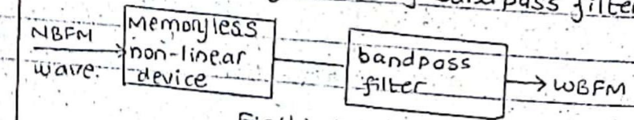
b) Generation of wide band FM

In this method first the narrowband FM is produced. This NBFM is passed to frequency multiplier to generate WBFM.

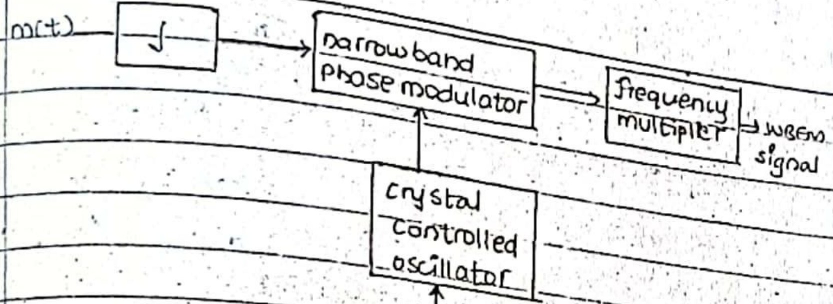



Fig(a)

- The frequency multiplier is nothing but memoryless non linear device followed by band pass filter.




Fig(b)




It is used as source of carrier for frequency stability.

The input-output relationship of non-linear device is given by.

$$s_2(t) = a_1 s_1(t) + a_2 s_1^2(t) - - - + a_n s_1^n(t) \quad ----(iii)$$

where $a_1, a_2 \ldots a_n$ are constant coff.

If we substitute the expression of NBFM as $s_1(t)$ in eqn (iii) we see that the output of the device will have frequency modulated waves with carrier frequencies $f_1, 2f_1 \ldots nf_1$ and frequency deviations $\Delta f_1, 2\Delta f_1 \ldots n\Delta f_1$ respectively.

The use of band pass filter is designed with two aims.

(i) To pass the FM signal centred at carrier frequency $nf_1$ & frequency deviation $n\Delta f_1$.

(ii) To supress all other F.M spectra.

Thus by passing NBFM wave through a frequency multiplier we can generate WBFM wave with carrier frequency $f_c = nf_1$ and frequency deviation $\Delta f = nf_1$.

Two stage WBFM Modulator. (Numerical asked).

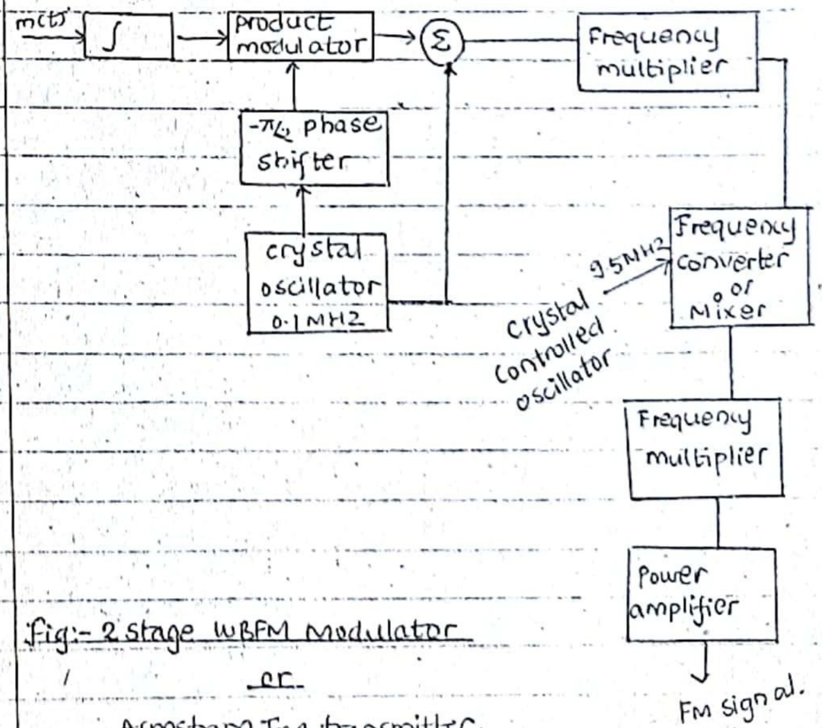
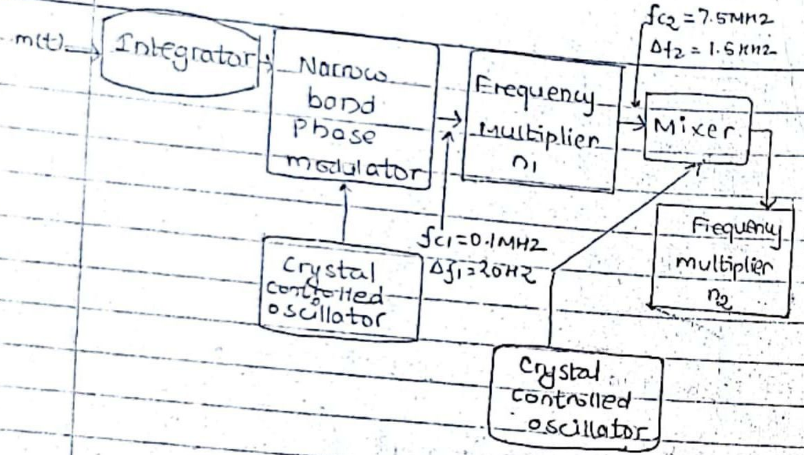



Fig:- 2 stage WBFM Modulator
or
Armstrong FM transmitter




The need for two stage WBFM signal is best explained through examples. Let us consider a FM transmitter which needs to transmit audio signals containing frequencies in the range of 100 Hz to 15 KHz. Let us consider that the narrow band phase modulator is supplied with the carrier signal of $f_1 = 0.1 MHz$ by a crystal controlled oscillator. The desired FM wave at the transmitter output has carrier frequency $f_{c4} = 100 MHz$ and frequency deviation $\Delta f_4 = 75 KHz$.

# For NBFM $\beta$ has to small ($\beta \leq 0.3$). here we consider $\beta = 0.2$, the 100 Hz frequency will limit the frequency $\Delta f_1 = 20Hz$ $\left(\beta = \frac{\Delta f}{f_m}\right)$.

# To produce a frequency deviation of $\Delta f_4 = 75$ KHz at the output of fm transmitter we need total frequency multiplication of $\frac{75000}{20} = 3750$.

# but the use of this multiplier will also shift a carrier to a much higher frequency than required value of 100 MHz. Therefore we need to use a 2 stage frequency multiplier with an intermediate stage of frequency mixer.

# let $n_1$ and $n_2$ be the respective frequency multiplier factor so

$$n_1 \cdot n_2 = \frac{\Delta f_4}{\Delta f_1} = \frac{75000}{20} = 3750 \quad \text{---(i)}$$

* If $f_2$ is the signal generated by 2nd crystal oscillator, then the output frequency of mixer be

$f_2 = n_1 f_1$

however,

the carrier frequency at the input of 2nd frequency multiplier is. $\frac{f_c}{n_2}$ equating these two frequencies we get.

$$(f_2 - n_1 f_L) = \frac{f_{c4}}{n_2}$$

or, $$9.5 - 0.1 n_1 = \frac{100}{n_2} \quad \text{------} \quad (ii)$$

Solving these two equations we get.

$n_1 = 75$

$n_2 = 50$.

using these multiplication factor we get values as.

| | at phase modulator o/p | 1st Freq multiplier o/p | Mixer o/p | 2nd Freq mult o/p. |
|---|---|---|---|---|
| carrier freq(fc) | 0.1MHz | 7.5MHz | 2.0MHz | 100 MHz |
| Freq devn(Δf). | 20Hz | 1.5KHz | 1.5KHz | 75 KHz |

**Demodulation of FM signals.**

(i) **Limiter discriminator method.**

The FM wave is given by

$$S_{FM}(t) = A_c \cos\left\{2\pi f_c t + 2\pi k_f \int_0^t m(t)\,dt\right\} \quad \text{-----} \quad (i)$$

The extraction of message signal from FM wave by this method involves 3 steps.

1. amplitude limiter
ii. discrimination (differentiator)
iii. envelope detector.

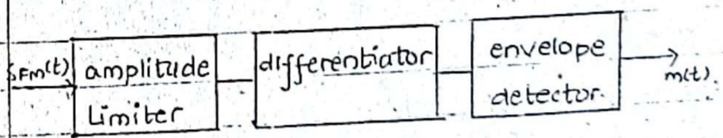



Fig: block diagram of Limiter discriminator.

i. amplitude limiter limits the amplitude of the signal to reduce the effect of fading & noise.

ii. The output of differentiator be.

$$\frac{d}{dt}\left[S_{FM}(t)\right] = \frac{d}{dt}\left[A_c \cos(2\pi f_c t + 2\pi k_f \int_0^t m(t)\,dt)\right]$$

$$= \frac{d}{dt}\left\{ A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(t)\,dt\right) \right\}$$
$$(2\pi f_c t + 2\pi k_f \int_0^t m(t)\,dt) \times \frac{d(2\pi f_c t + 2\pi k_f \int_0^t m(t)\,dt)}{dt}$$

$$= -A_c\left(2\pi f_c + 2\pi k_f m(t)\right) \cdot \sin\left(2\pi f_c t + 2\pi k_f \int_0^t m(t)\,dt\right)$$

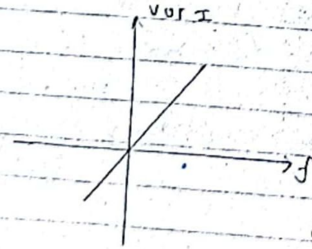
$$= -A_c\left(2\pi f_c + 2\pi k_f m(t)\right) \cdot \sin\left(2\pi f_c t + \phi_1(t)\right).$$

This is equivalent to standard AM signal. This process of differentiation in this case is known as FM to AM conversion. From this AM signal message m(t) can be recovered using envelope detector.

Implementation of differentiator.

→ Differentiator is a circuit that converts change in frequency into corresponding change in voltage or current.

→ It has following transfer characteristics.




$x(t) \xrightarrow{diff} Y(t)$

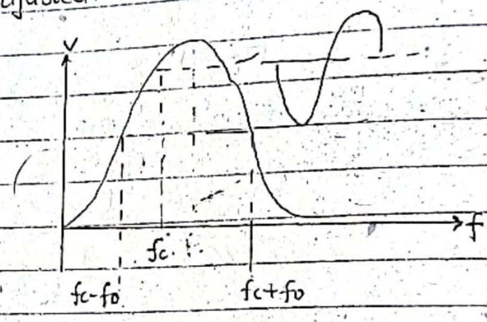
$y(t) \longrightarrow \frac{dx(t)}{dt}$

$y(f) = -j2\pi f\, x(t)$

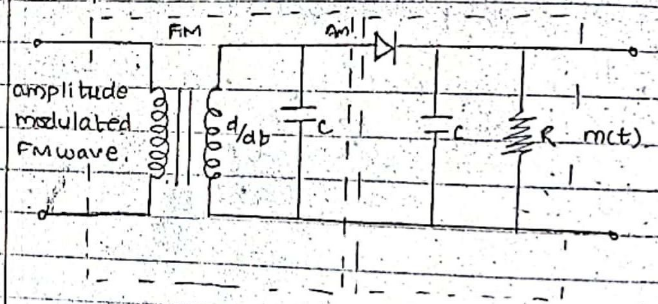
$|H(f)| = 2\pi f.$

Mathematically transfer function $H(f) = 2\pi f$.

a LC tank circuit tuned at frequency $f_c$ can be adjusted to have transfer function like below.




II). envelope detector.



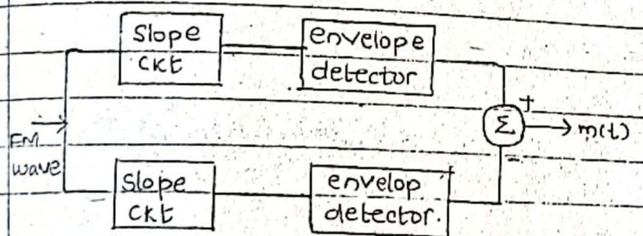

$S_d(t) = A_c\left\{2\pi f_c t \; 2\pi k_f m(t)\right\}$

or, $S_d(t) = A_c 2\pi f_c + A_c 2\pi k_f m(t)$

→ The drawback of this circuit is that it is not suitable to demodulate WBFM where peak frequency deviation is high.

→ The circuits non linear characteristics produces a harmonic distortion. The non linearity is obvious from the fact that slope is not same at each point of characteristics.

→ The better solution is the use of balance slope detector or ratio detector.



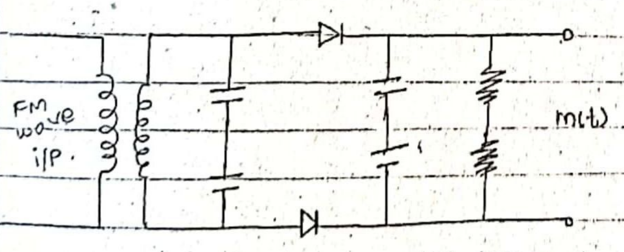
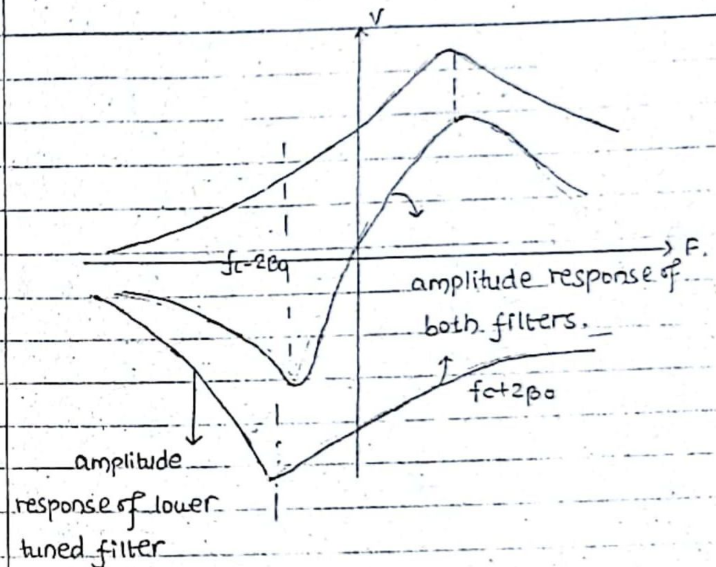




Fig: balance slope detector




as shown in the figure the combination of tuned circuits produce an acceptable linear region over which FM wave can be demodulated.

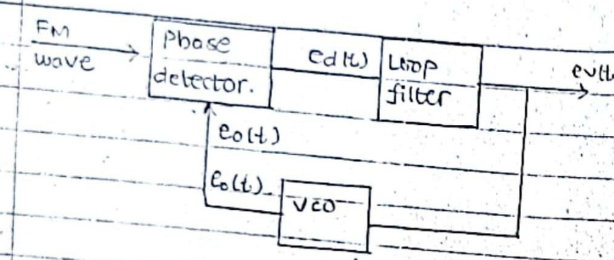
PLL as FM detector.

The PLL has 3 basic components.

(i) Phase detector.

(ii) loop filter (LPF)

(iii) VCO.




* We initially assume that control voltage to vco is zero

ie $e_v(t)=0$ which satisfies two conditions.

(i) Frequency of vco is precisely set to carrier frequency of incoming FM signal.

(ii) The vco output has 90° phase shift with respect to the unmodulated carrier wave.

→ Let the input signal applied to PLL be

$$S_{FM}(t) = A_c \cos\left[2\pi f_c t + \phi_1(t)\right] \quad \text{------(i)}$$

where,

$A_c$ = carrier amplitude

$$\phi_1(t) = 2\pi k_f \int_0^t m(t)\,dt.$$

Let, the vco output be

$$e_o(t) = A_v \sin\left[2\pi f_c t + \phi_2(t)\right] \quad \text{------(ii)}$$

here $\phi_2(t) = 2\pi K_v \int_0^t e_v(t)\,dt.$

$e_v(t)$ being control voltage applied to vco

$$e_d(t) = S_{FM}(t) * e_o(t).$$

$$= A_c A_v \cos(2\pi f_c t + \phi_1(t)) \cdot \sin(2\pi f_c t + \phi_2(t))$$

$$= \frac{A_c A_v}{2}\left\{\sin(4\pi f_c t + \phi_1(t) + \phi_2(t)) + \sin(\phi_2(t) - \phi_1(t))\right\}$$

→ here high frequency signal is supressed by loop filter. hence, input to the vco is given by.

$$e_v(t) = \frac{A_c \cdot A_v}{2} \sin\left\{\phi_2(t) - \phi_1(t)\right\}.$$

$$= \frac{A_c A_v}{2} \sin \phi_e(t). \quad \text{------(iii)}$$

where $\phi_e(t)$ = phase error.

When the PLL enters into the lock mode (ie phase & frequency match) the error voltage will be nearly equal to 0.

ie $e_v(t) \approx 0$

or, $\phi_e(t) \simeq 0$.

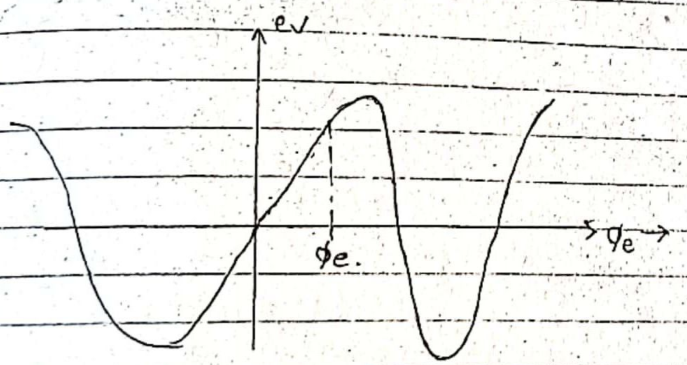
& $\phi_1(t) = \phi_2(t)$

or, $2K_f\pi \int_0^t m(t)\,dt = 2\pi K_v \int_0^t e_v(t)\,dt.$

or, $e_v(t) = \frac{K_f}{K_v} \cdot m(t).$

hence, the output of the PLL is nearly equal to the message signal. Hence PLL acts as demodulator of FM wave.

(•) PLL tracks the incoming frequency & phase.




Let us consider that incoming frequency suddenly increases from $f_c$ to $f_c + K$. Means the incoming signal becomes

$$s(t) = A_c \cos\left\{ 2\pi f_c t + 2\pi K t + \phi_1(t) \right\}$$

$$= A_c \cos\left\{ 2\pi f_c t + \phi_1'(t) \right\}.$$

From the curve we see that this increase in incoming phase results in increased phase error. The increase in phase error increases the control voltage which in turn increases the frequency of VCO output to match with the increase in input frequency.

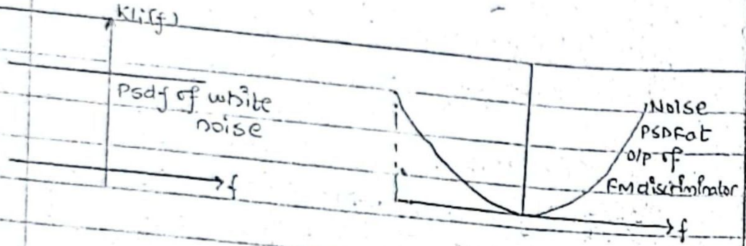
Uses of PLL.

(i) tracking the phase & frequency of incoming signal.
(ii) coherent detection of different types of signals.
(iii) demodulation under low SNR conditions.

Commercial FM Radio. (Preemphasis & deemphasis in broadcasting

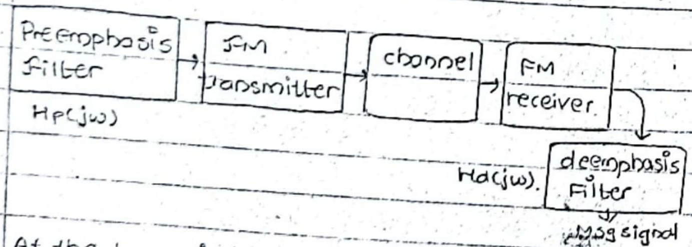
The following spectra shows the effect of passing white noise through FM detector.

$$S_{FM}(t) + N_i(t) \rightarrow \boxed{\text{FM detector}} \rightarrow m(t) + N_d(t).$$




It is seen that noise power is concentrated in the higher frequencies. But the PSDF of the message signal is concentrated at the lower frequencies. Thus noise psdf is concentrated strongly where message is weakest.

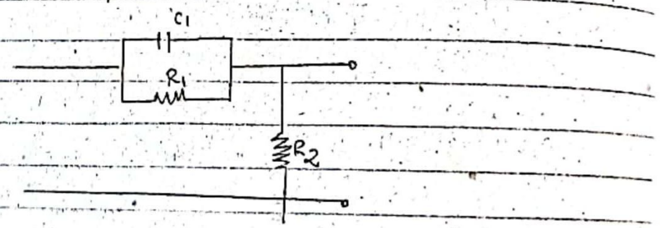
To compensate this effect the preemphasis circuit is used at the transmission end.




→ At the transmission end weaker high frequency components (beyond 2.1 kHz) of the audio signal m(t) are boosted before modulation by a pre-emphasis filter of transfer function Hp(jw). At the receiver the demodulator output is passed through a demphasis filter of transfer function $Hd(j\omega) = \frac{1}{Hp(j\omega)}$. The de-emphasis filter attenuates the high frequency components (beyond 2.1 kHz). The noise which enters at the channel & therefore has not been pre-emphasized (boosted). However as it passes through de-emphasis filter which attenuates higher frequencies components where most of the noise is concentrated. Thus the process of pre-emphasis & deemphasis leaves the desired signal untouched but reduces the noise power considerably.

- However broadcasting high frequency component of m(t) may increase its peak value which in turn increases frequency deviation & transmission bandwidth.

Pre-emphasis circuit




$$Hp(j\omega) = \frac{R_2}{(R_1+R_2)} \cdot \frac{(1+j\omega/\omega_1)}{(1+j\omega/\omega_2)}$$

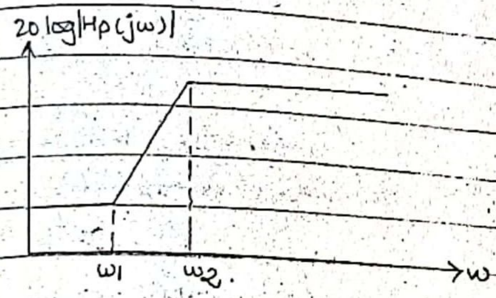
where,

$\omega_1 = 1/R_1C_1$

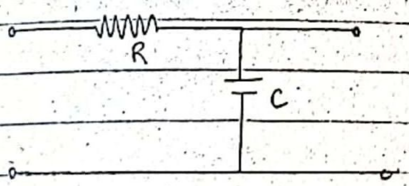
$\omega_2 = \dfrac{R_1+R_2}{R_1R_2C_1}$

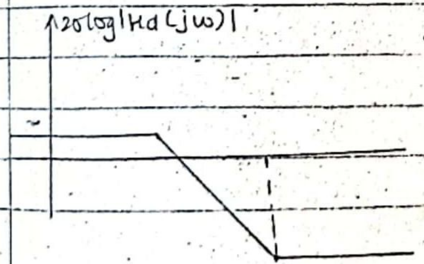
&

$\tau_1 = R_1C_1 = 75\ \mu s$.




de-emphasis circuit.




$Hd(j\omega) = \dfrac{1}{1+j\omega/\omega_1}$

where $\omega_1 = 1/RC$

& $\tau = RC = 75\ \mu s$.

Superheterodyne FM receiver (Monophonic).

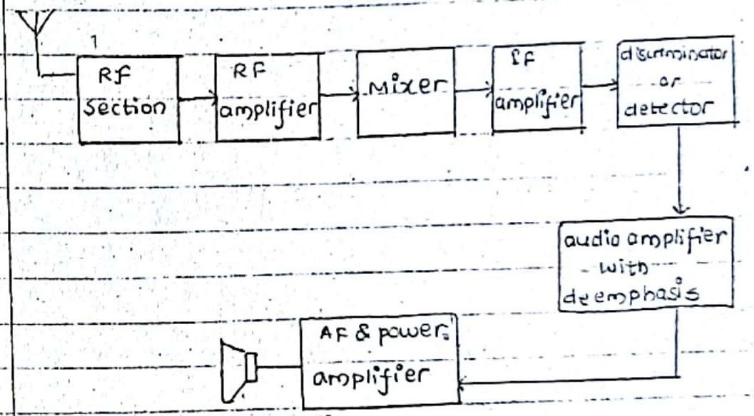



Fig: block diagram of S.H FM receiver.

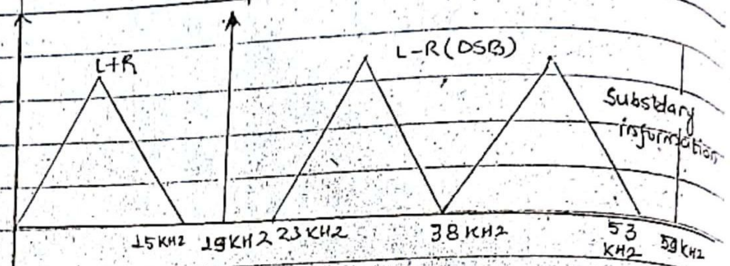
Limiter.

To remove all amplitude variations caused by noise at IF sections output. The resulting rectangular wave (by hard-limiter) is rounded off by BF filter that supresses harmonics of the carrier frequency. Thus the filter output is again sinusoidal

de-emphasis network.

It reduces the amplitude of high frequency in the audio signal which was earlier increased by the pre-emphasis network at transmitting station

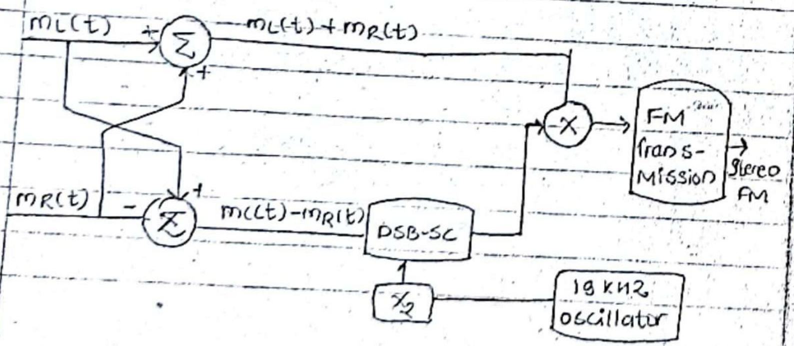
| | FM | AM |
|---|---|---|
| Range of frequency | 88-108 MHz | 545-1605 MHz |
| IF | 10.7 MHz | 455 KHz |
| Bandwidth | 200 KHz | 10 KHz. |




## Commercial stereo FM broadcasting & receiving.

The stereo FM signal consists of 2 signals simultaneously transmitted over the same FM wave.

- It is used in transmission of audio signals. Two signals being received would be then sent to the left & right speakers and designated as $M_L(t)$ & $M_R(t)$ respectively.




As shown in the figure, the signal $M_R(t)$ and $M_L(t)$ are applied to two summators. The sum component $M_L(t)+M_R(t)$ is left unprocessed in baseband form which is available for monophonic reception.

The difference component $M_L(t)-M_R(t)$ is DSB-SC modulated with the carrier of 38 KHz. This 38 KHz wave is generated from 19 KHz oscillator via the process of frequency multiplication.

The sum of the signals $m_L(t)+m_R(t)$, DSB-SC modulated wave & 19 KHz pilot carrier are simultaneously transmitted & together form the stereo FM.

The frequency spectrum is shown above in fig(b).

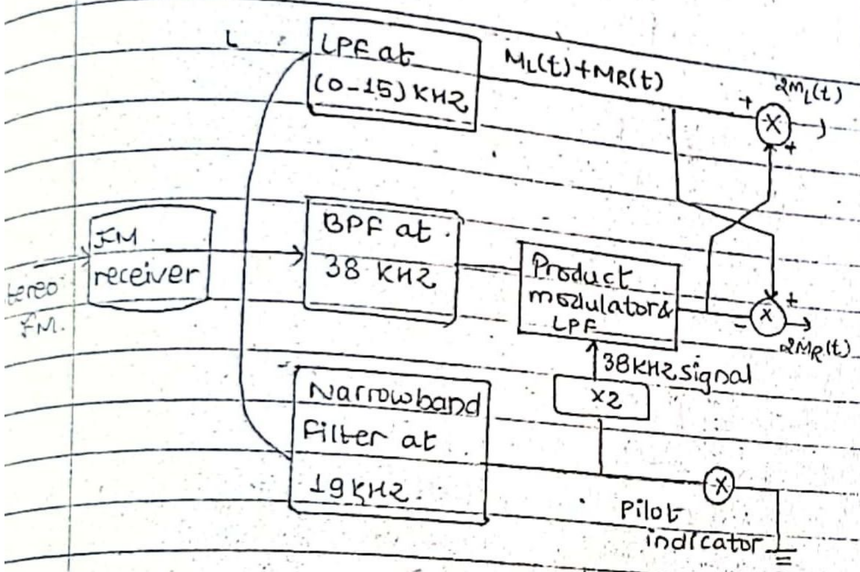



Fig:- Stereo FM receiver.

The stereo FM signal is passed through demultiplexing system. The three essential components of the stereo FM are extracted by the use of 3 filters.

- The recovered pilot carrier is frequency doubled to produce 38 KHz sub-carrier. The avaibility of sub-carrier enables the coherent demodulation of DSB-SC wave thereby, by receiving the difference component $[M_L(t) - M_R(t)]$.

- The LPF in the output path is designed to receive sum components $[M_L(t) + M_R(t)]$

Finally simple matrixer (or summator) reconstructs the left hand signal, $M_L(t)$ and right hand signal $M_R(t)$ and applied to the respective speakers.

chapter 7:

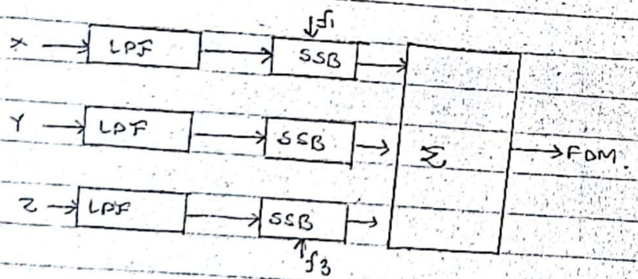
## Frequency division Multiplexing

### Frequency division multiplex. (FDM)

- Frequency division multiplexing is the system of multiplexing no of individual message signals over a common channel.
- In FDM available channel bandwidth is divided into no of overlapping frequency slots and each message signal is assigned a slot of frequency within the pass band of channel.

  Examples: composite TV signals.

- SSB is the most popular & effective means.




### FDM in telephony

FDM is extensively used in telephony to transmit no of telephone channels simultaneously over a channel of cable or microwave link.

- basic telephone channel is bandlimited to 300-3400 Hz
- Frequency slot of 4 KHz is assigned to each telephone channel so that there is guard band of 900 Hz for each channel.
- The first 3 channels are multiplexed at 12, 16, 20 KHz, to form a pregroup of 3 telephone channels.
- The multiplexing is SSB-USB.

### FDMA in satellite c.s.

- There are numerous artificially launched satellites for navigation, defense, broadcasting & other purposes.
- It may be domestic (INSAT indian satellite system) regional (used by 2 or more countries) and global (Intercontinental)
- In general the uplink and downlink frequencies are different to avoid interference.
- The height of the satellite should be greater than 600 km on the earth to balance centripetal force on the satellite and gravitational force on the earth.

## Demand Assigned Multiple access (DAMA).

- To increase the satellite utilization factor most of the satellite system uses DAMA system.
- In this system the earth station A desired to establish communication with another station B send request signal to satellite.
- The satellite assigns station A of the free channel, presently unoccupied to establish link between A and B.
- The assigment could be in FDM or in TDM (Time division multiplexing).

Example

* SPADE.

- It is a good example of FDMA.
- Single channel per carrier PCM Multiple access demand assigment equipment.
- In this a common pool of 800 channels are available to all ground stations having common signaling channels.
- If station A wishes to establish link with station B former selects free channel randomly & through signaling link send information on selected channel.
- When B confirms it the link is established.

## Filter and oscillator requirements in FDM

- FDM requires SSB filters (BPF) and frequency stability of oscillators. As in FDM the guard band between two adjascent frequency slots not very large.
- Frequency stability of oscillators should be very high to avoid overlapping.
- In practice: highly stable quart controlled oscillators with stability factor 10-5 is employed.
- Filter has to be tuned to its own frequency depending on frequency slot.
- Instability of tuning & drift in centre frequency due to environmental conditions would result in overlapping & cross talk.

$$= A\cos\omega_c t + \frac{A\cdot m_a}{2}[\cos(\omega_c+\omega_m)t + \cos(\omega_c-\omega_m)t]$$

or
$$s(t) = A\cos\omega_c t + \frac{A\cdot m_a}{2}\cos(\omega_c+\omega_m)t + \frac{A\cdot m_a}{2}\cos(\omega_c-\omega_m)t \quad ...(3.24)$$

Equation (3.24) reveals that the $AM$ signal has three frequency components as follow:

(i) carrier frequency $\omega_c$ having amplitude $A$

(ii) upper sideband $(\omega_c + \omega_m)$ having amplitude $\frac{m_a\cdot A}{2}$

(iii) Lower sideband $(\omega_c - \omega_m)$ having amplitude $\frac{m_a\cdot A}{2}$

With the help of these frequency components, we can plot the frequency-spectrum of single-tone amplitude modulated (AM) wave. Figure 3.4($a$) shows the one-sided frequency spectrum of single-tone AM wave.

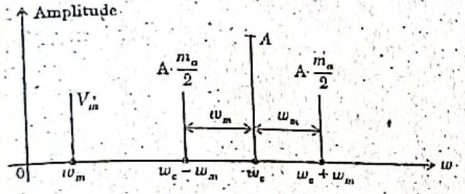



*Fig. 3.4(a). Single-sided frequency spectrum of single-tone AM wave.*

**EXAMPLE 3.1.** The tuned-circuit of the oscillator in an AM transmitter uses a 50 μH coil and a 1 nF capacitor. Now, if the oscillator output is modulated by audio frequencies upto 8 KHz, then find the frequency range occupied by the sidebands.

**Solution :** The oscillator in AM transmitter is used to generate high carrier frequency. Hence, the resonance frequency of the oscillator will be the carrier frequency.

Therefore,

Carrier frequency, $f_c = \frac{1}{2\pi\sqrt{LC}}$

Here given that $L = 50\ \mu H$

$L = 50\times 10^{-6}$ H

and $C = 1\ nF = 1\times 10^{-9}\ F$

Thus,
$$f_c = \frac{1}{2\pi\sqrt{50\times10^{-6}\times1\times10^{-9}}} = \frac{1}{2\pi\sqrt{5\times10^{-14}}}$$

$$= \frac{1}{2\pi\times10^{-7}\times\sqrt{5}} = 7.12\times10^5\ \text{Hz}$$

$$f_c = 712\ \text{KHz}$$

Now, it is given that the highest modulating frequency is 8 KHz.

*Therefore, the frequency range occupied by the sidebands will range from 8 KHz above to 8 KHz below the carrier frequency, extending from 7 to 4 KHz to 720 KHz.* Ans.



## 3.6. POWER CONTENT IN AM WAVE

(U.P. Tech. Sem. Exam., 2004-05) (05 marks)

It may be observed from the expression of $AM$ wave that the carrier component of the amplitude modulated wave has the same amplitude as unmodulated carrier. In addition to carrier component, the modulated wave consists of two sideband components. It means that the modulated wave contains more power than the unmodulated carrier. However, since the amplitudes of two sidebands depend upon the modulation index, it may be anticipated that the total power of the amplitude modulated wave would depend upon the modulation index also. In this section, we shall find the power contents of the carrier and the sidebands.

We know that the general expression of $AM$ wave is given as

$$s(t) = A\cos\omega_c t + x(t)\cos\omega_c t \quad ...(3.25)$$

The total power $P$ of the $AM$ wave is the sum of the carrier power $P_c$ and sideband power $P_s$.

**Carrier Power**

The carrier power $P_c$ is equal to the mean-square ($ms$) value of the carrier term $A\cos\omega_c t$ i.e.

$$P_c = \text{mean square value of } A\cos\omega_c t$$

$$P_c = [A\cos\omega_c t]^2 = \frac{1}{2\pi}\int_0^{2\pi} A^2\cos^2\omega_c t\, dt^* = \frac{A^2}{2} \quad ...(3.26)$$

**Sideband Power**

The sideband power $P_s$ is equal to the mean square value of the sideband term $x(t)\cos\omega_c t$, i.e.

$$P_s = \text{mean square value of } x(t)\cos\omega_c t$$

$$P_s = [x(t)\cos\omega_c t]^2 = \frac{1}{2\pi}\int_0^{2\pi} x^2(t)\cos^2\omega_c t\, dt$$

$$P_s = \frac{1}{2\pi}\int_0^{2\pi}\frac{1}{2}\left[2\cos^2\omega_c t\right]x^2(t)\, dt$$

or
$$P_s = \frac{1}{2\pi}\int_0^{2\pi}\frac{1}{2}x^2(t)\, dt + \frac{1}{2\pi}\int_0^{2\pi}x^2(t)\cos 2\omega_c t\, dt \quad ...(3.27)$$

In $AM$ generation, a Band pass filter ($BPF$) or a tuned circuit tuned to carrier frequency $\omega_c$ is used to filter out the second integral term.

Therefore,

$$P_s = \frac{1}{2\pi}\int_0^{2\pi}\left[\frac{1}{2}x^2(t)\right]dt$$

or
$$P_s = \text{mean square } (ms) \text{ value of } \frac{1}{2}x^2(t) = \frac{1}{2}\overline{x^2(t)} \quad ...(3.28)$$

However, the total sideband power $P_s$ is due to the equal contributions of the upper and lower sidebands. Hence, the power carried by the upper and the lower sidebands will be

$$P_{s(LSB)} = P_{s(USB)} = \frac{P_s}{2} = \frac{1}{4}\overline{x^2(t)} \quad ...(3.29)$$

* since period of the signal $A\cos\omega_c t$ is $2\pi$.

Therefore, the total power $P_t$ of the *AM* signal is the sum of the carrier power $P_c$ and sideband power $P_s$.

$$P_t = P_c + P_s = \frac{1}{2}A^2 + \frac{1}{2}\overline{x^2(t)}$$

$$P_t = \frac{1}{2}[A^2 + \overline{x^2(t)}] \quad ...(3.30)$$

## 3.7. TRANSMISSION EFFICIENCY OF AMPLITUDE MODULATED SIGNAL

We know that the total modulated power of an *AM* signal is expressed as

$$P_t = P_c + P_s = \frac{1}{2}[A^2 + \overline{x^2(t)}] \quad ...(3.31)$$

Out of this total power $P_t$, the useful message or baseband power is the power carried by the sidebands, *i.e.* $P_s$. The large carrier power $P_c$ is a waste from the transmission point of view because it does not carry any information or message. This large carrier power $P_c$ is transmitted alongwith the sideband power only for the convenient and cheap detection. Hence, $P_s$ is the only useful message power present in the *AM* wave.

**DO YOU KNOW?**

Despites its simplicity and effectiveness, AM is a highly inefficient method of modulation.

In AM wave, the amount of useful message power $P_s$ may be expressed by a term known as transmission efficiency $\eta$.

Hence transmission efficiency of *AM* wave may be defined as the percentage of total power contributed by the sidebands.

Mathematically,

$$\text{Transmission Efficiency,} \quad \eta = \frac{P_s}{P_t} \times 100 \quad ...(3.32)$$

or

$$\eta = \frac{\frac{1}{2}\overline{x^2(t)}}{\frac{1}{2}A^2 + \frac{1}{2}\overline{x^2(t)}} \times 100 = \frac{100\overline{x^2(t)}}{A^2 + \overline{x^2(t)}} \quad ...(3.33)$$

The maximum transmission efficiency of the *AM* is only 33.33%. This implies that only one-third of the total power is carried by the sidebands and the rest two-third is wasted.

## 3.8. POWER OF A SINGLE-TONE AMPLITUDE-MODULATED (AM) SIGNAL

*(MD University, Rohtak, 2004-05) (05 marks)*

In article 3.6, we have found the power content of the *AM* signal when modulating-signal is any random signal and may consist of several frequency components. Likewise we can find power content of single-tone Amplitude Modulated (AM) signal.

Let us consider that a carrier signal $A\cos\omega_c t$ is amplitude-modulated by a single-tone modulating signal $x(t) = V_m \cos\omega_m t$.

Then the unmodulated or carrier power

$$P_c = \text{mean square (ms value)}$$

$$P_c = \overline{(A\cos\omega_c t)^2} = \frac{A^2}{2}$$

The sideband power $\quad P_s = \frac{1}{2}\overline{x^2(t)} = \frac{1}{2}\overline{(V_m \cos\omega_m t)^2}$





$$P_s = \frac{1}{2}\frac{V_m^2}{2} = \frac{1}{4}V_m^2 \quad ...(3.34)$$

We know that the total modulated power $P_t$ is the sum of $P_c$ and $P_s$.

Therefore

$$P_t = P_c + P_s = \frac{A^2}{2} + \frac{1}{4}\cdot V_m^2$$

or

$$P_t = \frac{A^2}{2}\left[1 + \frac{1}{2}\left(\frac{V_m}{A}\right)^2\right]$$

But

$$\frac{V_m}{A} = \frac{\text{Maximum baseband amplitude}}{\text{Maximum carrier amplitude}} = m_a$$

$$= \text{modulation index for } AM$$

Hence

$$P_t = \frac{A^2}{2}\left[1 + \frac{1}{2}\cdot m_a^2\right]$$

But

$$\frac{A^2}{2} = P_c = \text{carrier power}$$

Therefore,

$$P_t = P_c\left(1 + \frac{m_a^2}{2}\right) \quad ...(3.35)$$

**EXAMPLE 3.2. A 400 watts carrier is modulated to a depth of 75 percent. Find the total power in the amplitude-modulated wave. Assume the modulating signal to be a sinusoidal one.**

**Solution :** We know that for a sinusoidal modulating signal, the total power is expressed as

$$P_t = P_c\left(1 + \frac{m_a^2}{2}\right)$$

where $\quad P_t$ = total power or modulated power

$P_c$ = carrier power or unmodulated power

$m_a$ = modulation index

Given that, $\quad P_c$ = 400 watts

$m_a$ = 75 percent = 0.75

Therefore,

$$P_t = P_c\left(1 + \frac{m_a^2}{2}\right) = 400\left(1 + \frac{0.75^2}{2}\right) = 512.5 \text{ watts} \quad \textbf{Ans.}$$

**EXAMPLE 3.3. An AM broadcast radio transfer radiates 10 K watts of power if modulation percentage is 60. Calculate how much of this is the carrier power.**

**Solution :** We know that the total power is expressed as

$$P_t = P_c\left(1 + \frac{m_a^2}{2}\right) \quad ...(i)$$

where $\quad P_t$ = total power or modulated power

$P_c$ = carrier power or unmodulated power

$m_a$ = modulation index

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Given that,

$$P_t = 10 \text{ K watts}$$

$$m_a = 60 \text{ percent} = 0.6$$

From equation (*i*), we get

$$P_c = \frac{P_t}{1+\frac{m_a^2}{2}} = \frac{10}{1+\frac{0.6^2}{2}} = \frac{10}{1.18} = 8.47 \text{ kW} \quad \textbf{Ans.}$$

## 3.9. CURRENT CALCULATION FOR SINGLE-TONE AM

In AM, it is generally more convenient to measure the AM transmitter current than the power. In this case, the modulation index may be calculated from the values of unmodulated and modulated currents in the AM transmitter.

Let $I_c$ be the *r.m.s.* value of the carrier or unmodulated current and $I_t$ be the *r.m.s.* value of the total or modulated current of an AM transmitter. Let $R$ be the antenna resistance through which these currents flow.

Now, we know that for a single-tone modulation the power relation is expressed as

$$P_t = P_c\left(1+\frac{m_a^2}{2}\right) \quad ...(3.36)$$

where $P_t$ = total or modulated power
$P_c$ = carrier or unmodulated power
$m_a$ = modulation index

From equation (3.36), we may write

$$\frac{P_t}{P_c} = 1+\frac{m_a^2}{2}$$

or
$$\frac{I_t^2 \cdot R}{I_c^2 \cdot R} = 1+\frac{m_a^2}{2}$$

or
$$\frac{I_t}{I_c} = \sqrt{1+\frac{m_a^2}{2}}$$

or
$$I_t = I_c\sqrt{1+\frac{m_a^2}{2}} \quad ...(3.37)$$

**EXAMPLE 3.4.** **The antenna current of an AM transmitter is 8 A if only the carrier is sent, but it increases to 8.93 A if the carrier is modulated by a single sinusoidal wave. Determine the percentage modulation. Also find the antenna current if the percent of modulation changes to 0.8.**

*(JNTU, Hyderabad, 2004-05) (06 marks)*

**Solution :** (*i*) The current relation for a single-tone amplitude modulation is expressed as

$$I_t = I_c\sqrt{1+\frac{m_a^2}{2}} \quad (i)$$

where $I_t$ = total or modulated current
$I_c$ = carrier or unmodulated current
$m_a$ = modulation index

Using equation (*i*), we get

$$\frac{I_t}{I_c} = \sqrt{1+\frac{m_a^2}{2}}$$



or
$$\left(\frac{I_t}{I_c}\right)^2 = 1+\frac{m_a^2}{2}$$

or
$$\frac{m_a^2}{2} = \left(\frac{I_t}{I_c}\right)^2 - 1$$

or
$$m_a^2 = 2\left[\left(\frac{I_t}{I_c}\right)^2 - 1\right]$$

or
$$m_a = \sqrt{2\left[\left(\frac{I_t}{I_c}\right)^2 - 1\right]}$$

Putting all the given values, we have

$$m_a = \sqrt{2\left[\left(\frac{8.93}{8}\right)^2 - 1\right]} = \sqrt{2[(1.116)^2 - 1]}$$

$$m_a = \sqrt{2(1.246-1)} = \sqrt{0.492}$$

$$m_a = 0.701 = 70.1\%$$

(*ii*) Since
$$I_t = I_c\sqrt{1+\frac{m_a^2}{2}}$$

Here, $I_c = 8\text{A}$
and $m_a = 0.8$

Therefore,
$$I_t = 8\times\sqrt{1+\frac{0.8^2}{2}} = 8\sqrt{1+\frac{0.64}{2}}$$

or
$$I_t = 8\sqrt{1.32} = 8\times 1.149 = 9.19 \text{ A} \quad \textbf{Ans.}$$

## 3.10. POWER CONTENT IN MULTIPLE-TONE AMPLITUDE MODULATION (AM)

A multiple-tone amplitude modulation is that type of modulation in which the modulating signal consists of more than one frequency components.

Let us consider that a carrier signal $A \cos \omega_c t$ is modulated by a baseband or modulating signal $x(t)$ which is expressed as

$$x(t) = V_1 \cos \omega_1 t + V_2 \cos \omega_2 t + V_3 \cos \omega_3 t \quad ...(3.38)$$

We know that the general expression for AM wave is

$$s(t) = A \cos \omega_c t + x(t) \cos \omega_c t \quad ...(3.39)$$

Putting the value of $x(t)$, we get

$$s(t) = A \cos \omega_c t + [V_1 \cos \omega_1 t + V_2 \cos \omega_2 t + V_3 \cos \omega_3 t] \cos \omega_c t$$

or
$$s(t) = A\left[1+\frac{V_1}{A}\cos\omega_1 t + \frac{V_2}{A}\cos\omega_2 t + \frac{V_3}{A}\cos\omega_3 t\right]\cos\omega_c t \quad ...(3.40)$$

But we know that

$$\frac{V}{A} = \frac{\text{Maximum amplitude of modulating signal}}{\text{Maximum amplitude of carrier signal}}$$

$$= \text{modulation index } m_a$$

Therefore,
$$s(t) = A[1 + m_1 \cos \omega_1 t + m_2 \cos \omega_2 t + m_3 \cos \omega_3 t] \cos \omega_c t \quad ...(3.41)$$

where

$m_1 = \frac{V_1}{A}$, $m_2 = \frac{V_2}{A}$ and $m_3 = \frac{V_3}{A}$, are the modulation indexes of the corresponding frequency components.

The expression for *AM* wave in equation (3.41) may further be expanded as

$$s(t) = A\cos\omega_c t + m_1 A\cos\omega_c t\cos\omega_1 t + m_2 A\cos\omega_c t\cos\omega_2 t + m_3 A\cos\omega_c t\cos\omega_3 t$$

Now we know that the total power in *AM* is given as

$$P_t = \text{carrier power} + \text{sideband power}$$

$$P_t = P_c + P_s \qquad \text{...(3.42)}$$

The carrier power $P_c$ is given as

$$P_c = \overline{(A\cos\omega_c t)^2} = \frac{A^2}{2}$$

Similarly, the sideband power is given as

$$P_s = \frac{1}{2}\overline{x^2(t)}$$

$$P_s = \frac{1}{2}[\overline{(V_1\cos\omega_1 t)^2} + \overline{(V_2\cos\omega_2 t)^2} + \overline{(V_3\cos\omega_3 t)^2}] \qquad \text{...(3.43)}$$

But we know that

$$m_1 = \frac{V_1}{A} \text{ so that } V_1 = m_1 A$$

$$m_2 = \frac{V_2}{A} \text{ so that } V_2 = m_2 A$$

and

$$m_3 = \frac{V_3}{A} \text{ so that } V_3 = m_3 A$$

Putting the values of $V_1$, $V_2$ and $V_3$ in equation (3.43), we get

$$P_s = \frac{1}{2}[\overline{(m_1 A\cos\omega_1 t)^2} + \overline{(m_2 A\cos\omega_2 t)^2} + \overline{(m_3 A\cos\omega_3 t)^2}]$$

$$P_s = \frac{1}{2}\left[\frac{(m_1 A)^2}{2} + \frac{(m_2 A)^2}{2} + \frac{(m_3 A)^2}{2}\right]$$

or

$$P_s = \frac{1}{4}A^2[m_1^2 + m_2^2 + m_3^2] \qquad \text{...(3.44)}$$

Now putting the value of $P_c$ and $P_s$ in equation (3.41), we get

$$P_t = P_c + P_s = \frac{A^2}{2} + \frac{1}{4}A^2[m_1^2 + m_2^2 + m_3^2]$$

or

$$P_t = \frac{A^2}{2}\left[1 + \frac{1}{2}(m_1^2 + m_2^2 + m_3^2)\right] = P_c\left[1 + \frac{m_1^2 + m_2^2 + m_3^2}{2}\right]$$

or

$$P_t = P_c\left[1 + \frac{m_1^2}{2} + \frac{m_2^2}{2} + \frac{m_3^2}{2}\right]$$

This expression may be extended upto to *n*-modulating terms *i.e.*

$$P_t = P_c\left[1 + \frac{m_1^2}{2} + \frac{m_2^2}{2} + \frac{m_3^2}{2} + \ldots + \frac{m_n^2}{2}\right] \qquad \text{...(3.45)}$$



### 3.10.1. Total or Net modulation Index for Multiple-Tone Modulation

Let us consider that $m_t$ is the total or net modulation indexes for a multiple-tone modulation.

We know that for a multiple-tone modulation, the total power is expressed as

$$P_t = P_c\left[1 + \frac{m_1^2}{2} + \frac{m_2^2}{2} + \frac{m_3^2}{2} + \ldots + \frac{m_n^2}{2}\right] \qquad \text{...(3.46)}$$

where $m_1, m_2 \ldots m_n$ are the modulation indexes for different modulating signals.

The power for *AM* wave is also expressed as

$$P_t = P_c\left(1 + \frac{m_t^2}{2}\right) \qquad \text{...(3.47)}$$

Comparing equations (3.46) and (3.47), we get

$$m_t^2 = m_1^2 + m_2^2 + m_3^2 + \ldots + m_n^2$$

or

$$m_t = \sqrt{m_1^2 + m_2^2 + m_3^2 + \ldots + m_n^2} \qquad \text{...(3.48)}$$

This is the desired expression for the total or net modulation index.

**EXAMPLE 3.5. An AM transmitter radiates 9 K watts of power when the carrier is unmodulated and 10.125 K watts when the carrier is sinusoidally modulated. Find the modulation index, percentage of modulation. Now, if another sine wave, corresponding to 40 percent modulation is transmitted simultaneously, then calculate the total radiated power.** **(Very Important)**

**Solution :** (*i*) We know that for a single-tone sinusoidal amplitude-modulation the total power is expressed as

$$P_t = P_c\left(1 + \frac{m_a^2}{2}\right) \qquad \text{...(i)}$$

where $P_t$ = modulated or total power

$P_c$ = unmodulated or carrier power

$m_a$ = modulation index

Given that, $P_t$ = 10.125 kW

and $P_c$ = 9 kW

Using equation (*i*), we get

$$1 + \frac{m_a^2}{2} = \frac{P_t}{P_c}$$

or

$$\frac{m_a^2}{2} = \frac{P_t}{P_c} - 1 = \frac{10.125}{9} - 1$$

or

$$\frac{m_a^2}{2} = 1.125 - 1 = 0.125$$

or

$$m_a^2 = 0.125 \times 2 = 0.250 = 0.50$$

(*ii*) We know that in case of modulation by two sinusoidal waves, the total modulation index $m_t$ is expressed as

$$m_t = \sqrt{m_1^2 + m_2^2}$$

Let $m_1 = m_a = 0.5$

Given that $m_2 = 0.4$

Therefore, $m_t = \sqrt{(0.5)^2 + (0.4)^2} = \sqrt{0.25 + 0.16} = \sqrt{0.41} = 0.64$

The total radiated power in this case will be

$$P_t = P_c\left(1 + \frac{m_t^2}{2}\right) = 9 \times \left(1 + \frac{0.64^2}{2}\right)$$

$$P_t = 9\,(1 + 0.205) = 10.84 \text{ kW} \qquad \text{Ans.}$$

## 3.11. GENERATION OF AMPLITUDE MODULATION (AM)

*(PTU, 2005)(10 marks)*

The device which is used to generate an Amplitude modulated (*AM*) wave is known as Amplitude Modulator.

The methods of *AM* Generation may be broadly classified as follow :

(*i*) Low-level AM Modulation

(*ii*) High-level AM Modulation

### 3.11.1. Low Level Amplitude Modulation (Low Level AM Transmission)

Figure 3.5(*a*) shows the block diagram of a low level AM modulation system. In a low-level amplitude modulation system, the modulation is done at low power level. At low power levels, a very small power is associated with the carrier signal and the modulating signal. Because of this, the output power of modulation is low. Therefore, the power amplifiers are required to boost the amplitude-modulated signals upto the desired output level. From block diagram in figure 3.5(*a*), it is clear that modulation is done at low power level. After this, the amplitude-modulated signal (*i.e.* a signal containing a carrier and two sidebands) is applied to a wide-band power amplifier. A wide-band power amplifier is used just to preserve the sidebands of the modulated signal. Amplitude modulated systems, employing modulation at low power levels are also called **low-level amplitude modulation transmitters.**

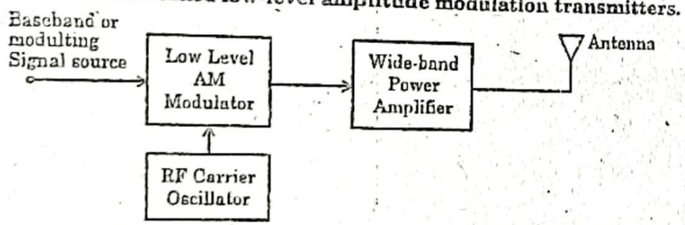


*Fig. 3.5(a) Block Diagram for Low level AM Modulation.*

Square-low diode modulation and switching modulation are examples of low-level modulation.

### 3.11.2. High Level Amplitude Modulation (High Level AM Reception)

Figure 3.5(*b*) shows the block diagram of a high level AM modulation system. In a high-level amplitude modulation system, the modulation is done at high power level. Therefore, to produce amplitude-modulation at these high power levels, the baseband signal and the carrier signal must be at high power levels. In block diagram of figure 3.5(*b*) the modulating signal and carrier signal are first power amplified and then applied to AM high-level modulator. For modulating signal, the wide-band power amplifier is

> High-level modulation produces the best type of AM, but it requires an extremely high-power modulator circuit. In fact, the power supplied by the modulator must be equal to one-half the total class *C* power amplifier rating for 100 percent modulation.



required just to preserve all the frequency components present in modulating signal. On the other hand, for carrier signal, the narrow-band power amplifier is required because it is a fixed-frequency signal. The collector modulation method is the example of high-level modulation.

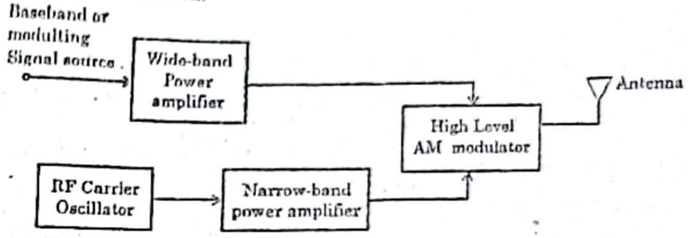


*Fig. 3.5(b). Block Diagram for High level AM Modulation.*

*Before we discuss low level and high level modulation methods in detail, we shall establish the fact that a non-linear resistance of a non-linear device can be made to produce Amplitude Modulation when two different frequencies are passed together through it.*

### 3.11.3. Non-linear resistance or Non-linear Circuits

We know that the relationship between voltage and current in a linear resistance is expressed as

$$i = bv \qquad \text{...(3.49)}$$

where

$v$ = voltage across the linear resistance

$i$ = current through linear resistance

and $b$ = any constant of proportionality

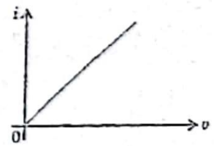


*Fig. 3.6. Linear resistance characteristics*

If equation (3.49) is applied to a resistor, then constant $b$ is clearly its conductance.

Also, if equation (3.49) is applied to the linear portion of the transistor characteristic then $i$ is the collector current and $v$ is the voltage applied to the base.

As more general, equation (3.49), may be written as

$$i = a + bv \qquad \text{...(3.50)}$$

where a is the d.c. component of the current.

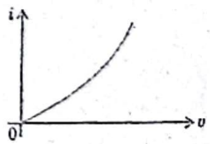


*Fig. 3.7. Non-linear resistance characteristics*

Now, let us consider a non-linear resistance. For a non-linear resistance the current-voltage characteristics will be non-linear as shown in figure 3.7.

The non-linear relationship between voltage and current may be expressed as

$$i = a + bv + cv^2 + dv^3 + \ldots \qquad \text{...(3.51)}$$

This means that due to non-linearity in the $v$–$i$ characteristics of a non-linear resistance, the current becomes proportional not only to voltage but also to the square, cube and higher powers of the voltage.

For simplicity, neglecting the higher terms in equation (3.51), we have

$$i = a + bv + cv^2 \qquad \text{...(3.52)}$$

first, and after the heterodyning operation it is impossible to distinguish the two. Note that the image frequency is separated from the desired signal by exactly twice the IF. Usually, the image frequency signal is attenuated by a selective RF amplifier placed before the mixer.

## MISCELLANEOUS SOLVED EXAMPLES

**EXAMPLE 3.9.** An audio signal given as 15 sin 2 π(1500*t*) amplitude modulates a carrier given as 60 sin 2 π (100,000*t*) determine the following:

(*a*) Sketch the audio signal.
(*b*) Sketch the carrier signal.
(*c*) Construct the modulated wave.
(*d*) Determine the modulation index and percent modulation.
(*e*) What are the frequencies of the audio signal and the carrier?
(*f*) What frequencies would present in a spectrum analysis of the modulated wave?

**Solution :** *Given that :* Audio signal = 15 sin 2 π (1500*t*)

Carrier = 60 sin 2 π (100,000*t*)

(*a*) audio signal or modulating signal is sketched in figure 3.52
(*b*) The carrier signal is sketched in figure 3.52
(*c*) To construct the modulated wave, first let us develop the envelope of the modulated wave in the following two steps:
(*i*) Locate the amplitude of the carrier (dashed line).
(*ii*) Using the amplitude of the carrier as an axis, lay in the audio signal.

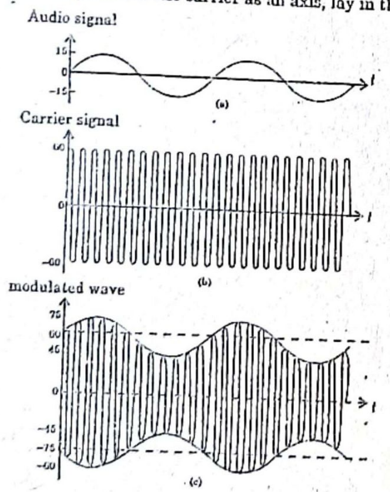


*Fig. 3.52. Waveforms for example 3.9.*

Now that the envelope has been determined, a signal having an amplitude defined by the envelope found above and having a frequency of the carrier is laid in within the envelope as shown in figure 3.52 (*c*).



(*d*) The modulation index is given by

$$m_a = \frac{\text{Maximum audio amplitude}}{\text{Maximum carrier amplitude}} = \frac{15}{60} = \frac{1}{4} = 0.25 \quad \textbf{Ans.}$$

Further, converting modulation index to percent modulation, we have

$$M = m_a \times 100 = 0.25 \times 100$$

$$M = 25\% \quad \textbf{Ans.}$$

(*e*) Since the expression for audio signal is given by

$$v_m = V_m \sin 2\pi f_m t = 15 \sin 2\pi\,(1500t)$$

Hence $f_m = 1500 Hz$ **Ans.**

Since the expression for carrier signal is given by

$$v_c = V_c \sin 2\pi f_c t = 60 \sin 2\pi\,(100{,}000)$$

Therefore, $f_c = 100{,}000\ Hz$ **Ans.**

(*f*) We know that the frequency spectrum of an amplitude-modulated wave consists of

$$f_c,\ f_c + f_m, \text{ and } f_c - f_m$$

$$f_c = 100{,}000 \text{ Hz}$$

$$f_c + f_m = 100{,}000 + 1500 = 101{,}500 \text{ Hz}$$

$$f_c - f_m = 100{,}000 - 1500 = 98{,}500 \text{ Hz}$$

Therefore, the frequency content of the modulated wave will be

100,000 Hz
101,500 Hz
98,500 Hz **Ans.**

**EXAMPLE 3.10.** The maximum power efficiency of an AM modulator is

(*a*) 25% (*b*) 50%
(*c*) 75% (*d*) 100% *(Gate Examination-1992)*

**Solution:** We know that $P_t = P_c\left(1 + \frac{m^2}{2}\right) = \frac{A_c^2}{2}\left(1 + \frac{m^2}{2}\right)$

Also, useful power $P_u = \frac{m^2}{2}\cdot\frac{A_c^2}{R}$ and $P_c = \frac{A_c^2}{R}$

Hence, efficiency $\eta = \frac{P_u}{P_c} = \frac{m^2}{2}$

But $m \leq 1$, therefore $\eta_{max} = \frac{1}{2}$ *i.e.*, 50%

Thus, option (*b*) is correct.

$\frac{A_c^2 m}{4}$

**EXAMPLE 3.11.** A 75 MHz carrier signal having an amplitude of 50 V is modulated by a 3 kHz audio signal having an amplitude of 20 V.

(*a*) Sketch the audio signal.
(*b*) Sketch the carrier signal
(*c*) Construct the modulated wave.
(*d*) Determine the modulation index and percent modulation.
(*e*) What frequencies would be there in a spectrum analysis of the modulated wave?
(*f*) Write trigonometric equation for the carrier and the modulating wave...

**Solution :** Given that :

$$f_c = 75 \text{ MHz}$$
$$V_c = 50 \text{ V}$$
$$f_m = 3 \text{ kHz}$$
$$V_m = 2 \text{ V}$$

(a) Audio signal or modulating signal is sketched in figure 3.53(a).

(b) The carrier signal is sketched in figure 3.53(b)

(c) The envelope of the carrier is fist developed by drawing the horizontal dashed line at the unmodulated carrier amplitude, both positive and negative. The audio signal is now sketched around the dashed line, providing the envelope within which the radio-frequency signal can be laid as shown in figure 3.53(c)

(d) Modulation index is given as

$$m_a = \frac{V_m}{V_c} = \frac{20}{50} = 0.4 \quad \textbf{Ans.}$$

Percent modulation may now be determined by multiplying the modulation index by 100. *i.e.*,

$$M = m_a \times 100 = 0.4 \times 100$$
$$M = 40\% \quad \textbf{Ans.}$$

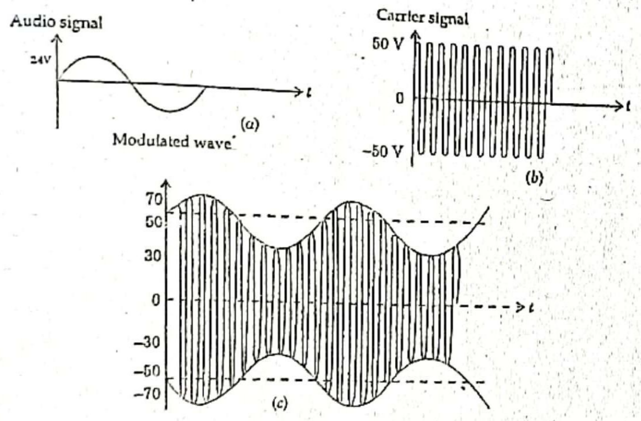


*Fig. 3.53.*

(e) The frequency content of an AM signal consists of the carrier frequency and the side band frequencies which result from adding the audio frequency to the carrier and from subtracting the audio frequency from the carrier frequency as under :

$$f_c = 75 \text{ MHz}$$
$$f_c + f_m = 75 \text{ MHz} + 3 \text{ kHz} = 75{,}000 \text{ kHz} + 3 \text{ kHz} = 75{,}003 \text{ kHz}$$
$$f_c - f_m = 75{,}000 \text{ kHz} - 3 \text{ kHz} = 74{,}997 \text{ kHz}$$

Thus, the frequency content of the AM wave will be

75.000 MHz

75.003 MHz

74.997 MHz **Ans.**



(f) The expression for modulating signal is given by

$$v_m = V_m \sin 2\pi f_m t$$

where $V_m = 20V$ and $f_m = 3000$ Hz. **Ans.**

Therefore, $v_m = 20 \sin 2\pi(3000)t = 20 \sin 6000\,\pi t$

The expression for carrier signal is given by

$$v_c = V_c \sin 2\pi f_c t$$

where $V_c = 50$ V and $f_c = 75$ MHz

$v_c = 50 \sin 2\pi (75 \times 10^6)t = 50 \sin 150 \times 10^6\,\pi t$ **Ans.**

*(Remember that $v_c$ represents the maximum amplitude of the carrier signal and $v_m$ the maximum amplitude of the modulating signal.*

**EXAMPLE 3.12.** How many AM broadcast stations can be accommodated in 100 kHz band width if the highest frequency modulating a carrier is 5 kHz ?

**Solution :** Given that :

$$\text{Total BW} = 100 \text{ kHz}$$
$$f_{m_{max}} = 5 \text{ kHz}$$

We know that, any station being modulated by a 5 – kHz signal will produce an upper-side frequency 5 kHz above its carrier and a lower-side frequency 5 kHz below its carrier, thereby requiring a bandwidth of 10 kHz. Thus, we have

Number of stations accommodated

$$= \frac{\text{tatal BW}}{\text{BW per station}} = \frac{100 \times 10^3}{10 \times 10^3}$$

Hence, number of stations accommodated = 10 stations **Ans.**

**EXAMPLE 3.13.** A bandwidth of 20 MHz is to be considered for the transmission of AM signals. If the highest audio frequencies used to modulate the carriers are not to exceed 3 kHz, how many station could broadcast within this band simultaneously without interfering with one another ?

**Solution :** Given that :

$$\text{Total BW} = 20 \text{ MHz}$$
$$f_{m_{max}} = 3 \text{ kHz}$$

We know that the maximum bandwith of each AM station is determined by the maximum frequency of the modulating signal.

$$\text{Station BW} = 2 f_{m_{max}} = 2 \times 3 \times 10^3 = 6 \times 10^3 = 6 \text{ kHz}$$

Thus, the number of stations that can be broadcasted simultaneously without interfering with one another will be

$$\frac{20 \times 10^6}{6 \times 10^3} = 3.333 \times 10^3$$

Number of station = 3333 **Ans.**

**EXAMPLE 3.14.** The total power content of an AM signal is 1000 W. Determine the power being transmitted at the carrier frequency and at each of the sidebands when the percent modulation is 100%.

**Solution :** Given that

$$P_t = 1000 \text{ W}$$
$$m = 100\% = 1$$

We know that the total power consists of the power at the carrier frequency, at the upper sideband frequency and at the lower sideband frequency, *i.e.*,

$$P_t = P_c + P_{USB} + P_{LSB}$$

Form the equation for total power, we have

$$P_t = P_c + \frac{m_a^2 P_c}{4} + \frac{m_a^2 P_c}{4} = P_c + \frac{m^2 P_c}{2}$$

Substituting, given values, we get

$$1000 = P_c + \frac{(1.0)^2 P_c}{2} = P_c + 0.5\,P_c = 1.5P_c$$

or $$\frac{1000}{1.5} = P_c$$

Solving, we get

$$P_c = 666.67 \text{ W} \qquad \textbf{Ans.}$$

The leaves $1000 - 666.67 = 333.33$ watts to be shared equally between upper and lower sidebands.

i.e., $$P_{USB} + P_{LSB} = 333.33 \text{ W}$$

But, $$P_{USB} = P_{LSB}$$

Thus, $$2P_{LSB} = 333.33$$

or $$P_{LSB} = P_{USB} = \frac{333.33}{2} = 166.66 \qquad \textbf{Ans.}$$

**EXAMPLE 3.15.** **Determine the power content of the carrier and each of the sidebands for an AM signal having a percent modulation of 80% and a total power of 2500 W.**

**Solution** : Given that

$$M = 80\%;\ m_a = 0.8$$
$$P_t = 2500 \text{ W}$$

We have to find $P_c$, $P_{USB}$, and $P_{LSB}$

We know that the total power of an AM signal is the sum of the power at the carrier frequency and the power contained in the sidebands i.e.

$$P_t = P_c + P_{USB} + P_{LSB}$$

Using the equation for total power, we write

$$P_t = P_c + \frac{m_a^2 P_c}{4} + \frac{m_a^2 P_c}{4}$$

But $$\frac{m_a^2 P_c}{4} + \frac{m_a^2 P_c}{4} = \frac{m_a^2 P_c}{2}$$

So, $$P_t = P_c + \frac{m_a^2 P_c}{2}$$

$$2500 = P_c + \frac{(0.8)^2 P_c}{2} = P_c + \frac{0.64}{2} P_c = 1.32P_c = 1.32\,P_c$$

$$P_c = \frac{2500}{1.32} = 1893.9 \text{ W} \qquad \textbf{Ans.}$$

The power in the two sidebands is the difference between the total power and the carrier power. i.e.,

$$P_{USB} + P_{LSB} = 2500 - 1893.9$$

or $$P_{USB} + P_{LSB} = 606.1 \text{ W}$$

or $$P_{USB} = P_{LSB} = \frac{606.1}{2} \text{ W}$$

or $$P_{USB} = P_{LSB} = 303.50 \text{ W} \qquad \textbf{Ans.}$$

**EXAMPLE 3.16.** **The power content of the carrier of an AM wave is 5 kilowatts. Determine the power content of each of the sidebands and the total power transmitted when the carrier is modulated with 75%.**



**Solution** : Given that

$$P_c = 5 \text{ kW}$$
$$M = 75\%;\ m_a = 0.75$$

We have to find $P_{USB}$, $P_{LSB}$, and $P_t$

Since, in an AM wave, the power in each of the sidebands is equal, therefore, we have

$$P_{USB} = P_{LSB} = \frac{m_a^2 P_c}{4} = \frac{(0.75)^2 (5000)}{4} = P_{LSB} = 703.13 \text{ W} \qquad \textbf{Ans.}$$

Now, the total power is the sum of the carrier power and the power in the two sidebands.

Thus, we have $$P_t = P_c + P_{USB} + P_{LSB} = 5000 + 703.13 + 703.13$$

or $$P_t = 6406.26 \text{ W} \qquad \textbf{Ans.}$$

**EXAMPLE 3.17.** **Which of the following demodulators can be used for demodulating the signal**

$$x(t) = 5(1 + 2\cos 2000\,\pi t)\cos 2000\,\pi t$$

**(a) Envelope demodulator**

**(b) Square-law demodulator**

**(c) Synchronous demodulator**

**(d) None of the above.** *(Gate Examination-1993)*

**Solution:** The given signal is

$$x(t) = 5(1 + 2\cos 2000\pi t)\cos 2000\,\pi t$$

Note that this is essentially an amplitude modulation (AM) double-sideband signal which can be demodulated by any of the 3 demodulators (a), (b) or (c), though envelope detection using a square law device (such as a diode) is most commonly used.

Hence, answers (a), (b) and (c) are correct.

**EXAMPLE 3.18.** **An amplitude-modulated wave has a power content of 800 W at its carrier frequency. Determine the power content of each of the sidebands for a 90% modulation.**

**Solution** : Given that

$$P_c = 800 \text{ W}$$
$$M = 90\%;\ m = 0.90$$

We have to find $P_{USB}$, $P_{LSB}$

Since, the power in each of the sidebands is equal to $\frac{m_a^2 P_c}{4}$, therefore, we get

$$P_{LSB} = P_{USB} = \frac{m_a^2 P_c}{4} = \frac{(0.9)^2\, 800}{4}$$

or $$P_{LSB} = P_{USB} = 162 \text{ W} \qquad \textbf{Ans.}$$

**EXAMPLE 3.19.** **Determine the percent modulation of an amplitude-modulated wave which has a power content at the carrier of 8 kW and 2 kW in each of its sidebands when the carrier is modulated by a simple audio tone.**

**Solution:** Given that :

$$P_c = 8 \text{ kW}$$
$$P_{USB} = P_{LSB} = 2 \text{ kW}$$

We have to find ...

Knowing the power content of the sidebands and the carrier, the relationship of sideband power can be used to determine the modulation index. Once the modulation index is known, merely multiplying it by 100 provides percent modulation.

$$P_{USB} = P_{LSB} = \frac{m_a^2 P_c}{4}$$

$$2 \times 10^3 = \frac{m_a^2(8 \times 10^3)}{4}$$

or $$m_a^2 = \frac{4 \times 2 \times 10^3}{8 \times 10^3} = 1.0$$

or $$m_a = 1.0$$

Also, $$M = m_a \times 100$$

$$M = 100\% \quad \text{Ans.}$$

**EXAMPLE 3.20. The total power content of an AM wave is 600 W. Determine the percent modulation of the signal if each of the sidebands contains 75 W.**

Solution : Given that

$$P_t = 600 \text{ W}$$

$$P_{USB} = P_{LSB} = 75 \text{ W}$$

We have to find $M$.

In order to determine the percent modulation, the power contained at the carrier frequency is first determined. Once $P_c$ is known, the relationship between $P_c$ and the sideband power will provide a means of determining the modulation index, from which the percent modulation is easily found.

Carrier power can be determined as follows :

$$P_t = P_c + P_{USB} + P_{LSB}$$

$$600 = P_c + 75 + 75$$

or $$P_c = 6 - 150 = 450 \text{ watts} \qquad \text{Ans.}$$

Now, using the relationship between sideband power and carrier power, we have

$$P_{USB} = P_{LSB} = \frac{m_a^2 P_c}{4}$$

$$75 = \frac{m_a^2(450)}{4}$$

or $$m_a^2 = \frac{4(75)}{450} = 0.667$$

or $$m_a = 0.816 \qquad \text{Ans.}$$

Converting modulation index to percent modulation, we get

$$M = m_a \times 100 = 0.816 \times 100$$

or $$M = 81.6\% \qquad \text{Ans.}$$

**EXAMPLE 3.21. Find the percent modulation of an AM wave whose total power content is 2500 W and whose sidebands each contains 400 W.**

Solution : Given that

$$P_t = 2500 \text{ W}$$

$$P_{USB} = P_{LSB} = 400 \text{ W}$$

We have to find $M$



First find the power contained at the carrier frequency. Then use the relationship between sideband power and carrier power to determine the modulation index. Once the modulation index is known, the percent modulation can easily be found merely by multiplying by 100

The power at the carrier frequency can be determined as follows :

$$P_t = P_c + P_{USB} + P_{LSB}$$

$$2500 = P_c + 400 + 400$$

$$P_c = 2500 - 800 = 1700 \text{ W}$$

$$P_{USB} = P_{LSB} = \frac{m_a^2 P_c}{4}$$

$$400 = \frac{m_a^2(1700)}{4}$$

or $$m_a^2 = \frac{400(4)}{1700} = \frac{1600}{1700} = 0.941$$

or $$m_a = 0.970$$

Thus, $$M = 0.970 \times 100$$

$$M = 97\% \qquad \text{Ans.}$$

**EXAMPLE 3.22. Determine the power content of each of the sidebands and of the carrier of an AM signal that has a percent modulation of 85% and contains 1200 W of total power.**

**Solution : Given that**

$$M = 85\%; \quad m_a = 0.85$$

$$P_t = 1200 \text{ W}$$

We have to find $P_c$, $P_{USB}$, $P_{LSB}$

Using the expression which relates the total power to carrier power, we have

$$P_t = P_c\left(1 + \frac{m_a^2}{2}\right)$$

Substituting, $$1200 = P_c\left[1 + \frac{(0.85)^2}{2}\right] = P_c\left[1 + \frac{0.7225}{2}\right]$$

$$= P_c\,[1 + 0.3613] = 1200 = 1.363 P_c$$

or $$P_c = \frac{1200}{1.3613} = 881.5 \text{ W} \qquad \text{Ans.}$$

The sum of carrier power and sideband power is equal to total power *i.e.,*

$$P_c + P_{SB} = P_t$$

$$881.5 + P_{SB} = 1200$$

$$P_{SB} = 1200 - 881.5 = 318.5 \text{ watts}$$

The total sideband power is made up equally of upper sideband power and lower sideband power *i.e.,*

$$P_{USB} = P_{LSB} = \frac{P_{SB}}{2} = \frac{318.5}{2} = 159.25 \qquad \text{Ans.}$$

**EXAMPLE 3.23. An AM signal in which the carrier is modulated upto 70% contains 1500 W at the carrier frequency. Determine the power content of the**

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upper and lower sidebands for this percent modulation. Calculate the power at the carrier and the power content of each of the sidebands when the percent modulation drops to 50%.

Solution : Given that

$$M = 70\%;\ m_a = 0.70$$
$$P_{c70} = 1500\ \text{W}$$

We have to find $P_{USB70}, P_{LSB70}, P_{c50}, P_{USB50}, P_{LSB50}$

We know that the power content of each of the sidebands is equal to $m_a^2 P_c/4$, thus, we have

$$P_{USB70} = P_{LSB70} = \frac{m_a^2 P_c}{4} = \frac{(0.7)^2 1500}{4} = 183.75$$
$$P_{USB70} = P_{LSB70} = 183.75\ \text{W} \qquad \textbf{Ans.}$$

Since in standard AM transmission, carrier power remains the same, regardless of percent modulation, thus, we have

$$P_{c50} = P_{c70} = 1500\ \text{W}$$
$$P_{USB50} = P_{LSB50} = \frac{m_a^2 P_c}{4} = \frac{(0.5)^2 1500}{4}$$
$$= 93.75\ \text{W} \qquad \textbf{Ans.}$$

**EXAMPLE 3.24. The percent modulation of an AM wave changes from 40% to 60%. Originally, the power content at the carrier frequency was 900 W. Determine the power content at the carrier frequency and within each of the sidebands after the percent modulation has risen to 60%.**

Solution : Given that

$$M_1 = 40\%;\ m_1 = 0.40$$
$$M_2 = 60\%;\ m_2 = 0.60 = P_{c40} = 900\ \text{watts}$$

We have to find $P_{c60}, P_{USB60}, P_{LSB60}$

The power content of the carrier of an AM signal remains the same regardless of percent modulation. Thus, we have

$$P_{c60} = P_{c40} = 900\ \text{W}$$

The power content of each of the sidebands is equal to $m_a^2 P_c/4$.

Thus, we have
$$P_{USB60} = P_{LSB40} = \frac{m^2 P_c}{4} = \frac{(0.60)^2 (900)}{4}$$
$$P_{USB60} = P_{LSB60} = 81.0\text{W} \qquad \textbf{Ans.}$$

**EXAMPLE 3.25. A single sideband (SSB) signal contains 1 kW. How much power is contained in the sidebands and how much at the carrier frequency?**

Solution : Given that

$$P_{SSB} = 1\ \text{kW}$$

We have to find $P_{SB}$, $P_c$

We know that in a signal side band transmission, the carrier and one of the two sidebands are eliminated. Therefore, all the transmitted power is transmitted at one of the sidebands regardless of percent modulation.

Thus,
$$P_{SB} = 1\ \text{kW}$$
and
$$P_c = 0\ \text{W} \qquad \textbf{Ans.}$$





**EXAMPLE 3.26. An SSB transmission contains 10 kW. This transmission is to be replaced by a standard amplitude modulated signal with the same power content. Determine the power content of the carrier and each of the sidebands when the percent modulation is 80%.**

Solution : Given that

$$P_{SSB} = 10\text{kW}$$
$$M = 80\%;\ m_a = 0.80$$

We have to find $P_c, P_{LSB}, P_{USB}$

Since the total power content of the new AM signal is to be the same as the total power content of the SSB signal, therefore, we have

$$P_t = P_{SSB} = 10\ \text{kW}$$

Solving for power contained at the carrier frequency, we get

$$P_t = P_c + P_{LSB} + P_{USB}$$

or
$$P_t = P_c + \frac{m_a^2 P_c}{4} + \frac{m_a^2 P_c}{4}$$
$$10{,}000 = P_c + \frac{(0.8)^2 P_c}{4} + \frac{(0.8)^2 P_c}{4}$$
$$10{,}000 = P_c + \frac{0.64 P_c}{2} = 1.32\ P_c$$

or
$$\frac{10{,}000}{1.32} = P_c$$
$$P_c = 7575.76\text{W} \qquad \textbf{Ans.}$$

The power content of the sidebands is equal to the difference between the total power and the carrier power, *i.e.*,

$$P_{SB} = P_t - P_c$$

The power content of the upper and the lower sidebands is equal, *i.e.*,

$$P_{LSB} + P_{USB} = 10{,}000 - 7575.76 = 242.24$$
$$P_{LSB} = P_{LSB} = \frac{2424.24}{2} = 1212.12\text{W}$$

Thus,
$$P_c = 7575.76\ \text{W}$$
$$P_{LSB} = P_{USB} = 1212.12\ \text{W} \qquad \textbf{Ans.}$$

**EXAMPLE 3.27. Determine the modulation index and percent modulation of the signal shown in figure 3.54.**

Solution : Given that an AM signal which is shown in figure 3.54.

We have to find $m_a$ and $M$

Using the equation relating maximum peak-to-peak amplitude and minimum peak-to-peak amplitude to modulation index, we have

$$m = \frac{\max p\text{-}p - \min p\text{-}p}{\max p\text{-}p + \min p\text{-}p}$$

From, figure 3.51, we get

$$\max p\text{-}p = 2(80) = 160$$
$$\min p\text{-}p = 2(20) = 40$$

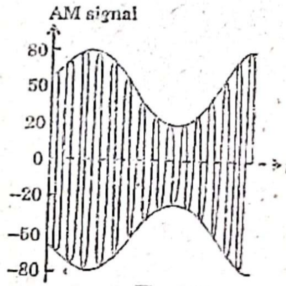



*Fig. 3.54*

Thus, $m_a = \dfrac{160-40}{160+40} = \dfrac{120}{200} = 0.6 = 0.6$

Hence, $M = m_a \times 100 = 0.6 \times 100 = 60\%$ **Ans.**

**EXAMPLE 3.28. Find the modulation index and the percent modulation of the signal shown in figure 3.55.**

**Solution :** Given that AM signal as shown in figure 3.55.

We have to find $m_a$ and $M$

Using $m_a = \dfrac{\max p\text{-}p - \min p\text{-}p}{\max p\text{-}p + \min p\text{-}p}$

and values from figure 3.52, we get

$\max p\text{-}p = 2(50) = 100$

$\min p\text{-}p = 2(15) = 30$

Thus, $m_a = \dfrac{100-30}{100+30} = \dfrac{70}{130} = 0.538$

Hence, $M = m_a \times 100 = 53.8\%$ **Ans.**

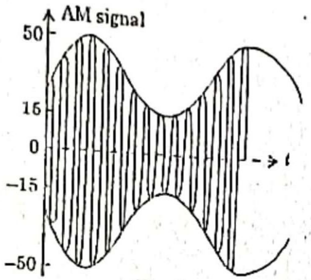



*Fig. 3.55.*

**EXAMPLE 3.29. Verify that the message signal $m(t)$ is recovered from a modulated DSB signal by first multiplying it by a local sinusoidal carrier and then passing the resultant signal through a low-pass filter (LPF), as shown in figure 3.56 (a) in the time domain and (b) in the frequency domain.**

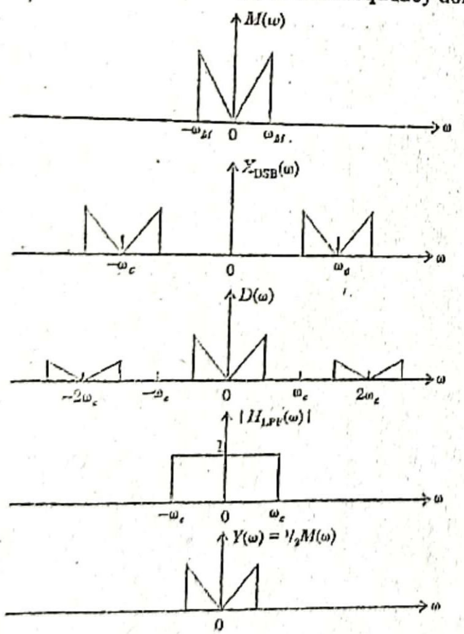



*Fig. 3.56. Demodulation of a DSB signal.*



**Solution :** The output of the multiplier is given by

$$d(t) = x_{DSB}(t)\cos\omega_c t$$

or $$d(t) = [m(t)\cos\omega_c t]\cos\omega_c t$$

or $$d(t) = m(t)\cos^2\omega_c t$$

or $$d(t) = \frac{1}{2}m(t) + \frac{1}{2}m(t)\cos 2\omega_c t$$

After low-pass filtering of signal $d(t)$, we get

$$y(t) = \frac{1}{2}m(t)$$

Thus, by proper amplification (multiplying by 2), we can recover the message signal $m(t)$.

(b) Demodulation of signal $x_{DSB}(t)$ in frequency domain has been illustrated in figure 3.56.

**EXAMPLE 3.30. Evaluate the effect of a phase error in the local oscillator on synchronous DSB demodulation.**

**Solution :** Let the phase error of the local oscillator be $\phi$. Then the local carrier is expressed as $\cos(\omega_c t + \phi)$. Now, we have

$$x_{DSB}(t) = m(t)\cos\omega_c t$$

and $$d(t) = [m(t)\cos\omega_c t]\cos(\omega_c t + \phi)$$

$$= \frac{1}{2}m(t)[\cos\phi + \cos(2\omega_c t + \phi)]$$

or $$d(t) = \frac{1}{2}m(t)\cos\phi + \frac{1}{2}m(t)\cos(2\omega_c t + \phi)$$

The second term on the right-hand side is filtered out by the low-pass filter (LPF), and thus we obtain

$$y(t) = \frac{1}{2}m(t)\cos\phi$$

This output is proportional to $m(t)$ when $\phi$ is constant. The output is completely lost when $\phi = \pm\pi/2$. Thus, the phase error in the local carrier cause attenuation of the output signal without any distortion as long as $\phi$ is constant and not equal to $\pm\pi/2$. If the phase error $\phi$ varies randomly with time, then the output also will vary randomly and is undersirable.

**EXAMPLE 3.31. A given AM broadcast station transmits a total power of 50 kW when the carrier is modulated by a sinusoidal signal with a modulation index of 0.707. Calculate:**

**(i) the carrier power**

**(ii) the transmission efficiency, and**

**(iii) the peak amplitude of the carrier assuming the antenna to be represented by a $(50 + j0)\ \Omega$ load.** *(Gate Examination, 1994)*

**Solution:** The total power transmitted by the AM broadcast station is given by

$$P_c(1 + P_x) = 50\text{ kW}$$

where $P_c$ is the carrier power and

$$P_x = \frac{1}{2}\times(0.707)^2 = 0.25$$

We know that the instantaneous frequency of the modulated signal is given as

$$\omega_i = \omega_c + k_f \cdot x(t) \qquad ...(4.29)$$

Putting the value of $x(t)$, we get

$$\omega_i = \omega_c + k_f \cdot V_m \cos \omega_m t$$

But we know that frequency deviation is given as

$$\Delta\omega = |k_f \cdot x(t)|_{max} = k_f |x(t)|_{max}$$

$$\Delta\omega = k_f \cdot V_m$$

Therefore,

$$\omega_i = \omega_c + \Delta\omega . \cos \omega_m t \qquad ...(4.30)$$

The total phase angle $\phi_i$ of the modulated wave is given as

$$\phi_i = \int \omega_i \, dt$$

Putting the value of $\omega_i$ from equation (4.30), we get

$$\phi_i = \int [\omega_c + \Delta\omega \cos \omega_m t]\, dt = \omega_c t + \frac{\Delta\omega}{\omega_m} \sin \omega_m t \qquad ...(4.31)$$

But, modulation index $m_f$ is given as

$$m_f = \frac{\text{frequency deviation}}{\text{modulating frequency}} = \frac{\Delta\omega}{\omega_m}$$

Therefore, putting the value of $m_f$ in equation (4.31) we obtain

$$\phi_i = \omega_c t + m_f \sin \omega_m t$$

Substituting this value of $\phi_i$ in equation (4.28), we get the expression for single-tone *FM* wave

$$s(t) = A \cos \phi_i = A \cos [\omega_c t + m_f \sin \omega_m t] \qquad ...(4.32)$$

which is the required mathematical expression for single tone FM wave.

**EXAMPLE 4.1.** **A single-tone FM is represented by the voltage equation as:**

$$v(t) = 12 \cos (6 \times 10^8\, t + 5 \sin 1250\, t)$$

**Determine the following :**

**(i) Carrier frequency .**

**(ii) modulating frequency**

**(iii) the modulation index**

**(iv) maximum deviation**

**(v) what power will this FM wave dissipate in 10Ω resistors.**

**Solution :** We know that the standard expression for a single-tone FM wave is given as

$$v(t) = A \cos (\omega_c t + m_f \sin \omega_m t) \qquad ...(i)$$

The given expression is

$$v(t) = 12 \cos (6 \times 10^8\, t + 5 \sin 1250\, t) \qquad ...(ii)$$

Comparing equations (i) and (ii), we get

(i) carrier frequency

$$\omega_c = 6 \times 10^8 \text{ rad/sec.}$$

or

$$f_c = \frac{6 \times 10^8}{2\pi} = 95.5 \text{ MHz} \qquad \text{Ans.}$$

(ii) modulating frequency

$$\omega_m = 1250 \text{ rad/sec.}$$



or

$$f_m = \frac{1250}{2\pi} = 199 \text{ Hz} \qquad \text{Ans.}$$

(iii)

$$m_f = 5 \qquad \text{Ans.}$$

(iv) maximum frequency deviation is given as

$$m_f = \frac{\Delta\omega}{\omega_m} = \frac{\Delta_f}{f_m}$$

or

$$\Delta_f = m_f . f_m = 5 \times 199 = 995 \text{ Hz} \qquad \text{Ans.}$$

(v) The power dissipated is

$$P = \frac{V_{rms}^2}{R} = \frac{(12/\sqrt{2})^2}{10} = \frac{72}{10} = 7.2 \text{ watts} \qquad \text{Ans.}$$

**EXAMPLE 4.2.** **A 107. 6 MHz carrier signal is frequency modulated by a 7 kHz sine wave. The resultant FM signal has a Frequency deviation of 50 kHz. Determine the following :**

**(i) the carrier swing of the FM signal.**

**(ii) the highest and the lowest frequencies attained by the modulated signal.**

**(iii) the modulation index of the FM wave.**

**Solution :** Given that

$$f_c = 107.6 \text{ MHz}$$

$$f_m = 7 \text{ kHz}$$

$$\Delta_f = 50 \text{ kHz}$$

(i) Carrier swing = 2 × frequency deviation = 2 × 50 = 100 kHz **Ans.**

(ii) The highest frequency attained by the modulated signal is equal to the resting or carrier frequency plus the frequency deviation, *i.e.*

$$f_H = f_c + \Delta_f = 107.6 \times 10^6 + 50 \times 10^3 = (107600 \times 10^3) + (50 \times 10^3)$$

or

$$f_H = 107650 \times 10^3 \text{ Hz} = 107.65 \text{ MHz} \qquad \text{Ans.}$$

Similarly, the lowest frequency attained by the modulated signal is equal to the resting or carrier frequency minus the frequency deviation i.e.

$$f_L = f_c - \Delta_f = 107.6 \times 10^6 - 50 \times 10^3)$$

$$= (107600 \times 10^3) - (50 \times 10^3)$$

or

$$f_L = 107550 \times 10^3 \text{ Hz} = 107.55 \text{ MHz} \qquad \text{Ans.}$$

(iii) The modulation index is given as

$$m_f = \frac{\text{frequency deviation}}{\text{modulating frequency}} = \frac{\Delta_f}{f_m} = \frac{50 \times 10^3}{7 \times 10^3} = 7.143 \qquad \text{Ans.}$$

**EXAMPLE 4.3.** **Determine the frequency deviation** and **carrier swing for a frequency-modulated (FM) signal which has a resting frequency of 105.00 MHz and whose upper frequency is 105.007 MHz when modulated by a particular wave. Find the lowest frequency reached by the FM wave.** *(Important)*

**Solution :** Given that

Carrier frequency $f_c$ = 105.000 MHz

$$f_H = 105.007 \text{ MHz}$$

we know that frequency deviation $\Delta_f$ is defined as the maximum change in frequency of the modulated signal away from the resting or carrier frequency *i.e.*,

$$\Delta_f = f_H - f_c = (105.007 - 105.000)\ 10^6$$

or

$$\Delta_f = 0.007 \times 10^6 \text{ Hz} = 7000 \text{ Hz} = 7 \text{ kHz} \quad \textbf{Ans.}$$

carrier swing is given by

carrier swing = 2 × frequency deviation = $2 \times \Delta_f = 2 \times 7 = 14$ kHz. **Ans.**

Lowest frequency reached by the modulated wave is given as

$$f_L = f_c - \Delta_f = (105.000 - 0.007) \text{ MHz} = 104.993 \text{ MHz.} \quad \textbf{Ans.}$$

**EXAMPLE 4.4. What is the modulation index of an FM signal having a carrier swing of 100 kHz when the modulating signal has a frequency of 8 kHz ?**

**Solution :** Given that

carrier swing = 100 kHz

modulating frequency $f_m$ = 8 kHz

modulation index is given as

$$m_f = \frac{\text{frequency deviation}}{\text{modulating frequency}} = \frac{\Delta_f}{f_m} \qquad ...(i)$$

But we know that

carrier swing = $2 \times \Delta_f$

$$\Delta_f = \frac{\text{carrier swing}}{2} = \frac{100}{2} = 50 \text{ kHz}$$

Now, using equation (i) we get

$$m_f = \frac{50}{8} = 6.25 \quad \text{Ans.}$$

**EXAMPLE 4.5. An FM transmission has a frequency deviation of 20 kHz.**

**(i) determine the percent modulation of this signal if it is broadcasted in the 88–108 MHz band.**

**(ii) Calculate the percent modulation if this signal is broadcasted as the audio portion of a television broadcast.**

**Solution :** Given that

$$\Delta_f = 20 \text{ kHz}$$

(i) Precent modulation for an FM wave is defined as

$$M = \frac{\Delta f_{actual}}{\Delta f_{max}} \times 100$$

$\Delta f_{actual}$ is given = 20 kHz

The maximum frequency deviation $\Delta f_{max}$ permitted in the FM broadcast band is 75 kHz.

Thus,
$$M = \frac{20 \times 10^3}{75 \times 10^3} \times 100 = 26.67\% \quad \textbf{Ans.}$$

(ii)
$$M = \frac{\Delta f_{actual}}{\Delta f_{max}} \times 100$$

$\Delta f_{actual}$ = 20 kHz

The maximum frequency deviation $\Delta f_{max}$ permitted for the FM audio portion of a TV broadcast is 25 kHz.

Thus
$$M = \frac{20 \times 10^3}{25 \times 10^3} \times 100 = 80\% \quad \textbf{Ans.}$$



## 4.6. PHASOR REPRESENTATION OF ANGLE MODULATED (i.e. FM and PM WAVES)

The expression for FM wave is

$$s(t)_{FM} = A\cos\left[\omega_c t + k_f \int_0^t x(t)\, dt\right] \qquad ...(4.33)$$

similarly, the expression for PM wave is

$$s(t)_{PM} = A\cos[\omega_c t + k_p \cdot x(t)] \qquad ...(4.34)$$

we may consider the signals $s(t)_{FM}$ and $s(t)_{PM}$ as real parts of phasors $C_{FM}$ and $C_{PM}$ respectively expressed as

$$C_{FM} = Ae^{j[\omega_c t + k_f \int x(t)\, dt]} \quad \text{and} \quad C_{PM} = Ae^{j[\omega_c t + k_p x(t)]}$$

Assuming $\int x(t)\, dt = y(t)$

So that
$$C_{FM} = Ae^{j[\omega_c t + k_f\, y(t)]} \qquad ...(4.35)$$

## 4.7. TYPES OF FREQUENCY MODULATION (FM)

*(U.P. Tech., Semester, Exam., 2003-04) (04 marks)*

In article 4.3, we discussed that the bandwidth of an *FM* signal depends upon the frequency deviation $\Delta\omega = k_f\, x(t)$.

Thus, as a matter of fact, if the frequency deviation is high, the bandwidth will be large. Similarly, if the frequency deviation is low, the bandwidth will be small.

Since deviation is given by $\Delta\omega = k_f\, x(t)$, it means that for any given $x(t)$, the frequency deviation and therefore the bandwidth will depend upon frequency sensitivity $k_f$. Hence, when $k_f$ is quite small then the bandwidth will be narrow and when $k_f$ is large then the bandwidth will be large.

Thus, depending upon the value of frequency sensitivity $k_f$, *FM* may be divided as under –

**(i) *Narrow band FM* :** In this case, $k_f$ is small and hence the bandwidth of *FM* is narrow.*

**(ii) Wideband *FM* :** In this case, $k_f$ is large and hence the *FM* signal has a wide bandwidth.**

### 4.7.1. Narrowband FM

We know that the general expression for *FM* wave in the phasor form is given by

$$C_{FM} = Ae^{j[\omega_c t + k_f\, y(t)]} \qquad ...(4.36)$$

However, for a narrowband *FM*, we have

$k_f \times y(t) << 1$ for all the values of $t$

Thus
$$e^{jk_f \cdot y(t)} \cong 1 + jk_f \cdot y(t)$$

Hence, the *FM* phasor expression, now, becomes

$$C_{FM} = Ae^{j\omega_c t} \cdot e^{jk_f \cdot y(t)} = A[1 + jk_f \cdot y(t)]\, e^{j\omega_c t}$$

* We shall observe, later on, that the BW of a narrowband FM is almost same as that of AM.

** We shall study, later on, that unlike AM, the bandwidth of wideband FM is too large.

Thus, for n sidebands the bandwidth of FM wave is given by

$$BW = 2\, n\omega_m \text{ radians/sec.}$$

or

$$BW = 2nf_m \text{ Hz.} \quad \text{...(4.55)}$$

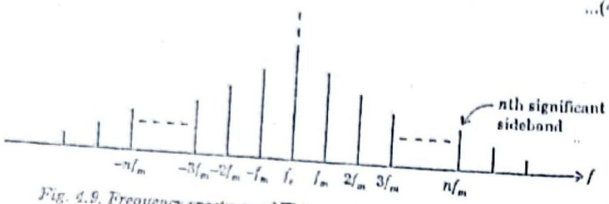



*Fig. 4.9. Frequency spectrum of FM wave for n significant sidebands.*

### Universal Curve

Schwartz developed a graph for determining the bandwidth of an FM signal if the modulation index is known. This graph has been shown in figure 4.10. Schwartz uses as his criterion the rule of thumb that any frequency component with a signal strength (voltage) less than 1% of that of the unmodulated carrier will be considered too small to be significant. This graph is also called universal curve which shows the variation of the Bandwidth B normalised with respect to $\Delta f$ against $m_f$.

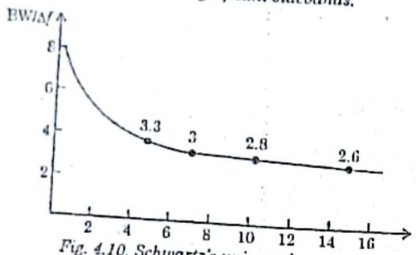



*Fig. 4.10. Schwartz's universal curve.*

### 4.10.1. Carson's Rule or Practical Bandwidth

*(U.P. Tech. Sem. Exam, 2002-03) (3 Marks)*

Carson's rule provides a thumb formula to calculate the bandwidth of a single-tone wideband FM. According to this rule, the FM bandwidth is given as twice the sum of the frequency deviation and the highest modulating frequency. However, it must be remembered that this rule is just an approximation.

Mathematically,

$$BW = 2\,(\Delta\omega + \omega_m)$$

But

$$m_f = \frac{\Delta\omega}{\omega_m}$$

Therefore,

$$BW = 2\,(m_f\, \omega_m + \omega_m)$$

$$BW = 2(1 + m_f)\omega_m \quad \text{...(4.56)}$$

Regarding Carson's rule, we can consider two special cases :

(i) $\because$ $BW = 2(1 + m_f)\, \omega_m$

If $\Delta\omega << \omega_m$ i.e. $m_f << 1$ as is the case for narrowband FM

then $BW = 2\, \omega_m$*

(ii) $\because$ $BW = 2(1 + m_f)\, \omega_m \quad \text{...(4.57)}$

* This case is equivalent to AM.



If $\Delta\omega >> \omega_m$ i.e. $m_f >> 1$ as is the case for wideband FM

then $BW = 2(m_f)\omega_m \quad \text{...(4.58)}$

But $m_f\, \omega_m = \Delta\omega$

Thus $BW = 2\Delta\omega$**

**EXAMPLE 4.6.** **Find the bandwidth of a commercial FM transmission if frequency deviation $\Delta f = 75$ kHz and modulating frequency $f_m = 15$ kHz.**

**Solution :** According to Carson's rule, the bandwidth is given by

$$BW = 2(\Delta_f + f_m) = 2(75 + 15)$$

$$BW = 2 \times 90 = 180 \text{ kHz} \quad \textbf{Ans.}$$

**EXAMPLE 4.7.** **Determine the bandwidth of a narrowband FM signal which is generated by a 4 kHz audio signal modulating a 125 MHz carrier.**

**Solution :** Given that

$$f_m = 4 \text{ kHz}$$

and

$$f_c = 125 \text{ MHz}$$

Since this is a narrowband FM signal the bandwidth is found merely by doubling modulating frequency as

$$BW = 2f_m = 2 \times 4 \times 10^3$$

$$BW = 8 \text{ kHz} \quad \textbf{Ans.}$$

**EXAMPLE 4.8.** **The maximum deviation allowed in an FM broadcast system is 75 kHz. If the modulating signal is a single-tone sinusoid of 8 kHz, determine the bandwidth of FM signal. What will be the bandwidth when modulating signal amplitude is doubled ?**

**Solution :** Given that

$$\Delta f = 75 \text{ kHz}$$

$$f_m = 8 \text{ kHz}$$

Bandwidth is given by

$$BW = 2(\Delta f + f_m) = 2\,(75 + 8)$$

$$BW = 166 \text{ kHz}$$

Now when the modulating signal amplitude is doubled, the frequency deviation $\Delta f$ becomes $2 \times 75 = 150$ kHz.

Therefore the bandwidth becomes

$$BW = 2\,(\Delta_f + f_m) = 2(150 + 8)$$

$$= 316 \text{ kHz} \quad \textbf{Ans.}$$

## 4.11. FM BANDWIDTH FOR AN ARBITRARY MODULATING SIGNAL X(T)

Till now, we have discussed the bandwidth for a single-tone FM. Now let us consider that the modulating signal $x(t)$ is an arbitrary signal *i.e.* it may consists of large

** For large values of $m_f$, this bandwidth relation has a very small error and hence can be assumed to be true bandwidth for all practical purposes.

number of frequency components. In this case to calculate the bandwidth, we first find a parameter known as deviation ratio $D$ defined as

$$D = \frac{\text{Peak frequency deviation corresponding to the maximum possible amplitude of } x(t)}{\text{Maximum frequency component present in the modulating signal } x(t)} \quad ...(4.59)$$

Note: *The deviation ratio D has same meaning to estimate the bandwidth as $m_f$ does for the single-tone FM. These for this case assuming deviation ratio D as equivalent to $m_f$, we can calculate the transmission bandwidth of FM using carson's rule.*

## 4.12. NARROWBAND FM VERSUS WIDEBAND FM

An examination of the Schwartz bandwidth curve of figure 4.11 shows that at high values of $m_f$ the curve tends towards a horizontal asymptote and at low values of $m_f$ it tends toward the vertical. Detailed mathematical study shows that the bandwidth of an FM signal for which $m_f$ is less than $\pi/2$ is dependent mainly upon the frequency of the modulating signal and is quite independent of frequency deviation. Further analysis shows that the bandwidth of an FM signal for which $m_f$ is less than $\pi/2$ is equal to twice the modulating frequency.

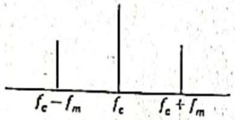


*Fig. 4.11.*

$$\text{Bandwidth} = 2f_m \quad \text{for } m_f < \pi/2$$

Just as with AM, and unlike the situation in which $m_f > \pi/2$, two side frequencies show up for each modulating frequency, one above and one below the frequency of the carrier, each spaced $f_m$ away from the carrier frequency. Because of the limited bandwidth of FM signals with $m_f < \pi/2$, such modulations are known as narrowband FM, and FM signals with $m_f > \pi/2$ are known as *wideband FM.*

Though the spectrum for an AM signal and a narrowband FM signal appear to be the same, a Fourier analysis shows that the magnitude and phase relationships for AM and FM are quite different. See figure 4.11 for the frequency spectrum of a narrowband FM signal.

Many of the advantages obtained with wideband FM, such as noise reduction, are not available with narrowband FM. Why, then, would one want to use narrowband FM rather than AM ? One reason is that with narrowband FM (as well as with wideband FM) the power content at the carrier frequency decreases as the modulation increases so that we have desirable situation of putting the power where the information is.

**EXAMPLE 4.9.** **A 5 kHz audio tone is used to modulate a 50 MHz carrier causing a frequency deviation of 20 kHz. Determine (*i*) the modulation index and (*ii*) the bandwidth of the FM signal.**

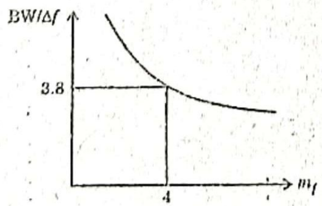


*Fig. 4.12.*

**Solution :** Given that $f_m = 5$ kHz,

$f_c = 50.0$ MHz,

$\Delta f = 20$ kHz

(*i*) Modulation index is defined as

$$m_f = \frac{\Delta f}{f_m} = \frac{20 \times 10^3}{5 \times 10^3} = 4 \quad \text{Ans.}$$



(*ii*) Referring to the Schwartz bandwidth curve, shown in figure 4.12, and entering on the horizontal axis with $m_f = 4$, it is found that

$$\frac{BW}{\Delta f} = 3.8$$

This is shown in figure 4.12. Substituting $20 \times 10^3$ for $\Delta f$ as given,

$$\frac{BW}{20 \times 10^3} = 3.8$$

Solving for BW, we have

$$BW = 3.8 \times 20 \times 10^3 = 76 \times 10^3$$

$$BW = 76 \text{ kHz} \quad \text{Ans.}$$

**EXAMPLE 4.10.** **Determine the frequency of the modulating signal which is producing an FM signal having a bandwidth of 50 kHz when the frequency deviation of the FM signal is 10 kHz.**

**Solution :** Given that BW = 50 kHz ;

$$\Delta f = 1 \text{ kHZ}$$

In order to find $f_m$, reference must be made to the Schwartz bandwidth curve, *i.e.*, figure 4.13.

In order to enter this curve, we have to determine BW/Δf as

$$\frac{BW}{\Delta f} = \frac{50 \times 10^3}{10 \times 10^3} = 5$$

From figure 4.13, we have

$$m_f = 2 = \frac{\Delta f}{f_m}$$

Therefore,
$$2 = \frac{10 \times 10^3}{f_m}$$

$$f_m = \frac{10 \times 10^3}{2} = 5 \text{ kHz Ans.}$$

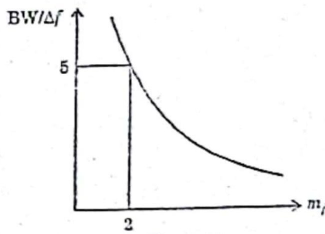


*Fig. 4.13.*

## 4.13. MULTIPLE FREQUENCY MODULATION

In article 4.5, we discussed a specific case of a single frequency-modulating signal. Now, let us extend these results to the case of multiple frequencies. First only two frequencies will be considered. It can then be generalized to any number of frequencies.

Let us consider

$$x(t) = a_1 \cos \omega_1 t + a_2 \cos \omega_2 t$$

$$\omega_i = \omega_c + k_f x(t) = \omega_c + k_f(a_1 \cos \omega_1 t + a_2 \cos \omega_2 t)$$

The maximum frequency deviation will be

$$\Delta\omega = (a_1 + a_2)k_f$$

and
$$\phi = \int \omega_i \, dt = \omega_c t + \frac{a_1 k_f}{\omega_1} \sin \omega_1 t + \frac{a_2 k_f}{\omega_2} \sin \omega_2 t$$

or
$$\phi = \omega_c t + m_1 \sin \omega_1 t + m_2 \sin \omega_2 t$$

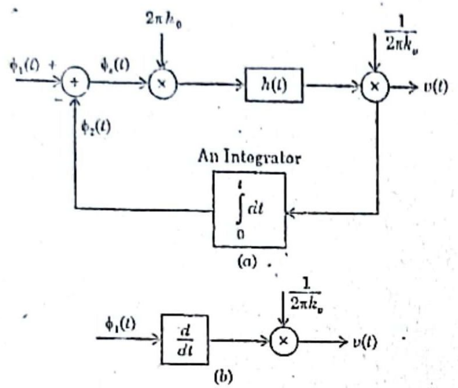



*Fig. 4.44. Illustration of an equivalent Model of PLL.*

Now, substituting the values of $H(f)$ from equation (4.110), in equation (4.109), we obtain

$$V(f) = \frac{k_o}{k_v}\left\{\frac{jf}{k_o}L(f)\right\}\Phi_e(f) = \frac{jf}{k_o}L(f)\Phi_e(f) \qquad ...(4.111)$$

Substituting the values of $\Phi_e(f)$ from equation (4.107), in equation (4.111), we have

$$V(f) = \frac{jf}{k_v}L(f)\Phi_e(f) = \frac{jf}{k_v}L(f)\frac{1}{1+L(f)}\Phi_1(f)$$

or

$$V(f) = \left(\frac{jf}{k_v}\right)\left(\frac{L(f)}{1+L(f)}\right)\Phi_1(f) \qquad ...(4.112)$$

If $|L(f)| >> 1$ then equation (4.112) may be approximated as

$$V(f) = \left(\frac{jf}{k_v}\right)[1]\,\Phi_1(f) = \left(\frac{jf}{k_v}\right)\Phi_1(f) \qquad ...(4.113)$$

The corresponding time-domain representation of equation (4.113) can be obtained by taking inverse Fourier transform of both sides of equation (4.113). Hence, we have

$$\text{Inverse FT }[V(f)] = \text{Inverse } FT\left[\left(\frac{jf}{k_v}\right)\Phi_1(f)\right]$$

or

$$v(t) = \frac{1}{2\pi k_v}\frac{d\phi_1(t)}{dt} \qquad ...(4.114)$$

**Conclusion**

Here, it can be concluded that if the magnitude of $L(f)$ is very large for all frequencies of interest, then PLL may be modeled as a differentiator with its output scaled by a factor $1/2\pi K_v$. It has been illustrated in figure 4.44 (b). The simplified model of PLL shown in figure 4.44 (b) may be used as an FM demodulator. This can be easily verified by substituting the value of $\phi_1(t)$ from equation $\phi_1(t) = 2\pi k_f \int_0^t x(t)\,dt$ into equation (4.114).



Therefore, we have

$$v(t) = \frac{1}{2\pi k_v}\frac{d\phi_1 t}{dt} = \frac{1}{2\pi k_v}\frac{d}{dt}\left\{2\pi k_f \int_0^t x(t)\,dt\right\}$$

or

$$v(t) = \frac{2\pi k_f}{2\pi k_v}\frac{d}{dt}\left\{\int_0^t x(t)\,dt\right\} = \frac{k_f}{k_v}x(t) \qquad ...(4.115)$$

Hence, we can say that output $v(t)$ of PLL is approximately same except for a scaling factor $k_f/k_v$, as the original base-band or modulating signal $x(t)$ and the frequency demodulation is performed.

**Note:**

*It may be noted that the incoming FM wave can have much wider bandwidth than that of the loop filter with transfer function $H(f)$. Here $H(f)$ is restricted to baseband. The control signal of the VCO has a bandwidth of the baseband signal whereas the output of the VCO is a wide-band FM wave. The instantaneous frequency of the WBFM wave tracks the incoming FM. The complexity of a PLL is obtained by the transfer function $H(f)$ of the loop filter. If $H(f) = 1$ then PLL is called simplest PLL. This means that there is no loop filter and the PLL is referred to as first-order PLL. The order of the PLL is determined by the order of the denominator polynomial of the closed loop transfer function. The transfer function $H(f)$ determines the output Fourier transform $V(f)$ in terms of input Fourier transform $\Phi_1(f)$. It is given by equation (4.113).*

## MISCELLANEOUS SOLVED EXAMPLES

**EXAMPLE 4.20.** Determine the permissible range in maximum modulation index for

(i) Commerical FM which has 30 Hz to 15 kHz modulating frequencies.

(ii) narrowband FM system which allows maximum deviation of 10 kHz and 100 Hz to 3 kHz modulating frequencies.

**Solution :** (i) The maximum deviation in commerical $FM$ is given as

$$\Delta f = 75 \text{ kHz}$$

Modulation index in FM is

$$m_f = \frac{\Delta f}{f_m} \qquad ...(i)$$

Modulation Index for commerical FM at $f_m = 30$ Hz is

$$m_f = \frac{\Delta f}{f_m} = \frac{75 \times 10^3}{30} = 2500$$

Modulation index for commerical FM at $f_m = 15$ kHz is

$$m_f = \frac{\Delta f}{f_m} = \frac{75 \times 10^3}{15 \times 10^3} = 5$$

Hence, the modulation index for commerical FM varies between 2500 and 5.

(ii) For a given Narrowband FM system, the maximum frequency deviation is given as $\Delta f =$ 10 kHz.

Hence, modulation index for a given NBFM system varies between

$$m_f = \frac{\Delta f}{f_m} = \frac{10\times 10^3}{100} = 100 \text{ and}$$

$$m_f = \frac{\Delta f}{f_m} = \frac{10\times 10^3}{3\times 10^3} = 3.33 \quad \text{Ans.}$$

**EXAMPLE 4.21.** A 100 MHz carrier wave has a peak voltage of 5 volts. The carrier is frequency modulated (FM) by a sinusoidal modulating signal or wave form of frequency 2 kHz such that the frequency deviation $\Delta f$ is 75 kHz. The modulated waveform passes through zero and is increasing at $t = 0$. Determine the expression for the modulated carrier waveform.

**Solution :** Because the frequency modulated carrier waveform passes through zero and is increasing at $t = 0$, therefore, the FM signal must be sine wave (signal). Thus

$$s(t) = A\sin\left[2\pi f_c t + m_f \sin(2\pi f_m t)\right] \quad ...(i)$$

where $m_f$ = modulation index of FM wave = $\frac{\Delta f}{f_m}$

Given that $f_c$ = Carrier wave frequency = $100 \times 10^6 = 10^8$ Hertz

$\Delta f$ = frequency deviation = 75 kHz = $75 \times 10^3$ Hertz

$f_m$ = modulating frequency = 2 kHz = $2 \times 10^3$ Hertz

$A$ = peak voltage of carrier wave = 5 volt.

Now, $$m_f = \frac{\Delta f}{f_m} = \frac{75\times 10^3}{2\times 10^3} = 37.5$$

Substituting all the above values in equation (i), we get

$$s(t) = 5\sin\left[2\pi\times 10^8 t + 37.5\sin(2\pi\times 2\times 10^3 t)\right]$$

or $$s(t) = 5\sin\left[2\pi\times 10^8 t + 37.5\sin(4\pi\times 10^3 t)\right] \quad \text{Ans.}$$

**EXAMPLE 4.22.** A carrier wave of frequency 1 GHz and amplitude 3 volts is frequency modulated (FM) by a sinusoidal modulating signal frequency of 500 Hz and of peak amplitude 1 volt. The frequency deviation $\Delta f$ is 1 kHz. The level of the modulating waveform (signal) is changed to 5 volt peak and the modulating frequency is changed to 2 kHz. Obtain the expression for the new modulated waveform (FM).

**Solution :** We know that the FM wave is given by the expression

$$s(t) = A\cos\left[2\pi f_c t + m_f \sin(2\pi f_m t)\right]$$

where $m_f$ = Modulation index of FM wave = $\frac{\Delta f}{f_m}$

and $\Delta f$ = frequency deviation = $k_f A_m$

$k_f$ = Sensitivity of Frequency modulator

$A_m$ = Amplitude of the modulating signal

Given that $f_c$ = 1 MHz = $1 \times 10^6$ Hz

$A$ = 3 volt

$A_m$ = 1 volt

and $\Delta f$ = 1 kHz



Therefore, $k_f$ can be found as

$$k_f = \frac{\Delta f}{f_m} = \frac{1\times 10^3}{1} = 10^3 \text{ Hz/volt}$$

Now, for the second case, we have

when, $A_m$ = 5 volt and $f_m$ = 2 kHz

Modulation Index will be

$$m_f = \frac{\Delta f}{f_m} = \frac{10^3 \times 5}{2\times 10^3} = 2.5$$

The desired FM signal can be expressed by

$$s(t) = A\cos\left[2\pi f_c t + m_f \sin(2\pi f_m t)\right]$$

Substituting all the values, we get

$$s(t) = 3\cos\left[2\pi 10^6 t + 2.5\sin(2\pi\times 2\times 10^3 t)\right]$$

or $$s(t) = 3\cos\left[2\pi 10^6 t + 2.5\sin(4\pi\times 10^3 t)\right] \quad \text{Ans.}$$

**EXAMPLE 4.23.** Given a signal

$$s(t) = \cos(2\pi f_c t) + 0.2\cos(2\pi f_m t)\sin(2\pi f_c t)$$

**(i) Prove that $s(t)$ is a combination of AM-FM signal**

**(ii) Draw the phasor diagram at $t = 0$.**

*(U.P. Tech., Sem., Exam., 2004-05) (05 marks)*

**Solution :** (i) The given signal $s(t)$ can be modified in the following form :

$$s(t) = \cos(2\pi f_c t) + 0.2\cos(2\pi f_m t)\sin(2\pi f_c t)$$

$$s(t) = \left[1 + \{0.2\cos(2\pi f_m t)\}^2\right]^{1/2} \times \cos\left[\left\{2\pi f_c t - \tan^{-1}(0.2\cos(2\pi f_m t))\right\}\right]$$

$$s(t) \simeq \left[1 + \frac{0.04}{2}\cos^2(2\pi f_m t)\right]\cos\left[(2\pi f_c t) - 0.2\cos(2\pi f_m t)\right]$$

or $$s(t) = \left[1 + 0.01\cos(4\pi f_m t)\right]\cos\left[(2\pi f_c t) - 0.2\cos(2\pi f_m t)\right] \quad ...(i)$$

*From equation (i), we know that the amplitude as well as the instantaneous phase angle of the given signal changes in accordance with the modulating or message signal. Hence, the given signal s(t) is a combination of AM-FM signal.*

(ii) For the construction of phasor diagram, we can express the signal $s(t)$ as under :

$$s(t) = \cos(2\pi f_c t) + 0.2\cos(2\pi f_m t)\sin(2\pi f_c t)$$

or $$s(t) = \cos(2\pi f_c t) + \frac{0.2}{2}\sin\left[2\pi(f_c + f_m)t\right] + \frac{0.2}{2}\sin\left[2\pi(f_c - f_m)t\right] \quad ...(ii)$$

Phasor diagram for equation (ii) has been shown in figure 4.45.

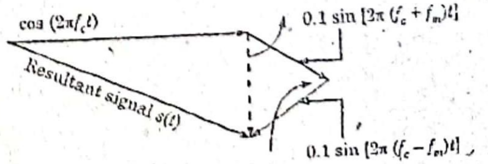



*Fig. 4.45. Illustration of Phasor diagram for example 4.19.*

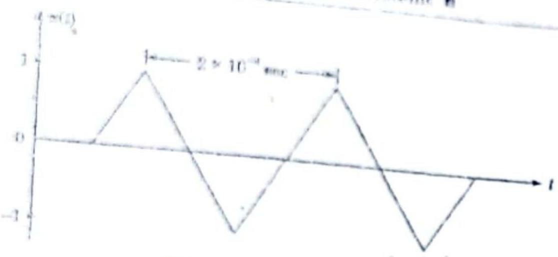



Fig. 4.47. for example 4.15.

**Solution :** We know that for FM, the instantaneous frequency is given by

$$f_i = f_c + k_f x(t) = f_c + 10^5 x(t)$$

or

$$[f_i]_{max} = f_c + 10^5 [x(t)]_{max} \quad ...(i)$$

From figure 4.30, we have $x(t)_{max} = 1$. ...(ii)

Substituting $[x(t)]_{max} = 1$ in equation (ii), we get

$$[f_i]_{max} = f_c + 10^5 [x(t)]_{max} = f_c + 10^5 \times 1$$

$$= f_c + 10^5 = 100 \times 10^6 + 10^5$$

or

$$[f_i]_{max} = 100.1 \times 10^6 \text{ Hertz} = 100.1 \text{ MHz}$$

Again, from figure 4.30, $[x(t)]_{min} = -1$

Substituting the $[x(t)]_{min} = -1$ in equation (i), we get

$$[f_i]_{min} = 100 \times 10^6 + 10^5(-1) = 100 \times 10^6 - 0.1 \times 10^6$$

$$= 99.9 \times 10^6 \text{ Hertz} = 99.9 \text{ MHz}$$

Frequency deviation $\Delta f$ can be found as

$$\Delta f = [f_i]_{max} - f_c = (101.1 - 100) \text{ MHz}$$

$$= 0.1 \text{ MHz} = 10^5 \text{ Hertz}$$

The bandwidth of the resulting signal may be calculated using Carson's rule as under :

$$BW = 2\Delta f\left(1 + \frac{f_m}{\Delta f}\right) = 2\Delta f\left(1 + \frac{1}{D}\right) \quad ...(iii)$$

where $D = \dfrac{\Delta f}{f_m}$

$f_m$ = Essential bandwidth of the modulating signal $x(t)$. This can be determined by expanding $x(t)$ shown in Fig. 4.30 in Fourier series.

The modulating signal $x(t)$ can be expanded as

$$x(t) = \sum_{n=-\infty}^{\infty} C_n \cos(2\pi n f_o t) \quad ...(iv)$$

where,

$$f_o = \frac{1}{T_o} = \frac{1}{2 \times 10^{-4}} = 5000 \text{ Hz} = 5 \text{ kHz}$$

and Fourier series coefficient

$$C_n = \begin{cases} \dfrac{8}{\pi^2 n^2} ; \text{for odd values of } n \\ 0 \quad ; \text{for even values of } n \end{cases} \quad ...(v)$$



Now, it may be observed that the amplitudes of the various harmonic frequency components present in $x(t)$ decreases rapidly with increasing $n$. The powers of third harmonic and fifth harmonic are 1.21% and 0.16% of the power of the fundamental frequency. It is thus reasonable to assume that the essential bandwidth of modulating signal $x(t)$ is $3f_o = 3 \times 5 \times 10^3 = 1.5 \times 10^4$ Hertz.

Hence, the bandwidth of frequency modulated (FM) wave is given by

$$BW = 2\Delta f\left(1 + \frac{f_m}{\Delta f}\right) = 2 \times 10^5\left(1 + \frac{1.5 \times 10^4}{2 \times 10^5}\right)$$

or

$$BW = 2 \times 10^5\left(1 + \frac{1.5}{20}\right) = 230 \times 10^3 \text{ Hz} = 230 \text{ kHz}$$

For the phase modulated case, we have

$$\theta_i(t) = 2\pi f_c t + k_p x(t)$$

Instantaneous frequency is found as

$$f_i = \frac{1}{2\pi}\frac{d\theta_i(t)}{dt} = \frac{1}{2\pi}\frac{d}{dt}\left[2\pi f_c t + k_p x(t)\right]$$

or

$$f_i = f_c + \frac{k_p}{2\pi}\frac{d}{dt}[x(t)] \quad ...(vi)$$

Maximum and minimum values of $f_i$ will be

$$[f_i]_{max} = 100 \times 10^6 + \frac{10\pi}{2\pi}\frac{d}{dt}[x(t)]\Big|_{max}$$

or $[f_i]_{max} = 100 \times 10^6 + 5 \times 20{,}000$

or $[f_i]_{max} = 100 \times 10^6 + 0.1 \times 10^6$

or $[f_i]_{max} = 100.1 \times 10^6$ Hertz

or $[f_i]_{max} = 100.1$ MHz

Further, $[f_i]_{max} = 99.9$ MHz

Hence, frequency deviation will be

$$\Delta f = [f_i]_{max} - f_c = [100.1 - 100] \text{ MHz} = 0.1 \text{ MHz} = 10^5 \text{ Hertz.}$$

**Note :**

The values of $\dfrac{d[m(t)]}{dt}\Big|_{max}$ has been obtained by noting that $\dfrac{d[m(t)]}{dt}$ switches back and forth between −20,000 and 20,000.

The bandwidth of the PM signal is given by

$$BW_{PM} = 2\Delta f\left(1 + \frac{f_m}{\Delta f}\right)$$

or

$$BW_{PM} = 2 \times 10^5\left(1 + \frac{1.5 \times 10^4}{2 \times 10^5}\right) = 230 \text{ kHz} \quad \textbf{Ans.}$$

**EXAMPLE 4.27.** Find the bandwidth of commerical FM transmission assuming frequency deviation $\Delta f$ = 75 kHz and bandwidth of modulating signal $x(t)$, $f_m$ = 15 kHz.

**Solution :** The deviation ratio for commerical FM transmission is given by

$$D = \frac{\Delta f}{...} = \frac{75 \text{ kHz}}{...}$$

Bandwidth of FM signal can be calculated by using Carson's rule as

$$BW = 2\Delta f\left(1+\frac{f_m}{\Delta f}\right) = 2\Delta f\left(1+\frac{1}{D}\right)$$

$$BW = 2\times 75\times 10^3\left(1+\frac{1}{5}\right) = 150\times 10^3\left(\frac{6}{5}\right)$$

or

$$BW = 180\times 10^3 \text{ Hertz} = 180 \text{ kHz}$$

or

Also, using universal curve, replacing $m_f$ by $D$, we get

$$BW = 3.2\times\Delta f \text{ for } D = 5 = 3.2\times 75\times 10^3$$

$$= 240\times 10^3 \text{ Hertz} = 240 \text{ kHz}$$

Now, we can find percentage of underestimation of bandwidth by using Carson's rule as

% under estimation of bandwidth will be

$$= \frac{240-180}{240}\times 100 = \frac{60}{240}\times 100 = 25\%$$

It means that Carson's rule underestimates the bandwidth by 25% as compared with the result obtained from the universal curve. **Ans.**

**EXAMPLE 4.28.** A rule of bandwidth for FM signal is sometimes used as

$$BW = (2m_f + 1)f_m$$

Find the fraction of the signal power that is included in that frequency band. Assume that $m_f = 1$ and 10.

**Solution :** The bandwidth for FM signal can be calculated on the basis of 98% power requirement given by Carson's rule as

$$BW = 2\Delta f\left(1+\frac{1}{m_f}\right) = 2(m_f+1)f_m \quad \text{...(i)}$$

The fraction of signal power included in the frequency band $B$ is

$$P = \frac{B}{BW}\times\left(\frac{98}{100}\right) \quad \text{...(ii)}$$

For $m_f = 1$

$$P = \frac{B}{BW}\times\left(\frac{98}{100}\right) = \frac{B}{BW}\times 0.98$$

or

$$P = \frac{(2m_f+1)f_m}{2(m_f+1)f_m}\times 0.98 = \frac{2\times1+1}{2(1+1)}\times 0.98$$

or

$$P = \frac{2\times1+1}{2(1+1)}\times 0.98 = \frac{3}{4}\times 0.98 = 73.5\%$$

for $m_f = 10$

$$P = \frac{B}{BW}\times 0.98 = \frac{(2m_f+1)f_m}{2(m_f+1)f_m}\times 0.98 = \frac{(2\times10+1)}{2(10+1)}\times 0.98$$

or

$$P = \frac{21}{22}\times 0.98 = 97.6\% \quad \textbf{Ans.}$$

**EXAMPLE 4.29.** A carrier is frequency modulated (FM) by a sinusoidal modulating signal $x(t)$ of frequency 2 kHz, it results in a frequency deviation $\Delta f$ of 5 kHz. Find the bandwidth occupied by the FM waveform. The amplitude of the modulating sinusoid is increased by a factor of 3 and its frequency lowered by 1 kHz. Find the new bandwidth.

**Solution :** Given that

$$f_m = 2 \text{ kHz and}$$

$$\Delta f = 5 \text{ kHz}$$



Hence, the bandwidth of the FM signal will be

$$BW = 2(\Delta f + f_m)$$

or

$$BW = 2(5+2)$$

or

$$BW = 14 \text{ kHz}$$

When the amplitude of the modulating signal is trippled, then the frequency deviation increases three times.

Therefore, $\Delta f = 3\times 5 \text{ kHz} = 15 \text{ kHz}$

Also, $f_m = 1 \text{ kHz}$

The new bandwidth will be

$$BW' = 2(\Delta f + f_m) = 2(15+1)$$

or

$$BW' = 32 \text{ kHz} \quad \textbf{Ans.}$$

**EXAMPLE 4.30.** Determine the relative power of the carrier wave and side frequencies when modulation index $m_f = 0.20$ for 10 KW FM transmitter.

**Solution :** For $m_f = 0.20$

$$J_0(m_f) = 0.99$$

(See the table 4.1 of Bessel Functions)

Thus, only significant side frequency pair is $f_c \pm f_m$ with relative amplitude,

$$J_1(m_f) = 0.099$$

Hence, the carrier power will be

$$P_c = J_0^2(m_f)\times \text{Power of FM transmitter}$$

$$P_c = (0.99)^2\times 10\times 10^3$$

$$P_c = 9.8\times 10^3 \text{ watt}$$

$$P_c = 9.8\,K \text{ watt} \quad \textbf{Ans.}$$

and the power in each side frequency is expressed as

$$\therefore P_{s1} = P_{s2}$$

$$= J_1^2(m_f)\times \text{Power of FM transmitter}$$

$$= (0.099)^2\times 10\times 10^3 = 98 \text{ watt.} \quad \textbf{Ans.}$$

**EXAMPLE 4.31.** In an FM, system a 7 kHz modulating (or baseband) signal modulates 107.6 MHz carrier wave so that the frequency deviation is 50 kHz. Find

(i) carrier swing in the FM signal and modulating index $m_f$

(ii) the highest and lowest frequencies attained by the FM signal.

**Solution :** (i) Given that frequency deviation

$$\Delta f = 50 \text{ kHz} = 50\times 10^3 \text{ Hertz}$$

Carrier swing in FM signal will be

$$= 2\Delta f$$

$$= 2\times 50\times 10^3 \text{ Hz} = 100 \text{ kHz}$$

Modulation Index of FM wave,

$$m_f = \frac{\Delta f}{f_m} = \frac{50\times 10^3 \text{ Hertz}}{7\times 10^3 \text{ Hertz}} = 7.143$$

(ii) The upper or highest frequency attained by FM signal will be

$$= f_c + \Delta f$$
$$= 107.6 \times 10^6 + 50 \times 10^3$$
$$= 107.65 \text{ MHz}$$

The lower or lowest frequency attained by FM signal will be

$$= f_c - \Delta f$$
$$= 107.6 \times 10^6 - 50 \times 10^3$$
$$= 107.55 \text{ MHz} \quad \text{Ans.}$$

**EXAMPLE 4.32.** **Determine the frequency deviation $\Delta f$ and carrier swing for an FM signal which has a carrier frequency of 100 MHz and whose upper frequency is 100.007 MHz when modulated by a particular modulating signal or wave. Also find the lowest frequency reached by the FM wave.**

**Solution :** We know that frequency deviation $\Delta f$ is defined as the maximum change in frequency of the modulated signal away from the carrier frequency $f_c$.

This means that

i.e., $\Delta f = f_u - f_c = [100.007 - 100.000]$ MHz

$$= 0.007 \text{ MHz} = 7 \times 10^3 \text{ Hz} = 7 \text{ kHz}$$

Carrier swing $= 2\Delta f = 2 \times 7 = 14$ kHz

The lowest frequency $f_L$ reached by the modulated FM wave is equal to the difference of the frequency deviation from the carrier frequency.

$$f_L = f_c - \Delta f = (100.00 - 0.007) \text{ MHz}$$

or $f_L = 99.993$ MHz  Ans.

**EXAMPLE 4.33.** **Determine the modulation index $m_f$ of an FM signal which is being broadcast in the 88 – 108 MHz band. This FM wave has a carrier swing of 125 kHz.**

**Solution :** We know that the frequency deviation is given by

$$\Delta f = \frac{\text{Carrier swing}}{2} = \frac{125 \times 10^3}{2} \text{ Hz}$$

or $\Delta f = 62.5 \times 10^3$ Hz $= 62.5$ kHz

Since, maximum Frequency deviation for the FM broadcast band is 75 kHz, therefore

$m_f$ = modulation index

$$m_f = \frac{\Delta f}{f_m} = \frac{62.5}{75} = 83.3\% \quad \text{Ans.}$$

**EXAMPLE 4.34.** **The modulating signal in an FM wave is 500 Hz with amplitude 3.2 volt and frequency deviation is 6.4 kHz. If the audio frequency voltage is now increased to 8.4 volt, determine the new frequency deviation and modulation index. If the audio frequency voltage is raised to 20 volts while the audio frequency is dropped to 200 Hz, determine the frequency deviation and modulation index.**

**Solution :** Given that $\Delta f = 64$ kHz

$f_m = 500$ Hz

$V_m = 3.2$ volt



We know that,

Frequency sensitivity, $k_f = \frac{\Delta f_1}{V_{m1}} = \frac{6.4 \text{ kHz}}{3.2 \text{ volt}} = 2$ kHz/volt

Frequency deviation for $V_m = 8.4$ volt will be determined as

$$\Delta f_2 = k_f V_{m2} = 2 \times 8.4 = 16.8 \text{ kHz.}$$

Frequency deviation for $V_m = 20$ volt will be expressed as

$$\Delta f_3 = k_f V_{m3} = 2 \times 20 = 40 \text{ kHz}$$

It may be observed that the change in modulating frequency made no difference to the frequency deviation because it is independent of the modulating frequency.

The modulation indices can be calculated as under :

$$m_1 = \frac{\Delta f_1}{f_{m1}} = \frac{6.4 \text{ kHz}}{0.5 \text{ kHz}} = 12.8$$

$$m_2 = \frac{\Delta f_2}{f_{m2}} = \frac{16.8 \text{ kHz}}{0.5 \text{ kHz}} = 33.6$$

$$m_3 = \frac{\Delta f_3}{f_{m3}} = \frac{40 \text{ kHz}}{0.2 \text{ kHz}} = 200 \quad \text{Ans.}$$

**EXAMPLE 4.35.** **An FM wave is given by**

**$s(t) = 20 \sin (6 \times 10^8 t + 7 \sin 1250t)$. Determine**

**(i) The carrier and modulating frequencies, the modulation index, and the maximum deviation.**

**(ii) Power dissipated by this FM wave in a 100 ohm resistor.**

**Solution :** (a) The standard expression for FM is

$$s(t) = A \sin [\omega_c t + m_f \sin (\omega_m t)] \quad \text{...(i)}$$

Given expression is

$$s(t) = 20 \sin [6 \times 10^8 t + 7 \sin 1250t] \quad \text{...(ii)}$$

On comparing equations (i) and (ii), we obtain

$$f_c = \frac{\omega_c}{2\pi} = \frac{6 \times 10^8}{2\pi} = 95.5 \text{ MHz}$$

$$f_m = \frac{\omega_m}{2\pi} = \frac{1250}{2\pi} = 199 \text{ Hertz}$$

and $m_f = 7$, $\Delta f = m_f f_m = 7 \times 199 = 1393$ Hz  Ans.

(ii) Power dissipated by the given FM wave in 100 ohm resistor can be calculated as under :

$$P = \frac{\left(\frac{A}{\sqrt{2}}\right)^2}{R} = \frac{\left(\frac{20}{\sqrt{2}}\right)^2}{100} = 2 \text{ watt.} \quad \text{Ans.}$$

**EXAMPLE 4.36.** **Find the instantaneous frequency in hertz of each of the following signals :**

**(i)** $10 \cos\left(200\pi t + \frac{\pi}{3}\right)$

**(ii)** $10 \cos(20\pi t + \pi t^2)$

**(iii)** $\cos 200\pi t \cos(5 \sin 2\pi t) + \sin 200\pi t \sin(5 \sin 2\pi t)$

**EXAMPLE 4.55.** An angle-modulated signal is given by

$$x_c(t) = 5\cos\,[2\pi\,(10^6)t + 0.2\,\pi\,t]$$

Can you identity whether $x_c(t)$ is a PM or and FM signal ?

**Solution :** For angle modulation, the modulated carrier is represented by,

$$x_c(t) = A\cos\,[\omega_c t + \phi(f)]$$

In case of PM, $\phi(t) = k_p m(y)$,

where $m(t)$ is the message signal and $k_p$ is phase duration constant.

and for FM, $\dfrac{d\phi(t)}{dt} = k_f m(t)$

where $k_f$ is frequency deviation constant.

Given $x_c(t) = 5\cos\,[2\pi\,(10^6 t) + 0.2\cos 200\,\pi t)$.

Now, if $m(t) = a_m \cos\,(200\,\pi t)$

then, $\phi(t) = K_p a_m \cos\,(200\,\pi t) = 0.2\cos\,(200\,\pi t)$

and then $x_c(t)$ = PM signal.

But, if $m(t) = a_m \sin\omega_m t$

then $\phi(t) = k_f \displaystyle\int_{t_0}^{t} m(\lambda)\,d\lambda + \phi(t_0) = k_f \int_{-\infty}^{t} m(\lambda)\,d\lambda + \phi(-\infty)$

or $\phi(t) = k_f \displaystyle\int_{-\infty}^{t} m(\lambda)\,d\lambda$ [At $t_0 = -\infty, \phi(t_0) =$ ]

or $\phi(t) = \dfrac{k_f a_m}{\omega_m}\cos\omega_m t$

and then $x_c(t)$ is a FM signal. Hence it can be either PM or an FM signal.

**EXAMPLE 4.56.** A baseband signal $m(t)$ modulates a carrier to produce the following angle modulated signal.

$A_c\cos\,[2\pi \times 10^8 t + k_p m(t)]$, where $m(t)$ is shown in the figure 4.53.

Determine the value of $k_p$ so that the peak-to-peak frequency deviation of the carrier is 100 kHz. *(Gate Examination-1999)*

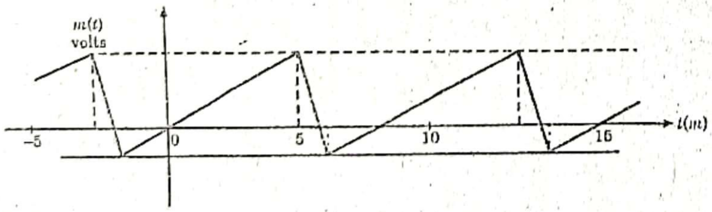



*Fig. 4.53.*

**Solution : Given**

$x(t) = A_c\cos[2\pi \times 10^8 t + k_p m(t)]$ as an angle modulated signal.

Since, phase deviation $\phi(t) = k_p m(t)$ *i.e.*, proportional to message signal, therefore, it is an phase-modulated signal.



Now, instantaneous frequency is given by

$$\omega_i = \omega_c + k_p \frac{dm(t)}{dt}$$

Then, maximum frequency deviation will be

$$|\omega_i - \omega_c|_{max} = k_p \left|\frac{dm(t)}{dt}\right|_{max}$$

or $$100 \times 2\pi = k_p \left|\frac{dm(t)}{dt}\right|_{max} \quad ...(i)$$

from the given figure, we have

$$\left|\frac{dm(t)}{dt}\right|_{max} = \frac{17.5}{7} = 2.5$$

then, from equation (i), we get

$$k_p = \frac{100 \times 2\pi}{2.5} = 80\,\pi \quad \textbf{Ans.}$$

**EXAMPLE 4.57.** An angle modulated signal with carrier frequency $\omega_c = 2 \times 10^6$ rad/sec is given by

$$s(t) = \cos 2\pi(2 \times 10^6 t + 30\sin 150t + 40\cos 150t)$$

then,

(*i*) Determine the maximum frequency deviation

(*ii*) Find maximum phase deviation, and

(*iii*) Find the bandwidth of $s(t)$.

**Solution :** (*i*) We know that instaneous frequency is given by

$$\omega_i = \frac{d}{dt}\theta(t) = \frac{d}{dt}[2\pi(2 \times 10^6 t + 30\sin 150t + 40\cos 150t)]$$

$$= 2\pi \times 10^6 + 60\pi \times 150\,.\cos 150t - 80\pi \times 150\,.\sin 150t$$

or $$\Delta\omega = \omega_i - \omega_c = 60\pi \times 150.\cos 150t - 80\pi \times 150.\sin 150t$$

$$= 3000\pi\,[3\cos 150t - 4\sin 150t]$$

$$\Delta\omega = 15000\pi\,.\cos\,(150t + \alpha),$$

**EXAMPLE 4.58.** An angle-modulated signal is given as below:

$$s(t) = 20\cos(\omega_c t + 4\sin\omega_m t)$$

Assuming as a PM signal and $f_m = 2$ kHz, calculate the modulation index and bandwidth when (*i*) $f_m$ is increased two times (*ii*) $f_m$ is decreased by half.

**Solution :** A PM signal is given as,

$$s(t) = A_m\cos[\omega_c t + k_p\,m(t)] \quad ...(i)$$

or $$s(t) = 20\cos[\omega_c t + 4\sin\omega_m t]$$

Hence, $$m(t) = a_m \sin\omega_m t \quad ...(ii)$$

then, $$s(t) = 20\cos[\omega_c t + k_p a_m \sin\omega_m t)$$

Comparing equations (*i*) and (*ii*), we get

$$\beta = k_p a_m = 4$$

Also, $f_B = 2(\beta + 1)f_m = 2(4 + 1).2 = 20$ kHz

(*i*) When, $f_{m1} = 2f_m = 4$ kHz

$f_B = 2(4 + 1)\cdot 4 = 40$ kHz **Ans.**

(i) When, $f_{m2} = \frac{f_m}{2} = 1$ kHz

$f_B = 2(\beta + 1) \cdot f_{m2} = 2(4+1).1 = 10$ kHz **Ans.**

**EXAMPLE 4.59. A carrier wave of frequency 100 MHz is frequency modulated by a sinusoidal wave of amplitude 20 V and frequency 100 kHz. The frequency sensitivity of the modulator is 25 kHz per volt. Determine approximate bandwidth of FM signal.** *(U.P. Tech, Sem. Exam.2002-03) (3 Marks)*

**Solution :** Given that $f_c = 100$ MHz, $V_m = 100$ kHz $k_f = 25$ kHz/volt.

We know that the approximate bandwidth is given by,

$$BW = 2[\Delta f + f_m]$$

But $\Delta f = k_f \times V_m = 25 \text{ kHz/V} \times 20 \text{ V}$ ... (i)

or $\Delta f = 500$ kHz or 0.5 MHz ... (ii)

or BW = 1200 kHz or 1.2 MHz **Ans.**

**EXAMPLE 4.60. A modulating signal $A_m$ sin $(2\pi f_m t)$ plus a bias $V_b$ is applied to a pair of varactor diode connected across the parallel combination of 200 μH inductor and 100 pF capacitor as**

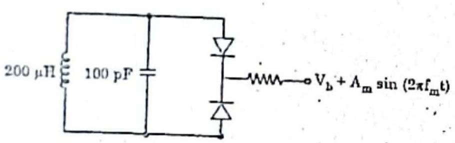



*Fig.4.54.*

**Shown in figure 4.54. The capacitor of each varactor diode is related to voltage (in volts) applied across its electrodes by**

$$C = 100\ V^{1/2} \text{ pF}$$

**The output is applied to a frequency multiplier to produce FM signal with a carrier frequency 64 MHz and modulation index of 5. Find the bias voltage $V_b$ and amplitude $A_m$ of modulating wave. Given that the unmodulated frequency of oscillation is 1 MHz and $f_m$ = 10 kHz.**

*(U.P. Tech, Sem. Exam. 2002-2003) (6.5 Marks)*

**Solution :** Figure 4.55 shows the equivalent circuit. Here, $C_1$ and $C_2$ are the capacitance values offered by the varactor diode. Let $C_1 = C_2 = C_V$

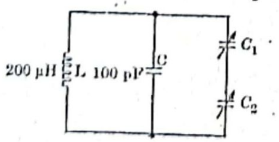



*Fig. 4.55.*

Now, we have

$$C_{eq} = C \parallel (C_1 \text{ series } C_2) = C + \frac{C_1 C_2}{C_1 + C_2}$$

But $C_1 = C_2 = C_V$



Therefore, $C_{eq} = C + \frac{C_V^2}{2C_V} = C + \frac{C_V}{2}$ ... (i)

Let the modulating signal $A_m$ sin $(2\pi f_m t)$ be absent

Here, varactor capacitance $C_V = 100\ V^{1/2} = 100\sqrt{V_b}$

Substituting in equation (i), we get

$$C_{eq} = C + \frac{100\sqrt{V_b}}{2} = C + 50\sqrt{V_b} \quad \text{... (ii)}$$

The unmodulated frequency is given by

$$f_c = \frac{1}{2\pi\sqrt{LC_{eq}}}$$

or
$$1 \times 10^6 = \frac{1}{2\pi\left[200\times10^{-6}\times(100\times10^{-12}+50\sqrt{V_b})\right]^{1/2}}$$

Squaring both sides, we have

$$1 \times 10^{12} = \frac{1}{4\pi^2\left[200\times10^{-6}\times(100\times10^{-12}+50\sqrt{V_b})\right]}$$

or $100 \times 10^{-12} + 50\sqrt{V_b} = 1.2665 \times 10^{-10}$

or $50\sqrt{V_b} = 2.6651 \times 10^{-11}$

or $V_b = 2.8412 \times 10^{-25}$ Volts

The required deviation is given by

$$\Delta f = m_f \times f_m = 5 \times 10 \text{ kHz} = 50 \text{ kHz}.$$

Hence, the maximum frequency is 1 MHz + 50 kHz *i.e.*, 1050 kHz

or
$$1050 \times 10^3 = \frac{1}{2\pi\sqrt{LC_{eq}}} = \frac{1}{2\pi\sqrt{200\times10^{-6}\times C_{eq}}}$$

or $C_{eq} = 114.876$ pF.

But $C_{eq} = C + \frac{C_V}{2}$

or $114.876 = 100 + \frac{C_V}{2}$

or $C_V = 7.437$ pF

But $C_V = 100\sqrt{V} = 100\sqrt{V_b + A_m}$

or $7.437 \times 10^{-12} = 100\sqrt{V_b + A_m}$

or $V_b + A_m = 5.53 \times 10^{-27}$

or $A_m = 5.53 \times 10^{-27} - 2.3412 \times 10^{-25}$

or $A_m = -2.7858 \times 10^{-25}$ volts. **Ans.**

**EXAMPLE 4.61. A carrier which attains a peak voltage of 5 V has a frequency of 100 MHz. This carrier is frequency modulated by a sinusoidal**

waveform of frequency 2 kHz to such an extent that frequency deviation from carrier frequency is 75 kHz. The modulated waveform passes through zero and is increasing at time $t = 0$. Write the expression for modulated carrier waveform.

**Solution :** Given that $V_c = 5$ V, $f_c = 100$ MHz, $f_m = 2$ kHz, $\Delta f = 75$ kHz.

We know that the standard expression for an FM wave is given by

$$s(t) = v_{PM} = V_c \sin [\omega_c t + m_f \sin \omega_m t]$$

But $\omega_c = 2\pi f_c = 2\pi \times 100 \times 10^6 = 0.6283 \times 10^9$ rad/s

and $\omega_m = 2\pi f_m = 2\pi \times 2 \times 10^3 = 12.566 \times 10^3$ rad/s

Also, $m_f = \dfrac{\Delta f}{f_m} = \dfrac{75}{2} = 37.5$

Therefore, $s(t) = 5 \sin [0.6283 \times 10^9 t + (37.5 \sin 12.566 \times 10^3 t)]$

This is the required expression.

**EXAMPLE 4.52.** Given an angle modulated signal $x(t) = 20 \cos [(10^6) \pi t + 10 \sin 2\pi (10^3) t]$. Find the maximum phase deviation and the maximum frequency deviation.

**Solution :** The standard expression for FM wave is given by

$$s(t) = V_c \cos [\omega_c t + m_f \sin \omega_m t]$$

Comparing with the given expression, we get

$$m_f = 10$$

The standard expression for PM wave is given by

$$s(t) = V_c \sin [\omega_c t + m_p \sin \omega_m t]$$

The given expression is

$$s(t) = 20 \cos [10^6 \pi t + 10 \sin 2\pi (10^3 t)]$$

or $$s(t) = 20 \cos [10^6 \pi t + 10 \sin 2\pi (10^3 t) + \pi/2]$$

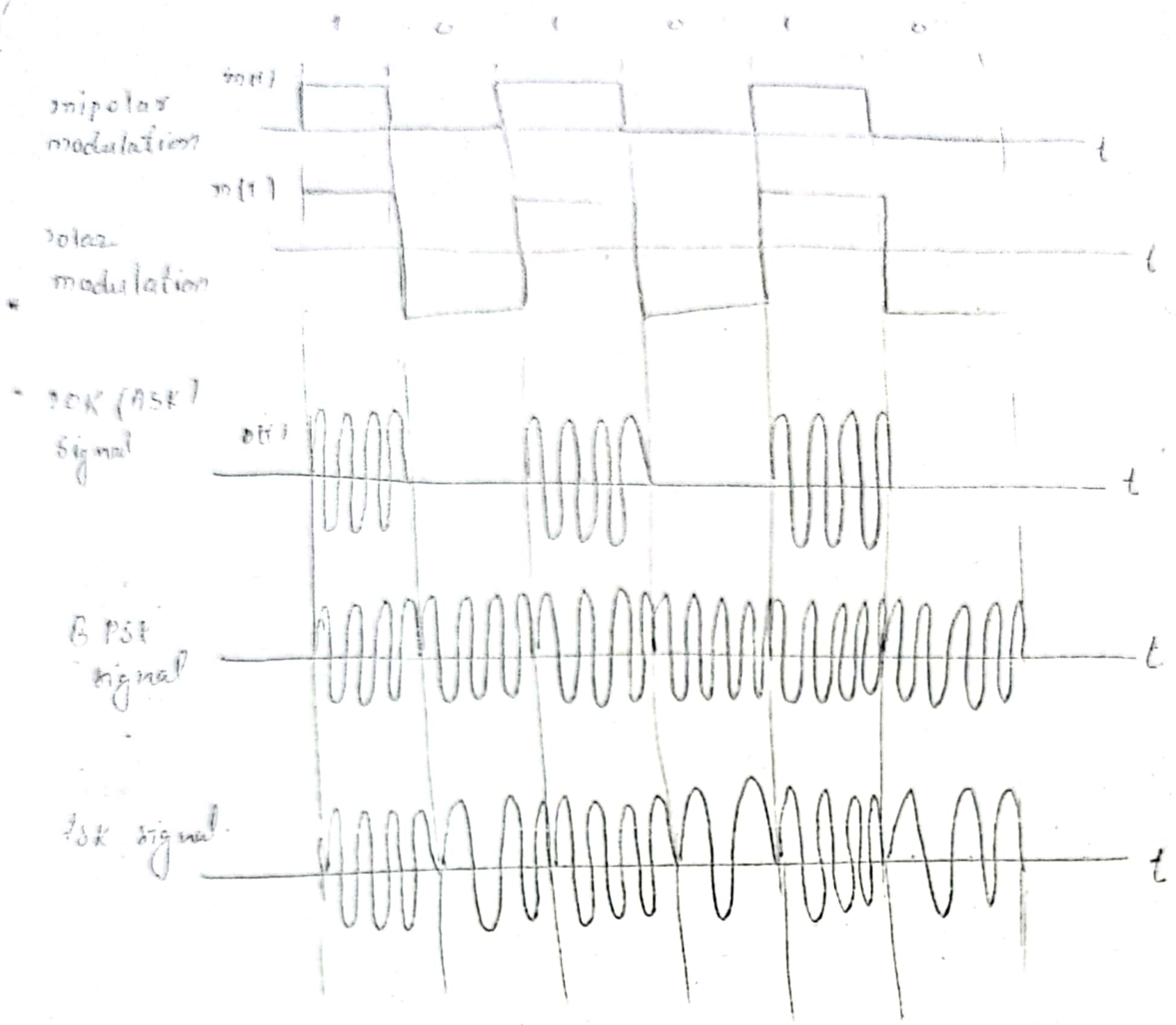
Therefore, maximum phase deviation is $(10 + \pi/2)$ rad = 11.57 rad. **Ans.**

## SUMMARY

1. Angle modulation may be defined as the process in which the total phase angle of a carrier wave is varied in accordance with the instantaneous value of the modulating or message signal while keeping the amplitude of the carrier constant.
2. We can vary this phase angle $\phi$ in two ways and thus there are two types of angle modulation as under :
   (i) Phase Modulation (PM)
   (ii) Frequency Modulation (FM).
3. Phase modulation (PM) is that type of angle modulation in which the phase angle $\phi$ is varied linearly with a baseband or modulating signal $x(t)$ about an unmodulated phase angle $(\omega_c t + \theta_0)$.
4. This means that in Phase Modulation, the instantaneous value of the phase angle is equal to the phase angle of the unmodulated carrier $(\omega_c t + \theta_0)$ plus a time-varying component which is proportional to modulating signal $x(t)$.



5. Frequency modulation is that type of angle modulation in which the instantaneous frequency $\omega_i$ is varied linearly with a message or baseband signal $x(t)$ about an unmodulated carrier frequency $\omega_c$. This means that the instantaneous value of the angular frequency $\omega_i$ will be equal to the carrier frequency $\omega_c$ plus a time-varying component proportional to the baseband signal $x(t)$.
6. $s(t) = A \cos \left[\omega_c t + k_f \int_0^t x(t)\, dt\right]$ which is the required general expression for FM wave.
7. The maximum change in instantaneous frequency form the average frequency $\omega_c$ is called frequency deviation.
8. This maximum change in instantaneous frequency $\omega_i$ from the average or carrier frequency $\omega_c$ depends upon the magnitude and sign of $k_f \cdot x(t)$.
9. Therefore,
   Frequency deviation $\Delta\omega = |k_f \cdot x(t)|_{max}$
10. To get FM by using PM, we first integrate the baseband signal and then apply to the phase modulator.
11. The total variation in frequency from the lowest to the highest point is called carrier swing. Obviously
    The carrier swing = 2 × frequency deviation
    = 2 × $\Delta\omega$
12. The amount of frequency deviation or variation depends upon the amplitude (loudness) of the modulating (audio) signal. This means that louder the sound, greater the frequency deviation and vice versa.
13. The frequency deviation is useful in determining the FM signal bandwidth. Since, a maximum frequency deviation of 75 KHz is allowed for commercial FM broadcast stations using a band of 88 MHz to 108 MHz, therefore approximate FM channel width is 2 × 75 = 150 KHz. Allowing a 25 KHz guardband on either side, the channel width becomes 2(75 + 25) = 200 KHz. This guardband is meant to prevent interference between adjacent channels.
14. In FM broadcast, the highest audio frequency transmitted is 15 KHz.
15. For FM, the modulation index is defined as the ratio of frequency deviation to the modulating frequency.
16. Modulation index, $m_f = \dfrac{\text{Frequency deviation}}{\text{modulation frequency}}$
    or $m_f = \dfrac{\Delta\omega}{\omega_m}$
17. This modulation index may be greater than unity.
18. The term "percent modulation" as it is used in reference to FM refers to the ratio of actual frequency deviation to the maximum allowable frequency deviation. Thus 100% modulation corresponds to 75 kHz for the commercial FM broadcast band and 25 kHz for the commercial FM broadcast band and 25 kHz for television.
19. Percent modulation $M = \dfrac{\Delta f_{actual}}{\Delta f_{max}} \times 100$.
20. When the value of modulation index $m_f$ is quite large, then in FM a large number of sidebands are produced and hence the bandwidth of FM is sufficiently large. This type of FM system is known as wideband FM.


1 0 1 0 1 0
m(t)
unipolar modulation
m(t)
polar modulation
OOK (ASK) signal
BPSK signal
FSK signal
t