Chapter 6: Non Linear Modulation

ANGLE MODULATION

- With Amplitude Modulation System, the amplitude of the carrier signal was varied in some way with the message but with angle modulation the amplitude of the signal would stay constant but varying factor would either be the **frequency** or the **phase** of the carrier.
- Angle modulation encompasses **phase modulation** (PM) and **frequency modulation** (FM). The phase angle of a sinusoidal carrier signal is varied according to the modulating signal.
- In angle modulation, the spectral components of the modulated signal are not related in a simple fashion to the spectrum of the modulating signal. Superposition does not apply and the bandwidth of the modulated signal is usually much greater than the modulating signal bandwidth.

- Angle Modulation is the process in which the frequency or the phase of the carrier signal varies according to the message signal.
- The standard equation of the angle modulated wave is:

$$\mathbf{s(t)=A_c\cos\theta_i(t)}$$

Where, Ac is the amplitude of the modulated wave, which is the same as the amplitude of the carrier signal and θi(t) is the angle of the modulated wave

$$\theta(t) = \omega_c t + \varphi(t) = 2\pi f_c t + \varphi(t).$$

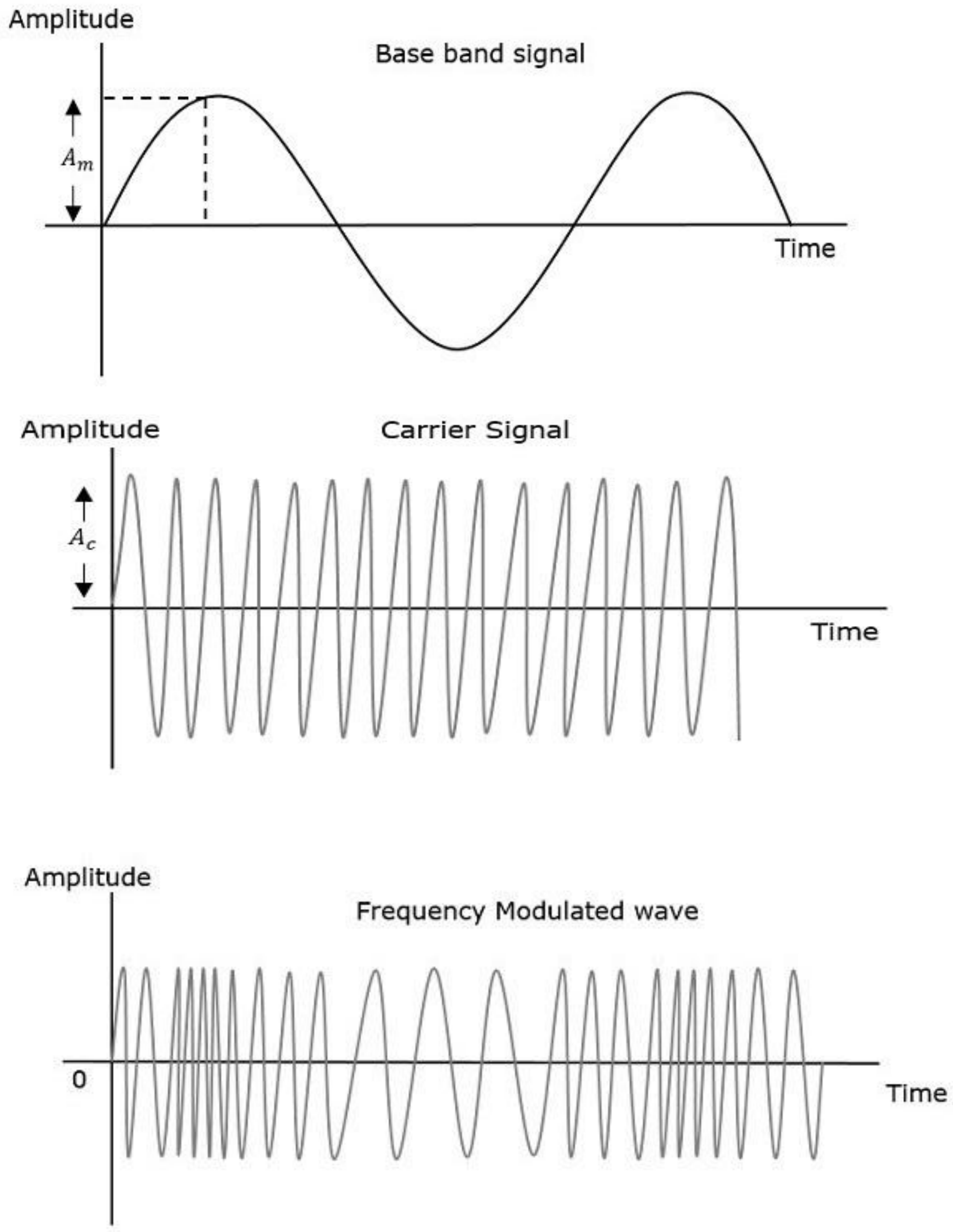
So angle modulation, we can write

$$\mathbf{s(t) = Ac\cos[2\pi f_c t + \varphi(t)]}$$

- Angle modulation is further divided into **frequency modulation** and **phase modulation**.
  - **Frequency Modulation** is the process of varying the frequency of the carrier signal linearly with the message signal.
  - **Phase Modulation** is the process of varying the phase of the carrier signal linearly with the message signal.

Frequency Modulation

- In amplitude modulation, the amplitude of the carrier signal varies. Whereas, in **Frequency Modulation (FM)**, the instantaneous frequency of the carrier signal linearly varies with the modulating signal.
- Hence, in frequency modulation, the amplitude and the phase of the carrier signal remains constant.



- The equation for instantaneous frequency f_i in FM modulation is:

$$\mathbf{f_i = f_c + k_f m(t)}$$

Where, f_c is the carrier frequency, k_f is the freq. sensitivity (Hz/V) and $m(t)$ is message signal

- We know the relationship between angular frequency ω_i and angle $\theta_i(t)$ as

$$\omega_i = d\theta_i(t)dt$$

$$2\pi fi = d\theta i(t)/dt$$

$$\Rightarrow \theta_i(t) = 2\pi \int f_i dt$$

Substitute, f_i value in the above equation.

$$\theta_i(t)=2\pi\int(f_c+k_f m(t))dt$$

$$\Rightarrow\theta_i(t)=2\pi f_c t+2\pi k_f\int m(t)dt$$

Therefore the standard equation of angle modulated wave is:

$$s(t)=A_c\cos(2\pi f_c t + 2\pi k_f\int m(t)dt)$$

Single Tone FM

Let us assume modulating signal is $m(t)=A_m\cos(2\pi f_m t)$,
Then the equation of FM wave will be:

$$s(t)=A_c\cos(2\pi f_c t + \beta\sin(2\pi f_m t))$$

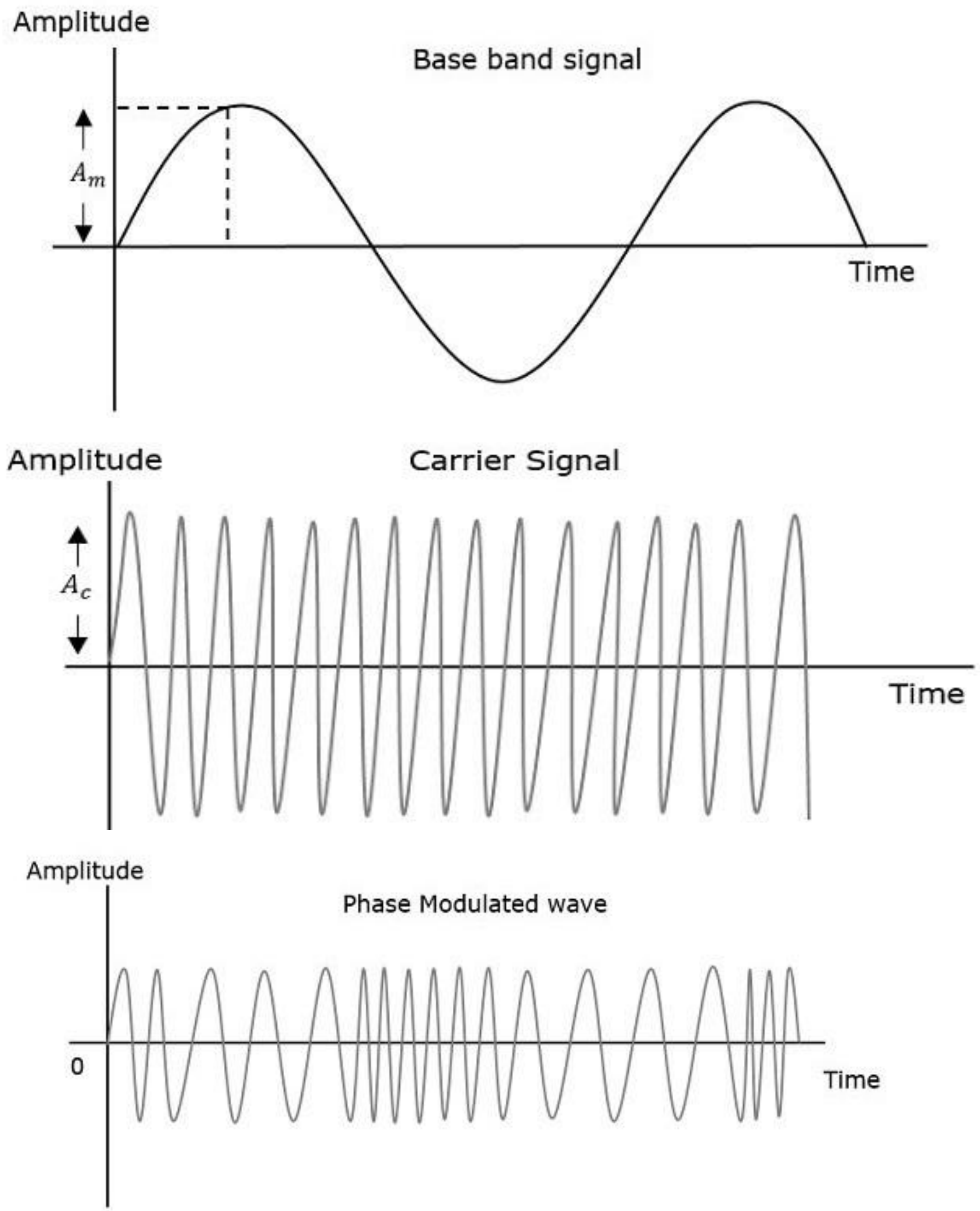
Where,

β= ***modulation index*** = $\Delta f/f_m = k_f A_m/f_m$

The difference between instantaneous frequency and normal carrier frequency is termed as **Frequency Deviation**. It is denoted by Δf, which is equal to the product of k_f and A_m.

PHASE MODULATION

- In frequency modulation, the frequency of the carrier varies. Whereas, in **Phase Modulation (PM)**, the phase of the carrier signal varies in accordance with the instantaneous amplitude of the modulating signal.
- So, in phase modulation, the amplitude and the frequency of the carrier signal remains constant.



- The equation for instantaneous phase ϕ_i in phase modulation is

$$\phi_i = k_p m(t)$$

Where, k_p is the phase sensitivity (radian/Volt),
m(t) is the message signal

- The standard equation of angle modulated wave is

$$s(t) = A_c \cos(2\pi f_c t + \phi_i)$$

Therefore equation of PM wave becomes:

$$s(t) = A_c \cos(2\pi f_c t + k_p m(t))$$

Single Tone PM

If the modulating signal, $m(t)=A_m\cos(2\pi f_m t)$

then the equation of PM wave will be

$$\mathbf{s(t)=A_c\cos(2\pi f_c t+\beta\cos(2\pi f_m t))}$$

Where,

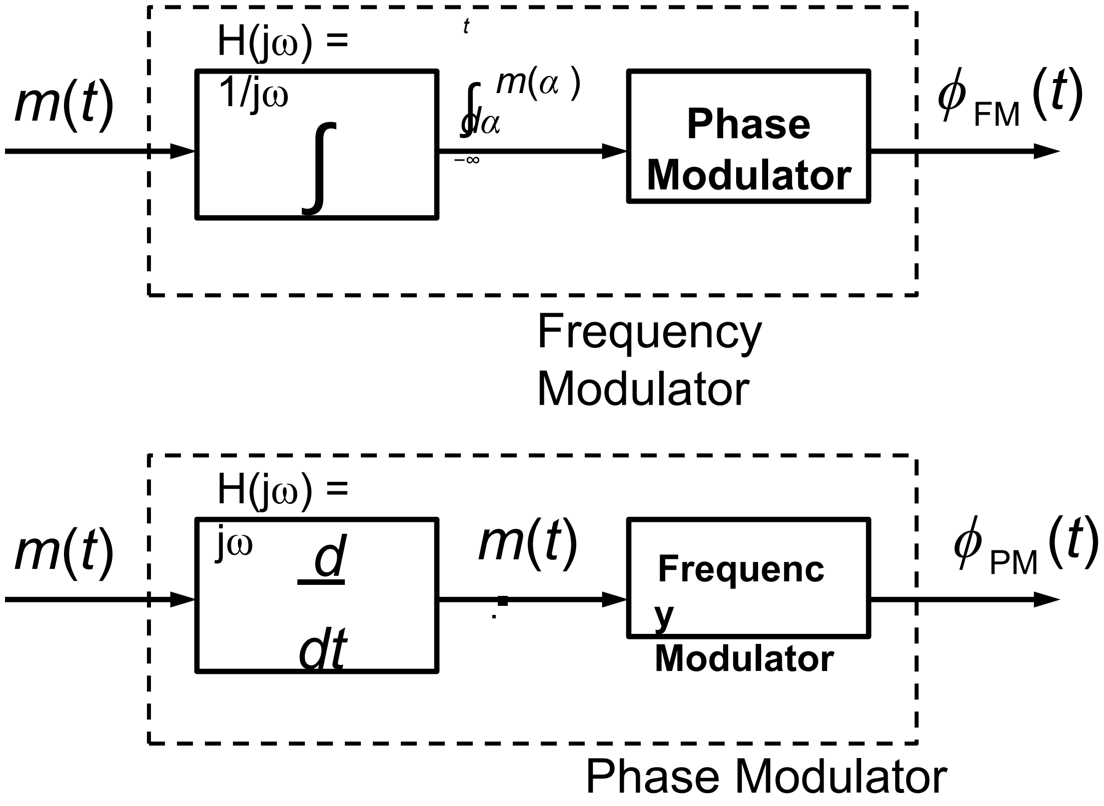
- β= ***modulation index*** = $\Delta\phi=k_pA_m$
- $\Delta\phi$ is phase deviation

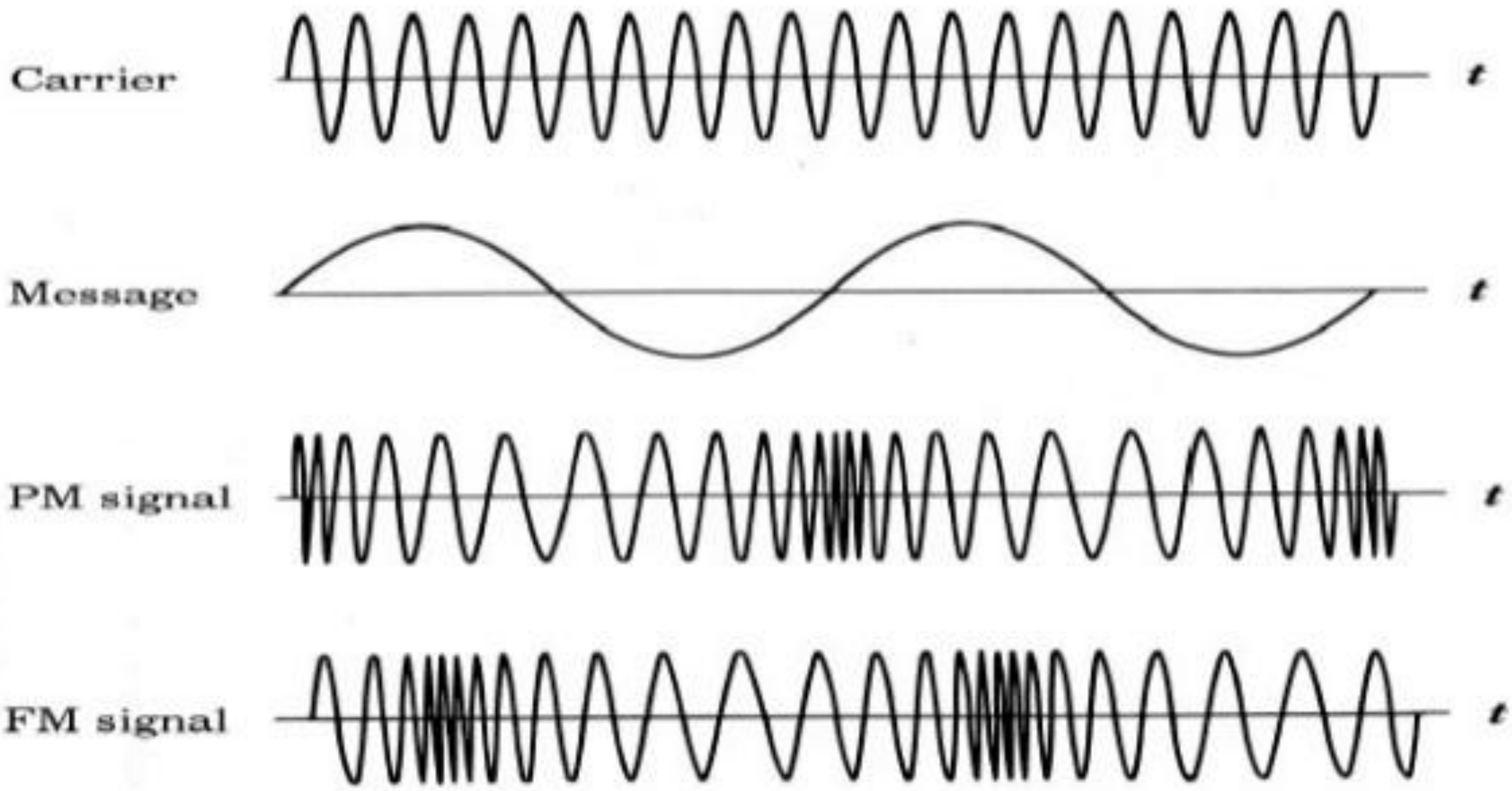
Phase modulation is used in mobile communication systems, while frequency modulation is used mainly for FM broadcasting.

Comparing Frequency Modulation to Phase Modulation

#	Frequency Modulation (FM)	Phase Modulation (PM)
1	Frequency deviation is proportional to modulating signal $m(t)$	Phase deviation is proportional to modulating signal $m(t)$
2	Noise immunity is superior to PM (and of course AM)	Noise immunity better than AM but not FM
3	Signal-to-noise ratio (SNR) is better than in PM	Signal-to-noise ratio (SNR) is not as good as in FM
4	FM is widely used for commercial broadcast radio (88 MHz to 108 MHz)	PM is primarily for some mobile radio services
5	Modulation index is proportional to modulating signal $m(t)$ as well as modulating frequency f_m	Modulation index is proportional to modulating signal $m(t)$

A Pictorial Way to View the Generation of FM and PM




Carrier
t
Message
t
PM signal
t
FM signal
t

Spectrum Analysis of Sinusoidal FM wave:

- It is not possible to evaluate the Fourier transform of a general FM signal, therefore, for the sake of simplicity, the case of a sinusoidal modulating signal is considered and given as:

$$s(t)=A_c\cos(2\pi f_c t+\beta \mathrm{Sin}(2\pi f_m t)) \quad \ldots\ldots\ldots\ldots\ldots(1)$$

- Expanding the equation using Trigonometric identity,

$$s(t) = A_c\cos(2\pi f_c t).\cos[\beta\sin(2\pi f_m t)] - A_c\sin(2\pi f_c t).\sin[\beta\sin(2\pi f_m t)] \;\ldots\ldots..(2)$$

$s_I(t)= A_c\cos[\beta\sin(2\pi f_m t)]$

$s_Q(t)= A_c\sin[\beta\sin(2\pi f_m t)]$

Therefor complex envelope of the FM wave is :

$$\widehat{s(t)} = s_I(t) + j\, s_Q(t)$$

- The complex envelope $\widehat{s(t)}$ retains complete information about the modulation process. Indeed we may readily express the FM wave s(t) in terms of the complex envelope $\widehat{s(t)}$ by writing:

$$\mathbf{s(t) = Re\,[\;Ac\,exp[j2\pi f_c t + j\,\beta sin(2\pi f_m t)]}$$

$$= \mathrm{Re}\,[\,\widehat{s(t)}\,\exp(j2\pi f_c t)]\;\ldots\ldots\ldots\ldots(4)$$

Since complex envelope is a periodic function of time with a fundamental freq. f_m we may therefore expand $\widehat{s(t)}$ in the form of complex Fourier series as follows:

$$\widehat{s(t)} = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_m t} \quad \ldots\ldots\ldots\ldots\ldots(5)$$

Where the complex Fourier coefficient is:

$$c_n = \frac{1}{T}\int_{-T/2}^{T/2} \widehat{s(t)}\, e^{-jn\omega_m t}\,dt\,.$$

- $$C_n = f_m \int_{-1/2fm}^{1/2fm} \widehat{s(t)} \exp[-j2\pi n f_m t]\, dt$$

$$= f_m A_c \int_{-1/2fm}^{1/2fm} \exp[j\boldsymbol{\beta}\sin 2\pi n f_m t - j2\pi n f_m t\ ]\, dt$$

For convenience we define the variable $x = 2\pi n f_m t$ So,

$$c_n = \frac{A_c}{2\pi} \int_{-\pi}^{\pi} \exp[\, j(\beta \sin x - nx)]\, dx \ \ldots\ldots\ldots\ldots(6)$$

The integral on the right side of above equation is recognized as the n^{th} order Bessel function of the first kind and argument **β.** This function is commonly denoted by the symbol $J_n(\beta)$.

$$J_n(\beta) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \exp[\, j(\beta \sin x - nx)]\, dx$$

Hence we may write above equation as:

$$c_n = A_c\, J_n(\beta)$$

- Now substituting the value of c_n in equation 5 we get

$$\widehat{s(t)} = \sum_{n=-\infty}^{\infty} A_c\ J_n(\beta)\exp(j2\pi n f_m t)\ \ldots\ldots\ldots\ldots\ldots(7)$$

Hence the equation $s(t) = \text{Re}\left[\widehat{s(t)}\exp(j2\pi f_c t)\right]$ becomes

$$s(t) = A_c\,\text{Real}[J_n(\beta)\exp(j2\pi n f_m t)\exp(j2\pi f_c t)]$$

$$s(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta)\cos[2\pi(f_c+f_m)t]$$

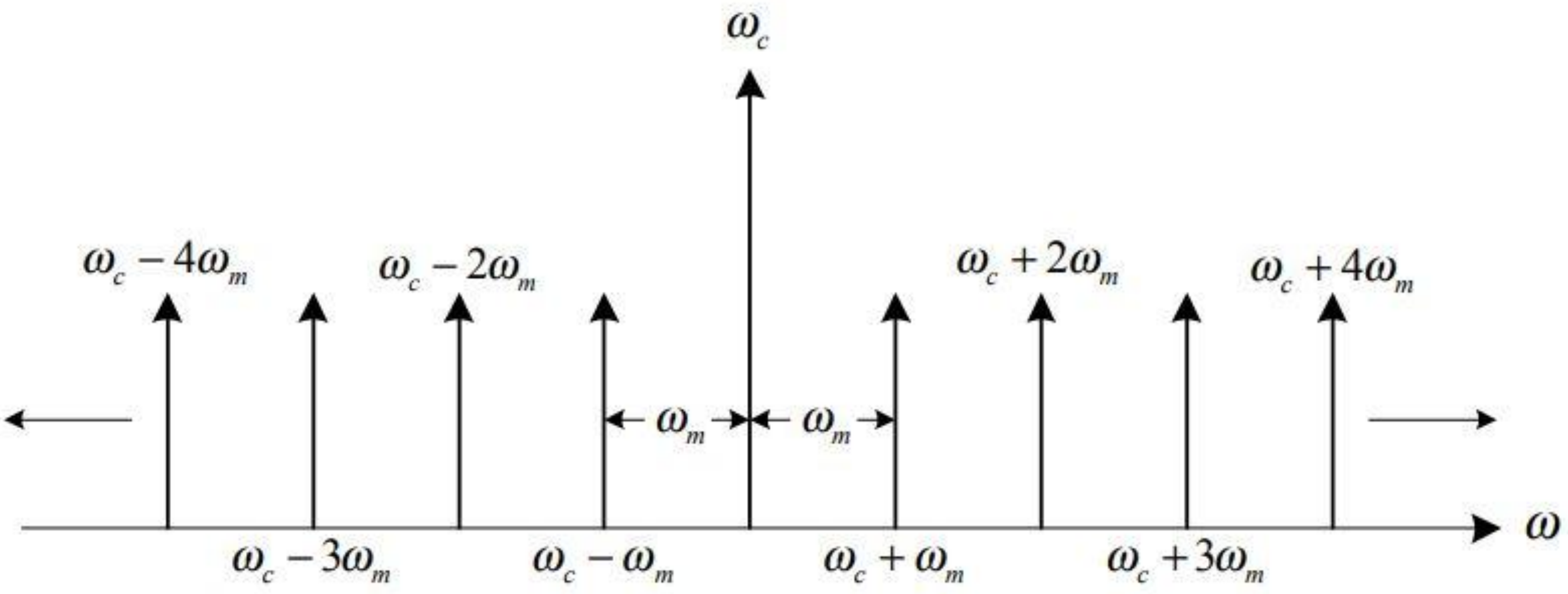
This is the desired form for the FS representation of the single tone FM wave s(t) for an arbitrary value of β.

The discrete spectrum of s(t) is obtained by taking the FT of both sides

$$\mathbf{S(f) = \frac{A_c}{2} \sum_{n=-\infty}^{\infty} J_n(\beta)\,[\delta(f-f_c-nf_m)+\delta(f+f_c+nf_m)]}$$

Spectrum of FM

$$S(f) = \frac{A_c}{2} \sum_{n=-\infty}^{\infty} J_n(\beta)\, [\delta(f-f_c-nf_m)+\delta(f+f_c+nf_m)]$$



Properties of Bessel function

1) $\sum_{n=-\infty}^{\infty} J_n^2(\beta)=1$

Therefore, the total average power of FM is

$$P_T = \sum_{n=-\infty}^{\infty} \frac{A^2}{2} J_n^2(\beta) = \frac{A_c^2}{2}$$

The average total power can be transmitted only when all sideband (infinite no. are transmitter) since it is practically limited side band upto number n are transmitter and in this case the transmitted average power is

$$P_n = \sum_{k=-n}^{n} \frac{A^2{}_c}{2} J_n^2(\beta)$$

In practice n is chosen in such a way that the actual transmitted power is at least 98% of the total theoretical average power.

2) $J_{-n}(\beta) = J_n(\beta)$ for n even.

3) $J_{-n}(\beta) = -J_n(\beta)$ for n odd.

4) $J_{n-1}(\beta) + J_{n+1}(\beta) = 2n/\beta \, . \, J_n(\beta)$

Bandwidth of FM

- Bandwidth $= 2\ n\ f_m$

$$n = \beta + 1$$

$$\approx 2\Delta f + 2f_m$$

$$= 2\Delta f(1+f_m/\Delta f)$$

$$= 2\Delta f\ (1+1/\beta)\ \ldots\ldots\ldots\ldots(i)$$

$= 2(\beta+1)f_m$

Where, β = modulation index $= \Delta f/f_m$

Narrowband FM

- For small value of modulation index(β) much small then 1.
- The FM wave is assumed to be narrow band form consisting a carrier, a upper sides frequency and lower size frequency the expression of NBFM is given by:

$$U_{NBFM}(t) = A_c \cos 2\pi f_c t + \beta A_c/2 \cos[2\pi(f_c+f_m)t] - \beta A_c/2 \cos[2\pi(f_c - f_m)t]$$

Wideband FM

- **Wide band FM(WBFM):**
- For large value of modulation index (β) compare to 1, FM contains a carrier on infinite no. of side frequency located symmetrically around the carrier such a FM wave is defined as wide band FM.
- For WBFM u(t) can be written as,

$$U_{WBFM}(t) = A_c \sum_{n=-\infty}^{\infty} J_n^{\,2}(\beta)\cos[2\pi fc + nfm)t]$$

Where, $J_n(\beta)$ =Bessel function (calculated from given table)

Difference between NBFM and WBFM

S.NO	WBFM	NBFM
1	Modulation index is greater than 1	Modulation index less than 1
2	Frequency deviation 75 KHz	Frequency deviation 5 KHz
3	Bandwidth 15 times NBFM	Bandwidth 2fm
4	Noise is more suppressed	Less suppressing of noise

FM generation

Two methods:

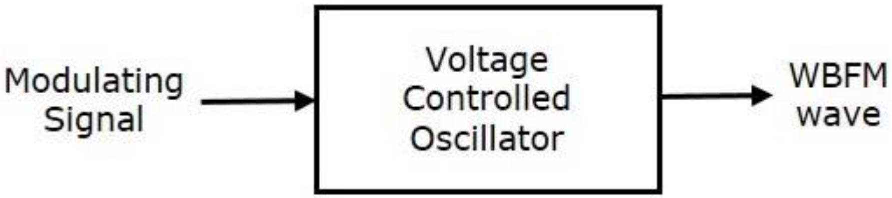
(i) Direct Method (Parameter Variation)

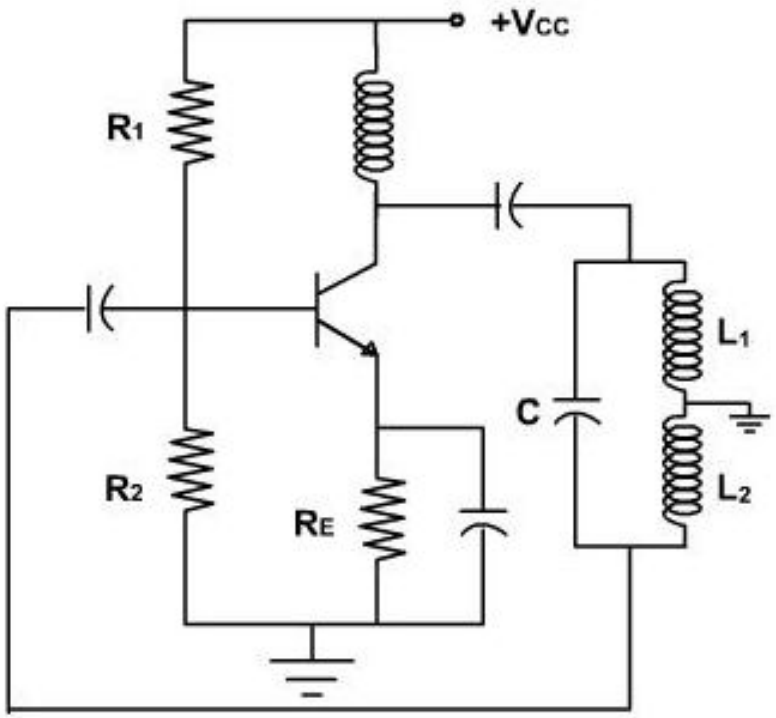
- Reactance Modulator
- Varactor Diode Modulator

(ii) Indirect Method (Armstrong's method)

Direct method

- In the direct method of FM generation the instantaneous frequency of carrier wave is varied directly in accordance with the message signal by mean of a device known as voltage controlled oscillator is an oscillator circuit having a Varactor diode in its frequency determining section as shown in fig.





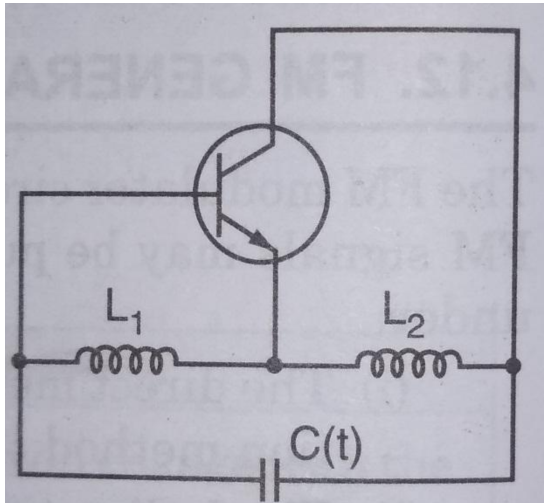


Fig. 4.20. Hartley oscillator

- The frequency of oscillation of the Hartely oscillator is given by:

$$F = \frac{1}{2\pi\sqrt{LC}}$$

- C(t) is the total capacitance of the fixed capacitor and variable-voltage capacitor.
- L = L1+L2, the effective series inductance of the inductors L1 and L2 in the tank circuit.
- Assume for modulating wave m(t) the capacitance is expressed as:

$$C(t) = C_0 - K_c\, m(t)$$

C_0 = Total capacitance in absence of modulation

K_c= Variable capacitors' sensitivity

$$f_i(t) = \frac{1}{2\pi\sqrt{(L1+L2)(C_0 - K_c\, m(t))}} = \frac{1}{2\pi\sqrt{(L1+L2)C_0 - (L1+L2)\, Kc\, m(t)}}$$

$$= \frac{1}{2\pi\sqrt{(L1+L2)C_0\, [1 - \frac{K_c\, m(t)}{C_0}]}}$$

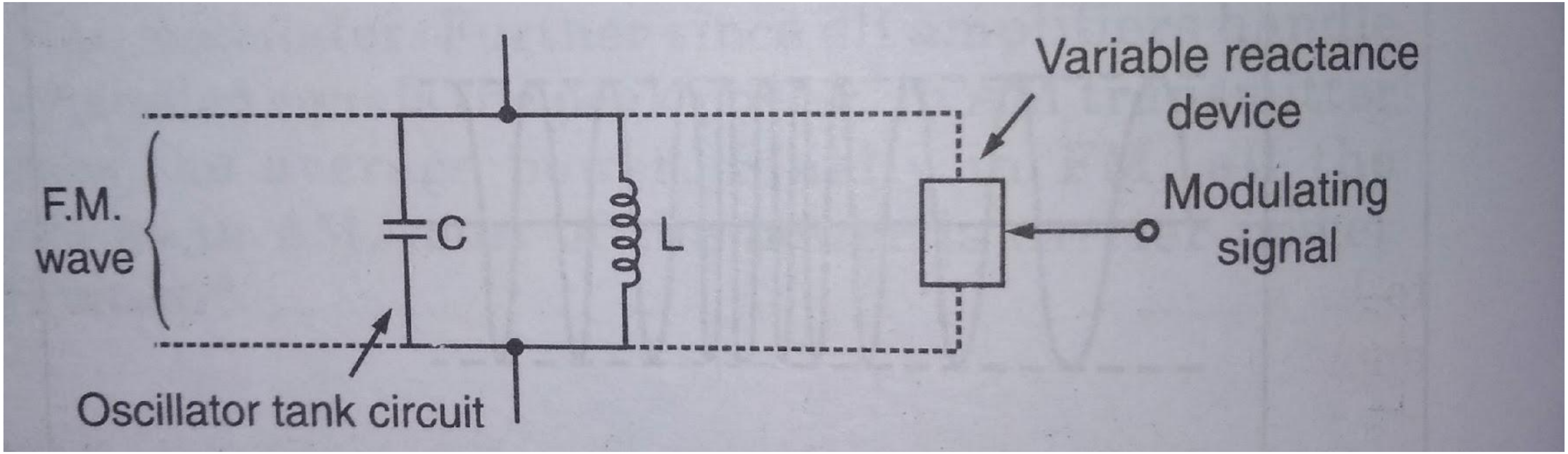
$$= f_0\, [1 - \frac{K_c\, m(t)}{C_0}\,]^{-1/2}$$

$$= f_0\, [1 + \frac{K_c\, m(t)}{2C_0}\,]$$

$$= f_0\ + \frac{f_0\, K_c}{2C_0}\, m(t)]$$

Therefore, $f_i(t) = f_0\ + K_f\, m(t)]$

$Where\ K_f\ is$ the frequency sensitivity of the m


Variable reactance
device
F.M.
wave
C
L
Modulating
signal
Oscillator tank circuit

Indirect method:

- When the β<<1 the J1 and J2 term together make total radiate power more then 98% of the total average power, in this case the expression for the FM can be written as

$$U_{NBFM}(t) = A_c\cos 2\pi f_c t + \beta A_c/2 \cos[2\pi(f_c+f_m)t] - \beta A_c/2 \cos[2\pi(f_c- f_m)t]$$

$$U_{FM}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta)\cos(2\pi(f_c+nf_m)t)$$

For 98% of radiation of average power expanding this expression for n= -1,0,and 1

For β<<1,

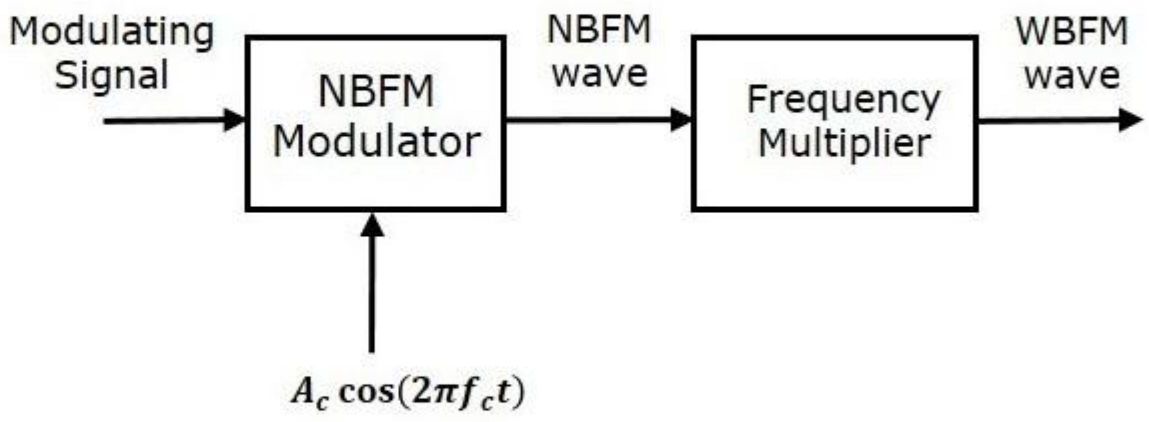
$$J_1(\beta) = \beta/2 \ , \ J_{-1}(\beta) = \beta/2$$

And $J_n(\beta) \approx 1$ then, the expression of FM will be ,

$$U_{FM}(t) \approx A_c\cos 2\pi f_c t + A_c/2\ \beta\cos 2\pi\ (f_c+f_m)t - A_c/2\ \beta \cos 2\pi\ (f_c-f_m)t$$

Indirect method or Armstrong method

- The indirect method of generating wideband FM was first proposed by E.H. Armstrong, so this method is popularly known as Armstrong method.
- This method is called as Indirect Method because we are generating a wide band FM wave indirectly.
- This means, first we will generate NBFM wave and then with the help of frequency multipliers we will get WBFM wave.
- The block diagram of generation of WBFM wave is shown in the following figure.



- This block diagram contains mainly two stages. In the first stage, the **NBFM wave will be generated** using NBFM modulator.
- The frequency stability of indirect method of FM generation is very high because crystal oscillators are used.
- We know that the modulation index of NBFM wave is less than one. Hence, in order to get the required modulation index (greater than one) of FM wave, choose the **frequency multiplier** value properly.

1. **Generate NBFM using modulating wave**
2. **Convert NBFM to WBFM using frequency multiplier**

- Consider first the generation of a NBFM wave. Expression for an FM wave s1(t) for the general case of a modulating wave m(t) is written in the form:

$$s_1(t) = A_1\, Cos[2\pi f_1 t + \boldsymbol{\phi}_1(t)] \text{..........(1)}$$

Where,

$$\boldsymbol{\phi}_1(t) = 2\pi k_1 \int_0^t m(t)dt \quad \dots\dots\dots\dots\dots (2)$$

Assume $\boldsymbol{\phi}_1(t)$< 1 for all value of t then

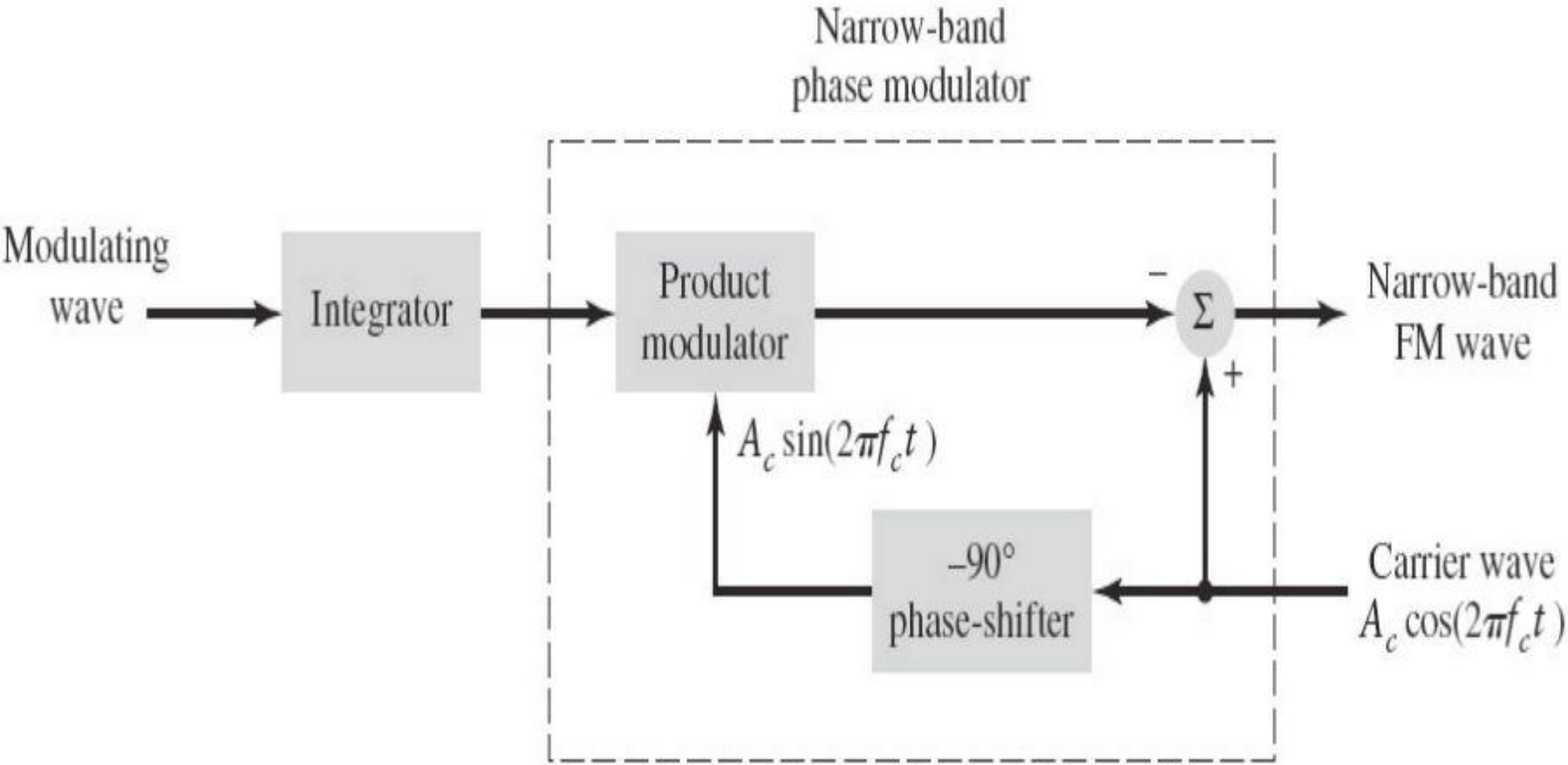
Cos $[\boldsymbol{\phi}_1(t)] \approx 1$

Sin $[\boldsymbol{\phi}_1(t)] \approx \boldsymbol{\phi}_1(t)$

So, equation 1 becomes,

$$s_1(t) = A_1\, Cos(2\pi f_1 t) - A_1\, Sin(2\pi f_1 t).\, \boldsymbol{\phi}_1(t)$$

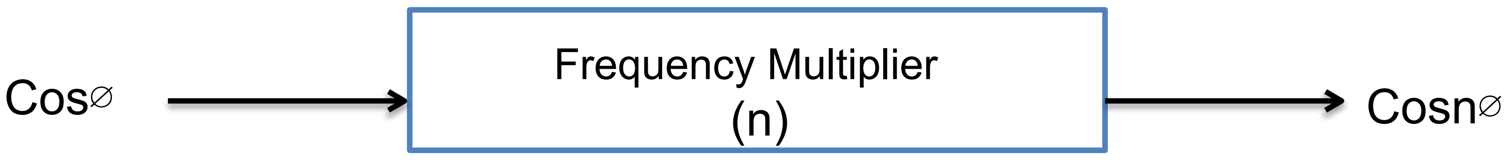
$$s_1(t) = A_1\, Cos(2\pi f_1 t) - 2\pi k_1 A_1\, Sin(2\pi f_1 t).\int_0^t m(t)dt \text{ ....(3)}$$



Narrow-band
phase modulator
Modulating
wave
Integrator
Product
modulator
−
Σ
+
Narrow-band
FM wave
$A_c \sin(2\pi f_c t)$
–90°
phase-shifter
Carrier wave
$A_c \cos(2\pi f_c t)$

Frequency Multiplier

- **Frequency multiplier** is a non-linear device, which produces an output signal whose frequency is 'n' times the input signal frequency. Where, 'n' is the multiplication factor.
- If NBFM wave whose modulation index β is less than 1 is applied as the input of frequency multiplier, then the frequency multiplier produces an output signal, whose modulation index is 'n' times β and the frequency also 'n' times the frequency of WBFM wave.
- Sometimes, we may require multiple stages of frequency multiplier and mixers in order to increase the frequency deviation and modulation index of FM wave.



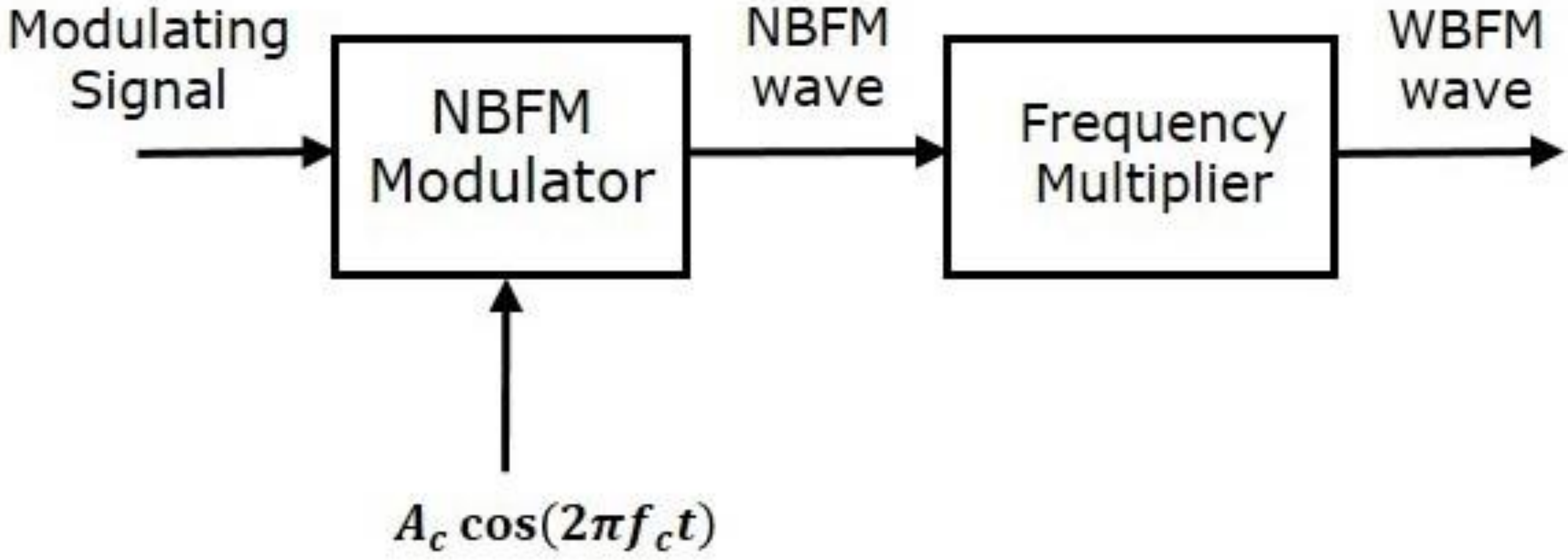
- When NBFM is passed through Frequency multiplier WBFM is obtained.
- The output of the NBFM oscillator is
- S_{NBFM}(t)=$A_c\cos[2\pi f_c t + \beta\ sin 2\pi f_m t]$
- Where β is very less than 1
- The output of frequency multiplier with multiplying factor n is a wideband FM that is
- S_{WBFM}(t)=$A_c\cos[n(2\pi f_c t + \beta\ sin 2\pi f_m t)]$

$\therefore$ S_{WBFM}(t)=$A_c\cos[2\pi n f_c t + n\beta\ sin 2\pi f_m t]$

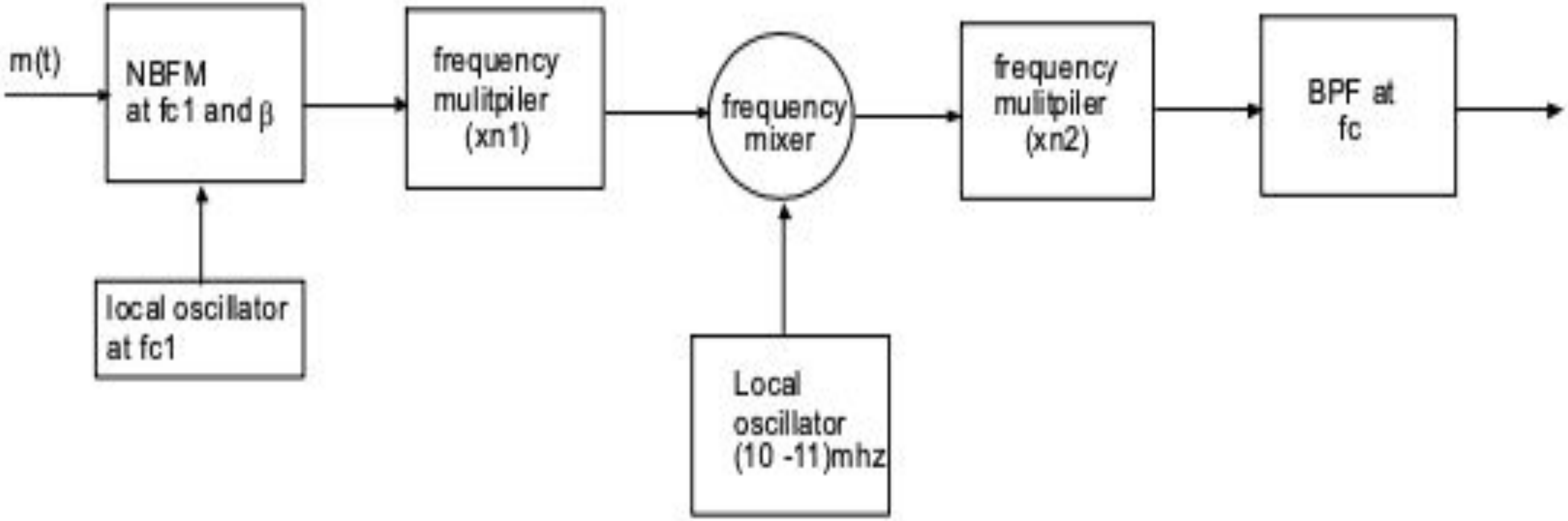
$\therefore$ S_{WBFM}(t)=$A_c\cos[2\pi f_c{}' t + \beta'\ sin 2\pi f_m t]$

Where $f_c' = nf_c$

And $\beta' = n\beta$ >1


Modulating Signal
NBFM Modulator
NBFM wave
Frequency Multiplier
WBFM wave
$A_c \cos(2\pi f_c t)$

- Use of freq. multiplier increases carrier freq. of FM wave as well as Modulation index. This may result very high carrier freq. in order to achieve a given modulation index. To solve this problem the mixer followed by BPF is used to control value of the carrier freq. f_c.



Parameters	Amplitude modulation	Frequency Modulation
Stands for	AM stands for Amplitude Modulation	FM stands for Frequency Modulation
Origin	AM method of audio transmission was first successfully carried out in the mid 1870s.	FM radio was developed in the United states in the 1930s, mainly by Edwin Armstrong.
Modulating differences	In AM, a radio wave known as the "carrier" or "carrier wave" is modulated in amplitude by the signal that is to be transmitted. The frequency and phase remain the same.	In FM, a radio wave known as the "carrier" or "carrier wave" is modulated in frequency by the signal that is to be transmitted. The amplitude and phase remain the same.
Pros and cons	AM has poorer sound quality compared with FM, but is cheaper and can be transmitted over long distances. It has a lower bandwidth so it can have more stations available in any frequency range.	FM is less prone to interference than AM. However, FM signals are impacted by physical barriers. FM has better sound quality due to higher bandwidth.
Frequency Range	AM radio ranges from 535 to 1705 KHz (OR) Up to 1200 bits per second.	FM radio ranges in a higher spectrum from 88 to 108 MHz. (OR) 1200 to 2400 bits per second.
Bandwidth Requirements	Twice the highest modulating frequency. In AM radio broadcasting, the modulating signal has bandwidth of 15kHz, and hence the bandwidth of an amplitude-modulated signal is 30kHz.	Twice the sum of the modulating signal frequency and the frequency deviation. If the frequency deviation is 75kHz and the modulating signal frequency is 15kHz, the bandwidth required is 180kHz.
Zero crossing in modulated signal	Equidistant	Not equidistant
Complexity	Transmitter and receiver are simple but synchronization is needed in case of SSBSC AM carrier.	Tranmitter and reciver are more complex as variation of modulating signal has to beconverted and detected from corresponding variation in frequencies.(i.e. voltage to frequency and frequency to voltage conversion has to be done).
Noise	AM is more susceptible to noise because noise affects amplitude, which is where information is "stored" in an AM signal.	FM is less susceptible to noise because information in an FM signal is transmitted through varying the frequency, and not the amplitude.

Demodulation of FM signals

- **Two methods**

(i) Direct or Limiter-discriminator method or Frequency Discrimination

(ii) Indirect of PLL Method or Phase Discrimination method

Limiter Discriminator method

It involves three steps:

(a) Amplitude limit: Limit any change due to noise.
(b) Discrimination : differentiator
(c) Envelope detection: achieve original message by detecting envelope.

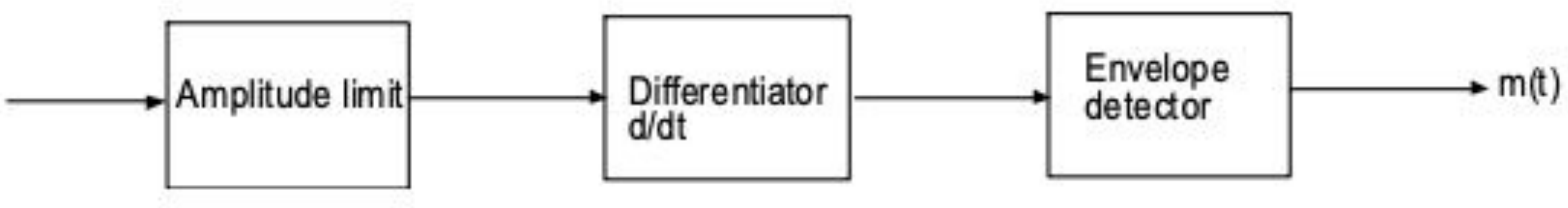


Fig.: Block diagram of limiter-discriminator demodulator

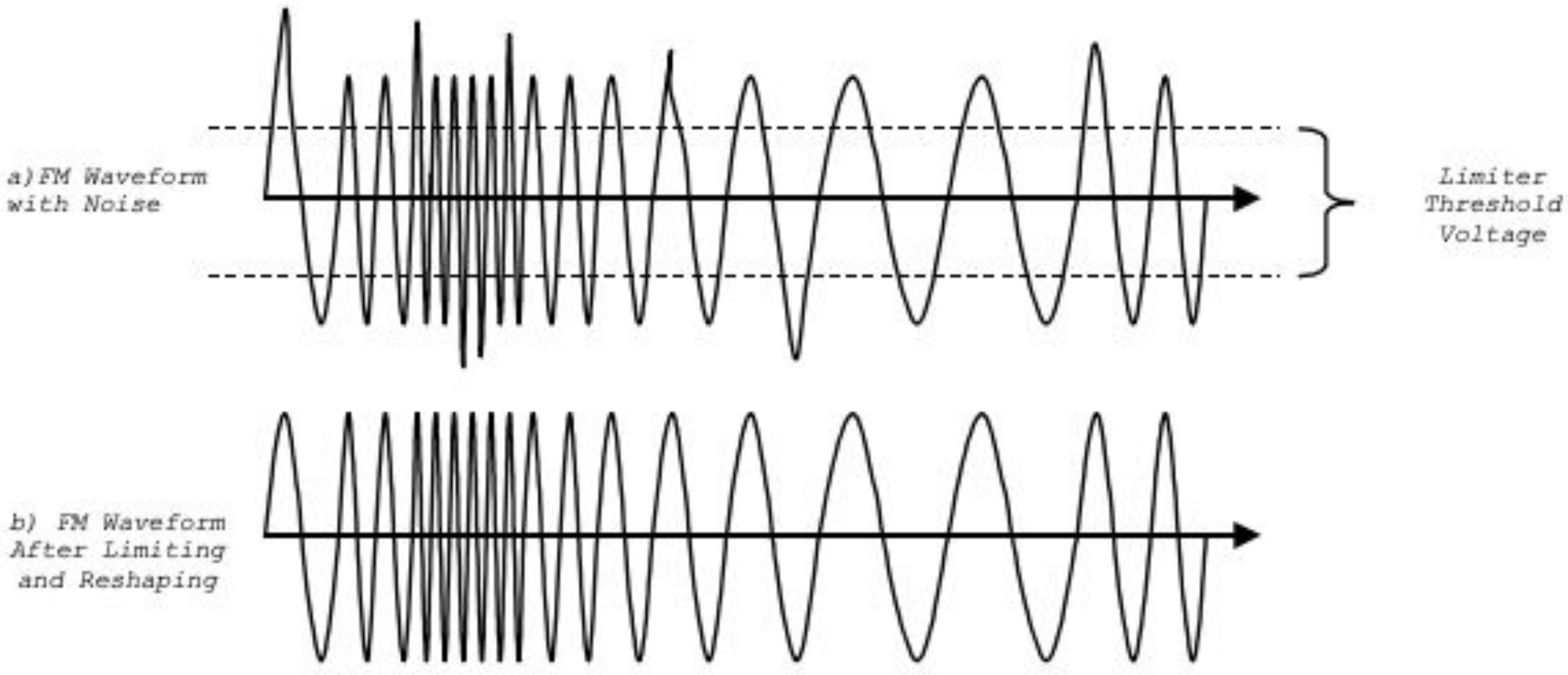


Figure 9-2: An FM signal Before and After Limiting

Consider $u_{FM}(t) = A_c \cos(2\pi f_c t + \phi(t))$

Where, $\phi(t) = 2\pi k_f \int_0^t m(t)\, dt$

(a) Amplitude limier limits the amplitude of the signal to reduce the effect of feeding and noise.

(b) The o/p of differenciator be,

$$\frac{d[u_{FM}(t)]}{dt} = \frac{d\{A_c \cos[2\pi f_c t + 2\pi k_f \int m(t)dt]\}}{dt}$$

$$\frac{d[u_{FM}(t)]}{dt} = \frac{A_c d\{\cos[2\pi f_c t + 2\pi k_f \int m(t)dt]\}}{d[2\pi f_c t + 2\pi k_f \int m(t)dt]} \times \frac{d[2\pi f_c t + 2\pi k_f \int m(t)dt]}{dt}$$

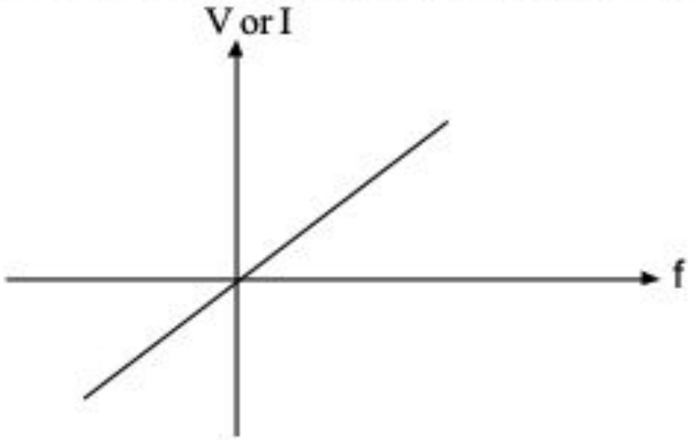
$=-A_c \sin[2\pi f_c + 2\pi k_f\, m(t)][2\pi f_c + 2\pi k_f m(t)]$

$=-A_c[2\pi f_c + 2\pi k_f m(t)] \sin[2\pi f_c t + 2\pi k_f m(t)]$

This is an equivalent as a standard AM signal, form which m(t) can be recovered by using envelop detector.

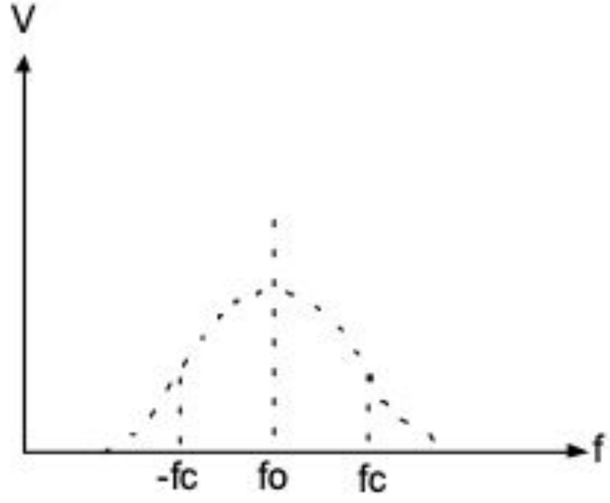
Implementation of differentiator: Differentiator is a ckt that convert change in frequency to change in voltage or current .

- It must have the following transfer characteristics.



Mathematically, the transfer function, $H(f) = j2\pi f$

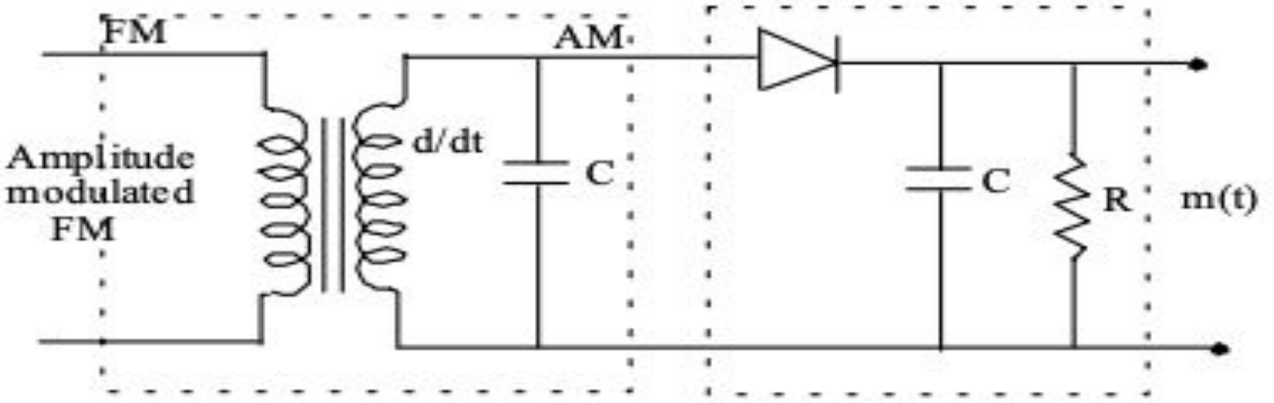
A LC tank ckt turned at f_o+-f_c have such transfer function.



(c) The last stage is envelop detector which gives,

$$U_d(t) = A_c[2\pi f_c + 2\pi k_f m(t)]$$
$$= A_c 2\pi f_c + A_c 2\pi k_f m(t)$$

The first component is the DC component and 2nd is the message signal multiplied by certain constant.

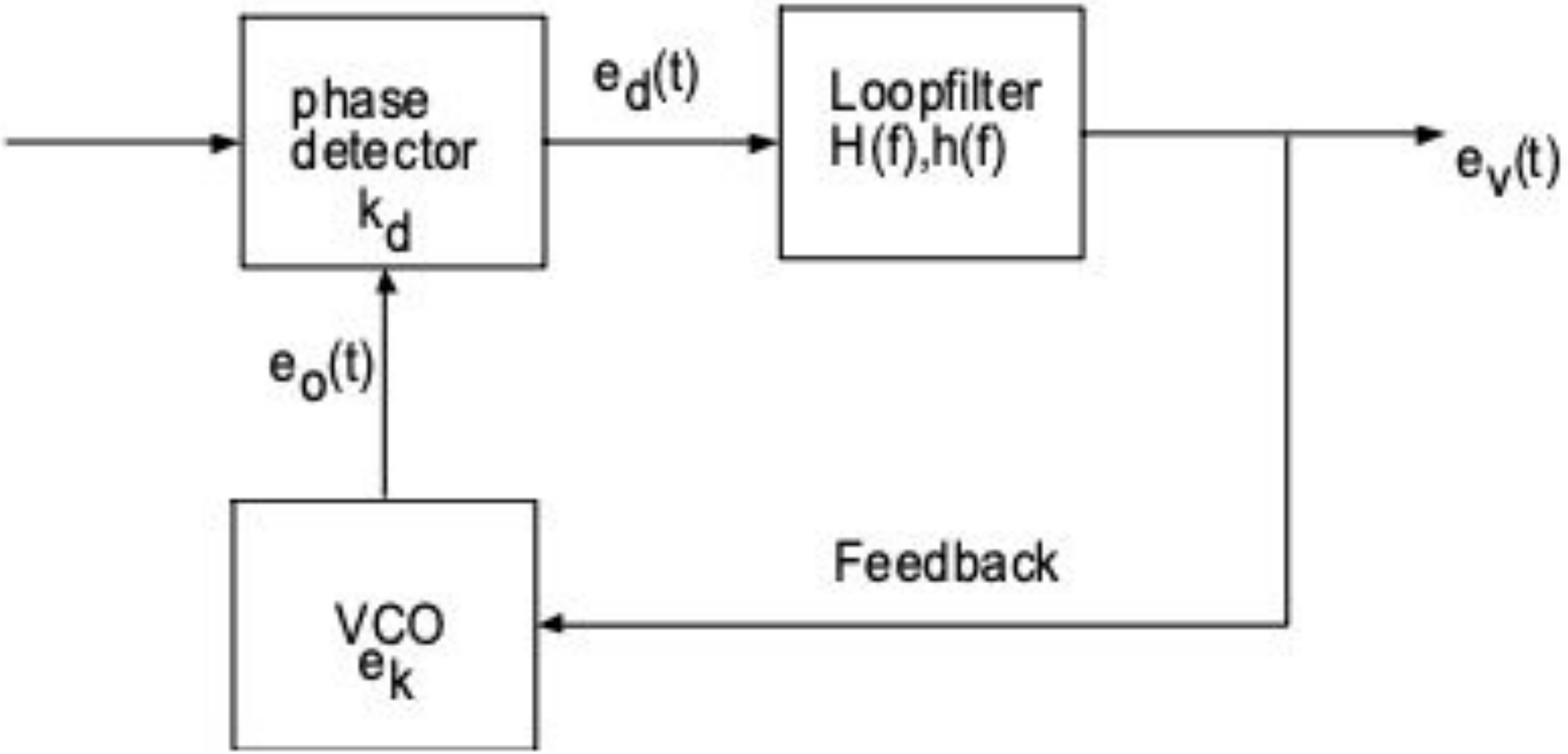


Phase Locked Loop (PLL) for FM demodulation

- PLL is a negative feedback system
- It consist of product modulator, VCO and filter.
- It is used in demodulator circuits due to effectiveness in large noise and low power environment.
- It is used in space vehicle where power, weight and reliability are premium.
- It is used in commercial FM receiver.

Basic terminology in PLL

- Free running range
- Capture range
- Lock range


phase
detector
k_d
$e_d(t)$
Loopfilter
H(f),h(f)
$e_v(t)$
$e_o(t)$
VCO
e_k
Feedback

Let us assume that, $e_v(t) = 0$ [i.e $u_{FM}(t) = 0$] at this stage the VCO is calibrated in such a way that , it's free running frequency of oscillation is equal to the carrier frequency of the incoming FM signal with constant phase shift of 90 . Then , the o/p of the VCO be $e_o(t) = A_v\cos(w_ct - \pi/2)$

$$= A_v\sin w_ct$$

Now , let us assume that $e_v(t) \neq 0$, then,

$E_0\,(t) = A_v\,\sin(w_ct+\phi_0(t))$

Where, $\phi_0(t) = 2\pi\,k_v \int_0^t e_v(t)\,dt$.

[$\phi_0\,(t)$ = signal proportional to the o/p of the PLL]

Where, $e_v(t)$ = is the control voltage applied to the VCO input and k_v is called VCO sensitivity.

Now let us assume that the incoming FM signal is expressed by the equation $U_{FM}(t) = A_c\cos[w_c t + \phi_i(t)]$

Where, $\phi_i(t) = 2\pi k_f \int_o^t m(t)\, dt$

Here, k_f = frequenecy sensitivity of modulator

Now,

Error voltage ,

$E_d(t) = U_{FM}(t) * e_o(t)$

$= A_c \cos[w_c t + \phi_i(t)] * A_v \sin[w_c t + \phi_o(t)] * k_d$

$E_d(t) = |_{LPF} = k_d A_c A_v/2. \sin[\phi_i(t) - \phi_0(t)]$

$= k_d\, A_c A_v/2. \sin[\phi_e(t)]$

Where, $\phi_e(t) = \phi_i(t) - \phi_o(t)$ is called phase errors.

When the PLL enters into the Lock mode (i.e when two frequency and phase match) the error voltage will be very low and nearly equal to zero.

i.e $\phi_e(t) = \phi_i(t) - \phi_o(t)$ nearly equal to o

or , $\phi_i(t) = \phi_o(t)$

$2\pi k_f \int_0^t m(t)\, dt = 2\pi k_v \int_o^t e_v(t) dt$

Or , $k_v e_v(t) = k_f m(t)$

$\therefore\ e_v(t) = k_f/k_v\ .\ m(t)$

Thus in Lock mode , the output voltage of the PLL is nearly equal to message signal.

Commercial FM
Pre- emphasis and De emphasis network

- In FM transmitter, we have seen the pre-emphasis network (High pass filter), which is present before FM modulator.
- This is used to improve the SNR of high frequency audio signal.
- The reverse process of pre-emphasis is known as **de-emphasis**. Thus, in this FM receiver, the de-emphasis network (Low pass filter) is included after FM demodulator.

Pre-emphasis: The noise suppression ability of FM decreases with the increase in the frequencies. Thus increasing the relative strength or amplitude of the high frequency components of the message signal before modulation is termed as Pre-emphasis. The Fig3 below shows the circuit of pre-emphasis.

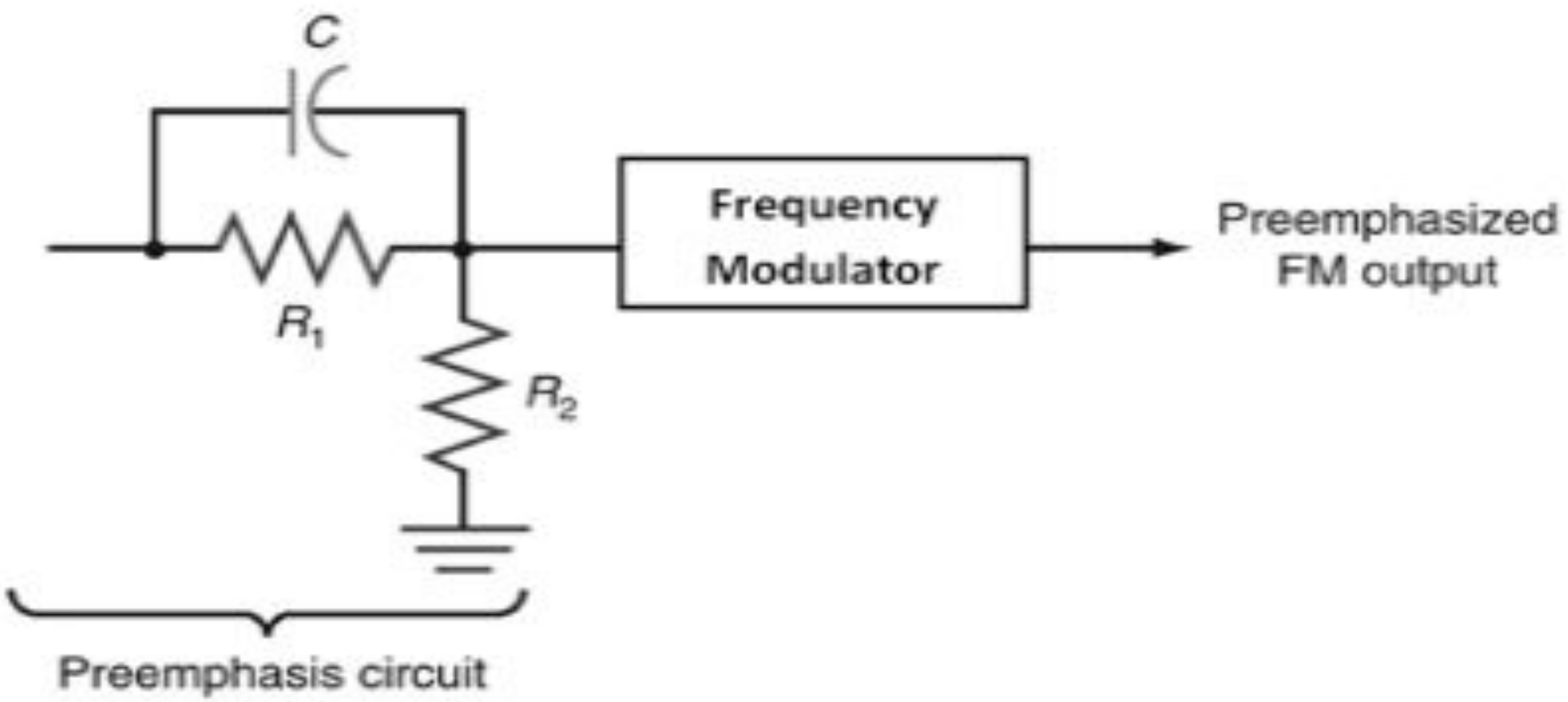


Fig3. Pre-emphasis circuit

De-emphasis: In the de-emphasis circuit, by reducing the amplitude level of the received high frequency signal by the same amount as the increase in pre-emphasis is termed as De-emphasis. The Fig4. below shows the circuit of de-emphasis.

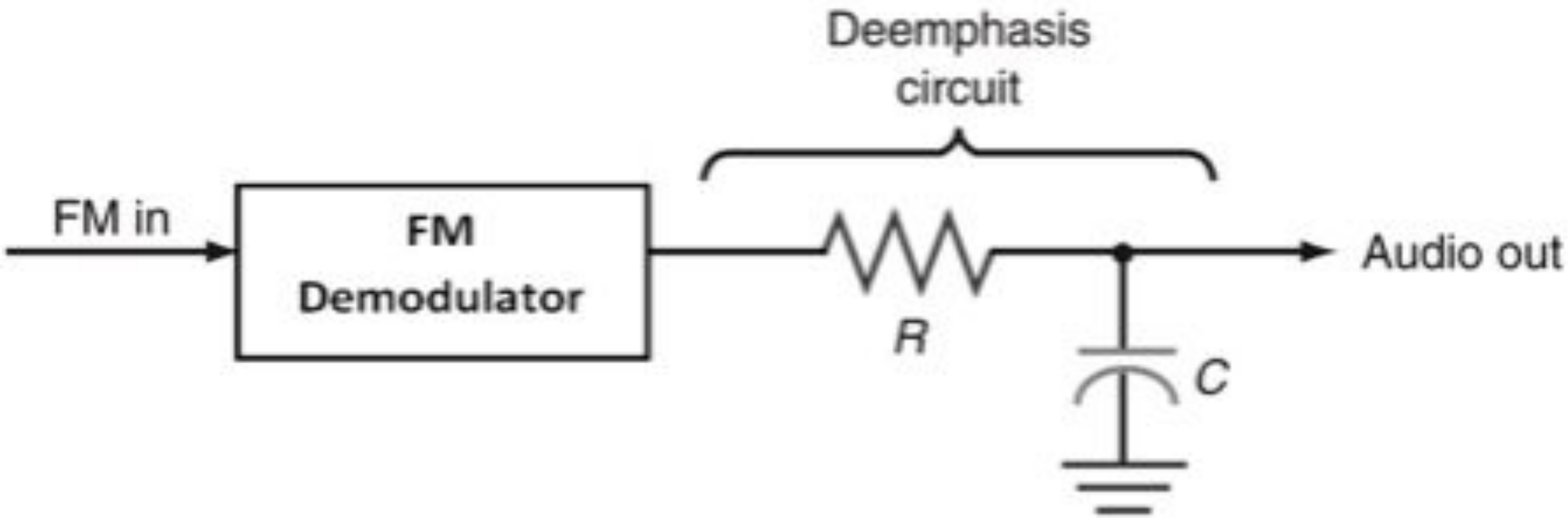


Fig4. De-emphasis circuit

The pre-emphasis process is done at the transmitter side, while the de-emphasis process is done at the receiver side.

Thus a high frequency modulating signal is emphasized or boosted in amplitude in transmitter before modulation. To compensate for this boost, the high frequencies are attenuated or de-emphasized in the receiver after the demodulation has been performed. Due to pre-emphasis and de-emphasis, the S/N ratio at the output of receiver is maintained constant.

The de-emphasis process ensures that the high frequencies are returned to their original relative level before amplification.

Pre-emphasis circuit is a high pass filter or differentiator which allows high frequencies to pass, whereas de-emphasis circuit is a low pass filter or integrator which allows only low frequencies to pass.

Radio Receiver

- It is and electronic device which pick the desired signal, rejected unwanted signals, amplify desired signals, demodulates the modulated wave to get back the original message signal.
- **Types of radio receiver:**

a) Depending on Application:

AM receiver

FM receiver

Communication receiver

Television receiver

Radar receiver

b) Depending upon fundamental aspect

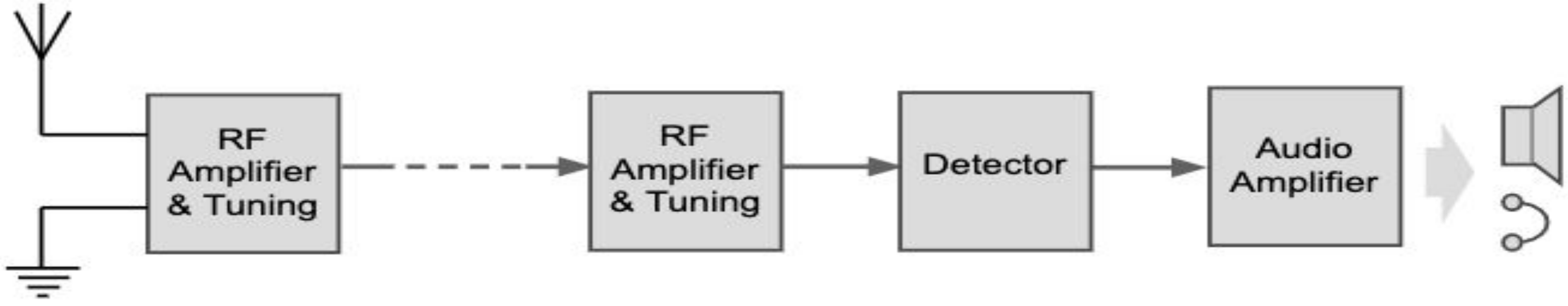
Tuned radio frequency receiver

Superheterodyne receiver

- The definition of the tuned radio frequency, TRF receiver is a receiver where the tuning, i.e. selectivity is provided by the radio frequency stages.
- In essence the simplest tuned radio frequency receiver is a simple crystal set.
- Tuning is provided by a tuned coil / capacitor combination, and then the signal is presented to a simple crystal or diode detector where the amplitude modulated signal, in this case, is recovered.
- This is then passed straight to the headphones.
- As vacuum tube / thermionic valve technology developed, these devices were added to provide more gain.

Typically a TRF receiver would consist of three main sections:

- ***Tuned radio frequency stages:*** This consisted of one of more amplifying and tuning stages. Early sets often had several stages, each proving some gain and selectivity.
- ***Signal detector:*** The detector enabled the audio from the amplitude modulation signal to be extracted. It used a form of detection called envelope detection and used a diode to rectify the signal.
- ***Audio amplifier:*** Audio stages to provide audio amplification were normally, but not always included.



Tuned radio frequency receiver, TRF, block diagram

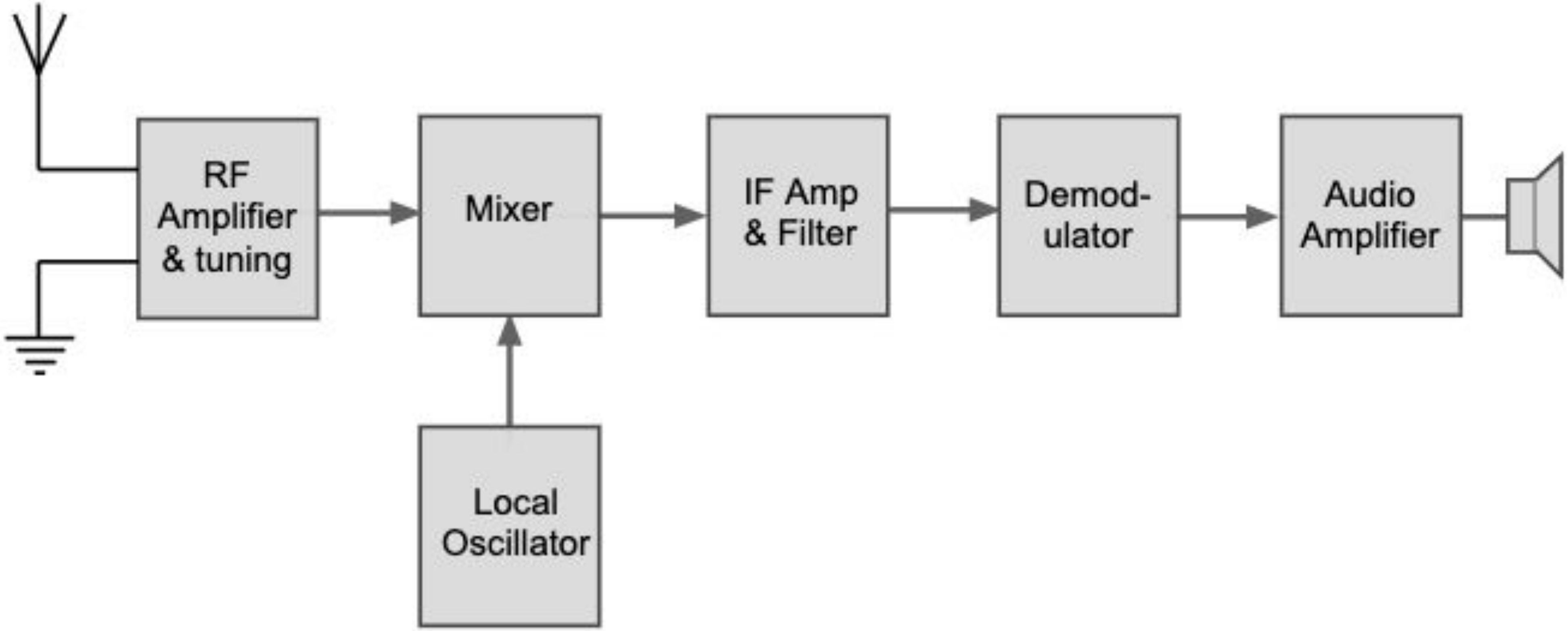
- The tuned radio frequency receiver was popular in the 1920s as it provided sufficient gain and selectivity for the receiving the broadcast stations of the day.
- However tuning took a little while as each stage in the early radios needed to be adjusted separately.
- Later ganged tuning capacitors were introduced, but by this time the **superheterodyne receiver** was becoming more widespread.

- **Limitations:**

- Instability of receiver
- Selectivity is poor
- Bandwidth variation over tuning is high

AM radio receiver(Super heterodyne receiver)

Professional superheterodyne receiver



Block diagram of a basic superheterodyne receiver

Operation:

Signals enter the receiver from the antenna and are applied to the RF amplifier where they are tuned to remove the image signal and also reduce the general level of unwanted signals on other frequencies that are not required.

- The signals are then applied to the mixer along with the local oscillator where the wanted signal is converted down to the intermediate frequency. Here significant levels of amplification are applied and the signals are filtered.
- This filtering selects signals on one channel against those on the next. It is much larger than that employed in the front end.
- The advantage of the IF filter as opposed to RF filtering is that the filter can be designed for a fixed frequency. This allows for much better tuning. Variable filters are never able to provide the same level of selectivity that can be provided by fixed frequency ones.
- Once filtered the next block in the superheterodyne receiver is the demodulator. This could be for amplitude modulation, single sideband, frequency modulation, or indeed any form of modulation. It is also possible to switch different demodulators in according to the mode being received.
- The final element in the superheterodyne receiver block diagram is shown as an audio amplifier, although this could be any form of circuit block that is used to process or amplified the demodulated signal.

There are some key circuit blocks within the design of the basic superheterodyne receiver.

Although more complicated receivers can be made, the basic RF circuit design is widely used –

Further blocks can add improved performance or additional functionality and their operation within the whole receiver is normally easy to determine once the basic block diagram is understood.

- ***RF tuning & amplification:*** This RF stage within the overall block diagram for the receiver provides initial tuning to remove the image signal. It also provides some amplification. There are many different approaches used within the RF circuit design for this block dependent its application.
- The RF circuit design presents some challenges. Low cost broadcast radios may have an amplifying mixer circuit that gives some RF amplification. HF radios may not want too much RF gain because some of the very strong signals received could overload later stages. The RF design may incorporate some amplification as well as RF attenuation to overcome this issue. Radios for VHF and above will tend to use more gain to have a sufficiently low noise figure to receive the signal.
- If noise performance for the receiver is important, then this stage will be designed for optimum noise performance. This RF amplifier circuit block will also increase the signal level so that the noise introduced by later stages is at a lower level in comparison to the wanted signal.

Local oscillator:

- Like other areas of the RF circuit design, the local oscillator circuit block within the superheterodyne radio can take a variety of forms.
- Early receivers used free running local oscillators. There was a considerable degree of RF circuit design expertise used with these oscillators in high performance superheterodyne radios to ensure the lowest possible drift. High Q coils, low drift circuit configurations, heat management (because heat causes drift), etc . .
- Today most receivers use one or more of a variety of forms frequency synthesizers. The most common approach in the RF circuit design is to use a phase locked loop approach. Single and multi-loop synthesizers are used. Direct digital synthesizers are also being used increasingly. Whatever form of synthesizer is used in the RF design, they provide much greater levels of stability and enable frequencies to be programmed digitally in a variety of ways, normally using some form of microcontroller or microprocessor system.

- ***Mixer:*** The mixer can be one of the key elements within the overall RF design of the receiver. Ensuring that the mixer performance matches that of the rest of the radio is particularly important.

Both the local oscillator and incoming signal enter this block within the superheterodyne receiver. The wanted signal is converted to the intermediate frequency.

- ***IF amplifier & filter:*** This superheterodyne receiver block provides the majority of gain and selectivity. Often comparatively little gain will be provided int he previous blocks of the RF circuit design of the radio. The IF stages are where the main gain is provided. Being fixed in frequency, it is much easier to achieve high levels of gain and overall performance.

Originally the IF stage might have included a number of different transistors, FETs or thermionic valves / vacuum tubes, but nowadays it is possible to obtain integrated circuits that contain a complete IF strip.

- ***Demodulator:*** The superheterodyne receiver block diagram only shows one demodulator, but in reality many radio RF designs may have one or more demodulators dependent upon the type of signals being receiver.

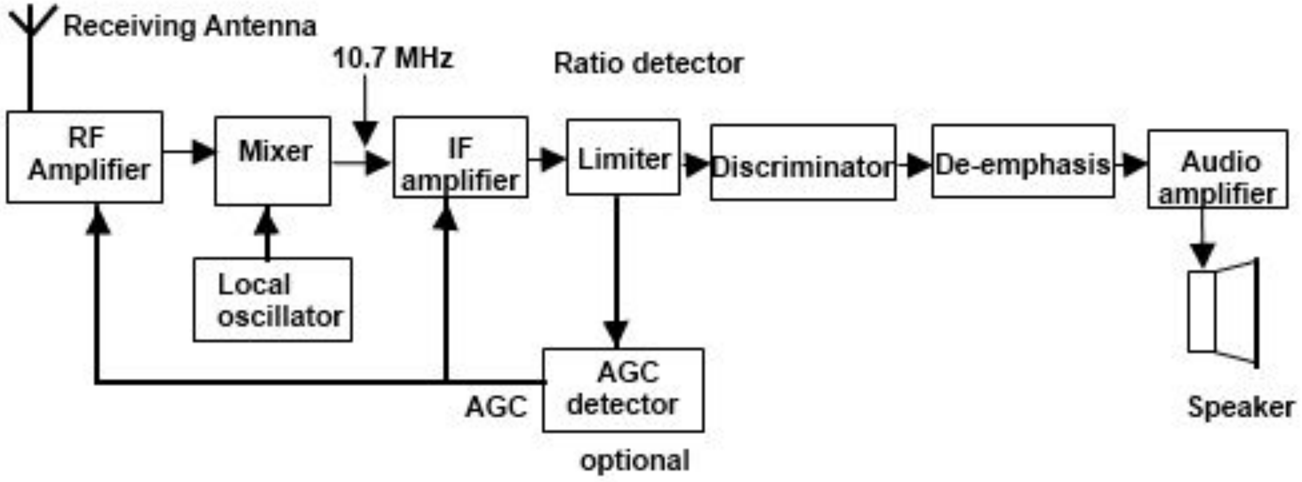
Even many broadcast radios will have AM and FM, but professional radios used for monitoring and two way radio communications may require a larger variety in some instances. Having a variety of demodulators will enable many different signal modes to be received and increase the capability of the radio.

- ***Automatic Gain Control, AGC:*** An automatic gain control is incorporated into most superheterodyne radio block diagrams. The function of this circuit block is to reduce the gain for strong signals so that the audio level is maintained for amplitude sensitive forms of modulation, and also to prevent overloading.

- ***Audio amplifier:*** Once demodulated, the recovered audio is applied to an audio amplifier block to be amplified to the required level for loudspeakers or headphones. Alternatively the recovered modulation may be used for other applications whereupon it is processed in the required way by a specific circuit block.
- In many ways, this circuit block within the superheterodyne radio is the most straightforward. For many applications, the audio amplifier will involved some straightforward electronic circuit design, especially if the audio is applied to simple headphones or a loudspeaker. For two way radio communication applications, the audio bandwidth may need to be limited to the "telecommunications" bandwidth of about 300 Hz to 3.3 kHz. Audio filters could be employed as well.
- For applications requiring a higher quality output, more thought may need to be applied during the electronic circuit design to achieving high fidelity performance.

Superheterodyne FM receiver

The block diagram of an FM receiver is illustrated in Figure (a). The RF amplifier amplifies the received signal intercepted by the antenna. The amplified signal is then applied to the mixer stage. The second input of the mixer comes from the local oscillator. The two input frequencies of the mixer generate an IF signal of 10.7 MHz. This signal is then amplified by the IF amplifier. Figure (a) shows the block diagram of an FM receiver.



Superheterodyne FM Receiver Block Diagram

The output of the IF amplifier is applied to the limiter circuit. The limiter removes the noise in the received signal and gives a constant amplitude signal. This circuit is required when a phase discriminator is used to demodulate an FM signal.

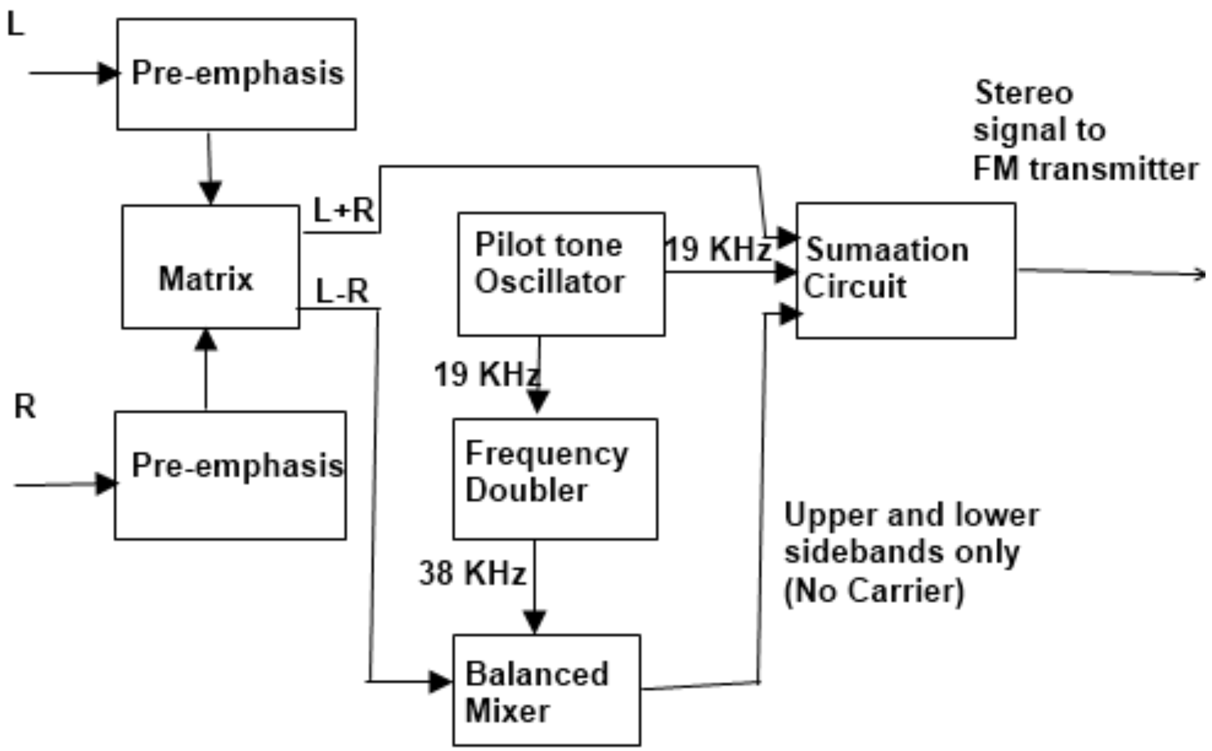
The output of the limiter is now applied to the FM discriminator, which recovers the modulating signal. However, this signal is still not the original modulating signal. Before applying it to the audio amplifier stages, it is de-emphasized. De-emphasizing attenuates the higher frequencies to bring them back to their original amplitudes as these are boosted or emphasized before transmission. The output of the de-emphasized stage is the audio signal, which is then applied to the audio stages and finally to the speaker.

It should be noted that a limiter circuit is required with the FM discriminators. If the demodulator stage uses a ratio detector instead of the discriminator, then a limiter is not required. This is because the ratio detector limits the amplitude of the received signal. In Figure (a) a dotted block that covers the limiter and the discriminator is marked as the ratio detector.

Commercial and stereo FM broadcasting and receiving technique.

- For better quality of signal reproduction (specially that of music signal at the receiving end , the output of microphones located at various positions of a concert hall are combined into two groups and the signal form each group is transmitted independently. The groups of signal are categorized at left and right channels. The system capable of transmitting both left and right channel signal and reproducing these two signal as separate signal at receiving end is called stereo system.

Transmitter:

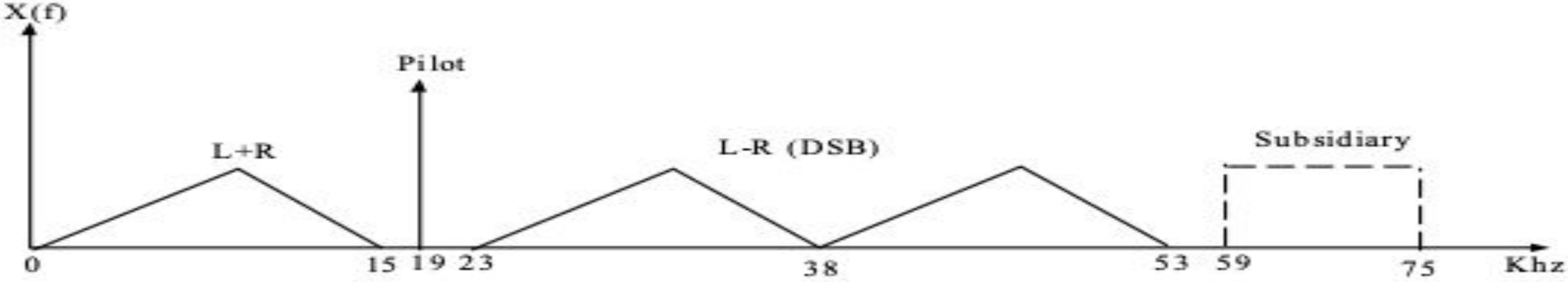


Form the matrix of resistor L+R and L-R signals are generated form L and R signal. Since L+R signal is directly passed to the modulator, only L-R signal undergo DSB-SC at 38khz. Some portion of the half of the carrier frequency (19khz) is also added to the DSB-SC for synchronizing purpose at the receiver. In this way for a audio signal L+R occupying bandwidth of 15 khz, the base band signal for Fm is 53khz. Since a maximum deviation of 75khz is allowed, the remaining frequency space of 59 to 75 khz can be used for subsidiary communication purpose.

In stereo FM broadcasting system to separate channel left(L) and right(R) are combined in the following to contributed the base band FM signal:

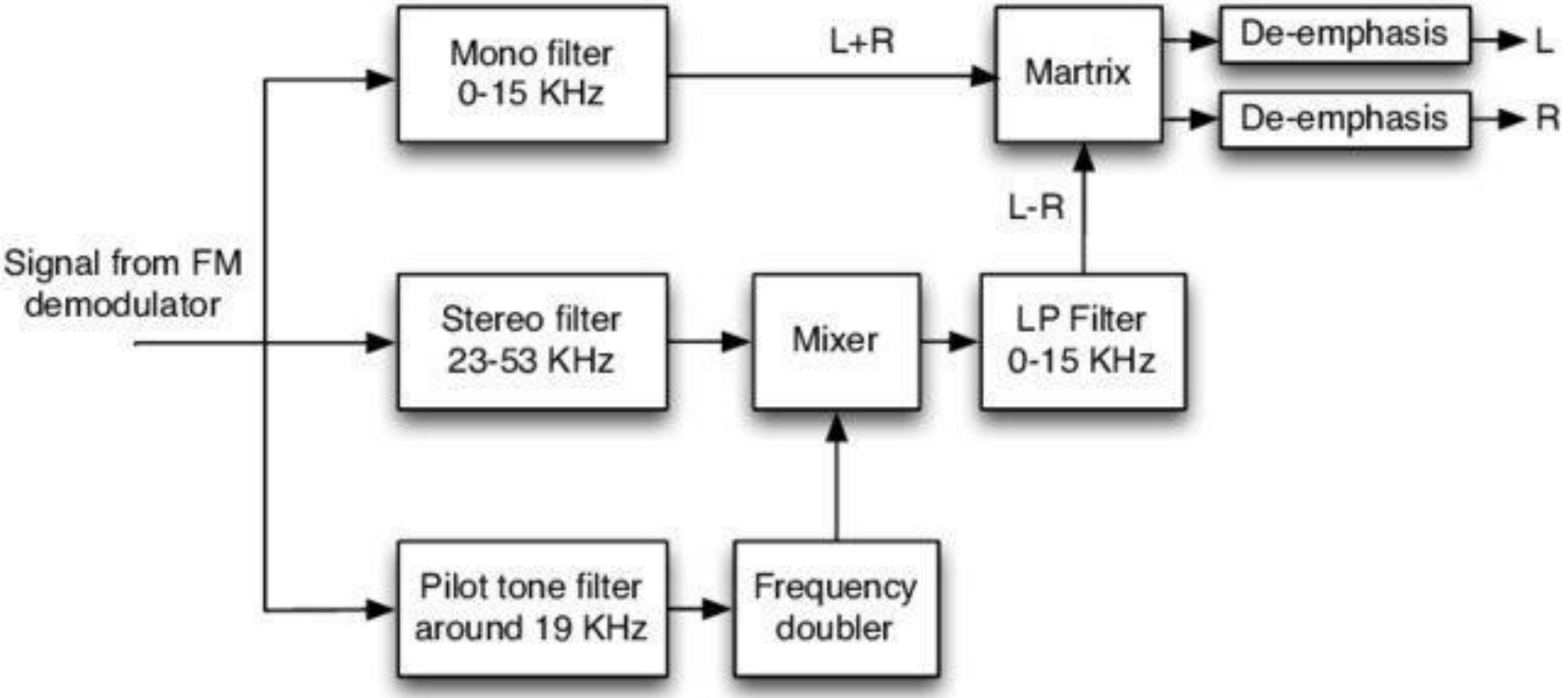
- Sum of L and R channels
- A pilot tone at 19khz as synchronizing signal and as indicator of stereo transmission system
- DSB-SC of difference of L and R (L-R) signals for the purpose of separation of L land R signal at the receiver

The base band spectrum of the FM stereo signal is shown below:



For FM stereo broadcasting channel in VHF band form 88 to 108 Mhz with spacing between channels 200khz are allocated. The allowable peak deviation per channel is 75khz.

Receiver:


Mono filter
0-15 KHz
L+R
Martrix
De-emphasis
L
De-emphasis
R
L-R
Signal from FM
demodulator
Stereo filter
23-53 KHz
Mixer
LP Filter
0-15 KHz
Pilot tone filter
around 19 KHz
Frequency
doubler

A non-stereo FM receiver can still receive stereo broadcast signal because there is baseband (L+R) signal in the stereo baseband spectrum. Similarly a stereo FM receiver can receive non-stereo FM broadcast signals. These two conditions are called forward and backward compatibility for FM stereo transmission. Therefore FM receiver having stereo decoder built in is called stereo FM receiver. A FM receiver available in the market may not contain stereo decoder. Such FM receiver will reproduce stereo FM signal in mono form.

Numerical:

1. Given $m(t) = 2\cos(2000\pi t) + \cos(3000\pi t)$ and
$c(t) = \sin(200000\pi t)$
write expression for FM and PM in time domain [take $k_f = 100$, $k_p = 200$]

→

For FM,
$$S_{FM}(t) = A_c \sin\left(2\pi f_c t + 2\pi k_f \int_0^t m(t)\,dt\right)$$
$$= \sin\left[2\pi f_c t + 2\pi \times 100 \int_0^t [2\cos(2000\pi t) + \cos(3000\pi t)]\,dt\right]$$
$$= \sin\left[200000\pi t + \frac{2\pi \times 100 \times 2}{2000\pi} \sin 2000\pi t + \frac{2\pi \times 100}{3000\pi} \sin 3000\pi t\right]$$
$$= \sin[200000\pi t + 0.2 \sin 2000\pi t + 0.0667 \sin 3000\pi t]$$

For PM,
$$S_{PM}(t) = A_c \sin[2\pi f_c t + k_p m(t)]$$
$$= \sin[2\pi f_c t + 200\{2\cos 2000\pi t + \cos 3000\pi t\}]$$
$$= \sin[200000\pi t + 400 \cos 2000\pi t + 200 \cos 3000\pi t]$$

Numerical:

\# If $V(t) = 10\cos(8*10^6 t + 4\sin 1000t)$, determine the following:

(i) Modulation index
(ii) Carrier frequency
(iii) Modulating frequency
(iv) Maximum deviation.

Solution:

$V(t) = 10\cos(8*10^6 t + 4\sin 1000t)$ ……..(i)

And we have,

$U_{FM}(t) = A_c\cos(2\pi f_c t + \beta\sin 2\pi f_m t)$

(i) $\beta = 4$

(ii) $2\pi f_c t = 8*10^6/2\pi$ hz.

(iii) $2\pi f_m t = 1000t$

$F_m = 1000/2\pi$

(iv) $\Delta f = ?$

$\beta = \Delta f / f_m$

$\Delta f = \beta f_m = 4 * 1000/2\pi$

A 107.6 Mhz carrier signal is frequency modulated by a 7khz sine wave. The resultant FM signal has a frequency deviation of 50 khz.

Determine:

(a) Carrier swing of FM signal.

(b) The highest and lowest frequency attenuates by the modulated signal.

(c) Modulation index of the FM signal.

$\Delta f = 50$ khz , $f_c = 107.6$ Mhz, $f_m = 7$khz.

Solution,

(a) Carrier swing $= 2\Delta f = 2*50 = 100$khz

(b) $f_h = f_c + \Delta f = 107.6$ Mhz + 50khz

$f_L = f_c - \Delta f = 107.6$Mhz – 50khz

(c) $\beta_{FM} = \Delta f / f_m = 50$khz/7khz

A sinusoidal modulating waveform of amplitude 5V and a frequency of 2KHz is applied to FM generator, which has a frequency sensitivity of 40Hz/volt. Calculate the frequency deviation, modulation index, and bandwidth.

Given, A_m=5V f_m=2KHz k_f=40Hz/volt

We know $\Delta f = k_f A_m$

$\Delta f = 40 \times 5 = 200$Hz

And modulation index (β) $=\Delta f / f_m$

$\beta = 200/(2 \times 1000) = 0.1$

The formula for Bandwidth of Narrow Band FM is the same as that of AM wave.

$BW = 2f_m$

$BW = 2 \times 2K = 4$KHz

The modulated signal is given by $x(t) = 40 \cos(100.1 * 10^6 * 2\pi t + 10\sin(15 * 10^3 * 2\pi t))$

- Determine which modulated signal is?
- Determine practical bandwidth
- Determine peak frequency deviation
- Determine frequency sensitivity
- Determine power deliver to 72 ohm impedance

2. A single-tone FM is represented by the voltage equation as:

$$v(t) = 12\cos[6\times10^8 t + 5\sin 1250t]$$

Determine: i) carrier frequency

ii) Modulating frequency

iii) the modulation index

iv) Maximum deviation

v) What power will this FM wave ~~dissipate~~ dissipate in 10 Ω resistors.

→ We know, for single tone FM.

$$S_{FM}(t) = A_c\cos[2\pi f_c t + \beta\sin 2\pi f_m t] \quad \text{——— (1)}$$

Comparing this with given equation.

i) Carrier frequency:

$$2\pi f_c = 6\times10^8 \text{ rad/sec}$$

$$f_c = \frac{6\times10^8}{2\pi} = 95.5\,\text{MHz}$$

ii) Modulating frequency (f_m) :

$$2\pi f_m = 1250 \text{ rad/sec}$$

$$f_m = \frac{1250}{2\pi} = 199 \text{ Hz}$$

iii) Modulation index (β):

$$\beta = 5$$

iv) Maximum frequency deviation (Δf) is :

As, $\beta = \frac{\Delta f}{f_m}$

$$\Rightarrow \quad \Delta f = \beta \cdot f_m$$

$$= 5 * 199$$

$$= 995 \text{ Hz}.$$

v) power dissipated (P) is:

$$P = \frac{A_c^2}{2R} = \frac{12^2}{2 \times 10}$$

$$\therefore \quad P = 7.2 \text{ Watt}.$$

3) How many AM broadcast stations can be ~~accomoda~~ accommodated in 100 KHz bandwidth if the highest frequency modulating a carrier is 5 KHz?

→ Bandwidth = 100 KHz.

Modulating frequency (f_m) = 5 KHz

Bandwidth of message signal (B) = $2 f_m$ = 10 KHz.

∴ No. of Station = $\dfrac{\text{Total Bandwidth}}{\text{Bandwidth per Station}} = \dfrac{100 \times 10^3}{10 \times 10^3}$

= 10

4) A 107.6 MHz Carrier signal is frequency modulated by a 7 KHz sine wave. The resultant FM signal has a frequency deviation of 50 KHz. Determine.

i) The carrier swing of the FM signal.

ii) The highest and the lowest frequencies attained by the modulated signal.

iii) The modulation index of the FM wave.

Given that

Carrier frequency (f_c) = 107.6 MHz

Message frequency (f_m) = 7 KHz

Frequency deviation (Δf) = 50 KHz.

i) Carrier swing = 2 × frequency deviation = $2 \times \Delta f$

= 2×50 = 100 KHz.

ii) The highest frequency and lowest frequency attained by the modulated signal is sum and difference of carrier frequency and frequency deviation.

i.e. Highest frequency (f_H) = $f_c + \Delta f$

$= 107.6 \times 10^6 + 50 \times 10^3$

$\therefore f_H = 107.65$ MHz

lowest frequency (f_L) = $f_c - \Delta f$

$= 107.6 \times 10^6 - 50 \times 10^3$

= 107.55 MHz.

iii) Modulation index (β) = $\frac{\Delta f}{f_m}$

$= \frac{50 \times 10^3}{7 \times 10^3} = 7.143$

$\beta = 7.143$.

5. What is modulation index of an FM signal having a carrier swing of 100 KHz, when the modulating signal has a frequency of 8 KHz?

Given that,

Carrier swing $(2\Delta f) = 100 KHz$

$\Rightarrow \quad \Delta f = 50 KHz$

Modulating frequency $(f_m) = 8 KHz$.

Therefore,

Modulation Index $(\beta) = \frac{\Delta f}{f_m} = \frac{50}{8}$

$= 6.25$

6. A baseband or modulating signal $x(t) = 5\cos 2\pi \times 15 \times 10^3 t$ angle modulates a carrier signal $A\cos\omega_c t$.

i) Determine the modulation index and bandwidth for FM and PM system.

ii) Find the change in the bandwidth and modulation index for both PM and FM if modulating frequency f_m is reduced by 5 KHz.

→ Assume $K_f = \cancel{K_p} =$ 15 KHz/volt and
K_p = 15 rad/volt

i) Given, $A_m = 5$
$f_m = 15$ KHz
$\Delta f = K_f \cdot A_m = 15 \times 5$
$= 75$ KHz.

Hence, modulation index $(\beta) = \dfrac{\Delta f}{f_m} = \dfrac{75}{15} = 5$

According to Carson's rule

Bandwidth $(BW) = 2(\Delta f + f_m) = 2(75 + 15)$
$BW = 180$ KHz

ii) For change in modulating frequency, $f_m = 5$ KHz.
$\Delta f = K_f \cdot A_m$
$= 75$ KHz

and, Bandwidth $(BW) = 2(\Delta f + f_m)$
$= 2(75 + 5)$
$= 160$ KHz.

Change in bandwidth $= 180 - 160$
$= 20$ KHz.

7) The maximum deviation allowed in an FM broadcast system is 75KHz. If the modulating signal is a single-tone sinusoidal of 8KHz, determine the bandwidth of FM signal. What will be the bandwidth when modulating signal amplitude is doubled?

→ Given, $\Delta f = 75 KHz$, $f_m = 8 KHz$

Bandwidth (BW) $= 2(\Delta f + f_m) = 2(75+8)$

$= 166 KHz$.

Now when the modulating signal amplitude is doubled the frequency deviation $\Delta f'$ becomes double.

Since, $\Delta f = K_f \cdot A_m$.

as amplitude double. $\Delta f' = 2 * \Delta f$

$= 150 KHz$.

$BW = 2(\Delta f' + f_m)$

$= 2(150+8)$

$= 316 KHz$.

Thank You