CHAPTER 1: **RADIATION AND ANTENNA FUNDAMENTALS**

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COURSE STRUCTURE

- RETARDED POTENTIALS: EM WAVE GENERATION WITH A CONDUCTION CURRENT, THE SHORT UNIFORM CURRENT DIPOLE, THE RADIATED ELECTRIC AND MAGNETIC FIELDS
- RADIATION PATTERNS AND INPUT IMPEDANCE OF THE SHORT UNIFORM CURRENT DIPOLE, THE SHORT DIPOLE AND LONG DIPOLE
- ANTENNA THEOREMS: RECIPROCITY, SUPERPOSITION, THEVENIN, MINIMUM POWER TRANSFER, COMPENSATION, EQUALITY OF DIRECTIONAL PATTERNS, EQUIVALENCE OF RECEIVING AND TRANSMITTING IMPEDANCES

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Phaser

The phaser in the electromagnetic field is analogous to the logarithm in the number field. As the log simplified the calculation for numbers,the phaser simplifies the calculation . the phaser simplifies the calculation involved the analysis of EM waves. The phaser is therefore a tool which makes a life easier in electromagnetics.

If the expression for varying x-component of electric field E_x is

$$E_x = E_{x0}\cos(wt + \varphi)$$

Then It's phaser representation is given by;

$$E_{xs} = E_{x0}e^{j\varphi}$$

To recover the original E_x from E_{xs} (like taking antilog in number field). We multiply E_{xs} by e^{jwt} and then take the real part i. e.

$$\begin{aligned} E_{xs}.\, e^{jwt} &= E_{x0}e^{j\varphi}.\, e^{jwt} \\ &= E_{x0}e^{j(wt+\varphi)} \\ &= E_{x0}[\cos(wt + \varphi) + j\sin(wt + \varphi)\,] \end{aligned}$$

$$Re\left[E_{xs}.\, e^{jwt}\,\right] = E_{x0}\cos(wt + \varphi)$$

Which is the original equation.

The phaser simplifies the differentiation of Ex as shown below:
We know,

$$E_x = E_{x0}\cos(wt+\varphi)$$

$$\frac{\delta E_x}{dt} = -wE_{x0}\sin(wt+\varphi)$$

But $Re\left[E_{xs}.\, jwe^{jwt}\right] = Re[jwE_{x0}e^{j(wt+\varphi)}]$

$= Re[jwE_{x0}\,[\cos(wt+\varphi) + j\sin(wt+\varphi)\,]\,]$

$= Re[jwE_{x0}\,\mathrm{wcos}(wt+\varphi) - wE_{x0}\sin(wt+\varphi)\,]$

$= -wE_{x0}\sin(wt+\varphi)$

Thus the multiplication of phaser quantity by jwe^{jwt} and taking its real part is equivalent to the differentiation of that quantity in time domain. Therefore the phaser has replaced the complex differentiation with the simple multiplication.

The wave equation :- The wave equation in E.M field(for electric field) is represented as

$E_x = E_{x0}\cos[w(t - z\sqrt{\mu \in})]$

Where $\sqrt{\mu \in} = \frac{1}{V} = \frac{1}{F\lambda} = \frac{T}{\lambda}$ is the phase velocity of the E.M wave in a medium having permeability μ and permittivity ϵ

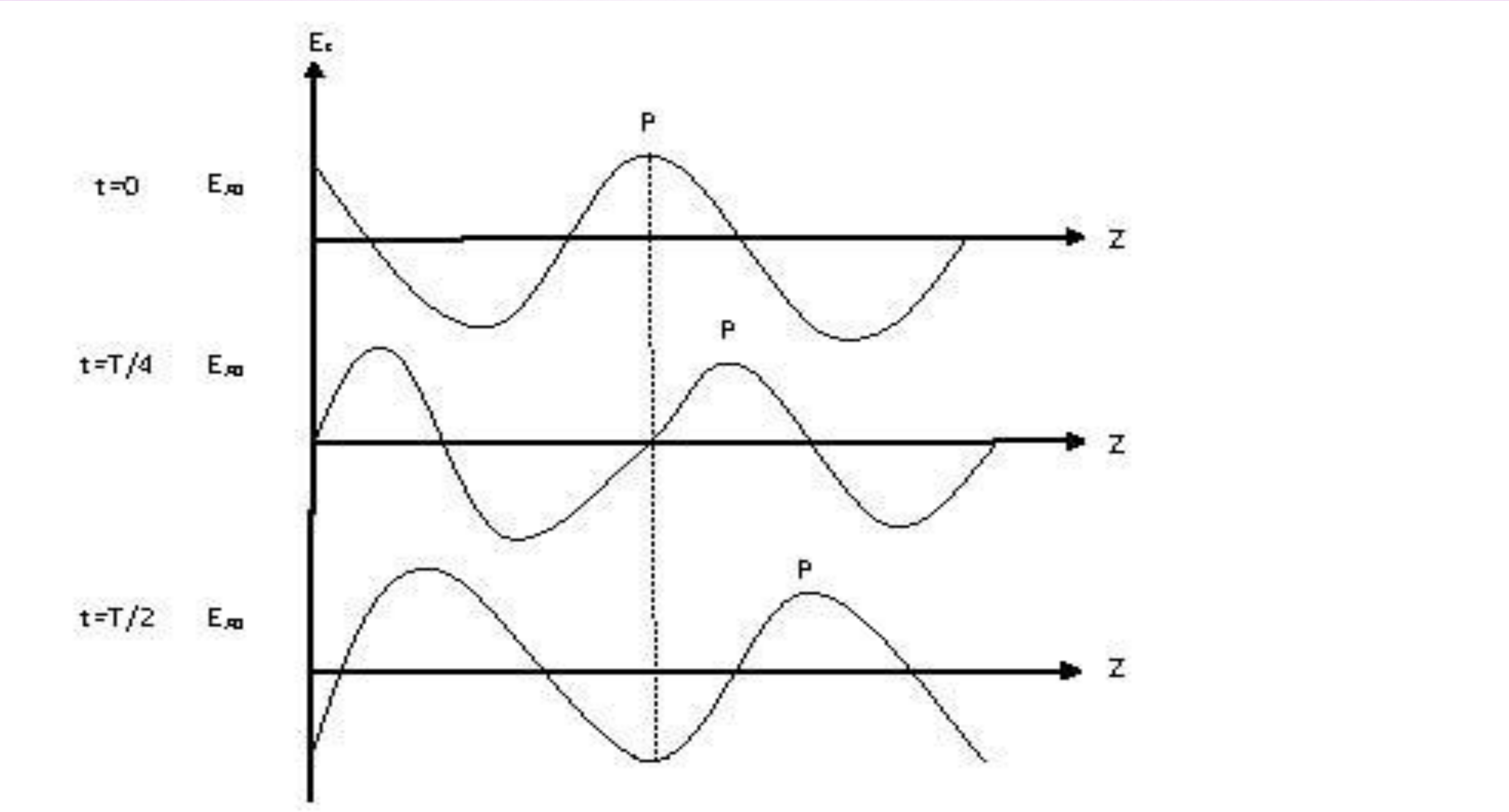
Direction travel:-

We have,

$$E_x = E_{x0}\cos[w(t - z\sqrt{\mu \in})]$$
$$= E_{x0}\cos[-(wz\sqrt{\mu \in} - \text{wt})]$$
$$= E_{x0}\cos[\frac{2\pi}{T}.z.\frac{T}{\lambda} - \frac{2\pi}{T}.t]$$

$\therefore E_x = E_{x0}\cos[\frac{2\pi}{\lambda}z - \frac{2\pi}{T}.t]$ ....(a)

$$for\ t = 0$$
$$E_x = E_{x0}\cos[\frac{2\pi}{\lambda}z\ ]$$
$$for\ t = \frac{T}{4}$$
$$E_x = E_{x0}\cos[\frac{2\pi}{\lambda}z - \frac{\pi}{2}]$$
$$E_x = E_{x0}\sin[\frac{2\pi}{\lambda}z\ ]$$
$$for\ t = \frac{T}{2}$$
$$E_x = E_{x0}\cos[\frac{2\pi}{\lambda}z - \pi]$$
$$E_x = E_{x0}\cos[\frac{2\pi}{\lambda}z\ ]$$



From the figure above it is clear that a point P which is under consideration on the wave is travelling in Z direction

When wave propagates in –ve Z direction then the wave equation becomes

$E_x = E_{x0} \cos[w(t + z\sqrt{\mu \in})]$

Wave Velocity

We have,

$E_x = E_{x0} \cos[w(t - z\sqrt{\mu \in})]$

The quantity $[w(t - z\sqrt{\mu \in})]$ is constant for any point on the wave provided the distance of the point is measured from a certain reference line. We assure the reference line at the point $for\ t = 0, z = 0$.

$Thus\ for\ t = 0, z = 0$

$[w(t - z\sqrt{\mu \in})] = 0$

Again for $t = \frac{T}{4}, Z = \frac{\lambda}{4}$

$[w(t - z\sqrt{\mu \in})] = [w\,(\frac{T}{4} - \frac{\lambda}{4}.\frac{T}{\lambda})\,] = 0$

Again for $t = \frac{T}{2}, Z = \frac{\lambda}{2}$

$[w(t - z\sqrt{\mu \in})] = 0$

And so on…..

The quantity $[w(t - z\sqrt{\mu \in})]$ is therefore a constant and is equal to zero.

$\therefore w(t - z\sqrt{\mu \in}) = 0$

Or $t - z\sqrt{\mu \in} = 0$

Or $t = z\sqrt{\mu \in}$

Or $\frac{z}{t} = \frac{1}{\sqrt{\mu \in}}$

Or $\frac{dz}{dt} = \frac{1}{\sqrt{\mu \in}} = \frac{1}{\sqrt{\mu_0 \mu_r \in_0 \in_r}}$ in air $\mu_r = \in_r = 1$

Here $\frac{dz}{dt}$ is the rate of change of distance w.r.t time, whish is the velocity of the wave.

Substituting the value of $\mu_0 = 4\pi x 10^{-7}$ and $\epsilon_0 = 8.854x10^{-12}\ f/m$

We get $\frac{dz}{dt} = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = \frac{1}{\sqrt{4\pi x10^{-7} X 8.854x10^{-12}}} = 3x10^8 m/s$

Which is equal to the velocity of light . The electromagnetic wave therefore travels with a velocity of light in free space or air .

Wave Impedance

The wave impedance is defined as

$$Wave\ Impedance = \frac{Electric\ Field\ Component}{Magnetic\ Field\ Component}$$

For TEM (Transverse Electromagnetic Wave) there exist only one component of each of the electric and magnetic fields, therefore we have only one wave impedance which is generally known as intrinsic impedance. i.e.

$$\eta = \frac{E_x}{H_y}\,\Omega$$

Where η =intrinsic impedance

E_x= X-component of EF

H_y=Y- component of magnetic field

For lossy medium (0<σ<∞)

$$\eta = \sqrt{\frac{jw\mu}{\sigma + jw\epsilon}} = \eta_m < \theta_\eta = \eta_m e^{j\theta}$$

For perfect dielectric(σ=0)

$$\eta = \sqrt{\frac{\mu}{\epsilon}}$$

For perfect conductor(σ→∞)

$$\eta = \sqrt{\frac{jw\mu}{\sigma}} = \eta_m < \theta_\eta$$

Here σ = conductance and w=angular frequency(rad/sec)

Poyinting vector

A pointing vector $\vec{p}$ is the cross product of $\vec{E}$(electric field) and $\vec{H}$(magnetic field). i.e.

$\vec{p} = \vec{E} X \vec{H}$

The magnitude of $\vec{p}$ represents the instantaneous power density (w/m^2) at a point and its direction indicates the direction of the power flow at that point and it is perpendicular to the plane containing $\vec{E}$ and $\vec{H}$.

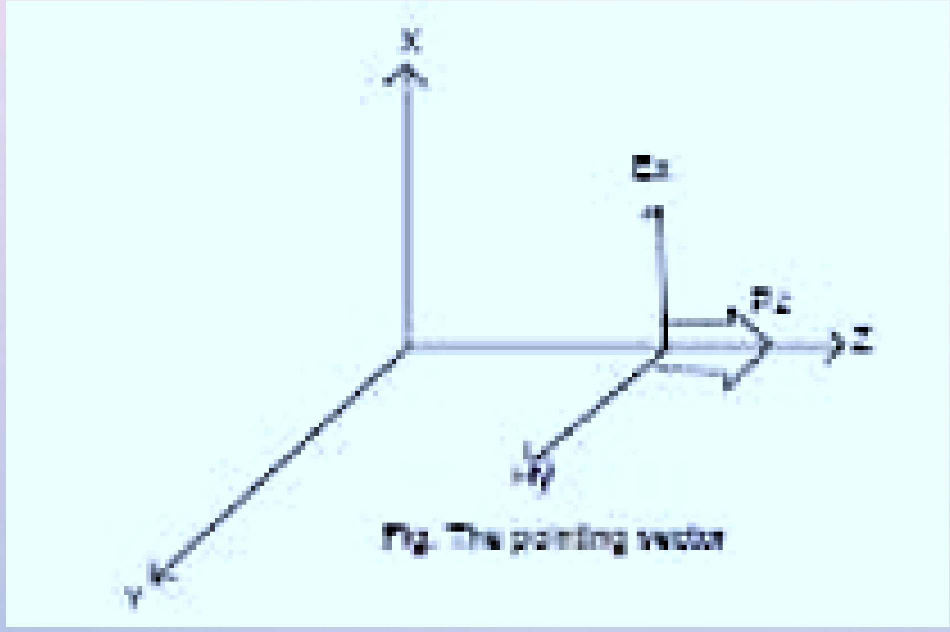


Fig. The pointing vector

Above figure illustrates a pointing vector P_z obtained for single component of the electric and the magnetic fields, that is E_X and H_y.

Following is the calculation of $\vec{p}$ for the TEM wave when it travels in different media.

Case 1: TEM wave travelling in a perfect dielectric(σ=0)

$$E_x = E_{x0}\cos[w(t - z\sqrt{\mu \in})] = E_{x0}\cos[wt - \beta z] \quad [\because w\sqrt{\mu \in} = \frac{2\pi}{T}.\frac{T}{\lambda} = \beta]$$

Where β is called phase constant.

For perfect dielectric

$$\eta = \frac{E_x}{H_y} = \sqrt{\frac{\mu}{\epsilon}}$$

$$\therefore H_y = \frac{E_x}{\eta} = \frac{1}{\eta}E_{x0}\cos[wt - \beta z]$$

And P$z = E_x H_y = \frac{1}{\eta}E^2{}_{x_0}\,cos^2[wt - \beta z]$

For convenience, an average power density Pz, av is therefore calculated as

P$z, av = \frac{1}{T}\int_0^T \frac{1}{\eta}E^2{}_{x_0}\,cos^2[wt - \beta z] = \frac{1}{2\eta}E^2{}_{x_0}\,w/m^2$

Case 2 : TEM wave is travelling in lossy medium (0< σ <∞)

The wave in lossy medium is found to be exponentially attenuated as distance Z increases. But the degree of attenuation depends upon the medium itself and is given α (dBm^{-1}) which is called the attenuation constant.

When these aspects are accommodated ,the wave equation turns out to be

$$E_x = e^{-\alpha z} E_{x0} \cos[wt - \beta z\ ]$$

Where $e^{-\alpha z}$ is called the attenuation coefficient.

For lossy medium $\frac{E_x}{H_y} = \eta_m < \theta_\eta = \eta_m e^{j\theta_\eta}$

$\therefore\ H_y = \frac{E_x}{\eta_m < \theta_\eta} = \frac{1}{\eta_m} e^{-\alpha z} E_{x0} \cos[wt - \beta z\ ] e^{-j\theta_\eta}$

Or $H_{ys} = \frac{1}{\eta_m} e^{-\alpha z} E_{x0} e^{-j(\beta z + \theta_\eta)}$

Or $H_y = \mathrm{Re}\{ \frac{1}{\eta_m} e^{-\alpha z} E_{x0} e^{-j(\beta z + \theta_\eta)} . e^{jwt} \}$

Or $H_y = \{ \frac{1}{\eta_m} e^{-\alpha z} E_{x0} \cos(\mathrm{wt} - \beta z - \theta_\eta) \}$

Hence

P$z = E_x H_y = \frac{1}{2\eta_m} e^{-2\alpha z} E^2{}_{x_0} [\cos\left(2\mathrm{wt} - 2\beta z - \theta_\eta\right) + \cos\theta_\eta]$ and

P$z, av = \frac{1}{2\eta_m} e^{-2\alpha z} \cos\theta_\eta$

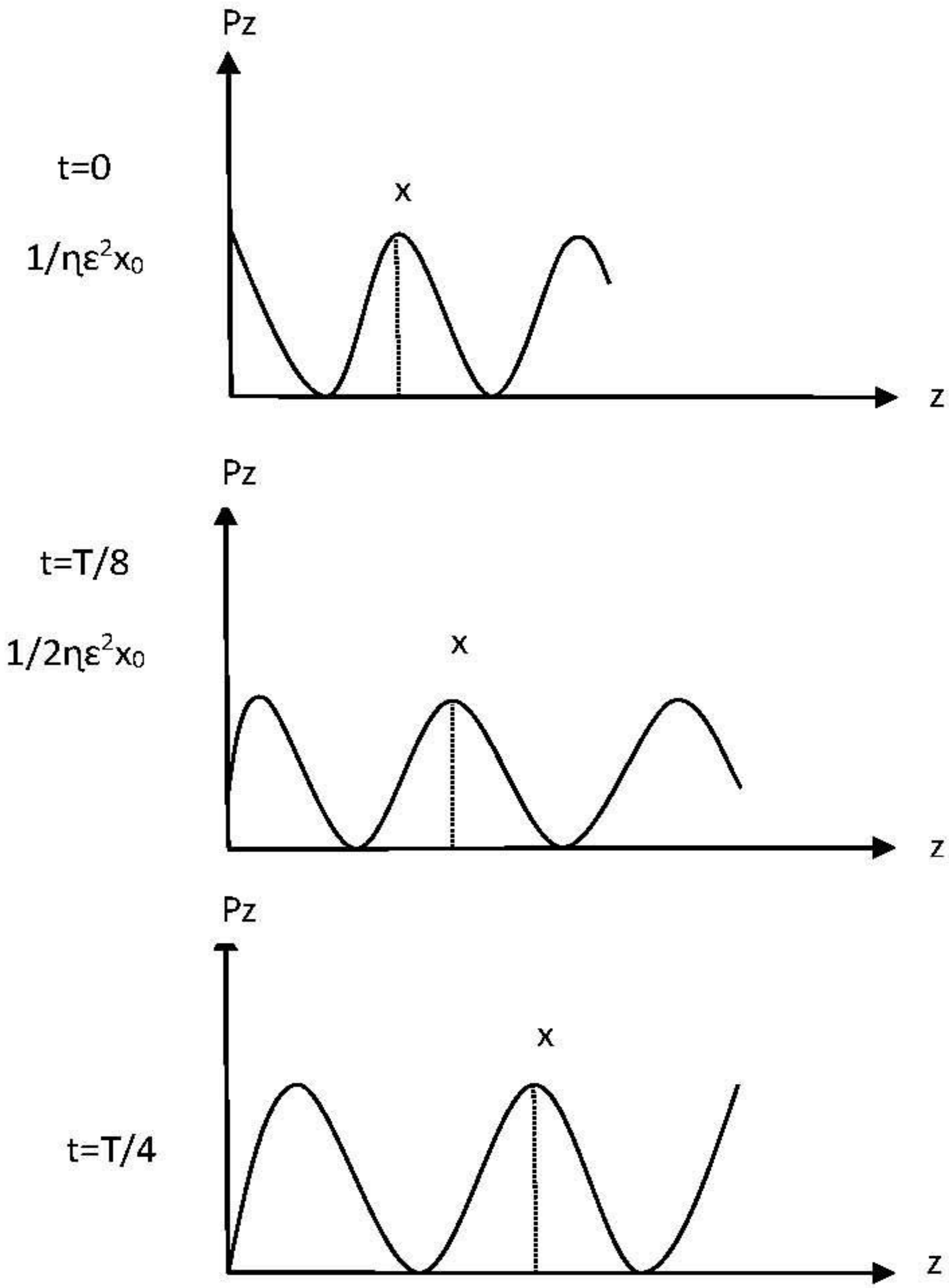


Fig: A power plot showing the motion of a point 'x' towards +Z direction which indicates the instantaneous power flow in that direction.

Retarded potential :-

The scalar electric potential at a point caused by a linear charge density is defined as

$V = \int \frac{[\rho_L d_L]}{4\pi\varepsilon r}$(V)………(i)

Where r is the distance between d_L and the point of interest.

Similarly, the vector magnetic potential is defined as

$\vec{A} = \int \frac{\mu I \overrightarrow{d_L}}{4\pi\varepsilon r}\ (wbm^{-1}) \ldots \ldots \ldots \ldots (ii)$

The direction of $\vec{A}$ is same as that of the current . In above equation(i) and (ii), ρ_L and I do not change with time and therefore V and $\vec{A}$ at the point of interest are fixed for all the time. But if ρ_L and I vary with time then their values seen at the time of measurement can not be used to calculate the V and $\vec{A}$ at a distant point . Because it takes time to reach the effect from the source to the point of interest, the values of ρ_L and I which actually contributed the effects have therefore already been changed to some other new values.

Therefore the above equation are modified as follows:

$$V = \int \frac{[\rho_L d_L]}{4\pi\varepsilon r}$$

And $\vec{A} = \int \frac{\mu[I]\overrightarrow{d_L}}{4\pi\varepsilon r}$

The V and $\vec{A}$ in above equation are respectively termed as retarted scalar electric potential and retarted vector magnetic potential. The symbol [.] represents that the corresponding quantity has been retarted in time in order to compensate the time elapsed in propagating the effect from the source to the points where two quantity is being calculated.

The sketch in figure (a) shows the effect propagating with the velocity of V from the source carrying the current I to the point of interest P at a distance r. The retarded current in this case is given by,

$$[I] = I_0 cosw(t - t`)$$

$$[I] = I_0 cosw(t - r/w)$$

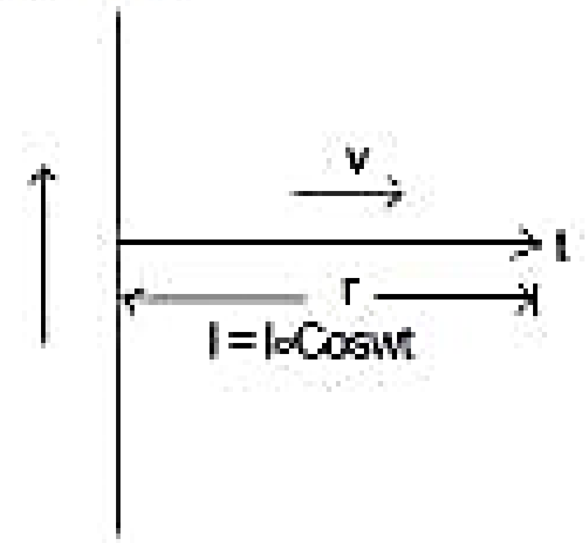


Figure (a)

THANK YOU