Chapter

# 5 BANDPASS (MODULATED) DATA COMMUNICATION SYSTEMS

## Introduction

In baseband digital system, the information bearing signals are transmitted without any changes in spectral components of the signal. Since the baseband signal power is localized at low frequencies, it would .require impractically large size antennas to radiate the low-frequency spectrum of the signal efficiently. Hence, for wireless communication the signal spectrum must be shifted to higher frequency range by employing modulation techniques.

In digital communication, the message signal (also called modulating signal) consists of digital data .If we change amplitude, frequency, phase of high frequency sinusoidal carrier signal according to the modulating signal, than the modulation is called Amplitude Shift Keying (ASK), Frequency Shift Keying (FSK) and Phase Shift Keying (PSK) respectively. The waveform of ASK, FSK and PSK is shown in Fig. 5.1 respectively.

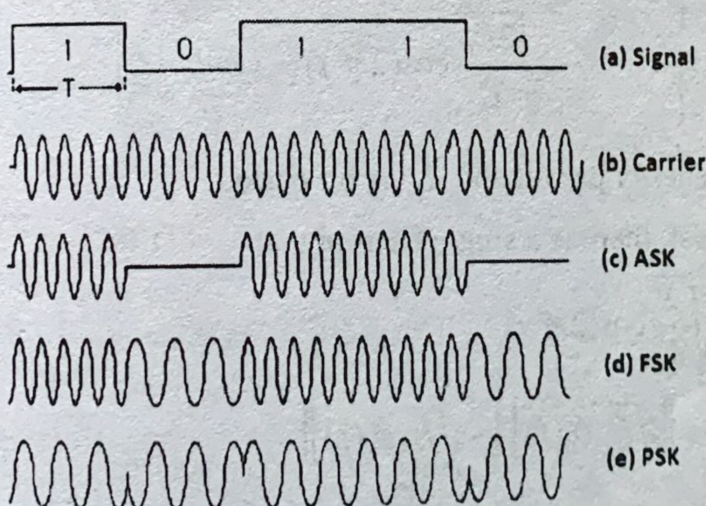


*Fig. 5.1: Waveform of (a) Binary signal, (b) Carrier signal, (c) ASK, (d) FSK and (e) PSK modulation*

In M-ary signaling, the modulator produces one of an available set of M (i.e., equal to $2^n$) distinct signals in response to n bits of source data at a time. Binary modulation is a special case of M-ary modulation with M equals to 2 (i.e., $2^1$).

At receiver, demodulation can be performed by either coherent or non-coherent detection methods as in case of analog modulation.

## 5.1 Geometric Representation and Constellation diagram of digital modulation

Digital modulation involves choosing a particular analog signal waveform $s_i(t)$ from a finite set S of possible signal waveforms based on the information bits applied to the modulator. For example, in binary modulation schemes, a binary information is directly mapped to a signal and set S contains only two signals, representing 0 and 1. Similarly, for M-ary scheme, set S contains more than two signals and each represent more than a single bit of information.

Any element of set S (i.e., $s_i(t)=\{s_1(t), s_2(t), \ldots s_M(t)\}$), can be represented as a point in a vector space whose coordinates are basis signal $\phi_i(t)$ (i.e., $f_1(t)$, $f_2(t)$ . . . . $\theta_N(t)$, such that function $\phi(t)$ are orthogonal over the interval $T$ is expressed as follows:

$$\int_T \phi_i(t)\, \phi_j(t) = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases} \qquad (5.1)$$

Now, $s_i(t)$ can be represented as a linear combination of the basis signals.

$$s_i(t) = \sum_{j=1}^{N} s_{ij}\phi_j(t) \qquad \begin{matrix} 0 \le t \le T \\ i = 1, 2, \ldots\ldots, M \end{matrix} \qquad (5.2)$$

**Example 5.1:** PSK Geometric representation

$$s_{BPSK}(t) = \begin{Bmatrix} \sqrt{\dfrac{2E_b}{T_b}}\cos(2\pi f_c t)\ ; & symbol\ 1 \\ -\sqrt{\dfrac{2E_b}{T_b}}\cos(2\pi f_c t); & symbol\ 0 \end{Bmatrix} \qquad (1)$$

Where, $E_b$ is energy per bit and $T_b$ is bit period.

For this signal set, there is a single basis signal

$$\phi_1(t) = \pm\sqrt{\frac{2}{T_b}}\cos(2\pi f_c t) \qquad (2)$$

$$s_{BPSK}(t) = \left\{\left[\sqrt{E_b}\phi_1(t)\right], \left[-\sqrt{E_b}\phi_1(t)\right]\right\} \qquad (3)$$

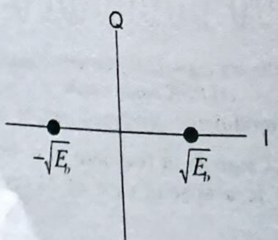


*Fig.1: Constellation Diagram for PSK signal.*



Constellation Diagram is a graphical representation of the complex envelope of each possible signal. In Fig.1 of Example.5.1, the x-axis represents the in-phase component and the y-axis represents the quadrature component of the complex envelope. The distance between signals 'd' on a constellation diagram relates to how different modulation waveforms are differentiated by the receiver from random noise present.

## 5.2 Amplitude Shift Keying (ASK)

In Amplitude Shift Keying (ASK) system, binary symbol '1' is represented by transmitting a sinusoidal carrier wave of fixed amplitude $A_c$ and fixed frequency $f_c$ for a bit duration of $T_b$ seconds. Whereas, binary symbol '0' is represented by switching off the carrier for $T_b$ seconds. This signal can be generated simply by turning the carrier ON and OFF for prescribed period, that is why it is called ON-OFF Keying (OOK).

Let the sinusoidal carrier be represented by

$$s_c(t) = A_c\cos(2\pi f_c t) \qquad (5.3)$$

Then ASK signal can be represented by

$$s(t) = \begin{Bmatrix} A_c\cos(2\pi f_c t)\ ; & symbol\ 1 \\ 0 & ;\ symbol\ 0 \end{Bmatrix} \qquad (5.4)$$

In more general form

$$s(t) = x(t)\cdot A_c\cos(2\pi f_c t) \qquad 0 \le t \le T_b \qquad (5.5)$$

where $x(t)$ is '1' or '0'.

This signal has power $P = \dfrac{A_c^2}{2}$

So, $A_c = \sqrt{2P}$

$$s(t) = \sqrt{2P}\cos(2\pi f_c t) \qquad 0 \le t \le T_b \qquad (5.6)$$

$$s(t) = \sqrt{PT_b}\sqrt{\frac{2}{T_b}}\cos(2\pi f_c t) \qquad 0 \le t \le T_b \qquad (5.7)$$

$$s(t) = \sqrt{E_b}\sqrt{\frac{2}{T_b}}\cos(2\pi f_c t) \qquad 0 \le t \le T_b \qquad (5.8)$$

where $E_b = PT_b$ is the energy contained in a bit duration

**Signal space diagram of ASK**

The ASK waveform of Eq.(5.8) for symbol '1' can be represented as,

$$s(t) = \sqrt{E_b}\sqrt{\frac{2}{T_b}}\cos(2\pi f_c t) \qquad 0 \le t \le T_b \qquad (5.9)$$

$$s(t) = \sqrt{E_b}\phi_1(t) \qquad (5.10)$$

This means that there is only one carrier function $f_1(t)$. The signal space diagram will have two points on x-axis. One will be at zero and other will be at $\sqrt{E_b}$. Fig.5.1 shows the constellation diagram of ASK signal. The distance between the two signal points is,

$$d = \sqrt{E_b} \tag{5.11}$$

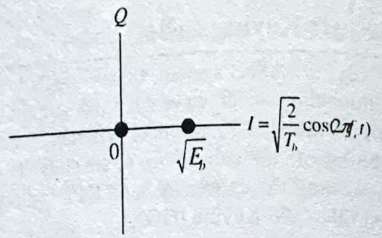


*Fig.5.2: Constellation diagram of ASK signal*

### Generation of ASK signal

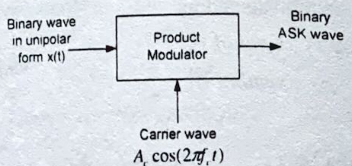


*Fig.5.3: ASK modulation*

ASK signal can be generated by applying the incoming binary data (should be in the unipolar format) and the sinusoidal carrier to the two inputs of the product modulator (Balanced Modulator) as shown in Fig.5.3. ASK signal allows the carrier signal for input message bit '1' and stops carrier signal for message bit '0'.

### Detection of ASK

**1. Coherent**

ASK signals can be detected coherentlyor non-coherently. In coherent detection, exact replicas (carrier waves) of the possible arriving signals are available at the receiver. This received signal is then cross related with each of the replicas of the received signal and decision is made based on comparison with preselected thresholds.

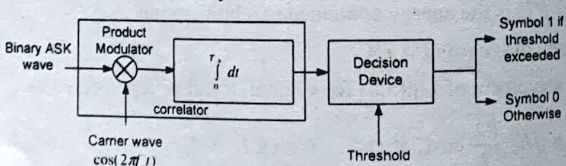


*Fig.5.4. ASK Coherent Demodulation*



The block of coherent detector of ASK signal is shown in Fig.5.4. As discussed above, cross-correlation of incoming ASK signal and locally generated un-modulated carrier signal is estimated with the help of product modulator and integrator block. The output of the integrator is then applied to the decision making device thatcompares the output with a preset threshold. The decision is made in favor of symbol '1' if the threshold is exceeded and in favor of symbol '0' otherwise. In this method,it is assumed that the local carrier is in perfect synchronization with the carrier used in the transmitter. This means the frequency and phase of the locally generated carrier is same as those of the carrier used in the transmitter.

**2. Non-coherent**

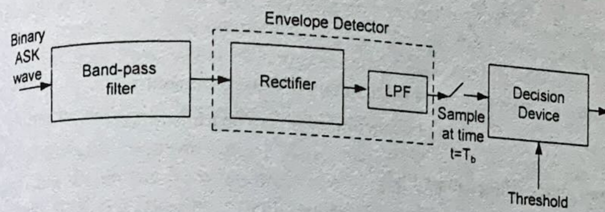


*Fig.5.5. ASK Non-coherent Demodulation*

In Non-coherent detection shown in Fig.5.5, the priorknowledge of the phase and frequency of carrier wave is not required. ASK can be demodulated using Non-coherent detector using simple envelope detector. Basically, if envelope is detected, it means bit'1' has been transmitted and no envelope means '0' bit.

## 5.3 Phase shift keying (PSK)

Phase Shift Keying (PSK) is more efficient than ASK, and is used for high bit rates data transmission. In Binary PSK (PSK) system, a sinusoidal carrier wave of fixed amplitude and fixed frequency $f_c$ is used to represent both symbols '1' and '0', except the carrier phase of each symbol differs by 180° (i.e., $\pi$ radians). Let the unmodulated carrier be represented by

$$s_c(t) = A_c\cos(2\pi f_c t) \tag{5.12}$$

Then the PSK signal can be represented by

$$s(t) = \begin{cases} A_c\cos(2\pi f_c t)\,; & symbol\ 1 \\ A_c\cos(2\pi f_c t + \pi)\,; & symbol\ 0 \end{cases} \tag{5.13}$$

This signal has power $P = \frac{A_c^2}{2}$. So, $A_c = \sqrt{2P}$

$$s(t) = \begin{cases} \sqrt{2P}\cos(2\pi f_c t)\,; & symbol\ 1 \\ -\sqrt{2P}\cos(2\pi f_c t)\,; & symbol\ 0 \end{cases} \tag{5.14}$$

As, $E_b = PT_b$ we have,

$$s(t) = \begin{cases} \sqrt{\dfrac{2E_b}{T_b}}\cos(2\pi f_c t); & symbol\ 1 \\ -\sqrt{\dfrac{2E_b}{T_b}}\cos(2\pi f_c t); & symbol\ 0 \end{cases} \tag{5.15}$$

Eq.(5.15) is further simplified to define PSK signal as,

$$s(t) = b(t)\sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t) \tag{5.16}$$

Where,

$$b(t) = \begin{cases} +1 \text{ when binary '1' is to be transmitted} \\ -1 \text{ when binary '0' is to be transmitted} \end{cases}$$

**Signal space diagram of PSK**

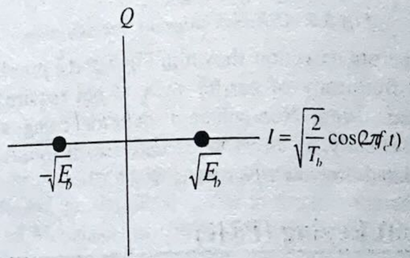


*Fig.5.6. Constellation diagram of PSK signal*

The signal space or constellation diagram of message signal is shown in Fig 5.6. The PSK signal is expressed in Eq.(5.16), is rearranging as

$$s(t) = b(t)\sqrt{E_b}\sqrt{\frac{2}{T_b}}\cos(2\pi f_c t) \tag{5.17}$$

Replacing b(t) with $\pm 1$

$$s(t) = \pm\sqrt{E_b}\sqrt{\frac{2}{T_b}}\cos(2\pi f_c t) \tag{5.18}$$

Where, $\phi_1(t) = \pm\sqrt{\frac{2}{T_b}}\cos(2\pi f_c t)$ represent the basis of carrier signal.



**Generation of PSK signal**

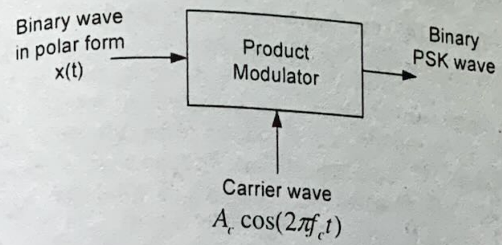


*Fig.5.7. PSK Modulation*

PSK signal may be generated by applying carrier signal to a product or balanced modulator. The binary data signal (0's and 1's) is converted into a polar form. The PSK signal PSK signal can be generated by using the same scheme as used in the generation of ASK. The only difference is that the incoming binary data should be in the polar form.

The bandwidth requirement of ASK and PSK is same. But the most significant difference between these two is that ASK in a linear modulation where as PSK is a non-linear modulation scheme.

**Detection of PSK**

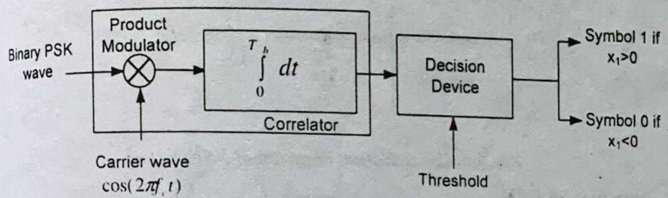


*Fig.5.8. PSK Coherent Demodulation*

The same coherent detector as in case of ASK can be used to detect the BPSK. The only difference is the threshold level. In ASK, the threshold level is set to d/2, whereas in BPSK the threshold level is 0.

In case of phase shift keying (PSK), the non-coherent demodulation is not possible as,the envelop detection will produce same level of signal at the output for both 1 and 0. However, there is a pseudo PSK technique known as differential phase shift keying (DPSK) which can be viewed as the non-coherent form of PSK.

## 5.4 Frequency Shift Keying(FSK)

In FSK system, two sinusoidal carrier waves of the same amplitude $A_c$, but with different frequencies $f_{c1}$ and $f_{c2}$ are used to represent binary symbol '1' and '0' respectively.

The FSK wave $s(t)$ may be viewed as

$$s(t)=\begin{cases} A_c\cos(2\pi f_{c_1}t); & symbol\ 1 \\ A_c\cos(2\pi f_{c_2}t); & symbol\ 0 \end{cases} \tag{5.19}$$

Generally it is represented by

$$s_i(t)=\begin{cases} \sqrt{\dfrac{2E_b}{T_b}}\cos(2\pi f_{c_i}t); & 0\le t\le T_b \\ 0 & ;elsewhere \end{cases} \tag{5.20}$$

Where, $i$=1 and 2, and $E_b$ is transmitted signal energy per bit.

The transmitted frequency is computed as.

$$f_{c_i}=\frac{n_c+i}{T_b} \text{ (for some fixed integer } n_c.)$$

Thus, symbol 1 is represented by $s_1(t)$ and symbol 0 by $s_2(t)$.

FSK system has signal space that is two dimensional with two message points. The signal constellation is shown in Fig.5.9.

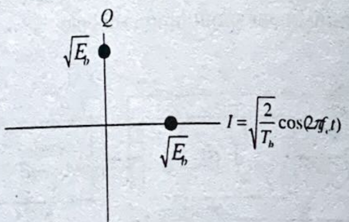


*Fig.5.9. Constellation diagram of FSK signal*

**Generation of FSK**

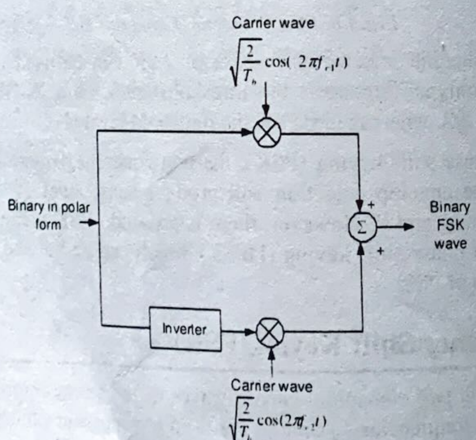


*Fig.5.10. Binary FSK Modulator*



The binary sequence is represented in polar form, with symbol 1 represented by a constant amplitude of $\sqrt{E_b}$ volts and symbol 0 represented by zero volts. When symbol 1 is at the input, the oscillator with frequency $f_{c1}$ in the upper channel is switched on, while the oscillator with the frequency $f_{c2}$ in the lower channel is switched off, resulting in frequency $f_{c1}$ being transmitted. Similarly, when symbol 0 is at the input, the oscillator in the upper channel is switched off, and the oscillator in the lower channel is switched on, resulting in frequency $f_{c2}$ being transmitted as shown in Fig.5.10.

As with analog FM, the bandwidth requirement of FSK is higher than that of ASK and PSK.

**Detection of FSK signal**

1. **Coherent**

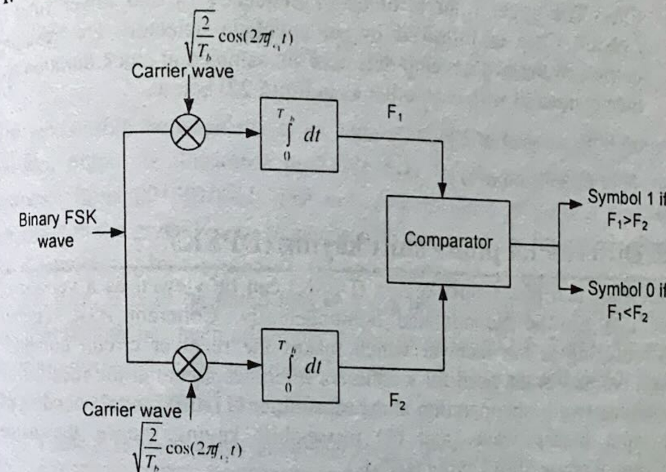


*Fig.5.11. Binary FSK Demodulator*

The coherent detector arrangement for demodulation of binary FSK is as shown in Fig.5.11. It consists of two correlators with a common binary FSK signal input. They are correlated with locally generated coherent reference signal $\sqrt{\frac{2}{T_b}}\cos(2\pi f_{c1}t)$ and $\sqrt{\frac{2}{T_b}}\cos(2\pi_{c2}t)$. The decision making device is essentially a comparator which compares output $F_1$ (in the upper path) and output $F_2$ (in the lower path). The decision logic is.

$$\left.\begin{matrix} F_1>F_2 \text{ so symbol '1'} \\ F_1<F_2 \text{ so symbol '0'} \end{matrix}\right\} \tag{5.21}$$

2. **Non-coherent**

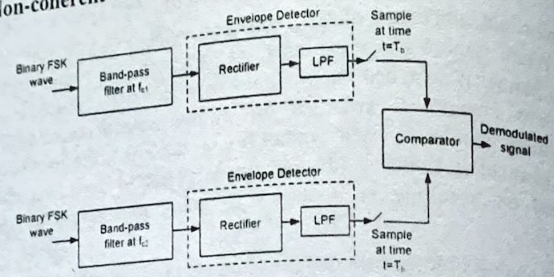


*Fig.5.12. Non-coherent Demodulator of binary FSK*

The arrangement for non-coherent demodulation of FSK is shown in Fig.5.12. The received FSK signal is applied to two band-pass filters.The upper filter is tuned to frequency $f_{c1}$ and lower filter to $f_{c2}$.Each filter is followed by an envelope detector. The resulting outputs of the two envelop detectors are sampled at clock duration and then compared with each other as in Eq.(5.22) below.

$$\left.\begin{aligned} F_1 > F_2 \text{ so symbol '1'} \\ F_1 < F_2 \text{ so symbol '0'} \end{aligned}\right\} \tag{5.22}$$

## 5.5 Differential phase shift keying (DPSK)

The differential phase-shift keying (DPSK) can be viewed as a version of PSK that can be demodulated non-coherently. Coherent PSK requires synchronizing at the receiver which makes the receiver circuit complex. DPSK eliminates the need for a coherent reference signal at the receiver by combining two basic operation at the transmitter (1) differential encoding of the input binary wave, and (2) phase-shift keying, hence the name differential phase-shift keying (DPSK).

The information content of the binary data is encoded in terms of signal transitions. The symbol 0 is used to represent transition of current signal waveform by 180° and symbol 1 to indicate no transition.

**Generation of DPSK**

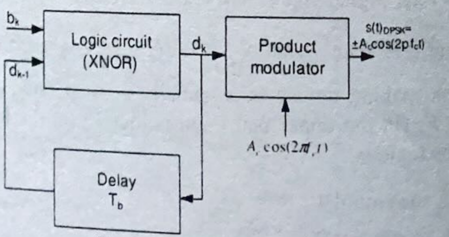


*Fig.5.13. DPSK generator*



The block diagram for the generation of DPSK signal is as shown in Fig.5.13. The binary data stream $b_k$ is applied to the input of the encoder. The output of the encoder is applied to the product modulator. To the other input of product modulator, a sinusoidal carrier of fixed amplitude and frequency (i.e., $A_c \cos(2\pi f_c t)$) is applied. A one-bit delay element is used to convert an input binary sequence $b_k$ into a differentially encoded sequence $d_k$.

$$s_{DPSK}(t) = \begin{cases} A_c \cos(2\pi f_c t)\,; & symbol\ 1 \\ A_c \cos(2\pi f_c t + \pi); & symbol\ 0 \end{cases} \tag{5.23}$$

Thus, similar to PSK

$$s_{DPSK}(t) = \begin{cases} \sqrt{\dfrac{2E_b}{T_b}} \cos(2\pi f_c t)\,; & symbol\ 1 \\ -\sqrt{\dfrac{2E_b}{T_b}} \cos(2\pi f_c t); & symbol\ 0 \end{cases} \tag{5.24}$$

The relationship between the input binary sequence and its differentially encoded output is illustrated in Table 5.1, for an assumed input data sequence $b_k$ is 0010010011. The encoding is done in such a way that transition in the given binary sequence with respect to the previous encoded bit is represented by a symbol 0 and no transition by symbol 1 i.e., using a XNOR gate. It is to be noted that an extra bit (symbol 1) in differentially encoded data $d_k$ has been arbitrarily added as an initial bit. This is essential to determine the encoded sequence. The phase of the generated DPSK signal has been shown in the 3rd row of Table 5.1.

**Table 5.1:** Generation and detection of DPSK signal.

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | Binary data $[b_k]$ | | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 1 |
| 2 | Differentially encoded data $[d_k]$ | 1* | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 1 |
| 3 | Phase of DPSK | 0 | π | 0 | 0 | π | 0 | 0 | π | 0 | 0 | 0 |
| 4 | Shifted differentially encoded data $[d_{k-1}]$ | | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 |
| 5 | Phase of shifted DPSK | | 0 | π | 0 | 0 | π | 0 | 0 | π | 0 | 0 |
| 6 | Phase comparison output | | - | - | + | - | - | + | - | - | + | + |
| 7 | Detected binary sequence | | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 1 |

*Arbitrary starting reference bit.

### Detection of DPSK

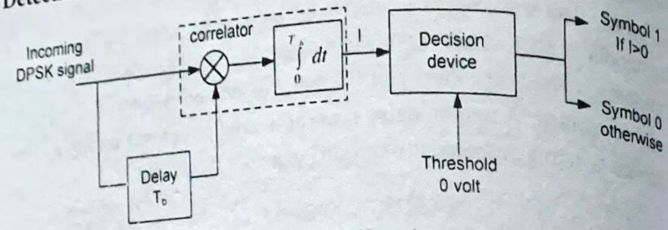


*Fig.5.14. DPSK receiver*

For detection of the differentially encoded PSK (i.e., DPSK), the receiver arrangement as shown in Fig. 5.14 can be used. The received DPSK signal is applied as one input to the correlator and a delayed version of the received DPSK signal by the time interval $T_b$ is applied to another input. The delayed version of the received DPSK signal (in the absence of channel noise) has been shown in the 4th row of the table. The output of the difference is proportional to cos ($\phi$), here $\phi$ is the difference between the carrier phase angle of the received DPSK signal and its delayed version, measured in the same bit interval. The phase angles of the DPSK signal and its delayed version have been shown in 3rd and 5th rows respectively. The phase difference between the two sequences for each bit interval is used to determine the sign of the phase comparator output. When $\phi = 0$, the integrator output is positive whereas when $\phi = \pi$, the integrator output is negative. By comparing the integrator output with a decision level of zero volt, the decision device can reconstruct the binary sequence by assigning a symbol 0 for negative output and a symbol 1 for positive output. The reconstructed binary data is shown in the last row (i.e., 7$^{th}$ row) of Table 5.1.

The advantage of DPSK is that no synchronous carrier is necessary at the receiver, and the disadvantage is that an error occurs in pairs, because one erroneous bit affects the decision in two successive bit intervals. Hence, in DPSK, the error rate is greater than in PSK.

## 5.6 M-ary Signaling

In an M-ary signaling scheme, we may send one of M possible signals, $s_1(t)$, $s_2(t)$ . . . $s_M(t)$, during each signaling interval of duration $T$ seconds. The number of possible signals i.e., $M$ is given by $M = 2^n$, where $n$ is an integer. The symbol duration is $T = nT_b$, where $T_b$ is the bit duration. These signals are generated by changing the amplitude, phase or frequency of a carrier in $M$ discrete steps.



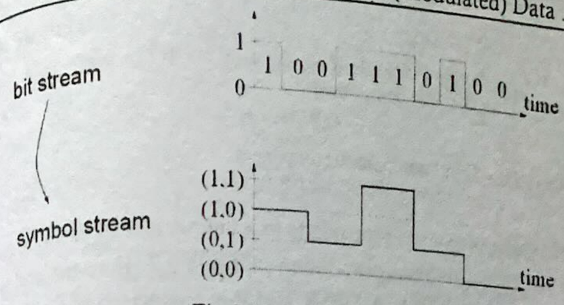


*Fig.5.15. M-ary signal*

The equivalent data rate $R_{min}$ M-ary signaling is $\log_2 M$ times faster than the binary signaling. The bandwidth requirement in M-ary is same as in binary but the power requirement is higher.

### 5.6.1 Quadriphase Shift Keying(QPSK)

All the modulation techniques studied so far are not much efficient in bandwidth. This section deals with two bandwidth efficient modulation schemes for transmission of binary data. They are both examples of quadrature carrier multiplexing system, which produces a modulated wave described as follows:

$$s(t) = s_I(t)\cos(2\pi f_c t) - s_Q(t)\sin(2\pi f_c t)$$

$s_I(t)$ = In phase component

$s_Q(t)$ = Quadrature phase component of modulated wave

The QPSK system is an example of M-ary PSK with $M = 4$. In the PSK; the basic idea is to transmit the information in the phase change of carrier. In QPSK, the phase of carrier takes on one of the four equally spaced values such as $\pi/2$, $3\pi/4$, $5\pi/4$, $7\pi/4$ as shown by

$$s_i(t) = \begin{cases} \sqrt{\dfrac{2E}{T}}\cos[2\pi f_c t + \varphi_i(t)] & 0 \le t \le T \\ 0 & elsewhere \end{cases} \tag{5.25}$$

Where, phase $\phi_i(t) = (2i - 1)\dfrac{\pi}{4}$ for $i = 1, 2, 3, 4$. Now the equation can be expressed as

$$s_i(t) = \begin{cases} \sqrt{\dfrac{2E}{T}}\cos(\varphi_i(t))\cos(2\pi f_c t) - \sqrt{\dfrac{2E}{T}}\sin(\varphi_i(t))\sin(2\pi f_c t) & 0 \le t \le T \\ 0 & elsewhere \end{cases} \tag{5.26}$$

Here,

$$\sqrt{\frac{2E}{T}}\cos(\varphi_i(t)) = \text{In phase component}$$

$$\sqrt{\frac{2E}{T}}\sin(\varphi_i(t)) = \text{Quadrature phase component}$$

Each possible value of phase corresponds to a unique pair of two bits called dibits. Here we have complete description in Table 5.2. Here the codes are gray codes 10, 00, 01, and 11. A QPSK signal is characterized by having a two-dimensional signal constellation and four message points, as shown in Fig.5.16.

**Table 5.2:** Constellation points of QPSK.

| Input dibit | Phase of QPSK | Coordinates of message points | |
|---|---|---|---|
| | | $s_{i1}$ | $s_{i2}$ |
| 10 | $\pi/4$ | $+\sqrt{E/2}$ | $-\sqrt{E/2}$ |
| 00 | $3\pi/4$ | $-\sqrt{E/2}$ | $-\sqrt{E/2}$ |
| 01 | $5\pi/4$ | $-\sqrt{E/2}$ | $+\sqrt{E/2}$ |
| 11 | $7\pi/4$ | $+\sqrt{E/2}$ | $+\sqrt{E/2}$ |

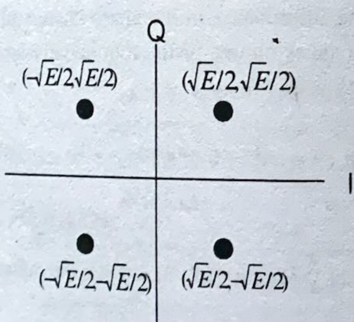


*Fig.5.16. Constellation diagram QPSK*



## QPSK Modulator

The block diagram of QPSK generation is shown in Fig 5.17.

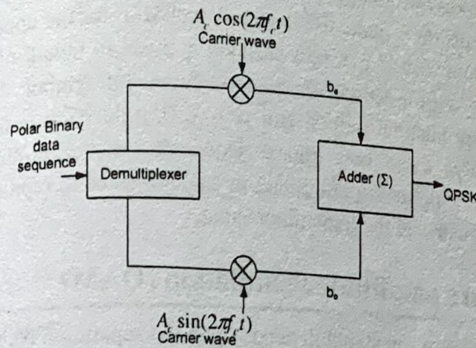


***Fig 5.17:** QPSK Modulator*

The input binary sequence to the QPSK modulator is represented in polar form. This binary sequence is divided into odd and even numbered bits using a demultiplexer. Now, even numbered bits (representing the in-phase component of input ($b_e$)) is multiplied by $\cos(2\pi f_c t)$. Odd numbered bits (representing the out of phase component of input($b_o$)) is multiplied by $\sin(2\pi f_c t)$. Then these two streams are added to get QPSK signal. Note that the symbol duration $T$ of a QPSK wave is twice as long as the bit duration $T_b$, of the input binary wave. That is, for a given bit rate $1/T_b$, a QPSK wave requires half of the transmission bandwidth. Alternately for given transmission bandwidth, a QPSK wave carriers twice as many bits of information as the corresponding binary PSK wave.

### QPSK Demodulator

The QPSK signal can be detected by using a pair of correlators (Multiplier-followed by integrator) in parallel. The circuit diagram has shown in Fig. 5.18.

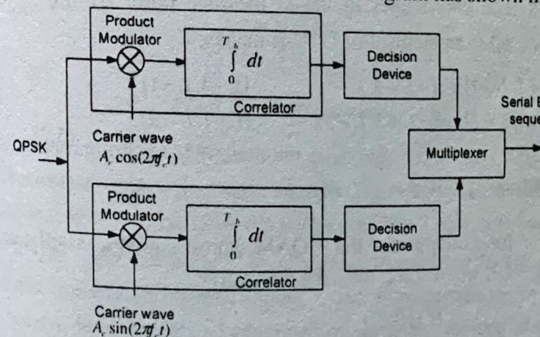


***Fig.5.18.** QPSK Demodulation*

The correlator in the upper path computes the cosine of the carrier wave where as the lower path computes the sine of the carrier wave.

The correlator outputs $x_1$ and $x_2$ are each compared with a threshold of zero volt. If $x_1 > 0$, a decision is made in favour of symbol '1' for the upper 'or' in phase channel output. But if $x_1 < 0$, a decision is made in favour of symbol '0'. Similarly, if $x_2 > 0$, a decision is made in favour of symbol '1' for the lower or quadrature channel output. But if $x_2 < 0$, a decision is favour of symbol '0'. Finally, these two binary sequences at the in-phase and quadrature channel outputs are combined in a multiplexer to reproduce the original binary sequence at the transmitter input.

## 5.7 Quadrature amplitude modulation (QAM)

The correct detection of the signal depends upon the separation between the signal points in the signal space. In case of M-PSK systems all points lie on the circumference of the circle in the signal space diagram. This is because M-PSK signal has constant amplitude throughout. If amplitude of the signal is also varied along with phase, then the points will lie inside the circle also on the signal space diagram. This further increases the noise immunity of the system. Such system involves phase as well as amplitude shift keying and is called quadrature amplitude modulation (QAM).

The general form of M-ary QAM is defined by

$$s_i(t) = \begin{cases} \sqrt{\dfrac{2E_0}{T}}\, a_i \cos(2\pi f_c t) + \sqrt{\dfrac{2E_0}{T}}\, b_i \sin(2\pi f_c t) & 0 \le t \le T \\ 0 & elsewhere \end{cases} \qquad (5.27)$$

Where, $E_0$ denotes the energy of the lowest amplitude signal, and $a_i$ and $b_i$ are pair of independent integers chosen according to the position of the message points. The coordinates of the $i^{th}$ message point are $a_i\sqrt{E_o}$ and $b_i\sqrt{E_o}$, where $(a_i, b_i)$ is an element of L-by-L matrix:

$$(a_i, b_i) = \begin{bmatrix} (-L+1, L-1) & (-L+3, L-1) & \cdots & (L-1, L-1) \\ (-L+1, L-3) & (-L+3, L-3) & \cdots & (L-1, L-3) \\ \vdots & \vdots & \vdots & \vdots \\ (-L+1, -L+1) & (-L+3, -L+1) & \cdots & (L-1, -L+1) \end{bmatrix}_{L\times L} \qquad (5.28)$$

Where, $L = \sqrt{M}$ for example, for the 4-QAM whose signal constellation is depicted in Fig.5.19, where L=2, we have the matrix.

$$(a_i, b_i) = \begin{bmatrix} (-1,1) & (1,1) \\ (-1,-1) & (1,-1) \end{bmatrix}_{2\times 2} \qquad (5.29)$$



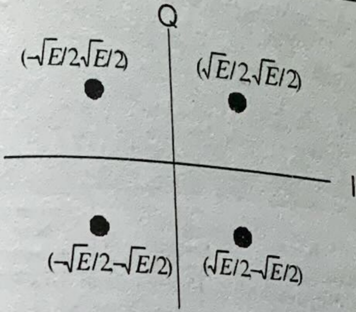


***Fig.5.19.*** *Constellation diagram of 4-QAM*

**QAM modulator**

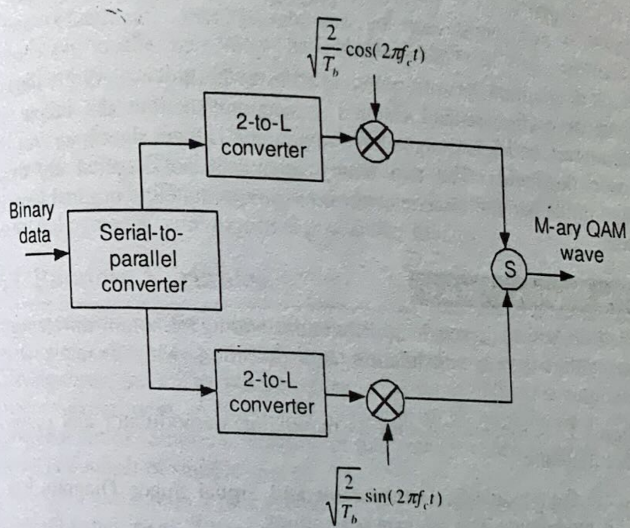


***Fig.5.20.*** *QAM modulator*

The block diagram of QAM transmitter is shown in Fig.5.20. The serial-to-parallel converter binary sequence at a bit rate $R_b=1/T_b$ to two parallel binary sequences whose bit rates are $R_b/2$ each. The 2-to-L level converters, where $L = \sqrt{M}$ generate polar L-level signals in response to the respective in-phase and quadrature channel inputs. Quadrature-carrier multiplexing of the two polar L-level signals so generated produces the desired M-ary QAM signals.

**QAM demodulator**

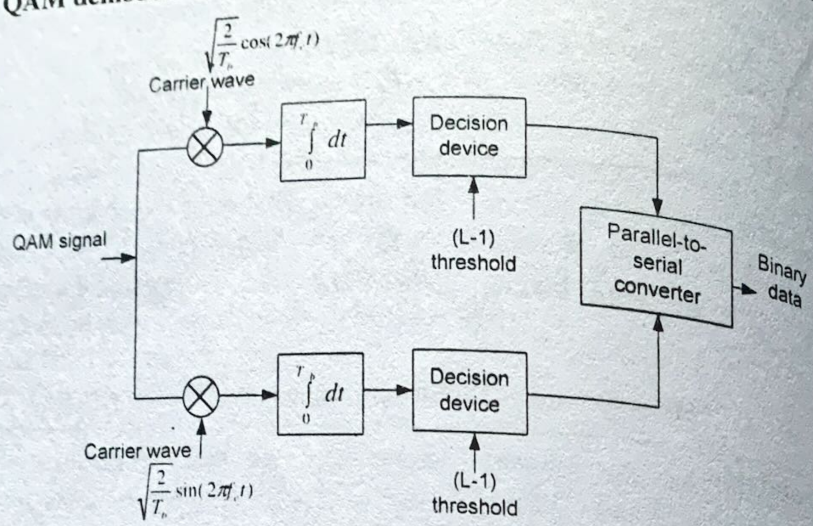


*Fig.5.21. QAM Demodulator*

The block diagram of the corresponding receiver is shown Fig.5.21. shows. Decoding of each baseband channel is accomplished at the output of decision circuit, which is designed to compare the L-level signals against L-1 decision thresholds. The two binary sequences so detected are then combined in the parallel-to-series converter to reproduce the original binary sequence.

## Previous Exam Questions

1. What do you understand by differential coding? Explain differential phase shift keying modulation and detection with example and diagrams.
2. Discuss FSK modulation with its modulator, demodulator and signal space diagrams.
3. Explain the Modulator, Demodulator and Signal Space Diagram for QPSK modulation with relevant derivations.

✡✡✡

Chapter 6

# RANDOM SIG NOISE

## Introduction

In previous chapters we dealt entirely with dete signals were function of time indicating that t known or determined for all time. This chapte whose exact behavior cannot be described i occur in communication both as unwanted nois bearing waveforms. If a message to be recei known beforehand), then it contains no u information to the receiver. The noise sig otherwise they can simply be subtracted at the r Due to lack of detailed knowledge of the time the mathematical model based in terms of properties is used. Thus, this chapter deals with of random signals. The major topics include pr statistical averages, and important probability n

## 6.1 Random Variables

An experiment whose outcome cannot be e random experiment (e.g. tossing of a coin, dra playing cards, etc.). The sample space is the s random experiment. A particular outcome c sample point or sample. Collection of outcome event is a subset of sample space.

A random variable is a real valued function d random experiment. Thus, the random variab maps the sample points into real number. It variable, or random function or stochastic func random variables are denoted by upper-case the value assumed by them is denoted by lov such as $x_1$, $x_2$, $y_1$, $y_2$, etc. The random variable

1. **Discrete Random variables**

   A random variable whose outcome take is known as a discrete random variable. space S contains a countable number of discrete random variable. For example, t a fair coin, there are eight (i.e., $2^2$=