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PULSE MODULATION SYSTEMS.

Unlike analog modulation systems, where sinusoidal carriers were used, in pulse modulation systems, the carrier is no longer a continuos sinusoid but consists of a pulse train.

Now, by ~~se~~ varying ~~x~~ some of the parameters of these pulse train with respect to modulating signal (i.e. message signal), we derive any required modulation. so, we basically get two type of pulse modulation system as,

1. Pulse analog modulation
2. Pulse digital modulation.

1. Pulse analog modulation:

In such modulation scheme, only time is expressed in the digital form, i.e. we get discrete time signals and any one of the pulse parameters is varied in a continuous manner with respect to the message signal.

The various pulse parameters

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can be the pulse amplitude, position or duration (i.e. time).

So, we can again have two differe[nt] pulse analog modulations, namely,

a. Pulse amplitude modulation.
b. Pulse time modulation.

a. Pulse amplitude modulation (PAM).

In pulse amplitude modulation, the amplitudes of regularly spaced rectangula[r] pulses vary according to the instantaneous value of the modulating signal.

Thus, PAM can be regarded as sampling methods (Recall natural or flat-top) So, basically, the output of a flat top sampling process is a PAM signal.

So, when we ~~ta~~ mention PAM it is generally a flat-top sampling process.

Generation of PAM signal (Flat-top)

A sample and hold circuit is used to produce a flat-top PAM signal. A sample and hold circuit consists of two switches and a capacitor.

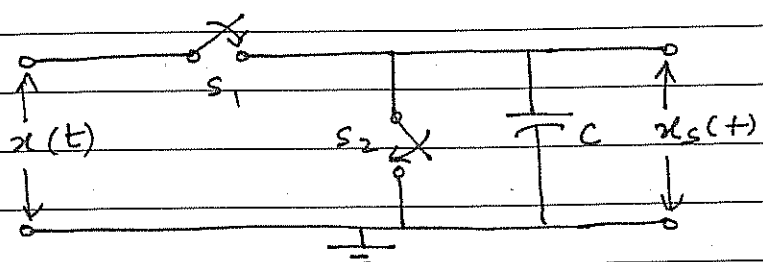

fig. Sample and hold (S/H) circuit

In the figure above, S_1 is the sampling switch and S_2 is the discharge switch. C represents a capacitor.

Now, S_1 is closed for a short duration of time. During this period, the capacitor 'C' is quickly charged upto the voltage equal to the instantaneous sample value of the signal $x(t)$.

As the switch is opened after the short duration of time, the capacitor still holds the charge and thus a voltage equal to the sampled-instant is recieved as $x_s(t)$.

At some time 'τ', the discharge switch 'S_2' is closed such that the capacitor is discharged to zero volts. Now as the switch 'S_2' is opened, the capacitor holds no voltage. And we ~~repeat~~ again close the switch 'S_1' at some time 'T_s'.

In this manner we get the output of sample and hold circuit as a sequence of flat top samples.

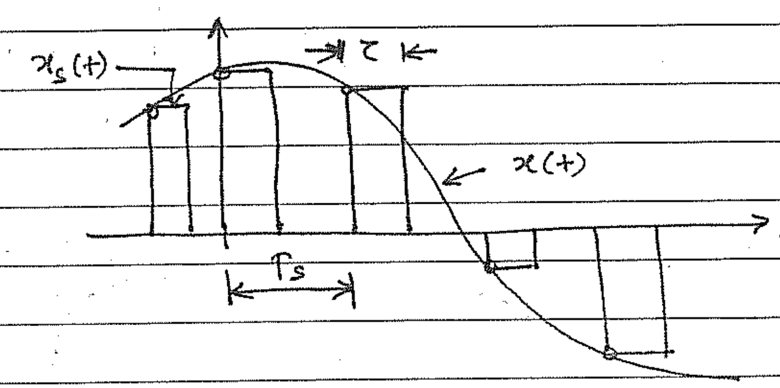

fig. flat-top PAM signal.

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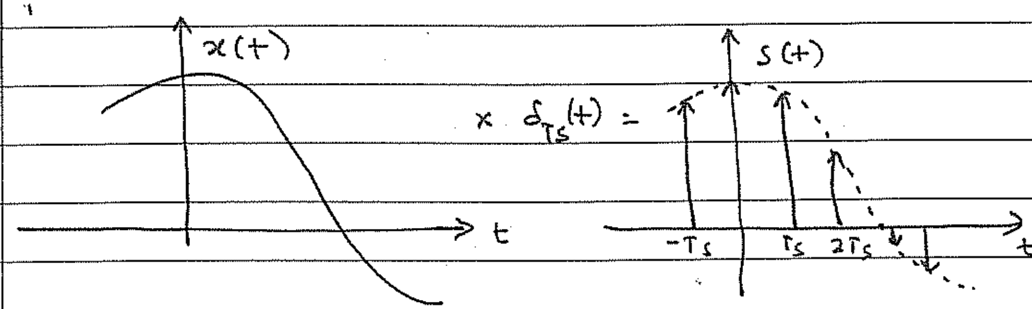
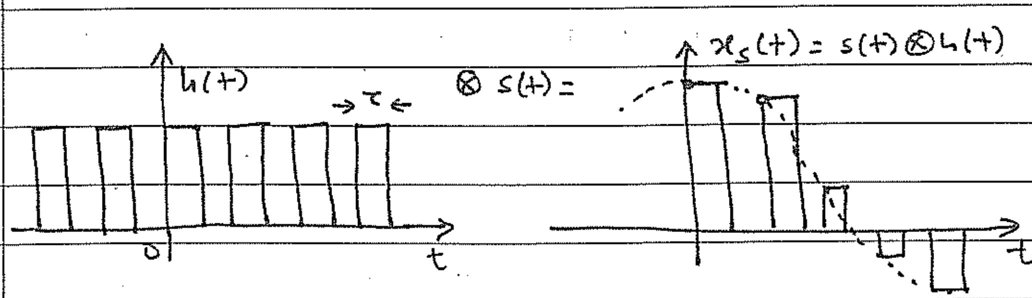
Mathematically, the flat-top PAM signal can be described as the convolution of instantaneous sample and a pulse train.

i.e.

For a message signal $x(t)$, if $s(t)$ is its instantaneous sample then,

$$x_s(t) = s(t) \otimes h(t). \quad \left[s(t) = x(t) \times \delta_{T_s}(t) \right]$$





Now we have,

$$\delta_{T_s}(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

and

$$s(t) = x(t) \cdot \delta_{T_s}(t)$$

$$= \sum_{n=-\infty}^{\infty} x(nT_s)\,\delta(t - nT_s)$$

And,

$$x_s(t) = s(t) \otimes h(t)$$

$$= \int_{-\infty}^{\infty} s(\tau)\, h(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} x(nT_s)\,\delta(\tau - nT_s)\, h(t-\tau)\, d\tau$$

$$= \sum_{n=-\infty}^{\infty} x(nT_s) \int_{-\infty}^{\infty} \delta(\tau - nT_s)\, h(t-\tau)\, d\tau$$

$$\therefore x_s(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\, h(t - nT_s) \quad \left[\because \int f(t)\,\delta(t-t_0)\,dt = f(t_0) \right]$$

Also, $X_s(f) = S(f) \cdot H(f)$

But, $S(f) = f_s \sum_{n=-\infty}^{\infty} X(f - nf_s)$

$$\boxed{\therefore X_s(f) = f_s \sum_{n=-\infty}^{\infty} X(f - nf_s) \cdot H(f)}$$

spectrum of flat-top PAM signal.

And for pulse train $h(t)$, its Fourier transform $H(f)$ is given as,

$$H(f) = \tau \, \text{sinc}(f.\tau) \, e^{-j\pi f \tau}$$

Such that,

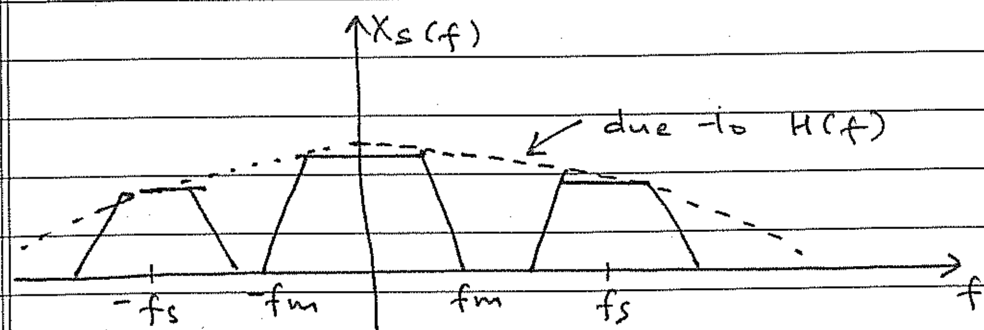


Fig. Spectrum of PAM signal.

Similarly ~~for~~ if natural sampling were used,

$$X_s(f) = \tau/T_s \cdot \sum_{n=-\infty}^{\infty} \text{sinc}(n f_s \tau) \cdot X(f_s - n f_s)$$

& ~~$X_s(f)$~~ $x_s(t) = \tau/T_s \sum_{n=-\infty}^{\infty} x(t) \, \text{sinc}(n f_s \tau) \, e^{j2\pi n f_s t}$

And for ideal PAM signal,

$$x_s(t) = \sum_{n=-\infty}^{\infty} x(nT_s) . \delta(t - nT_s)$$

& $X_s(f) = f_s \sum_{n=-\infty}^{\infty} X(f - nf_s)$.

⊞ Transmission bandwidth in PAM.
i.e. Bandwidth requirements.

In PAM, the pulse duration 'τ' is considered to be very small in comparison to time period, T_s, between any two samples.

i.e. pulse duration << sampling time
or $\tau << T_s$.

Now, if the highest frequency present in a signal $x(t)$ is f_m then we have sampling frequency,

$$f_s \geq 2 f_m$$

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Since, $f_s = 1/T_s$,

So, $\frac{1}{T_s} \geq 2f_m$.

or $T_s \leq \frac{1}{2f_m}$

As, $\tau << T_s$

Therefore, $\tau << T_s \leq \frac{1}{2f_m}$

i.e. $\tau << \frac{1}{2f_m}$

or $f_m << \frac{1}{2\tau}$

Now, suppose that the 'ON' and 'OFF' time of the PAM pulse is same i.e.

$T_{on} = T_{off} = \tau$.

In such case we attain the highest frequency for the PAM signal. i.e. The maximum frequency of the PAM signal,

$f_{max} = \frac{1}{\tau+\tau} = \frac{1}{2\tau}$

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Now, the bandwidth required for the transmission of such PAM signal is atleast equal to the maximum frequency present in PAM signal.

so, bandwidth

$BW \geq f_{max}$

$\boxed{\text{or } BW \geq \frac{1}{2\tau}}$

~~but~~ Also, $f_m << \frac{1}{2\tau}$

$\therefore BW \geq \frac{1}{2\tau} >> f_m$

Therefore the transmission bandwidth required ~~for~~ PAM signal is very very greater than the maximum frequency present at the message signal.

$\boxed{\text{So, } B.W >> f_m}$

$\boxed{Tx.\ B.W \geq \frac{1}{2\tau}}$

(#) For a pulse-amplitude modulated (PAM) transmission of voice signal having max. frequency, 'fm' equal to 3 KHz, calculate the transmission bandwidth. It is given that the sampling frequency, $f_s = 8$ KHz and the pulse duration, $\tau = 0.1\ T_s$.

Given:

$f_m = 3$ KHz.

$f_s = 8$ KHz

$\tau = 0.1\ T_s$.

Now,

$$T_s = \frac{1}{f_s} = \frac{1}{8\text{ KHz}} = 0.125 \times 10^{-3} \text{ sec.}$$

or $T_s = 125\ \mu$ seconds.

So,

$$\tau = 0.1 \times T_s$$
$$= 0.1 \times 125\ \mu\text{ sec}$$
$$= 12.5\ \mu\text{ sec}$$

And,

$$BW \geq 1/2\tau \text{ or } BW \geq \frac{1}{2 \times 12.5\ \mu sec}$$

$$\therefore BW \geq \frac{10^6}{25} \text{ or } B.W \geq 40 \text{ KHz.}$$

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Reconstruction of original message signal from PAM signal.

The retrieval of message signal $x(t)$ from PAM signal $x_s(t)$ is done using a holding circuit and a low pass filter.

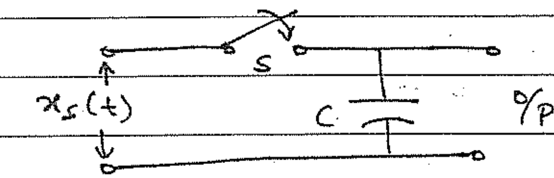

fig. Holding circuit

As the switch 'S' is closed $x_s(t)$ is allowed in through the holding circuit for a time duration 'τ' of the individual sample pulses in $x_s(t)$, ~~After this~~ and then the switch is opened. When switch is closed, capacitor 'C' gets charged to the voltage value equal to the pulse amplitude value.

Now, this capacitor holds this voltage till ~~next~~ the switch is closed again.

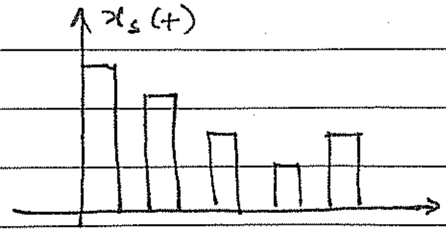

Fig. PAM signal

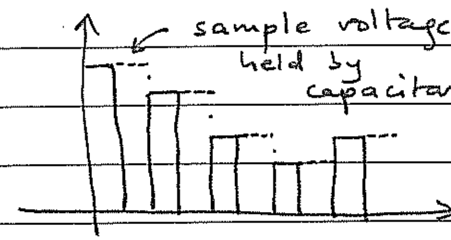

Fig. Samples held.

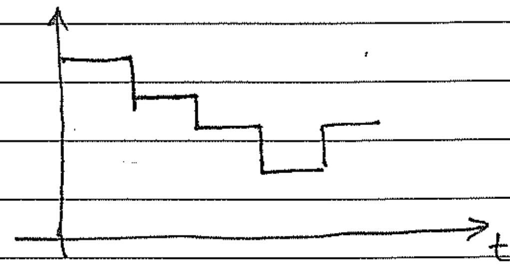

Fig Holding circuit output.

The holding circuit output is smoothened by passing the signal through a low pass filter such that the original message signal is recovered.

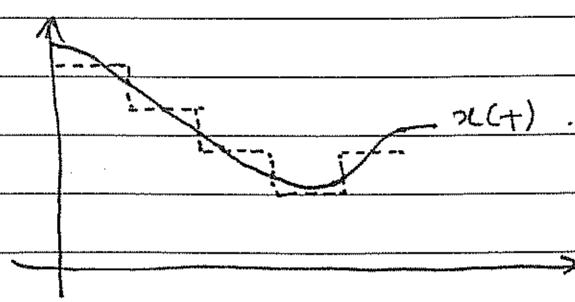

Fig. LPF output.

It should be noted that due to aperture effect there will always be distortion and attenuation so it is better to apply an equalizer to the reconstruction filter.

As the duration of 'τ' increases, aperture effect ~~has~~ results in greater distortion and thus equalizer must be used to compensate this effect.

Now, we have for equalizer

$$|H_{eq}(f)| = \frac{K}{|H(f)|}$$

$$\text{or } |H_{eq}(f)| = \frac{K}{\tau \operatorname{sinc}(f\tau)}$$

$$\because H(f) = \tau \operatorname{sinc}(f\tau)\, e^{-j\pi f\tau}$$

So, the equalizer with transfer f^n

$$\boxed{|H_{eq}(f)| = \frac{K}{\tau \cdot \sin(f\tau)}}$$

is ~~added~~ multiplied to the PAM signal before reconstruction.

b. Pulse time modulation (PTM).

In such modulation, the timing of the pulses of the ~~ca~~ carrier train is varied.

There are two types of PTM,

i) Pulse width Modulation (PWM).

ii) Pulse position Modulation (PPM).

i) Pulse width modulation (PWM)

In PWM, the width of the pulses of the carrier pulse train is varied in accordance with the message signal $x(t)$.

In such modulation technique, the amplitude of the modulated pulses remain constant whereas the width ~~of th~~ varies in proportion with the amplitude of the message signal $x(t)$.

Thus it can be said that the information regarding the message signal is contained in the width of the modulated signal. So the variation ~~of~~ in the width describes the information of $x(t)$.

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Since information is contained in the width variation it is unaffected by the additive noise which affects the amplitude of the modulated pulses. PWM can ~~thus~~ be seen as FM ~~and has~~ and is more immune to noise than PAM signals.

Generation of PWM signal.

A PWM signal is generated using a sawtooth signal as a sampling signal, and a modulating signal $x(t)$.

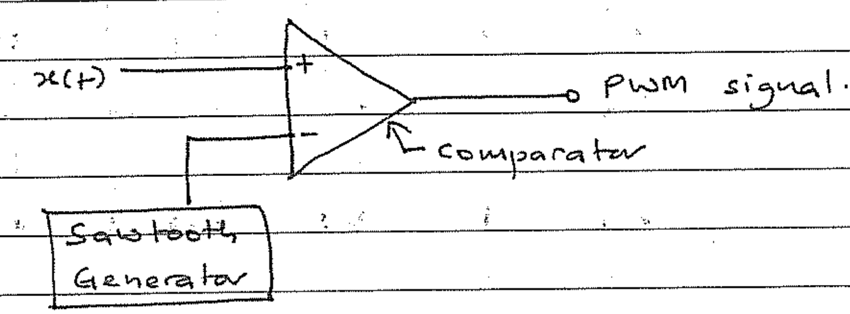

Fig. PWM generator.

In the figure above, a sawtooth generator produces a sawtooth signal of frequency fs which is the sampling frequenc

This sawtooth or ramp signal is applied to the inverting terminal of a comparator. A modulating signal $x(t)$ is applied to the non-inverting terminal of the same comparator.

Now the comparator output will remain high as long as the instantaneous amplitude of $x(t)$ is higher than the ramp signal. This gives rise to PWM signal at the comparator output.

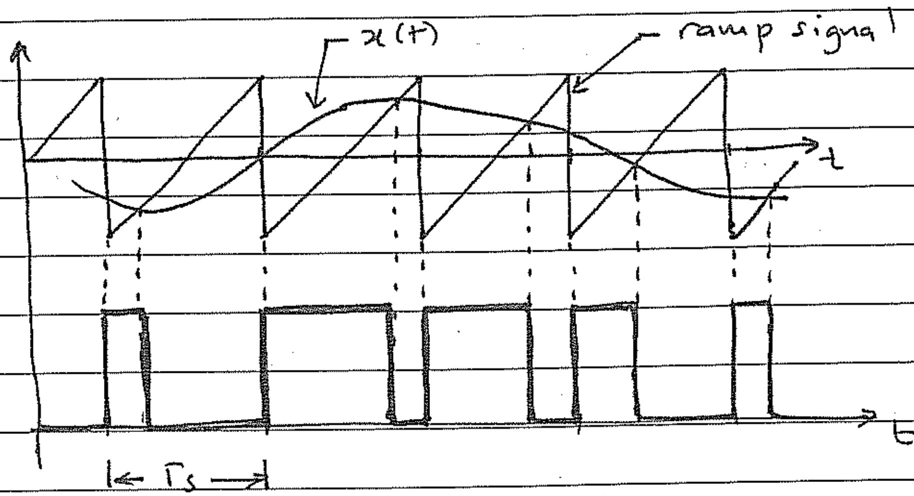

Fig PWM waveform

In the figure above, we can see that the rising or leading edges of PWM waveform coincide with the falling edges of ramp signal. So, the leading edges are always generated at fixed interval of time 'Ts'. But the trailing edges occur at the instantaneous amplitude of $x(t)$. Thus this PWM signal is said to be trail edge modulated PWM.

ii) Pulse position modulation (PPM).

As the name states, the information of the message signal is contained in the variations in the position of pulses of the modulated signal.

So, in PPM, the position of the pulses of the carrier pulse train is varied with respect to the modulating signal $x(t)$. Here, the pulse width as well as the pulse amplitude remains constant.

Generation of PPM.

Just like PWM, a PPM signal can be generated using a sawtooth signal as sampling signal where the PWM signal generated is passed through a monostable multivibrator.

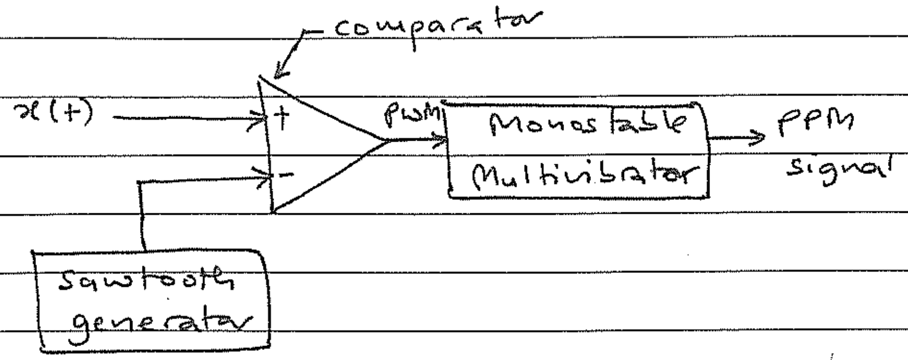

fig. Generation of PPM signal.

As, for the generation of PPM signals, the PWM output from the comparator is fed into a ~~not~~ monostable multivibrator.

The monostable multivibrator is negative edge triggered such that corresponding to each trailing edge of PWM signal, the output goes high.

This output remains high for a fixed time with respect to its RC components. So, as trailing edge of PWM signals keep shifting with respect to instantaneous amplitude of $x(t)$, PPM pulses also keep shifting but with constant pulse width and amplitude.

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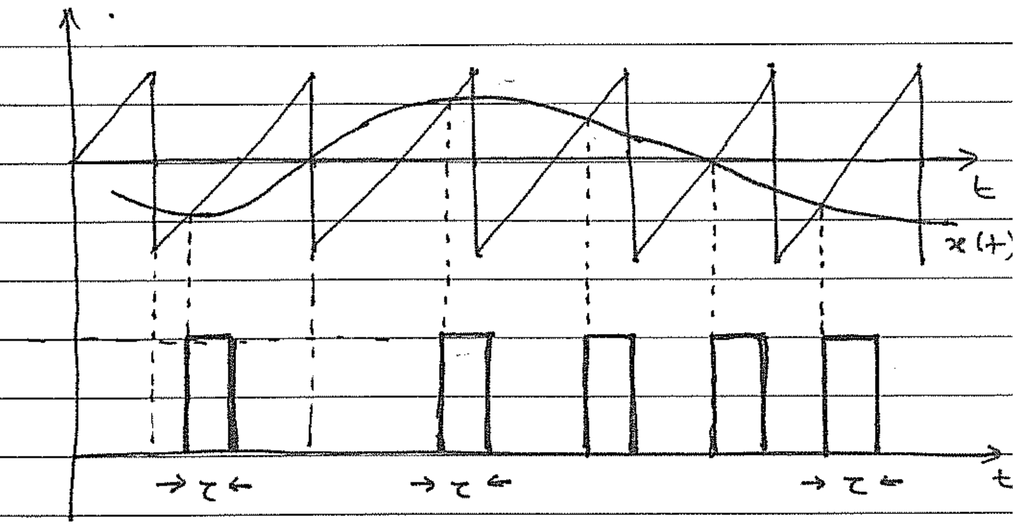

fig. PPM waveform

$\boxed{\tau = 1.1\,RC}$

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2. Pulse digital modulation.

In pulse digital modulation, the time occur in discrete form and the pulse parameter (usually amplitude) occur in digital coded form. Thus pulse digital modulation is a technique which converts the analog signal to its corresponding digital form. PCM is one of its type.

i. Pulse code Modulation (PCM).

Pulse code modulation (PCM) is a technique by which analog signals are converted into digitally encoded signals. In PCM, the message signal is sampled and the amplitude of each sample is approximated to the nearest one of a finite set of discrete levels. Thus we can realize both time and amplitude in discrete form.

A PCM ~~th~~ technique has three basic and essential operations. They are,

a. Sampling
b. Quantizing
c. ~~end~~ encoding

So, basically pulse code modulation requires that a message signal is first sampled i.e. a flat-top PAM signal is generated.

This PAM signal is then quantized to a finite set of discrete levels. And finally these discrete levels ~~an~~ are encoded into bit stream.

Thus the sampling, ~~quat~~ quantizing and encoding process act together ~~as~~ ~~an~~ to perform as an analog to digital converter.

a. Sampling :

→ ideal

→ natural

→ flat-top

$$x_s(t) = x(t) \cdot \delta_{T_s}(t) = x[nT_s]$$

or

$$x_s(t) = x(t) \cdot h(t) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} \text{sinc}[nf_s\tau] e^{j2\pi n f_s \tau} \cdot x(t)$$

or

$$x_s(t) = x[nT_s] \circledast h(t) = \sum_{n=-\infty}^{\infty} x[nT_s] \cdot h[t - nT_s]$$

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b. Quantization :

Let us consider the sampled signal as $x[nT_s]$.

The process of comparing the discrete time input $x[nT_s]$ with a fixed level of voltage is known as quantization. So, the sampled signal $x[nT_s]$ is assigned one of the digital levels from the fixed digital levels.

So, quantization is the process of representing the analog sample amplitude by a finite set of levels, thus providing discrete amplitude values.

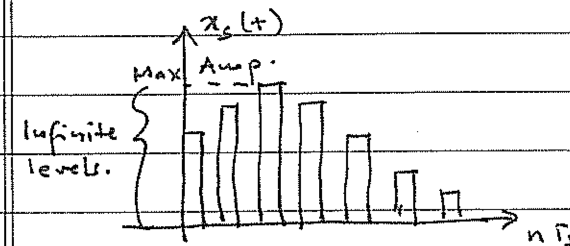


Fig. Sampled signal

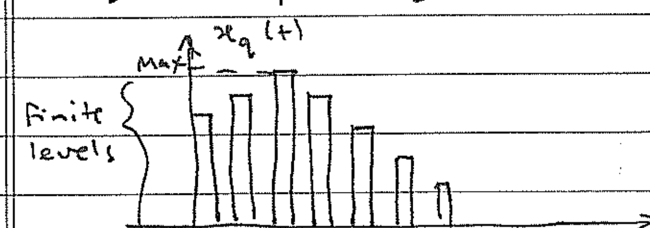


Fig. Quantized signal

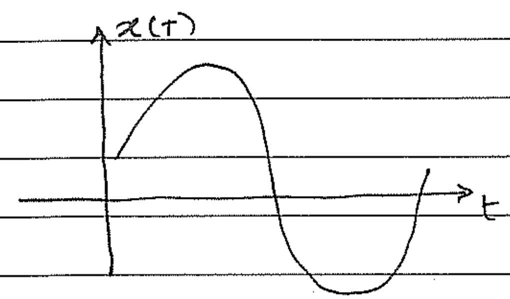
Quantization process may be classified as,

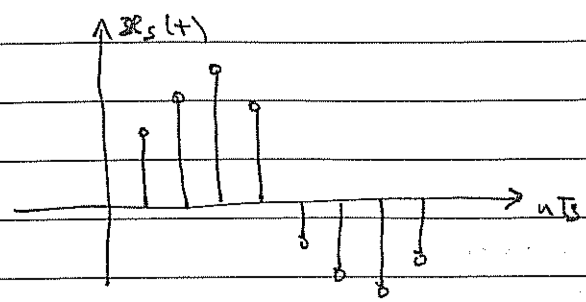
i) Uniform Quantization
ii) Non-uniform quantization.

i) Uniform quantization.

It is the process of converting the analog sample amplitude to a finite set of discrete amplitude in the range of $-X_{max}$ to X_{max}.

So, the total range of the amplitude is $2X_{max}$. This '$2X_{max}$' range is divided into 'N' number of levels.





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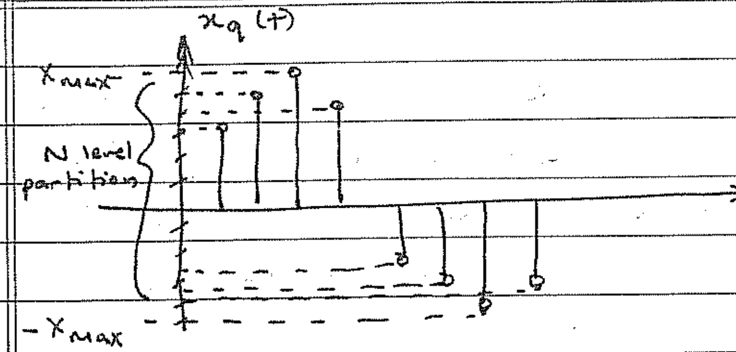
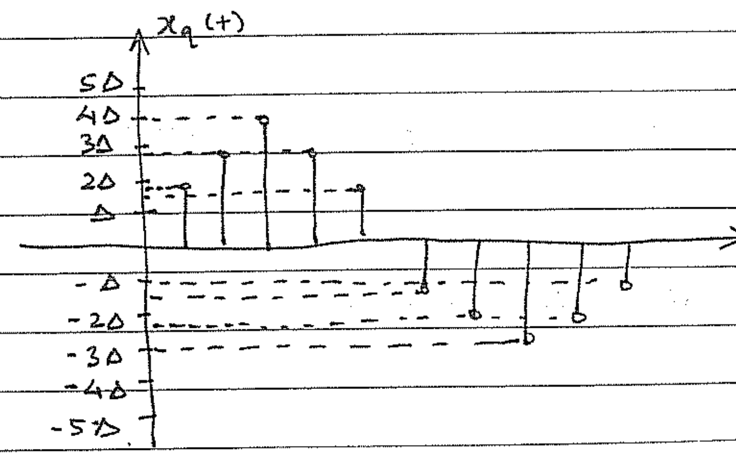


Now, each partition will have an magnitude represented by $\Delta = \frac{2X_{max}}{N}$

Δ is also called 'quantum' or stepsize and is generally measured in volts.

So, for uniform quantizer this step size 'Δ' remains same through out the input range.



Now, these discrete voltage levels can be represented in sequence of bits and such process is known as encoding.

So, with 'N' discrete levels, we need

$b = \log_2 N$ bits to represent 'N' levels.

Also,

$$\Delta = \frac{2X_{max}}{N}$$

or $\quad N = \frac{2X_{max}}{\Delta}$

$$\therefore \quad b = \log_2\left(\frac{2X_{max}}{\Delta}\right)$$

And, $\quad N = 2^b$.

There are two types of uniform quantizer, namely,

a. Mid tread type
b. Mid Rise type.

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a. Mid tread :

Here, the origin lies on the middle of tread of staircase.

b. Mid rise :

Here the origin lies on the middle of the rising part of the staircase.

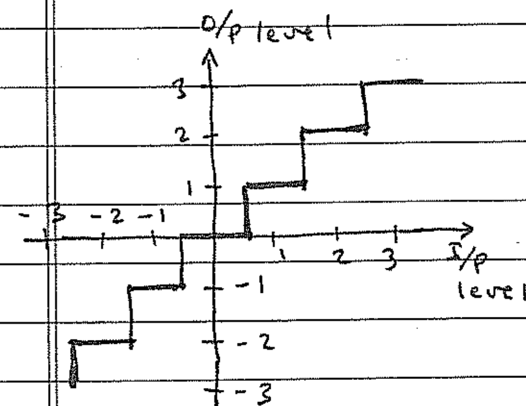


Fig. Mid tread type

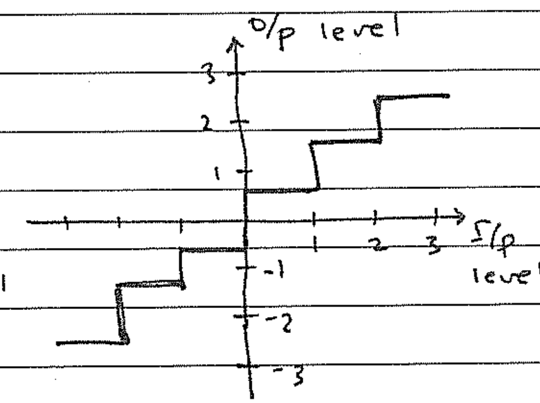


Fig. Mid rise type.

Fig. Input output characteristics.

let us take the maximum input voltage be +4 V and minimum be -4 V.

i.e.

range : -4 V to 4 V

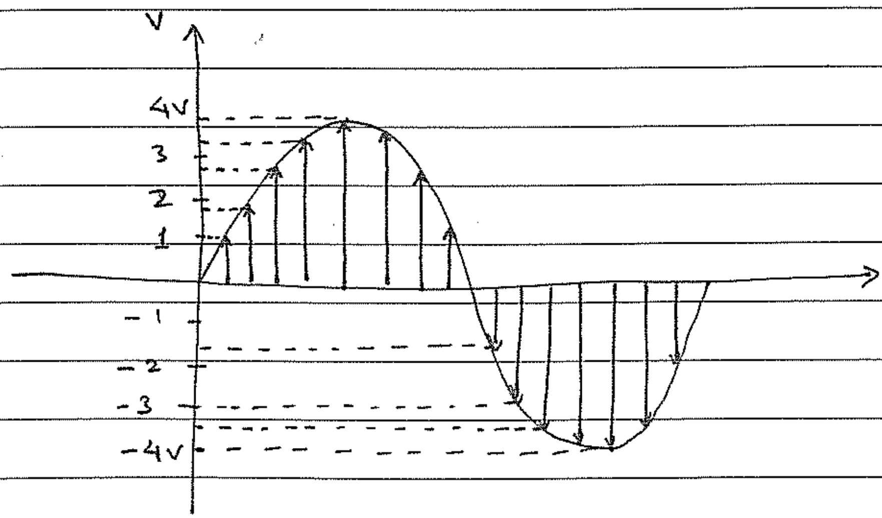
let us have 8 levels of partition such that,

$$N = 8.$$

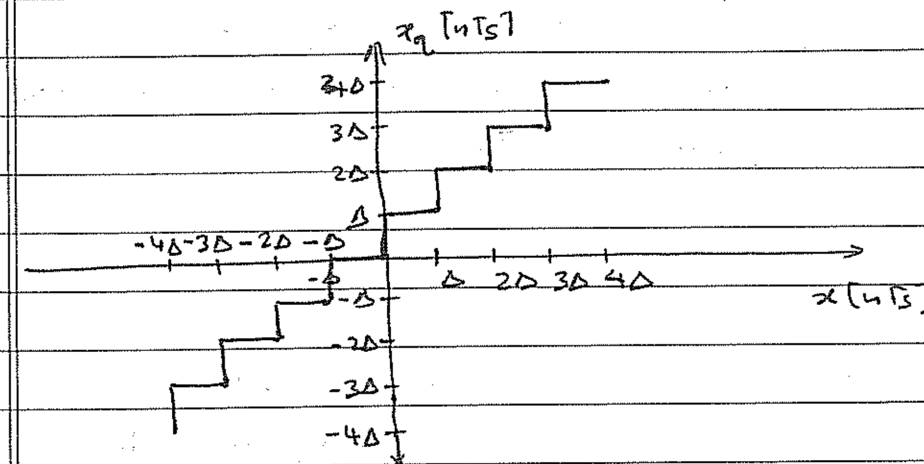
Now,

$$\Delta = \frac{2 \cdot X_{max}}{N} = \frac{2 \times 4}{8} = 1 \text{ Volts.}$$

i.e. we divide the voltage range into 8 equal 1 volts.



let us now round off the range 0-1 volts to 1 volts of finite levels, 1-2 volts to 2 volts, 2-3 volts to 3 volts and 3-4 volts to 4 volts. This can be represented as,



In the figure above, 'Δ' represents step size equal to 1 volts.

$x[nT_s]$ represents the sampled voltage output at different instances such that the range of voltages were almost infinite. But $x_q[nT_s]$ which represents quantized output level has finite level of voltage spaced 'Δ' volts apart.

So, for an input $x[nT_s]$ we get a quantized set of voltage in $x_q[nT_s]$ as output through a quantizer.

Now, as the digital output is the approximation ~~of~~ the input value to any nearest finite voltage level it is evident that ~~that~~ there will be some ~~var~~ difference between $x[nT_s]$ and $x_q[nT_s]$.

This difference is known as the quantization error.

i.e.

$$\varepsilon = x_q[nT_s] - x[nT_s]$$

So we see that when $x[nT_s] = 0$,

$x_q[nT_s] = \Delta$

and when $x[nT_s] = \Delta$

$x_q[nT_s] = \Delta$

so, for $x[nTs]$ in the range 0 to Δ, we have error,

$$\varepsilon = \Delta - 0 \text{ to } \Delta - \Delta$$

$$= \Delta \text{ to } 0.$$

$\therefore$ Highest error $= \Delta$.

$\sim$ Maximum error $= |\Delta|$

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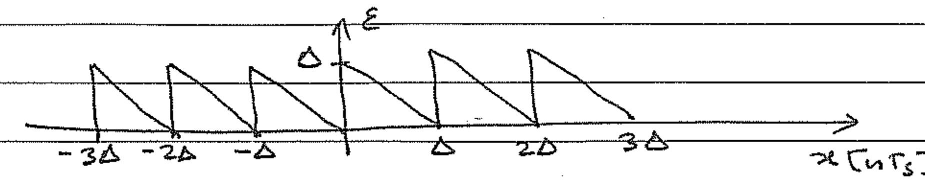
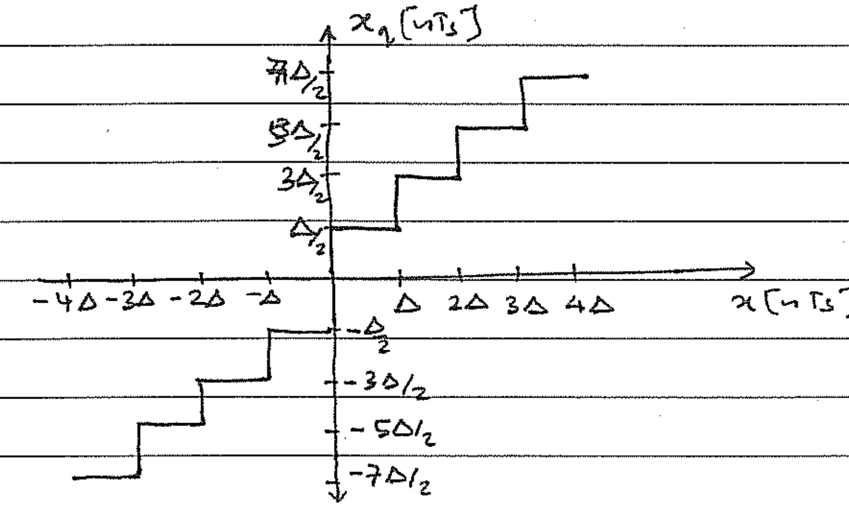


$\Delta =$ max quantization error

Now, for the same range ~~for~~ of $x[nT_s]$ in $-4V$ to $4V$, let the fixed digital levels available are,

$\pm \Delta/2$, $\pm \frac{3\Delta}{2}$, $\pm 5\Delta/2$, $\pm 7\Delta/2$ i.e.



So,

at $x[nT_s] : 4\Delta$, $x_q[nT_s] : 7\Delta/2$

$x[nT_s] : 3\Delta$, $x_q[nT_s] : 5\Delta/2$

$\therefore \varepsilon = 7\Delta/2 - 4\Delta$, $\varepsilon = 7\Delta/2 - 3\Delta = \Delta/2$

$= -\Delta/2$

i.e. $\varepsilon = \pm \Delta/2$

or $\varepsilon_{max} = |\Delta/2|$

or $-\frac{\Delta}{2} \le \varepsilon \le \frac{\Delta}{2}$

Now for the interval $(-\frac{\Delta}{2}, \frac{\Delta}{2})$, the quantization noise (ε) may be assumed as an uniformly distributed random variable such that the probability density function for 'ε' can be defined as,

$$f(\varepsilon) = \begin{cases} 0 & \text{for } \varepsilon \le -\Delta/2 \\ \frac{1}{\Delta} & \text{for } -\frac{\Delta}{2} < \varepsilon \le \frac{\Delta}{2} \\ 0 & \text{for } \varepsilon > \frac{\Delta}{2} \end{cases}$$

with zero mean square value.

And the ~~noise of~~ ~~pri~~ quantization noise power is given by,

$$P_q = \frac{V^2_{noise}}{R} = V^2_{noise} \quad [\text{Taking } R = 1\Omega]$$

V^2_{noise} : mean square voltage

or $P_q = \int_{-\infty}^{\infty} \varepsilon^2 \cdot f(\varepsilon) \cdot d\varepsilon$

or $P_q = \frac{1}{\Delta} \int_{-\infty}^{\infty} \cancel{\Delta}\, \varepsilon^2 \cdot d\varepsilon$

$$= \frac{1}{\Delta} \int_{-\Delta/2}^{+\Delta/2} \varepsilon^2 \cdot d\varepsilon$$

$$= \frac{1}{\Delta} \left| \frac{\varepsilon^3}{3} \right|_{-\Delta/2}^{\Delta/2}$$

~~Now, magnitude of $\varepsilon = \Delta/2$~~,

$$\therefore P_q = \frac{1}{\Delta} \left[\frac{(\Delta/2)^3}{3} - \frac{(-\Delta/2)^3}{3} \right]$$

$$= \frac{1}{3 \cdot \Delta} \left[\frac{\Delta^3}{8} + \frac{\Delta^3}{8} \right]$$

$$= \frac{2\Delta^3}{8} \times \frac{1}{3\Delta}$$

$$P_q = \frac{\Delta^2}{12}$$

$\therefore$ The quantization noise power ~~for~~ at 1Ω resistor,

$$P_q = \frac{\Delta^2}{12}$$

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Signal to noise (quantization) ratio for uniform quantization.

We have, $\frac{S}{N} = \frac{\text{Signal power}}{\text{Quantized noise power}}$

Taking $R = 1\Omega$, we have, $\left[\text{Power} = \frac{V^2}{R}\right]$

$$\frac{S}{N} = \frac{\text{Normalized signal power}}{P_q}$$

taking normalized signal power as 'P',

$$S/N = P/P_q$$

$$= \frac{P}{\Delta^2/12} = \frac{P}{\Delta^2} \times 12.$$

Now, we have,

$$\Delta = \frac{2\,X_{max}}{N}$$

and $N = 2^b$

so, $\Delta = \frac{2\,X_{max}}{2^b}$

$$\therefore \frac{S}{N} = \frac{P}{\left(\frac{2 \cdot X_{max}}{2^b}\right)^2} \times 12$$

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or $\frac{S}{N} = \frac{P}{\frac{4\,X^2_{max}}{2^{2b}}} \times 12$

$$\therefore \boxed{\frac{S}{N} = \frac{3\,P \cdot 2^{2b}}{X^2_{max}}}$$

The above expression shows that the signal to noise power ratio increases exponentially with increasing bits per sample.

Now normalizing $x(t)$ i.e. $X_{max} = 1$, we have,

$$\frac{S}{N} = 3\,P \cdot 2^{2 \cdot b}$$

In the above relation, 'P' is generally regarded as the received power at the receiver end. And normalizing 'P' we get,

$$P \leq 1$$

Such that

$$\frac{S}{N} \leq 3 \times 2^{2 \cdot b}$$

or $\left(\frac{S}{N}\right) dB = 10 \log_{10} \left(\frac{S}{N}\right) dB \leq 10 \log_{10} \left[3 \times 2^{2b}\right] dB$

$$\therefore \left(\frac{S}{N}\right) dB \leq (4.8 + 6b)\, dB.$$

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Transmission bandwidth in a PCM system.

Let the quantizer use 'b' number of binary digits to represent each level such that,

$N = 2^b$ where, N = number of levels.

if $b = 2$, $N = 4$

$b = 4$, $N = 16$

We thus have each samples is converted to 'b' number of binary bits.

So,

b = number of bits per sample

Now, from Nyquist sampling rate,

$f_s \geq 2f_m$ is number of samples per second.

Therefore,

Number of bits per second

= no. of bits per sample × no. of samples per second

Number of bits per second is also known as signalling rate

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Thus,

signalling rate 'r' = $b \cdot f_s$

$\therefore$ $r = b \cdot f_s$

Now, the bandwidth required for PCM transmission is given by half the signalling rate,

i.e. $B.W \geq \frac{r}{2}$

or $BW \geq \frac{b \cdot f_s}{2}$

or $BW \geq \frac{b \cdot 2 \cdot f_m}{2}$

Therefore

$$\boxed{BW \geq b \cdot f_m}$$

But in practice, $BW = (1+\rho)\, b \cdot f_m$

where, ρ = roll-off factor.

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So, for any arbitrary voltage signal $x(t)$, we have normalized SQNR as,

$$\frac{S}{N} \leq 3 \cdot \cancel{P} \cdot 2^{2b}$$

or $\left(\frac{S}{N}\right) dB \leq (4.8 + 6b)\ dB$.

Now, if signal $x(t)$ is a sinusoidal voltage signal with peak amplitude 'A_m' then we have signal power as,

$$P = \frac{V^2}{R} = \frac{(A_m/\sqrt{2})^2}{R}$$

And for $R = 1\Omega$,

$$P = A_m^2/2$$

Now, S/N w.r.t. 'P' is,

$$\frac{S}{N} = \frac{3P}{X^2_{Max}} \times 2^{2 \cdot b}$$

or $$\frac{S}{N} = \frac{3 \times A_m^2/2 \times 2^{2b}}{A_m^2} \quad \left[\because X_{max} = A_m\right]$$

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or $$\frac{S}{N} = \frac{3}{2} \times 2^{2b}$$

$$= 1.5 \times 2^{2b}$$

$$\therefore \left(\frac{S}{N}\right) dB = 10 \log_{10} (1.5 \times 2^{2b})$$

$$= 1.76 + 6.01b$$

$$\simeq 1.8 + 6b.$$

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(4) Non-uniform quantization.

We have for uniform quantization,

$$\varepsilon_{max} = \left|\frac{\Delta}{2}\right| = \text{maximum quantization error}$$

And,

$$\Delta = \frac{2X_{max}}{N}$$

If $X_{max} = 1$ i.e. normalized, then,

$$\Delta = 2/N$$

where,

$$N = 2^b$$

If we take $b = 4$, we get,

$$N = 2^4 = 16$$

So, $\Delta = 2/16 = 1/8$

and

$$\varepsilon_{max} = \left|\frac{\Delta}{2}\right| = 1/16$$

So, $\varepsilon_{max} = 1/16^{th}$ part of full voltage range available.

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So, for a voltage range of 16 volts i.e. +8V to -8V, quantization error will be 1 volt.

If we have higher voltage signals i.e. 15V or 16V, this 1V of quantization error can be considered reasonably small but for low signal amplitudes, say 3 or 2 volts, the maximum error is almost 30% to 50% respectively.

So, if we have a signal that consists of more of the low level signal amplitudes ~~that~~ than high signal amplitudes then obviously, the average signal to noise ratio will decrease.

One such signal that comprises of low signal ~~level~~ amplitudes is speech or music signal.

[of speech signal]

Therefore, in uniform quantization, where the noise power is constant ($\Delta^2/12$), ~~there will~~ most of time signal level remains small leading to lower SQNR.

SQNR : Signal to quantization noise ratio.

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Necessity of non-uniform quantization for speech signal.

Speech and music signals are characterised by high crest factor.

$$\text{crest factor} = \frac{\text{peak value}}{\text{rms value}} \gg 1 \quad \text{for speech \& music signal.}$$

Now,

$$S/N = 3 \times P \times 2^{2b}$$

where, b = no. of bits per sample

P = signal power

Now,

$$P = \frac{V^2_{signal}}{R} = \frac{\text{mean square value of signal voltage}}{R}$$

$$= \frac{\overline{x^2(t)}}{R}$$

and Normalized

$$P = \overline{x^2(t)}$$

$\therefore$ rms $= \sqrt{P}$ and peak value $= X_{max}$

$$\therefore \text{Crest factor} = \frac{\text{peak value}}{\text{rms value}} = \frac{X_{max}}{\sqrt{P}}$$

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If $X_{max} = 1$, then,

$$\text{crest factor} = 1/\sqrt{P}$$

For speech signal, this crest factor should be very high, thus signal power is very small.

i.e.

$P \ll 1$ for large crest factor

But we have,

$\frac{S}{N}$ dB $= (4.8 + 6b)$ dB for $P = 1$,

Therefore,

$$4.8 * \left|3 \times 2^{2b} \times P\right|_{P \ll 1} \lll \left|3 \times 2^{2b} \cdot P\right|_{P=1}$$

i.e the actual signal to quantization noise ratio would be significantly less than the value when $P = 1$.

This is due to the low signal power and relative high quantization noise power, P_q, where $P_q = \Delta^2/12$ is directly proportional to the step size.

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So, in order to improve the signal to quantization noise ratio for music & speech signal with high crest factor, we must reduce the step size of the quantizer for ~~small~~ low level signal input whereas increase the step size for high level signal input.

i.e $\frac{S}{N}$ ~~goo~~ $= \frac{S}{\Delta^2/12}$

i.e. As, Δ goes low, $\frac{S}{N}$ goes high.

So, non-uniform quantization is a process in which the step size is not constant but variable and is dependent on the amplitude of input signal. So, the quantizer characteristic is non-linear.

The direct implementation of varied step size with respect to input signal is difficult to realize thus a technique called companding is utilized for the improvement of SQNR.

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Companding:

Since the direct realization of varied step size is difficult we use a different approach where the weak signals are first amplified and the strong signals attenuated. The resulting signal is then applied to a uniform quantizer.

This process of amplifying the low level signals and attenuating the high level signals is known as compression. Compression is done at the transmitter side.

Now, at the receiver side, exactly opposite is performed i.e. ~~too~~ the amplified signal is attenuated and the attenuated signal is amplified to obtain the original signal. This process is known as expansion.

So, the process of compression of signal at the transmitter and the consequent expansion of at the receiver is combinely known as companding i.e,

compressing + expanding = companding.

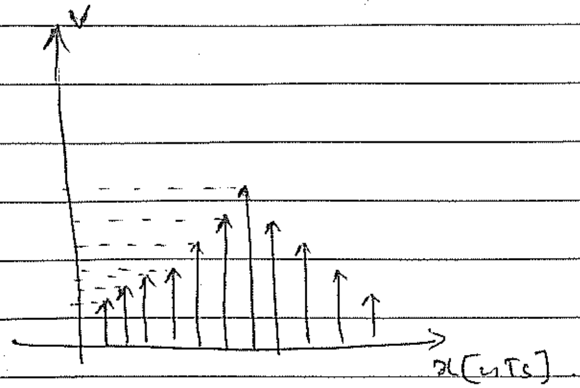

Fig. Sampled values,

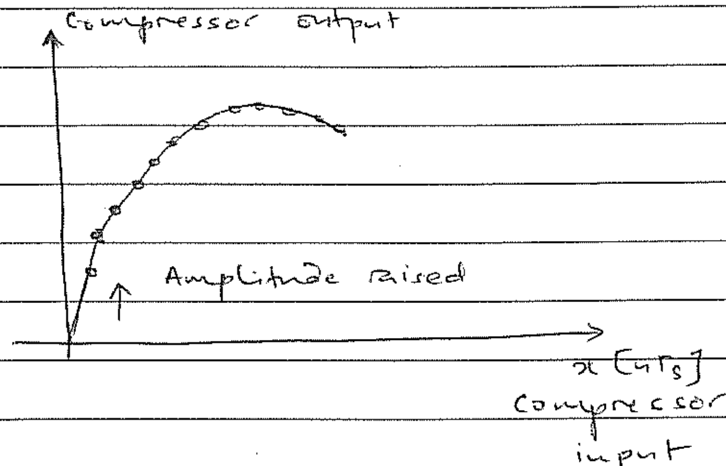

Fig. Compressor characteristics.

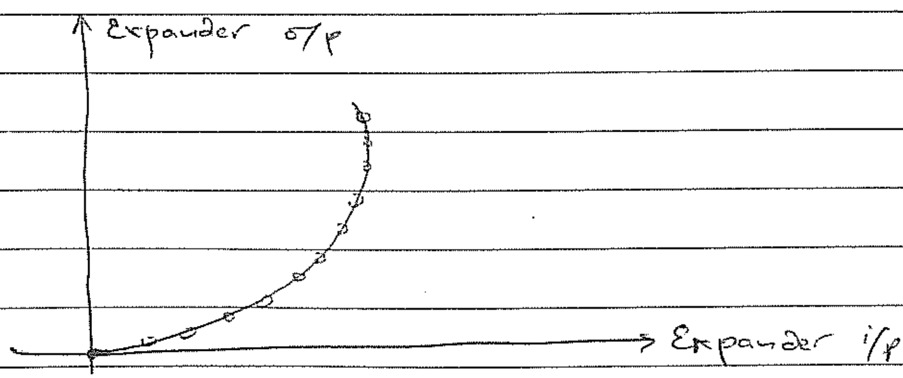

Fig. Expander characteristics.

There are different types of companding techniques used worldwide for PCM system.

We can see from the figure in the preceeding page that the characteristic curve follows logarithmic pattern thus we have two basic companding technique,

i) μ- law companding
ii) A - law companding.

i) μ- law

This is a companding technique that is used for PCM telephone systems in USA, Canada and Japan. It is the same technique used in Nepal too.

For μ- law, the compressor output is given as,

$Z(x) =$ / (8G5)

P.T.O.

$$z(x) = [\text{sgn } x] \frac{\ln\left[1 + \mu \, |x|/x_{max}\right]}{\ln\left[1+\mu\right]}$$

where,

$0 \leq \left| x / x_{max} \right| \leq 1$ is normalized input

$z(x)$ = compressor ~~input~~ output

$\frac{|x|}{x_{max}}$ = compressor input

$[\text{sgn } x]$ = + or - according to the input

μ = companding parameter.

Now, for small value of μ, the output characteristic is almost linear whereas for higher value of 'μ', the output characteristic is logaritmic.

The practically used value of μ = 255.

For μ - law, we get ~~an~~ a gain advantage of 24 dB for μ = 255.

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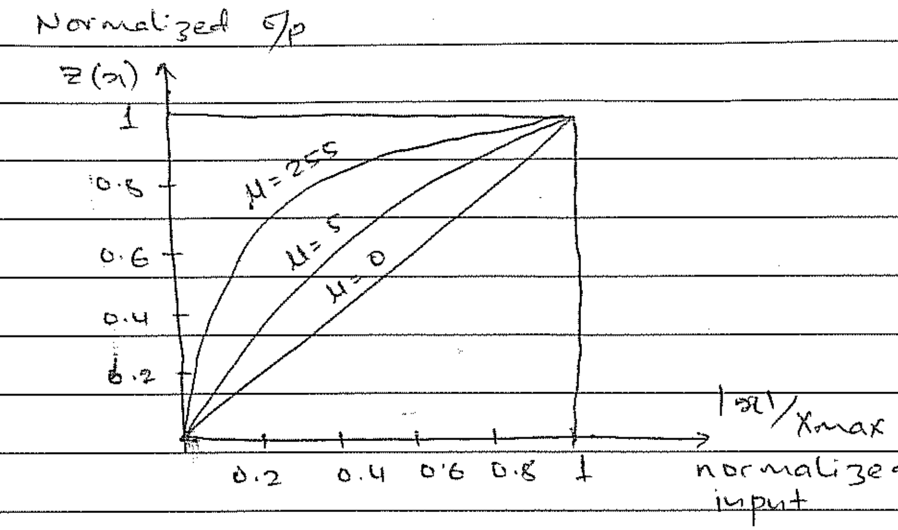

Fig. Compressor characteristics of μ-law compressor.

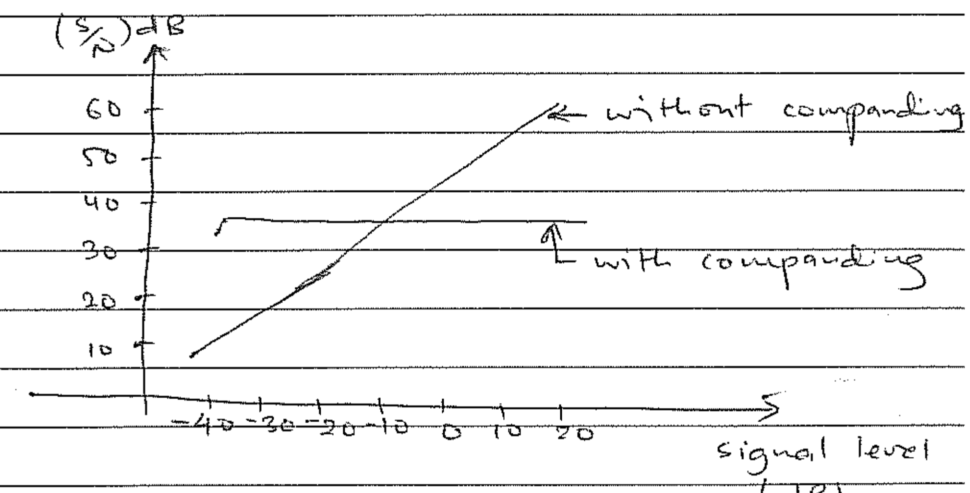

Fig. PCM performance with & without μ-law companding

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ii) A- law companding:

This type of companding is used for PCM telephony in Europe. The output of the compressor unit is given by,

$$\frac{z(x)}{x_{max}} = \begin{cases} \dfrac{A \cdot |x|/x_{max}}{1+\ln A} & \text{for } 0 \le \dfrac{|x|}{x_{max}} \le \dfrac{1}{A} \\ \dfrac{1+\ln [A \cdot |x|/x_{max}]}{1+\ln A} & \text{for } \dfrac{1}{A} \le \dfrac{|x|}{x_{max}} \le 1. \end{cases}$$

where,

$\frac{z(x)}{x_{max}}$ = normalized compressor output

$\frac{|x|}{x_{max}}$ = normalized compressor input

A = companding parameter

A practically used value of this type of companding is,

$A = 87.56$

From, the equation it is clear that for lower value of A, the output characteristic is linear where as for higher value of A, the output is logarithmic.

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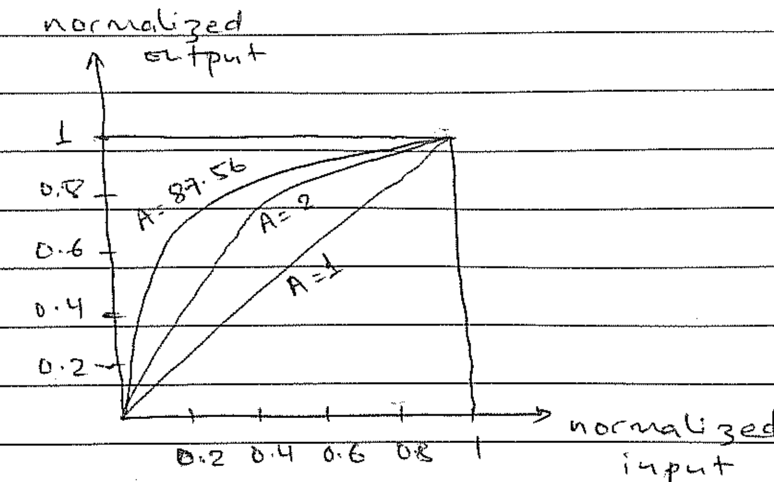


fig. Compressor characteristic of A. law compressor.

It has been deduced that for A = 87.56, the SQNR is improved by 25dB.

So, a functional PCM system should ~~incorp~~ incorporate compressor and an expander to derive a constant SQNR for efficient transmission and reception of speech and music signal. i.e. The output of the compressor is now fed to the uniform quantizer.

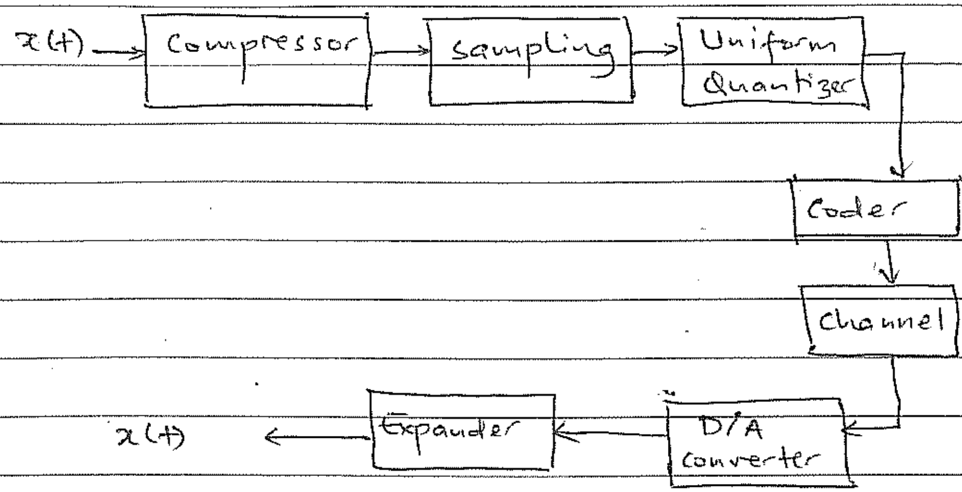


fig. functional block diagram of complete PCM system.

Time division Multiplexing (TDM).

Multiplexing is the process of simultaneously transmitting two or more individual signals over a single communication channel.

i.e.

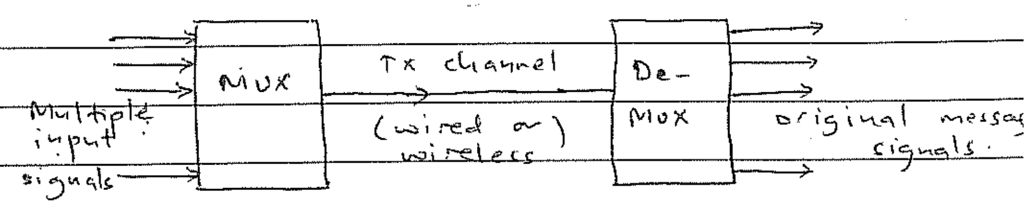
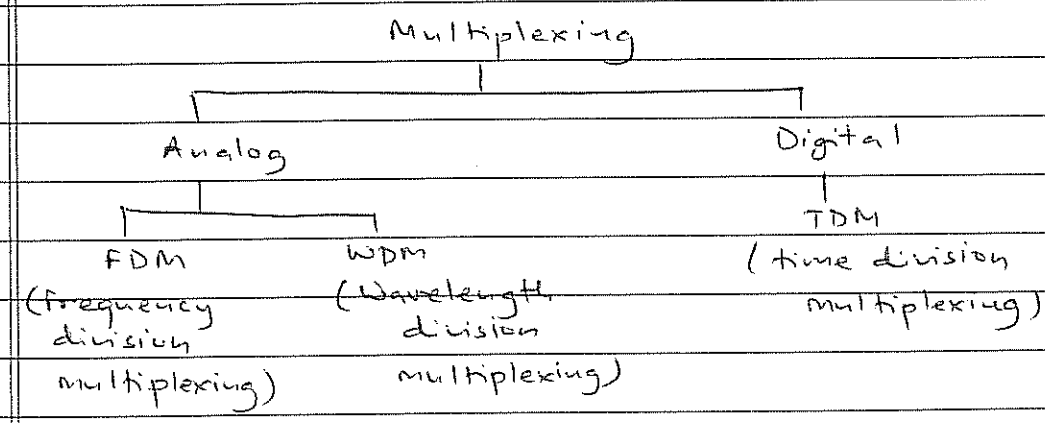


fig. Concept of multiplexing.



Time division multiplexing (TDM).

→ It is a technique used to utilize common transmission medium to transmit more than one signal simultaneously.

Sampling theorem states that the samples are taken at some fixed interval of time. TDM makes use of this ~~as~~ aspect.

There will be some interval of time between adjacent samples of any individual signal. We can thus place sample of another message signal in ~~between~~ ~~th~~ between the adjacent samples of the other signals.

So, taking the time interval between the adjacent samples into consideration, we can multiplex as many signal samples from number of ~~inp~~ message signals. This process of multiplexing number of ~~messa~~ sampled message signals taking time interval into account is known as time division multiplexing.

So in TDM, sample of each individual message signal is transmitted for a very short time.

Once all of the ~~sa~~ samples for each individual message signal is transmitted, we ~~call~~ say one frame is completed.

let us suppose there are three sources of signal, A, B & C.

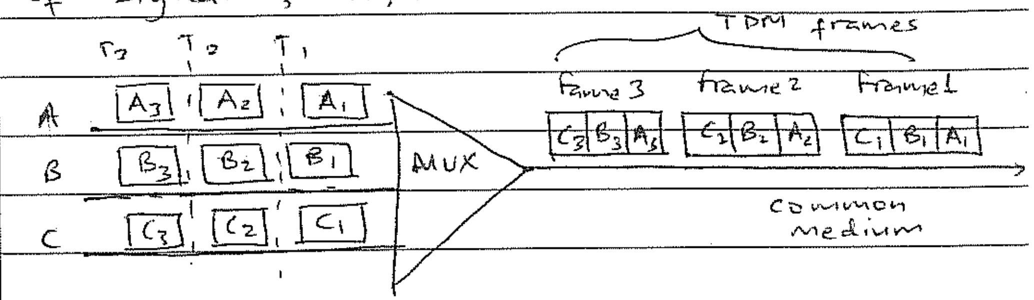


fig. TDM system

from the figure above, at any time T_1, samples A_1, B_1 & C_1 are multiplexed into one TDM frame and transmitted. At another time T_2, A_2, B_2 and C_2 are multiplexed and transmitted through common medium. This process goes on until all the message signals are transmitted.

PAM / TDM system.

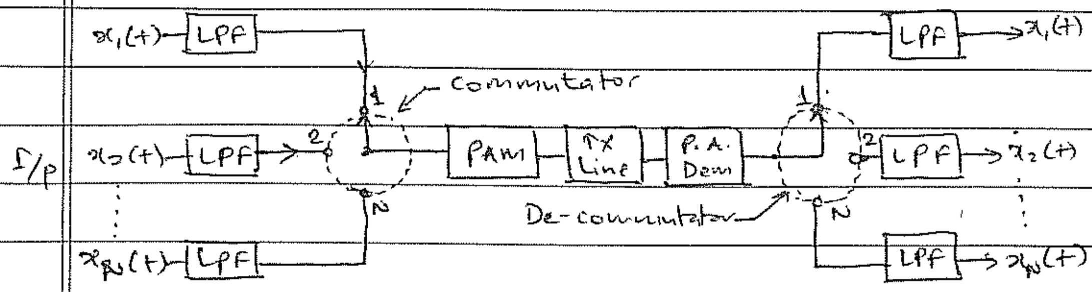


fig. Block diagram of PAM / TDM system.

In the figure above the message signals are bandlimited using LPF. A commutator or a single pole rotating switch acts as a multiplexer.

This commutator can be mechanical or electrical switch, which rotates at f_s rotations per second.

As the commutator rotates anticlockwise, it makes contact to the points 1, 2, to N for a short period of time. And thus the signals $x_1(t)$, $x_2(t)$ to $x_N(t)$ are fed to pulse amplitude modulator.

So, in one rotation of the commutator, 'N' numbers of ~~input signals~~ samples are fed to the transmission line, thus a frame is completed in one ~~sec~~ rotation i.e. T_s seconds.

A commutator thus,

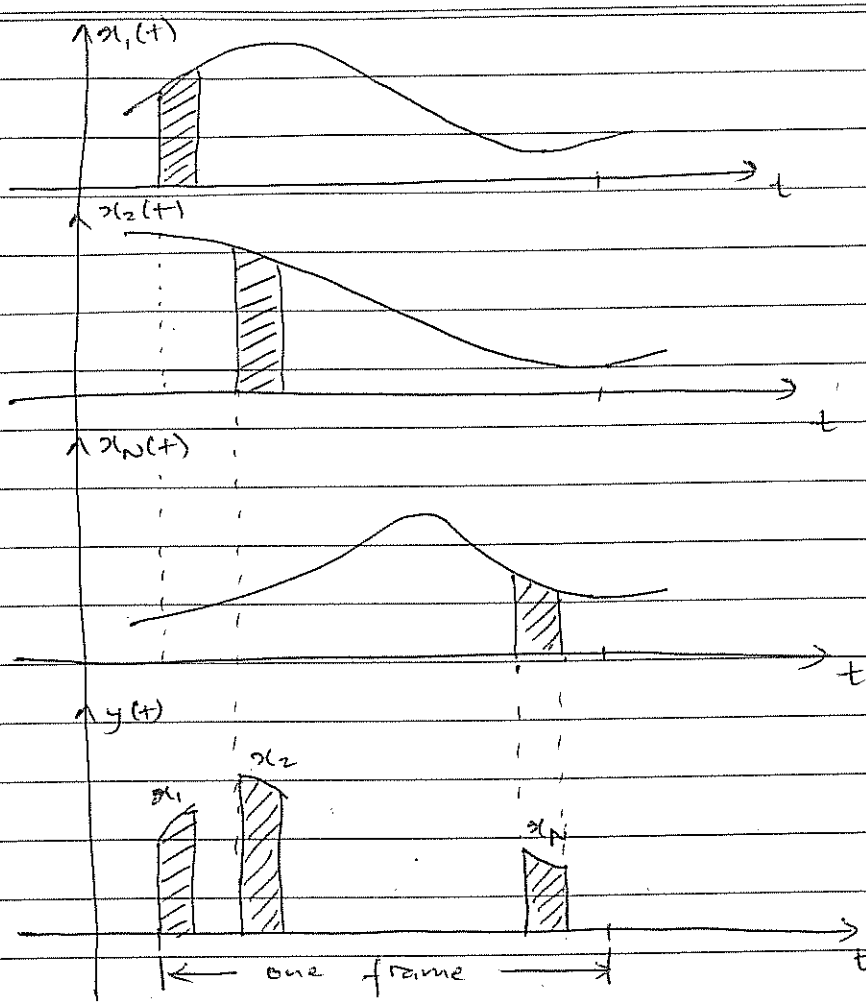
i) takes narrow sample of each input message at a rate f_s which ~~is~~ is higher than $2f_m$.

ii) provides 'N' samples inside the interval $T_s = \frac{1}{f_s}$.

For the complete TDM system, it must regenerate the original signal at the receiver.

So, the multiplexed signal are first fed to pulse amplitude modulator and then transmitted through transmission line. The signal at the receiver end must thus be fed to demodulator first. The demodulated signal to 'N' number of low pass filters through a rotating switch called decommutator.

This decommutator, as in figure, rotates in clockwise direction and is synchronized with the commutator.



$T_s = 1/f_s$

fig. Multiplexed PAM signal

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signalling rate of PAM / TDM system.

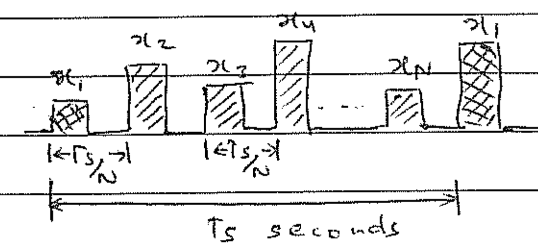
Let f_m be the maximum frequency present for the input signals x_1 to x_N.

As the commutator is rotating at f_s rotations per second, we have sampling frequency as f_s.

Now as per ~~Nyqu~~ Nyquist criterion,

$f_s \geq 2 f_m$,

So, the commutator must rotate at f_s rotations per second atleast.



As shown in figure, one revolution of commutator corresponds to one fram which contains sample from each input.

∴

1 rev = 1 frame = N pulses = T_s seconds

∴ Pulse to pulse spacing $= \frac{T_s}{N} = \frac{1}{N \cdot f_s}$

And number of pulses per second,

$$= \frac{N}{T_s} = N \cdot f_s$$

which is equal to the signaling rate for PAM.

i.e.

$$r = N \cdot f_s \text{ pulses / second}$$

Also, $f_s \geq 2f_m$

$\therefore$ Signaling rate $(r) \geq N \cdot 2f_m$.

Transmission bandwidth

$$B.W = \frac{1}{2} \times \text{Signalling rate}$$

$$\geq \frac{1}{2} \times N \times 2f_m$$

$\therefore BW \geq N \cdot f_m$

Therefore minimum bandwidth required

$$BW_{min} = N \cdot f_m.$$

N = no. of individual signal samples.

PCM / TDM system.

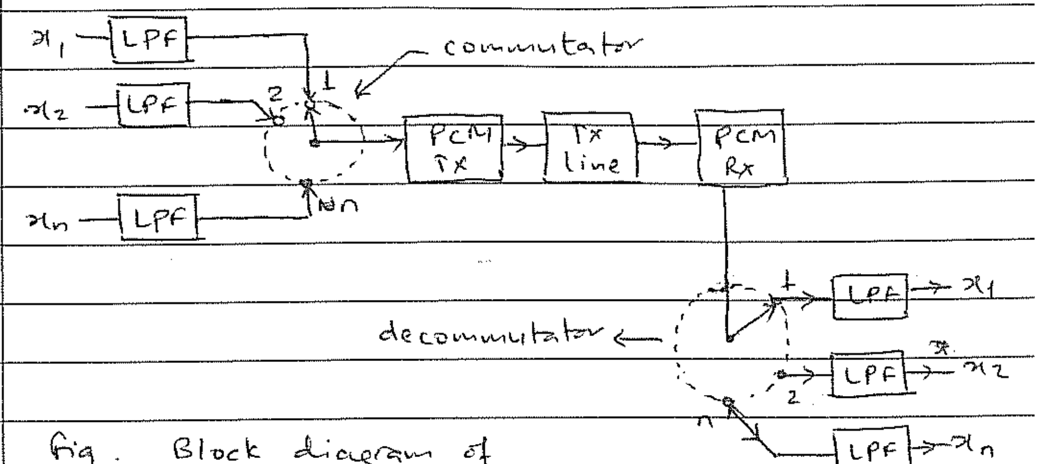


fig. Block diagram of PCM / TDM system.

In the above figure, x_1 to x_n are voice signals bandlimited to 3.3 KHz at the LPF. The commutator rotates at the rate 8 KHz i.e. f_s = 8 KHz [$f_s \geq 2f_m$].

Thus the input voice signals are sampled at the rate f_s = 8 KHz.

So, the commutator samples individual voice signals and feeds to the PCM transmitter where each sample is converted to an 8 bit codeword.

So, for each rotation of the commutator, 'n' samples are fed to the

PCM transmitter resulting in $n \times 8$ bit codewords multiplexed through a common transmission line.

At the PCM receiver, the codewords are converted to the analog form and the decommutator demultiplex the signals to desired x_1 to x_n voice signals.

It should be noted that the commutator and decommutator are always synchronized.

Now, if we take the number of voice channels, $n = 24$, then the system is known as T_1 carrier system.

Such that,

1 revolution = 1 frame = 24 channelsamples

Since, each sample is encoded by 8 bits,

$$1 \text{ frame} = 24 \times 8 = 192 \text{ bits.}$$

x_1	x_2	x_3	x_4	----	x_{23}	x_{24}
8 bits	8 bits	8 bits	8 bits	----	8 bits	8 bits

$\longleftarrow$ $24 \times 8 = 192$ bits / frame $\longrightarrow$

Now, for the purpose of synchronization, an extra bit is added preceeding the 192 bits. This bit is termed as frame synchronization bit 'F'.

Thus the total number of bits per frame becomes 193 bits.

So, for T_1 system,

$$f_s = 8\ \text{KHz} = \frac{1}{T_s}$$

And 1 revolution = 1 frame = T_s

$$\therefore\ 1 \text{ frame} = T_s = \frac{1}{8000} = 125\ \mu\text{sec}$$

Hence,

193 bits are transmitted in $125\ \mu$sec,

and

$$\text{number of bits in 1 sec} = \frac{193}{125 \times 10^{-6}}$$

$$= 1.544 \times 10^{6}$$

$\therefore$ Signalling rate or bit rate of T_1 Carrier $= 1.544 \times 10^{6}$

And Minimum BW $= \dfrac{\text{Bit rate}}{2} = \dfrac{1.544 \times 10^{6}}{2} = 772$ KHz

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Now, if we take no. of voice channels equal to 30, we get E_1 system, where additional 2 channels are used for signaling and controlling.

i.e. 1 frame = 32 channel samples.

= 256 bits ~~/frame~~

but,

$$f_s = 8\,KHz \Rightarrow T_s = \frac{1}{8000} = 125 \times 10^{-6}\,sec$$

or 256 bits transmitted in 125 μsec.

$$\text{Bit rate} = \frac{256}{125\,\mu sec} = 2.048\ \text{M bits per sec.}$$

$$= 2.048\ \text{Mbps}.$$

and $BW_{min} = \frac{2.048}{2} = 1.024\ MHz.$

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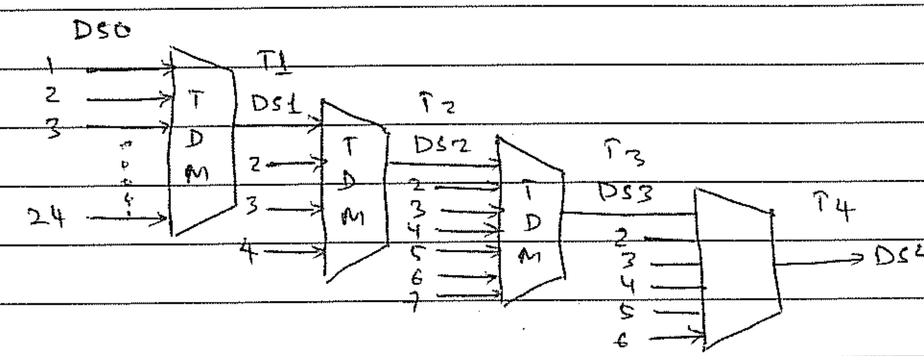
T1 Heirarchy

S.No.	Line	Rate (Mbps)	Number of voice channels.
1	T1	1.544	24
2	T2	6.312	96
3	T3	44.736	672
4	T4	274.176	4032



DS1 = 24 DS0 i/p = T1

DS2 = 4 DS1 = 4 × 24 DS0 i/p = T2

DS3 = 7 DS2 = 7 × 4 DS1 = 7 × 4 × 24 DS0 i/p = T3

DS4 = 6 DS3 = 42 DS2 = 4032 DS0 i/p = T4.

fig. T1 heirarchy

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E1 Heirarchy

S.No.	Line	Rate (Mbps)	Number of voice channels.
1	E1	2.048	30
2	E2	8.448	120
3	E3	34.368	480
4	E4	139.264	1920

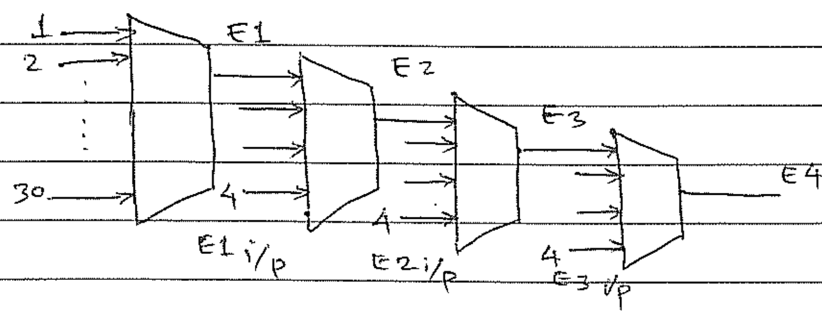

Fig. E1 Heirarchy

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$S = \frac{1}{3}$

$T1 \times 4 \times \frac{49}{48} \times \frac{288}{288-S}$

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Differential PCM.

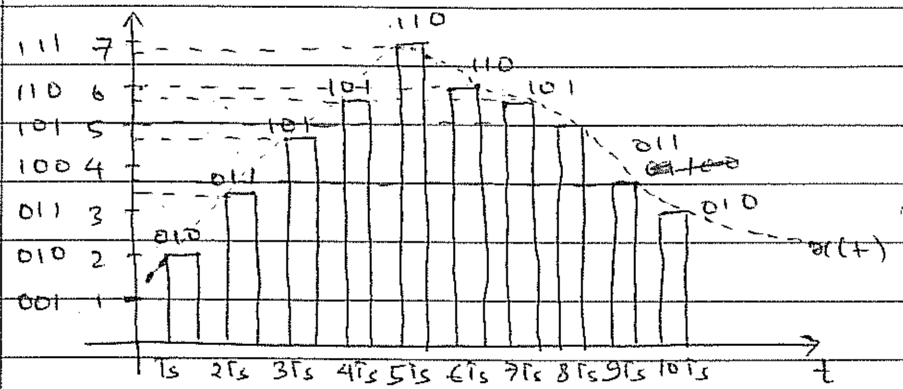

fig. Illustration of redundant information in PCM.

A speech signal can be characterized by high correlation. i.e the samples of a speech signal are highly correlated with each other such that the sampled amplitude very minimally.

So, when these samples are encoded, the resulting encoded signal contains same codewords resulting in redundant informations.

From the figure above, we can see that at $3T_s$ & $4T_s$, the codeword is (101). Also, for other sampling time,

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at least one of the bits ~~are~~ is same for two adjacent samples. Thus we are transmitting redundant (same) information over each sampling time for PCM. ~~The~~ i.e. we are using bits unnecessary making the bandwidth wider.

So, instead of quantizing ~~the~~ each of the samples, if we quantize the difference between adjacent samples, we can reduce the number of bits required to encode the information.

This process is known as differential pulse code modulation (DPCM).

Encoder DPCM.

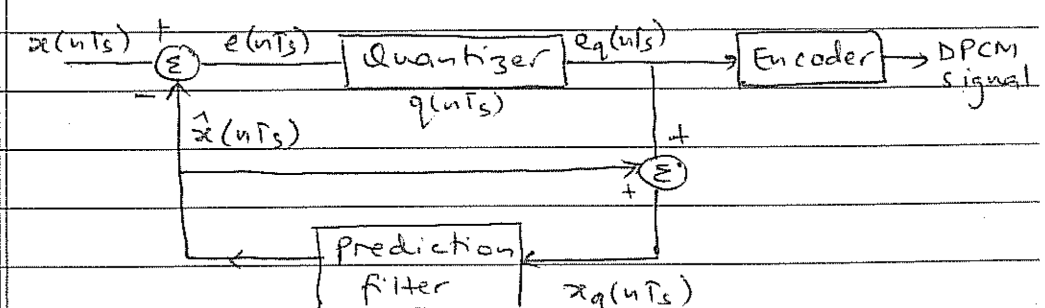

fig. DPCM Encoder (Transmitter).

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A differential pulse code modulation works on the principle of prediction. i.e. the value or estimate of present sample is predicted from the past samples.

In the figure given, a DPCM transmitter is shown where $x[nT_s]$ is the sampled input signal and the predicted signal is the estimate of previous value of samples given by $\hat{x}(nT_s)$.

This estimate of the sampled signal $\hat{x}(nT_s)$ is produced by a prediction filter.

Now, at the comparator, the difference of $x(nT_s)$ & $\hat{x}(nT_s)$ is found. i.e.

$$x(nT_s) - \hat{x}(nT_s) = e(nT_s) \quad -(i)$$

Here,

$e(nT_s)$ = prediction error

This prediction error $e(nT_s)$ is then fed to a quantizer such that the quantizer output is $e_q(nT_s)$.

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Now, the quantized prediction error $e_q(nT_s)$ is added to the previous prediction and ~~added~~ fed to the prediction filter.

This makes the prediction ~~more~~ closer to the actual sample signal. Now the signal fed to prediction filter be $x_q(nT_s)$.

Also, the quantizer output is taken as,

$$e_q[nT_s] = e(nT_s) + q(nT_s)$$

where,

$q[nT_s]$ = quantization error

Also,

$$x_q(nT_s) = \hat{x}(nT_s) + e_q(nT_s)$$

or $x_q(nT_s) = \hat{x}(nT_s) + e(nT_s) + q(nT_s)$

or $x_q(nT_s) = x(nT_s) + q(nT_s)$, which shows that the quantized version of $x(nT_s)$ is sum of original value and quantization error $q(nT_s)$. and thus doesn't depend on prediction filter characteristics.

Now, it may be observed that the quantized error $e_q(nT_s)$ is very small and

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thus can be encoded by using small number of bits and hence the transmission bandwidth can be reduced.

DPCM Decoder (Receiver).

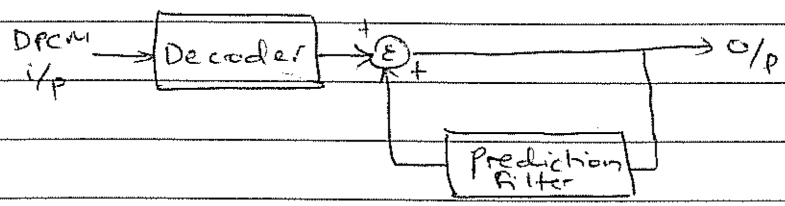

Fig. DPCM decoder.

A decoder first reconstructs the quantized error signals from the incoming DPCM binary signal.

A prediction filter then gives an estimate of possible input message signal. This estimate is added to the quantized error signal to give the quantized version of original signal.

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So, the signal at the receiver differs from the original signal by the quantization error $\epsilon_q(nT_s)$. This error is introduced permanently in the reconstructed signal.

Delta modulation (DM)

A delta modulation, like DPCM is a predictive waveform coding technique and can be considered as a special case of DPCM. It uses a two level (one bit) quantizer.

In DM, the analog signal is highly over-sampled in order to increase the adjacent sample correlation.

DM- encoder :

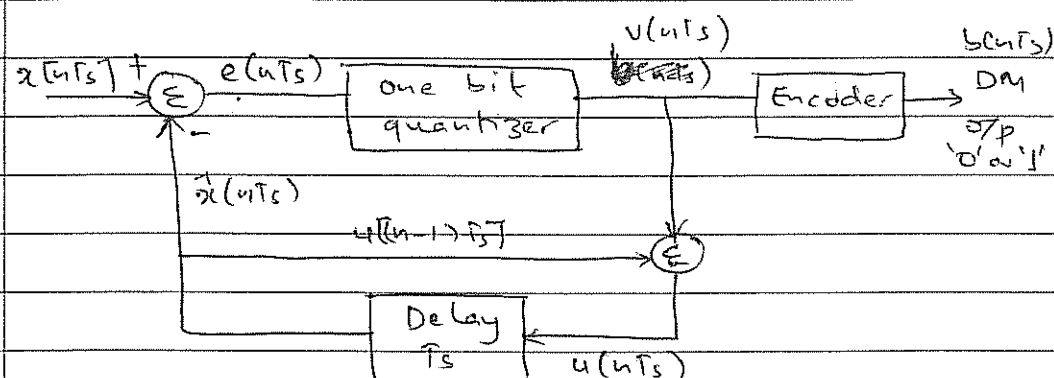

fig. Delta modulator Transmitter.

In the figure above, the DM encoder approximates an input time function by a

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series of linear segments of constant slope.

A sampled signal is fed to the comparator where it is compared to the approximation of signal '$\hat{x}(nT_s)$'.

So, the difference between $x(nT_s)$ & $\hat{x}(nT_s)$ is the error incorporated such that,

$$e(nT_s) = x(nT_s) - \hat{x}(nT_s) \quad : \text{prediction error}$$

Now, this prediction error is confined to two levels i.e. $+\Delta$ & $-\Delta$.

Now, if the difference is positive, the approximated signal is increased by 'Δ' and if it is negative then the approximated step signal is decreased by 'Δ'.

This step size '2Δ' is thus kept constant.

~~So, if step size is reduced then~~

So, if the step signal goes to '$-\Delta$', then a bit '0' is transmitted or if it goes to '$+\Delta$' then 1 is transmitted.

$$\left[\text{Step size} = \begin{array}{c} +\Delta \\ -\Delta \end{array} = 2\Delta\right]$$

let us assume that $u[nT_s]$ is the present sample approximation of the staircase output, then, from figure,

$$u[(n-1)T_s] = \hat{x}(nT_s)$$

and $u(nT_s) = u[(n-1)T_s] + v(nT_s)$

$$= u[(n-1)T_s] + [\pm\Delta]$$

Because, $v(nT_s)$ is $= \pm\Delta$

i.e. $v(nT_s) = +\Delta$ if $x(nT_s) \geq \hat{x}(nT_s)$

$= -\Delta$ if $x(nT_s) < \hat{x}(nT_s)$

Also,

$$e(nT_s) = x(nT_s) - \hat{x}(nT_s)$$

$$= x(nT_s) - u[(n-1)T_s]$$

and

$$v(nT_s) = \Delta \operatorname{sgn}[e(nT_s)].$$

$$\therefore u(nT_s) = u[(n-1)T_s] + v(nT_s)$$

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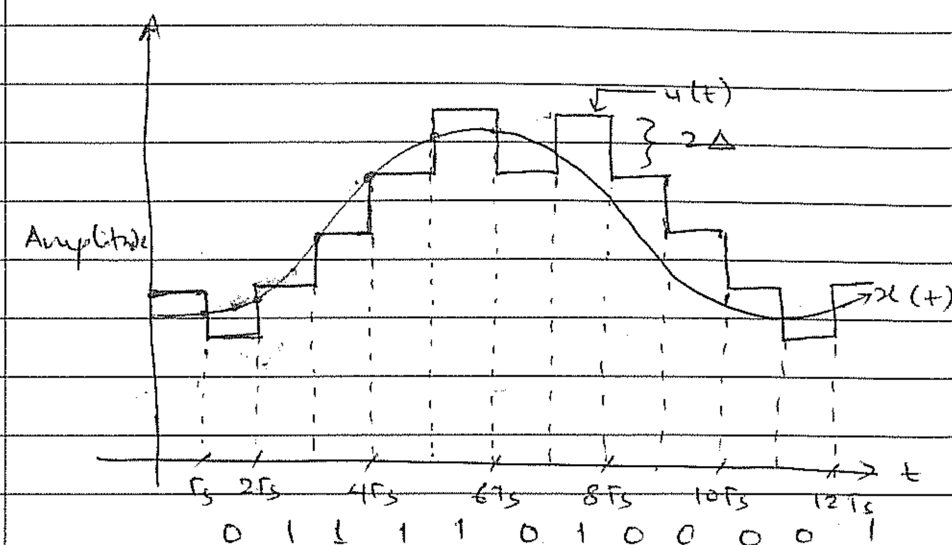

So, with $v(nT_s) = \pm\Delta$, the DM output is '1' for $b(nT_s) = +\Delta$ and '0' for $v(nT_s) = -\Delta$.

Thus, for a given input signal $x(nT_s)$, we get a 1 bit o/p. Hence, the ~~bam~~ transmission bandwidth is reduced for DM as compared to PCM.

DM decoder (Receiver).

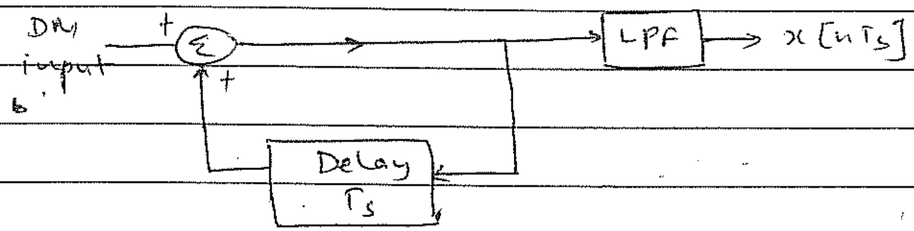

Fig. DM decoder.

The demodulation of DM signal can be made using an accumulator ckt and a low pass filter.

As shown in the figure, the accumulator generates the staircase approximated signal output which is delayed by one sampling period 'T_s'. This delayed signal is added with the input DM signal.

So, if the DM input is '1' then it adds '$+\Delta$' step to the previous output. And if the binary input is '0' then '$-\Delta$' step is added to the delayed output.

Now, the LPF has a cut off frequency equal to the highest frequency in $x(t)$ such that the LPF finally

smoothes the staircase signal to reconstruct the original signal $x(nT_s)$.

Noises in DM.

When converting the input signal to two bit DM signals, if the approximation do not properly engulf the message signal then quantization errors are bound to occur. These errors are termed as noise.

There are basically two kinds of noise is delta modulation.

i) ~~Sa~~ Slope overload distortion

ii) Granular or idle noise.

i) Slope overload distortion.

This type of distortion occurs when the input signal has large dynamic range such that the step by step accumulation process may not ~~ch~~ catch up with the rate of change of input signal.

So, when the delta modulation is done in such case, the modulated signal

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output may not be similar to the input signal. So, at the receiver, the quality of received signal differs from the original message. Hence, the received signal are said ~~said~~ to ~~be~~ suffer from slope overload distortion.

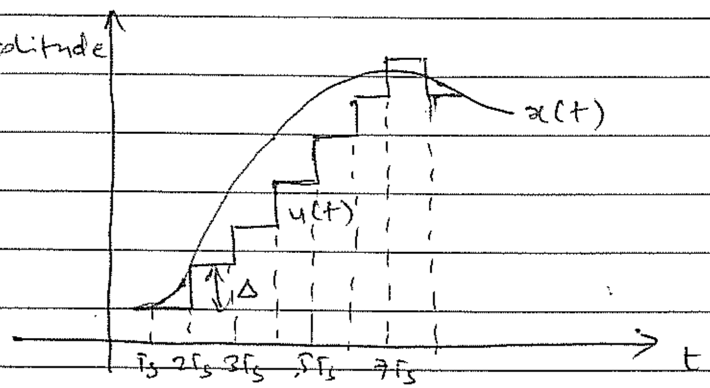

fig. slope overload.

We can see from the figure, the signal $x(t)$ is always greater than $u(t)$ till $t = 7T_s$. But the step size being 'Δ', it can only step up by 'Δ' amplitude and thus cannot engulf the message signal. So, the received signal will differ significantly from the message signal.

Remedy will be ~~to~~ increase step size 'Δ'.

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ii) Granular noise

When the step size is too large compared to the small variations in the input signal then granular noise or idle noise occurs. This can happen when the slope of ~~samp~~ input signal is low i.e. almost constant w.r.t. time and the step size 'Δ' is relatively high. In such case, the ~~approxim~~ approximation starts swinging from −Δ to Δ causing high noise level.

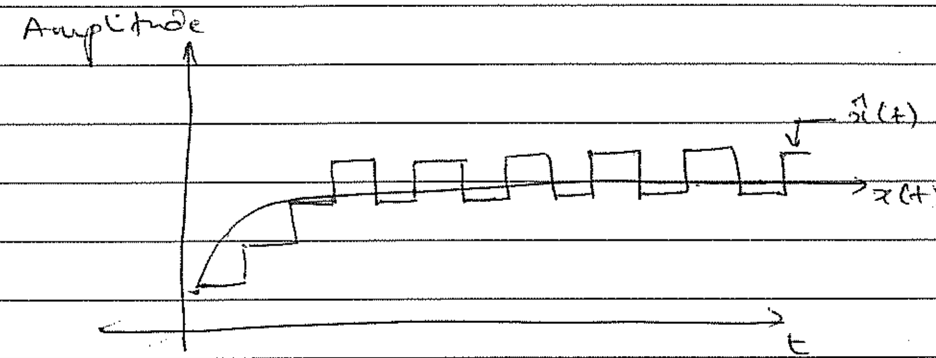

fig. Granular noise.

It is evident that the granular noise can be minimized by reducing the step size.

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Conditions for avoiding slope-overload.

For delta modulation we have '1' bit representation for each sample.

Thus quantization level,

$$L = 2^1 = 2$$

Thus there will be two levels between $+\Delta$ & $-\Delta$ such that step size $= \Delta$.

Now, if the input signal changes more than 'Δ' within sampling interval then there will be slope-overload distortion.

Therefore the desired limiting condition on the input signal $x(t)$ for avoiding slope overload is,

$$\left.\frac{d\,x(t)}{dt}\right|_{max} \le \frac{\Delta}{T_s}$$

let us consider a harmonic signal as an input to DM i.e.

$$x(t) = A_m \cos 2\pi f_m t$$

where, f_m = max. frequency of $x(t)$.

A_m = peak amplitude

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Then,

$$\left.\frac{d\,x(t)}{dt}\right|_{max} = \frac{d}{dt}(-A_m \cos 2\pi f_m t)$$

$$= \left|-A_m\, 2\pi f_m \cdot \sin(2\pi f_m)\right|_{max}$$

$$= A_m \cdot 2\pi f_m$$

Now, ~~fo~~

for no ~~sp~~ slope overload,

$$A_m\, 2\pi f_m \le \frac{\Delta}{T_s}$$

$$\text{or} \quad A_m \le \frac{\Delta}{2\pi f_m \cdot T_s}$$

$$\text{or} \quad A_m \le \frac{\Delta \cdot f_s}{2\pi f_m}$$

So, for no slope overload, the amplitude A_m ~~shd~~ should ~~at least~~ be ~~eq~~ at most be equal to $\frac{\Delta \cdot f_s}{2\pi f_m}$.

Here, $f_s \gg 2 f_m$.

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(#) Signal to quantization noise in DM.

We have condition for no slope overload as,

$$A_m\Big|_{max} = \frac{\Delta \cdot f_s}{2\pi f_m},$$

So the signal power,

$$P = \left(\frac{A_m}{\sqrt{2}}\right)^2 = \frac{A_m^2}{2}$$

$$= \frac{\Delta^2 \cdot f_s^2}{4\pi^2 f_m^2 \times 2}$$

$$\therefore \quad P = \frac{\Delta^2 \cdot f_s^2}{8\pi^2 f_m^2}$$

Also,

$e_q(nT_s) = \Delta \, sgn \{ e(nT_s) \}$

such that $e_q(nT_s)$ lies in the interval $+\Delta$ & $-\Delta$.

i.e. total swing $= 2\Delta$.

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So the normalized noise power,

$$N_q = \int_{-\alpha}^{\alpha} \varepsilon^2 f_\varepsilon(\varepsilon)\, d\varepsilon$$

$$= \frac{1}{2\Delta}\int_{-\Delta}^{\Delta} \varepsilon^2\, d\varepsilon$$

$$= \frac{1}{2\Delta}\left[\frac{\varepsilon^3}{3}\right]_{-\Delta}^{\Delta} = \frac{1}{2\Delta}\left[\frac{\Delta^3 + \Delta^3}{3}\right]$$

$$= \frac{\Delta^2}{3}$$

Therefore,

$$\frac{S}{N} = \frac{P}{N_q} = \frac{\Delta^2 \cdot f_s^2 / 8\pi^2 f_m^2}{\Delta^2/3}$$

$$= \frac{3 f_s^2}{8\pi^2 f_m^2}$$

Which is the SQNR for the transmitting side.

Now, for the receiver part, the received signal is passed through a low pass filter.

Let f_M be to cut off frequency of LPF such that,

$f_M \geq f_m$.

and $f_M < f_s$.

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Now, if N_q is distributed uniformly over to f_s, the output quantization noise for LPF with cut off frequency f_M is given by.

$$N_q' = \frac{\Delta^2}{3} \times \frac{f_M}{f_s}$$

So, the SQNR at receiver is given as,

$$\left(\frac{S}{N}\right)_{o/p} = \frac{P}{N_q'}$$

$$= \frac{\frac{\Delta^2 f_s^2}{8\pi^2 f_m^2}}{\frac{\Delta^2}{3} \times \frac{f_M}{f_s}}$$

$$\left(\frac{S}{N}\right)_{o/p} = \frac{3 f_s^3}{8\pi^2 \cdot f_m^2 \cdot f_M}$$

If for LPF, $f_M = f_m$, then,

$$(SQNR)_{o/p} = \frac{3 \cdot f_s^3}{8\pi^2 f_m^3}$$

$$= \frac{3}{8} \cdot \frac{1}{\pi^2} \cdot \left(\frac{f_s}{f_m}\right)^3$$

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⊕ Parametric speech coding ~~codi~~ (Vocoders).

Analog signals such as speech and video signals can be encoded by a digital method where a particular value is predicted by a linear function of past values of the signal.

Under normal circumstances i.e. in PCM the speech is sampled at 8000 samples/sec with 8 bits to represent each sample. So, the bitrate is 64000 bps.

Now, using linear prediction coding, this bitrate can be reduced to 2400 bps with acceptable quality. this is possible due to analysis-synthesis method.

So, a vocoder (voice ~~codec~~ encoder) is a device that analyzes and synthesizes the human voice signal for data compression. For parametric speech coding, an ~~channel~~ LPC vocoder is used.

A vocoder takes natural speech as their input and use that speech to generate various types of acoustic parameters which take up less transmission bandwidth

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than the original speech.

~~So, in LPC vo~~

LPC vocoders.

(Linear predictive coefficient) vocoders.

In such device, the speech signal is first modeled (i.e. analyzed) and the parameters of the model is extracted.

These parameters are then transmitted using PCM and at the receiver end, the original voice signal is predicted or synthesized from those parameters.

So, such technique of speech coding is called linear predictive coding.

A speech signal can now be characterized by certain parameters,

i) voiced-unvoiced information
ii) pitch
iii) gain parameter (G)
(iv) filter coefficients

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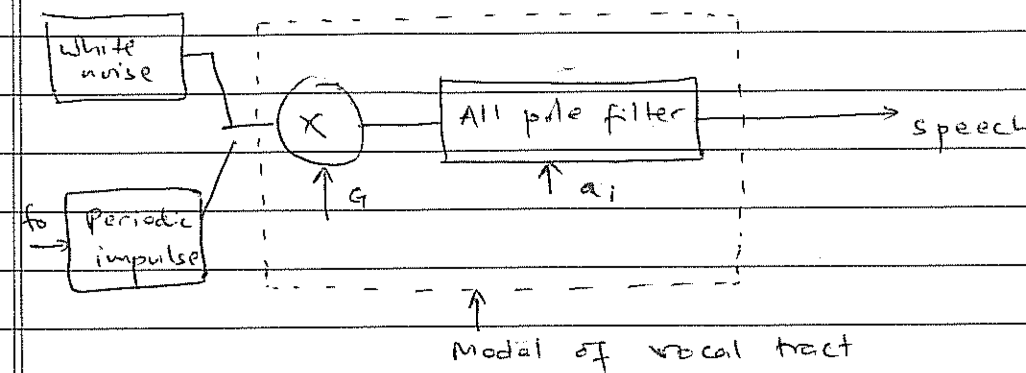

Fig. Synthesizer.

It is known that speech signal remain constant for a short period of 20-30ms. It means that within this time frame, the filter coefficients can be assumed to be constant. We can therefore analyze or estimate the value of f_0, a_i and G for these period and transmit these parameters using PCM.

Then at the receiving end, using same prediction filter, we can reconstruct the original speech signal.

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Suppose a signal is filtered at 3KHz and sampled at 8000 samples/sec. Now, a block of 160 ~~samp~~ samples corresponding to 20 ms interval is considered to be a frame for prediction.

i.e.

$f_s = 8000\,Hz,$

$T_s = 0.125\ ms.$

$\therefore$ For,

$20\,ms \Rightarrow \frac{20\,ms}{0.125\,ms} = 160$ samples.

The average filter coefficient a_i, G and periodic impulse at f_0 is estimated and converted into PCM.

The bitwise representation of each parameter ~~is~~ are as follows,

voiced-unvoiced information	1 bit
pitch ($1/f_0$)	6 bits
gain parameter (G)	5 bits
filter coefficient	8-10 bits

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Now if 1^{st} order filter is used, then, total number of bits to represent a frame of 20ms would be 22 bits (max).

Now, frame rate is ~~160 frames~~ 160 samples/sec.

160×22 bits $= 3520$ bits/sec.

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4) A sinusoidal voice signal $x(t) = \cos(6000\pi t)$ is to be transmitted using either PCM or DM. The sampling rate for PCM is 8 KHz and for transmission with DM, the step size is decided to be 31.25 mV. The slope overload distortion is to be avoided. Assume that the number of quantization levels for PCM system is 64. Determine the signaling rates of the both of these systems.

Given, $f_m = 3$ KHz, $A = 1$

for PCM,

$f_s = 8$ KHz

$L = 64$

or $2^b = 64 \Rightarrow b = 6$

Now,

rate, $r = n \cdot f_s$

$= 6 \times 8$

$= 48$ Kbps

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for DM,

condition for no slope overload,

$$A \leq \frac{\Delta \cdot f_s}{2\pi f_m}$$

$$\text{or } f_s \geq \frac{2\pi f_m \cdot A}{\Delta}$$

$$\text{or } f_s \geq \frac{2\pi \cdot 3 \times 10^3 \cdot 1}{31.25 \times 10^{-3} V}$$

or $f_s \geq 603.18$ KHz

$\therefore$ Signaling rate $= n \cdot f_s$

$= 1 \times 603.18$

$= 603.18$ Kbps.