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Communication System I

Kathford college

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4. AMPLITUDE MODULATION (AM)

Modulation is a process in which some characteristic of a carrier signal is varied according to the value of the modulating signal.

The modulating signals contain the information that needs to be transmitted, and are also known as baseband signals.

If the carrier waveform is continuous in nature then the modulation process is known as continuous wave (CW) modulation. For CW modulation we have amplitude modulation (AM) and angle modulation.

Amplitude modulation (AM)

Amplitude modulation is a process in which the amplitude of a higher frequency carrier signal $c(t)$ is varied according to the change in modulating signal $m(t)$.

let $c(t) = A_c \cos(2\pi f_c t + \theta)$

where, A_c = maximum amplitude of $c(t)$

f_c = carrier frequency

θ = phase shift of carrier signal.

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For convenience let us assume that $\theta = 0$, i.e.

$$c(t) = A_c \cos 2\pi f_c t$$

So for any arbitrary message signal $m(t)$ and any carrier signal $c(t) = A_c \cos 2\pi f_c t$, we can have three general types of amplitude modulation.

1) Double side band AM or full carrier [DSB-AM or DSB-FC].
2) Double sideband - supressed carrier [DSB-SC]
3) Single side band - Supressed carrier [SSB-SC].

(1) DSB-AM or DSB-FC

The standard form or equation for DSB-AM is given as,

$$s(t) = A_c [1 + k_a m(t)] \cos 2\pi f_c t$$

where, k_a = amplitude sensitivity of the modulator

$m(t)$ = message or modulating signal

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expanding $s(t)$ we get,

$$s(t) = A_c \cos 2\pi f_c t + k_a \cdot A_c \cdot m(t) \cos 2\pi f_c t$$

Now, $\cos 2\pi f_c t \xleftrightarrow{F.T.} \dfrac{\delta(f - f_c) + \delta(f + f_c)}{2}$ &

$$m(t) \cos 2\pi f_c t \xleftrightarrow{F.T} \frac{M(f - f_c) + M(f + f_c)}{2}$$

$\therefore$

$$S(f) = \frac{A_c}{2}[\delta(f - f_c) + \delta(f + f_c)] + \frac{k_a \cdot A_c}{2}[M(f - f_c) + M(f + f_c)]$$

So,

$$s(t) = A_c [1 + k_a m(t)] \cos 2\pi f_c t$$

→ time domain AM signal.

$$S(f) = \frac{A_c}{2}[\delta(f - f_c) + \delta(f + f_c)] + \frac{k_a A_c}{2}[M(f - f_c) + M(f + f_c)]$$

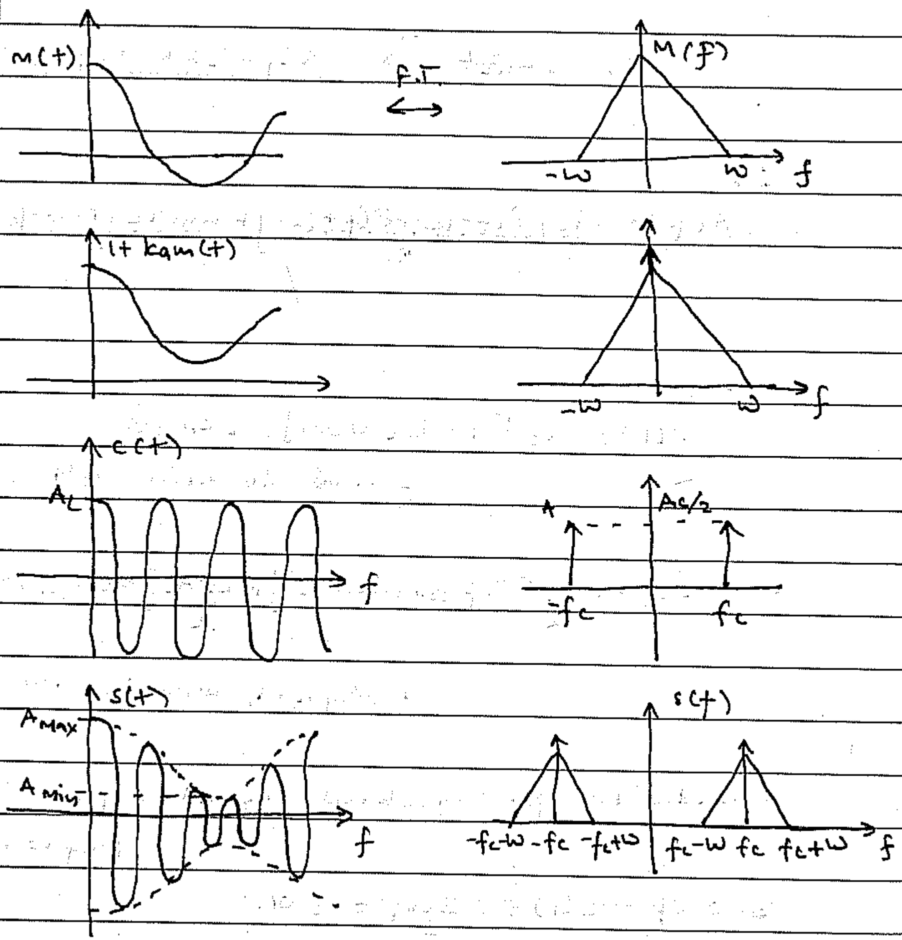
→ frequency domain AM signal.

Bandwidth for baseband signal = f_m = highest frequency content

B.W of $s(t) = 2f_m = 2W$.

We see that $s(t)$ contains a term $A_c[1 + k_a m(t)]$, which is the envelope of the modulated signal.

Now, for graphical representation,



Now, for single tone modulation,

$m(t) = A_m \cos 2\pi f_m t$

therefore,

$$s(t) = A_c[1 + k_a \cdot A_m \cos 2\pi f_m t] \cdot \cos 2\pi f_c t$$

$$= A_c[1 + \mu \cos 2\pi f_m t] \cos 2\pi f_c t$$

where, $\mu = k_a \cdot A_m$ = modulation index

Here,

$A_c[1 + \mu \cos 2\pi f_m t]$ represents the envelope of the AM signal $s(t)$.

such that $A_{max} = A_c(1+\mu)$ &

$A_{min} = A_c(1-\mu)$

$$\therefore \mu = \frac{A_{max} - A_{min}}{A_{max} + A_{min}}$$

Also, $A_{max} = A_c + A_m$, $A_{min} = A_c - A_m$

$$\therefore \frac{A_{max}}{A_{min}} = \frac{A_c + A_m}{A_c - A_m}$$

or $A_{max} \cdot A_c - A_m \cdot A_{max} = A_c \cdot A_{min} + A_m \cdot A_{min}$

or $A_c \cdot A_{max} - A_c \cdot A_{min} = A_m \cdot A_{max} + A_m \cdot A_{min}$

or $A_c[A_{max} - A_{min}] = A_m[A_{max} + A_{min}]$

or $\dfrac{A_{max} - A_{min}}{A_{max} + A_{min}} = \dfrac{A_m}{A_c} = \mu.$

Now,

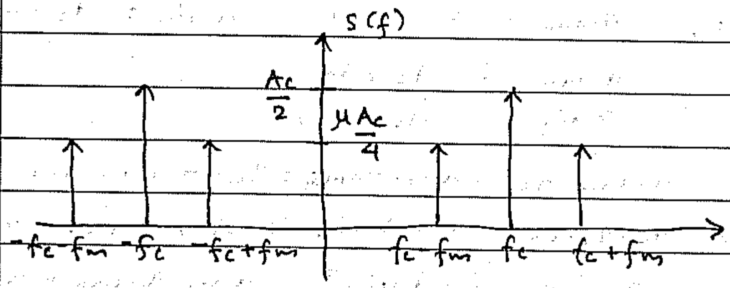
$$s(t) = A_c[1 + \mu \cos 2\pi f_m t] \cos 2\pi f_c t$$

$$= A_c \cos 2\pi f_c t + \mu \cdot A_c \cos 2\pi f_m t \cdot \cos 2\pi f_c t$$

$$= A_c \cos 2\pi f_c t + \frac{\mu A_c}{2}\left[\cos[2\pi(f_c+f_m)t] + \cos[2\pi(f_c-f_m)t]\right]$$

$$= A_c \cos 2\pi f_c t + \frac{\mu A_c}{2} \cos[2\pi(f_c+f_m)t] + \frac{\mu A_c}{2}\cos[2\pi(f_c-f_m)t]$$

$$\therefore S(f) = \frac{A_c}{2}[\delta(f-f_c) + \delta(f+f_c)] + \frac{\mu A_c}{4}[\delta(f-f_c-f_m) + \delta(f+f_c+f_m)] + \frac{\mu A_c}{4}[\delta(f-f_c+f_m) + \delta(f+f_c-f_m)]$$



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Now,

$$S(f) = \frac{A_c}{2}[\delta(f-f_c) + \delta(f+f_c)] \rightarrow \text{carrier component}$$

$$+ \frac{\mu A_c}{4}[\delta(f-f_c-f_m) + \delta(f+f_c+f_m)] \rightarrow \text{Upper side band component}$$

$$+ \frac{\mu A_c}{4}[\delta(f-f_c+f_m) + \delta(f+f_c-f_m)] \rightarrow \text{lower side band component}$$

As for the power of AM signal,

We have,

$$P = (V_{rms})^2/R = \frac{V^2}{2R}$$

For $R = 1\Omega$,

Power of carrier, $P_c = \frac{A_c^2}{2}$

Power of USB, $P_{USB} = \left(\frac{\mu A_c}{2}\right)^2/2 = \frac{\mu^2 A_c^2}{8}$

Power of LSB, $P_{LSB} = \frac{\mu^2 A_c^2}{8}$

$\therefore$ Total power,

$$P_t = P_c + P_{LSB} + P_{USB}$$

$$= \frac{A_c^2}{2} + \frac{\mu^2 A_c^2}{8} + \frac{\mu^2 A_c^2}{8} = \frac{A_c^2}{2}\left[1 + \frac{\mu^2}{2}\right]$$

Efficiency of DSB-AM.

Efficiency can be defined as the ratio of useful power at the output to total power consumed.

In DSB-AM the useful power is the power contained in sidebands only

i.e.

$$\eta = \frac{P_{LSB} + P_{USB}}{P_t} = \frac{\mu^2 A_c^2/8 + \mu^2 A_c^2/8}{\frac{A_c^2}{2}\left[1 + \frac{\mu^2}{2}\right]}$$

$$= \frac{\mu^2 A_c^2/4}{A_c^2/2\left[1 + \frac{\mu^2}{2}\right]}$$

$$\therefore \eta = \frac{\mu^2}{2 + \mu^2}$$

Now,

for $\mu = 1$ [modulation index = 1]

$$\eta = \frac{1}{3} = 0.33$$

& $\eta_{\%} = 33\% =$ Maximum efficiency.

So, we have modulation index,

$$\mu = \frac{A_{max} - A_{min}}{A_{max} + A_{min}} = \frac{A_m}{A_c} = \frac{\text{amplitude of } m(t)}{\text{amplitude of } c(t)}$$

and percent modulation is $\mu \times 100\% = \frac{A_m}{A_c} \times 100\%$

Also, the envelope of $s(t)$ has the same shape as the message $m(t)$ provided that two conditions are satisfied

i) $|k_a \cdot m(t)| < 1$ or

or $|k_a \cdot A_m| < 1$

i.e. $\mu < 1$

ii) The carrier frequency f_c should be much greater than the highest frequency component f_m of the message signal $m(t)$

i.e. $f_c >> f_m$

As long as the modulation index is less than or equal to 1, the envelope has the same shape as the message signal. Such case is known as Linear modulation.

Now if $\mu > 1$, then there is some distortion in envelope and such case is known as over modulation.

So, depending upon the value of μ, there are two types of modulation,

i) Linear Modulation

a) Under modulation

$\mu < 1$

b) 100% modulation

$\mu = 1$

ii) over Modulation

$\mu > 1$.

→ envelope distortion evident

→ phase reversal in envelope can be seen.

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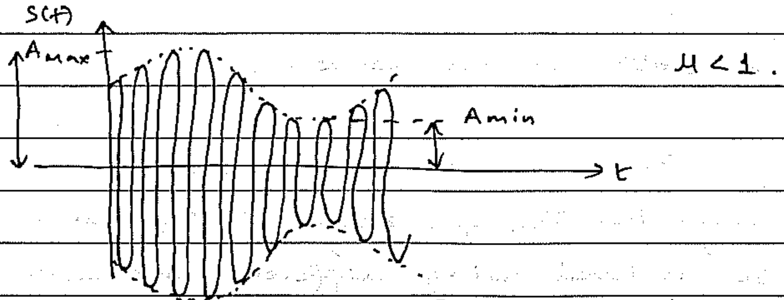


fig. under modulation

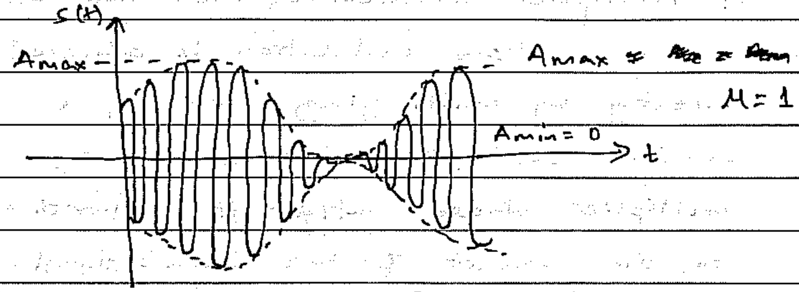


fig. 100% modulation

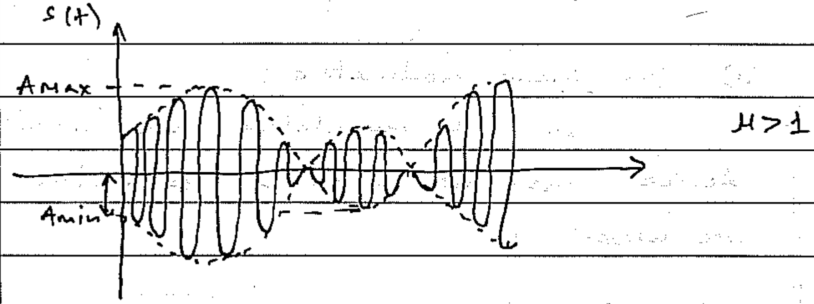


fig. over modulation

Generation of AM waves.

Modulation can be achieved in several ways. i.e. the generation of AM waves can be achieved using different modulators.

Some commonly used modulators are,

i) Multiplier modulators (Linear modulators)

Here modulation is achieved directly by multiplying $m(t)$ by a carrier $\cos 2\pi f_c t$ using an analog multiplier whose output is proportional to the product of two inputs signal. But such modulators are expensive and the linearity is difficult to maintain.

ii) Non linear modulators:

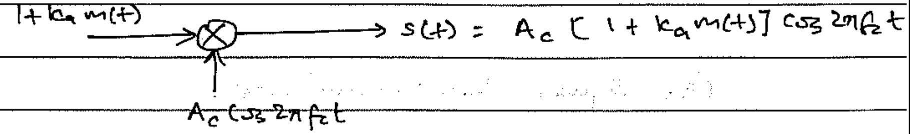
In such modulators, non-linear devices such as diode or transistors are used.

Now, using these modulators, DSB-FC wave can be generated in two ways,

A. Generation of DSB-FC AM waves.

i) direct method:

In this method, the level shifted modulated signal i.e. $[1 + k_a m(t)]$ is multiplied by a carrier signal.



or.

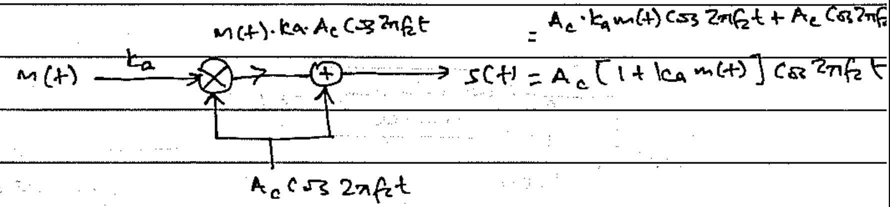


Fig. Direct method for AM generation

Direct method of DSB-FC generation is the simplest method but are not suitable for high frequency applications.

And since it used ~~produ~~ multiplier modulators, the linearity is difficult to maintain.

ii) Indirect method :

This method requires the use of non-linear devices. Basically we can use two such circuits to generate DSB-FC waves.

(a) Square law modulator :

A square law modulator can be shown as,

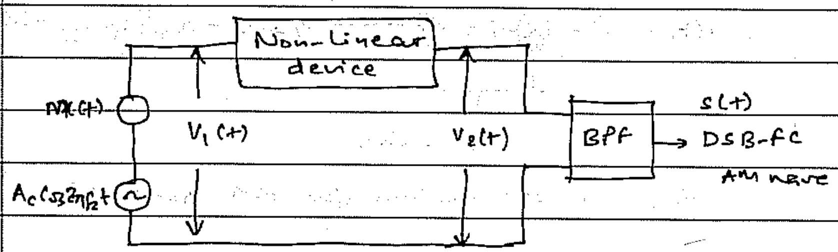


Fig. Square Law modulator

A square law modulator consists of,

1. modulating and carrier signal
2. non-linear device
3. band-pass filter.

Now, when the output of a device is not directly proportional to input throughout the operation, such device is known as non-linear device. The input output relation of a non-linear device can be expressed as,

$$V_o = a_0 + a_1 V_{in} + a_2 V_{in}^2 + a_3 V_{in}^3 + a_4 V_{in}^4 + \cdots$$

And

when the input is very small, the higher power can be neglected, and considered only upto the square of the input and hence the name square law modulator. Neglecting the dc components a_0 as its frequency = 0, we have for square law modulator,

$$V_o = a_1 V_{in} + a_2 V_{in}^2$$

Now, from figure,

$$V_{in} = V_1(t) = A_c \cos 2\pi f_c t + m(t)$$

and

$$V_2(t) = V_o = a_1 \left[A_c \cos 2\pi f_c t + m(t) \right] + a_2 \left[A_c \cos 2\pi f_c t + m(t) \right]^2$$

or $v_2(t) = a_1[A_c \cos 2\pi f_c t + m(t)] + a_2[A_c \cos 2\pi f_c t + m(t)]^2$

or $v_2(t) = a_1 A_c \cos 2\pi f_c t + a_1 m(t) + a_2[A_c^2 \cos^2 2\pi f_c t + 2\,m(t) \cos 2\pi f_c t \cdot A_c + m^2(t)]$

or $v_2(t) = a_1 A_c \cos 2\pi f_c t + a_1 m(t) + a_2 A_c^2 \cos^2 2\pi f_c t + a_2 m^2(t) + 2a_2 m(t) \cdot A_c \cos 2\pi f_c t$

$= a_1 A_c \cos 2\pi f_c t + 2a_2 m(t) A_c \cos 2\pi f_c t + a_1 m(t) + a_2 m^2(t) + a_2 A_c^2 \cos^2 2\pi f_c t$

$= a_1 A_c \left[1 + \frac{2a_2}{a_1} m(t)\right] \cos 2\pi f_c t + a_1 m(t) + a_2 m^2(t) + a_2 A_c^2 \cos^2 2\pi f_c t$

Now, when $v_2(t)$ is passed through ~~a~~ the bandpass filter centred around f_c, we get

$$s(t) = a_1 A_c \left[1 + \frac{2a_2}{a_1} m(t)\right] \cos 2\pi f_c t$$

Now we have general AM eqn as,

$$s(t) = A_c [1 + k_a m(t)] \cos 2\pi f_c t$$

$\therefore k_a = \frac{2a_2}{a_1}$ = modulator sensitivity

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Also, if $m(t) = A_m \cos 2\pi f_m t$, then,

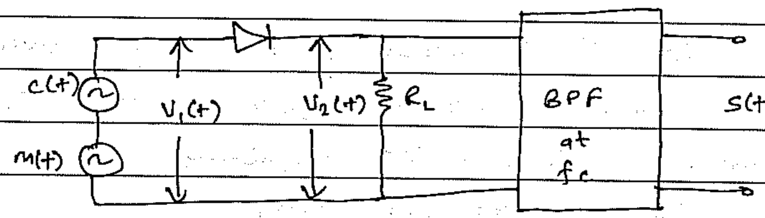
$$s(t) = a_1 A_c \left[1 + \frac{2a_2}{a_1} \cdot A_m \cos 2\pi f_m t\right] \cos 2\pi f_c t$$

or $\mu = \frac{2a_2}{a_1} \cdot A_m$.

The AM signal thus generated is free from distortion and attenuation only when $f_c - f_m > 2f_m$ or $f_c > 3f_m$.

(b) Switching modulator:

The functional diagram of switching modulator is given below,



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Following assumptions are made while analyzing the modulator,

→ The diode is assumed to be working as a switch. i.e. it acts as a closed switch when forward biased i.e. $v_1(t) > 0$, and as an open switch when $v_1(t) < 0$.

→ $|m(t)| << |c(t)|$ such that $m(t)$ alone cannot forward bias the diode.

→ The +ve half cycle of $c(t)$ can forward bias the diode.

Now, we can write,

$$v_2(t) = v_1(t) = m(t) + c(t) \quad \text{when } c(t) > 0$$
$$= 0 \quad \text{when } c(t) < 0$$

i.e. $v_2(t)$ varies periodically between the values $v_1(t)$ and zero at the rate equal to the carrier frequency.

Thus we can write,

$$v_2(t) = v_1(t) \cdot g_p(t)$$

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where,

$$g_p(t) = 1 \quad \text{when } c(t) > 0$$
$$0 \quad \text{when } c(t) < 0$$

Now, as $g_p(t)$ is periodic it can be expressed in Fourier series as,

$$g_p(t) = \frac{1}{2} + \frac{2}{\pi}\cos(2\pi f_c t) + \underbrace{\text{odd harmonic components}}_{gh_o(t)}$$

Thus, the output of diode can be written as,

$$v_2(t) = v_1(t) \cdot g_p(t)$$
$$= [A_c \cos 2\pi f_c t + m(t)]\left[\frac{1}{2} + \frac{2}{\pi}\cos(2\pi f_c t) + gh_o(t)\right]$$
$$= \frac{1}{2}A_c \cos 2\pi f_c t + \frac{1}{2}m(t) + \frac{2}{\pi}A_c \cos^2 2\pi f_c t$$
$$+ \frac{2m(t)}{\pi}\cos 2\pi f_c t + gh_o(t) \cdot m(t)$$
$$+ gh_o(t) \cdot A_c \cos 2\pi f_c t$$

or $v_2(t) = \frac{A_c}{2}\cos 2\pi f_c t + \frac{1}{2}m(t) + \frac{2A_c}{\pi}\cos^2 2\pi f_c t$

$+ \frac{2m(t)}{\pi}\cos 2\pi f_c t + g_{ho}(t)[m(t) + A_c\cos 2\pi f_c t]$

When, $v_2(t)$ is passed through a BPF with centre frequency f_c, we get the desired AM wave as,

$$s(t) = \frac{A_c}{2}\cos 2\pi f_c t + \frac{2}{\pi}m(t)\cos 2\pi f_c t$$

$$= \frac{A_c}{2}\left[1 + \frac{4}{\pi \cdot A_c}m(t)\right]\cos 2\pi f_c t$$

comparing with

$s(t) = A_c[1 + k_a m(t)]\cos 2\pi f_c t$,

we get,

$k_a = \frac{4}{\pi A_c}$ is the modulator sensitivity of switching modulator.

(2) Double-sideband suppressed carrier (DSB-SC)

In DSB-FC we can see that the carrier wave is transmitted along with the sidebands that contains the message. Thus this addition carrier wave represents the waste of power when all we actually require is transmission of message.

To overcome this shortcoming we use a modulation scheme where the carrier is suppressed and thus not transmitted along with the message including sidebands. Thus DSB-SC increases the efficiency of the transmission.

The general equation for DSB-SC can be shown as,

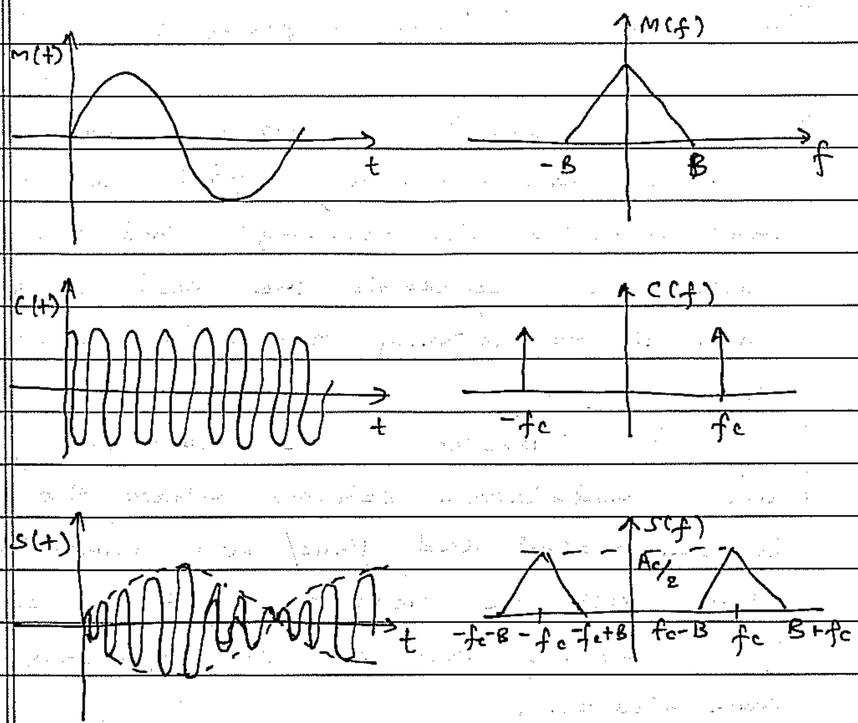
$s(t) = m(t) \cdot c(t)$

$= A_c \cdot m(t)\cos 2\pi f_c t$,

And its frequency domain representation is,

$$s(f) = \frac{A_c}{2}[M(f-f_c) + M(f+f_c)]$$

Graphically,



From the above figure,

i) Except for the scaling factor, the modulation process simply translates the spectrum of the message signal by $\pm f_c$. It is also clear that impulses at $\pm f_c$ are missing which indicates that the carrier term is suppressed with two sidebands present.

ii) Considering the positive time,

USB frequency $= f_c + B$

LSB frequency $= f_c - B$

$\therefore$ Tx bandwidth, $BW = f_c + B - f_c + B$

$= 2B$

i.e. BW = Double the bandwidth of message signal.

Also, considering −ve time,

USB ~~LSB~~ frequency $= -f_c - B$

LSB ~~USB~~ frequency $= -f_c + B$

$\therefore$ Tx $BW = -f_c + B + f_c + B$

$= 2B$

And the total power is DSB-SC,

$P_t = P_{LSB} + P_{USB}$

$= 2P_{LSB} = 2P_{USB}$

as $P_{LSB} = P_{USB}$.

⊕ Single tone DSB-SC modulation.

Here we have,

$m(t) = A_m \cos 2\pi f_m t$

then,

$s(t) = m(t) \cdot c(t)$

$= A_m \cos 2\pi f_m t \cdot A_c \cos 2\pi f_c t$

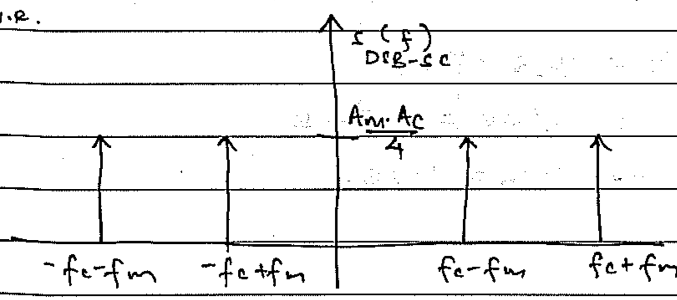
$= \frac{A_m \cdot A_c}{2} \cdot 2 \cos 2\pi f_m t \cdot \cos 2\pi f_c t$

$\therefore \; s(t) = \frac{A_m \cdot A_c}{2} \left[\cos 2\pi (f_c + f_m)t + \cos 2\pi (f_c - f_m)t \right]$

And

$$S(f) = \frac{A_m \cdot A_c}{4} \left[\delta(f + f_c + f_m) + \delta(f - f_c - f_m) + \delta(f - f_c + f_m) + \delta(f + f_c - f_m) \right]$$

i.e.



Freq. transform of DSB-SC

$\therefore$ BW $= 2f_m$

Power content of DSB-SC, single tone,

We have,

$P_t = P_{LSB} + P_{USB}$

$= \left(\frac{A_m \cdot A_c}{2}\right)^2 / 2 + \left(\frac{A_m \cdot A_c}{2}\right)^2 / 2$

$= \frac{A_m^2 \cdot A_c^2}{8} + \frac{A_m^2 \cdot A_c^2}{8}$

$= \frac{2 \cdot A_m^2 \cdot A_c^2}{8}$

$= \frac{A_m^2 \cdot A_c^2}{4}$

$= A_c^2 \cdot \frac{A_m^2}{A_c^2} \cdot \frac{1}{4} \cdot A_c^2 = \frac{A_c^4 \cdot \mu^2}{4}$

$\left[\because \frac{A_m}{A_c} = \text{modulation index}\right]$

$\therefore \; P_t = \frac{\mu^2 \cdot A_c^4}{4}$

$= \frac{\mu^2}{2} \cdot \left(\frac{A_c^2}{2}\right) = \frac{\mu^2}{2} \cdot P_c^2$ where, P_c = carrier power.

Generation of DSB-SC AM wave.

i) Balanced Modulator.

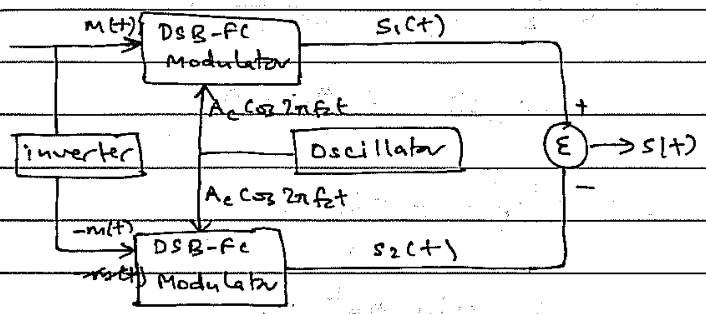


Fig. Balanced modulator.

The figure above consists of two DSB-FC modulators arranged in balanced configuration such that they suppress the carrier completely.

The message signal applied to the two modulators are 180° phase shifted version of one-another.

The oscillator produces the required carrier signal.

The outputs for two DSB-FC modulators can thus be written as

$$s_1(t) = A_c\,[1 + k_a m(t)] \cos 2\pi f_c t \quad \&$$
$$s_2(t) = A_c\,[1 - k_a m(t)] \cos 2\pi f_c t$$

Finally, the balanced modulator output,

$$s(t) = s_1(t) - s_2(t)$$
$$= A_c\,[1 + k_a m(t)] \cos 2\pi f_c t - A_c\,[1 - k_a m(t)] \cos 2\pi f_c t$$
$$= A_c \cos 2\pi f_c t - A_c \cos 2\pi f_c t + A_c \cdot k_a m(t) \cos 2\pi f_c t + A_c k_a m(t) \cos 2\pi f_c t$$
$$s(t) = 2\,k_a \cdot m(t) \cdot A_c \cos 2\pi f_c t$$

Comparing with standard DSB-SC signal,

$$s(t) = A_c \cdot m(t) \cos 2\pi f_c t,$$

we get, the standard DSB-SC signal from balanced modulator with a scaling factor $2k_a$.

ii) Ring Modulator :

A ring modulator is shown below with four diodes arranged in a ring form.

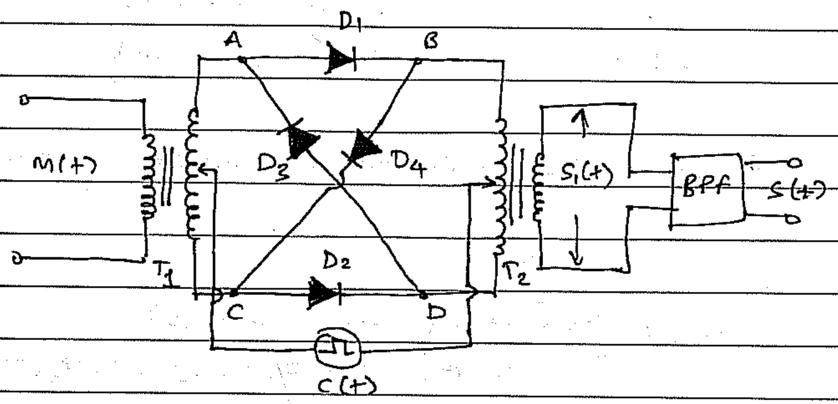


Fig. Ring modulata (Using diodes).

In the figure above, the operation of diodes are controlled by a square wave carrier with frequency 'fc' applied by means of two centre tapped transformers T_1 & T_2.

Following assumptions are made while analysing the operation of the ring modulator.

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→ diodes are ideal

→ transformers are perfectly balanced and centre tapped

→ $|m(t)| << |c(t)|$ and $m(t)$ alone cannot forward bias the diodes.

Now, when $c(t)$ is +ve, D_1 and D_2 are ON and D_3 & D_4 are OFF.

In such condition,

$$s_1(t) = m(t)$$

And, when $c(t)$ is -ve, D_3 & D_4 are ON and D_1 & D_2 are OFF,

such that,

$$s_1(t) = -m(t).$$

Therefore,

$$s_1(t) = \begin{cases} m(t) & \text{when } c(t) > 0 \\ -m(t) & \text{when } c(t) < 0 \end{cases}$$

Thus we can write,

$$s_1(t) = m(t) \cdot g_p(t)$$

where,

$$g_p(t) = \begin{cases} 1 & \text{for } c(t) > 0 \\ -1 & \text{for } c(t) < 0 \end{cases}$$

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Now $g_p(t)$ represents the periodic square wave carrier signal and can be expressed in fourier series as,

$$g_p(t) = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \left[\cos 2\pi f_c t (2n-1) \right]$$

$$= \frac{4}{\pi} \cos 2\pi f_c t + \text{higher harmonics}$$

Then,

$$s_1(t) = m(t) \cdot g_p(t)$$

$$= m(t) \left[\frac{4}{\pi} \cos 2\pi f_c t + \text{higher order harmonics} \right]$$

$$\therefore s_1(t) = m(t) \cdot \frac{4}{\pi} \cos 2\pi f_c t + m(t) \cdot \text{higher order harmonics.}$$

When $s_1(t)$ is passed through a BPF centred at f_c, then,

$s(t) = \frac{4 \cdot m(t)}{\pi} \cos 2\pi f_c t$ is the required DSB-SC AM wave

where, $A_c = \frac{4}{\pi}$.

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(3) Single Sideband Modulation (SSB)

We have so far seen that for DSB-FC and DSB-SC modulation, the transmission bandwidth required is twice the message bandwidth. Also, the upper side bands and lower side bands have symmetric property and contain the message in them.

So, it can be seen that even if we transmit either of the side bands, the information can be efficiently transmitted, and thus the concept of single side band suppressed carrier arises.

In SSB-SC, the transmission of one sideband and suppression of carrier and other sideband at the transmitter is performed. This results in better power efficiency and reduced transmission bandwidth.

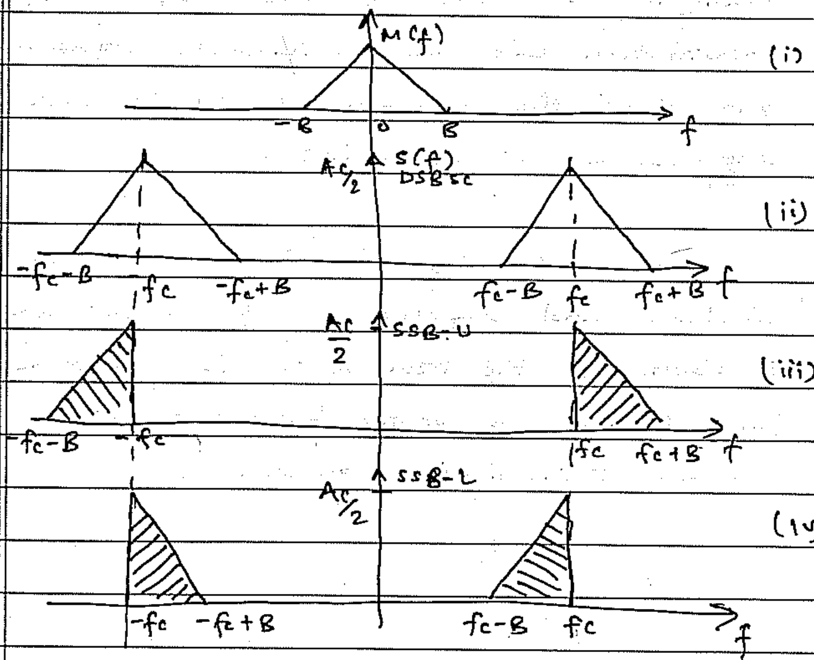
Frequency domain analysis for SSB.

Let us consider a message signal $m(t)$ having spectrum $M(f)$ band limited to $-B \le f \le B$.

Also, the spectrum for DSB-SC is given as,

$$S_{DSB-SC}(f) = \cancel{\frac{A_m A_c}{2}} \; \frac{A_c}{2}\left[M(f-f_c) + M(f+f_c)\right]$$

i.e.



We can see that figure (ii) being the spectrum of DSB-SC, contains both LSBs and USBs. Thus, if we could remove, either of the sideband pairs, we result in SSB. i.e. in figure (iii), we get SSB with USB only whereas in figure (iv) we get SSB with LSB only.

And the resulting transmission bandwidth for SSB is,

$$\left.\begin{aligned} f_c + B - f_c &= B \\ \text{or} \quad -f_c - (f_c - B) &= B \\ \text{or} \quad f_c - (f_c - B) &= B \\ \text{or} \quad -f_c + B - (-f_c) &= B \end{aligned}\right\} \text{Tx BW} = B = \text{bandwidth of msg.}$$

Now the time domain representation of SSB can be shown as,

$$S_{SSB}(t) = \frac{A_c}{2} m(t) \cos 2\pi f_c t \pm \frac{A_c}{2} \hat{m}(t) \sin 2\pi f_c t$$

or $S_{USB}(t) = A_c/_2 \, m(t) \cos 2\pi f_c t - A_c/_2 \, \hat{m}(t) \sin 2\pi f_c t$

& $S_{LSB}(t) = A_c/_2 \, m(t) \cos 2\pi f_c t + A_c/_2 \, \hat{m}(t) \sin 2\pi f_c t$

where,

$\hat{m}(t)$ = Hilbert transform of $m(t)$

Single tone SSB modulation.

let $m(t) = A_m \cos 2\pi f_m t$ &

$c(t) = A_c \cos 2\pi f_c t$

then $\hat{m}(t)$ = Hilbert transform of $m(t)$

$= A_m \cos[2\pi f_m t - 90°]$

or

$\hat{m}(t) = A_m[\cos 2\pi f_m t \cdot \cos(-90°) + \sin 2\pi f_m t \cdot \sin(-90°)]$

$\therefore \hat{m}(t) = A_m \sin 2\pi f_m t$

Now,

$$s_{USB}(t) = \frac{A_c}{2}[m(t)\cos 2\pi f_c t - \hat{m}(t)\sin 2\pi f_c t]$$

$$= \frac{A_c}{2}[A_m \cos 2\pi f_m t \cdot \cos 2\pi f_c t - A_m \sin 2\pi f_m t \cdot \sin 2\pi f_c t]$$

$$= \frac{A_c \cdot A_m}{2}[\cos 2\pi f_m t \cdot \cos 2\pi f_c t - \sin 2\pi f_m t \cdot \sin 2\pi f_c t]$$

$$s_{USB}(t) = \frac{A_c \cdot A_m}{2}[\cos 2\pi(f_c + f_m)t]$$

similarly,

$$s_{LSB}(t) = \frac{A_c \cdot A_m}{2}[\cos 2\pi(f_c - f_m)t]$$

Generation of SSB

A. Frequency discrimination method (Filtering Method)

In filtering method, the message signal m(t) and carrier signal c(t) are applied to product modulator to generate DSB-SC wave which is then passed through a bandpass filter such that only one of the sidebands is filtered out.

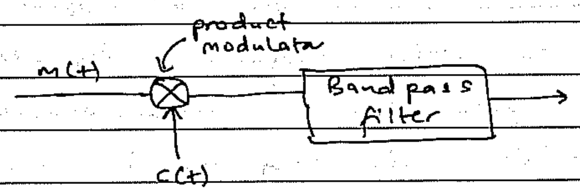
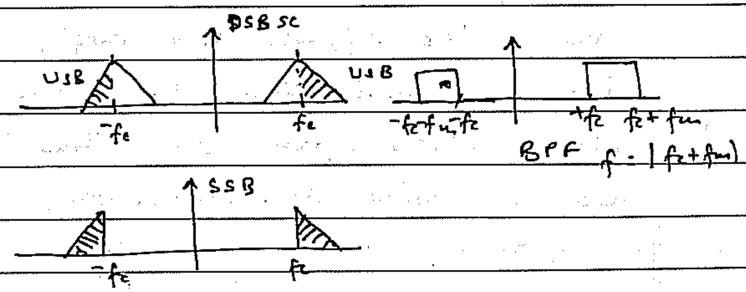


Fig. SSB modulator.

i.e.



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It is quite evident that to implement this method following conditions should be satisfied.

i) The USB and LSB are non-overlapping and are separated by significant frequency gap. This is because the bandpass filters needs to have very sharp change over from attenuation to pass band and vice-versa.

ii) The baseband signal must be appropriately related to carrier frequency. The design of band pass filter becomes quited difficult if carrier frequency is quite higher than bandwidth of baseband signal.

Thus, SSB signals are rarely used in video communication where the baseband signal starts from DC. Thus it is more used in voice communication.

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B. Phase shift method (Hartley method)

A phase shift method makes use of two product modulators and two 90° phase shifting networks to supress either of the sidebands. The block diagram can be shown as,

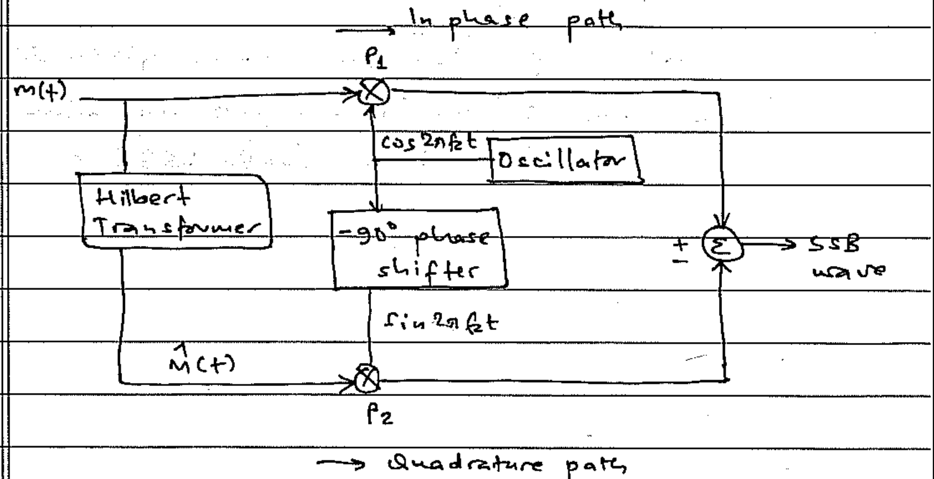

fig. & Hartley method

In the figure above, the inphase path consists of product modulator P_1 such that its output is $m(t) \cdot \cos 2\pi f_c t$ whereas the quadrature path consists of a Hilbert

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transformer and product modulator P_2 such that its output is $\hat{m}(t) \sin 2\pi f_c t$.

Now, the output of both of the modulators are fed to an adder such that the overall output is given as,

$$s(t) = m(t) \cos 2\pi f_c t \pm \hat{m}(t) \sin 2\pi f_c t$$

where if negative ~~-ve~~ sign used gives SSB-USB and if positive ~~+ve~~ sign used in adder, gives SSB-LSB.

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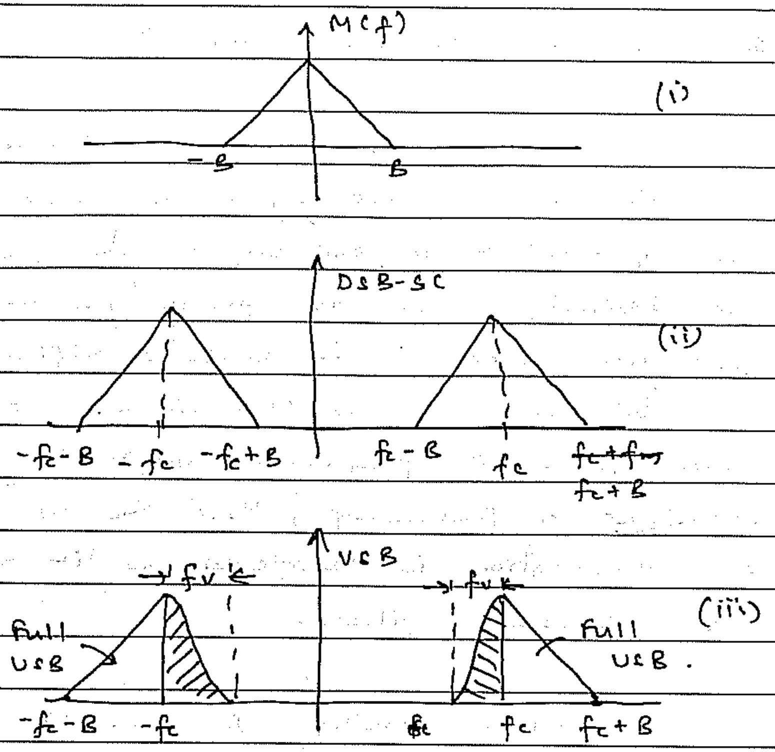
Vestigial side band Modulation,

SSB modulation is well suited for the transmission of voice signal as most of the information is carried in the frequency of 3400 Hz. i.e. the energy gap between zero and 3400 Hz is quite significant.

But in video ~~commu~~ signals, the signals range from low frequency approaching DC to higher ~~fr~~ frequency, thus the use of SSB modulation is restricted as it requires sharp selective filters.

So, another method of modulation called vestigial sideband modulation (VSB) is preferred. In VSB modulation, one side band is completely passed ~~and~~ and just a trace of another sideband, known as vestige, is transmitted.

VSB modulation has become a standard for transmission of television and similar signals where good phase characteristics and transmission of low frequency components are important.



From figure (iii) we can see that the transmission bandwidth now is,

$BW = (B + f_v)\ Hz$

or for singletone message signal with frequency f_m,

$BW = (f_m + f_v)\ Hz$.

where, f_v = width of vestigial sidebands.

So, the bandwidth ~~of~~ required for the transmission of ~~USB~~ VSB is greater than SSB which is typically about 25% greater than SSB.

(iii) Generation of VSB ~~modulated~~ wave,

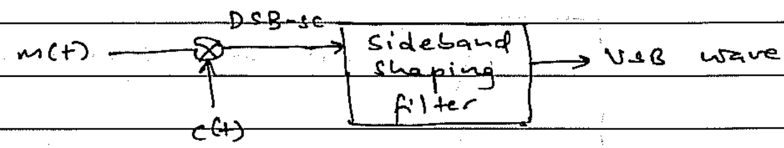


Fig. VSB modulator.

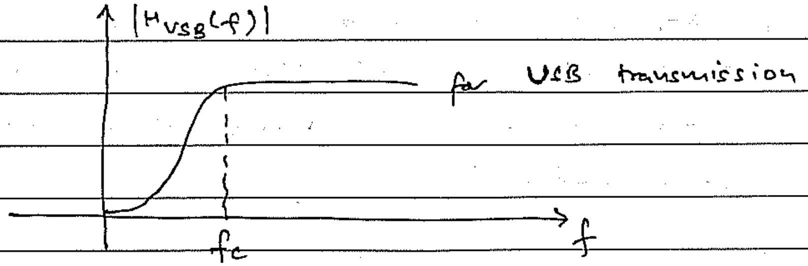
$$s_{VSB}(t) = A_c\, m(t) \cos 2\pi f_c t \circledast h_{VSB}(t)$$

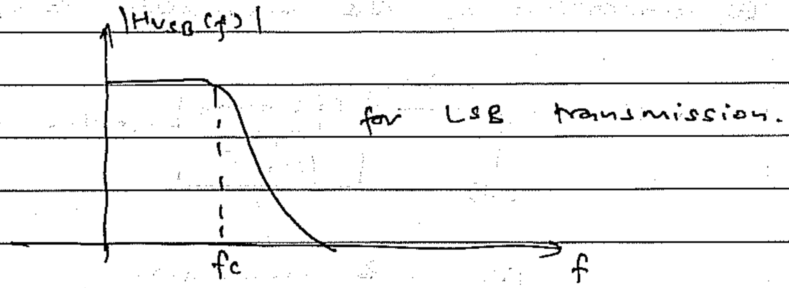
where, $h_{VSB}(t)$ is the impulse response of VSB filter.

And,

$$S_{VSB}(f) = \cancel{A_c\, m(t) \cos 2\pi f_c t}$$

$$= \frac{A_c}{2}\left[M(f-f_c) + M(f+f_c)\right] \cdot H_{VSB}(f).$$





The generation of USB modulated wave thus can be achieved by producing a DSB-SC wave first by using product modulator to modulate message signal m(t) and carrier ~~wt~~ c(t).

This DSB-SC wave is then passed through a sideband shaping filter. This filter generates one of the sidebands and a trace or vestige of other sideband.

Independent sideband modulation (ISB)

We have seen for SSB that the carrier and one of the sidebands are removed from the modulated signal.

Thus it is possible to replace the removed sideband with another sideband of information created by modulating a different input signal on the same carrier giving independent sideband or ISB.

In such ~~a~~ case both input signals have frequencies in the same audio spectrum range but in the transmitted signal each signal occupies a different group of frequencies.

Thus ISB can be taken as a DSB-SC system that transmits two message signal simultaneously.

Generation of ISB.

Two independent message signals are fed to two product modulators separately which is then modulated with carriers to generate two DSB-SC waves.

These DSB-SC waves are filtered out using a LSB suited filter and USB suited filter respectively, resulting in two independent lower side band and upper side band.

Finally, the output of the ~~so~~ SSB filters are summed to give out ISB wave. The block diagram can be shown as,

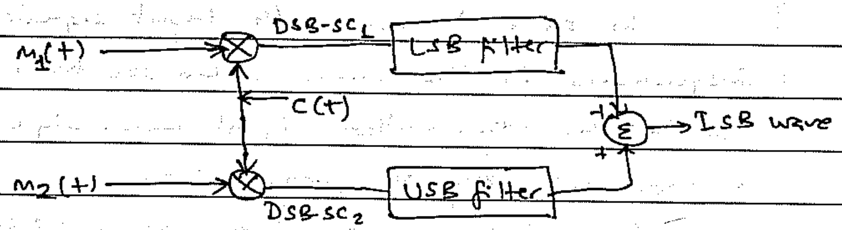
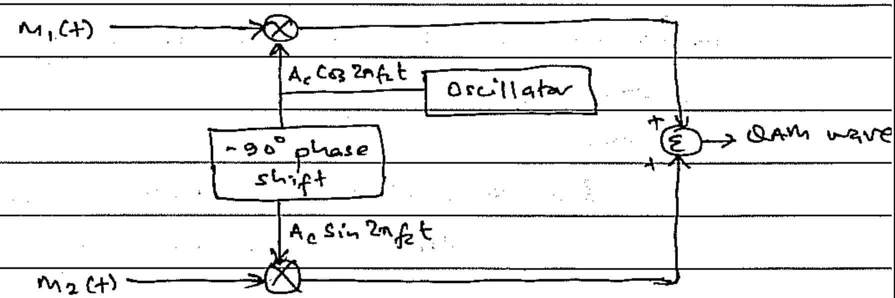


Fig. ISB wave modulator.

Quadrature amplitude modulation (QAM).

A quadrature amplitude modulation (QAM) enables two DSB-SC modulated waves resulting from the application of two independent message signals, to occupy the same transmission bandwidth and allows for the separation of two message signals at the receiver output. It is therefore a bandwidth conservation scheme.

The generation of QAM wave can be achieved with following block diagram,



The block diagram above consists of two DSB-SC modulators which are supplied with two different message signals and two carrier waves of same frequency but quadrature in phase with each other.

Therefore the output of two product modulators are $A_c m_1(t) \cos 2\pi f_c t$ and $A_c m_2(t) \sin 2\pi f_c t$ respectively.

Finally, these two DSB-SC signals are added to generate QAM signals such that,

$$s(t) = \underbrace{A_c m_1(t) \cos 2\pi f_c t}_{\text{inphase}} + \underbrace{A_c m_2(t) \sin 2\pi f_c t}_{\text{Quadrature}}$$

components.

This QAM signal $s(t)$ occupies transmission bandwidth of $2B$ centred at carrier frequency f_c.

where,

B = bandwidth of $m_1(t)$ or $m_2(t)$ whichever is the Largest.

5. Demodulation of AM signals.

5.1 Demodulation of DSB-FC, DSB-SC and SSB using synchronous detection.

Synchronous detection or coherent detection is a process where the local carrier generated at the receiver is phase locked with the carrier at the transmitter. ie. the phase and frequency of the carrier at the transmitter is exactly synchronized (same) to the phase and frequency of the locally generated carrier.

Basic block diagram for the demodulation of AM signals implementing synchronous detection can be shown as,

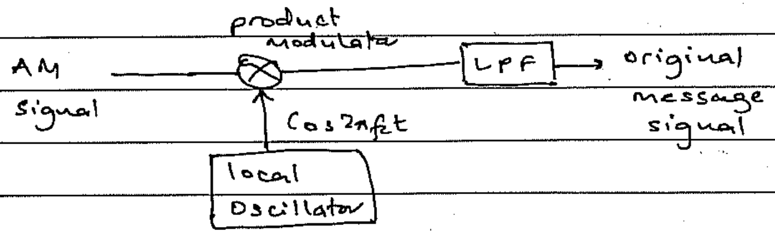
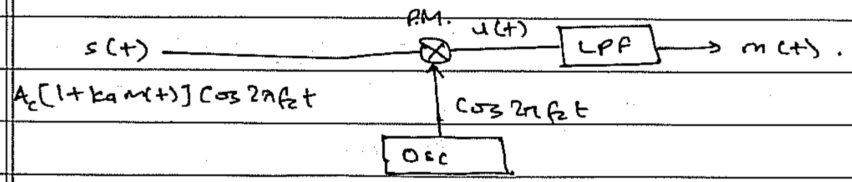


Fig. Block diagram of synchronous demodulation

Synchronous detection of DSB-FC



from the figure above,

$$u(t) = s(t)\cdot c(t)$$

$$= A_c[1+k_a m(t)]\cos 2\pi f_c t \cdot \cos 2\pi f_c t$$

$$= A_c[1+k_a m(t)]\cos^2 2\pi f_c t$$

$$= A_c[1+k_a m(t)]\cdot\left[\frac{1+\cos 4\pi f_c t}{2}\right]$$

$$\therefore u(t) = \frac{A_c}{2}[1+k_a m(t)] + \frac{A_c}{2}[1+k_a m(t)]\cos 4\pi f_c t$$

Now, when u(t) is passed through a LPF centred around the frequency of message signal we get LPF o/p as,

$$\frac{A_c}{2} + \frac{A_c}{2}k_a\cdot m(t)$$

Since $A_c/2$ is DC component it is omitted and the demodulator output can be recovered as $\frac{k_a\cdot A_c}{2}\, m(t)$.

where $\frac{k_a\cdot A_c}{2}$ = scaling factor.

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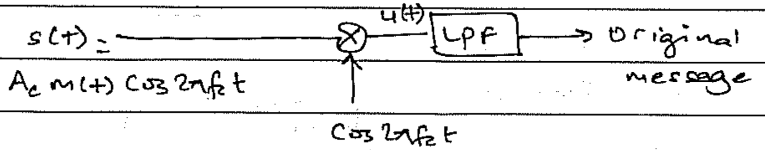
⊕ Synchronous demodulation of DSB-SC.



~~Aag~~ Again,

$u(t) = A_c \cdot m(t) \cos 2\pi f_c t \cdot \cos 2\pi f_c t$

$= A_c m(t) \cos^2 2\pi f_c t$

$= A_c m(t) \left[\frac{1 + \cos 4\pi f_c t}{2} \right]$

$\therefore u(t) = \frac{A_c m(t)}{2} + \underbrace{\frac{A_c m(t) \cos 4\pi f_c t}{2}}_{\to \text{ Filtered}}$

When, passed through LPF, we recover the original message as $\frac{A_c}{2} m(t)$

$\therefore$ We recover original message signal $m(t)$ with a scaling factor $\frac{A_c}{2}$.

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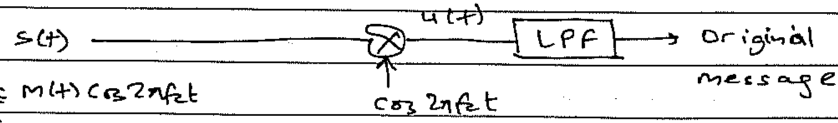
⊕ Synchronous detection of SSB signal.



$= \frac{A_c}{2} m(t) \cos 2\pi f_c t$

$\pm \frac{A_c}{2} \hat{m}(t) \sin 2\pi f_c t$

Again,

$u(t) = \left[\frac{A_c}{2} m(t) \cos 2\pi f_c t \pm \frac{A_c}{2} \hat{m}(t) \sin 2\pi f_c t \right] \cdot \cos 2\pi f_c t$

$= \frac{A_c}{2} m(t) \cos 2\pi f_c t \cdot \cos 2\pi f_c t$

$\pm \frac{A_c}{2} \hat{m}(t) \sin 2\pi f_c t \cdot \cos 2\pi f_c t$

$= \frac{A_c}{2} m(t) \cdot \frac{1}{2} \left[1 + \cos 4\pi f_c t \right]$

$\pm \frac{A_c}{2} \hat{m}(t) \cdot \frac{1}{2} \sin 4\pi f_c t$

$= \frac{A_c}{4} m(t) \left[1 + \cos 4\pi f_c t \right]$

$\pm \frac{A_c}{4} \hat{m}(t) \sin 4\pi f_c t$

$\therefore u(t) = \frac{A_c}{4} m(t) + \frac{A_c}{4} m(t) \cos 4\pi f_c t$

$\pm \frac{A_c}{4} \hat{m}(t) \sin 4\pi f_c t$

When $u(t)$ is passed through an LPF, the higher frequency terms gets filtered and only the term containing the message is passed.

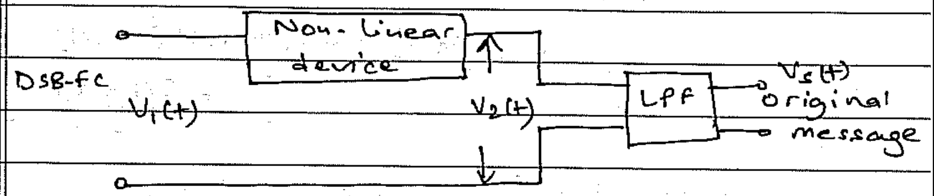
i.e. LPF o/p is $\frac{A_c}{4} m(t)$.

Thus we recover the message signal $m(t)$ with scaling factor $\frac{A_c}{4}$ wrt using synchronous detection.

5.2 Square law and envelop detection of DSB-FC.

Square law detector (Demodulator)

A block diagram of square law detector for demodulation of DSB-FC signal is shown below;



In the figure above, a DSB-FC signal is applied to a non-linear device which is a square law device such that its output consists the highest power of two.

Now, input voltage,

$$v_1(t) = A_c[1 + k_a m(t)] \cos 2\pi f_c t$$

where,

$m(t)$ = message signal

$c(t)$ = carrier signal = $A_c \cos 2\pi f_c t$

k_a = amplitude sensitivity of modulator

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Now, for square law non-linear device,

$$v_2(t) = a_1 v_1(t) + a_2 v_1^2(t)$$

$$= a_1 A_c [1 + k_a m(t)] \cos 2\pi f_c t + a_2 A_c^2 [1 + k_a m(t)]^2 \cos^2 2\pi f_c t$$

$$= a_1 A_c [1 + k_a m(t)] \cos 2\pi f_c t + a_2 A_c^2 [1 + 2 \cdot k_a m(t) + k_a^2 m^2(t)] \cos^2 2\pi f_c t$$

or

$$V_2(t) = a_1 A_c [1 + k_a m(t)] \cos 2\pi f_c t + a_2 A_c^2 [1 + 2 k_a m(t) + k_a^2 m^2(t)] \times \left[\frac{1 + \cos 4\pi f_c t}{2}\right]$$

or

$$v_2(t) = a_1 A_c [1 + k_a \cdot m(t)] \cos 2\pi f_c t + \frac{a_2 A_c^2}{2} [1 + 2 k_a m(t) + k_a^2 m^2(t)] + \frac{a_2 A_c^2}{2} [1 + 2 k_a m(t) + k_a^2 m^2(t)] \cos 4\pi f_c t$$

Now, when $V_2(t)$ is passed through LPF we get,

$$V_s(t) = \frac{a_2 A_c^2}{2} [1 + 2 k_a m(t) + k_a^2 m^2(t)]$$

$$\therefore V_s(t) = \frac{a_2 A_c^2}{2} + a_2 A_c^2 \cdot k_a m(t) + \frac{a_2 A_c^2 k_a^2 m^2(t)}{2}.$$

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In $V_s(t)$,

i) $\frac{a_2 A_c^2}{2}$ → is a dc term and only shifts the message signal.

ii) $a_2 A_c^2 k_a m(t)$ → contains the required message signal

iii) $\frac{a_2 A_c^2 k_a^2 m^2(t)}{2}$ → is the unwanted message signal also termed as noise.

So, the ratio of wanted signal to unwanted signal, also known as SNR is given as,

$$SNR = \frac{S}{N} = \frac{a_2 A_c^2 k_a m(t)}{\frac{a_2 A_c^2 k_a^2 m^2(t)}{2}}$$

$$= 2/k_a \cdot m(t)$$

Now for good reception of signal, SNR should be as large as possible. i.e. SNR should be maximized to minimize any distortion. To achieve that $|m(t)|$ should be compared ~~as~~ to unity (1) for all values of t. But then, AM signal will have to have weak message signal $m(t)$.

Envelope detector for DSB-FC.

An envelope detector is a simple and very efficient device which is suitable for the detection of a narrowband AM signal. A narrowband AM signal here, means the signal that has the carrier frequency 'fc' much higher than the highest bandwidth of the modulating signal. An envelope detector as the name applies, produces an output signal that follows the envelope of the input AM signal exactly. It is widely used in all of the commercial AM radio receivers.

The circuit diagram of envelope detector can be shown as,

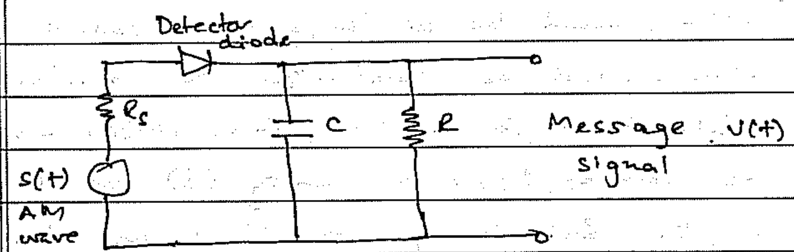

Fig.. Envelope detector

The circuit for envelope detection consists of a diode and an RC filter.

The standard AM wave (DSB-FC) is applied at the input of the detector. The diode ~~th~~ in the circuit acts as a rectifier, and is considered to be ideal i.e. it presents zero resistance during forward bias and infinite resistance during reverse bias. We also assume that the AM wave applied to the input ~~to~~ of the detector is supplied by a source having internal resistance R_s.

Now in the positive half cycle of the input signal, the diode is forward biased. Now, with the diode 'ON', the capacitor connected across load resistor R, charges to the peak value of the input signal. As soon as the capacitor charges to the peak value, the diode stops conducting, i.e. the diode is reverse biased. The capacitor will now discharge through R until the next positive half cycle. When, the input signal becomes greater than the capacitor voltage,

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the diode conducts again and the process repeats itself.

Here it should be noted that the capacitor charges through diode 'D' and R_s when the diode is on 'ON' and it discharges through R when the diode is 'OFF'.

The charging time constant R_sC should be short compared to carrier period $1/f_c$

i.e. $R_s \cdot C << 1/f_c$

But the discharging time RC should be long enough as the capacitor discharges slowly through Load R and should not be too long which will not allow capacitor voltage to discharge at the maximum rate of change of modulating wave, such that,

$$\frac{1}{f_c} << RC \leq \frac{1}{B}$$

where, B = message bandwidth

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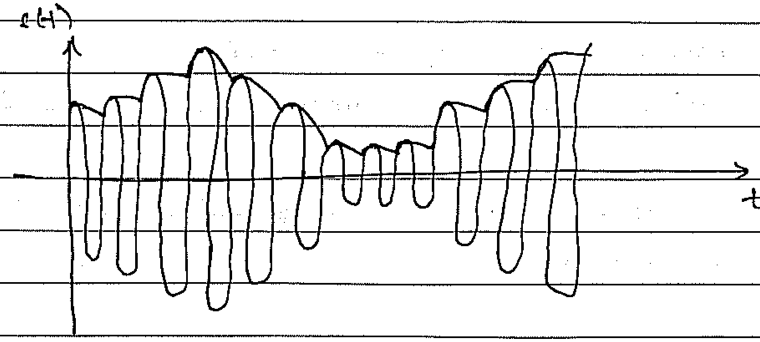

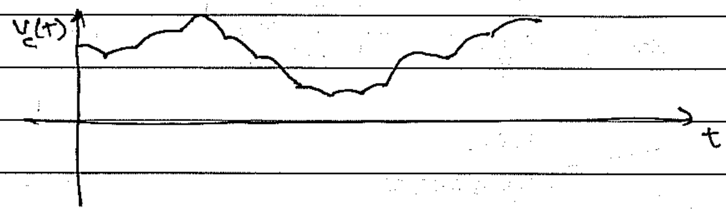

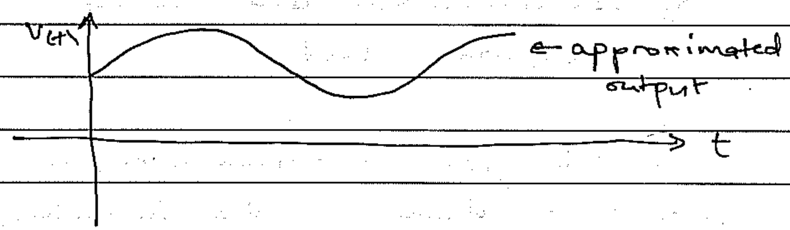

So, the envelope detector provides the waveform output similar to the original message signal.

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5.3 Demodulation of SSB using carrier reinsertion and carrier recovery circuits.

(A) Demodulation of SSB using carrier reinsertion.

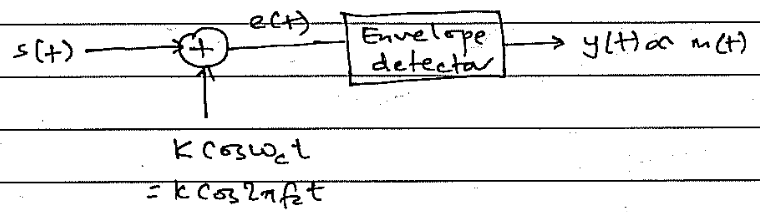

Fig. SSB demodulator using carrier insertion method.

In this type of demodulation, a carrier is added to the ~~incoming~~ received SSB signal and passed through an envelope detector.

Now from the above figure,

$$e(t) = s(t) + k\cos 2\pi f_c t$$

$$= \left[\frac{A_c}{2} m(t)\cos 2\pi f_c t \pm \frac{A_c}{2}\hat{m}(t)\sin 2\pi f_c t\right] + k\cos 2\pi f_c t$$

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or $e(t) = \left[\frac{A_c}{2} m(t) + k\right] \cos 2\pi f_c t$

$$\pm \frac{A_c}{2}\hat{m}(t)\sin 2\pi f_c t$$

where $\left[\frac{A_c}{2} m(t) + k\right]$ is inphase component

& $\frac{A_c}{2}\hat{m}(t)$ is quadrature component

Now, the output of envelope detector is the resultant of inphase and quadrature components such that

$$y(t) = \sqrt{\left[\frac{A_c}{2} m(t) + k\right]^2 + \left[\frac{1}{2}\frac{A_c}{2}\hat{m}(t)\right]^2}$$

Now, if $K >> 1$ then,

$$\left[\frac{A_c}{2} m(t) + k\right]^2 >> \left[\frac{A_c\hat{m}(t)}{2}\right]^2$$

Then,

$$y(t) = \sqrt{\left[\frac{A_c}{2} m(t) + k\right]^2 + \left[\frac{A_c}{2}\hat{m}(t)\right]^2}$$

$$= \sqrt{\left[\frac{A_c}{2} m(t) + k\right]^2}$$

$\therefore y(t) = \frac{A_c}{2} m(t) + K \Rightarrow y(t) \propto m(t)$

Hence m(t) is recovered.

Carrier recovery circuits.

We can see that using coherent detection requires the exact knowledge of incoming carrier signal at receiver such that an exact replica of the incoming carrier is generated at the local oscillator of the receiver.

i.e both frequency and phase of locally generated carrier are same to that of incoming carrier.

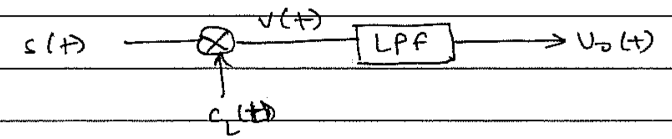
Now let us consider that the output of local oscillator drifts by 'ϕ' phase and 'Δf' frequency, then,

$$c_L(t) = \cos[2\pi(f_c+\Delta f)t + \phi]$$

where, Δf = difference in frequency

ϕ = difference in phase from incoming carrier.

Now, if we use this local carrier to demodulate DSB-SC then we get,



$$= \cos[2\pi(f_c+\Delta f)t + \phi]$$

$$v(t) = s(t)\cdot c_L(t)$$

$$= m(t)\cdot c(t)\cdot c_L(t)$$

$$= A_c m(t) \cos 2\pi f_c t \cdot \cos[2\pi(f_c+\Delta f)t + \phi]$$

$$= \frac{A_c}{2}\left[2\cos 2\pi f_c t \cdot \cos[2\pi(f_c+\Delta f)t + \phi]\right] m(t)$$

$$= \frac{A_c}{2}\left[\cos[2\pi f_c t + 2\pi f_c t + 2\pi\Delta f t + \phi] + \cos[2\pi f_c t - 2\pi f_c t - 2\pi\Delta f t - \phi]\right] m(t)$$

$$= \frac{A_c}{2}\left[\cos[4\pi f_c t + 2\pi\Delta f t + \phi] + \cos[-2\pi\Delta f t - \phi]\right] m(t)$$

$$= \frac{A_c}{2}\left[\cos[4\pi f_c t + 2\pi\Delta f t + \phi] + \cos[2\pi\Delta f t + \phi]\right] m(t)$$

considering $\Delta f = 0$,

$$v(t) = \frac{A_c}{2}\left[\cos[4\pi f_c t + \phi] + \cos\phi\right] m(t)$$

$$= \frac{A_c}{2}\cdot\cos\phi\cdot m(t) + \frac{A_c}{2}\cos[4\pi f_c t + \phi]\cdot m(t)$$

When $V(t)$ is passed through LPF, then higher order frequency will be eliminated such that,

$$V_o(t) = \frac{A_c}{2} m(t) \cos \phi.$$

Thus we see that the demodulated signal $V_o(t)$ is proportional to message signal $m(t)$ as long as the phase error 'ϕ' is constant.

And the amplitude of demodulated signal is maximum when $\phi = 0$

i.e. $V_o(t) = \frac{A_c}{2} m(t)$, and

~~$V_o(t) = 0$ [illegible] when $\phi = \pm \pi/2$~~

$$V_o(t) = 0 \quad \text{when } \phi = \pm \pi/2$$

This demodulated amplitude value of 0 volts for $\phi = \pm \pi/2$ represents the quadrature null effect of the coherent detector.

So, as long as the phase error 'ϕ' is constant, the detector output is scaled but undistorted version of message signal $m(t)$.

But in real practice the factor $\cos \phi$ at ~~the~~ the receiver varies randomly ~~with~~ with time which is undesirable for the coherent detection.

Thus there is a need for circuitry at receiver that maintains local oscillator in perfect synchronization in both phase and frequency with the carrier of the transmitter.

One of such circuitry to recover synchronized carrier is Costas loop. Another method is signal squaring method.

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(i) Costas loop :

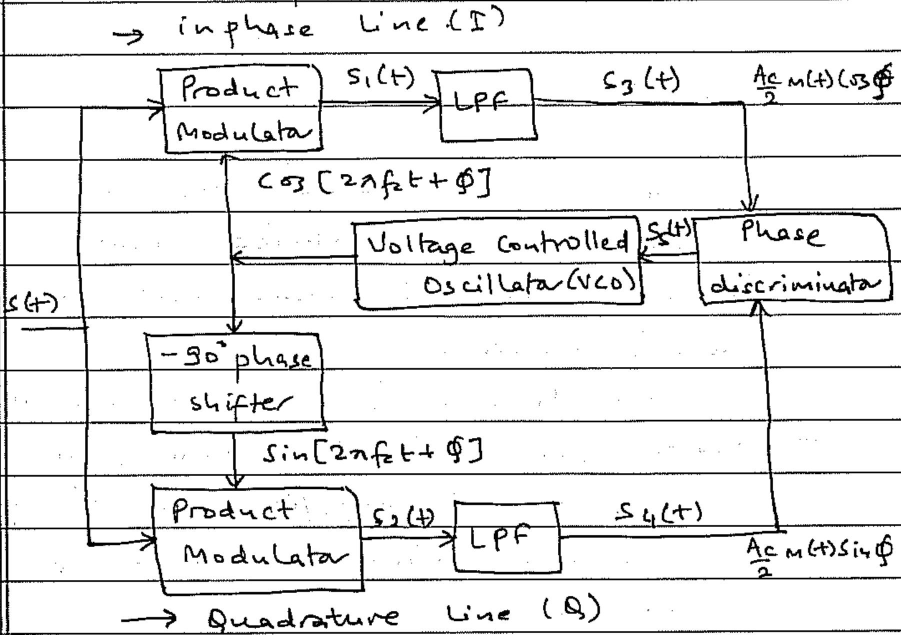

Fig. Costas loop

A Costas loop as shown above consists of two product modulators supplied with same received waveform $s(t)$ and to inphase and quadrature carrier respectively.

Let the frequency generated by the VCO be f_c but has slight phase error ϕ.

i.e. output of VCO = $\cos[2\pi f_c t + \phi]$

Now, $s_1(t) = s(t) \cdot \cos[2\pi f_c t + \phi]$ represents inphase line

or $s_1(t) = s(t) \cos[2\pi f_c t + \phi]$

let the incoming waveform $s(t)$ be DSB-SC such that $s(t) = m(t) \cdot c(t) = m(t) A_c \cos 2\pi f_c t$

$$\therefore \; s_1(t) = A_c m(t) \cos 2\pi f_c t \cdot \cos[2\pi f_c t + \phi]$$

$$= \frac{A_c}{2} m(t) \left[\cos\phi + \cos[4\pi f_c t + \phi] \right]$$

$$s_1(t) = \frac{A_c}{2} m(t) \cos\phi + \frac{A_c}{2} m(t) \cos(4\pi f_c t + \phi)$$

When $s_1(t)$ is passed through LPF,

$$s_3(t) = \frac{A_c}{2} m(t) \cos\phi$$

Similarly,

$$s_2(t) = A_c m(t) \cos 2\pi f_c t \cdot \sin(2\pi f_c t + \phi)$$

$$= \frac{A_c}{2} m(t) \left[\sin\phi + \sin(4\pi f_c t + \phi) \right]$$

$$= \frac{A_c}{2} m(t) \sin\phi + \frac{A_c}{2} m(t) \sin(4\pi f_c t + \phi)]$$

And

$$s_4(t) = \frac{A_c}{2} m(t) \sin\phi \; .$$

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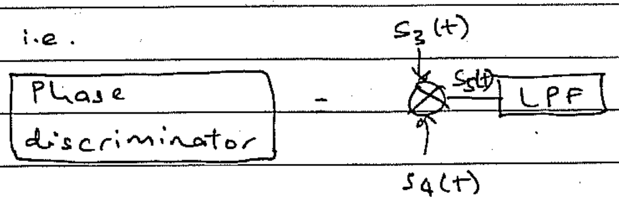
Now, the I & Q channel outputs are fed to the phase discriminator which consists of another product ~~multiplier~~ modulator and a LPF.

i.e.



Now,

$$S_5(t) = \frac{A_c}{2} m(t) \cos\phi \cdot \frac{A_c}{2} m(t) \sin\phi$$

$$= \frac{A_c^2}{8} m^2(t) \sin 2\phi$$

Here no matter what the message $m(t)$, the square of it will be positive and contain a DC component which can be filtered off.

Now the LPF o/p $= \frac{A_c^2}{8} \sin 2\phi$

$= K \sin 2\phi$.

Here, K = DC component of $\frac{A_c^2}{8} m^2(t)$.

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Now, 'K' is the control signal to the VCO which is a function of phase error ϕ. This control voltage change sign according to the magnitude of ϕ.

Providing the loop is stable, the tendency will be to shift the phase of VCO until ϕ is reduced to zero, since only then will the VCO come to rest.

ii) Signal squaring method :

In a signal squaring method, the incoming signal is squared and then passed through a narrowband bandpass filter tuned to $4f_c$. The output of this filter is the sinusoid ~~K sin~~ $K \cos 2\pi f_c t$, with some residual unwanted signal.

This signal is then applied to a PLL to obtain a cleaner sinusoid of twice the carrier frequency. This signal is then fed to a 2:1 frequency divider to obtain a local carrier.

This method is generally used for

DSB-SC detection.

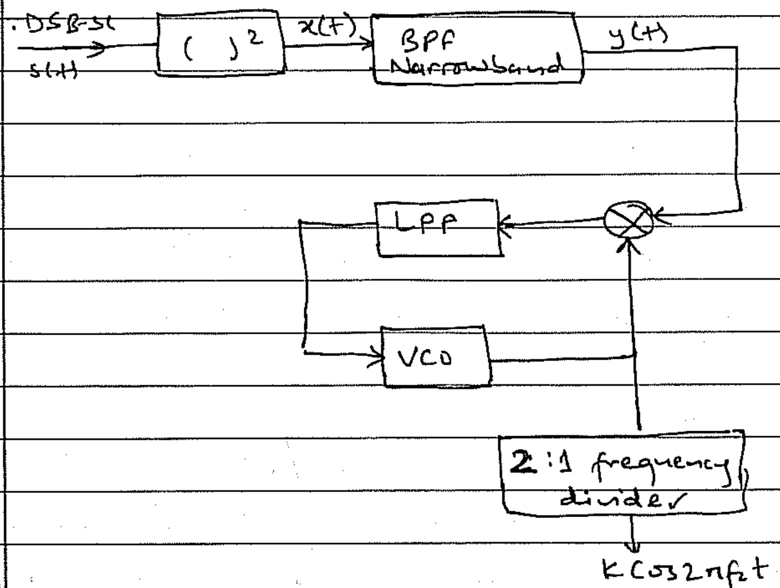

Fig. signal squaring circuit for DSB-SC demodulation.

From the above figure,

$s(t) = m(t)\cos 2\pi f_c t$

When $s(t)$ is passed through squaring device,

$x(t) = [s(t)]^2$

or $x(t) = m^2(t)\cos^2 2\pi f_c t$

$= \frac{m^2(t)}{2}\left[1 + \cos 4\pi f_c t\right]$

$= \frac{m^2(t)}{2} + \frac{m^2(t)}{2}\cos 4\pi f_c t$

When $x(t)$ is passed through N.B. BPF, we get

$y(t) = \frac{m^2(t)}{2}\cos 4\pi f_c t$

~~Now, y(t) is fed to the phase locked loop~~ or $y(t) = K\cos 4\pi f_c t + \phi(t)\cos 4\pi f_c t$

where, K = dc component of $\frac{m^2(t)}{2}$

$\phi(t)$ = zero mean baseband signal.

Since, the BPF is narrowband, its actual output,

$y(t) = K\cos 4\pi f_c t + \phi(t)\cos 4\pi f_c t$

Now, the PLL tracks and refines $y(t)$ such that PLL output is $K\cos 4\pi f_c t$. Finally the 2:1 frequency divider yields the required carrier signal as $K\cdot\cos 2\pi f_c t$.

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Phase locked loop (PLL)

PLL is ~~at~~ another method for carrier recovery and is widely used in communication system.

A PLL consists of a product modulator, a voltage controlled oscillator and a LPF.

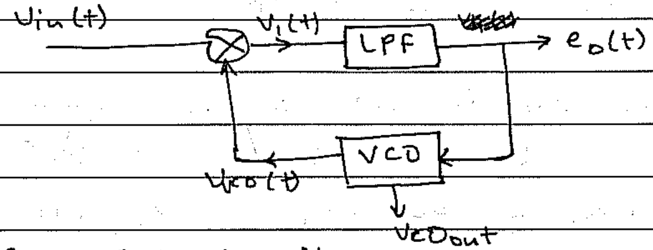

fig. PLL circuit

Now, let the incoming voltage signal

$V_{in}(t) = A_c \cos(2\pi f_c t + \phi_c)$

and

$V_{co}(t) = A \sin(2\pi f_c t + \theta_c)$

Here, it is assumed that the frequency of the two signals is same but differs in phase only.

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Now, the output of the multiplier will be,

$V_1(t) = A \cdot A_c \cos[2\pi f_c t + \phi_c] \cdot \sin[2\pi f_c t + \theta_c]$

or $V_1(t) = \frac{A \cdot A_c}{2} \left\{ \left[\sin(4\pi f_c t + \theta_c + \phi_c) \right] + \sin(\theta_c - \phi_c) \right]$

When $V_1(t)$ is passed through LPF, we get

$e_o(t) = \frac{A_c \cdot A}{2} \sin(\theta_c - \phi_c)$

This $e_o(t)$ is termed as error voltage which acts as the control voltage to the VCO. Therefore the control voltage applied to VCO is proportional to the phase difference between two signals.

Now, if the phase difference is small then,

$e_o(t) \approx \frac{A_c \cdot A}{2} \cdot (\theta_c - \phi_c)$

also, $e_o(t) = \underset{\text{min}}{0}$ when both signals are of same phase

and $e_o(t) = \underset{\text{Max}}{\frac{A \cdot A_c}{2}}$ when two signals are in quadrature phase.

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So, the basic idea of a PLL is to make the phase difference between two signals equal to zero such that the output of VCO is the exact replica of the input signal.

It should be noted that the ~~ip.~~ input signal and VCO output signal have always a 90° phase difference.

A PLL now can be defined as a feedback system combining a voltage controlled oscillator (VCO) and a phase comparator (multiplier + LPF) so connected that the oscillator maintains a constant phase angle relative to a reference (incoming) signal.

The phase locked loop works in such a manner that the phase difference between θ_c & ϕ_c becomes as small as possible ~~ue~~ or equal to zero, i.e.

$\theta_c = \phi_c$.

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⊕ Demodulation of AM using PLL.

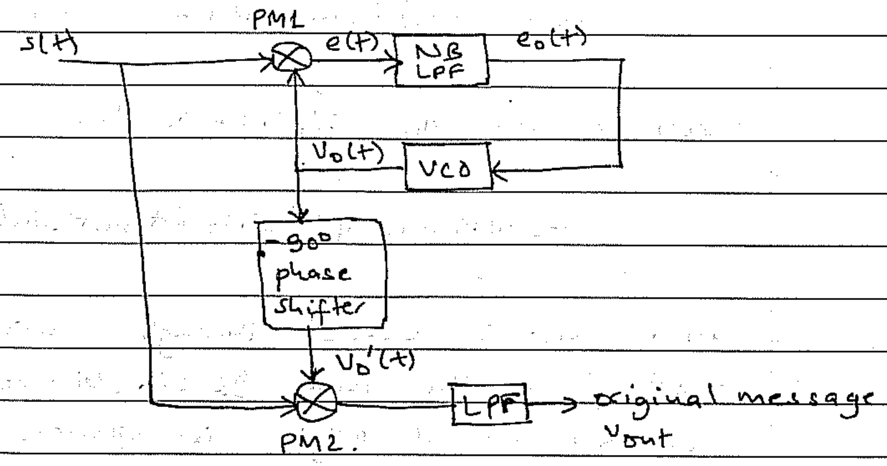

Fig. PLL for demodulation of AM.

We now know that a PLL can be used to recover the carrier from the incoming batch of AM input signal.

Here, ~~let~~ $s(t) = A_c[1 + k_a m(t)] \cos(2\pi f_c t + \theta_i)$

let $k_a = 1$, such that

$s(t) = [A_c + A_c m(t)] \cos(2\pi f_c t + \theta_i)$

Also,

VCO output is quadrature of ~~cos~~ carrier,

i.e. $V_o(t) = \sin(2\pi f_c t + \theta_o)$

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So,

$$e(t) = [A_c + A_c m(t)] \cos(2\pi f_c t + \theta_i) \cdot \sin(2\pi f_c t + \theta_o)$$

$$= \frac{[A_c + A_c m(t)]}{2} \cdot [\sin(4\pi f_c t + \theta_i + \theta_o) + \sin(\theta_i - \theta_o)]$$

or $$e(t) = \frac{A_c}{2} \sin(4\pi f_c t + \theta_i + \theta_o) + \frac{A_c}{2} \sin(\theta_i - \theta_o)$$

$$+ \frac{A_c}{2} m(t) \sin(4\pi f_c t + \theta_i + \theta_o) + \frac{A_c}{2} m(t) \sin(\theta_i - \theta_o)$$

when $e(t)$ is passed through Narrowband LPF, only the term $\frac{A_c}{2} \sin(\theta_i - \theta_o)$ centred around origin is allowed to pass forward.

i.e.

$$e_o(t) = \frac{A_c}{2} \sin(\theta_i - \theta_o)$$

$$= \frac{A_c}{2} \sin \theta_e$$

where $\theta_e = \theta_i - \theta_o$ = phase error

Now as the phase locked loop works in such a way that the phase error becomes as small as possible tending to zero.

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Once the PLL is locked then

$$v_o(t) = \sin(2\pi f_c t + \theta_i) \quad [\because \theta_o = \theta_i]$$

Now, as $v_o(t)$ is passed through a $-90°$ phase shifter,

$$v_o'(t) = \cos(2\pi f_c t + \theta_i)$$

Now, the output of PM2 is,

$$s(t) \cdot v_o'(t) = A_c[1 + m(t)] \cos(2\pi f_c t + \theta_i) \cdot \cos(2\pi f_c t + \theta_i)$$

$$= A_c[1 + m(t)] \cos^2(2\pi f_c t + \theta_i)$$

$$= A_c[1 + m(t)] \left[\frac{1 + \cos(4\pi f_c t + 2\theta_i)}{2}\right]$$

$$s(t) \cdot v_o'(t) = \frac{A_c[1 + m(t)]}{2} + \frac{A_c}{2}[1 + m(t)] \cos(4\pi f_c t + 2\theta_i)$$

When the above signal is passed through a LPF centred around frequency of $m(t)$, we get,

$$V_{out}(t) = \frac{A_c}{2}[1 + m(t)] = \frac{A_c}{2} + \frac{A_c}{2} m(t).$$

Hence $V_{out}(t) \propto m(t)$.