COMMUNICATION SYSTEM I

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2.

Representation of Signal and systems in communication

2.1 Review of Signals.

Signal :

It is defined as a function of one or more variables (time, pressure, position, temperature, ~~voltage~~ ~~current~~ distance etc) which contains some information about any particular phenomenon.

eg. music, speech, picture, video, voltage etc.

Signals can furthermore classified as,

i) Continuous time and discrete time signals

A continuous time signal is defined continuously in the time domain, such that a signal $x(t)$ is continuous signal if 't' is a continuous variable. A continuous time signal can also be termed as analog signals.

CTS : continuous time signal
DTS : discrete time signal

On the other hand, if the time 't' is defined for some discrete value, the signal thus attained is discrete time signal. So, for discrete time signal we can get the signal reading at fixed interval of time. We can realize a ~~disce~~ discrete time signal from continuous time signal with the help of sampling.

CTS → sampling → DTS

And when a DTS is quantized and encoded we get a digital signal.

ii) Periodic & Aperiodic signals.

A periodic signal exhibits a definite pattern and repeats itself over and over again at a certain interval of time, such that,

$$x(t) = x(t+T) \quad \text{for } -\infty < t < \infty$$

Here, 'T' is the period of the signal. The smallest value for 'T' that satisfies above equation is called its fundamental period, and is a positive constant.

Now, a signal is said to be aperiodic if it doesn't repeat itself at regular interval of time.

i.e. it doesn't follow $x(t) = x(t+T)$.

iii) Deterministic and random signals.

Deterministic signals are those type of signals which can be completely specified in time. Such signals can be characterized mathematically and the parameters of the signals can be predicted with ease.

Random signals on other hand are unpredictable such that there is uncertainty before the actual occurance of signal. We cannot predict the value of signal at any particular instant but go for some probable value. Thus random signals are better suited by probability theory.

iv)

iv) Even and odd signals.

An even signal is one which exhibits symmetry in time domain, i.e. it satisfies the condition,

$$x(t) = x(-t) \quad \text{for all } 't'.$$

i.e. even signals are symmetrical about vertical axis.

An odd signal is one which shows anti-symmetric behaviour. this type of signals are mirror image ~~identical~~ to their ~~its~~ negative.

i.e.

$$x(t) = -x(-t) \quad \text{for all } 't'.$$

v) Causal and non-causal signals.

A signal is said to be causal if its value at ~~time~~ negative time is zero.

i.e. $x(t) = 0$ for $t < 0$.

And a signal is anti-causal if the signal reading is zero for all positive time.

i.e. $x(t) = 0$ for $t > 0$.

A non-causal signal is one that has non-zero value in both positive and negative time.

i.e. $x(t) \neq 0$ for $'-t'$ to $'t'$.

vi) Energy and power signals.

An energy signal is one which has finite energy and zero average power.

whereas a power signal has finite value of average power and infinite energy.

So, for any signal $x(t)$,

energy, and Power

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt \qquad P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$$

or $E = \lim_{T \to \infty} \int_{-T}^{T} |x(t)|^2 dt$

so, if $0 < E < \infty \rightarrow$ energy signal

and

if, $0 < P < \infty \rightarrow$ power signal

⊕ There are some signals that are neither energy nor power signals.

Apart from these signal classifications, some special types of signal used to describe ideal phenomenon are given as,

i) Harmonic signal.

It is a periodic signal expressed in terms of sinusoidal function defined for $-\alpha \leq t \leq \alpha$.

A signal,

$x(t) = A\cos(2\pi f t + \theta)$ is a harmonic signal.

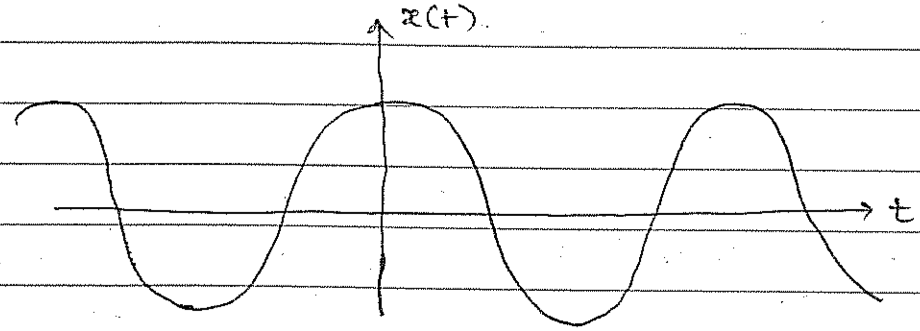


Fig. Harmonic signal

ii) Unit step signal:

A signal which exists only for the positive ~~sig~~ side of the time domain and has an amplitude of '1' is defined as unit step signal. It is represented as '$u(t)$'.

i.e. $u(t) = \begin{cases} 0 & \text{for } t < 0 \\ 1 & \text{for } t \geq 0 \end{cases}$

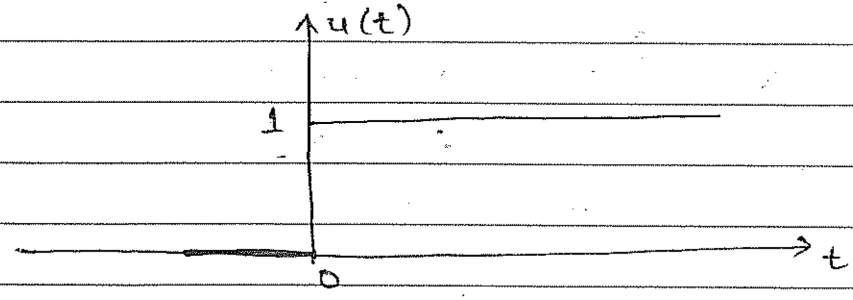


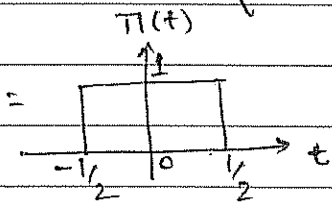
Fig. unit step f^n.

iii) Rectangular pulse signal:

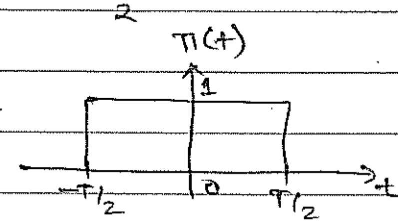
A rectangular pulse signal or a unit pulse signal is defined as,

$$\text{rect}(t) = \Pi(t) = \begin{cases} 1 & \text{for } -\frac{1}{2} < t \leq \frac{1}{2} \\ 0 & |t| > \frac{1}{2} \end{cases}$$

i.e. $\Pi(t) =$



or



Also, for a defined time period 'T',

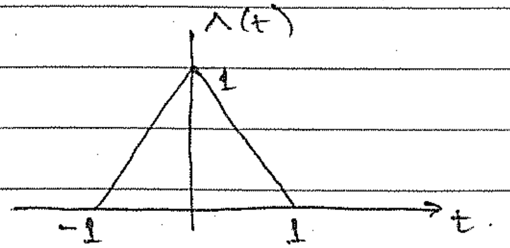
$\text{rect}(t/T) = 1$ for $-T/2 < t \leq T/2$

$= 0$ elsewhere.

iv) Triangular pulse function / signal.

A triangular pulse signal is defined as,

$$\Lambda(t) = \begin{cases} t+1 & \text{for } -1 \le t \le 0 \\ -t+1 & \text{for } 0 \le t \le 1 \\ 0 & \text{elsewhere} \end{cases}$$



v) Sinc signal.

A sinc signal is defined as

$$\text{sinc}(t) = \frac{\sin \pi t}{\pi t}.$$

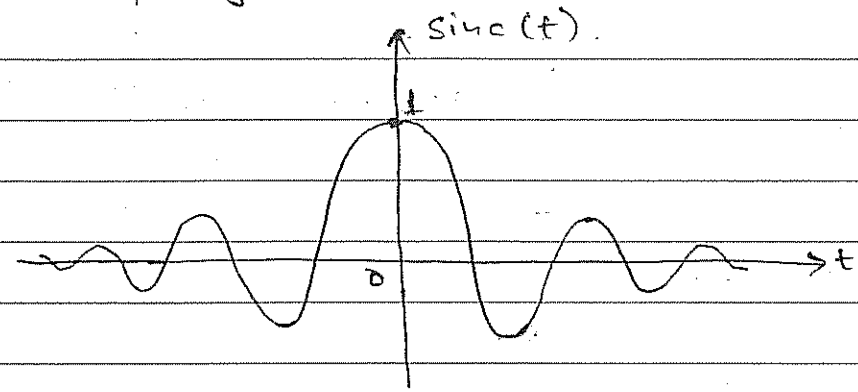
for, time $t=0$, we have $\frac{\sin \pi t}{\pi t} = 1$,

$\therefore$

$$\text{Sinc}(t) = \frac{\sin \pi t}{\pi t} \quad \text{for } t \neq 0$$

$$= 1 \quad \text{for } t = 0.$$

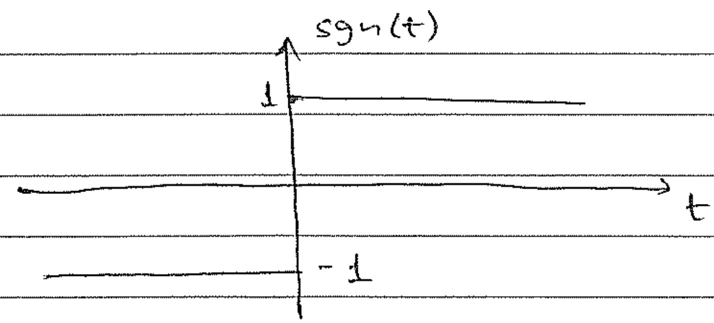
So, a sinc signal has its maximum value equal to '1' at $t=0$ and gradually it alternatively tends to zero ~~for~~ as 't' tends to infinity.



vi) Signum signal :

It is a signal used to define the polarity of a signal as per the time axis, i.e.

$$\text{sgn}(t) = 1 \quad \text{for } t > 0$$
$$= 0 \quad \text{for } t = 0$$
$$= -1 \quad \text{for } t < 0$$



vii) Delta or impulse function / signal or unit impulse function.

It is the mathematical model to represent the physical phenomenon that takes place in a very short period of time. It is one of the most widely used elementary signal for the analysis of communication system. It is defined as,

$$\delta(t) = 1 \quad \text{for } t = 0$$
$$= 0 \quad \text{for } t \neq 0$$

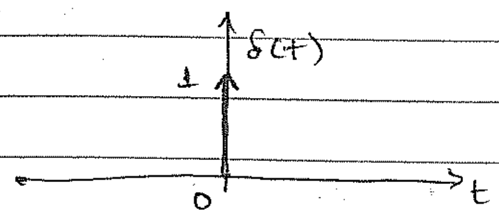


fig. delta signal

vii) Delta or impulse signal.

It is the mathematical model to represent the physical phenomenon for a very short period of time. It is one of the most widely used elementary function for the analysis of communication system. It is denoted by $\delta(t)$ and is expressed as,

$$\delta(t) = 0 \qquad \text{for } t \neq 0$$

and
$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1$$

which implies that the area under delta f^n is equal to 1.

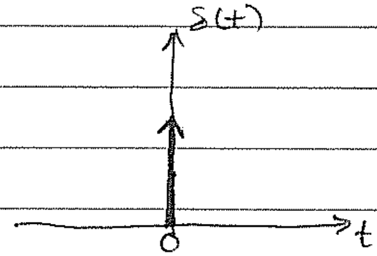


Fig. delta signal

(#) Now, the convolution of any function with delta function gives the value of that function, i.e.

$$x(t) \circledast \delta(t) = x(t)$$
$$x(t) \circledast \delta(t - t_0) = x(t - t_0)$$

also,
$$\delta(at) = \frac{1}{|a|}\,\delta(t) \quad \text{for all } a \neq 0.$$

Proof for $\delta(at) = \frac{1}{|a|}\,\delta(t)$

We have, $\int_{-\infty}^{\infty} \delta(t)\, dt = 1$,

let $t = at$ then $dt = a\,dt$

Thus, $\int_{-\infty}^{\infty} \delta(t)\, dt = \int_{-\infty}^{\infty} \delta(at)\, a\, dt = 1$

or $a \int_{-\infty}^{\infty} \delta(at)\, dt = 1$

or $\int_{-\infty}^{\infty} \delta(at)\, dt = \frac{1}{a}$; $a > 0$

& $\int_{-\infty}^{\infty} \delta(at)\, dt = -\frac{1}{a}$; $a < 0$

Thus $\int_{-\infty}^{\infty} \delta(at)\, dt = \frac{1}{|a|}$

or $\int_{-\infty}^{\infty} \delta(at)\, dt = \frac{1}{|a|} \int_{-\infty}^{\infty} \delta(t)\, dt$

$\therefore \delta a(t) = \frac{\delta(t)}{|a|}$

2.2 Systems.

A system may be defined as the set of elements or functional blocks connected together which produces an output in response to an input signal.

So, basically a system processes on an input signal and produces any desired output signal. The response or the output of the system depends on the transfer function of the system.

Some basic types of systems are,

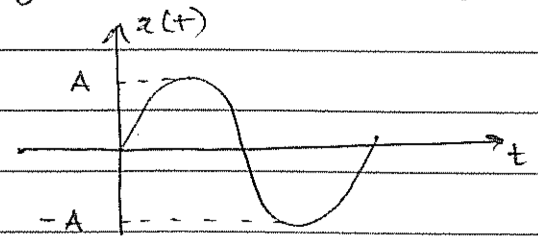
i) Linear system:

Any system is termed linear system if the principle of superposition can be applied. let $x_1(t)$ and $x_2(t)$ be the inputs to ~~two~~ the systems with $y_1(t)$ and $y_2(t)$ the corresponding output signals

Representation of signals.

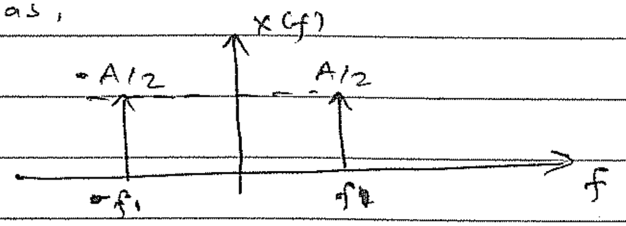
i) Time domain representation.

In time domain representation, the signal is a time varying quantity.



ii) frequency domain representation.

Since working with the time domain representation is a tedious task, frequency domain representation of signal is more preferred. In doing so, a signal is represented by its frequency spectrum. So, for the signal above, its frequency spectrum can be shown as,



2.2 Systems.

A system may be defined as the set of elements or functional blocks connected together which produces an output in response to an input signal.

A system thus basically processes on an input signal and produces any desired output signal. The response or the output of the system thus depends on the transfer function of the system or the impulse responce of the system.

$$x(t) \longrightarrow \boxed{\text{system}} \longrightarrow y(t)$$

$h(t)$ or $H(f)$

where,

$x(t)$ = input signal ; $y(t)$ = output signal

$h(t)$ = impulse response ; $H(f)$ = transfer function.

Impulse response :

The output of a system when input is impulse or delta function is called impulse response of the system, ie.

$$\delta(t) \longrightarrow \boxed{\phi} \longrightarrow h(t).$$

or $h(t) = \phi\,\delta(t)$

$\phi \rightarrow$ system operator

Transfer function: $H(f)$

The Fourier transform of impulse response $h(t)$ is called transfer function.

i.e.

$$h(t) \xrightarrow{F.T} H(f).$$

or

$$H(f) = \int_{-\infty}^{\infty} h(t)\, e^{-j2\pi f t}\, dt$$

and

$$h(t) = \int_{-\infty}^{\infty} H(f)\, e^{j2\pi f t}\, df.$$

$$\therefore\ h(t) \rightleftharpoons H(f).$$

Now, some basic types of system are,

i) Linear system

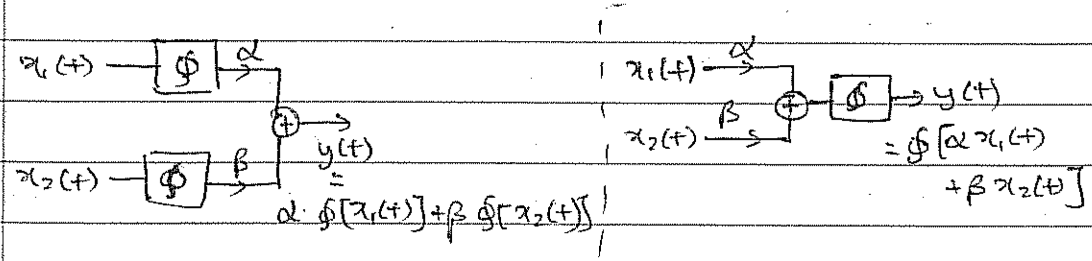
A system is termed linear system if the principle of superposition can be applied. i.e. If $x_1(t)$ and $x_2(t)$ are two inputs to a system with operator 'ϕ' such that the corresponding outputs $y_1(t) = \alpha\phi\, x_1(t)$ & $y_2(t) = \beta\phi\, x_2(t)$, then the output for the combination of two inputs must yield

$$y(t) = \alpha\, \phi\, x_1(t) + \beta\, \phi\, x_2(t) = \phi[\alpha x_1(t) + \beta x_2(t)]$$

where α, & β are scaling factors.

i.e. A system is linear if

$$\phi[\alpha x_1(t)] + \phi[\beta x_2(t)] = \phi[\alpha x_1(t) + \beta x_2(t)]$$



i.e. $y(t) = \alpha y_1(t) + \beta y_2(t)$

ii) Non-linear system.

Any system not satisfying the principle of superposition is termed as non-linear system.

iii) Time invariant system:

A system is said to be time invariant if the response of the system does not depend on any shift in time for input signal $x(t)$.

i.e.

if $y(t) = \phi[x(t)]$

then $y(t-t_0) = \phi[x(t-t_0)]$

iv) Time variant:

If the response of the system varies with the shift in time at input signal then such such system is a time variant system.

Linear time invariant (LTI) system

Two basic properties namely, linearity and time invariance play a vital role in the analysis of signals and system. A system which is both linear and time invariant, then such system is known as LTI system. An LTI system is characterized by its impulse response. And the complete characterization of any LTI system in terms of its impulse response is performed by convolution integral in continuous time case and by convolution sum in discrete time case.

Convolution between two signals $x_1(t)$ and $x_2(t)$ is defined as,

$$y(t) = x_1(t) \otimes x_2(t) = \int_{-\infty}^{\infty} x_1(\tau)\, x_2(t-\tau)\, d\tau$$

$$y(t) = \int_{-\infty}^{\infty} x_2(\tau)\, x_1(t-\tau)\, d\tau$$

$\otimes \rightarrow$ symbol of convolution

In discrete time, $y[n] = \sum_{k=-\infty}^{\infty} x_1[k]\, x_2[n-k]$

Signal transfer in LTI system.

$$x(t) \longrightarrow \boxed{\phi[\cdot]} \longrightarrow y(t)$$

We have defined that an impulse response $h(t)$ is the output of the system for $\delta(t)$ as input. i.e. $h(t) = \phi\, \delta(t)$

Now, we can express $x(t)$ in terms of $\delta(t)$ as,

$$x(t) = \int_{-\infty}^{\infty} x(\tau)\, \delta(t-\tau)\, d\tau$$

Now,
$$y(t) = \phi[x(t)]$$
$$= \phi \int_{-\infty}^{\infty} x(\tau)\, \delta(t-\tau)\, d\tau$$
$$= \int_{-\infty}^{\infty} x(\tau) \cdot \phi\, \delta(t-\tau)\, d\tau$$
$$= \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau \quad [\because h(t) = \phi\, \delta(t)]$$

$$\therefore\ y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

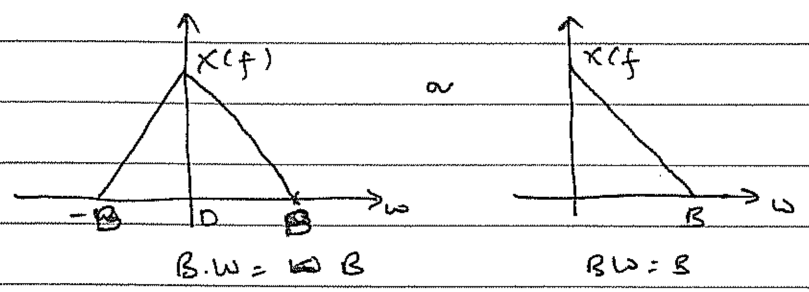
or, $y(t) = x(t) \otimes h(t)$

Properties of LTI system.

i) Commutative property
ii) Distributive property
iii) Associative property
iv) Static and dynamic LTI system
v) Invertibility of LTI system
vi) Stability of LTI system
vii) Causality of LTI system
viii) Unit-step response of LTI system.

2.3 low pass and band pass signals/systems

A low pass signal is the signal that has its significant spectral content centered around the origin. For a low pass signal, the bandwidth is defined as one half of the total width of the main spectral lobe. Basically audio, video signals are low pass signals in unmodulated form.



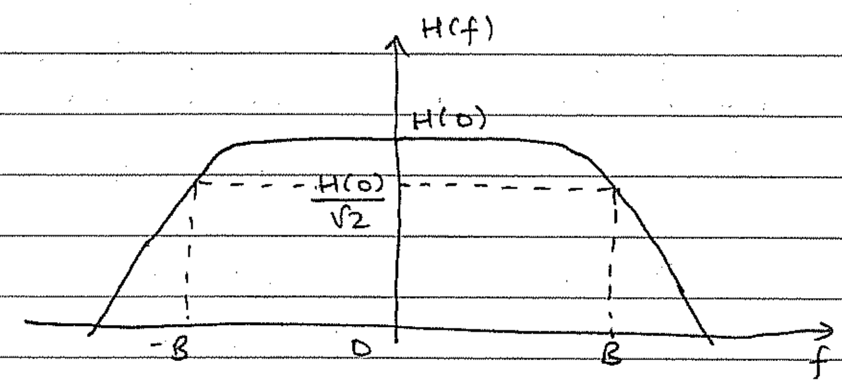
Whereas, a signal is said to be bandpass if its significant spectral content lies around a frequency 'fc' which is a non-zero frequency.

The bandwidth for such signals is defined as the width of the main lobe for positive frequencies.

Now, the system that processes on these low pass and high pass signals are known as Low pass system and high pass system respectively.

⊕ System bandwidth:

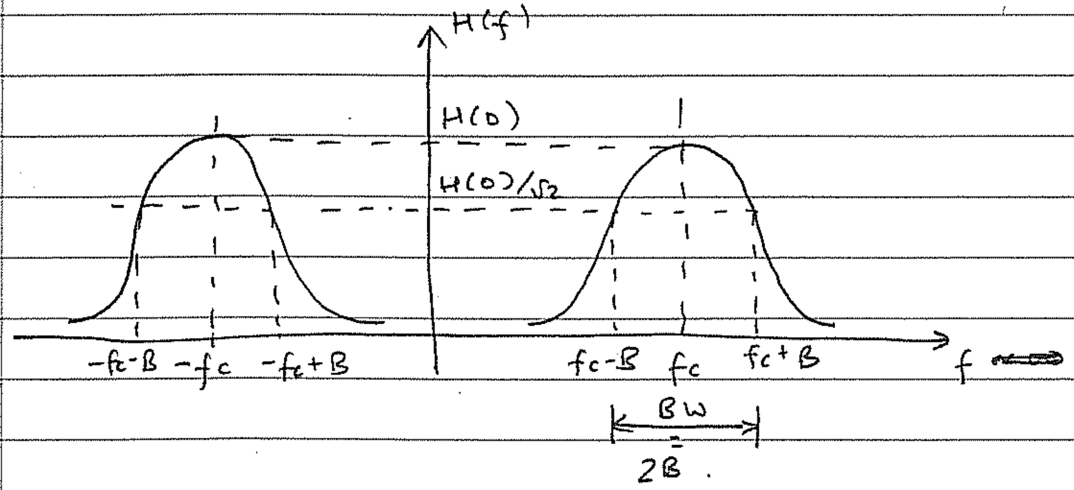
In case of low pass system, the 3dB bandwidth is defined as the difference between zero frequency when the amplitude is highest and the frequency when the amplitude drops by $1/\sqrt{2}$.



In the above figure, at zero frequency system response attains ~~higher~~ peak amplitude of H(0). Now at frequency 'B', the amplitude drops to $H(0)/\sqrt{2}$.

Thus the system bandwidth = 'B'.

In the case of bandpass system, the 3dB bandwidth is defined as the difference between the frequencies at which the amplitude response drops $1/\sqrt{2}$ times the peak amplitude centered around any frequency 'f_c' other than '0'.



In the above figure, bandwidth is the difference of f_c+B and f_c-B as their amplitudes are $H(0)/\sqrt{2}$.

i.e.

$$B.W = f_c + B - f_c + B = 2B.$$

Distortionless transmission.

The transmission of a signal through a system is said to be distortionless if the output signal is the exact replica of the input signal.

If $x(t)$ is the input signal passed through a system without distortion, then the output $y(t)$ should be defined by,

$$y(t) = k\,x(t-t_0) \qquad -(i)$$

where k : scaling factor

t_0 : delay in transmission

Taking the fourier transform of equation (i),

$$Y(f) = k\,X(f)\,e^{-j2\pi f t_0}$$ such that,

transfer function, $H(f) = \dfrac{Y(f)}{X(f)} = k e^{-j2\pi f t_0}$

and impulse response, $h(t) = k \cdot \delta(t-t_0)$

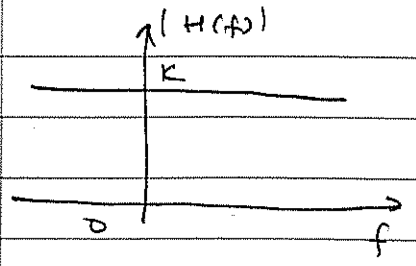
So it is apparent that in order to achieve the distortionless transmission through a system, the transfer function of system must satisfy two conditions.

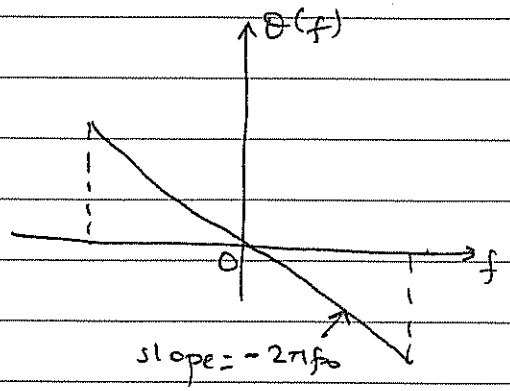
i) The amplitude response of $H(f)$ is constant for all frequencies.

i.e. $|H(f)| = K$

ii) The phase of $H(f)$ is linear with frequencies, passing through origin.

i.e. $\measuredangle H(f) = -2\pi f t_0 = \theta(f)$.





⊕ Hilbert transformation :

Hilbert transform (HT) is an operator which adds a $-90°$ phase shift to all positive frequencies and a $+90°$ phase shift to all negative frequencies of the input signal, but the amplitude ~~range~~ response of the system is constant for entire frequency range of interest. The Hilbert transform of a signal $x(t)$ is given as $\hat{x}(t)$.

And the device that performs the Hilbert transform is known as Hilbert transformer. The impulse response of such Hilbert transformer is,

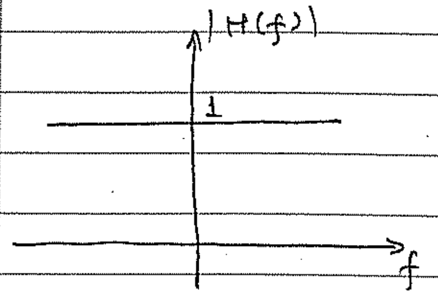
$$h(t) = 1/\pi t$$

The frequency response of H.T. is given by.

$$H(f) = -j\,\mathrm{sgn}(f) = \begin{cases} -j & \text{for } f > 0 \\ 0 & f = 0 \\ j & f < 0 \end{cases}$$

and phase response as,

$$\cancel{\measuredangle H(f)} \quad \theta(f) = \begin{cases} -\pi/2 & ; f > 0 \\ 0 & ; f = 0 \\ \pi/2 & ; f < 0 \end{cases}$$



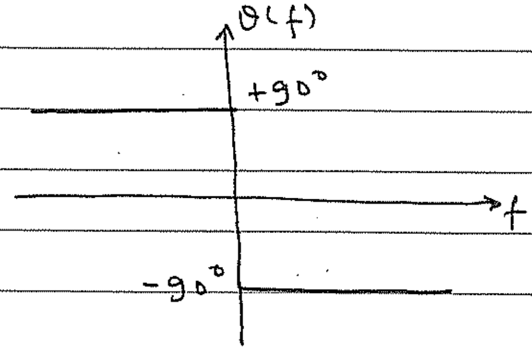


fig. Amplitude response fig. Phase response.

for H.T.

Now, we have $h(t) = 1/\pi t$

Also, $x(t) \longrightarrow \boxed{h(t)} \longrightarrow \hat{x}(t)$ (H.T.)

$\therefore \hat{x}(t) = x(t) \otimes h(t)$

$$= \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} x(\tau) \cdot \frac{1}{\pi(t-\tau)}\, d\tau$$

$$\therefore \hat{x}(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{x(\tau)}{(t-\tau)}\, d\tau$$

And the inverse of $\hat{x}(t)$ leads to $x(t)$,

i.e.

$$x(t) = -\frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\hat{x}(\tau)}{t-\tau}\, d\tau$$

Properties of H.T.

(i) The energy content in both $x(t)$ and $\hat{x}(t)$ are same

i.e. $|\hat{X}(f)|^2 = |X(f)|^2$

ii) $x(t)$ and $\hat{x}(t)$ are orthogonal and therefore the cross correlation between them is zero.

i.e. $\int_{-\infty}^{\infty} x(t) \cdot \hat{x}(t)\, dt = 0$

iii) If $x(t)$ is a high frequency sinusoidal signal and $m(t)$ is a low frequency sinusoidal signal then the Hilbert transform of product of $x(t)$ and $m(t)$ is equal to the product of $m(t)$ and $\hat{x}(t)$.

i.e.

$$x(t) \cdot m(t) \xrightarrow{H.T} m(t) \cdot \hat{x}(t)$$

\# Uses of H.T.

i) Generation of SSB signals

ii) Designing minimum phase type filters.

iii) Representation of bandpass signals.