(12 marks)

Chapter-4: BaseBand Data Communication Systems

4.1 Introduction to Information Theory

Information theory is the mathematical modelling and analysis of Communication system rather than with physical Sources and physical channel.

In other words we can say as a branch of applied probability theory to study the Communication system.

Information here can be defined as the occurence of an event, source produces event called symbol or letters whose group or set of symbols is known as source Alphabet.

A Discrete Memoryless Source (DMS) is characterized by the list of symbols and the assignment of the probability to these symbols.

According to the Information Theory, the information content or amount of information of a symbol (x_i) denoted by $I(x_i)$ is inversely proportional to the probability of occurence $p(x_i)$ satisfying

1) $I(x_i)$ approaches 0 as $p(x_i)$ approaches infinity.
For message "sun will rise in the east". This message doesnot Contain any information since sun will always rise in east with probability '1'.

2) If message contains information the Information content $I(x_i)$ is a non-negative quantity.

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3) For higher probability of occurence, it has less information content and viceversa.

* Information source

Information source produces the message signal.

* symbol Rate :→

It is the rate at which the information source generates source Alphabet or symbol. It is represented in symbols/sec.

* Entropy :→

Entropy is defined as the average information content per symbol. Let DMS source generates one of M symbols $(x_1, x_2, x_3 \ldots x_m)$ with corresponding probability of occurence $P_1, P_2, P_3 \ldots P_m$ then Entropy is given as

$$\boxed{H(x) = \sum_{i=1}^{m} p(x_i) \log_2\left(\frac{1}{p(x_i)}\right)}$$

Proof :→ Consider source emits 'm' possible symbols $x_1, x_2, x_3 \ldots x_m$ whose probabilities are $P_1, P_2, P_3 \ldots P_m$ respectively. If a long sequence of 'N' symbols, occurence of symbol x_i is $N p(x_i)$.

Information Content for first symbol is

$$I(x_1) = \log_2\left(\frac{1}{p(x_i)}\right)$$

Then total Information content of N sequence of M symbol is

$$I_{total} = \sum_{i=1}^{M} N\, p(x_i) \log_2\left(\frac{1}{p(x_i)}\right)$$

The Average Information content is

$$I_{average} = H = \frac{I_{total}}{N} = \sum_{i=1}^{m} p(x_i) \log_2\left(\frac{1}{p(x_i)}\right)$$

$$\therefore \boxed{\text{Entropy}(H) = \sum_{i=1}^{m} p(x_i) \log_2\left(\frac{1}{p(x_i)}\right)}$$

* Information Rate :→

Information Rate is defined as the maximum rate of transmission of errorless data. Information Rate can be achieved by multiplication of symbol Rate and Entropy.

$$\boxed{\text{Information Rate} = \text{Symbol Rate} \times \text{Entropy}}$$

whose unit is bits/sec.

1. A source produces one of four possible symbols during each interval having probabilities $p(x_1) = \frac{1}{2}$, $p(x_2) = 1/4$, $p(x_3) = p(x_4) = 1/8$. Obtain the information content of each of these symbols.

$$I(x_1) = \log_2\left(1/p(x_1)\right) = \log_2\left(\frac{1}{1/2}\right)$$

$$I(x_1) = \log_2(2) = \times \frac{\log(2)}{\log(2)} =$$

$$I(x_2) = \log_2\left(\frac{1}{p(x_2)}\right) = \log_2(4) = \frac{\log 4}{\log 2} = 2$$

$$I(x_3) = \log_2\left(\frac{1}{p(x_3)}\right)$$

$$I(x_4) = \log_2\left(\frac{1}{p(x_4)}\right)$$

2) An Analog signal is band limited to f_m Hz and sampled at Nyquist rate. The samples are quantized into 4 levels. Each level represent one symbol. Thus there are 4 symbols. The probabilities of occurence of these 4 levels (symbol) are $p(x_1) = p(x_4) = 1/8$ and $p(x_2) = p(x_3) = 3/8$. Obtain information rate of the source.

Given, $f_s = 2f_m$ $\quad v = 2$

Entropy

$$H(x) = \sum_{i=1}^{4} p(x_i) \log_2\left(\frac{1}{p(x_i)}\right)$$

$$H(x) = p(x_1)\log_2\left(\frac{1}{p(x_1)}\right) + p(x_2)\log_2\left(\frac{1}{p(x_2)}\right) + \cdots$$

$$H(x) = \frac{1}{8}\log_2(8) + \frac{3}{8}\log_2\left(\frac{8}{3}\right) + \frac{3}{8}\log_2\left(\frac{8}{3}\right) + \frac{1}{8}\log_2(8)$$

$H(x) = 1.8$ bits/symbol

since the signal is sampled at Nyquist rate

$f_s \geq 2f_m$ samples/sec

Symbol rate $r = 2f_m$ symbols/sec since every sample generates one source symbol.

$\therefore$ Information Rate (R) $= r\,H(x)$

$= 2f_m \times 1.8 = 3.6\, f_m$ bits/sec.

3) A signal of bandwidth 4.5 KHZ is sampled at the double rate given by Nyquist, the signal is quantized in 8 levels, the probability of occurrence of the level are 0.1, 0.15, 0.15, 0.05, 0.2, 0.05, 0.18, 0.12. Find the minimum number of bits per sample and information rate.

$f_m = 4.5$ KHZ

Entropy

$$H(x) = \sum_{i=1}^{8} p(x_i)\log_2\left(\frac{1}{p(x_i)}\right)$$

$$H(x) = p(x_1)\log_2\frac{1}{p(x_1)} + p(x_2)\log_2\frac{1}{p(x_2)} + p(x_3)\log_2\frac{1}{p(x_3)} + p(x_4)\log_2\frac{1}{p(x_4)}$$

$$+ p(x_5)\log_2\left(\frac{1}{p(x_5)}\right) + p(x_6)\log_2\frac{1}{p(x_6)} + p(x_7)\log_2\frac{1}{p(x_7)} + p(x_8)\log_2\frac{1}{p(x_8)}$$

$$H(x) = 0.1\log_2\left(\frac{1}{0.1}\right) + 0.15\log_2\left(\frac{1}{0.15}\right) + 0.15\log_2\left(\frac{1}{0.25}\right) + 0.05\log_2\left(\frac{1}{0.05}\right)$$

$$+ 0.2\log_2\left(\frac{1}{0.2}\right) + 0.05\log_2\left(\frac{1}{0.05}\right) + 0.18\log_2\left(\frac{1}{0.18}\right) + 0.12\log_2\left(\frac{1}{0.12}\right)$$

$$= 0.33 + 0.41 + 0.41 + 0.21 + 0.46 + 0.21 + 0.44 + 0.36$$

$$= 2.33 \text{ bits/symbol}$$

since the signal is sampled at double rate given by Nyquist

symbol rate $(r) = 2 \times 2f_m = 4 \times 4.5 = 18000$ symbols/sec

Information Rate (R) $= r\,H(x)$

$= 18000 \times 2.33$ bits/sec

$= 41940$ bps

4.2 Shannon Hartley channel capacity theory. Implication of theorem and theoritical limits.

In Shannon Hartley channel capacity theory, the channel capacity is defined as the maximum rate at which information may be transmitted without error through the channel which is given as

$$\boxed{C = B\log_2(1+SNR)} \quad \text{bit/second}$$

Where

$C \Rightarrow$ channel capacity $B \Rightarrow$ Channel Bandwidth

$SNR \Rightarrow$ Received signal to Noise Ratio (SNR).

* Implications

1 Indicates the upper limit of data transmission for reliable communication.

2 Trade-off between Bandwidth (B) and signal to Noise Ratio (SNR) for given channel capacity (C). as they are exchangeable. Here, channel capacity can be increased either increase of Bandwidth or signal power which is clearly illustrated with the expression.

3 Bandwidth Compression

Shannon channel capacity theorem indicate that it is possible to transmit signal with highest frequency fmax through a channel having bandwidth less than fmax.

* Theoritical Limits

1 As the Bandwidth of the channel 'B' tend to infinity ($B\to\infty$), the channel capacity reaches upper limit (C_{max}). This is because Noise power is related to the Bandwidth $B\to\infty$, $C\to\infty$, $N\to 0$

2) Noise less channel refered as Ideal channel has zero noise. Subsequently the channel capacity 'C' will tend to infinity.

Let us take a chanel consisting of white noise, the channel capacity is given as

$$C = B\log_2(1+SNR) = B\log_2(1+S/N) \quad \text{—} (1)$$

PSDF of white Noise can be calculated as

$$P_N = \int_{-B}^{B} S_{WN}(f)\,df$$

$$P_N = \int_{-B}^{B} \frac{N_0}{2}\,df = BN_0$$

putting value of N in eq^n (1) we get

$$C = B\log_2(1 + S/BN_0)$$

$$C = \cancel{B}\cdot\frac{S}{N_0\cancel{B}}\cdot\log_2\left(1+\frac{S}{BN_0}\right)^{BN_0/S}$$

Let $\frac{S}{BN_0} = x$ when $B\to\infty$, $x\to 0$

$$C = \frac{S}{N_0}\log_2(1+x)^{1/x}$$

$$C = \frac{S}{N_0} \lim_{x \to 0} \log_2 (1+x)^{1/x}$$

$$= \frac{S}{N_0} \log_2 e \qquad \left[\lim_{x \to 0} (1+x)^{1/x} = e\right]$$

$$\boxed{C = 1.44 \frac{S}{N_0}}$$

* Line Codes

Line code is the process in which the output of the Multiplexer of digital signal are coded into electric pulse or waveform for transmission over channel or medium. There are different types of Line Codes. They are

1. Unipolar RZ :→

Unipolar RZ format has a single polarity. For unipolar RZ 'A' volt is transmitted for $T_b/2$ and returns to '0' for another half $T_b/2$ when '1' is to be transmitted. Similarly wave form has zero value when symbol '0' is transmitted for complete symbol duration.

Mathematically, $x(t) = A$ for $0 \le t < T_b/2$ (Half Interval)

$= 0$ for $T_b/2 \le t < T_b$ (Half Interval)

If symbol '0' is transmitted

$x(t) = 0$ for $0 \le t \le T_b$ (complete Interval)

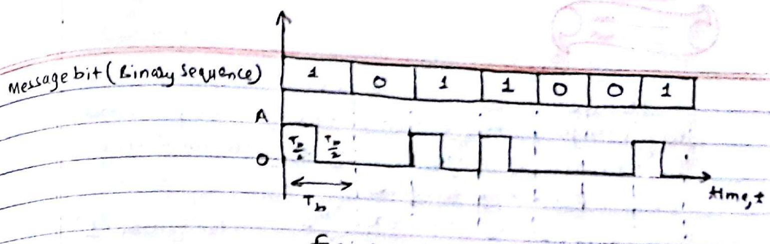


fig → Unipolar RZ format

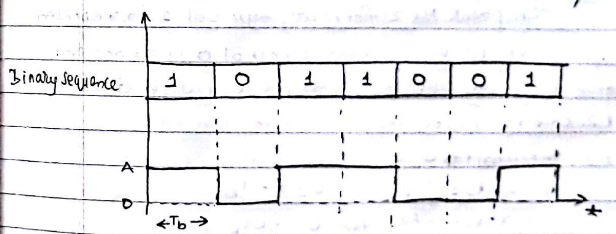
2. Unipolar NRZ :→ waveform and Expression.

Unipolar Not Return to zero (NRZ) has a single polarity. Here 'A' volt is transmitted for full duration when symbol 1 is to be transmitted and '0' volt for symbol 0.

It can be expressed as

$x(t) = A$ for $0 \le t < T_b$ (Complete Interval)

$x(t) = 0$ for $0 \le t < T_b$ (Complete Interval)



3) Polar RZ :→

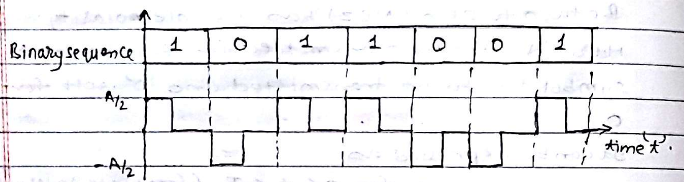
In this format, symbol 1 is represented by positive voltage polarity and symbol 0 is represented by negative voltage polarity only for half duration.

Polar RZ is expressed as

$$x(t) = \begin{cases} A/2 & \text{for } 0 \le t \le T_b/2 \\ 0 & \text{for } T_b/2 \le t < T_b \end{cases} \quad \text{(For symbol 1)}$$

For transmission of symbol 0

$$x(t) = \begin{cases} -A/2 & \text{for } 0 \le t < T_b/2 \\ 0 & \text{for } T_b/2 \le t < T_b \end{cases}$$



4) Polar NRZ :→

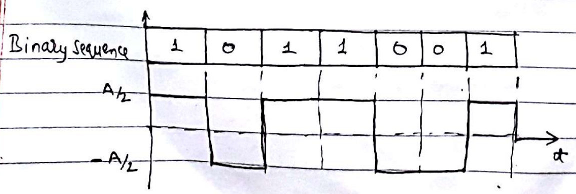
In polar NRZ format, symbol 1 is represented by positive polarity and symbol 0 is represented by negative polarity over the complete pulse duration.

Polar NRZ is expressed as

symbol 1 is transmitted

$$x(t) = A/2 \quad \text{for } 0 \le t < T_b$$

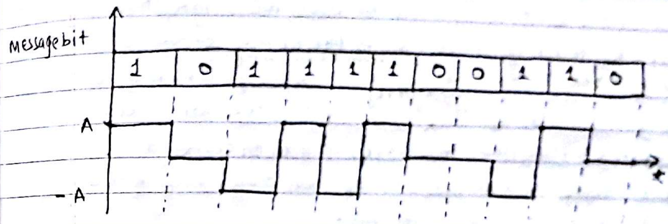
$$x(t) = -A/2 \quad \text{for } 0 \le t < T_b$$



Both polar RZ and NRZ are Bipolar format.

5) Bipolar NRZ [Alternate Mark Inversion (AMI)]

AMI is also known as Bipolar NRZ. In this format, the successive '1' are represented by pulses with alternate polarity and '0' are represented by no pulses.



6) Split phase Manchester Format :→

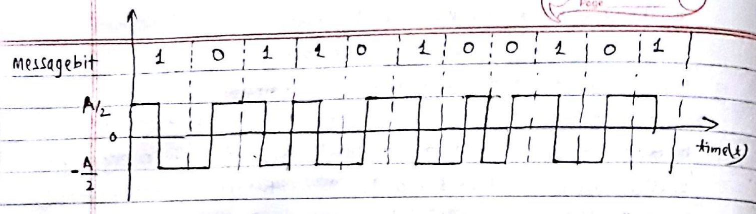
In split phase Manchester format symbol 1 is transmitted by a positive half interval pulse followed by a negative half interval pulse. For symbol '0' negative half interval pulse is followed by a positive half interval pulse.

Symbol 1 is transmitted for

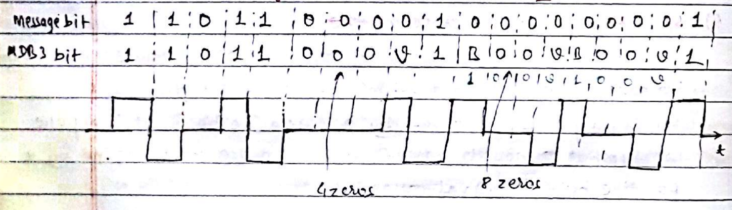
$$x(t) = \begin{cases} A/2 & \text{for } 0 \le t < T_b/2 \\ -A/2 & \text{for } T_b/2 \le t < T_b \end{cases}$$

For symbol 0

$$x(t) = \begin{cases} -A/2 & \text{for } 0 \le t < T_b/2 \\ \frac{A}{2} & \text{for } \frac{T_b}{2} \le t < T_b \end{cases}$$



7. HDB3 Code :→ It is also known as High Density Bipolar coding and is an European standard. It is denoted by HDBN. The most widely used format is HDB3. In AMI coding scheme, if long sequence of 0's (N+1) are transmitted will cause problem in synchronization. To overcome this problem special sequences of code 000V and B00V are inserted which is shown below
[Note → value of B & V are considered '1']

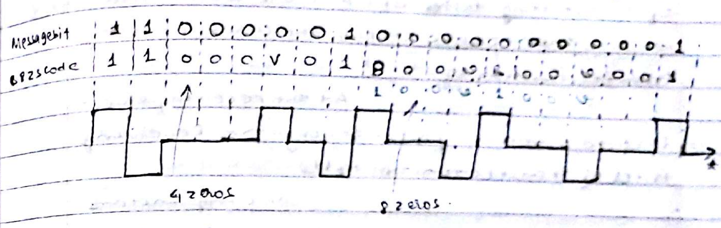


8) B8ZS Line Code :→

B8ZS is known as Binary 8-zero suppression and is U.S standard.

B8ZS coding also overcomes the synchronization problem caused by the successive transmission of 0's in AMI.

Special sequence of code 000V and B00V whose both value B & V = 1 are inserted in the group of four 0's and eight 0's to ensure proper synchronization



4.4 Base Band Data Communication Systems

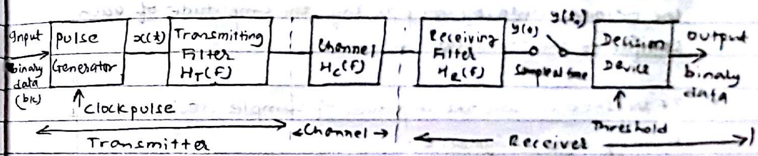


fig :→ Baseband Binary PAM system

Above figure shows the typical arrangement of a baseband binary PAM system. The input binary data is the sequence of data represented by 'b_k' with duration of time 'T_b' seconds is in the form of 1 and 0.

Output of pulse Generator is in the pulse waveform and given by

$$x(t) = \sum_{k=-\infty}^{\infty} A_k\, g(t - kT_b)$$ where $g(t)$ is unmodulated pulse of rectangular shape.

A_k ⇒ Amplitude of the pulse waveform which is

$A_k = +a$ If the input (b_k) is symbol 1.

$A_k = -a$ If the input (b_k) is symbol 0.

The PAM signal $x(t)$ then passes through the transmitting filter having transfer function of $H_T(f)$. The output of transmitting filter defines the transmitted signal which is then passed through the channel having transfer function of $H_c(f)$.

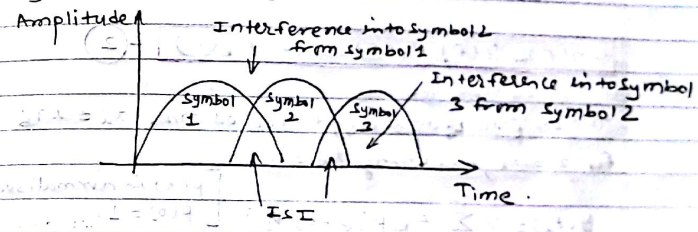
At the receiving side Received signal is passed through the Receiving filter of transfer function $H_e(f)$

Sampling Instants are extracted in the receiving filter and are fed to the decision device. Decision device then reconstructs the original data by compairing the amplitude of each sample to the threshold.

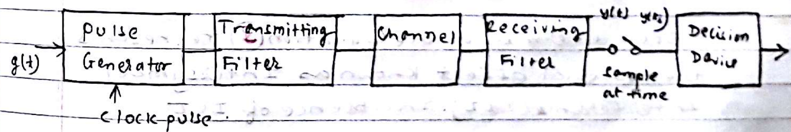
- If threshold is above the amplitude of sample, device decides incoming data is symbol 1.
- If threshold is below the amplitude of sample, device decides incoming data is symbol 0.
- If threshold is equal to the amplitude of sample, any one symbol 1 or 0 is chosen without affecting the overall performance.

* Inter Symbol Interference (ISI) :-

Inter symbol Interference (ISI) is a form of distortion of a signal in which one symbol interferes with subsequent symbol due to dispersion. This spreading and covering of symbols to each other (Interference) will be interpreted incorrectly at the receiving side. Therefore ISI is the major limiting factor for any communication system.



Considering noise less channel, distortion can also be occured due to the dispersion of pulse. For Base Band Binary PAM system.



If $g(t)$ is unmodulated pulse of rectangular shape fed to the pulse generator. $A_k\, g(t)$ is the excitation pulse to the transmitting filter producing response of $\mu A_k p(t)$.

where, $\mu \Rightarrow$ scaling factor. $A_k \Rightarrow$ Amplitude of pulse waveform.

$p(t) \Rightarrow$ Transmitted pulse.

$i = 0, \pm 1, \pm 2, \pm 3$

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Therefore Receiving filter output is

$$\boxed{y(t) = \mu \sum_{k=-\infty}^{\infty} A_k\, p(t - kT_b - t_d)} \quad -(1)$$

$t_d \Rightarrow$ time delay introduced in the channel

$T_b \Rightarrow$ Bit duration of 'T_b' second

$A_k \Rightarrow$ Amplitude depends on the identity of b_k (0 or 1)

For Frequency domain we can write

$$\boxed{\mu\, p(f) = G(f)\, H_T(f)\, H_c(f)\, H_R(f)} \quad -(2)$$

The output of Receiving filter at time $t_i = iT_b$ for $t = t_i$, assuming $t_d = 0$

$$y(t_i) = \mu \sum_{k=-\infty}^{\infty} A_k\, p[(i-k)T_b] \qquad \left[\begin{array}{l} p(t) \text{ in normalised form} \\ p(0) = 1 \end{array}\right]$$

$$\boxed{y(t_i) = \mu A_i + \mu \sum_{\substack{k=-\infty \\ k \neq i}}^{\infty} A_k\, p[(i-k)T_b]} \quad -(3)$$

Second term in above equation (3) represents the residual effect known as Intersymbol Interference (ISI). In absence of ISI

$\boxed{y(t_i) = \mu A_i}$ is known as Nyquist condition for zero ISI which is then decoded correctly at the receiver.

(1) which illustrates

$$p(i-k)T_b = \begin{cases} 1 & \text{for } i = k \\ 0 & \text{for } i \neq k \end{cases} \quad -(4)$$

* Frequency Domain representation

Let us consider sequence of sample represented by $p(nT_b)$ where $n = 0, \pm 1, \pm 2 \ldots$ after the process of extraction. The frequency domain representation of sequence of sample $p(nT_b)$ is

$$P_\delta(f) = R_b \sum_{n=-\infty}^{\infty} p(f - nR_b) \quad -(1) \qquad \left[\begin{array}{l} \text{where } R_b = \text{bit rate} \\ \text{given as } R_b = \frac{1}{T_b} \end{array}\right]$$

As $P_\delta(f)$ also represents the fourier transform of an infinite periodic sequence of unit impulse for respective sample value of $p(t)$.

$$P_\delta(f) = \int_{-\infty}^{\infty} \sum_{m=-\infty}^{\infty} [p(mT_b)\, \delta(t - mT_b)]\, e^{-j2\pi ft}\, dt \quad -(2)$$

Let $m = i - k$, If $i = k$, $m = 0$

and If $i \neq k$ $m \neq 0$

Again, For $i = k$

$$P_\delta(f) = \int_{-\infty}^{\infty} p(0)\, \delta(t)\, e^{-j2\pi ft}\, dt$$

$P_\delta(f) = p(0)$ [For shifting property of delta function)

$P_\delta(f) = 1$ (Since $p(0) = 1$ for normalization)

Therefore substituting this value in eqn (1) we get

$$1 = R_b \sum_{n=-\infty}^{\infty} p(f - nR_b)$$

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$$\sum_{n=-\infty}^{\infty} P(f - nR_b) = \frac{1}{R_b} = T_b$$

$$\boxed{\sum_{n=-\infty}^{\infty} P(f - nR_b) = T_b}$$ is called the Nyquist Criterion for distortionless baseband transmission in the absence of noise.

* Methods to reduce ISI (Ideal Solution)

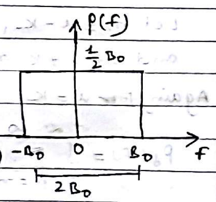
For the sample whose frequency function P(f) extends has range of frequencies $-B_0$ to B_0.

where B_0 is the Nyquist bandwidth equal to the minimum transmission bandwidth needed for zero Intersymbol Interference (ISI) given as

$$\boxed{B_0 = \frac{R_b}{2}}$$ $R_b \Rightarrow$ Bit Rate

P(f) can be defined as

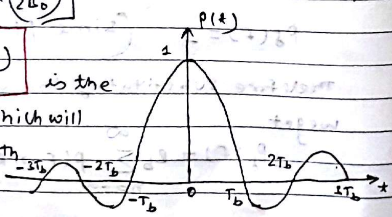
$$\boxed{P(f) = \frac{1}{2B_0}\, rect\left(\frac{f}{2B_0}\right)}$$



Taking Inverse Fourier Transform (IFT) we get

$$p(t) = F^{-1}\left[\frac{1}{2B_0}\, rect\left(\frac{f}{2B_0}\right)\right]$$

$$\boxed{p(t) = sinc(2B_0 t)}$$ is the Nyquist pulse shaping which will reduce ISI to zero with minimum possible bandwidth.



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However, there are number of practical difficulties.

1) For ideal pulse shape, the frequency response P(f) is needed to be flat in the range $-B_0$ to $+B_0$ and zero elsewhere. This may not be achieved practically as abrupt transition may occur.

2) practically no margin of errors is considered in sampling times of the receiver.

* Raised Cosine spectrum :→

The practical difficulties ~~achieved~~ perceived by Nyquist pulse shaping (Ideal Nyquist channel) can be overcome by increasing the bandwidth to the adjustable value between B_0 to $2B_0$.

To satisfy the above limitation the overall frequency response P(f) consisting of flat portion and roll of portion can be defined with raised cosine function given as

$$P(f) = \begin{cases} \frac{1}{2B_0} & 0 \le |f| \le f_1 \\ \frac{1}{4B_0}\, 1 - \sin\left(\frac{\pi(|f| - B_0)}{2B_0 - 2f_1}\right) & f_1 \le f \le 2B_0 - f_1 \\ 0 & |f| \ge 2B_0 - f_1 \end{cases}$$

The frequency f_1 and the Nyquist bandwidth are related as

$$\boxed{\alpha = 1 - \frac{f_1}{B_0}}$$

where α is the rolloff factor which shows abrupt variation for minimum bandwidth.

For $\alpha = 0$, we get $f_1 = B_0$ which shows abrupt variation and the frequency response of $p(f)$ normalized by multiplying it by $2B_0$ is plotted for three values of α

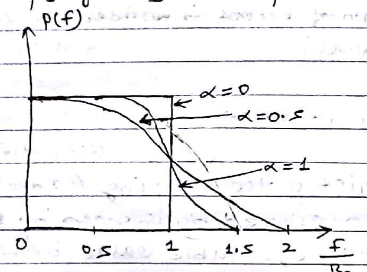


fig:→ Frequency Response

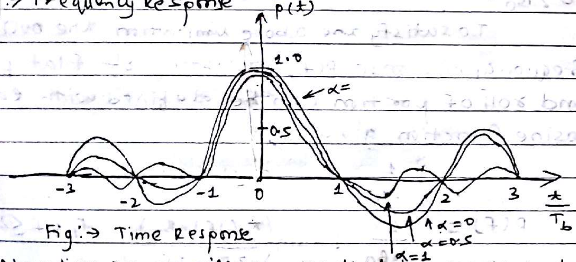


Fig:→ Time Response

Above time response illustrates that the amplitude of side lobe increases with the reduction of the rollfactor (α). The Sinc Shape (time response) passes through 0 at $\pm T_b, \pm 2T_b, \pm 3T_b$.

As we have seen in Ideal solution the bandwidth required is minimum and is equal to B_0 (Nyquist Bandwidth). To overcome the difficulties the frequency spectrum $p(f)$ by the use of raised cosine spectrum is limited from 0 to $2B_0 - f_1$ for positive frequency.

Therefore transmission bandwidth required is

$$B = 2B_0 - f_1 \qquad \because \alpha = 1 - \frac{f_1}{B_0}$$

$$B = 2B_0 - B_0 + \alpha B_0$$

we obtain

$$f_1 = (1-\alpha)B_0$$

$\boxed{\text{Tx Bandwidth } (B) = B_0(1+\alpha)}$ $B_0 \Rightarrow$ Nyquist bandwidth

Expression clearly shows that transmission bandwidth required for raised cosine solution exceeds ideal solution by amount 'αB_0'.

.5 - Correlative Coding techniques

Intersymbol Interference (ISI) is viewed as undesirable phenomenon that produces a degradation in the system performance.

However, ISI can also be used in controlled manner to achieve signalling rate higher than the bandwidth of the channel. Therefore Correlative Coding which is also known as partial response signaling is a technique for transmitting signal at rate of $2B_0$ symbols/second in a channel of bandwidth 'B_0' Hz with no practical difficulties encountered in Ideal solution. The simplest form of Correlative coding techniques are

1. Duobinary Encoders :→
2. Modified Duobinary Encoder :→

1 Duobinary Encoders :→

Duobinary Encoders uses Duobinary signaling - is the simplest type of Correlative Coding which doubles the transmission capacity as compared to the ordinary binary system.

Consider a binary input sequence 'b_K' of uncorrelated digits of duration 'T_b' seconds. Let symbol 1 be represented by a pulse of amplitude +1V and symbol 0 by a pulse amplitude -1V.

When this sequence is applied to duobinary encoder, it is converted into a three level of output of -2V, 0V and +2V. The output of duobinary encoder is expressed as

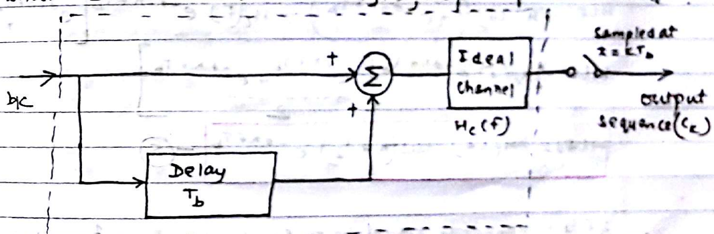
$$C_K = b_K + b_{K-1} \quad — (1)$$

$b_K \Rightarrow$ present binary digit

$b_{K-1} \Rightarrow$ previous binary digit.

$$C_K = \begin{cases} 2V & \text{for } b_K = b_{K-1} = 1 \\ 0 & \text{for } b_K \neq b_{K-1} \\ -2V & \text{for } b_K = b_{K-1} = 0 \end{cases} \quad — (2)$$

Which clearly illustrates that C_K is the correlated digits, output of the Encoder.



Duo Binary Conversion Filter H(f)

fig:→ Block Diagram of Duo Binary Encoder

Above figure shows the block diagram of Duo Binary Encoder. The DuoBinary encoder consist of direct path and parallel path with ideal element producing delay of duration 'T_b' to the Adder. It is then fed to the Ideal channel of transfer function of $H_c(f)$ whose output is sampled uniformly in every 'T_b' second thereby producing duobinary encoded sequences 'C_K'.

Now, overall transfer function of Duobinary Encoder is

$H(f) = H_d(f)\, H_c(f)$ —③ $H_d(f) \Rightarrow$ Transfer function of Loop Filter.

Again, Transfer function of Loop filter is

$$H_d(f) = 1 + \exp(-j2\pi f T_b) \quad -④$$

Note:→ Time shifting property of Fourier transform. [Ideal Delay Element] transfer function

$$H(f) = H_c(f)\left[1 + \exp(-j2\pi f T_b)\right]$$

$$H(f) = H_c(f)\left(1 + e^{-j2\pi f T_b}\right) = H_c(f)\, e^{-j\pi f T_b}\left[e^{j\pi f T_b} + e^{-j\pi f T_b}\right]$$

$$H(f) = 2H_c(f)\, e^{-j\pi f T_b}\left[\frac{e^{j\pi f T_b} + e^{-j\pi f T_b}}{2}\right]$$

$$\boxed{H(f) = 2H_c(f) \cos(\pi f T_b)\, e^{-j\pi f T_b}} \quad -⑤$$

If the channel is Ideal with Bandwidth $B_0 = \frac{1}{2T_b}$ then

$$H_c(f) = 1 \quad \text{for } |f| \le \frac{1}{2T_b}$$
$$= 0 \quad \text{otherwise}$$

Overall frequency response of the duobinary encoder is

$$H(f) = 2\cos(\pi f T_b) \exp(-j\pi f T_b) \quad |f| \le \frac{1}{2T_b}$$
$$= 0 \quad \text{otherwise.}$$

Again taking Inverse Fourier transform of eqn ⑤ we get

$$\boxed{h(t) = \frac{T_b^2 \sin(\pi t / T_b)}{\pi t (T_b - t)}}$$

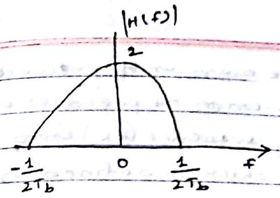


fig:→ Frequency Response

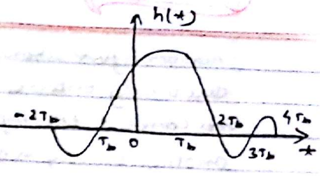


fig:→ Impulse Response

At the receiving side the original data 'b_k' can be recovered from the duobinary encoded data 'c_k' by substracting the previous decoded binary digit from the currently received digit. Let $\hat{b}_k$ represent the estimated data 'b_k' at $t = kT_b$ at the reciever.

$$\boxed{\hat{b}_k = c_k - \hat{b}_{k-1}}$$

$\hat{b}_{k-1}$ is the estimate at $t = (k-1)T_b$ which has to be very precised. If one bit has error, then it will effect all the other bits and will introduce propagation of error. To overcome this error precoder is used before Duobinary Encoders.

* Precoder :→

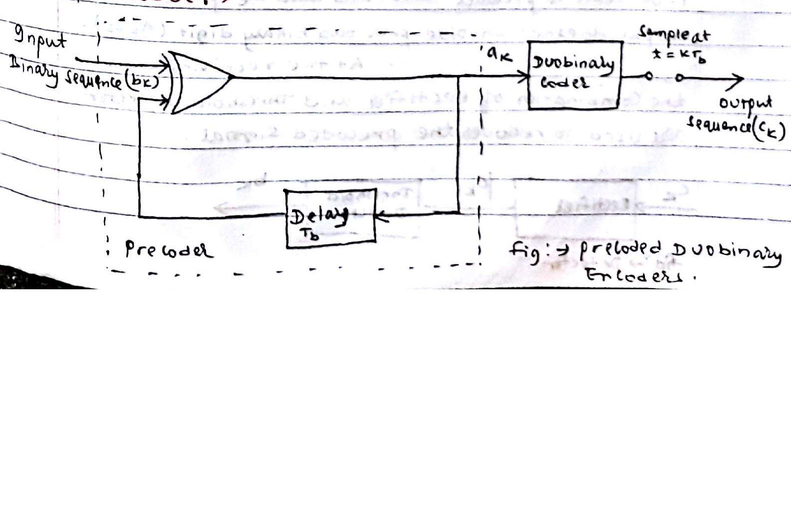


fig:→ Precoded Duobinary Encoders.

Above figure shows the block diagram of precoded duobinary Encoders. The function of precoder is to convert the input binary sequence (b_k) into another binary sequence a_k which is defined as

$$\boxed{a_k = b_k \oplus a_{k-1}} \quad -(1)$$

Note :→
$\oplus$ Modulo two addition
x-OR operation

Here extra bit is needed in precoded sequence (a_k). Extra bit may be 1 or 0.

The precoded output is then passed to the Duobinary Coder there by producing the sequence (c_k) that is related to a_k as

$$\boxed{c_k = a_k \oplus a_{k-1}} \quad -(2)$$

b_k	a_{k-1}	c_k
0	0	0
0	1	1
1	0	1
1	1	0

which verifies as

$$\boxed{\begin{aligned} c_k &= \pm 2V \quad \text{if } b_k = 0 \\ c_k &= 0 \quad \text{if } b_k = 1 \end{aligned}} \quad -(3)$$

which clearly

Truth table for precoder illustrates that the precoded duobinary output doesnot involve previous binary digit (a_{k-1}).

At the receiving side the Combination of Rectifier and Threshold Detector are used to recover the precoded signal.

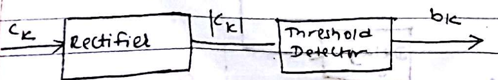


fig:→ Detector

The precoded signal c_k is fed to the Rectifier and the output of rectifier is compared to the threshold voltage of 1V to recover the original binary sequence.

$$\boxed{\begin{aligned} \hat{b}_k &= \text{symbol } 0 \quad \text{if } |c_k| > 1V \\ &= \text{symbol } 1 \quad \text{if } |c_k| < 1V \end{aligned}} \quad -(4)$$

* Consider the following input binary sequence applied to a precoded duobinary system [Num Example of Duobinary
b_k = 01011010
Show that the original binary sequence can be detected in the absence of noise irrespective of the choice of extra bit.

Input binary sequence (b_k)	0 1 0 1 1 0 1 0
Precoded binary sequence (a_k)	1 1 0 0 1 0 0 1 1
Voltage Representation of (a_k)	+1 +1 -1 -1 +1 -1 -1 +1 +1
Voltage Representation of duobinary Coder output c_k	2 0 -2 0 0 -2 0 2
Decoded Binary sequence $\hat{b}_k$	0 1 0 1 1 0 1 0

← -ve for
→ look eqn (1)

Table for precoded duobinary sequence for 1 as extra bit:

b_K		0	1	0	1	1	0	1	0
a_K	0	0	1	1	0	1	1	0	0
Voltage Representation a_K	-1	-1	+1	+1	-1	+1	+1	-1	-1
$\hat{c}_K$		-2	0	2	0	0	+2	0	-2
$\hat{b}_K$		0	1	0	1	1	0	1	0

* Modified Duobinary Encoders :→

Duobinary signalling is appropirate for channel which has good d.c response. Similarly the transfer function $H(f)$ and the power spectral density (PSD) of the transmitted pulse in the system is not zero at the origin.

Modified Duobinary Encoders overcome the limitation of duobinary encoder and can work very well for channel of a.c response. It is best suited for channels that donot support transmission of ac component.

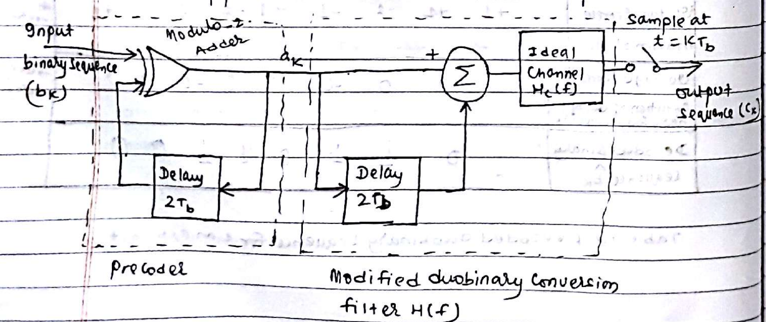


fig:→ Modified Duobinary signalling scheme

Above figure shows the block diagram of modified duobinary encoder along with an appropirate precoder and ideal bandlimited channel.

Consider a binary sequence b_K of uncorrelated digits of duration $2T_b$ seconds. Let symbol 1 be represented by a pulse of amplitude +1V and symbol 0 by a pulse amplitude -1V. When this sequence is applied to the Modified duobinary Encoders, it is converted into a three level of output of 2, 0 and -2V.

The output of precoder is fed to the channel and further sampled uniformly for every T_b seconds to produce the modified duobinary encoded sequence 'c_K'

$$\boxed{c_K = a_K - a_{K-2}} \quad \text{--- (1)}$$

$$\begin{aligned} c_K &= 2V \quad \text{for } a_K = a_{K-2} = 1 \\ &= 0 \quad \text{for } a_K \neq a_{K-2} \\ &= -2V \quad \text{for } a_K = a_{K-2} = 0 \end{aligned} \quad \text{--- (2)}$$

Let $H(f)$ be the transfer function of modified duobinary conversion filter, $H_d(f)$ delay line filter and $H_c(f)$ Ideal channel respectively. The overall transfer function of the Modified duobinary filter is

$$\begin{aligned} H(f) &= H_d(f)\, H_c(f) \\ &= H_c(f)\left[1 - \exp(-j4\pi f T_b)\right] \\ &= H_c(f)\left[e^{j2\pi f T_b} - e^{-j2\pi f T_b}\right] e^{-j2\pi f T_b} \end{aligned}$$

Scanned by CamScanner

$$H_c(f) = 2j\,H_c(f)\left[\frac{e^{j2\pi f T_b} - e^{-j2\pi f T_b}}{2j}\right] e^{-j2\pi f T_b}$$

$$H_c(f) = 2j\,H_c(f)\,\sin(2\pi f T_b)\, e^{-j2\pi f T_b}$$

$$\boxed{H(f) = 2j\,H_c(f)\,\sin(2\pi f T_b)\exp(-j2\pi f T_b)} \quad -(3)$$

For an ideal channel of Bandwidth $B_0 = \frac{1}{2T_b}$

$$H_c(f) = 1 \qquad |f| \le \frac{1}{2T_b}$$

$$= 0 \quad \text{otherwise}$$

$$\therefore \boxed{\begin{aligned} H(f) &= 2j\sin(2\pi f T_b)\exp(-j2\pi f T_b), \quad |f| \le \frac{1}{2T_b} \\ &= 0 \quad \text{otherwise} \end{aligned}} \quad -(4)$$

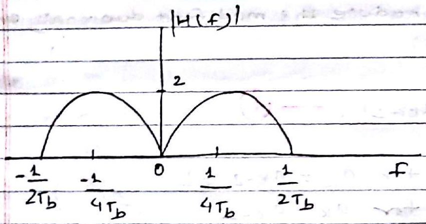


fig:→ Frequency response of modified duobinary Encoder.

Again, Taking Inverse Fourier Transform (I.F.T) of eqn (3) we get

$$\boxed{h(t) = \frac{2T_b^2 \sin(\pi t/T_b)}{\pi(2T_b - t)}} \quad -(5)$$

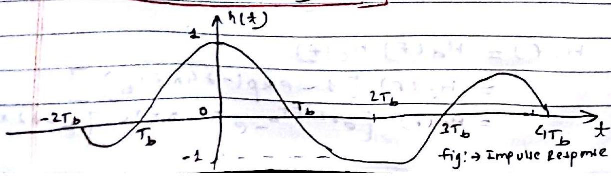


fig:→ Impulse Response

In order to eliminate the possibility of error propagation in the modified duobinary system, precoder are used which is clearly illustrated in above figure.

Prior to the input of Modified duobinary, precoder converts the binary input sequence (b_k) into another binary sequence given as

$$\boxed{a_k = b_k \oplus a_{k-2}} \quad -(6)$$

The sequence 'a_k' is the input of the final modified duobinary filter. Here the output c_k is defined as

$$\boxed{\begin{aligned} c_k &= 0 \\ &= \pm 2V \end{aligned}} \quad \begin{aligned} &\text{if } b_k \text{ is represented by symbol 0} \\ &\text{〃 〃 〃 〃 〃 Symbol 1} \end{aligned} \quad -(7)$$

Now, the decoded digit $\hat{b}_k$ at the receiver output are extracted from c_k by

$$\boxed{\begin{aligned} \hat{b}_k &= \text{Symbol 1} \\ &= \text{Symbol 0} \end{aligned}} \quad \begin{aligned} &\text{if } |c_k| > 1V \\ &\text{if } |c_k| \le 1V \end{aligned} \quad -(8)$$

The decoded sequence are exactly same as that of original binary sequence in the absence of noise in the channel. However two extra bits are added to the precoded sequence a_k in Modified duobinary Encoders.

Example :→

Consider a binary sequence b_k = 01001101 applied to the input of a precoded modified duobinary sequence. Determine

a. sequence a_k at the precoder output

b. Voltage representation of a_k assuming a symbol 1 to be represented by +1 V and 0 by -1 V

c. The sequence c_k at the output of the modified duobinary filter, assuming the additional bit to be 11 at the begining of the precoded sequence a_k

d. The decoded sequence $\hat{b}_k$ at the receiver output. Compare the sequence with the original binary sequence 'b_k'.

Input binary sequence b_k			0	1	0	0	1	1	0	1
Precoded binary sequence a_k	1	1 1	0	1	0	0	1	0	0	
Voltage representation of a_k	+1	+1	+1	-1	+1	-1	-1	+1	-1	-1 ← consider +, or - for + -
" " duobinary coder o/p c_k			0	-2V	0	0	-2V	2V	0	-2V ← Look eq^n (7)
Decoder binary sequence $\hat{b}_k$			0	1	0	0	1	1	0	1

Repeat with 00

Input binary sequence b_k			0	1	0	0	1	1	0	1
Precoded binary sequence a_k	0	0	0	1	0	1	1	0	1	1
Voltage representation of a_k	-1	-1	-1	+1	-1	+1	+1	-1	+1	+1
" " duobinary coder o/p c_k			0	2V	0	0	2V	-2V	0	2V
Decoder binary sequence $\hat{b}_k$			0	1	0	0	1	1	0	1

Type of Line coding: several ways of representing waveforms for digital data.

4.6 M-ary signaling, Comparision with binary signaling

1) Binary signaling :→

In Binary signaling the output of the pulse generator can have one or of two possible levels. For simplicity two symbols is used, in which the code is 'ON' for binary representation of signal 1 and their is no pulse or no transmission of signal for value '0'. eg :→ Line codes (Unipolar RZ, polar RZ ... AMI all of them)

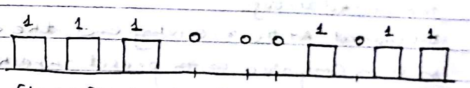


fig :→ Binary signaling

2) Multilevel signaling (M-ary signaling) :→

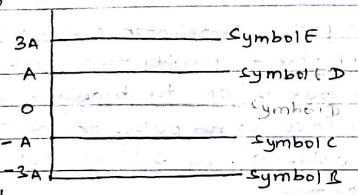
As we have studied, the output of pulse generator have any one of two possible amplitude levels which is very clearly shown by Line codes.

In M-ary version of the system, the output of pulse generator takes any one of the M possible amplitude level with M > 2. Here, the blocks of n-message bits are represented by an M-level waveform with

$$\boxed{M = 2^n} \quad \text{—— (1)}$$

For example for the input bits [0 0 10 1101] can be represented by various line codes which we have studied previously.

(quaternary system)
Here, $M=4$, The input bits are grouped in blocks of two using four amplitude levels to represent four possible combinations 00, 01, 10, and 11. The duration of quaternary system is $T = 2T_b$. This levels are then encoded and transmitted through the channel.



At the receiving end, the decoded output is then compared with preset threshold value and decision is made.

Forme
(PAM) Let us consider an M-ary signaling system having signaling alphabet that contains M symbols. Let the symbol duration be 'T' seconds. The signaling rate is given by

$$r = \frac{1}{T} \text{ symbols/sec or Bauds} \quad -(2)$$

∴ for M-ary coding each pulse corresponds to a block of n message bits given by

$$n = \log_2 M \quad -(3) \qquad \boxed{T = T_b \log_2 M}$$

similarly, M-ary signaling rate is

$$\boxed{r_s = r_b \log_2 M} \quad -(4)$$

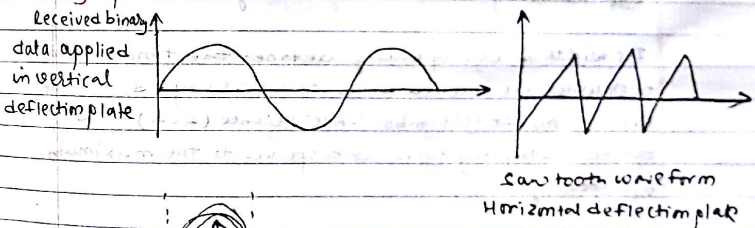
$r_s \Rightarrow$ signaling rate of M-ary signaling
$r_b \Rightarrow$ signaling rate of binary system.

Above eqn clearly shows that for a given channel bandwidth, it is possible to transmit data at a rate of $\log_2 M$ times faster than the corresponding binary system.

* Eye pattern :→

It is the pattern displayed on the screen of oscilloscope to study the performance of baseband signal. It is the practical way to study Intersymbol Interference (ISI) and its effect on PCM.

Here the distorted received wave is applied to the vertical deflection plate and the saw tooth wave at the rate equal to the transmitted symbol rate $1/T$ in Horizontal deflection plate. The waveforms in successive symbols interval are then translated into one interval on the oscilloscope display producing the pattern resemblance to human eye known as Eye pattern.



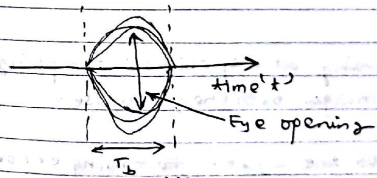


fig:→ pattern seen on oscilloscope

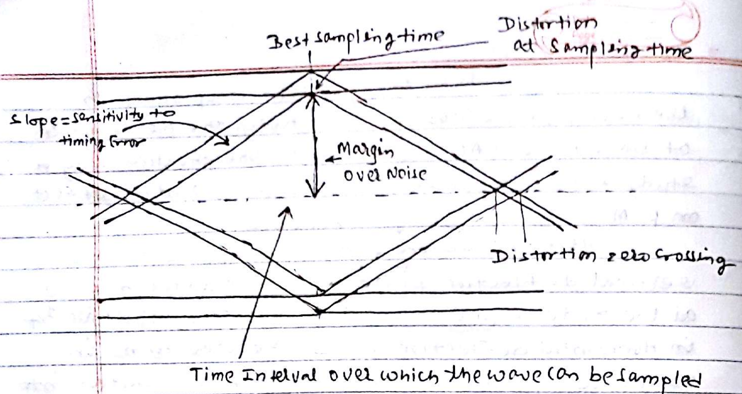


fig:⇒ Interpretation of Eye pattern.

Eye pattern provides the following information

a. The width of eye opening defines the time interval over which the received wave can be sampled without error from Intersymbol Interference (ISI). The optimum sampling time corresponds to the maximum eye opening.

b. The height of opening at a specified sampling time is a measure of margin over channel noise.

c. The sensitivity of the system to timing error is determined by the rate of closure of the eye of the eye pattern as the sampling time is varied.

d. Any non linear transmission distortion will be postured to assymetric eye pattern.

e. When the effect of ISI is excessive, traces from the upper portion of the eye pattern crosses the traces from lower portion resulting complete close of the pattern.

1) A source with BW 4000Hz is sampled at Nyquist rate. Assuming that the resulting sequence can be approximated by a DMS with alphabets (a, b, c, d, e) with probability 1/2, 1/4, 1/8, 1/16. Determine the rate of source in bits/sec

Entropy is given as

$$H(X) = \sum_{i=1}^{m} P_i \log_2\left(\frac{1}{P_i}\right) = \frac{15}{8} \text{ bits/symbol}$$

$f_m = 4000Hz$

Sampling frequency $(f_s) = 2 \times 4000Hz = 8000Hz$. $r = 8000$ symbol/sec

Information Rate (R) = Entropy × Symbol rate (r)

Information Rate (R) = $\frac{15}{8} \times \overset{1000}{\cancel{8000}} = 15000$ bps

2) An event has six possible outcomes with probabilities $P_1 = 1/2$, $P_2 = 1/4$, $P_3 = 1/8$, $P_4 = 1/16$, $P_5 = 1/32$, $P_6 = 1/32$. Find the entropy of the system and find the rate of information if there are 16 outcomes per second.

Symbol Rate $r = 16$ symbols/sec

Information Rate (R) = Entropy × Symbol Rate

3) A discrete source emits one of five symbol once every millisecond with probabilities $1/2, 1/4, 1/8, 1/16, 1/16$ respectively. Find the source entropy and information rate.

Given:→

$p(x_1) = 1/2 \quad p(x_2) = 1/4 \quad p(x_3) = \frac{1}{8} \quad p(x_4) = \frac{1}{16} \quad p(x_5) = \frac{1}{16}$

Entropy $H(x) = \frac{1}{2}\log_2(2) + \frac{1}{4}\log_2(4) + \frac{1}{8}\log_2(8) + 2\times\frac{1}{16}\log_2(16)$

Entropy $H(x) = 1.875$ bits/symbols

Sampling period $(T_s) = 1\ ms = 10^{-3}$ second

Symbol Rate $(r) = \frac{1}{T_s} = \frac{1}{10^{-3}} = 1000$ symbols/sec

Information rate (R) = Entropy × Symbol Rate = 1875 bits/sec

4. An Analog Signal bandlimited to 10KHz is quantized in 8 levels of a PCM system with probabilities of $1/4, 1/5, 1/5, 1/10, 1/10, 1/20, 1/20$ and $1/20$ respectively. calculate the entropy and the rate of information.

Given:→ $f_m = 10\,KHz$, $p(x_1) = 1/4$, $p(x_2) = 1/5$, $p(x_3) = 1/5$, $p(x_4) = \frac{1}{10}$

$p(x_5) = 1/10$, $p(x_6) = 1/20$ $p(x_7) = 1/20$ $p(x_8) = 1/20$

∴ Entropy $H(x) = p(x_1)\log_2\left(\frac{1}{p(x_1)}\right) + \cdots$

Entropy $H(x) = \frac{1}{4}\log_2(4) + \frac{2}{5}\log_2(5) + \frac{2}{10}\log_2(10) + \frac{3}{20}\log_2(20)$

Entropy $H(x) = 2.84$ bits/symbol

Sampling frequency $(f_s) = 2 f_m = 20\,KHz$

Symbol rate $(r) = 20000$ symbols/sec

Rate of Information (R) = Entropy $H(x)$ × symbol rate (r)
or Information rate
$= 2.84 \times 20000$
$= 56800$ bits/sec.