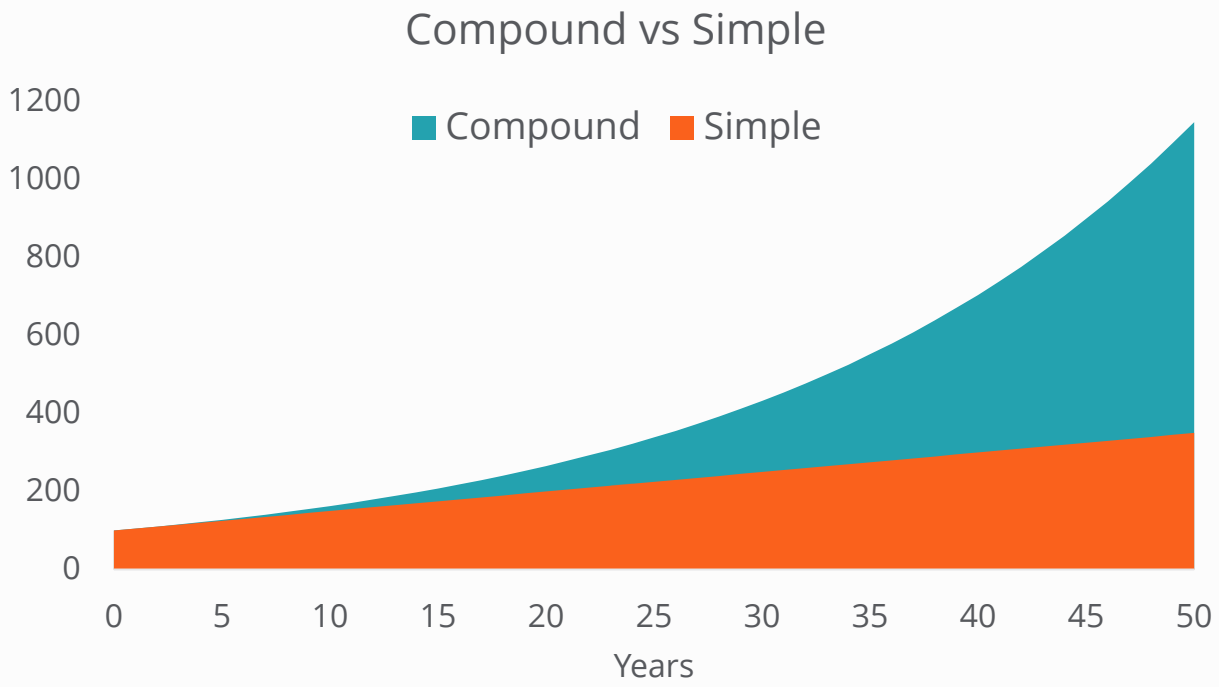
Math Fundamentals for Capital Markets



Nominal Interest vs Effective Interest

$$r_{eff} = \left(1 + \frac{r_{nom}}{f}\right)^{f} - 1$$

Year	Discount Factor
1	0.9524
2	0.9070
3	0.8638
4	0.8227
5	0.7835
Total	**4.3295**

Annuity Factor

$$PV = \frac{FV}{\left(1 + \frac{i}{f}\right)^{n \times f}}$$

$$FV = PV \times \left(1 + \frac{i}{f}\right)^{n \times f}$$

Learning Objectives

Calculate simple and compound interest rates

Convert values today into a future value and vice-versa

Understand how to calculate an annuity and what discounted cash flow is

Math Warm Up

Practice the **basic concepts of mathematics**.

X^2

Exponents / Roots

%

Percentages

$2X + 1 = Y$

Solving Equations

Simple Interest

Simple Interest Earned Each Period = Rate of Interest x Principal

Example: **Principal:** $100

Interest Rate: 5% per annum

Total Periods: 5 years

Year	Interest	Principal
1	$5	
2	$10	
3	$15	
4	$20	
5	$25	$100
Total	**$125**	

Future Value and Present Value

The value of a dollar **today** and the value of a dollar **a year from now** are different.

Opportunity cost of not having that dollar now

Inflation – purchasing power

Uncertainty – default risk

Time Value of Money

Future Value in a Simple Interest World

$$FV = PV \times (1 + i \times n)$$

FV: Future value

PV: Present value

i: Interest per period

n: Number of periods

Example: If you invest $100 today with a 5% interest rate per year. How much will you have in five years?

PV = $100

i = 5%

n = 5 years

FV = $100 x (1 + 5% x 5) = **$125**

Present Value in a Simple Interest World

$$PV = \frac{FV}{(1 + i \times n)}$$

PV: Present value

FV: Future value

i: Interest per period

n: Number of periods

Example: If you are going to receive $100 five years from now, how much is that worth today, assuming 5% annual interest?

FV = $100

i = 5%

n = 5 years

PV = $100 / (1 + 5% x 5) = **$80**

Compound Interest

Compound Interest Earned Each Period = **Rate of Interest** x (**Principal** + **Previously earned interest**)

Example:

Principal: $100

Interest Rate: 5% per annum

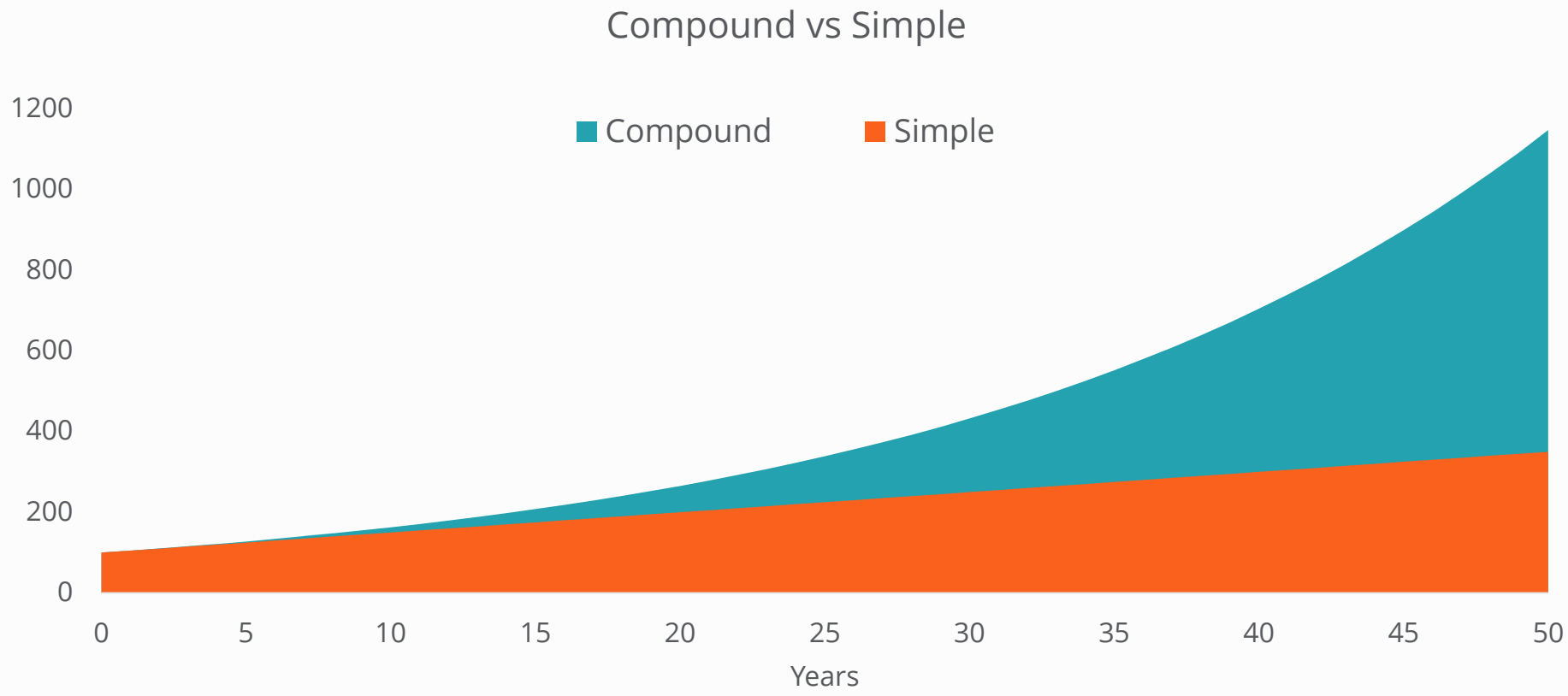
Total Periods: 5 years

Year	Total Interest	Principal + Interest	
0	$0	$100	X 5% = $5
1	$5	$105	X 5% = $5.25
2	$ 10.25	$ 110.25	X 5% = $5.51
3	$ 15.76	$ 115.76	X 5% = $5.79
4	$ 21.55	$ 121.55	X 5% = $6.08
5	$ 27.63	$ 127.63	
Total			

Simple Interest Versus Compound Interest

Example: **Principal:** $100

Interest Rate: 5% per annum



Compound Interest Frequency

Annual Compounding

Year	Total Interest	Principal + Interest
0	$0	$100
1	$5	$105
2	$10.25	$110.25
3	$15.76	$ 115.76

Semi-annual Compounding

Year	Total Interest	Principal + Interest
0	$0	$100
0.5	$2.5	$102.5
1	$5.06	$105.06
1.5	$7.69	$107.69
2	$10.38	$110.38
2.5	$13.14	$113.14
3	$15.97	$115.97

Nominal Rate and Effective Rate

Nominal Interest

Stated interest rate without compounding

Effective Interest

How much interest would be earned if the interest were compounding

Formula for converting **Nominal Rates (r_{nom})**, compounding at ***f*** times a year into an **Effective Rate (r_{eff})** is:

$$r_{eff} = \left(1 + \frac{r_{nom}}{f}\right)^{f} - 1$$

Math of Compounding

$$r_{eff} = \left(1 + \frac{r_{nom}}{f}\right)^{f} - 1$$

Example: 5% Nominal Rate would equal:

	Formula	Effective Rate
Compounded annually	$(1 + 0.05 / 1)^{1} - 1$	5%
Compounded semi-annually	$(1 + 0.05 / 2)^{2} - 1$	5.06%
Compounded quarterly	$(1 + 0.05 / 4)^{4} - 1$	5.09%
Compounded monthly	?	?
Compounded daily	$(1 + 0.05 / 365)^{365} - 1$	5.13%

The Language of Compounding

01

Compounding period not given

- 5% per year
- 1% per month

Effective Rate

02

Compounding period given, nominal vs. effectove not stated

- 8% per year, compounded semi-annually

Nominal Rate

03

Interest rate stated as effective rate

- Effective 5% per year compounded monthly

Effective Rate

The Language of Compounding

Lending

Nominal Rate (Annual Percentage Rate)

Credit Card

- 18% per year, compounded daily

Effective rate is higher than 18%

Deposit

Effective Rate (Annual Percentage Yield)

Term Deposit

- Effective 3.5% per year, compounded semi-annually

Nominal rate is lower than 3.5%

Future Value with Compounding

$$FV = PV \times \left(1 + \frac{i}{f}\right)^{n \times f}$$

FV: Future value

PV: Present value

i: Nominal rate per year

f: Frequency

n: Number of years

Example: If you invest $100 today with an 5% nominal rate compounded semi-annually. How much will you have in five years?

PV = 100

i = 5%

f = 2

n = 5

$$FV = 100 \times \left(1 + \frac{5\%}{2}\right)^{5 \times 2} = 128.01$$

Present Value with Compounding

$$PV = \frac{FV}{\left(1 + \frac{i}{f}\right)^{n \times f}} \quad \text{OR} \quad = FV \times \left(1 + \frac{i}{f}\right)^{-n \times f}$$

PV: Present value

FV: Future value

i: Nominal rate per year

f: Frequency

n: Number of years

Example: If you are going to receive $100 five years from now, how much is that worth today, assuming a 5% nominal rate per year?

FV = 100

i = 5%

f = 1

n = 5

$$PV = 100 \times \left(1 + \frac{5\%}{1}\right)^{-5 \times 1} = \mathbf{78.35}$$

Discount Factor

Discounting: Reducing future cash flows to their value currently.

Example: If you are going to receive $100 five years from now, how much does that worth today, assuming a 5% annual interest?

$$PV = \frac{FV}{\left(1 + \frac{i}{f}\right)^{n \times f}} \quad \text{OR} \quad = FV \times \left(1 + \frac{i}{f}\right)^{-n \times f}$$

$$PV = 100 \times \left(1 + \frac{5\%}{1}\right)^{-5 \times 1} = \mathbf{78.35}$$

Discounted Value: PV

Discounted Value = 78.35

Discount Rate / Internal Rate of Return (IRR): i

Discount Rate / IRR = 5%

Discount Factor: $\left(1 + \frac{i}{f}\right)^{-n \times f}$

Discount Factor $= \left(1 + \frac{5\%}{1}\right)^{-5 \times 1} = 0.7835$

Annuity

A series of equal payments

At equal time periods

For a fixed number of years

Annuity factor: The total of the discount factors in each period.

Example: If you are going to receive 5% annual interest for 5 years, what is the annuity factor?

Year	Discount Factor	
1	0.9524	$(1 + 5\%)^{-1}$
2	0.9070	$(1 + 5\%)^{-2}$
3	0.8638	$(1 + 5\%)^{-3}$
4	0.8227	$(1 + 5\%)^{-4}$
5	0.7835	$(1 + 5\%)^{-5}$
Total	**4.3295**	**Annuity Factor**

Annuity table: Present value of $1 per year for each of *n* years = $[1 - (1 + r)^{-n}]/r$

No. of years	Interest rate per year														
	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%
1	.990	.980	.971	.962	.952	.943	.935	.926	.917	.909	.901	.893	.885	.877	.870
2	1.970	1.942	1.913	1.886	1.859	1.833	1.808	1.783	1.759	1.736	1.713	1.690	1.668	1.647	1.626
3	2.941	2.884	2.829	2.775	2.723	2.673	2.624	2.577	2.531	2.487	2.444	2.402	2.361	2.322	2.283
4	3.902	3.808	3.717	3.630	3.546	3.465	3.387	3.312	3.240	3.170	3.102	3.037	2.974	2.914	2.855
5	4.853	4.713	4.580	4.452	4.329	4.212	4.100	3.993	3.890	3.791	3.696	3.605	3.517	3.433	3.352
6	5.795	5.601	5.417	5.242	5.076	4.917	4.767	4.623	4.486	4.355	4.231	4.111	3.998	3.889	3.784
7	6.728	6.472	6.230	6.002	5.786	5.582	5.389	5.206	5.033	4.868	4.712	5.564	4.423	4.288	4.160
8	7.652	7.325	7.020	6.733	6.463	6.210	5.971	5.747	5.535	5.335	5.146	4.968	4.799	4.639	4.487
9	8.566	8.162	7.786	7.435	7.108	6.802	6.515	6.247	5.995	5.759	5.337	5.328	5.132	4.946	4.772
10	9.471	8.983	8.530	8.111	7.722	7.360	7.024	6.710	6.418	6.145	5.889	5.650	5.426	5.216	5.019
11	10.37	9.787	9.253	8.760	8.306	7.887	7.499	7.139	6.805	6.495	6.207	5.938	5.687	5.453	5.234
12	11.26	10.58	9.954	9.385	8.863	8.384	7.943	7.536	7.161	6.814	6.492	6.194	5.918	5.660	5.421
13	12.13	11.35	10.63	9.986	9.394	8.853	8.358	7.904	7.487	7.103	6.750	6.424	6.122	5.842	5.583
14	13.00	12.11	11.30	10.56	9.899	9.295	8.745	8.244	7.786	7.367	6.982	6.628	6.302	6.002	5.724
15	13.87	12.85	11.94	11.12	10.38	9.712	9.108	8.559	8.061	7.606	7.191	6.811	6.462	6.142	5.847
16	14.72	13.58	12.56	11.65	10.84	10.11	9.447	8.851	8.313	7.824	7.379	6.974	6.604	6.265	5.954
17	15.56	14.29	13.17	12.17	11.27	10.48	9.763	9.122	8.544	8.022	7.549	7.120	6.729	6.373	6.047
18	16.40	14.99	13.75	12.66	11.69	10.83	10.06	9.372	8.756	8.201	7.702	7.250	6.840	6.467	6.128
19	17.23	15.68	14.32	13.13	12.09	11.16	10.34	9.604	8.950	8.365	7.839	7.366	6.938	6.550	6.198
20	18.05	16.35	14.88	13.59	12.46	11.47	10.59	9.818	9.129	8.514	7.963	7.469	7.025	6.623	6.259
25	22.02	19.52	17.41	15.62	14.09	12.78	11.65	10.67	9.823	9.077	8.422	7.843	7.330	6.873	6.464
30	25.81	22.40	19.60	17.29	15.37	13.76	12.41	11.26	10.27	9.427	8.694	8.055	7.496	7.003	6.566

Discounted Cash Flow (DCF) and Annuities

Discounted Cash Flow (DCF): The present value of an investment based on **expected** future cash flows.

Example: A company invested in a plant that is expected to:

Earn (FV): $1 million per year

Expected life: 20 years

Discount rate: 5%

Look for the **annuity factor** in the attached Annuity Table and calculate the **present value of the plant**.

No. of Year	Interest Rate / Year
20	5%
Annuity Factor	**12.4622**

PV of the Plant = $1,000,000 x 12.4622

= $12,462,200

Net Present Value (NPV)

Net Present Value (NPV) is the value of **all future cash flows** over the entire life of an investment discounted to the present **minus the initial investment**.

Example

- **Initial investment**: $1,000
- **Annual cash flow**: $400
- **Discount rate**: 5%
- **Total period**: 3 Years

Discount Factor = $(1 + 5\%)^{-n}$

Year	0	1	2	3
Cash Flow (FV)	-1,000	400	400	400
Discount Factor		0.9524	0.9070	0.8638
PV Year 1	380.95			
PV Year 2	362.81			
PV Year 3	345.54			
NPV	**89.30**			

Positive NPV

NPV Decision Rule

Companies may look at the cost-benefit of a project by using the **NPV decision rule**.

Projects with a **positive NPV** return enough cash to more than cover the cost of the project.

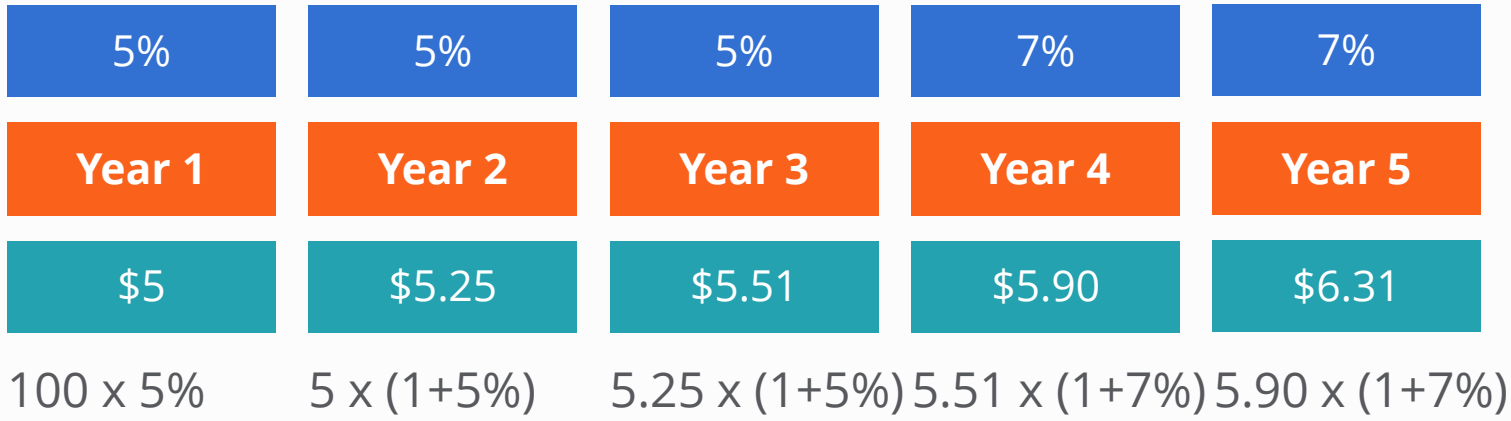
PROCEED

Projects with a **negative NPV** fail to return enough cash to cover the of the project.

REJECT

Changing Rates of Interest

Where the **rate of interest changes over time**, compound interest at the old rate up to the date of the change and then use the new rate.



Example:

Principal: $100

Interest Rate:

- 5% per annum for the first 3 years
- 7% per annum for the last 2 years

Changing Rates of Interest

Where the **rate of interest changes over time**, compound interest at the old rate up to the date of the change and then use the new rate.

5%	5%	5%	7%	7%
Year 1	**Year 2**	**Year 3**	**Year 4**	**Year 5**
$5	$5.25	$5.51	$5.90	$6.31
	$5	$5.25	$5.62	$6.01
	100 x 5%	5 x (1+5%)	5.25 x (1+7%)	5.62 x (1+7%)

Example:

Principal: $100

Interest Rate:

- 5% per annum for the first 3 years
- 7% per annum for the last 2 years

Changing Rates of Interest

Where the **rate of interest changes over time**, compound interest at the old rate up to the date of the change and then use the new rate.

Example:

Principal: $100

Interest Rate:

- 5% per annum for the first 3 years
- 7% per annum for the last 2 years

5%	5%	5%	7%	7%
Year 1	**Year 2**	**Year 3**	**Year 4**	**Year 5**
$5	$5.25	$5.51	$5.90	$6.31
	$5	$5.25	$5.62	$6.01
		$5	$5.35	$5.72
			$7	$7.49
				$7
		Total Interest		$32.54
		Total		$132.54