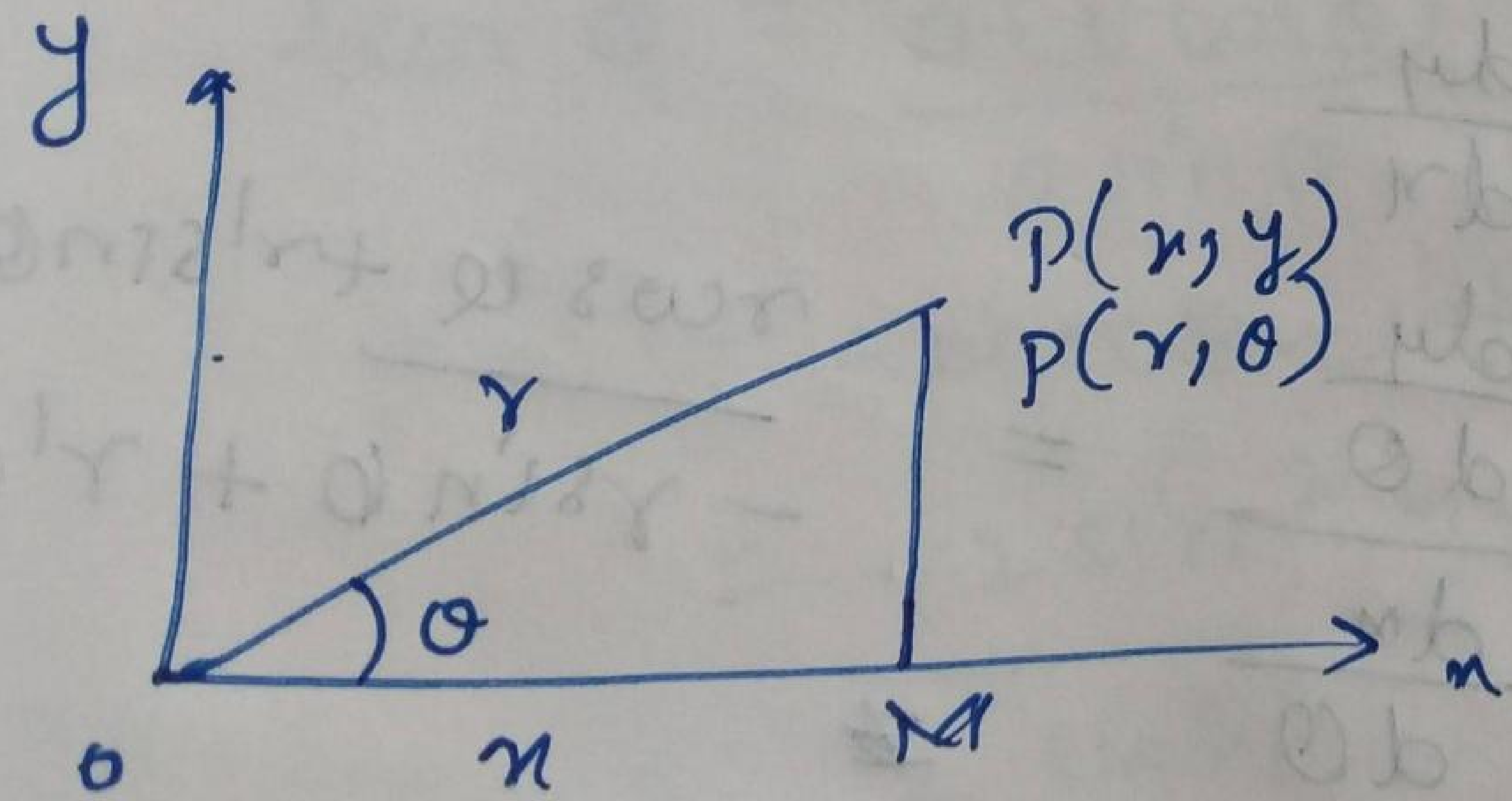
(18) $\cos\theta = \dfrac{e^{i\theta} + e^{-i\theta}}{2}$ (19) $\dfrac{e^{i\theta} - e^{-i\theta}}{2} = \sin\theta$

Polar curves

Relationship b/w cartesian co-ordinates (x, y) & polar co-ordinates (r, θ)

$x \to$ initial line $O \to$ pole



$x = r\cos\theta$ — (1)

$y = r\sin\theta$ — (2)

Squaring & adding

$$\boxed{x^2 + y^2 = r^2}$$

① ÷ ②

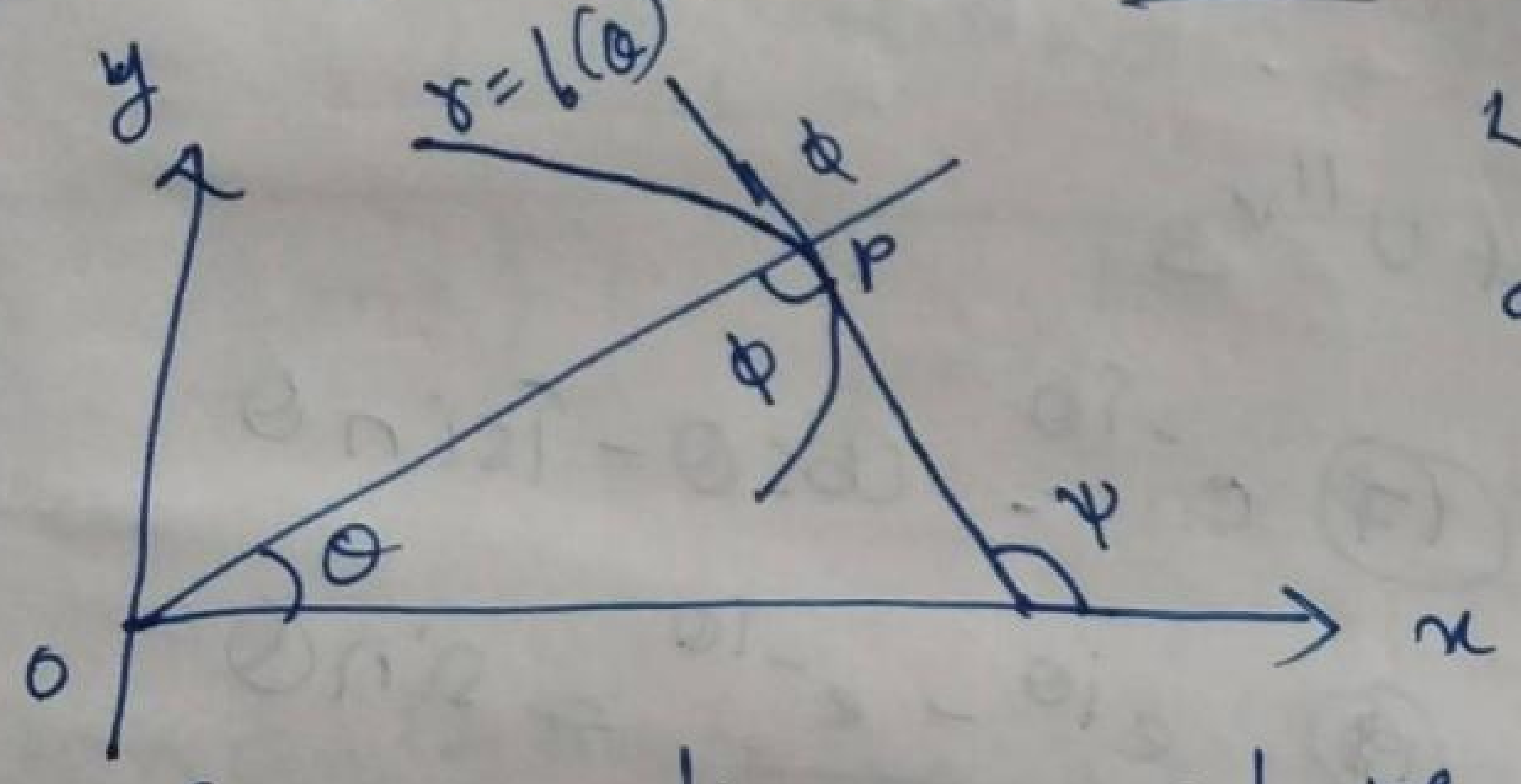
$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

(1) & (2) determine cartesian co-ordinates in terms of polar co-ordinates.

(3) & (4) determine polar co-ordinates in terms of cartesian co-ordinates.

It is evident that r is function of θ & equ^n in the form of $r = f(\theta)$ is called equ^n of the curve in polar form.

<u>Angle b/w radius vector & tangent</u>



Let $P(r, \theta)$ be any point at the curve

$XOP = \theta \quad OP = r$

from figure we have $\psi = \theta + \phi$

$\tan\psi = \tan(\theta + \phi)$

$$= \frac{\tan\theta + \tan\phi}{1 - \tan\theta\tan\phi} \quad \text{— ①}$$

Let (x, y) be the cartesian co-ordinates of P so that $x = r\cos\theta$ $y = r\sin\theta$

slope $\tan\psi = \frac{dy}{dx}$

$$= \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{r\cos\theta + r'\sin\theta}{-r\sin\theta + r'\cos\theta}$$

Divide num & den by $r'\cos\theta$

$$= \frac{\dfrac{r\cos\theta}{r'\cos\theta} + \dfrac{r'\sin\theta}{r'\cos\theta}}{\dfrac{-r\sin\theta}{r'\cos\theta} + \dfrac{r'\cos\theta}{r'\cos\theta}}$$

$$= \frac{\tan\theta + \dfrac{r}{r'}}{1 - \dfrac{r}{r'}\tan\theta} \quad -(2)$$

By (1) & (2)

$$\tan\phi = \frac{r}{r'} = \frac{r}{\dfrac{dr}{d\theta}} = \frac{r\,d\theta}{dr}$$

(1) Find angle b/w radius vector & tangent

(i) $r = a(1-\cos\theta)$ (ii) $r^2\cos 2\theta = a^2$

(iii) $\frac{l}{r} = 1 + e\cos\theta$ (iv) $r^m = a^m(\cos m\theta + \sin m\theta)$

(i) $r = a(1-\cos\theta)$

$\cancel{\frac{r}{a} = 1-\cos\theta}$

$\cancel{\cos\theta = 1 - \frac{r}{a}}$

$$\tan\phi = \frac{r\,d\theta}{dr}$$

$$\frac{dr}{d\theta} = a\sin\theta$$

$$\tan\phi = \frac{a(1-\cos\theta)}{a\sin\theta}$$

$= \cancel{\operatorname{cosec}\theta - \cot\theta}$

$$= \frac{2\sin^2\theta/2}{2\sin\theta/2\cos\theta/2}$$

$$= \tan\theta/2$$

$$\boxed{\phi = \theta/2}$$

(ii) $r^2\cos 2\theta = a^2$

$$\tan\phi = \frac{r\,d\theta}{dr}.$$

$$r = \sqrt{\frac{a^2}{\cos 2\theta}}$$

$$= \frac{a}{\sqrt{\cos 2\theta}} = a\sqrt{\sec 2\theta}$$

$$\frac{dr}{d\theta} = \frac{a}{\cancel{2}}(\sec 2\theta)^{3/2}\sec 2\theta\tan 2\theta$$

$$\tan\phi = (a\sqrt{\sec 2\theta})(a(\sec 2\theta)^{3/2}\tan 2\theta).$$

$$= a^2\sec^2 2\theta\tan 2\theta$$

$\log(r^2\cos 2\theta) = \log a^2$

$2\log r + \log(\cos 2\theta) = 2\log a$

$\frac{\cancel{2}}{r}\frac{dr}{d\theta} + \frac{1}{\cos 2\theta}(-\cancel{2}\sin 2\theta) = 0$

$\frac{1}{r}\frac{dr}{d\theta} = \frac{\sin}{\cos 2\theta}$

$\cot\phi = \overset{\tan}{\cancel{\cot}}\, 2\theta$

$\cancel{\phi = 2\theta}$ $\qquad \cot\phi = \cot\left(\frac{\pi}{2} - 2\theta\right)$

$\boxed{\phi = \pi/2 - 2\theta}$

(iii) $\frac{l}{r} = 1 + e\cos\theta$

$\log l - \log r = \log(1 + e\cos\theta)$

$-\frac{1}{r}\frac{dr}{d\theta} = \frac{1}{1+e\cos\theta}(-e\sin\theta)$

$\cot\phi = \frac{e\sin\theta}{1+e\cos\theta}$

$\phi = \cot^{-1}\left(\frac{e\sin\theta}{1+e\cos\theta}\right)$

(iv) $r^m = a^m(\cos m\theta + \sin m\theta)$

$m\log r = m\log a + \log[\cos m\theta + \sin m\theta]$

$\frac{\cancel{m}}{r}\frac{dr}{d\theta} = \frac{1}{\cos m\theta + \sin m\theta}(-\cancel{m}\sin m\theta + \cancel{m}\cos m\theta)$

$\frac{1}{r}\frac{dr}{d\theta} = \frac{\cos m\theta - \sin m\theta}{\cos m\theta + \sin m\theta}$ $\qquad \frac{1 - 2\cos m\theta\sin m\theta}{\cos}$

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$$\frac{\cot m\theta - 1}{\cot m\theta + 1} = \cot\phi$$

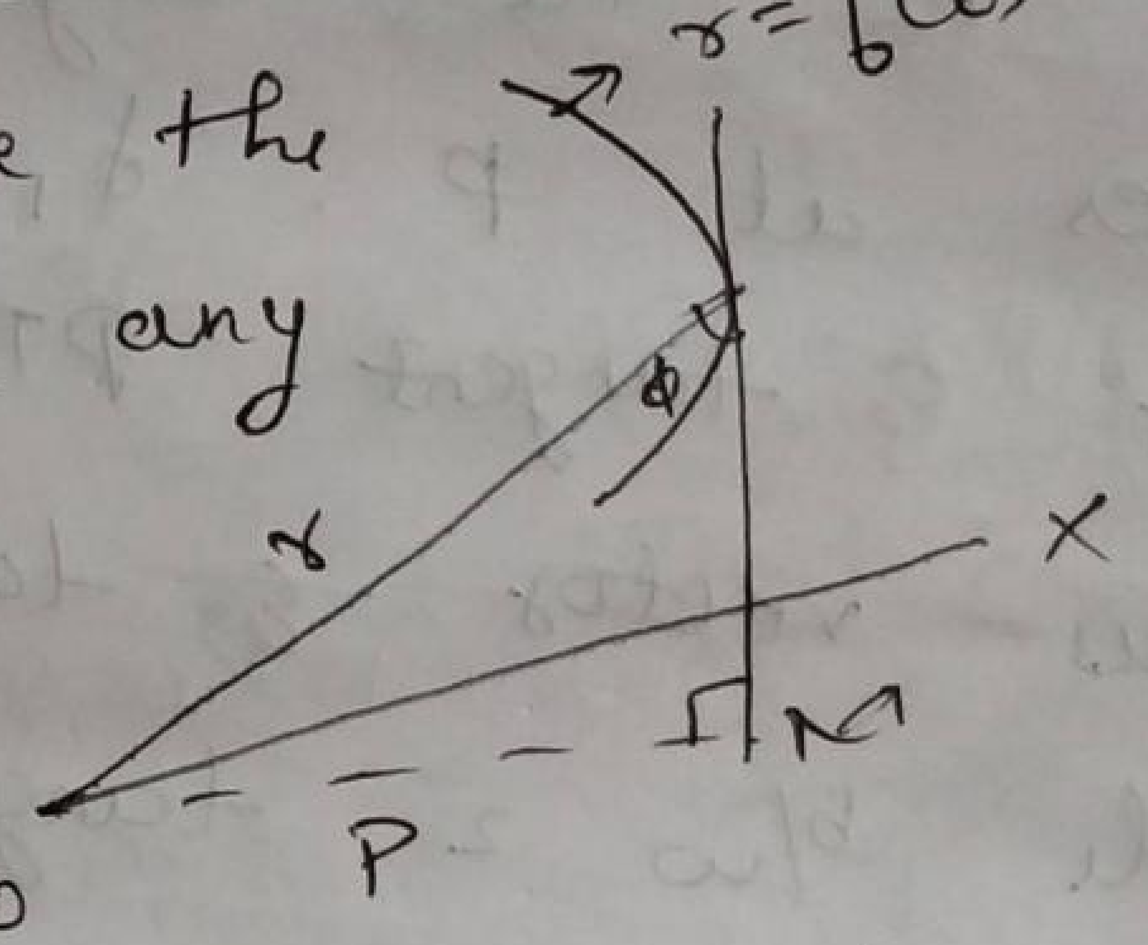
$$\phi = \cot^{-1}\left(\frac{\cot m\theta - 1}{\cot m\theta + 1}\right) //$$

$$\cancel{\frac{1-\tan m\theta}{1+\tan m\theta}}$$

$$= \cot\left(\frac{\pi}{4} + m\theta\right)$$

$$\boxed{\theta = \frac{\pi}{4} + m\theta}$$

Length of perpendicular from pole to the tangent



Let O be the pole & OX be the initial line, Let $P(r,\theta)$ be any pt. on the curve & hence $OP = r$, $XOP = \theta$. Draw $OM = p$ perpendicular from pole the tangent at P. Let ϕ be the angle b/w radius vector & tangent.

From ΔOMP

$$\sin\phi = \frac{OM}{OP} = \frac{p}{r}$$

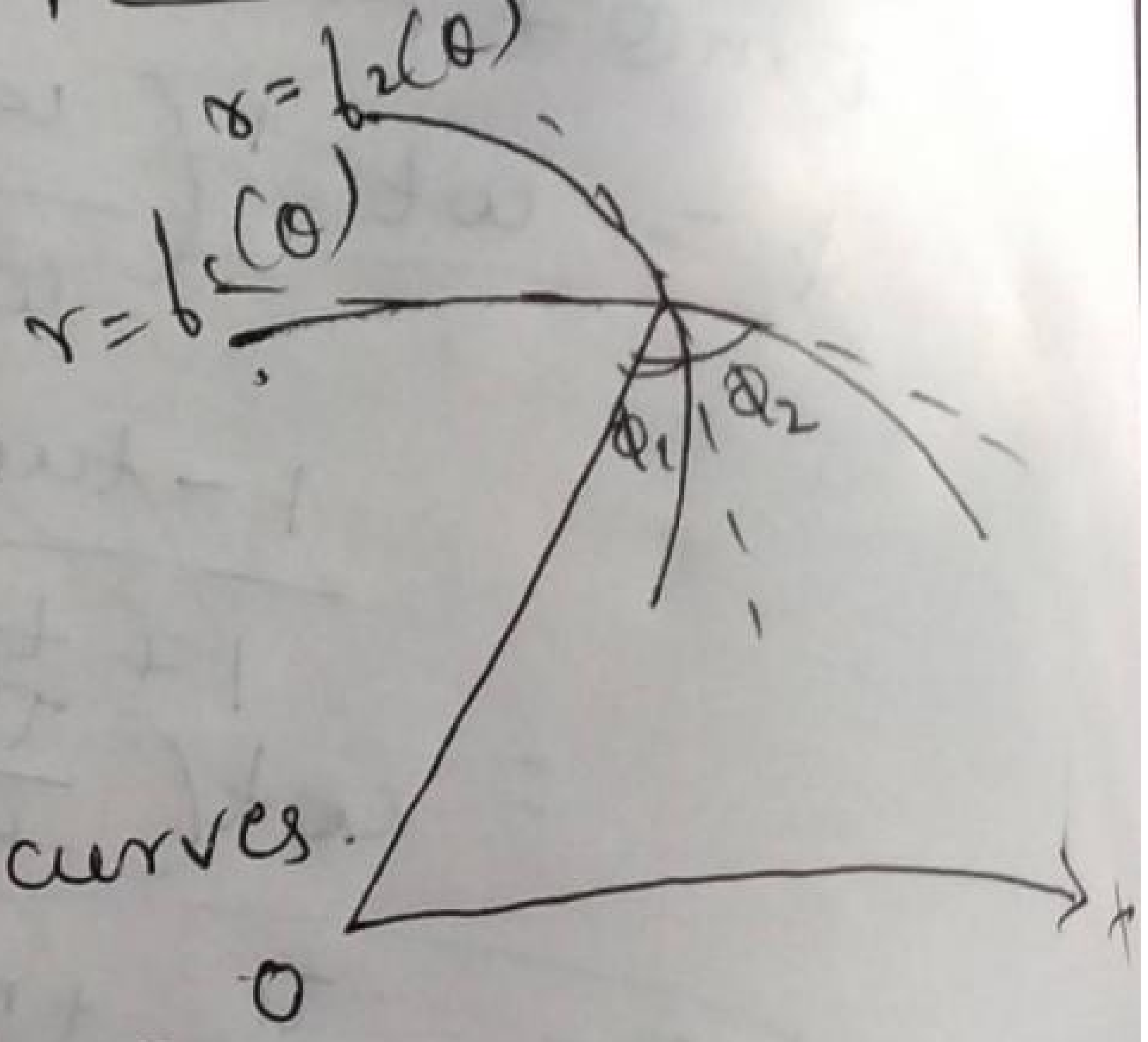
$$\boxed{p = r\sin\phi}$$

$$\frac{1}{p^2} = \frac{1}{r^2\sin^2\phi} = \frac{1}{r^2}(\operatorname{cosec}^2\phi)$$

$$= \frac{1}{r^2}(1+\cot^2\phi)$$

$$\boxed{\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2}$$

Angle of intersection of 2 polar curves



Angle of intersection of any 2 curves is equal to the angle b/w the tangents drawn at the pt. of intersection of 2 curves.

Let $r = f_1(\theta)$ & $r = f_2(\theta)$ be two polar curves intersect at the point P. PT_1 & PT_2 be the tangents drawn to the curves at P. ϕ_1 is the angle b/w radius vector & tangent PT_1 & ϕ_2 be the angle b/w radius vector & tangent PT_2.

Angle b/w 2 tangents is $= |\phi_2 - \phi_1|$

If $\phi_2 - \phi_1 = \pi/2 \rightarrow$ 2 curves intersect orthogonally

$$\phi_2 = \phi_1 + \pi/2$$

$$\tan\phi_2 = \tan(\phi_1 + \pi/2)$$

$$= -\cot\phi_1$$

$$\tan\phi_1 \tan\phi_2 = -1$$

* Find angle b/w radius vector & tangent & also find slope of tangent as indicated for the following curves.

1. $r = a(1+\cos\theta)$ at $\theta = \pi/3$
2. $r = a(1+\sin\theta)$ at $\theta = \pi/2$

(3) $\frac{2a}{r} = 1 - \cos\theta$ at $\theta = \frac{2\pi}{3}$

(4) $r\cos^2(\theta/2) = a$ at $\frac{2\pi}{3} = \theta$

(1) $r = a(1+\cos\theta)$

$$\frac{dr}{d\theta} = a(-\sin\theta)$$

$$r\frac{d\theta}{dr} = -\frac{r\,\mathrm{cosec}\,\theta}{a}$$

$$= \frac{-(1+\cos\theta)}{\sin\theta}$$

$$= \cancel{\cot\theta - \mathrm{cosec}\,\theta}$$

$$= \frac{-\cancel{2}\cos^2\theta/2}{\cancel{2}\sin\theta/2\cos\theta/2}$$

$$= -\cot\theta/2 = \tan(\pi/2 + \theta/2)$$

slope $= 0$

$$\psi = \theta + \phi$$

$$= \frac{\pi}{3} + \frac{2\pi}{3}$$

$$= \pi$$

$$\phi = \frac{\pi}{2} + \frac{\pi}{6}$$

$$= \frac{2\pi}{3}$$

(2) $r = a(1+\sin\theta)$

$$r.\frac{d\theta}{dr} = \frac{a(1+\sin\theta)}{a(\cos\theta)}$$

$$= \frac{1+\sin\theta}{\cos\theta}$$

$$\cot\phi = \frac{\cos\theta}{1+\sin\theta}$$

$$= \frac{\cos^2\theta/2 - \sin^2\theta/2}{\sin^2\theta/2 + \cos^2\theta/2 + 2\sin\theta/2\cos\theta/2}$$

$$= \frac{\cos\theta/2 - \sin\theta/2}{\cos\theta/2 + \sin\theta/2}$$

$$= \frac{1 - \tan\theta/2}{1 + \tan\theta/2}$$

$\tan\phi = \tan(\pi/4 + \theta/2)$

$$\boxed{\phi = \frac{\pi}{4} + \theta/2}$$

$\cos 2\theta = 2\cos^2\theta - 1$
$1 - 2\sin^2\theta$

$\phi = \frac{\pi}{4} + \frac{\pi}{4}$

$= \frac{\pi}{2}$

$\psi = \theta + \phi = \pi$

$\tan\pi = 0$

(3) $\frac{2a}{r} = 1 - \cos\theta$ at $\theta = \frac{2\pi}{3}$.

$\log 2a - \log r = \log(1-\cos\theta)$

$-\frac{1}{r}\frac{dr}{d\theta} \quad \frac{1}{1-\cos\theta}(\sin\theta)$

$\cot\phi = \frac{\sin\theta}{\cos\theta - 1}$

$= \frac{\sin\theta}{-2\sin^2\theta/2}$

$= \frac{\cancel{2}\sin\theta/2\cos\theta/2}{-\cancel{2}\sin^2\theta/2}$ $\qquad \pi - \theta/2$

$= -\cot\theta/2$

$\phi = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

$\phi = +\pi/3$ $\qquad \pi + \frac{\pi}{3}$

$\psi = \frac{2\pi}{3} + \frac{2\pi}{3} \qquad \frac{4\pi}{3}$

$=$

slope $= \sqrt{3}$

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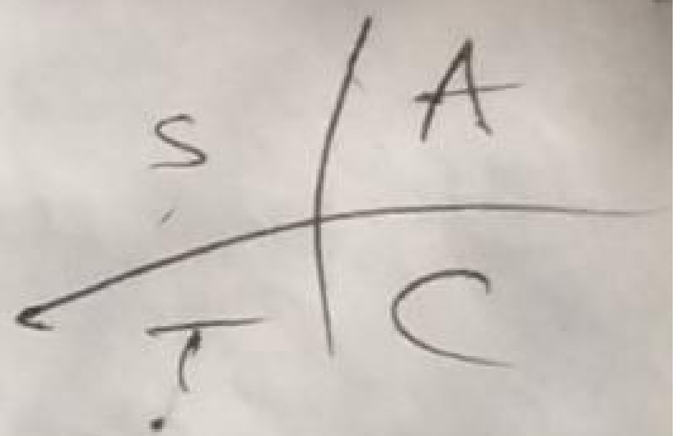
④ $r\cos^2\theta/2 = a$



$\log r + 2\log(\cos\theta/2) = \log a$

$\frac{1}{r}\frac{dr}{d\theta} = \frac{-\cancel{2}}{\cos\theta/2}(-\sin\theta/2)$

$= \tan\theta/2$

$\cot\phi = \cot\left(\frac{\pi}{2} + \theta/2\right)$

$\phi = \frac{\pi}{2} \mp \theta/2$

$= \frac{\pi}{2} \mp \frac{\cancel{2}\pi}{3}$

$= \frac{\cancel{2}\pi}{6}$ $\qquad \cancel{2}\pi - \frac{\pi}{6}$

$\psi = \frac{2\pi}{3} + \frac{\cancel{2}\pi}{6}$ $\qquad \cos 2\theta = 2\cos^2\theta - 1$

$= \frac{5\cancel{4}\pi}{6}$

$\tan\frac{11\pi}{6} = \tan\left(\cancel{2}\pi - \frac{\pi}{6}\right)$

$= -\frac{1}{\sqrt{3}}$

show that the following pair of curves intersect orthogonally

① $r = a(1+\cos\theta)$ & $r = b(1-\cos\theta)$

② $r^n = a^n\cos n\theta$ $\quad r^n = b^n\sin n\theta$

③ $r = ae^\theta$ & $re^\theta = b$

① $\frac{dr}{d\theta} = \frac{a(-\sin\theta)}{\cancel{\quad}}$ $\qquad \tan\phi = \frac{-a(1+\cos\theta)}{a\sin\theta}$

$= \frac{-2\cos^2\theta/2}{2\sin\theta/2\cos\theta/2}$

$= -\cot\theta/2 \quad$ — ①

$= \cancel{\tan\left(\frac{\pi}{2} + \theta/2\right)}$ $\boxed{\phi_1 - \phi_2 = \pi/2}$ Prove

$\cancel{\phi = \frac{\pi}{2} + \theta/2}$

$r = b(1-\cos\theta)$

$\frac{b(1-\cos\theta)}{b\sin\theta} = \frac{2\sin^2\theta/2}{2\sin\theta/2\cos\theta/2} = \tan\theta/2$ — (2)

(1) × (2)

$-\cot\theta/2 \tan\theta/2 = -1$

Hence pair of curves are orthogonal.

(2) $r^n = a^n \cos n\theta$ $\qquad r^n = b^n \sin n\theta$

$n\log r = n\log a + \log\cos n\theta$ $\qquad n\log r = n\log b + \log\sin n\theta$

$\frac{n}{r}\frac{dr}{d\theta} = \frac{n}{\cos n\theta} - \sin n\theta$ $\qquad \frac{n}{r}\frac{dr}{d\theta} = \frac{n}{\sin n\theta}\cos n\theta$

$\cot\phi = -\tan n\theta$ $\qquad \cot\phi = \cot n\theta$

$\tan\phi = -\cot n\theta$ — (1) $\qquad \tan\phi = \tan n\theta$ — (2)

(1) × (2)

$(-\cot n\theta)(\tan n\theta) = -1$

(3) $r = ae^\theta$ $\qquad re^\theta = b$

$\frac{ae^\theta}{ae^\theta} = \tan\phi$ $\qquad \cancel{r = \frac{b}{e^\theta}}$

$\tan\phi_1 = 1$ — (1) $\qquad \log r + \theta = \log b$

$\frac{1}{r}\frac{dr}{d\theta} + 1 = 0$

(1) × (2) $\qquad \cot\phi = -1$

$(\tan\phi_1)(\tan\phi_2) = -1$ $\qquad \tan\phi_2 = -1$ — (2)

Angle b/w 2 curves

(1) $r = \sin\theta + \cos\theta$ & $r = 2\sin\theta$

$$r\frac{d\theta}{dr} = \frac{(\sin\theta + \cos\theta)}{\cos\theta - \sin\theta}$$

$$= \frac{1 + \tan\theta}{1 - \tan\theta}$$

$$= \tan\left(\frac{\pi}{4} + \theta\right)$$

$$\boxed{\phi = \frac{\pi}{4} + \theta}$$

$$r\frac{d\theta}{dr} = \frac{2\sin\theta}{2\cos\theta}$$

$$\phi = \theta$$

$$\phi_2 - \phi_1 = \frac{\pi}{4}$$

$a^x = t$

$x\log a = \log t$

$\log a = \frac{1}{t}\frac{dt}{dx}$

$a^x \log a$

$$a\log\theta = \frac{a}{\log\theta}$$

$$\boxed{\theta = e}$$

(2) $r = a\log\theta$ $\qquad r = \frac{a}{\log\theta}$

$$\frac{dr}{d\theta} = \frac{a}{\theta}$$

$$\frac{a\log\theta}{a} = \theta\log\theta$$

$$\tan\phi_1 = \theta\log\theta$$

$$\frac{dr}{d\theta} = a\theta$$

$$\tan\phi_2 = \frac{a}{\theta a \log\theta} = \frac{1}{\theta\log\theta}$$

$$\frac{1}{r}\frac{dr}{d\theta} = -\frac{1}{\log\theta}\frac{1}{\theta}$$

$$\cot\phi_2 = \frac{-1}{\theta\log\theta}$$

$$\tan\phi_2 = -\theta\log\theta$$

$$\tan(\phi_1 - \phi_2) = \frac{\tan\phi_1 - \tan\phi_2}{1 + \tan\phi_1\tan\phi_2}$$

$$\phi_1 - \phi_2 = \tan^{-1}\left(\frac{2e}{1-e^2}\right) = \frac{2\theta\log\theta}{1-(\theta\log\theta)^2}$$

(3) $r = a(1+\cos\theta)$ & $r^2 = a^2\cos 2\theta$

$\tan\phi_1 = -\cot\theta/2$

$= \tan\left(\frac{\pi}{2} + \theta/2\right)$

$\phi_1 = \frac{\pi}{2} + \theta/2$

$2\log r = 2\log a + \log\cos 2\theta$

$\frac{\cancel{2}}{r}\frac{dr}{d\theta} = \frac{\cancel{2}\ -\sin 2\theta}{\cos 2\theta}$

$\tan\phi_2 = -\cot 2\theta$

$= \tan\left(\frac{\pi}{2} + 2\theta\right)$

$\phi_2 - \phi_1 = \frac{\pi}{2} + 2\theta - \frac{\pi}{2} - \theta/2$

$= \frac{3\theta}{2}$

$\cos 2\theta = 1 - 2\sin^2\theta$

$\cancel{a^2}(1+\cos\theta)^2 = \cancel{a^2}\cos 2\theta$

$1 + \cos^2\theta + 2\cos\theta = \cos 2\theta$

$= 2\cos^2\theta - 1$

$\cos^2\theta - 2\cos\theta - 2 = 0$

$\cancel{x^2 - 2x - 2 = 0}$

$\cancel{2 \pm \sqrt{4 \cdot 2}}$ $\frac{2 \pm \sqrt{4+8}}{2}$

$\frac{2 \pm \sqrt{12}}{2}$

$1 \pm \sqrt{3}$

$1 - \sqrt{3}$

$\theta = \cos^{-1}(1 - \sqrt{3})$

Angle $\rightarrow \frac{3}{2}\cos^{-1}(1 - \sqrt{3})$

(4) $r = a(1-\cos\theta)$ $r = 2a\cos\theta$

$\frac{dr}{d\theta} = a\sin\theta$

$\phi_1 = \theta/2$

$\frac{\cancel{a}(1-\cos\theta)}{\cancel{a}\sin\theta}$ $\frac{2\sin^2\theta/2}{2\sin\theta/2\cos\theta/2} = \tan\theta/2$

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$$\frac{2a\cos\theta}{-2a\sin\theta} = \tan\phi_2$$

$$-\cot\theta = \tan\phi_2$$

$$\tan\left(\frac{\pi}{2}+\theta\right) = \tan\phi_2$$

$$\phi_2 = \frac{\pi}{2}+\theta$$

$$\frac{\pi}{2}+\theta-\frac{\theta}{2} = \frac{\pi}{2}+\frac{\theta}{2}$$

$$\cancel{a}(1-\cos\theta) = 2\cancel{a}\cos\theta$$

$$1-\cos\theta = 2\cos\theta$$

$$3\cos\theta = 1$$

$$\theta = \cos^{-1}\left(\frac{1}{3}\right)$$

Angle b/w $= \frac{\pi}{2} + \frac{1}{2}\cos^{-1}\left(\frac{1}{3}\right)$

Find the angle of intersection of the following curves

$\frac{a}{\theta}$ $-\frac{\theta^2}{a}$

(1) $r = a\theta$ & $r = \frac{a}{\theta}$

$$\frac{\cancel{a}\theta}{\cancel{a}} = \tan\phi_1 \qquad \tan\phi_2 = -\frac{\cancel{a}}{\theta^2}\ \ -\theta$$

$$\tan(\phi_1-\phi_2) = \frac{\theta+\frac{a^2}{\theta^3}}{1+\frac{a^2}{\theta^2}} = \frac{\theta^4+a^2}{\theta^2+a^2}\times\frac{\theta^2}{\theta^3} = \frac{\theta^4+a^2}{\theta(\theta^2+a^2)}$$

$$\boxed{\phi_2-\phi_1 = \pi/2}$$

(2) $r = \frac{a\theta}{1+\theta}$ & $r = \frac{a}{1+\theta^2}$

$$\log r = \log a + \log\theta - \log(1+\theta)$$

$$\cot\phi_1 = \frac{1}{\theta} - \frac{1}{1+\theta} = \frac{1+\theta-\theta}{\theta(1+\theta)} = \frac{1}{\theta(1+\theta)}$$

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$$r = \frac{a}{1+\theta^2}$$

$$\cot\phi_2 = -\frac{2\theta}{1+\theta^2}$$

$$\tan\phi_1 = \theta(1+\theta)$$

$$\tan\phi_2 = -\left(\frac{1+\theta^2}{2\theta}\right)$$

$$= -1$$

$$\frac{\cancel{a}\theta}{1+\theta} = \frac{\cancel{a}}{1+\theta^2}$$

$$\theta(1+\theta^2) = 1+\theta$$

$$\cancel{\theta} + \theta^3 = 1 + \cancel{\theta}$$

$$\theta^3 = 1$$

$$\boxed{\theta = 1}$$

$$\tan(\phi_1 - \phi_2) = \frac{2+1}{1-2} = \frac{3}{-1} = -3$$

$$\underbrace{\phi_1 - \phi_2 = \tan^{-1}(-3)}$$

Pedal equation of polar curve.

Pedal equation of polar curve

The equation of the given curve $r = f(\theta)$ expressed in terms of p and r is called pedal equation or p-r equation. Find pedal equation of the following curves.

① $\frac{2a}{r} = 1+\cos\theta$ ③ $r^2 = a^2 \sec 2\theta$

② $r^n = a^n \cos n\theta$ ④ $\frac{1}{r} = 1+e\cos\theta$

① $\frac{2a}{r} = 1+\cos\theta$

$p = r\sin\phi$

$\log 2a - \log r = \log(1+\cos\theta)$

$-\frac{1}{r}\frac{dr}{d\theta} = \frac{(-\sin\theta)}{1+\cos\theta}$

$\cot\phi = \tan\theta/2$

$= \cot\left(\frac{\pi}{2} - \theta/2\right)$

$\phi = \frac{\pi - \theta}{2}$

$p = r\sin\phi$

$= r\sin\left(\frac{\pi}{2} - \theta/2\right)$

$= r\cos\theta/2$

From $\frac{\cancel{2}a}{r} = 1+\cos\theta$

$= \cancel{2}\cos^2\theta/2$

$\sqrt{\frac{a}{r}} = \cos\theta/2$

~~$\theta = 2\cos^{-1}\sqrt{\frac{a}{r}}$~~

$p = \sqrt{\frac{a}{r}}\,(r)$ $\boxed{p^2 = ar}$

② $n\log r = n\log a + \log\cos n\theta$

$\frac{\cancel{n}}{r}\frac{dr}{d\theta} = \frac{\cancel{n}(-\sin n\theta)}{\cos n\theta}$

$\cot\phi = -\tan n\theta$

$= \cot\left(\frac{\pi}{2} + n\theta\right)$

$\boxed{\phi = \frac{\pi}{2} + n\theta}$

$p = r\sin\left(\frac{\pi}{2} + n\theta\right)$

$= +r\cos n\theta =$

$r^n = a^n\cos n\theta$

$= a^n\left(\frac{p}{r}\right)$

$\boxed{\frac{r^{n+1}}{a^n} = p}$

(3) $r^2 = a^2 \sec 2\theta$

$2\log r = 2\log a + \log \sec 2\theta$

$\frac{\cancel{2}}{r}\frac{dr}{d\theta} = \frac{\cancel{2}}{\cancel{\sec 2\theta}} \cancel{\sec 2\theta} \tan 2\theta$

$\cot\phi = \tan 2\theta$

$= \cot\left(\frac{\pi}{2} - 2\theta\right)$

$\phi = \frac{\pi}{2} - 2\theta$

$p = r \cos 2\theta$

$\because \cos 2\theta = \left(\frac{a}{r}\right)^2$

$p = r\left(\frac{a}{r}\right)^2$

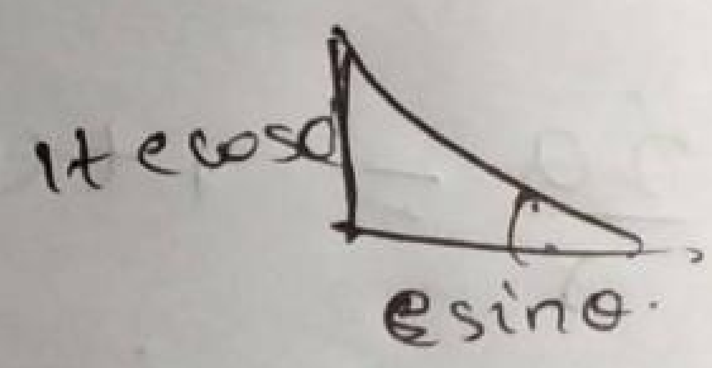
$\boxed{p = \frac{a^2}{r}}$

(4) $\frac{1}{r} = 1 + e\cos\theta$

$-\log r = \log(1+e\cos\theta)$

$-\frac{1}{r}\frac{dr}{d\theta} = \frac{1}{1+e\cos\theta}(-e\sin\theta)$

$-\cot\phi = \frac{-e\sin\theta}{1+e\cos\theta}$



$\tan\phi = \frac{1+e\cos\theta}{e\sin\theta}$

$(1+e\cos\theta)^2 + e^2\sin^2\theta$

$1 + e^2\cos^2\theta + e^2\sin^2\theta + 2e$

$1 + e^2 + 2e\cos\theta$

$p = r\left(\frac{1+e\cos\theta}{\sqrt{1+e^2+2e\cos\theta}}\right)$

$$p = \frac{r \times \frac{1}{\sqrt{r}}}{\sqrt{1+e^2+2e\cos\theta}} = \boxed{\sqrt{\frac{\sqrt{r}}{1+e^2+2e\cos\theta}} = p}$$

$$\sqrt{\frac{r}{1+e^2+2e\left(\frac{l}{r}-1\right)}}$$

$$\sqrt{\frac{r^2}{r+e^2r+2l-2r}}$$

$$p^2 = \frac{r^2}{r+e^2r+2l-2r} \qquad \frac{1}{p^2} = \frac{1}{r} + \frac{e^2}{r} + \frac{2l}{r^2} - \frac{2}{r}$$

$$\cot\phi = \frac{e\sin\theta}{1+e\cos\theta}$$

$$\frac{1}{p^2} = \frac{1}{r^2}\left(1+\frac{1}{r^2}\left(\frac{dr}{d\theta}\right)^2\right)$$

$$= \frac{1}{r^2}\left(1+\cot^2\phi\right)$$

$$= \frac{1}{r^2}\left(1+\left(\frac{e\sin\theta}{1+e\cos\theta}\right)^2\right) \quad -(1)$$

$$1+e\cos\theta = \frac{l}{r}$$

$$(e\sin\theta)^2 = e^2\sin^2\theta$$

$$= e^2(1-\cos^2\theta)$$

$$= e^2 - e^2\cos\theta$$

$$= e^2 - \left(\frac{l}{r}-1\right)^2$$

$$= \frac{e^2r^2-(l-r)^2}{r^2}$$

$\frac{1}{p^2} = \frac{1}{r^2}\cot^2\theta$

$\sin(90+30)$

$\cos 30$

$\frac{\sqrt{3}}{2}$

From (1) $$\frac{1}{p^2} = \frac{1}{r^2}\left(1+\frac{\frac{e^2r^2-(l-r)^2}{r^2}}{l^2/r^2}\right)$$

$$= \frac{1}{r^2}\left(1 + \frac{e^2r^2 - (l-r)^2}{l^2}\right)$$

$$= \frac{e^2-1}{l^2} + \frac{2}{lr}$$

(5) $r = ae^{\theta\cot\alpha}$

$\log r = \log a + \theta\cot\alpha$

$\cot\phi = \cot\alpha$

$\phi = \alpha$

$$\boxed{p = r\sin\alpha}$$

(6) $r^m = a^m(\cos m\theta + \sin m\theta)$

$m\log r = m\log a + \log(\cos m\theta + \sin m\theta)$

$$\frac{1}{r}\frac{dr}{d\theta} = \frac{-\sin m\theta + \cos m\theta}{\cos m\theta + \sin m\theta}$$

$$\cot\phi = \frac{\cos m\theta - \sin m\theta}{\cos m\theta + \sin m\theta}$$

$$= \frac{1 - \tan m\theta}{1 + \tan m\theta}$$

$$= \tan\left(\frac{\pi}{4} - m\theta\right)$$

$$= \cot\left(\frac{\pi}{2} - \frac{\pi}{4} + m\theta\right)$$

$$\boxed{\phi = \frac{\pi}{4} + m\theta}$$

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$$\boxed{p = r\sin\left(\frac{\pi}{4} + m\theta\right)}$$

$$= r\left(\frac{1}{\sqrt{2}}\cos m\theta + \frac{1}{\sqrt{2}}\sin m\theta\right)$$

$$= \frac{r}{\sqrt{2}}\left(\frac{r^m}{a^m}\right)$$

$$\boxed{p = \frac{r^{m+1}}{\sqrt{2}\,a^m}}$$

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① $r = a\theta$

$\tan\phi_1 = \theta$

$r = \frac{a}{\theta}$

$\tan\phi_2 = \frac{a}{\theta}\left(\frac{-\theta^2}{a}\right)$

$= -\theta$

$$\tan(\phi_1 - \phi_2) = \frac{2\theta}{1+\theta^2}$$

$$\phi_1 - \phi_2 = \tan^{-1}\left(\frac{2\theta}{1+\theta^2}\right)$$