(*) Find the length of perpendicular from pole to the tangent for the following curves.

① $r = a(1-\cos\theta)$ at $\theta = \pi/2$

$p = r\sin\phi$

~~$\cot\phi = a\sin\theta$~~

$\log r = \log a + \log(1-\cos\theta)$

$\cot\phi = \dfrac{\sin\theta}{1-\cos\theta}$

$= \dfrac{2\sin\theta/2\cos\theta/2}{2\sin^2\theta/2}$

$= \cot\theta/2$

$\boxed{\phi = \theta/2}$

$\phi = \dfrac{\pi}{4}$

$p = r\sin\dfrac{\pi}{4}$

$= \dfrac{r}{\sqrt{2}}$

$= \dfrac{a(1-\cos\theta)}{\sqrt{2}}$

$\boxed{p = \dfrac{a}{\sqrt{2}}}$

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(2) $r^2 = a^2 \cos 2\theta$ $\quad \theta = \pi/...$

$2 \log r = \log a^2 + \log \cos 2\theta$

$\cancel{2} \cot\phi = \dfrac{-\cancel{2}\sin 2\theta}{\cos 2\theta}$

$= -\tan 2\theta$

$= \cot\left(\frac{\pi}{2} + 2\theta\right)$

$\phi = \frac{\pi}{2} + 2\theta$

$p = r\sin\phi$

$= r\sin\left(\frac{\pi}{2} + 2\pi\right)$

$= r\sin\frac{5\pi}{2}$

$\boxed{p = r}$

$= a\sqrt{\cos 2\theta}$

$\boxed{p = a}$

(4) Length from $(a, \pi/2)$ on the curve $r = (1-\cos\theta)$

$\rightarrow \frac{a}{\sqrt{2}}$

(3) $\frac{2a}{r} = 1 - \cos\theta$ $\quad \theta = \pi/2$

$0 - \cot\phi = \dfrac{(\sin\theta)}{1-\cos\theta}$

$-\cot\phi = \dfrac{2\sin\theta/2 \cos\theta/2}{2\sin^2\theta/2}$

$= \cot\theta/2$

$\cot\phi = -\cot\theta/2$

$= \cot(\pi - \theta/2)$

$\phi = \pi - \theta/2$

$= \pi - \frac{\pi}{4}$

$= \frac{3\pi}{4}$

$p = r\sin\frac{3\pi}{4}$

$= r\sin\left(\pi - \frac{\pi}{4}\right)$

$= \frac{r}{\sqrt{2}} = \left(\frac{2a}{1-\cos\theta}\right)\frac{1}{\sqrt{2}}$

$= \frac{2a}{1}$

$\boxed{p = \sqrt{2}a \text{ units}}$

Determine length of perpendicular from pole to tangent at the pt. $\theta = \pi/3$ on the curve $r = a\sec^2\theta/2$

$\cancel{a\sec^2\theta/2}$ $\cot\phi = 2$

$\log r = \log a + 2\log\sec\theta/2$

$\cot\phi = \dfrac{2}{2}\dfrac{\sec\theta/2 \tan\theta/2}{\sec\theta/2}$

$= \tan\theta/2$

$= \cot\left(\frac{\pi}{2} - \theta/2\right)$

$\phi = \frac{\pi}{2} - \theta/2$

$\phi = \frac{\pi}{2} - \frac{\pi}{6}$

$\boxed{\phi = \frac{\pi}{3}}$

$p = r\sin\phi$

$= \frac{r}{\sqrt{2}} \frac{\sqrt{3}}{2}$

$= \frac{a\sec^2\theta/2}{\sqrt{2}} = a\sec^2\pi/6$

$\cancel{\boxed{p = 2\sqrt{3}2}}$

$\boxed{p = \frac{4a}{3} \cdot \frac{\sqrt{3}}{2}} = \frac{2a}{\sqrt{3}}$ //

Find ϕ for the curve

$a\theta = \sqrt{r^2 - a^2} - a\cos^{-1}(a/r)$

$\left[a\theta + a\cos^{-1}\left(\frac{a}{r}\right)\right]^2 = r^2 - a^2$

$2\log\left[a\theta + a\cos^{-1}\left(\frac{a}{r}\right)\right] = 2\log r - 2\log a$

$\cancel{\frac{2\left(a - \frac{a}{\sqrt{1-\frac{a^2}{r^2}}}\left(-\frac{a}{r^2}\right)\right)}{a\theta + a\cos^{-1}\left(\frac{a}{r}\right)} = r\cot\phi}$

$\cancel{\frac{\frac{a^2}{r^2\sqrt{1-\frac{a^2}{r^2}}}}{a\theta + a\cos^{-1}\left(\frac{a}{r}\right)} = \cot\phi}$

$\cancel{\frac{a}{r^2\sqrt{\frac{r^2-a^2}{r^2}}} = a\left[\theta + \cos^{-1}\left(\frac{a}{r}\right)\right]\cot\phi}$

$\cancel{\frac{a}{r\sqrt{r^2-a^2}} = \left[\theta + \cos^{-1}\left(\frac{a}{r}\right)\right]\cot\phi}$

$\cancel{\frac{1 + \frac{a}{r\sqrt{r^2-a^2}}}{\theta + \cos^{-1}\left(\frac{a}{r}\right)} = \cot\phi}$

$a\theta = \sqrt{r^2-a^2} - a\cos^{-1}\left(\frac{a}{r}\right)$

$a = \frac{1}{2\sqrt{r^2-a^2}} 2r\frac{dr}{d\theta} - a\left(\frac{-1}{\sqrt{1-\left(\frac{a}{r}\right)^2}}\right)\left(\frac{-a}{r^2}\right)\frac{dr}{d\theta}$

$= \frac{r}{\sqrt{r^2-a^2}}\frac{dr}{d\theta} - \frac{a^2}{r^2}\frac{r}{\sqrt{r^2-a^2}}\frac{dr}{d\theta}$

$= \frac{r}{\sqrt{r^2-a^2}}\frac{dr}{d\theta}\left(1 - \frac{a^2}{r^2}\right)$

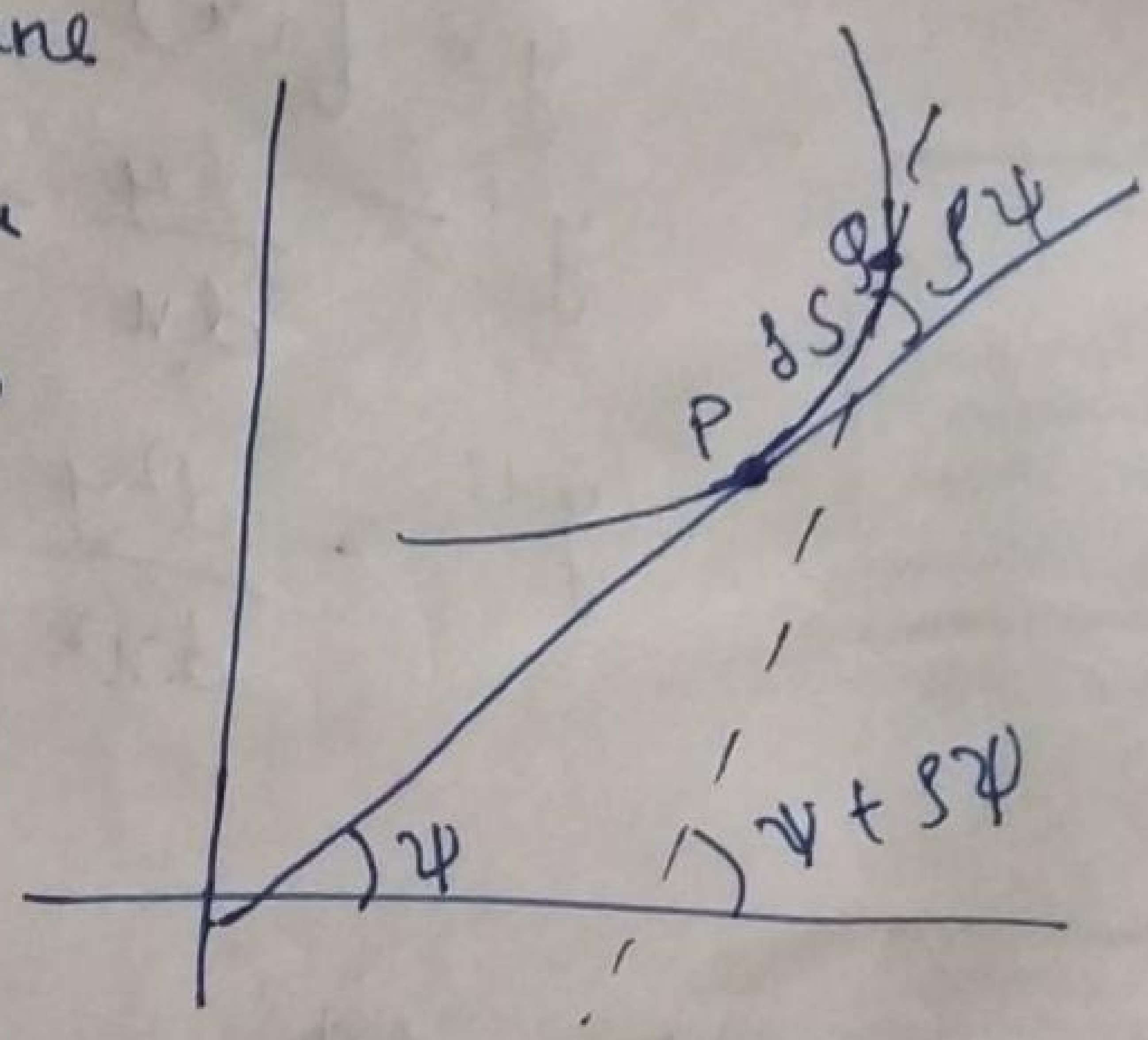
$= \frac{r}{\sqrt{r^2-a^2}}\frac{dr}{d\theta}\left(\frac{r^2-a^2}{r^2}\right)$

$\frac{r\,d\theta}{dr} = \frac{\sqrt{r^2-a^2}}{a}$

$\phi = \tan^{-1}\frac{\sqrt{r^2-a^2}}{a}$

Curvature & radius of curvature

Consider a curve C in xy plane & pt. P on it. Let ψ be the angle which the tangent at P to C makes with x axis. Consider pt. Q near to P. Let $\psi + \delta\psi$ be the angle which the tangent at Q to C making with x axis. Also δs be arc length from P to Q. Then $\delta\psi$ represents change in the angle ψ corresponding to the change δs in the arc length measured along C. Geometrically, a change in ψ represents the bending of the curve C. The ratio $\frac{\delta\psi}{\delta s}$ represents ratio of bending of C b/w points P & Q.



Rate of bending of C at P is given by

$$\frac{d\psi}{ds} = \lim_{Q \to P} \frac{\delta\psi}{\delta s}$$

This rate is called curvature of curve at P.

Thus $\frac{d\psi}{ds} = K$.

If $K \neq 0$ then $\frac{1}{K}$ is called radius of curvature and is denoted by ρ.

$$\rho = \frac{1}{K} = \frac{ds}{d\psi}.$$

Radius of curvature in Castenian form

$$y = f(x)$$

$$y' = \frac{dy}{dx} = \tan\psi$$

$$y'' = \frac{d^2y}{dx^2} = \sec^2\psi \frac{d\psi}{dx} = (1+\tan^2\psi)\frac{d\psi}{ds}\frac{ds}{dx}$$

$$= (1+(y')^2)\frac{d\psi}{ds}\frac{ds}{dx} \quad \text{— (1)}$$

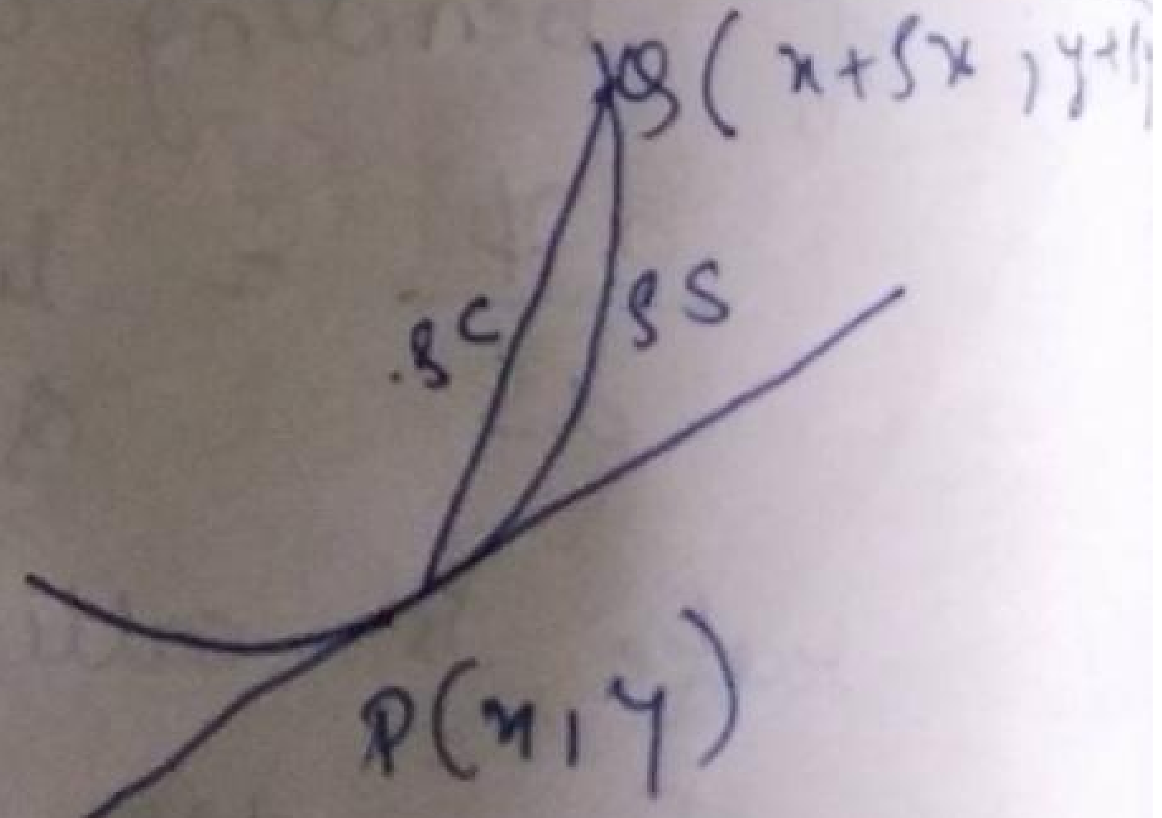
But $\frac{d\psi}{ds} = \frac{1}{\rho}$ — (2)

$$\frac{ds}{dx} = \left[1+(y')^2\right]^{1/2} = \cancel{[1+(y')^2]} \quad \text{— (3)}$$

Using (2) & (3) in (1)

$$y'' = \{1+(y')^2\}^{3/2}\frac{1}{\rho}$$

or $$\boxed{\rho = \frac{\{1+(y')^2\}^{3/2}}{y''}}$$



$$(\delta c)^2 = (\delta x)^2 + (\delta y)^2$$

$$\left(\frac{\delta c}{\delta x}\right)^2 = 1 + \left(\frac{\delta y}{\delta x}\right)^2$$

or $$\frac{\delta c}{\delta x} = \left[1+\left(\frac{\delta y}{\delta x}\right)^2\right]^{1/2}$$

$$\frac{\delta c}{\delta x} \doteq \frac{\delta s}{\delta c}\cdot\frac{\delta c}{\delta x} = \frac{\delta s}{\delta c}\left\{1+\left(\frac{\delta y}{\delta x}\right)^2\right\}^{1/2}$$

$\delta x \to 0$, $Q \to P$

taking limits on both sides as $\delta x \to 0$

$$\frac{ds}{dx} = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{1/2}$$

$\frac{27}{18}$ $\overline{45}$

(*) Find radius of curvature at the pt. (x, y) of each of the foll. curves

① $y = a \cosh(x/a)$

① $x^3 + y^3 = 3axy$ at the pt. $\left(\frac{3a}{2}, \frac{3a}{2}\right)$

$3x^2 + 3y^2 y' = 3axy' + 3ay$

$$\frac{\cancel{3}x^2 - \cancel{3}ay}{\cancel{3}y^2 + \cancel{3}ax} = y'$$

$$\frac{3\left(\frac{3a}{2}\right)^2 - 3ay}{3\left(\frac{3a}{2}\right)^2 - 3a\left(\frac{3a}{2}\right)}$$

$$\frac{\frac{27a^2}{4} - 3a\left(\frac{3a}{2}\right)}{\frac{27a^2}{4} - \frac{9a^2}{2}}$$

$$\frac{27a^2 - 18a^2}{\cancel{4}} \times \frac{\cancel{4}}{45a^2}$$

$$\frac{9a^2}{9\cancel{45}a^2} = \frac{1}{5} = y'$$

$$y' = \frac{x^2 - ay}{y^2 + ax}$$

$$y'' = \frac{(y^2 + ax)(2x - ay') - (x^2 - ay)(2yy' + a}{(y^2 + ax)^2}$$

$x^3+y^3=3axy$ at $\left(\frac{3a}{2}, \frac{3a}{2}\right)$

$3x^2+3y^2y' = 3axy' + 3ay$

$$y' = \frac{3x^2-3ay}{3ax-3y^2}$$

$$= \frac{x^2-ay}{ax-y^2}$$

$$= \frac{\left(\frac{3a}{2}\right)^2 - a\left(\frac{3a}{2}\right)}{a\left(\frac{3a}{2}\right) - \left(\frac{3a}{2}\right)^2}$$

$$= \frac{\frac{9a^2}{4} - \frac{3a^2}{2}}{\frac{3a^2}{2} - \frac{9a^2}{4}} = \frac{3a^2}{4} \times \frac{4}{-3a^2}$$

$$= -1$$

$$y'' = \frac{ax-y^2(2x-ay') - (x^2-ay)(a-2yy')}{(ax-y^2)^2}$$

$$= \frac{\left(a\left(\frac{3a}{2}\right) - \left(\frac{3a}{2}\right)^2\right)\left[\cancel{2}\left(\frac{3a}{\cancel{2}}\right) - a(-1)\right] - \left(\frac{9a^2}{4} - \frac{3a^2}{2}\right)\left(a - \cancel{2}\left(\frac{3a}{\cancel{2}}\right)-1\right)}{\left[a\left(\frac{3a}{2}\right) - \frac{9a^2}{4}\right]^2}$$

$$= \frac{\left[\frac{3a^2}{2} - \frac{9a^2}{4}\right]\left[3a+a\right] - \left[\frac{3a^2}{\cancel{4}}\right]\left[\cancel{4}a\right]}{\left(\frac{3a^2}{4}\right)^2}$$

$$\frac{\frac{-3a^2}{\cancel{4}} \times \cancel{4}a - 3a^3}{\frac{9a^4}{16}} = \frac{-\cancel{6}^{2}a^3 \times 16}{3\cancel{9}a^4}$$

$$= \frac{-32}{3a} //.$$

$$\rho = \frac{[1+(y')^2]^{3/2}}{y''} = \frac{(1+1)^{3/2}}{\left(-\frac{32}{3a}\right)} = 2^{3/2} \frac{(3a)}{-32}$$

$$= \frac{\cancel{2}\sqrt{2}(3a)}{-\cancel{32}\,16}$$

$$= \frac{-3a\sqrt{2}}{8 \times 2} = \frac{-3a}{8\sqrt{2}} //$$

2. Find radius of curvature at $ay^2 = x^3$

3. $ay^2 = a^3 - x^3$ at $(a, 0)$ on the curve.