1) Find the radius of curvature for the curve whose intrinsic equation is $s = a \log \tan(\pi/4 + \psi/2)$

$s = a \log \tan(\pi/4 + \psi/2)$

$$\frac{ds}{d\psi} = \frac{a \times \sec^2(\pi/4+\psi/2)}{\tan(\pi/4+\psi/2)} \times \frac{1}{2}$$

$$= \frac{a}{2} \frac{\cos(\pi/4+\psi/2)}{\sin(\pi/4+\psi/2)} \times \frac{1}{\cos^2(\pi/4+\psi/2}$$

$$= \frac{a}{2\sin(\pi/4+\psi/2)\cos(\pi/4+\psi/2)} = \frac{a}{\sin(\pi/2+\psi)} = \frac{a}{\cos\psi}$$

$$\frac{ds}{d\psi} = a\sec\psi \Rightarrow \rho = a\sec\varphi.$$

2) S.t the radius of curvature for the catenary of uniform strength $y = a \log \sec(x/a)$ is $a\sec(x/a)$

$$\rho = \frac{(1+y_1^2)^{3/2}}{y_2}$$

$y = a \log \sec(x/a)$

$$\frac{dy}{dx} = \frac{a \times \sec x/a \tan x/a}{\sec x/a} \times \frac{1}{a}$$

$$= \tan x/a$$

$$y_2 = \sec^2 x/a \times 1/a$$

$$\rho = \frac{(1+\tan^2(x/a))^{3/2}}{\sec^2(x/a) \times 1/a}$$

$$= \frac{(\sec^2(x/a))^{3/2}}{\sec^2(x/a)} \times a = a\sec(x/a)$$

classmate

Date

Page

3) S.t for the catenary $y = c\cosh(x/c)$, the radius of curvature is equal to y^2/c.

$$\rho = \frac{(1+y_1^2)^{3/2}}{y_2}$$

$$\frac{dy}{dx} = c\sinh(x/c) \times \frac{1}{c} = \sinh(x/c)$$

$$y_2 = \cosh(x/c) \times \frac{1}{c}$$

$$\rho = \frac{(1+\sinh^2(x/c))^{3/2}}{\cosh(x/c)} \times c$$

$$\rho = \frac{(\cosh^2(x/c))^{3/2}}{\cosh(x/c)} \times c = c\cosh^2(x/c) \quad -①$$

$$\frac{y^2}{c} = \frac{c^2\cosh(x/c)}{c} \quad -②$$

① = ② Hence proved.

$$\frac{d}{dx}(\sinh x) = \cosh x$$

$$\frac{d}{dx}(\cosh x) = \sinh x$$

$$\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$$

$$\frac{d}{dx}(\operatorname{cosech} x) = -\operatorname{cosech} x \coth x$$

$$\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x.$$

$$\frac{d}{dx}(\coth x) = -\operatorname{cosech}^2 x.$$

$$\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{1+x^2}} \qquad \frac{d}{dx}(\operatorname{cosech}^{-1} x) = \frac{-1}{x\sqrt{1+x^2}}$$

$$\frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2-1}}. \qquad \frac{d}{dx}(\operatorname{sech}^{-1} x) = \frac{-1}{x\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tanh^{-1} x) = \frac{1}{1-x^2} \qquad \frac{d}{dx}(\coth^{-1} x) = \frac{1}{1+x^2}$$

$$\int \cosh x \, dx = \sinh x$$

$$\int \sinh x \, dx = \cosh x$$

$$\int \operatorname{sech}^2 x \, dx = \tanh x$$

$$\int \operatorname{cosech} x \coth x \, dx = -\operatorname{cosech} x$$

$$\int \operatorname{sech} x \tanh x \, dx = \operatorname{sech} x$$

$$\int \operatorname{cosech}^2 x \, dx = -\coth x$$

$$\int \frac{1}{\sqrt{1+x^2}} \, dx = \sinh^{-1} x$$

$$\int \frac{dx}{\sqrt{x^2-1}} = \cosh^{-1} x$$

$$\int \frac{dx}{1-x^2} = \tanh^{-1} x$$

$$\int \frac{dx}{x\sqrt{1+x^2}} = -\operatorname{cosech}^{-1} x$$

$$\int \frac{dx}{x\sqrt{1-x^2}} = -\operatorname{sech}^{-1} x$$

$$\int \sqrt{a^2+x^2} \, dx = \frac{x}{2}\sqrt{a^2+x^2} + \frac{a^2}{2}\sinh^{-1}\left(\frac{x}{a}\right)$$

$$\int \sqrt{a^2-x^2} \, dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\cosh^{-1}\left(\frac{x}{a}\right)$$

Find the radius of curvature for the curve

$y^2 = \frac{4a^2(2a-x)}{x}$ where the curve meets the x-axis.

$0 = \frac{4a^2(2a-x)}{x}$ $(\because$ meets x-axis$)$

$\Rightarrow x = 2a$.

$(2a, 0)$ is the point on the curve at which ρ should be found.

The given eq^n can be put in the form:

$$y^2 = \frac{8a^3}{x} - 4a^2.$$

$2yy_1 = \frac{-8a^3}{x^2} \Rightarrow y_1 = \frac{-4a^3}{x^2 y}$

At $(2a, 0)$, $y_1 = \infty$ and hence we have to consider $\frac{dx}{dy}$.

$x_1 = \frac{dx}{dy} = -\frac{x^2 y}{4a^3}$. $x_1 = 0$ at $(2a, 0)$

$x_2 = \frac{-1}{4a^3}(2xx_1 y + x^2)$

At $(2a, 0)$, $x_2 = \frac{-1}{a}$

$\rho = \frac{(1+x_1^2)^{3/2}}{x_2}$

$= \frac{1}{-1/a} = -a$ $|\rho| = a$.

Find radius of curvature for the curve

$x^2 y = a(x^2+y^2)$ at $(-2a, 2a)$

$2xy + x^2 y_1 = a(2x + 2yy_1)$

$2xy - 2ax = y_1(2ay - x^2)$

$y_1 = \frac{2(xy - ax)}{2ya - x^2}$ at $(-2a, 2a)$, $y_1 = \infty$

$x_1 \Rightarrow 2xx_1y + y^2 = a(2xx_1 + 2y)$

$x_1(2xy - 2ax) = 2ay - x^2$

$$x_1 = \frac{2ay - x^2}{2xy - 2ax}$$

at $(-2a, 2a)$, $x_1 = 0$.

$$x_2 \to \frac{(2xy - 2ax)(2a - 2xx_1) - (2ay - x^2)(2x + 2yx_1 - 2ax_1)}{(2xy - 2ax)^2}$$

at $(-2a, 2a)$, $\dfrac{(-8a^2 + 4a^2)(2a) - \cancel{(4a^2 - 4a^2)}\,(\quad)}{(-8a^2 + 4a^2)^2}$ $\to 0$

$$= \frac{2a}{-4a^2} = -\frac{1}{2a}$$

$$\rho = \frac{(1 + x_1^2)^{3/2}}{x_2} = \frac{1}{-1/2a} = -2a$$

$|\rho| = 2a$.