



**Revus**

Journal for Constitutional Theory and Philosophy of  
Law / Revija za ustavno teorijo in filozofijo prava

**29 | 2016**

**Models of Legislative Authority, Interpretation,  
Realism, and Defeasibility**

---

## How deontic logic contributes to the analysis of legal systems

Review of Navarro & Rodríguez, *Deontic Logic and Legal Systems* (CUP 2014)

Jan Woleński

---



**Electronic version**

URL: <http://journals.openedition.org/revus/3518>

ISSN: 1855-7112

**Publisher**

Klub Revus

**Printed version**

Date of publication: 10 October 2016

Number of pages: 119-122

ISSN: 1581-7652

**Electronic reference**

Jan Woleński, « How deontic logic contributes to the analysis of legal systems », *Revus* [Online],  
29 | 2016, Online since 08 May 2016, connection on 19 April 2019. URL : [http://  
journals.openedition.org/revus/3518](http://journals.openedition.org/revus/3518)

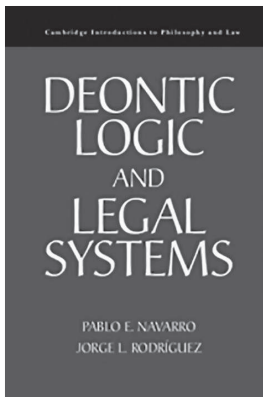
---

All rights reserved

Jan Woleński\*

## How deontic logic contributes to the analysis of legal systems

Book review of *Deontic logic and legal systems* by Pablo E. Navarro and Jorge L. Rodríguez (Cambridge UP, 2014)



The book under review consists of two parts closely related to its title: I Introduction to Deontic Logic, II Logic and Legal Systems. Each part is divided into chapters. Part I brings the following units:

1. The Language of Logic and the Possibility of Deontic Logic; 2. Paradoxes and Shortcomings of Logic; 3. Norm-propositions, Conditional Norms, and Defeasibility, and Part II the following: 4. Legal Systems and Legal Validity; 5. Legal Indeterminacy: Normative Gaps and Conflicts of Norms; 6. Legal Dynamics. Generally speaking, the reader might expect that the chapters of Part II use (or apply) the formal tools of deontic logic to analyse selected topics concerning legal systems.

Deontic logic is a branch of so-called philosophical logic. Although the concept of philosophical logic is fuzzy, a variety of modal logic in a wide sense (alethic modal logic, epistemic logic, doxastic logic, deontic logic, etc.) unquestionably falls within its scope. Accordingly, I consider some of the worries put forward by authors (section 1.5) concerning the possibility of deontic logic to be mistaken. The situation was clearly different 50 years ago, when Peter Geach

deliberated over some interpretative questions concerning normative utterances. However, as it appears today, in 2014, deontic logic is well-defined both syntactically and semantically, and has explicit metalogical properties. It belongs to so-called non-normal modal systems. In other words, it lacks the axiom (\*)  $\Box A \Rightarrow A$ , where the symbol  $\Box$  refers to a necessity-like operator (e.g., “is alethically necessary”, “knows that”, “is true”, “is obligatory”, etc.). It is perhaps interesting to observe that (\*) is valid only for a limited number of necessity-like operators, in particular operators expressing alethic necessity and truth, but not for “know that” (unless the classical definition of knowledge as justified true belief is adopted) or “it is obligatory that”. This observation goes against the claim made by the authors that (\*) (and its dual, namely the formula  $A \Rightarrow \Box A$ , where the symbol  $\Box$  refers to possibility-like operators) holds true for “most interpretations of modal concepts” (Navarro and Rodríguez 2014: 24). In fact, these axioms are exceptionally valid.

Although Part I is comprehensive and contains many interesting observations and comparisons, the way in which deontic logic is introduced by the authors raises some doubts. They begin (after a rudimentary presentation of parts of non-modal logic, that is, propositional calculus, predicate logic and syllogistic) with observations about analogies between deontic operators (“it is obligatory”, “it is permitted”, etc.), alethic modalities (“it is necessary”, “it is possible”, etc.) and quantifiers (“for any”, “there is”), and show the way in which these settings can be shaped by the traditional square of oppositions. In fact, this logical diagram

produces something which can be considered to be the minimal (or initial) portion of deontic logical dependencies. However, it is unclear which system Navarro and Rodríguez consider to be basic deontic logic. They mention a few proposals, and finally decide that the system called *KD* is basic. However, it does not settle the problem whether basic deontic logic has the axiom  $O_t$  (where  $t$  is a propositional tautology) or  $P_t$ . The first formula means that tautologies are obligatory, while the second that tautologies are permitted. Sometimes these formulas are expressed by  $O(A \vee \neg A)$  and  $P(A \vee \neg A)$  respectively. This, however, can lead to misunderstandings because it automatically directs attention to the principle of the excluded middle. Now, the formula  $O_t$ , and, *a fortiori*,  $O(A \vee \neg A)$  means that something is obligatory. Therefore, the formula is valid in a world in which obligations exist, but not in an anarchistic reality in which everything is permitted. On the other hand, one can prove that the formula  $\neg P_t$  is always contradictory and that makes the formula  $P_t$  valid on purely logical grounds. If we have a clear account of basic deontic logic, we can define its various extensions. Although the authors show various possibilities (Navarro and Rodríguez 2014: 30–33), I believe that their considerations are, to some extent, incomplete.

The other problem concerns the scope of modal logic and, *a fortiori*, deontic logic. To begin with alethic modal logic, we have the following situation. Propositional calculus serves as basic logic. Then modal operators are added. Thus, any modal logic appears to be an extension of propositional calculus. Similarly, we could begin with predicate calculus and obtain its modal extensions. Now, if we look at the semantics of alethic modal logic, the accessibility relation associated with the system *K* has no special conditions of reflexivity, symmetry, etc. Similar considerations concern deontic logic. For instance, the accessibility relation for its semantics is nonreflexive (it is related to the axiom  $(*)$ ). The question is whether such conditions are logical or extralogical. Note that logic, in a strict sense, does not distinguish any extralogical content. If so, special constraints on the accessibility relation are not purely logical. One could reply, however, that the same concerns the assumption of bivalence or admitting only non-empty individual constants. Moreover,

such facts are fixed in metalogic, not logic itself. A moral derived from this short discussion seems to be that the border between the logical and the extralogical is vague to some extent, and must be settled by a convention (which was already observed by Tarski in the 1930s).

Nevertheless, the question remains of how liberal such a convention might be. Why does this issue matter? Even if we agree that the deontic counterparts of *K*, *S4* or *S5* are systems of logic in the proper sense, what about the conditional obligations formalised by the formula  $O(A/B)$ , where the symbol  $/$  means “provided that” ( $B$  is obligatory provided that condition  $A$  is obtained)? Is  $/$  a logical constant or not? Define  $P'$  (its declared meaning is “strongly permitted” or “permitted with respect to a free-choice”) as  $P'(A \vee B) \Leftrightarrow P(A \wedge B)$ . Is  $P'$  a logical or extralogical concept? On the one hand, it is a logical concept due to its definition in purely logical terms (provided, of course, that  $P$  expresses a logical constant). On the other hand, however, having permission captured by  $P'$  seems to deal with a special situation. It is unclear to me when (it concerns some cases) Navarro and Rodríguez speak about purely logical issues, and when about applying logical notions to extralogical topics. Take, for instance, the problem of defeasibility, a highly favoured problem of today. Let us assume that we have a prohibition  $F$  of, for example, killing people. However, there is an exception, namely killing someone in self-defence. We say that the prohibition in question is defeasible in accordance with the given exception. However, we have a very simple device to describe this situation without any need to appeal to defeasibility. It is sufficient to use predicate calculus and say that the prohibition “for any  $x$ ,  $x$  is prohibited from killing any other person” be interpreted in such a way that the scope of the universal quantifier is restricted (first-order logic or propositional logic with quantifiers allows so-called restricted quantification).

The authors address several considerations of controversial problems of deontic logic. I have already made mention of conditional obligations and permissions expressed by  $P'$ . The paradoxes of deontic logic, iterations of deontic modalities, as in the formula  $OO(A \Rightarrow B)$ , or mixed formulas, such as  $A \Rightarrow OB$ , are further examples. Let me refer to the paradoxes. When Alf Ross in-

vented the paradox captured by the formula  $OA \Rightarrow O(A \vee B)$ , it was considered to have plagued the logic of normative discourse. Today, due to possible world semantics, it is clear that Ross's paradox is apparent and the authors share this view. It should be added that obeying the consequence of explicit obligations is not sufficient for conforming to Grice-like maxims. For obligations, the following rule is applicable: 'Obeying duties must consist in conforming to the most generally accessible obligations, unless otherwise allowed,' while for questions: 'An answer to a question must be logically equivalent to *datum quaestionis*.' Let me also mention that because  $B$  in Ross's paradox is arbitrary, we can have either  $OA \Rightarrow O(A \vee B)$  or  $OA \Rightarrow O(A \vee \neg B)$ . Now, given that formula  $A$  is equivalent to the conjunction  $(A \vee B) \wedge (A \vee \neg B)$ , we have a simple argument that Ross's paradox is apparent. The authors rightly observe that the Good Samaritan paradox poses a more serious challenge. However, it should be added that this paradox essentially depends on allowing formulas of the  $A \Rightarrow OB$  type. Of course, I do not claim that the way out consists in excluding mixed formulas. My intention is rather to point out that we need some syntactic decisions to perform formalizations of deontic logic.

The authors devote much attention to the relations between deontic propositions and norms. In fact, the first stage of the development of the logic of normative discourse is dominated by this issue. The so-called Jørgensen dilemma, which is extensively discussed by Navarro and Rodríguez (2014: 50–61), concerns the relations between deontic sentences, norms and norm-propositions. Unfortunately, the problem raised by Jørgensen is not reconstructed properly in the book. He observed that there are imperative inferences which look as though they are logically correct, but, because norms are neither true nor false, normative reasoning cannot be formally justified as having no proper semantic basis.<sup>2</sup> Hence, various attempts to build the logic of norms have been made. The authors, following Carlos Alchourrón and Eugenio Bulygin, consider the logic of norms (or normative propositions) to be necessary for grounding deontic logic. My proposal to solve this problem is radical—a non-linguistic theory

of norms.<sup>3</sup> The very idea of the logic of norms presupposes that norms are linguistic utterances. In my view, norms are decisions expressed by deontic (first-person or impersonal) sentences. In fact, the non-linguistic theory of norms is similar to the expressive conception of norms proposed by Alchourrón and Bulygin,<sup>4</sup> although the former does not treat norms as linguistic entities. Hence, deontic logic is the only logic of normative discourse. There is an *experimentum crucis* for the controversy over the logic of norms, namely the problem of permissive norms. Alchourrón, Bulygin, Rodríguez and Navarro maintain that such norms exist. On the other hand, the non-linguistic theory of norms rejects this view.<sup>5</sup> In order to legitimise normative inferences, one should either accept the view that norms are true or false (this view must face several epistemological and ontological problems) or abandon seeing norms as linguistic items and stay with deontic sentences as the basic units of normative discourse. I do not deny that several notions, such as the concept of competence, require fairly complicated investigations in terms of obligation, prohibition and (weak) permission, but my working hypothesis is that such treatment is possible. Let me add that Alchourrón's attempt to operate with the abstract notion of logical consequence (mentioned in Navarro and Rodríguez 2014: 64) fails, because the said properties of  $Cn$  (i.e., inclusion, idempotence and monotonicity) do not generate any logic.

Part II is surprising given the fact that almost nothing from the formal machinery developed in Part I is employed in the analysis of concrete legal topics. Consequently, if the reader's expectations are different (see the beginning of the review in hand), they are likely to be disappointed. This observation is not intended to underestimate the authors' treatment of legal validity, gaps and conflicts in legal systems or legal dynamics, because they are interesting and inspiring. However, this part of the book raises the question of how far deontic logic can be used in legal theory. I have no answer to this question. Does Part II of

3 See Woleński 1982: 66–73 for a more detailed account.

4 Alchourrón and Bulygin 1981.

5 See Opalek and Woleński 1973, 1986 and 1991.

2 Jørgensen 1937.

Navarro and Rodríguez's book motivate a scepticism of sorts, one perhaps not actually intended by the authors? Or could perhaps one relatively easily supplement Part II by a closer application of deontic logic to the classical problems of jurisprudence?

Let me illustrate the point by an example. The authors deny that we can define a legal system as a set of consequences of explicitly stated (enacted) obligations. The example they give is as follows. Let  $A = \{OA, O\neg A, OB\}$  and  $B = \{OA, O\neg A, O\neg B\}$  be two sets of initial normative bases. Since  $CnA = CnB$ , both systems are equivalent, although they are mutually inconsistent. The authors conclude that so-called dynamic normative systems cannot be defined by  $Cn$ , because such a definition entails that we have only one momentary system (Navarro and Rodríguez 2014: 214–232). However, this conclusion seems erroneous. The sets  $CnA$  and  $CnB$  are actually (and trivially) equivalent due to their internal inconsistency. We can only say that the hierarchical order of normative systems exceeds standard logical tools. But this was to be expected. Once again, the expressive power of deontic logic for an analysis of legal systems is still an open problem. Additionally, the tradition of ordinary language philosophy offers an alternative to investigations employing formal tools. The two perspectives should certainly not be viewed as disjoint, because several reasons support their mutual complementarity.

I shall finish with two general observations. First, the book contains several historical notes. Unfortunately, these are too limited and incomplete. The reader should be informed that the di-

chotomy of *is* and *ought* appeared already in Aristotle's *De Interpretatione* (since reviews are not monographs I skip more detailed bibliographical details; this concerns also contemporary authors). An even more serious historical flaw is omitting Hume as the inventor of Hume's guillotine (which is both the origin of the claim that the formula  $A \Rightarrow OA$  is not valid in deontic logic and an inspiration for the above mentioned formula (\*)). Poincaré should have also been included as the first author who observed that an imperative conclusion can be derived from a set  $X$ , provided that  $X$  contains at least one imperative. Jerzy Kalinowski elaborated on the first system of deontic (or normative) logic independently of Georg H. von Wright. I am pleased that my name is mentioned, but am surprised that the authors neglected to refer to the work of Kazimierz Opalek, Kazimierz Świrydowicz, Jerzy Wróblewski, Zdzisław Ziemia and Zygmunt Ziemiński (many of them have been published in English, German, French or Italian). This attitude diminishes attempts at international collaboration in the field of legal philosophy. Second and last, I have some concerns over whether the book under review should have been published in the Cambridge Introductions to Philosophy and Law book series. Its content is more advanced rather than introductory, particularly Part I, and requires a considerable degree of logical maturity not frequently possessed by lawyers, including those specialised in legal philosophy. However, the book at hand should generate considerable interest with everyone professionally interested in the relations between logic and law.

## References

Carlos E. ALCHOURRÓN & Eugenio BULYGIN, 1981: The Expressive Conception of Norms. In *New Studies in Deontic Logic*, ed. Risto Hilpinen, 95–124. Dordrecht: Reidel. Page numbers in my text refer to the reprint in *Normativity and Norms*, ed. Bonnie Litschewski-Paulson and Stanley Paulson, Chapter 21, 383–410. Oxford: Oxford University Press 1998.

Jørgen JØRGENSEN, 1937: Imperatives and Logic, *Erkenntnis* 7: 288–296.

Kazimierz OPAŁEK and Jan WOLEŃSKI, 1973: On Weak and Strong Permissions, *Rechtstheorie* IV (1973): 369–384.

Kazimierz OPAŁEK and Jan WOLEŃSKI, 1986: On Weak and Strong Permissions Once More, *Rechtstheorie* XVII (1986): 83–88.

Kazimierz OPAŁEK and Jan WOLEŃSKI, 1991: Normative Systems, Permission and Deontic Logic, *Ratio Juris* IV (1991): 334–348.

Jan WOLEŃSKI, 1982: Deontic Sentences, Possible Worlds and Norms, *Reports on Philosophy* 6 (1982): 66–73.